

Testing Stationarity

Dickey-Fuller test

(t-test)

$$y_t = y_{t-1} + \epsilon_t$$

$$(i) \quad \Delta y_t = \alpha \cdot y_{t-1} + \epsilon_t$$

$$y_t - y_{t-1}$$

$$y_t = (1 + \alpha) y_{t-1} + \epsilon_t$$

$$H_0: \alpha = 0$$

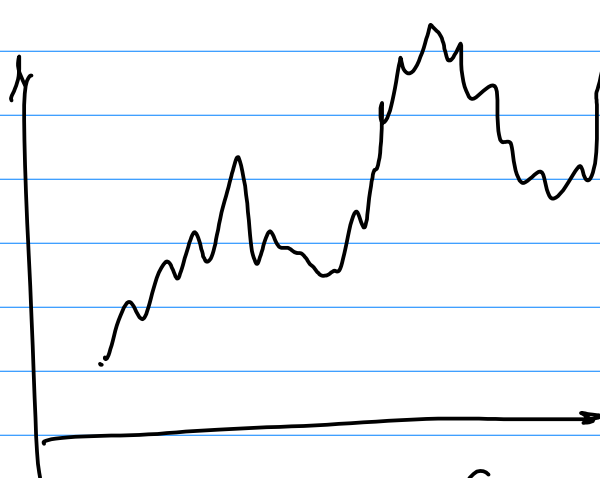
y_t - non-stationary (RW)

$$H_a: \alpha < 0$$

y_t - stationary

$$(ii) \quad \Delta y_t = \mu + \alpha \cdot y_{t-1} + \epsilon_t$$

y_t - non-stationary \Rightarrow RW with drift

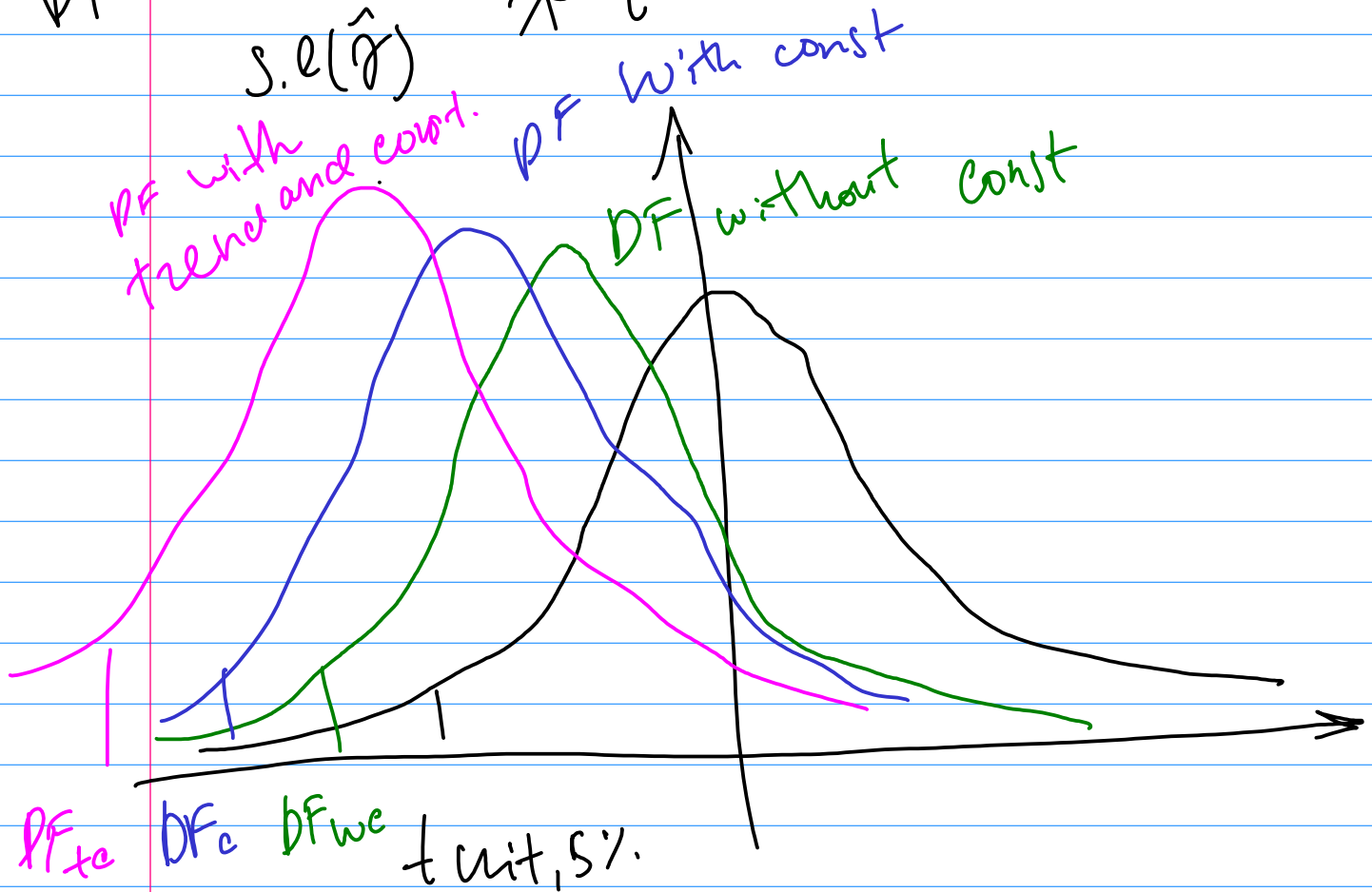


$$(iii) \quad \Delta y_t = \mu + \alpha \cdot y_{t-1} + \beta t + \epsilon_t$$

y_t - non-stationary \Rightarrow Trend-stationary

$$\Delta y_t = \alpha \cdot y_{t-1} + \epsilon_t$$

$$DF = \frac{\hat{\alpha}}{s.e(\hat{\alpha})} \quad * t$$

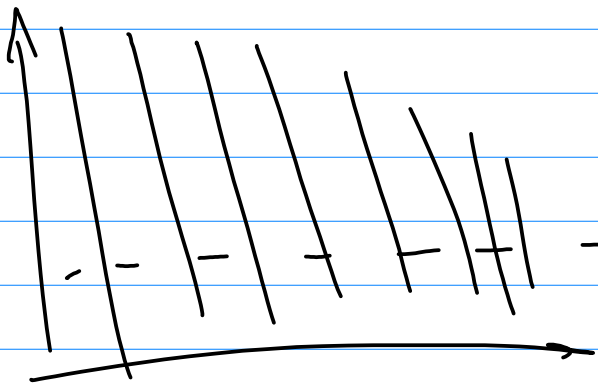


Augmented DF-test

AR(p)

$$y_t = \mu + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + \varepsilon_t$$

$$\Delta y_t = \mu + \alpha y_{t-1} + \beta_1 \Delta y_{t-1} + \dots + \beta_{p-1} \Delta y_{t-(p-1)} + \varepsilon_t$$



$$y_t = \alpha + \beta t + \varepsilon_t$$

$$y_t - \hat{\alpha} - \hat{\beta} t = \hat{\varepsilon}_t$$

Problem 10;

$N=183$

$$(1) \quad \hat{\Delta P}_t = 160.58 - 0.02P_{t-1} \quad R^2 = 0.01$$

(134.00) (0.014)

$$(2) \quad \hat{\Delta DP}_t = -0.97DP_{t-1} \quad R^2 = 0.487$$

(0.075)

$$(3) \quad \hat{\Delta VOL}_t = 1.48 \cdot 10^8 - 0.144VOL_{t-1} - 0.224\Delta(VOL_{t-1}) + 91320.24t \quad R^2 = 0.14$$

(871445.3) (0.045) (0.073) (871445.3)

$$(4) \quad \hat{\Delta VOL}_t = 1.55 \cdot 10^8 - 0.143VOL_{t-1} - 0.224\Delta(VOL_{t-1}) \quad R^2 = 0.14$$

(65210866) (0.044) (0.073)

$$(1) \quad P_t = \beta_1 + \beta_2 P_{t-1} + \varepsilon_t$$

$$P_t - P_{t-1}$$

$$\Delta P_t = \beta_1 + (\beta_2 - 1)P_{t-1} + \varepsilon_t$$

$$H_0: \beta_2 - 1 = 0 \quad \Leftrightarrow \quad \beta_2 = 1$$

$$DF = \frac{-0.02}{0.014} = -1.43 > -2.88$$

P_t - non-stationary

$$(2) \quad H_0: \beta_1 - 1 = 0$$

$$DF = \frac{-0.97}{0.075} = -12.93 < -2.88$$

$$< -3.48$$

DP_t - Stationary

$$(3) \quad y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \gamma_t + \varepsilon_t$$

$$\Delta y_t = \beta_1 + (\beta_2 - 1)y_{t-1} + \beta_3 y_{t-2} + \gamma_t + \varepsilon_t$$

$$\Delta y_t = \beta_1 + (\beta_2 + \beta_3 - 1)y_{t-1} - \beta_3 \Delta y_{t-1} + \gamma_t + \varepsilon_t$$


$$H_0: \beta_2 + \beta_3 - 1 = 0$$

$$DF = - \frac{0,144}{0,045} = -3,2 > -3,44$$

VOL_t - non-stationary

for γ :

$$t = \frac{91 \cdot 10^3}{871 \cdot 10^3} \approx 0,1 \quad t_{5\%} = 1,976$$

γ - non sig. 

$$DF = \frac{-0,143}{0,044} = -3,25 < -2,88$$

VOL_t - stationary