Breusch - Godfrey Test

$$y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + u_t, \quad t = \overline{1,T}$$

u_t — AR(p) process

$$u_t = \rho_1 u_{t-1} + \dots + \rho_p u_{t-p} + \varepsilon_t \quad (1)$$

Est. aux. regression

$$\hat{u}_t = \alpha_0 + \alpha_1 X_{1t} + \alpha_2 X_{2t} + \rho_1 \hat{u}_{t-1} + \dots + \rho_p \hat{u}_{t-p} + \varepsilon_t$$

$\hookrightarrow R^2_{aux}$

$$BG = n R^2_{aux} \sim \chi^2_p$$

$$n = T - p$$

Advantages:

1. No lim. on structure of the model
2. Exact critical values (unlike DW-test)
3. Always aplicable (unlike h-test Durbin's)
4. Applicable for AC of higher order
5. Applicable for MA form of error term

$$u_t = \varepsilon_t + \mu_1 \varepsilon_{t-1} + \dots + \mu_q \varepsilon_{t-q}$$

E.g.: MA(1) $\quad u_t = \varepsilon_t + \mu_1 \varepsilon_{t-1}$

$$u_{t-1} = \varepsilon_{t-1} + \mu_1 \varepsilon_{t-2}$$

$$\mathrm{cov}(u_t, u_{t-1}) = \mu_1 \sigma_\varepsilon^2$$

Disadvantage: 1. Asymptotic test (only for big samples)

Fighting Autocorrelation

1) Correct specification
2) AR(1) - transformation
(Cochrain-Orcutt trans.
GLS trans.
↳ generalized LS)
3) MA(1) - transform
4) Adding lagged dep. var.
5) More complex ADL(p,q) - form

Cochrain - Orcutt Transf.

AR(1)

(1) $y_t = \beta_1 + \beta_2 X_t + u_t$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

(2) $y_t = \beta_1 + \beta_2 X_t + \rho u_{t-1} + \varepsilon_t$

- Lag (1) $\cdot\ \rho$:

(3) $\rho y_{t-1} = \rho \beta_1 + \rho \beta_2 X_{t-1} + \rho u_{t-1}$

- Subtract (2) - (3)

$$\underbrace{y_t - \rho y_{t-1}}_{y_t^*} = \beta_1(1-\rho) + \beta_2 (\underbrace{X_t - \rho X_{t-1}}_{X_t^*}) + \varepsilon_t$$

Est. $\hat{\rho}$ (using formula from DW test)

$DW \to 2(1-\rho)$

Est. $y_t^* \mid X_t^* \to$ est $\hat{\rho}_{New}$

$\to$ calc. y_t^*, X_t^* New .....

Iterate until $\hat{\rho}$ converges

Price - Winston correction for 1^{st} obs.

To add 1^{st} obs. to sample

$$(x_1, y_1) \cdot \sqrt{1-\rho^2}$$

3) MA(1) - transform

OLS: $\hat{\beta}_{OLS} = (X'X)^{-1} X'y$

GLS: $\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1}y$

$\hat{Var}(\hat{\beta}_{GLS}) = (X'\hat{\Omega}^{-1}X)^{-1}$

$$\Omega = \begin{pmatrix} cov(\varepsilon_1, \varepsilon_1) & & \\ & \ddots & \\ & & cov(\varepsilon_n, \varepsilon_n) \end{pmatrix}$$

cov. matrix error

$\hat{\Omega}$ - est. cov. matrix Ω

FGLS: $\hat{\beta}_{GLS} = (X'\hat{\Omega}^{-1}X)^{-1} X'\hat{\Omega}^{-1}y$

No AC:

$$\Omega = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \cdot \sigma_\varepsilon^2$$

MA(1) $\quad u_t = \varepsilon_t + \rho\, \varepsilon_{t-1}$

$$\Omega = \begin{pmatrix} 1 & \rho & 0 & \cdots & 0 \\ \rho & \ddots & \ddots & & \vdots \\ & \ddots & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & \rho \\ 0 & \cdots & & \rho & 1 \end{pmatrix} \sigma_\varepsilon^2$$

AR(1) $\quad \mathrm{Cov}(u_t, u_{t-k}) = \sigma_\varepsilon^2 \cdot \dfrac{\rho^k}{1-\rho^2}$

$$\Omega = \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & \ddots & & & \vdots \\ \rho^2 & & \ddots & & \vdots \\ \vdots & & & \ddots & \\ \rho^{n-1} & \cdots & & & 1 \end{pmatrix} \cdot \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

ADL

Common Factor test
for test AR(1)

From CO - transform:

$$y_t - \rho y_{t-1} = \beta_1(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + \varepsilon_t$$

ADL(1, 1) with <u>non-linear</u> restrictions

$$\gamma_3 = -\rho\beta_2$$

$$(R)\ y_t = \underbrace{\beta_1(1-\rho)}_{\gamma_0} + \underbrace{\rho}_{\gamma_1} y_{t-1} + \underbrace{\beta_2}_{\gamma_2} X_t \underbrace{- \rho\beta_2}_{\gamma_3} X_{t-1} + \varepsilon_t$$

$$CF = n \log\left({RSS_R}/{RSS_{UR}} \right) \sim \chi^2_1$$

Alternative: Wald - test for

non-lin. restr.: $\gamma_3 = -\gamma_1 \cdot \gamma_2$