Breuch - Godfrey Test

$$t = \overline{1, T}$$

$$y_t = \beta_1 + \beta_2 X_{1t} + \beta_3 X_{2t} + u_t$$

$$u_t = \rho_1 u_{t-1} + \dots + \rho_p u_{t-p} + \varepsilon_t$$

estimate aux. regression

$$\hat{u}_t = \alpha_1 + \alpha_1 X_{1t} + \alpha_2 X_{2t} + \rho_1 \hat{u}_{t-1} + \dots + \rho_p \hat{u}_{t-p} + \varepsilon_t$$

$\hookrightarrow R^2_{aux}$

$$BG = n R^2_{aux} \sim \chi^2_p$$

$$n = T - p$$

Advantages:

1. No limitation on the structure
2. No uncertainty zones (unlike DW)
3. Always applicable (unlike h-Durbin when neg. values under root)
4. Test any order of AC.

5. Also aplicable for MA - process Errors

$$u_t = \varepsilon_t + \mu_1 \varepsilon_{t-1} + \dots + \mu_q \varepsilon_{t-q}$$

Disadvantage

1. Large samples are required

Fight Autocorrelation

1) Correct specification

2) Cochrain-Orchutt transformation
AR(1) - transform
(GLS)
generalized least squares

3) MA(1) - transform

4) Add lagged exp. variable

5) More complex ADL (p,q)

CO - transformation

(0) $y_t = \beta_1 + \beta_2 X_t + u_t$

$u_t = \rho u_{t-1} + \varepsilon_t \quad (*)$

(1) $y_t = \beta_1 + \beta_2 X_t + \rho u_{t-1} + \varepsilon_t$

ρ. lagged (0) $\qquad \hat{\rho}$ from DW - test

(2) $\rho y_{t-1} = \rho \beta_1 + \rho \beta_2 X_{t-1} + \rho u_{t-1}$

subtract (1) - (2)

$$\underbrace{y_t - \rho y_{t-1}}_{y_t^*} = \beta(1-\rho) + \beta_2 \underbrace{(X_t - \rho X_{t-1})}_{X_t^*} + \varepsilon_t \quad (3)$$

given $\hat{\rho}$ from previous step

est: OLS $y_t^* \mid X_t^* \Rightarrow \hat{\rho}_{new}$

Iterate until $\hat{\rho}$ converges

Price-Winsten correction:

add 1st obs $\cdot \sqrt{1-\rho^2}$

3) MA(1)

$$y_t = \beta_1 + \beta_2 X_t + u_t$$

$$u_t = \varepsilon_t + \rho \varepsilon_{t-1}$$

$$u_{t-1} = \varepsilon_{t-1} + \rho \varepsilon_{t-2}$$

GLS - method

Ω - correlation matrix of errors

$$\hat{\beta}_{GLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y$$

$$\hat{V}(\hat{\beta}_{GLS}) = (X' \hat{\Omega} X)^{-1}$$

$$\hat{\beta}_{OLS} = (X'X)^{-1} X' y$$

$\hat{\Omega}$ given some assump.

If no AC:

$$\underset{T\times T}{\hat{\Omega}} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \sigma_\varepsilon^2 = I \cdot \sigma_\varepsilon^2$$

If MA(1)

$$\hat{\Omega} = \begin{bmatrix} 1 & \rho & 0 & & \\ \rho & \rho & \ddots & \ddots & \\ 0 & 0 & \ddots & \ddots & \ddots \\ & & \ddots & \ddots & \ddots \\ & & 0 & \rho & 1 \end{bmatrix} \sigma_\varepsilon^2$$

If AR(1).

$$\operatorname{cov}(u_t, u_{t-k}) = \rho^k \cdot \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

$$\hat{\Omega} = \frac{\sigma_u^2}{1-\rho} \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \\ & & & & 1 \end{bmatrix}$$

4) Adding lagged exp. variable

How to test AR(1) vs ADL(1,0)

CO - form:

$$y_t - \rho y_{t-1} = \beta(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + \varepsilon_t \quad (3)$$

$$(1)\ y_t = \overset{\gamma_0}{\beta(1-\rho)} + \overset{\gamma_1}{\underline{\rho}}\, y_{t-1} + \overset{\gamma_2}{\underline{\beta_2}}\, X_t - \overset{\gamma_3}{\rho\beta_2}\, X_{t-1} + \varepsilon_t$$

ADL(1,1) with non-lin. restr.

$$\gamma_3 = -\gamma_1 \cdot \gamma_2$$

Test AR(1) form by

Common Factor test

$$n \log\left(\frac{RSS_R}{RSS_{UR}}\right) \sim \chi^2_1$$

Test Wald for $\theta_3 = \theta_1 \cdot \theta_2$