ADL(p, q)

$$y_t = \beta_1 + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} +$$
$$\gamma_0 x_t + \gamma_1 x_{t-1} + \ldots + \gamma_q x_{t-q} + \varepsilon_t$$

$$\varepsilon_t \sim N(0,1)$$

$ADL(1,0) \sim ADL(0,\infty)$

↗ Geometrically Distributed (Koych's) Lags

↘ Polynomial (Almond's) Lags

Koych's Lag Model

DL-form:

$$y_t = \alpha + \beta(1-\lambda)\sum_{j=0}^{\infty} \lambda^j X_{i-j} + \varepsilon_i$$

$$= \alpha + (\beta - \beta\lambda)(X_t + \lambda x_{t-1} + \ldots) + \varepsilon_i$$

Short run effect : $\beta(1-\lambda)$

AR-form:

$$y_t = \alpha(1-\lambda) + \beta(1-\lambda)X_t + \lambda y_{t-1} + \varepsilon_t - \lambda\varepsilon_{t-1}$$

$$= \alpha_0 + \beta_0 X_t + \lambda y_{t-1} + v_i$$

Long run effect : $\beta_0(1-\lambda)$

Problem 8. (UoL and ICEF Exam problem).

An econometrician having **quarterly data** for 12 years (plus current values 49 observations total) believes that current total consumption expenditure C_t is dependent not only on current value of disposable personal income Y_t and current price index P_t, but also on the last **two** years values of disposable personal income Y_{t-k}. She estimates using OLS the equation:

$$\hat{C}_t = 99 + 0.9Y_t - 0.2Y_{t-1} - 0.4Y_{t-2} - 0.2Y_{t-3} - 0.1Y_{t-4} + 0.04Y_{t-5} + 0.1Y_{t-6} + 0.4Y_{t-7} - 0.02Y_{t-8} + 0.4P_t \quad R^2 = 0.99$$

(91) (0.32) (0.28) (0.29) (0.29) (0.28) (0.30) (0.33) (0.33) (0.30) (0.31)

(a) What econometric phenomena can be observed in the equation above? What econometric problems (if any) are likely connected with these phenomena?

(b) Explain how you would test the hypothesis that consumption is dependent on disposable income for the last year only against the alternative hypothesis that consumption is dependent on the last two years. Give details of the information you need for this test.

(c) A colleague suggests that you should use an infinite lag model instead of the model above in **(a)**. How would you estimate this model on the basis of Koyck distribution;

(d) How would you estimate the same model on the basis of Koyck transformation?

(c) $C_t = \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^j Y_{t-j} + \gamma P_t + u_t$

$|\lambda| < 1$

Estimation

1 using NLS

- fix λ
- calculate $\sum_{j=0}^{\infty} \lambda^j Y_{t-j}$
- using OLS estimating β_0, β, γ
- calc. RSS

=> change λ to find λ^*

which min RSS

(d) 2. Estimation using Koyck's transformation

(1) $$C_t = \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^j y_{t-j} + \gamma P_t + u_t$$

Lagging Equation by one period (+)

multipl. by λ

(2) $$\lambda C_{t-1} = \lambda \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^{j+1} y_{t-j-1} + \lambda \gamma P_{t-1} + \lambda u_{t-1}$$

(1) - (2)

$$C_t = \lambda C_{t-1} + (1-\lambda)\beta_0 + \beta y_t + \gamma P_t - \lambda \gamma P_{t-1} + u_t - \lambda u_{t-1}$$

ADL (1,1) with restrictions

=> can't use OLS

=> nonlinear estimation

(by min RSS)

Polynomial Almon's Lag Model

$$y_t = \alpha + \sum_{j=0}^{q} \beta_j x_{t-j} + \varepsilon_i$$

$$\beta_j = \gamma_0 + \gamma_1 j + \gamma_2 \cdot j^2 + \dots + \gamma_p j^p = \sum_{k=0}^{p} \gamma_k j^k$$

$(p \leq q)$

$$y_t = \alpha + \sum_{k=0}^{p} \gamma_k z_{tk}$$

$$z_{tk} = \sum_{j=0}^{q} j^k x_{t-j}$$

short-term effect : $\beta_0 = \gamma_0$

Long-term effect $\sum_{j=0}^{q} \beta_j = \sum_{k=0}^{p} \gamma_k \sum_{j=0}^{q} j^k$

Economic Models that generate Geom. Lag Behavior

1. Partial Adjustment Model
2. Adaptive Expectation Model

Partial Adjustment Model

- $y_t^* = \alpha + \beta x_t$

$y_t \neq y_t^*$

- $\Delta y_t = (1-\lambda)(y_t^* - y_{t-1}) + \varepsilon_t$

$0 < \lambda < 1$

$$y_t = \alpha(1-\lambda) + \lambda y_{t-1} + \beta(1-\lambda) x_t + \varepsilon_t$$

$$= \gamma + \lambda y_{t-1} + \pi x_t + \varepsilon_t$$

$$\frac{\partial y_t}{\partial x_t} = \pi = \beta(1-\lambda) \leqslant \beta$$

↑ short-run effect

$$\frac{\partial y_t^*}{\partial x_t} = \beta \leftarrow \text{long-run effect}$$

(PAM)

$$y_t^* = \beta_1 + \beta_2 X_t + u_t$$

(*) $$y_t - y_{t-1} = \lambda(y_t^* - y_{t-1})$$

$$y_t - y_{t-1} = \lambda\beta_1 + \lambda\beta_2 \cdot X_t + \lambda u - \lambda y_{t-1}$$

(**) $$y_t = \lambda\beta_1 + \underbrace{\lambda\beta_2}\, X_t + (1-\lambda) y_{t-1} + \lambda u_t$$

$\Rightarrow$ ADL(1, 0)

from (*) $$y_t = \lambda y_t^* + (1-\lambda) y_{t-1}$$

↳ weighted sum

λ - speed of adjustment

Example:

from (**) $1-\lambda = 0{,}89$

$\Rightarrow \lambda = 0{,}11$

short-term effect: $[\lambda\beta_2 = 0{,}013]$

$\beta_2 = 0{,}12$

Long-term effect $\lambda\beta_2 / \lambda = \beta_2 = 0{,}12$