Autocorrelation

(1) $y_t = \beta_1 + \beta_2 X_t + u_t$

$Cov(u_t, u_s) \neq 0 \qquad t \neq s$

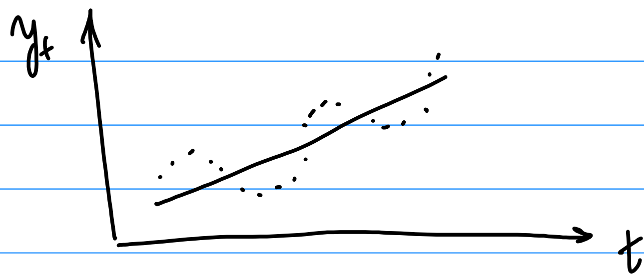
<u>First-order autocorrelation</u>

$u_t = \rho u_{t-1} + \varepsilon_t$

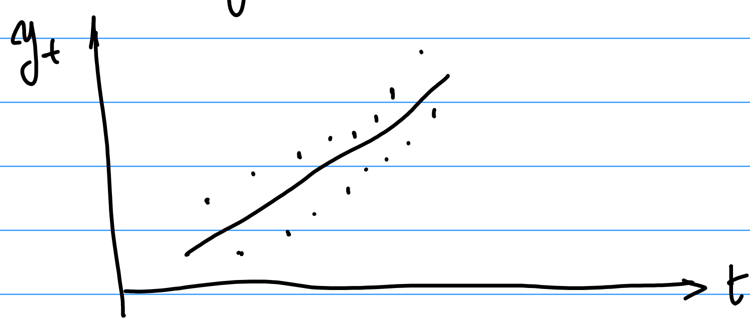
Satysfies GM cond.:
- $E(\varepsilon_t) = 0$
- $E(\varepsilon_i^2) = \sigma_\varepsilon^2$
- $E(\varepsilon_t, \varepsilon_s) \neq 0, \; t \neq s$

$-1 < \rho < 1$ 1^{st} order autocor. coef.

$\rho > 0$ Positive autocor.



$\rho < 0$ Negative autocor.



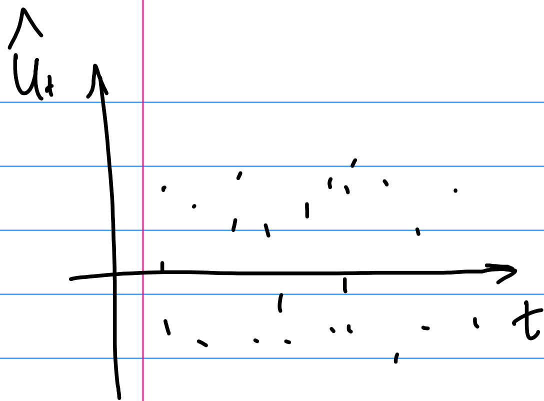
<u>Second-order autocorrelation:</u>

$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$

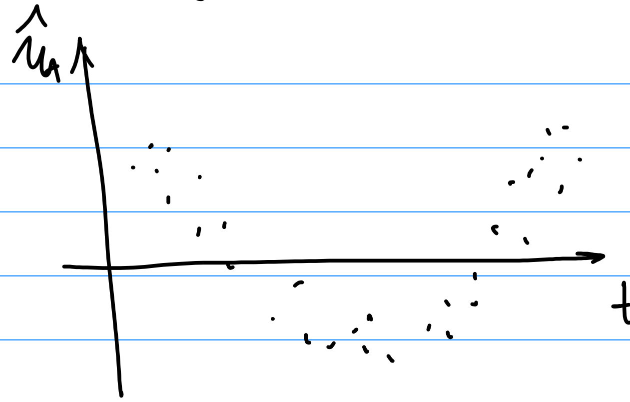
<u>Higher-order autocorrelation:</u>

often in case quarterly | monthly data

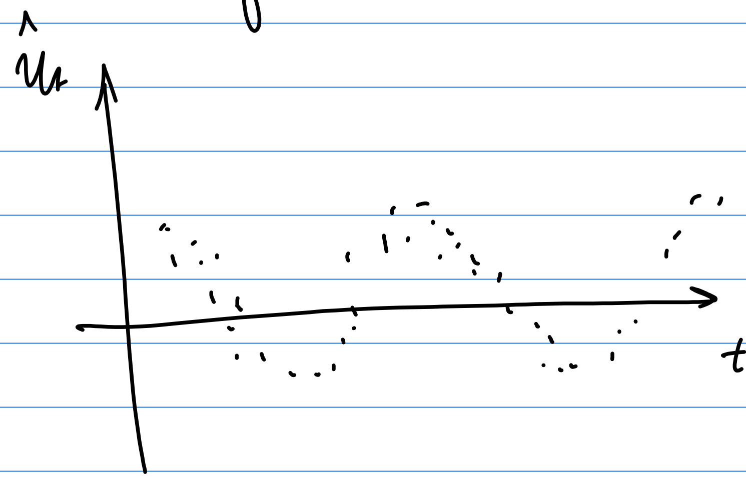
Detecting Autocorrelation



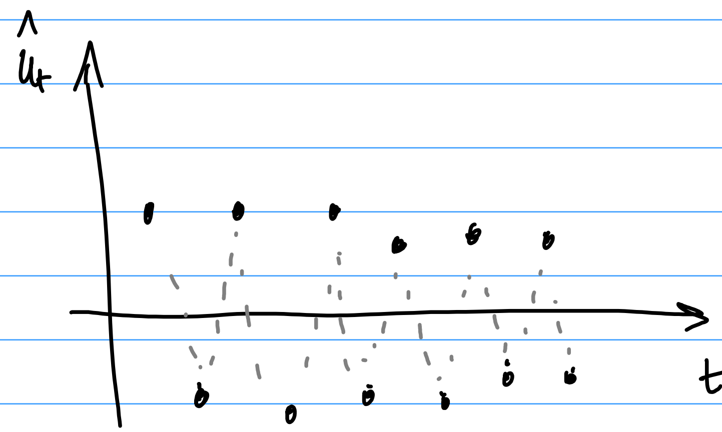
$\rho = 0$



$\rho > 0$



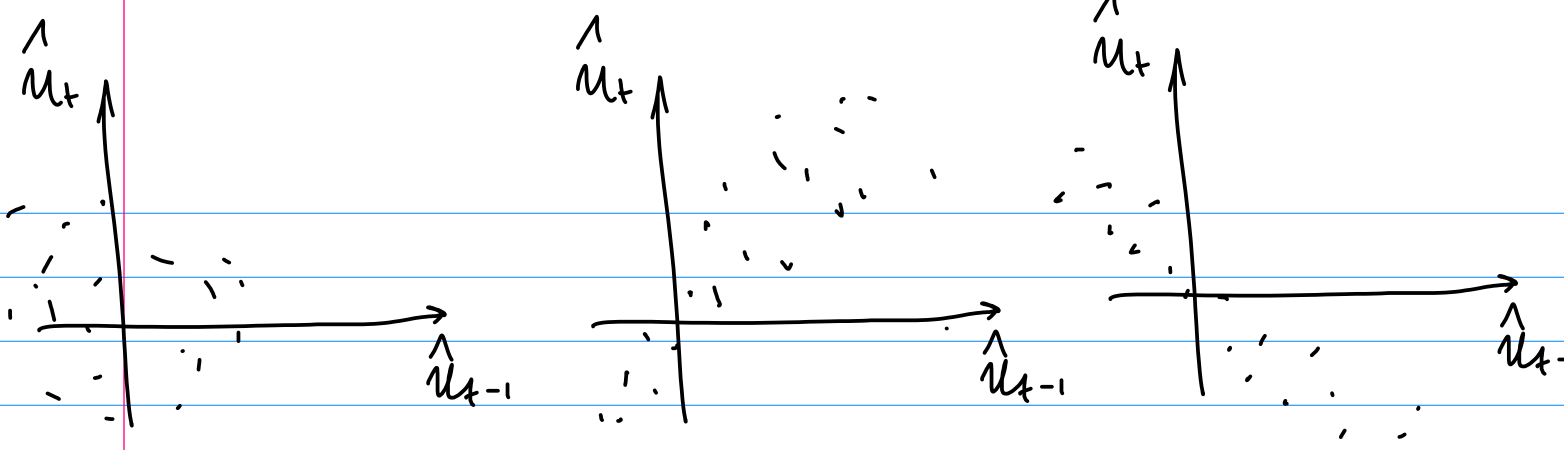
$\rho > 0$



$\rho < 0$

often (when small delta)

similar to
Normally distr. u_t



$\rho = 0$ $\qquad$ $\rho > 0$ $\qquad$ $\rho < 0$

True AC vs AC caused by misspecification

$u_t = \rho u_{t-1} + \varepsilon_t$

1. Omitted variable

True: $Y_t = \beta_1 + \beta_2 X_{1t} + \beta_3 X_{2t} + u_t$

Est: $Y_t = \beta_1 + \beta_2 X_{1t} + u_t^*$

$$u_t^* = \beta_3 X_{2t} + u_t$$

If X_{2t} – autocorrelated var.

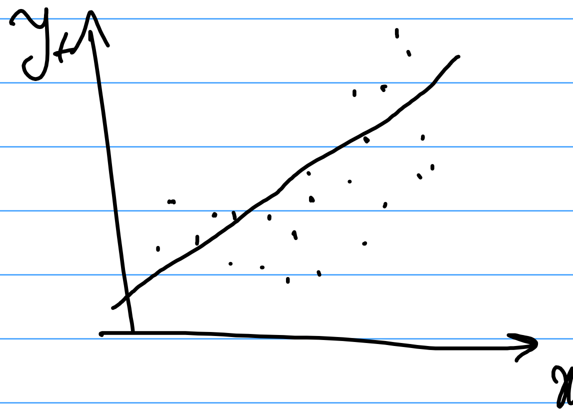
u_t – relatively small

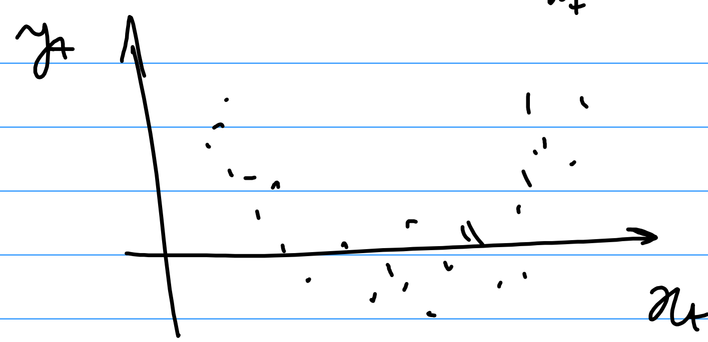
$\Rightarrow$ u_t^* – autocorrelated

② Wrong functional form

True : $\ln Y_t = \beta_1 + \beta_2 \ln X_t + u_t$

Est : $Y_t = \beta_1 + \beta_2 X_t + u_t^*$





Consequences of Autocorrelation (no lagged Y_t)

1) true AC => no bias in regr. est.

2) Pos. AC => underest. s.e. of coef. est.

=> t stats are inflated

tests are invalid

DW test

(for 1st order AC)

Assumptions:

1. Only for 1st order AC
2. Model should cont. intercept
3. No lagged explained variable in the model

$\hookrightarrow$ h-DW statistics.

$$DW = \frac{\sum (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2}$$

$H_0: \rho = 0$

$H_a: \rho > 0$

$(\rho < 0)$

$$DW = \frac{\sum (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2} = \frac{\sum \hat{\varepsilon}_t^2}{\sum \hat{\varepsilon}_t^2} - \frac{2 \sum \hat{\varepsilon}_t \hat{\varepsilon}_{t-1}}{\sum \hat{\varepsilon}_t^2} + \frac{\sum \hat{\varepsilon}_{t-1}^2}{\sum \hat{\varepsilon}_t^2}$$

$$\xrightarrow{p} \quad 1 - 2\rho + 1 = 2(1-\rho)$$

$$\rho = \frac{Cov(\varepsilon_t, \varepsilon_{t-1})}{Var(\varepsilon_t)}$$

$\rho = 1 \qquad DW = 0$

$\rho = 0 \qquad DW = 2$

$\rho = -1 \qquad DW = 4$

$-1 < \rho < 1 \qquad 0 < DW < 4$

Alt.:

$DW^* = 4 - DW$

$H_a: \rho > 0$

$H_a: \rho < 0$

$DW > d_u$ $\quad H_0$ do not rej => no AC

$DW < d_L$ $\quad H_0$ rej => $\rho > 0$

$d_L < DW < d_u$ $\quad$ can't reject / not reject

Durbin h-test

→ If model includes lagged dep. variable

$$h = \hat{\rho} \cdot \sqrt{\frac{n}{1 - n \, Var(\hat{\beta}_{y_{t-1}})}} \sim N(0,1)$$

$$DW \rightarrow 2(1 - \rho)$$

$$\hat{\rho} = 1 - 0{,}5 \, DW$$

$$h = (1 - 0{,}5 \, DW) \sqrt{\frac{n}{1 - n \, S^2_{\hat{\beta}_{y_{t-1}}}}} \sim N(0,1)$$