The International College of Economics and Finance.
Elements of Econometrics. 2021-2022.
Class 20. Non-Stationarity.

Problem 1. Explain what time series is called stationary. Explain what time series is called non-stationary.

Problem 2. Give some examples of stationary and non-stationary processes (no proof required).

Problem 3. Prove that time trend $X_t = \alpha + \beta t + \varepsilon_t$ is not stationary unless $\beta = 0$.

Problem 4. Prove that random walk (without drift) $X_t = X_{t-1} + \varepsilon_t$ is not stationary

Problem 5. Demonstrate that the AR(1) process, with $0<\beta_2 < 1$, is stationary for finite samples if X_0 is generated as a random variable with appropriate mean and variance, namely if $E(X_0) = 0$ and $\sigma^2_{X_0} = \frac{1}{1-\beta_2^2}\sigma^2_\varepsilon$.

Problem 6. Demonstrate that the AR(1) process, with $0<\beta_2 < 1$, is stationary for sufficiently large (potentially infinite) samples.

Problem 7. Demonstrate that the MA(1) process $X_t = \varepsilon_t + \alpha_1\varepsilon_{t-1}$ is stationary.

Problem 8. Show that MA(2) process $X_t = \varepsilon_t + \alpha_1\varepsilon_{t-1} + \alpha_2\varepsilon_{t-2}$ is stationary.

Problem 9 . Suppose that a series is generated as

$$X_t = \beta_2 X_{t-1} + \varepsilon_t$$

with β_2 is equal to $1 - \delta$, where δ is small. Investigate if (assuming that δ is small enough that terms involving δ^2 may be neglected) the variance may be approximated as

$$\sigma^2_{X_t} = (1-[2t-2]\delta) + ... + (1-2\delta) + 1)\sigma^2_\varepsilon$$
$$= (1-[t-1]\delta)t\sigma^2_\varepsilon$$

and draw your conclusions concerning the properties of the time series.

Problem 10. Why is it important to know whether the time series used in the regression model are stationary? Consider two variables Y_t and X_t, where

$$Y_t = \alpha + Y_{t-1} + \varepsilon_t$$
$$X_t = \beta + X_{t-1} + v_t.$$

ε_t and v_t are unrelated white noise processes. A researcher regresses Y_t on X_t and tests the significance of the slope coefficient. Discuss in detail the result he will get.

Problems from UoL and ICEF exams

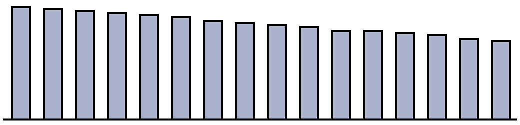
Problem 11. (ICEF Exam)
Consider the following three time series processes:

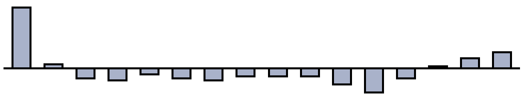
1) $X_t = u_t + \alpha_2 u_{t-1}$; $t = 1, 2, ..., T$;

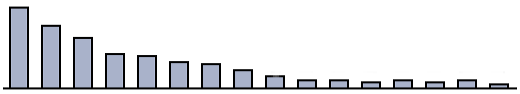
2) $X_t = \beta_2 X_{t-1} + u_t$; $t = 1, 2, ..., T$;

3) $X_t = X_t + u_t$; $t = 1, 2, ..., T$;

where u_t is IID – independent and identically distributed – with zero mean and finite variance, $0 < \beta_2 < 1$ and three graphs representing sample correlogramms of the processes (these graphs were obtained for the values of parameters $\alpha_2 = \beta_2 = 0.7$):

A) 

B) 

C) 

(a) What is autocorrelation function? Derive mathematical expression for autocorrelation function of the process 1) $X_t = u_t + \alpha_2 u_{t-1}$; $t = 1, 2, ..., T$;.

(b) Derive the mathematical expression for autocorrelation function of the process 2) $X_t = \beta_2 X_{t-1} + u_t$; $t = 1, 2, ..., T$.

(c) Which of the figures A-C above may correspond to each of these processes 1-3? Explain your choice. Which of the processes 1)-3) corresponds to the stationary time-series, and which of them corresponds to the non-stationary time series (no mathematical proof expected).

Problem 12 (UoL Exam).
Consider a model

$$y_t = u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2}; \ t = 1, 2, ..., T$$

$E(u_t) = 0$; $E(u_t^2) = \sigma^2$ and $E(u_s u_t) = 0$ if $s \neq t$ for all $s, t = 1, 2, ..., T$.

(i) Is y_t stationary? Explain in detail.

(ii) Calculate the autocorrelation function of y_t.