Adaptive Expectations Model

potential or equilibrium level of X

$$y_t = \alpha + \beta x_t^* + \varepsilon_t \qquad (1)$$

Adaptive Exp Hypothesis: $\quad 0 < \lambda < 1$

$$X_t^* - X_{t-1}^* = \lambda (X_t - X_{t-1}^*) \qquad (2)$$

observed value of X

$$X_t^* = \lambda X_t + (1-\lambda) X_{t-1}^* = \qquad (3)$$

$\{$ lag (3) and plug in (3) $\}$ $= \lambda (X_t + (1-\lambda) X_{t-1} + (1-\lambda)^2 X_{t-2} + \dots)$

$$= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j}$$

Plug (2) in (1):

$$y_i = \alpha + \beta (\lambda X_t + (1-\lambda) X_{t-1}^*) + \varepsilon_t$$

$$= \alpha + \beta \sum_{j=0}^{\infty} w_j X_{t-j} + \varepsilon_i$$

$$w_j = \lambda (1-\lambda)^j$$

ADL form for Koyck lag model

$$y_i = \alpha_0 + \beta_0 X_t + (1-\lambda) y_{t-1} + v_t$$

$\alpha_0 = \alpha\lambda \qquad \beta_0 = \beta\lambda \qquad v_t = \varepsilon_t - (1-\lambda)\varepsilon_{t-1}$

SR effect : $\beta_0 = \beta\lambda$

LR effect: $\bar{y} = \alpha\lambda + \beta\lambda\bar{X} + (1-\lambda)\bar{y}$

$\lambda\bar{y} = \lambda\alpha + \lambda\beta\bar{X} \quad | : \lambda$

$\beta = \beta_0 / \lambda \qquad \bar{y} = \alpha + \beta\bar{X}$

AEM:

$$X_t^*$$

$$y_t = \alpha + \beta X_{t+1}^e + \varepsilon_t$$

$$\Delta X_{t+1}^e = \lambda (X_t - X_t^e)$$

revision of expectation

coef of exp or speed of adjustment

error of forecast

$\lambda \approx 0$ slow revision

[P6-7]

$$\begin{cases} y_t = \beta_1 + \beta_2 X^e_{t+1} + u_t & (3) \\ X^e_{t+1} - X^e_t = \lambda\left(X_t - X^e_t\right) & (4) \end{cases}$$

from (4) $\quad X^e_{t+1} = X^e_t + \lambda\left(X_t - X^e_t\right) \quad (*)$

plug (4) in (3) $\quad y_t = \beta_1 + \lambda\beta_2 X_t + (1-\lambda)\beta_2 X^e_t + u_t$

$$\ldots = \beta_1 + \beta_2\lambda X_t + \beta_2\lambda(1-\lambda)X_{t-1} + \beta_2\lambda(1-\lambda)^2 X_{t-2} + \ldots$$

(5) $\quad \ldots + \beta_2\lambda(1-\lambda)^{s-1} X_{t-s+1} + \beta_2(1-\lambda)^s X^e_{t-s+1} + u_t$

(I) Estimation using non-linear technique:

Approximate of 5 – dropping $\beta_2(1-\lambda)^s$

1. fix $\bar{\lambda}$

2. $y_t = \beta_1 + \beta_2\underbrace{\left(\bar{\lambda}X_t + \bar{\lambda}(1-\bar{\lambda})X_{t-1} + \ldots\right)}_{X^*} + v_t$

$y_t = \beta_1 + \beta_2 X^* + u_t$

Est. using OLS $\Rightarrow \beta_1, \beta_2$, RSS

Repeat until RSS is min.

(II) Estimation using Koyck Transformation (with Koyck distr.)

From (5) :

$$y_t = \beta_1 + \beta_2 \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j} + u_t \qquad (6)$$

KT: $(6) - \text{lag}(6) \cdot (1-\lambda)$

$$y_{t-1} = \beta_1 + \beta_2 \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j-1} + u_{t-1} \quad \Big| \cdot (1-\lambda)$$

$$y_t - (1-\lambda) y_{t-1} = \lambda \beta_1 + \beta_2 \lambda X_t + u_t - (1-\lambda) u_{t-1}$$

ADL (1, 0)

$$y_t = (1-\lambda) y_{t-1} + \lambda \beta_1 + \beta_2 \lambda X_t + u_t - (1-\lambda) u_{t-1}$$

$$1 - \hat{\lambda} = 0{,}85$$

$$\hat{\lambda} = 0{,}15$$

II Estimation using Koych Transformation
(without Koych distr.)

$$\begin{cases} y_t = \beta_1 + \beta_2 X^e_{t+1} + u_t & (3) \\ X^e_{t+1} - X^e_t = \lambda (X_t - X^e_t) & (4) \end{cases}$$

KT: lag (3) $\quad y_{t-1} = \beta_1 + \beta_2 X^e_t + u_{t-1}$

$$\underline{\underline{\beta_2 X^e_t}} = -\beta_1 + y_{t-1} - u_{t-1}$$

$$y_t = \beta_1 + \beta_2 \lambda X_t + (1-\lambda)\underline{\underline{\beta_2 X^e_t}} + u_t =$$

$$= \beta_1 + \beta_2 \lambda X_t + (1-\lambda)(-\beta_1 + y_{t-1} - u_{t-1}) + u_t$$

$$= \lambda\beta_1 + \beta_2 \lambda X_t + (1-\lambda) y_{t-1} + u_t - (1-\lambda)u_{t-1}$$

Error Correction Model

(7) $y_t^* = \alpha + \beta X_t + \varepsilon_t$

EC hypothesis:

(8) $y_t - y_{t-1} = (1-\gamma)\underbrace{(y_t^* - y_{t-1}^*)}_{\substack{\text{change in}\\ \text{potential lev.}\\ \text{of } y}} + \underbrace{(1-\lambda)}_{=\tilde{\lambda}}\underbrace{(y_{t-1}^* - y_{t-1})}_{\substack{\text{previous}\\ \text{disequilibrium}}}$

$0 < 1-\gamma < 1 \qquad 0 < 1-\lambda < 1$

$\gamma = \lambda \Rightarrow$ ECM is PAM

Plug (7) in (8):

$y_t - y_{t-1} = (1-\gamma)(\beta(X_t - X_{t-1}) + \varepsilon_t - \varepsilon_{t-1})$
$+ (1-\lambda)(\alpha + \beta X_{t-1} + \varepsilon_{t-1} - y_{t-1})$

$y_t = \alpha_0 + \beta_0 X_t + \beta_1 X_{t-1} + \lambda y_{t-1} + v_t$

$\alpha_0 = (1-\lambda)\alpha \qquad \beta_1 = (\gamma - \lambda)\beta$

$\beta_0 = (1-\gamma)\beta \qquad v_t = (1-\gamma)\varepsilon_t + (\gamma-\lambda)\varepsilon_{t-1}$

ADL(1, 1)

SR effect: $\beta_0 = (1-\gamma)\beta$

LR effect: $\dfrac{\beta_0 + \beta_1}{1-\lambda} = \dfrac{\beta - \beta\gamma + \beta\gamma - \lambda\beta}{1-\lambda} = \beta$