The International College of Economics and Finance
Econometrics - 2020. First Semester Exam, December 24.

Part 1. (30 minutes). In each of 12 multiple choice tests indicate the correct answer. One point is given for the correct answer, penalty of 0.25 points is given for an incorrect one.

1. For the Model $Y_i = \beta_1 + \beta_2 X_i + u$ (X_i are non-stochastic, the Model A assumptions satisfied), the following 3 estimators of β_2 are proposed:

$$b_1 = \frac{\overline{Y}}{\overline{X}}\ ,\ b_2 = \frac{\sum\left(X_i - 2\overline{X}\right)\left(Y_i - 2\overline{Y}\right)}{\sum\left(X_i - 2\overline{X}\right)^2}\ ,\ b_3 = \frac{\sum X_i Y_i}{\sum X_i^{\,2}}\ .$$

The following is correct for these estimators:

1) All the estimators b_1, b_2 and b_3 are unbiased;
2) All the estimators b_1, b_2 and b_3 are biased;
3) The estimator b_2 is unbiased, while b_1 and b_3 are biased;
4) The estimators b_1 and b_2 are unbiased, while b_3 is biased;
5) The estimators b_2 and b_3 are unbiased, while b_1 is biased.

2. For the Model $Y_i = \beta_1 + \beta_2 X_i + u$ (X_i are stochastic, the Model B assumptions satisfied) the estimator $b = \dfrac{\sum_{i=2}^{n}(Y_i - Y_{i-1} + Y_{i-2} - Y_{i-3})}{\sum_{i=2}^{n}(X_i - X_{i-1} + X_{i-2} - X_{i-3})}$ is generally speaking:

1) biased but consistent estimator of β_2;
2) unbiased and consistent estimator of β_2, though inefficient;
3) unbiased but inconsistent estimator of β_2;
4) biased and inconsistent estimator of β_2;
5) unbiased, consistent and efficient estimator of β_2.

3. If you have estimated the parameters of the following model using the OLS directly (Model B assumptions satisfied), $Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + (\beta_1+\beta_2+\beta_3+\beta_4)X_3 + u$, then:

1) you can get an unbiased estimate of β_3;
2) you can not get an unbiased, or biased but consistent estimate of β_3;
3) you can not get an unbiased estimate of β_3, but can easily get a consistent estimate of it;
4) you can not get any estimate of β_3;
5) none of the above.

4.For a linear regression model without intercept $Y_i = \beta X_i + u_i$, estimated as $Y_i = bX_i + e_i$ using OLS ($\hat{Y}_i = bX_i$), the following is always correct:

1) $\sum_{i=1}^{n} e_i = 0$; 2) $b\sum_{i=1}^{n} X_i^2 = \sum_{i=1}^{n} X_i Y_i$; 3) $TSS = ESS + RSS$;

4) $\overline{Y} = \overline{\hat{Y}}$; 5) None of the above.

5.Multiple linear regression model with 2 explanatory variables is estimated for a sample with 70 observations. There is heteroscedasticity in the model where the standard deviation of disturbance term is likely of the following form: $\sigma_{ui}=\gamma_1 x_{1i}^2 + \gamma_2 x_{1i} + \gamma_3 x_{1i} x_{2i}$. Not knowing that, a student tries to do a test for heteroscedasticity. Which test is the most appropriate for that:

1) Goldfeld-Quandt test; 2) White test without cross terms;

3)White test with cross terms; 4)Likelihood Ratio test; 5) None of the above.

6. If the OLS is used in simple regression model in the case of heteroscedasticity, the population variance of slope coefficient is $\text{var}(b_2) = \frac{\sum_{i=1}^{n} x_i^2 \sigma_i^2}{(\sum_{i=1}^{n} x_i^2)^2}$ (1). The standard formula for homoscedasticity case is $\text{var}(b_2) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2}$ (2). Let $\sigma_i^2=\sigma^2 k_i$, where k_i are unknown weights *($\Sigma k_i=n$, the number of observations).* Then:

1) The expression (1) is always greater than (2);
2) The expression (1) is always less than (2);
3) The expression (1) is greater or equal to (2);
4) The expression (1) is less or equal to (2);
5) The expression (1) can be greater, less or equal to (2), depending on the nature of relationship between σ_i and x_i;

7. A student did estimate by the OLS the production function $y=\gamma_1+\alpha k+\beta l+u$ (1) , where y is the output growth rate, k is the capital growth rate, and l is the labour growth rate. Then he decided to estimate by the OLS the function $y\text{-}k\text{-}l=\gamma_2+\mu k+\rho l+u$ (2). Which statement of the following ones is incorrect?

1) $\hat{\mu} = \hat{\alpha} - 1$ 2) $\hat{\rho} = \hat{\beta} - 1$ 3) $\hat{\gamma}_1 = \hat{\gamma}_2$ 4) $RSS_1=RSS_2$ 5) $TSS_1=TSS_2$

8. The intercept was removed in the model with three groups of dummy variables A, B and C, and a new variable (corresponding to the reference category in the group A) was included in the model. Generally speaking, the following is **correct**:

1) all the estimated dummies' coefficients for the groups B and C have changed while the non-dummies' slope coefficients stayed the same;
2) all the estimated dummies' coefficients for the groups B and C and the non-dummies' slope coefficients stayed the same;
3) all the estimated dummies' coefficients for the groups B and C and the non-dummies' slope coefficients have changed;

4) all the estimated dummies' coefficients for the groups B and C stayed the same while the non-dummies' slope coefficients have changed;
5) the estimated coefficients for all the group A's variables stayed the same.

9. For the simultaneous equations model with 5 equations, 5 endogenous variables and 7 exogenous variables, the following statement is true for any its equation:
1) an equation is likely to be underidentified if 5 variables are missing from it;
2) an equation is likely to be exactly identified if 4 variables are missing from it;
3) an equation is likely to be overidentified if 3 variables are missing from it;
4) an equation is likely to be exactly identified if 6 variables are missing from it;
5) an equation is likely to be exactly identified if 5 variables are missing from it.

10.When MLE method is applied to the model $Y_i = \beta_1 + \beta_2 X_{2i} + ... + \beta_k X_{ki} + u_i$ estimation, the Likelihood Ratio is calculated using the formula (where L is the Likelihood function value in the output; L_0 is the likelihood function for the model with $\beta_2 = \beta_3 = ... = \beta_k = 0$):

1) $LR = 2\dfrac{\log L_0}{\log L}$. 2) $LR = 2\log\dfrac{L_0}{L}$.

3) $LR = 2\log\dfrac{L}{L_0}$. 4) $LR = 2\dfrac{\log L}{\log L_0}$. 5)None of the above.

11. Logit estimation of the model describing the probability to get a loan $F(Z_i)=p(Loan_i=1|\, X_i,\beta)$ has given the result $Z=-23.2+0.9\cdot X$, where X is the debt to income ratio. Increase of the probability $p(Loan_i=1)$ under one point increase of X, for $X=0.85$ *is*:

1) $\dfrac{\exp(23.2-0.9\cdot 0.85)}{(1+\exp(23.2-0.9\cdot 0.85))^2}\cdot 0.9$; 2) $\dfrac{-\exp(23.2-0.9\cdot 0.85)}{(1+\exp(23.2-0.9\cdot 0.85))^2}\cdot 0.9$;

3) $\dfrac{\exp(23.2-0.9\cdot 0.85)}{(1+\exp(23.2-0.9\cdot 0.85))^2}$; 4) $\dfrac{\exp(-23.2+0.9\cdot 0.85)}{(1+\exp(23.2-0.9\cdot 0.85))^2}\cdot 0.9$;

5)None of the above.

12. Function F is the distribution function of the standardised normal distribution, and the function f is its probability density function. If there is a lower bound Y_L for the dependent variable Y in the tobit model, but no upper bound, then the Log Likelihood function is:

1) $l(\beta) = \log L = \sum\limits_{i:Y_L<Y_i} \log(f((Y_i - \beta_1 - \beta_2 \cdot X_i)/\sigma)) + \sum\limits_{i:Y_i=Y_L} \log(F((Y_L - \beta_1 - \beta_2 \cdot X_i)/\sigma)) \to \max$

2) $l(\beta) = \log L = \sum\limits_{i:Y_L<Y_i} \log(F((Y_i - \beta_1 - \beta_2 \cdot X_i)/\sigma)) + \sum\limits_{i:Y_i=Y_L} \log(f((Y_L - \beta_1 - \beta_2 \cdot X_i)/\sigma)) \to \max$

3) $l(\beta) = \log L = \sum\limits_{i:Y_L<Y_i} \log(f((Y_i - \beta_1 - \beta_2 \cdot X_i)/\sigma)) + \sum\limits_{i:Y_i=Y_L} \log(1 - F((Y_L - \beta_1 - \beta_2 \cdot X_i)/\sigma)) \to \max$

4) $l(\beta) = \log L = \sum\limits_{i:Y_L<Y_i} \log(F((Y_i - \beta_1 - \beta_2 \cdot X_i)/\sigma)) + \sum\limits_{i:Y_i=Y_L} \log(1 - F((Y_L - \beta_1 - \beta_2 \cdot X_i)/\sigma)) \to \max$

5)None of the above.