Adaptive Expectation Model

$$y_t = \alpha + \beta X^*_t + \varepsilon_t \qquad (1)$$

Adaptive exp. hyp.:

$$\underbrace{X^*_t - X^*_{t-1}}_{\Delta X^*_t} = \lambda(X_t - X^*_{t-1}), \quad 0 < \lambda < 1$$

$$X^*_t = \lambda X_t + (1-\lambda) X^*_{t-1} = \qquad (2)$$

{lag (2) and plug it in}

$$\{X^*_t = \lambda X_{t-1} + (1-\lambda) X^*_{t-2}\}$$

$$= \lambda X_t + (1-\lambda)\lambda X_{t-1} + (1-\lambda)(1-\lambda) X^*_{t-2} =$$

$$= \lambda\left(X_t + (1-\lambda) X_{t-1} + (1-\lambda)^2 X_{t-2} + \ldots\right)$$

$$= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j}$$

Plug (2) in (1)

$$y_t = \alpha + \beta(\lambda X_t + (1-\lambda) X^*_{t-1}) + \varepsilon_t$$

$$= \alpha + \beta \sum_{j=0}^{\infty} w_j X_{t-j} + \varepsilon_t$$

$$w_j = \lambda(1-\lambda)^j$$

In Koyck model ADL form :

$$y_t = \alpha_0 + \beta_0 x_t + (1-\lambda) y_{t-1} + v_t$$

$$\alpha_0 = \alpha\lambda$$

$$\beta_0 = \beta\lambda$$

$$v_t = \varepsilon_t - (1-\lambda)\varepsilon_{t-1}$$

SR - effect : $\beta_0 = \beta\lambda$

LR - effect : $\beta = \beta_0 / \lambda$

(AEM)

$$y_t = \alpha + \beta x^e_{t+1} + \varepsilon_t \quad \longleftrightarrow x^*_t$$

↳ unobs. expectations

$$\Delta x^e_{t+1} = \lambda (x_t - x^e_t) \qquad 0 < \lambda \leq 1$$

↑ revision of x

λ - speed of adjustment

$\lambda \approx 1 \Rightarrow$ revision is fast

$\lambda \approx 0 \Rightarrow$ revision is slow

PG-7 ADL form:

$$\begin{cases} y_t = \beta_1 + \beta_2 X^e_{t+1} + \varepsilon_t & (3) \\ X^e_{t+1} - X^e_t = \lambda(X_t - X^e_t) & (4) \end{cases}$$

from (4) $X^e_{t+1} = X^e_t + \lambda(X_t - X^e_t)$

plug in (3) $y_t = \beta_1 + \beta_2(X^e_t + \lambda(X_t - X^e_t)) + \varepsilon_t$

$$= \beta_1 + \beta\lambda X_t + \beta_2(1-\lambda)X^e_t + \varepsilon_t =$$

$$\ldots = \beta_1 + \beta_2\lambda X_t + \beta_2\lambda(1-\lambda)X_{t-1} + \ldots$$

$$\ldots + \beta_2\lambda(1-\lambda)^{s-1}X_{t-s+1} + \underbrace{\beta_2(1-\lambda)^{s-1}X^e_{t-s+1}}_{} + \varepsilon_t$$

Approximation:

$$y_t = \beta_1 + \beta_2\lambda X_t + \beta_2\lambda(1-\lambda)X_{t-1} + \ldots + \beta_2\lambda(1-\lambda)^{s-1}X_{t-s+1} + \varepsilon_t$$

I Estimation: 1. fix $\bar\lambda$

2. $y_t = \beta_1 + \beta_2\underbrace{(\bar\lambda X_t + \bar\lambda(1-\bar\lambda)X_{t-1} \ldots)}_{X^*} + \varepsilon_t$

Estimate OLS $\Rightarrow \beta_1, \beta_2$; calc. RSS

Repeat to min RSS

II Estimation using Koyck transformation

(with Koyck Distr.)

$$y_t = \beta_1 + \beta_2 \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j} + \varepsilon_t \qquad (5)$$

KT : $(5) - \text{lag}(5) \cdot (1-\lambda)$

$$y_t - (1-\lambda) y_{t-1} = \lambda \beta_1 + \beta_2 \lambda X_t + \varepsilon_t - (1-\lambda)\varepsilon_{t-1}$$

ADL(1,0):

$$y_t = \underbrace{(1-\lambda)} y_{t-1} + \lambda \beta_1 + \beta_2 \lambda X_t + \underbrace{\varepsilon_t - (1-\lambda)\varepsilon_{t-1}}_{u_t}$$

1- speed of adj

$1 - \hat{\lambda} = 0.85 \Rightarrow \hat{\lambda} = 0,15$

II Estimation using Koyck transformation (without Koch Distr.)

$$\begin{cases} y_t = \beta_1 + \beta_2 X_{t+1}^e + \varepsilon_t & (3) \\ X_{t+1}^e - X_t^e = \lambda (X_t - X_t^e) & (4) \end{cases}$$

KT: 1) Lag (3)

$$y_{t-1} = \beta_1 + \beta_2 X_t^e + \varepsilon_{t-1}$$

2) $$\beta_2 X_t^e = y_{t-1} - \beta_1 - \varepsilon_{t-1}$$

3) $$y_t = \beta_1 + \beta_2 (\lambda X_t + (1-\lambda) X_t^e) + \varepsilon_t =$$

$$= \beta_1 + \beta_2 \lambda X_t + (1-\lambda)(y_{t-1} - \beta_1) + \varepsilon_t - (1-\lambda)\varepsilon_{t-1}$$

$$= \lambda \beta_1 + \beta_2 \lambda X_t + (1-\lambda) y_{t-1} + u_t$$

Error Correction Model

(5) $$y_t^* = \alpha + \beta x_t + \varepsilon_t$$

EC hypothesis:

(6) $$\Delta y_t = y_t - y_{t-1} = (1-\gamma)\underbrace{(y_t^* - y_{t-1}^*)}_{\Delta y_t^* \text{ change of potential levels}} + (1-\lambda)\underbrace{(y_{t-1}^* - y_{t-1})}_{\text{previous disequilibrium}}$$

$0 < 1-\gamma < 1$, $\quad 0 < 1-\lambda < 1$

$\gamma = \lambda \Rightarrow$ ECM is PAM

Plug (5) in (6)

$$y_t - y_{t-1} = (1-\gamma)\left(\beta(x_t - x_{t-1}) + \varepsilon_t - \varepsilon_{t-1}\right) + (1-\lambda)\left(\alpha + \beta x_{t-1} + \varepsilon_{t-1} - y_{t-1}\right)$$

$$y_t = \alpha_0 + \beta_0 x_t + \beta_1 x_{t-1} + \lambda y_{t-1} + v_t$$

$\alpha_0 = (1-\lambda)\alpha \qquad \beta_1 = (\gamma - \lambda)\beta$

$\beta_0 = (1-\gamma)\beta \qquad v_t = (1-\gamma)\varepsilon_t + (\gamma-\lambda)\varepsilon_{t-1}$

ADL(1,1)

SR effect: $\beta_0 = (1-\gamma)\beta$

LR effect: $\dfrac{\beta_0 + \beta_1}{1-\lambda} = \dfrac{(1-\gamma)\beta - (\gamma-\lambda)\beta}{1-\lambda} = \beta$