



**РАНХиГС**

РОССИЙСКАЯ АКАДЕМИЯ НАРОДНОГО ХОЗЯЙСТВА  
И ГОСУДАРСТВЕННОЙ СЛУЖБЫ  
ПРИ ПРЕЗИДЕНТЕ РОССИЙСКОЙ ФЕДЕРАЦИИ



РАНХиГС

экономический  
факультет

РОССИЙСКАЯ АКАДЕМИЯ НАРОДНОГО ХОЗЯЙСТВА И  
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# ПРОГНОЗИРОВАНИЕ ИЕРАРХИЧЕСКИХ ВРЕМЕННЫХ РЯДОВ

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ЭО-15-01

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## Цель:

- ▶ Используя модели учитывающие иерархическую структуру данных улучшить прогнозы агрегированного временного ряда.

## Задачи:

- ▶ Сбор данных с трехуровневой иерархической структурой
- ▶ Выбор модели для прогнозирования агрегированного ряда
- ▶ Прогнозирование рядов второго и третьего уровня по выбранной модели, сравнение суммы и оптимальной комбинации этих прогнозов с прогнозом агрегированного временного ряда.

# Данные:

Трехуровневая структура данных:

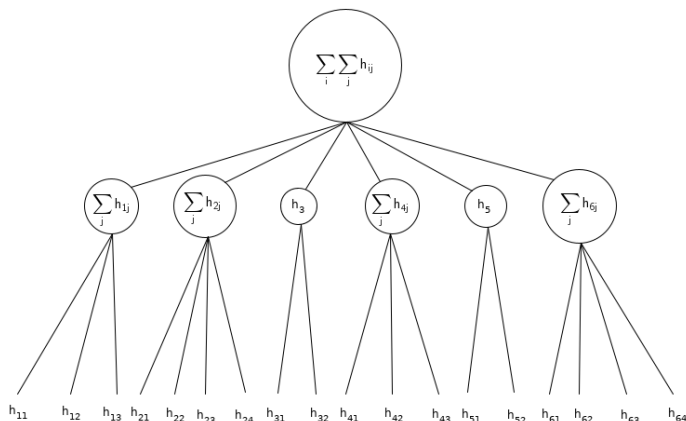
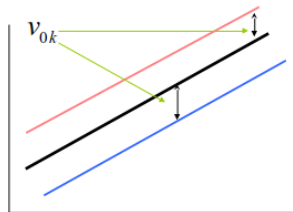


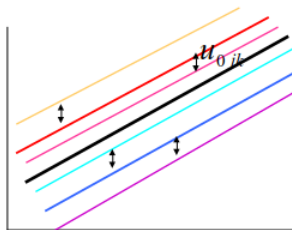
Рис.: Иерархическая структура временных рядов, необходимых для анализа

# Данные:

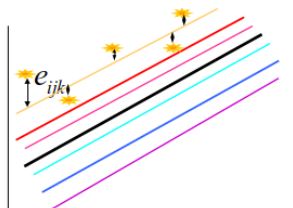
Трехуровневая структура данных:



Level 3 residuals



Level 2 residuals



Level 1 residuals

# Данные:

## Выбор подходящих наборов данных:

Для анализа были выбраны три набора данных с иерархической структурой с разной сезонностью:

- ▶ **квартальные (ЕС):**

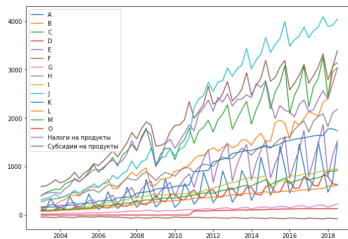
- ⇒ ВВП по 28 странам Европейского союза (включая Великобританию) в разбивке по 10 основным отраслям
- ⇒ Данные собраны за период с 2000-Q1 по 2018-Q3
- ⇒ Источник: Eurostat

- ▶ **квартальные сезонно сглаженные (США):**

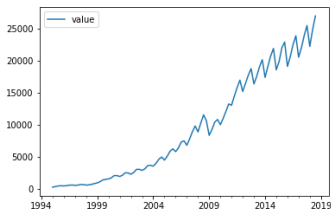
- ⇒ Данные по ВВП США (млн. долл., базовый год 2012) для каждого из 50 штатов с разбивкой по 21 основной отрасли
- ⇒ Данные собраны за период с 2005-Q1 по 2018-Q2
- ⇒ Источник: FRED

- ▶ **месячные (РФ):**

- ⇒ Данные по смертности и рождаемости в каждом регионе, дающие в сумме естественный прирост населения РФ ежемесячно
- ⇒ Данные собраны за период с 2006-01 по 2019-01
- ⇒ Источник: ЕМИСС



ВВП квартальный (производственный метод):



ВВП квартальный



Разница (не более 0.00006 %)

# Данные:

ЕС:

Квартальные данные по ЕС по секторам:  
2000-01-01 по 2018-07-01

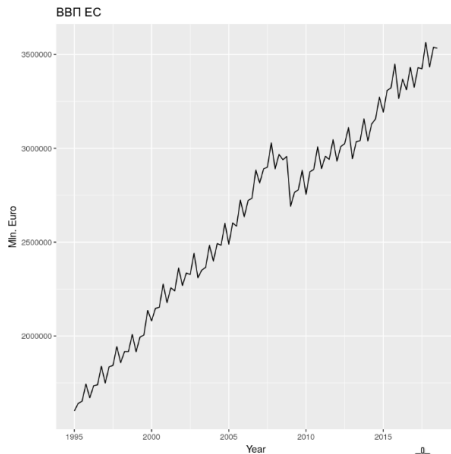


Рис.: ВВП ЕС28

# Forecasting :

## Aggregated TS:

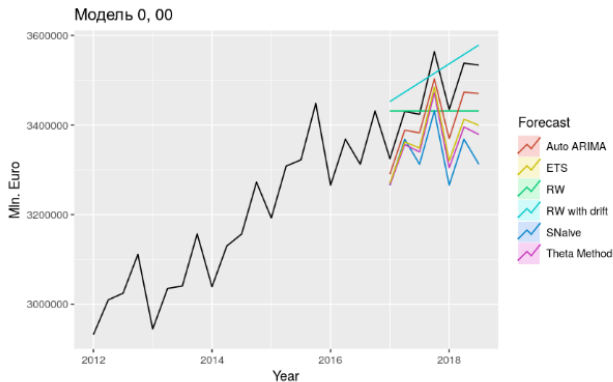


Рис.: Сравнение прогнозов



# Forecasting :

## Aggregated TS:

```
accuracy(gdp_rwf, test)
accuracy(gdp_rfwd, test)
accuracy(gdp_snaive, test)
accuracy(gdp_theta, test)
accuracy(gdp_arima, test)
accuracy(gdp_ets, test)|
```

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Thell's U
Training set	21038.66	93246.38	79262.43	0.8102572	3.103855	0.8108200	-0.66298186	NA
Test set	32817.03	85139.78	65623.51	0.8959751	1.880483	0.6712998	0.04778851	0.8211688

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Thell's U
Training set	9.099129e-10	90841.96	75477.66	-0.04039074	2.964913	0.7721034	-0.6629819	NA
Test set	-5.133762e+04	73862.83	65102.72	-1.51299717	1.899265	0.6659724	-0.4636484	0.6069265

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Thell's U
Training set	80232.22	107828.1	97755.89	3.245085	3.871968	1.00000	0.8540931	NA
Test set	132158.14	143232.6	132158.14	3.791442	3.791442	1.35192	0.5292738	1.532192

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Thell's U
Training set	18640.08	59189.53	44033.26	0.7022174	1.756951	0.450441	-0.3429432	NA
Test set	105149.04	110592.60	105149.04	3.0218262	3.021826	1.075629	0.5889082	1.176769

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Thell's U
Training set	496.5713	27093.45	18745.11	0.04021781	0.7075616	0.1917542	-0.006078177	NA
Test set	53108.4852	54480.16	53108.49	1.52742685	1.5274268	0.5432766	0.574894662	0.5760742

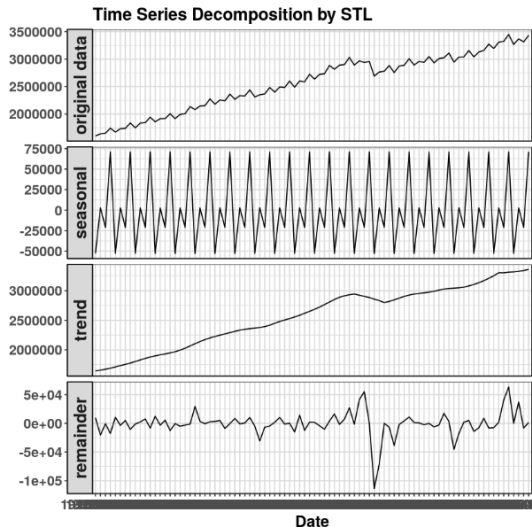
  

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Thell's U
Training set	-1002.636	28131.63	19893.79	-0.03510539	0.7573732	0.2035047	0.2537223	NA
Test set	93065.844	97402.36	93065.84	2.67558565	2.6755857	0.9520229	0.5815305	1.034584

Рис.: Сравнение прогнозов

# Forecasting :

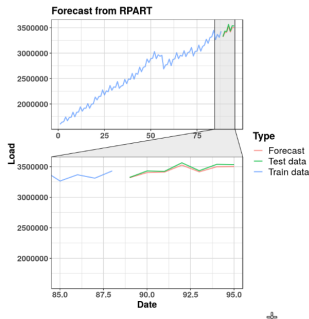
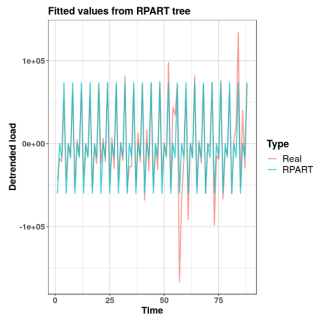
Aggregated TS:



# Forecasting :

## Aggregated TS:

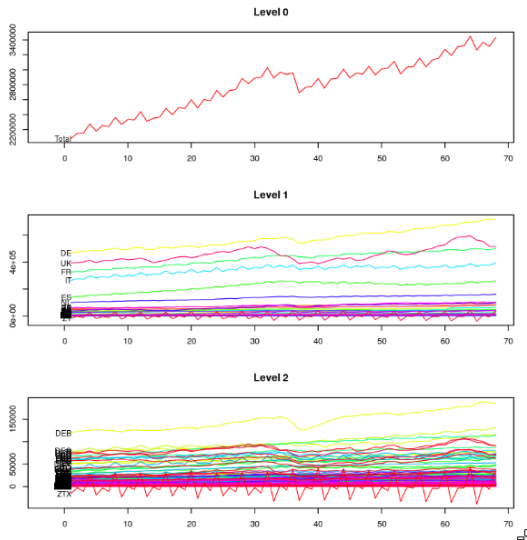
RPART, or CART (Classification and Regression Trees) is recursive partitioning type of binary tree for classification or regression tasks. It performs a search over all possible splits by maximizing an information measure of node impurity, selecting the covariate showing the best split.



	ME	RMSE	MAE	MPE	MAPE	ACF1	Theil's U
Test set	-24051.12	26940	24051.12	-0.6930563	0.6930563	-0.2798248	0.3608793

# Forecasting :

## Disaggregated TS



# Grouped Time Series

## Forecasting hierarchical time series

The assumption upon which many of these models are built on, is that by grouping series that behave in a similar way, the idiosyncratic errors within groups will tend to offset each other while the more relevant individual dynamics will be retained to be modelled.

Key idea: forecast reconciliation

- ▶ Ignore structural constraints and forecast every series of interest independently.
- ▶ Adjust forecasts to impose constraints.

Existing methods:

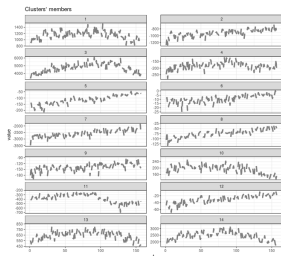
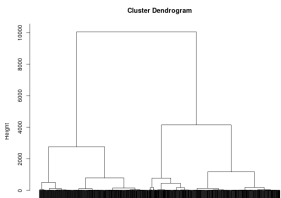
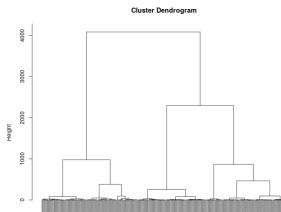
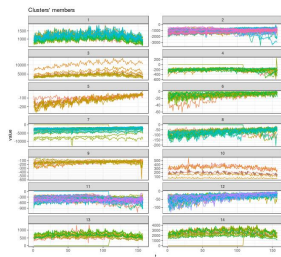
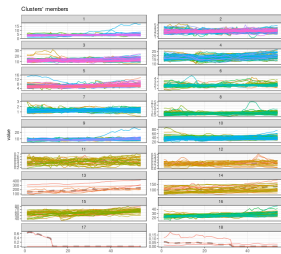
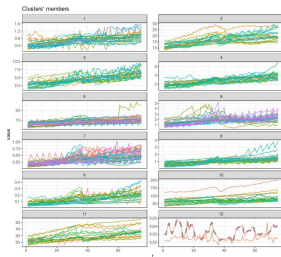
- ▶ Bottom-up
- ▶ Top-down
- ▶ Middle-out

An “optimal combination” approach can be advanced by proposing two new estimators based on WLS.

Both now implemented in the hts package

# Прогнозирование по рядам второго уровня

## Кластеризация временных рядов



# Сравнение прогнозов

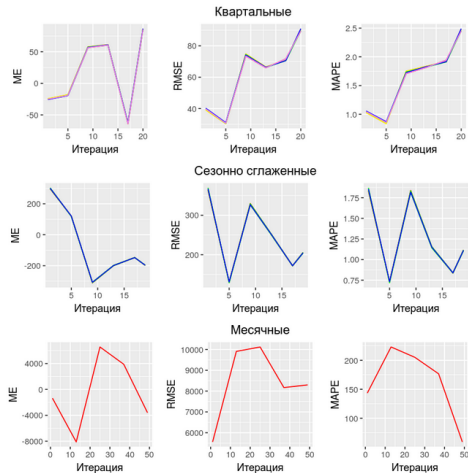


Рис.:

# Сравнение прогнозов

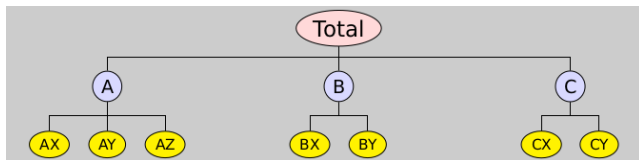


Рис.: Hierarchical structure

$Y_t$  : observed aggregate of all series at time  $t$ .

$Y_{X,t}$  : observation on series X at time  $t$ .



# Сравнение прогнозов

For the hierarchical structure we can write:

$$\begin{bmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{AA,t} \\ y_{AB,t} \\ y_{AC,t} \\ y_{BA,t} \\ y_{BB,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{AA,t} \\ y_{AB,t} \\ y_{AC,t} \\ y_{BA,t} \\ y_{BB,t} \end{bmatrix}$$

or in more compact notation, where  $y_t$  is an  $n$ -dimensional vector of all the observations in the hierarchy at time  $t$ ,  $S$  is the summing matrix.

$$y_t = S b_t$$

$b_t$  :  $m$ -dimensional vector of all series at bottom level in time  $t$ .

# Сравнение прогнозов

	ME	RMSE	MAE	MPE	MAPE
Training set	9.099129e-10	90841.96	75477.66	-0.04039074	2.964913
Test set	-5.133762e+04	73862.83	65102.72	-1.51299717	1.899265

	ME	RMSE	MAE	MPE	MAPE
Training set	80232.22	107828.1	97755.89	3.245085	3.871968
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Training set	18640.08	59189.53	44033.26	0.7022174	1.756951
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	ME	RMSE	MAE	MPE	MAPE
Training set	496.5713	27093.45	18745.11	0.04021781	0.7075616
Test set	53108.4852	54480.16	53108.49	1.52742685	1.5274268

Total	
ME	-47804.571429
RMSE	71937.089305
MAE	62579.114286
MAPE	1.826455
MPE	-1.411862

Total	
ME	1.321581e+05
RMSE	1.432326e+05
MAE	1.321581e+05
MAPE	3.791442e+00
MPE	3.791442e+00

Total	
ME	1.058988e+05
RMSE	1.118594e+05
MAE	1.058988e+05
MAPE	3.041683e+00
MPE	3.041683e+00

Total	
ME	81657.333946
RMSE	83547.346262
MAE	81657.333946
MAPE	2.348416
MPE	2.348416

# Сравнение прогнозов

## ARIMAX

	ME	RMSE	MAE	MPE	MAPE
<b>Training set</b>	496.5713	27093.45	18745.11	0.04021781	0.7075616
<b>Test set</b>	53108.4852	54480.16	53108.49	1.52742685	1.5274268

Total	
<b>ME</b>	81657.333946
<b>RMSE</b>	83547.346262
<b>MAE</b>	81657.333946
<b>MAPE</b>	2.348416
<b>MPE</b>	2.348416

Total	
<b>ME</b>	81627.482399
<b>RMSE</b>	83516.597308
<b>MAE</b>	81627.482399
<b>MAPE</b>	2.347558
<b>MPE</b>	2.347558

# Сравнение прогнозов

## Forecasting grouped time series

$$\begin{bmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{X,t} \\ y_{Y,t} \\ y_{AA,t} \\ y_{AB,t} \\ y_{AC,t} \\ y_{BA,t} \\ y_{BB,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{AA,t} \\ y_{AB,t} \\ y_{AC,t} \\ y_{BA,t} \\ y_{BB,t} \end{bmatrix}$$

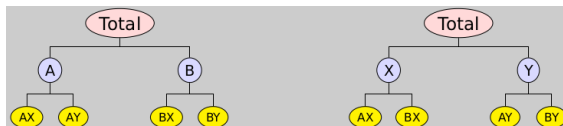


Рис.: Grouped structure

# Сравнение прогнозов

## Optimal forecasts

$$\hat{Y}_n(h) = S\beta_n(h) + e_h$$

$\hat{Y}_n(h)$  - a vector of initial  $h$ -step forecasts, made at time  $n$ , stacked in same order as  $Y_t$ .

$$\beta_n(h) = E[B_{n+h}|Y_1, \dots, Y_n]$$

$e_h$  has zero mean and covariance  $\Sigma_h$ .

$$\tilde{Y}_n(h) = S\hat{\beta}_n(h) = SP\hat{Y}_n(h) = S(S'\Sigma_h^+S)^{-1}S'\Sigma_h^+\hat{Y}_n(h)$$

$\tilde{Y}_n(h)$  - revised forecasts,  $\hat{Y}_n(h)$  - initial forecasts

$\Sigma_h^+$  - generalized inverse of  $\Sigma_h$ .

Optimal  $P = (S'\Sigma_h^+S)^{-1}S'\Sigma_h^+$

Problem:  $\Sigma_h$  hard to estimate.

# Сравнение прогнозов

## Approximate optimal forecasts

$$\tilde{Y}_n(h) = S(S'\Sigma_h^+S)^{-1}S'\Sigma_h^+\hat{Y}_n(h)$$

### ► OLS

$e_{B,h}$  is the forecast error at bottom level.

Assume  $e_h \approx Se_{B,h}$  then  $(S'\Sigma^+S)^{-1}S'\Sigma^+ = (S'S)^{-1}S'$

$$\tilde{Y}_n(h) = S(S'S)^{-1}S'\hat{Y}_n(h)$$

### ► Rescaling

Suppose we approximate  $\Sigma_h$  by its diagonal:  $\Lambda = \text{diag}(\hat{\Sigma}_1)^{-1}$ , which contain inverse one-step ahead in-sample forecast error variances.

$$\tilde{Y}_n(h) = S(S'\Lambda S)^{-1}S'\Lambda\hat{Y}_n(h)$$

### ► Averaging

If the bottom level error series are approximately uncorrelated and have similar variances, then  $\Lambda$  is inversely proportional to the number of series contributing to each node:  $\Lambda = \text{diag}(S \times 1)^{-1}$  is the inverse row sums of  $S$

# Forecast-error clustering

## 5. Forecast-error clustering

Ignoring the common factor and interdependencies will tend to make forecasting errors cluster instead of cancelling out. The dissimilarity measure the correlations of the out-of-sample forecasting errors for the most recent periods. Specifically, for each component  $i$  we fit  $x_{i,t-p+1} = a_i + \rho x_{i,t-p} + e_{i,t}$ , where  $p$  is the number of periods that are evaluated for the measure. With the model, we generate forecasts from  $t - p + 1$  to  $t$  and calculate the corresponding forecasting errors as  $\hat{x}_{i,s|s-1} - x_{i,s}$  for  $s = t - p + 1$  to  $t$  and collect them in  $\hat{e}_i^t$ . With this, the dissimilarity measure is defined as:

$$FC_{x_i, x_j} = 1 - \text{abs} \left( \frac{\text{cov}(\hat{e}_i^t, \hat{e}_j^t)}{\sigma_{\hat{e}_i^t} \sigma_{\hat{e}_j^t}} \right)$$

└

**Forward modelling:** with noise properties we can predict the Sampling Distribution (the probability for a general set of data)

In easy cases, the effect of the prior is simple. As experiment gathers more data, the likelihood tends to get narrower, and the influence of the prior diminishes.

Rule of thumb: if changing your prior to another reasonable one changes the answers significantly, you need more data



# Bayesian Methods

## Why is the Bayesian Inference controversial?

1. Bayesians claim that the parameters are random so that their credible interval is a valid probability argument, though it also depends on the prior, which is usually hard to obtain.
2. When the likelihood and prior is complicated, the inference has to rely on the MCMC sampling, which can be really slow in most of the real-world cases.
3. The biggest controversy about Bayesian inference is that you must quantify your prior knowledge about the question at hand. This makes it possible to actually influence your results, either accidentally or on purpose.

# Bayesian Methods

## Bayesian Modelling and Markov chain Monte Carlo

Bayesian method provides an intuitive way for us to fill the gaps left by small or incomplete data sets.

To calculate the Bayesian predictive distribution,  $\pi(x|D)$ , given some data  $D$ , we simply multiply the density function of the classical solution  $\ell(D|x)$ , with the density function produced by our prior knowledge  $\pi(x)$ . This is a direct application of Bayes' theorem. Unfortunately, this product will not integrate to one. To overcome this, we multiply the density function by a constant  $Z$ , which rescales the density so that it does integrate to one. The resulting Bayesian distribution defined over the  $n$ -dimensional parameter space  $S$  is

$$\pi(x|D) = \frac{1}{Z} \pi(x) \ell(D|x)$$

$$Z = \int_S \pi(x) \ell(D|x) dx$$

In one dimension it is easy to use numerical quadrature to calculate  $Z$ . However as the dimension becomes large, this method quickly becomes impractical. So we turn to a class of statistical algorithms known as Markov chain Monte Carlo (MCMC) methods, which can tackle these high dimensional parameter spaces.

# Bayesian Methods

## Bayesian Shrinkage

**Shrinkage** is implicit in Bayesian inference and penalized likelihood inference, and explicit in James–Stein-type inference. In contrast, simple types of maximum-likelihood and least-squares estimation procedures do not include shrinkage effects, although they can be used within shrinkage estimation schemes.

# Bayesian Methods

## Stein's paradox

**Stein's paradox**, in decision theory and estimation theory, is the phenomenon that when three or more parameters are estimated simultaneously, there exist combined estimators more accurate on average (that is, having lower expected mean squared error) than any method that handles the parameters separately.

*The best guess about the future is usually obtained by computing the average of past events. Stein's paradox defines circumstances in which there are estimators better than the arithmetic average*

Stein's paradox \*\*modern generalization, the Bayesian hierarchical model\*\*.

Bayesian hierarchical model can improve overall estimation accuracy, thereby improving our confidence in the assessment results, especially for standard compliance assessment of waters with \*\*small sample sizes.\*\*

# Small Sample

Most of the inferential procedures available in the analysis of time series data are asymptotic

Although analytic small sample results are available in a few cases, there is currently, no widely applicable and easily accessible method that can be used to make small sample inferences.

The bootstrap technique introduced by Efron (1979) could possibly be a potential alternative in estimation and inference from time series models in finite samples. However, in time series regressions, the standard bootstrap resampling method designed for independent and identically distributed (IID) errors is not applicable because in most situations the assumption of IID errors is violated.

The basic bootstrap approach consists of drawing repeated samples (with replacement). The simplest assumption for that method is that observations should be IID. But in time series models IID assumption is not satisfied. Thus the method needs to be modified.

## BS for TS:

- ▶ Estimating Standard Errors: if “Small Sample Size” distribution is normal then we can get a BS distribution to Estimate SE (same as asymptotic distribution for SE)
- ▶ Confidence Interval statements: Using BS distribution to Estimate CI we can get different result for CI (from asymptotic distribution), for example, because of BS distribution skewness

# Bootstrap

## The Recursive BS for stationary AR(p) model

Consider AR(p) process:

$$y_t = \sum_{i=1}^p a_i y_{t-i} + e_t, e_t \sim N(0, \sigma^2)$$

We estimate coefficients with OLS and get:

$$(\hat{a}_1, \dots, \hat{a}_p), \hat{e}_t$$

Define the centered and scaled residuals:

$$\tilde{e}_t = \left( \hat{e}_t - \frac{1}{n} \sum \hat{e}_t \right) \left( \frac{n}{n-p} \right)^{1/2}$$

Resample  $\tilde{e}_t$  with replacement to get the BS residuals  $e_t^*$

Construct the BS sample recursively using  $y_t^* = y_t$ :

$$y_t^* = \sum_{i=1}^p \hat{a}_i y_{t-i}^* + e_t^*$$

# Bootstrap

## A comparison of four different block bootstrap methods

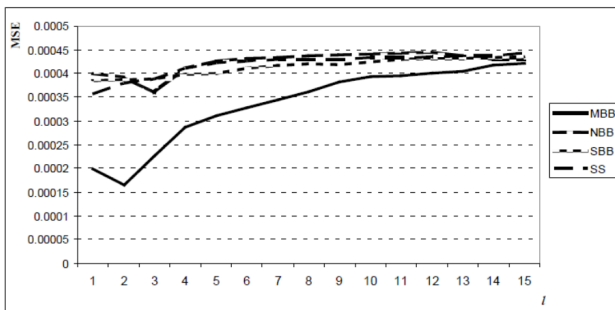


Figure 1: Preliminary comparison of block bootstrap methods

Рис.: MBB – Moving block bootstrap, NBB – Non-overlapping block bootstrap, SBB – Stationary block bootstrap, SS - Subsampling



# Bootstrap

## Bootstrapping vs. Posterior sampling

“Bootstrapping” is a framework that aims to improve simple but approximate frequentist methods:

- ▶ Parametric bootstrap: Improve asymptotic behavior of estimates for a trusted model: reduce bias of estimates, provide more accurate coverage of confidence regions
- ▶ Nonparametric bootstrap: Provide results that are approximately accurate with weak modeling assumptions

Most common approach uses Monte Carlo to simulate an ensemble of data sets related to the observed one, and use them to recalibrate a simple method.

Parametric bootstrap has a step producing an ensemble of estimates that looks like a set of posterior samples. Can they be thought of this way?