ADL(p,q)

$$y_t = \beta_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} +$$
$$\gamma_0 x_t + \gamma_1 x_{t-1} + \dots + \gamma_p x_{t-q} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$ADL(1,0) \longrightarrow ADL(0,\infty)$$

(1) Geometrically Distributed (Koyck's) Lags

(2) Polynomially Distributed (Almond's) Lags

Koyck Lag Model

$0 < \lambda < 1$

DL-form:

$$y_t = \alpha + \beta(1-\lambda)\sum_{j=0}^{\infty} \lambda^j X_{t-j} + \varepsilon_i$$

Short-run effect $\beta(1-\lambda)$

AR-form:

$$y_t = \alpha(1-\lambda) + \overbrace{\beta(1-\lambda)}^{\beta_0} X_t + \lambda y_{t-1} + \overbrace{\varepsilon_t - \lambda\varepsilon_{t-1}}^{u_t}$$

Long-run effect $\beta_0 / 1-\lambda = \beta$

Polynomial (Almon) Lag Model

$$y_t = \alpha + \sum_{j=0}^{q} \beta_j X_{i-j} + e_i$$

$$\beta_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2 + \dots + \gamma_p j^p = \sum_{k=0}^{p} \gamma_k j^k$$

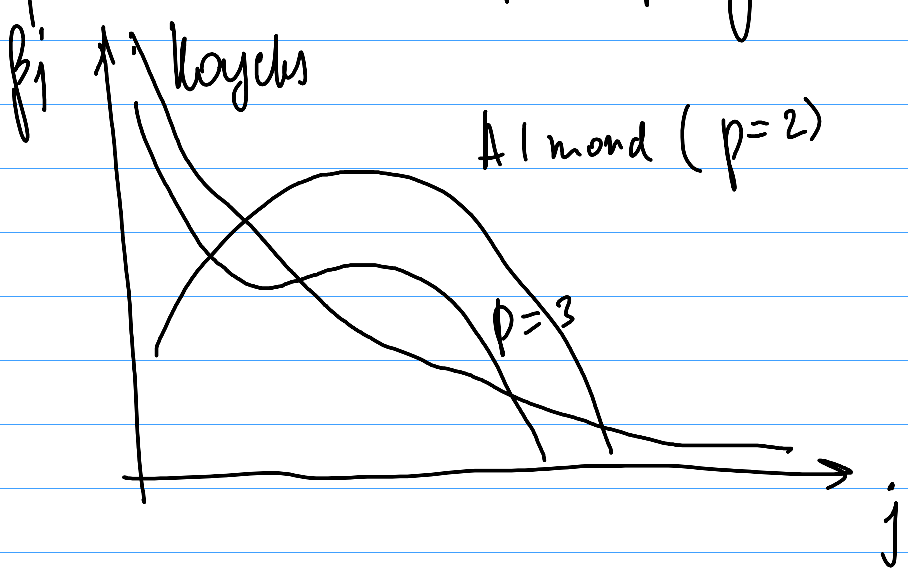
$$y_t = \alpha + \sum_{k=0}^{p} \gamma_k z_{tk} + e_i$$

$$z_{tk} = \sum_{j=0}^{q} j^k X_{t-j}$$

Short-run effect : $\beta_0 = \gamma_0$

Long-run effect : $\beta = \sum_{j=0}^{q} \beta_j = \sum_{k=0}^{p} \gamma_k \sum_{j=0}^{q} j^k$

p - order of polynoms



Problem 8. (UoL and ICEF Exam problem).

An econometrician having **quarterly data** for 12 years (plus current values 49 observations total) believes that current total consumption expenditure C_t is dependent not only on current value of disposable personal income Y_t and current price index P_t, but also on the last **two** years values of disposable personal income Y_{t-k}. She estimates using OLS the equation:

$$\hat{C}_t = 99 + 0.9Y_t - 0.2Y_{t-1} - 0.4Y_{t-2} - 0.2Y_{t-3} - 0.1Y_{t-4} + 0.04Y_{t-5} + 0.1Y_{t-6} + 0.4Y_{t-7} - 0.02Y_{t-8} + 0.4P_t \quad R^2 = 0.99 \tag{1}$$

$$(91)\ (0.32)\ (0.28)\ (0.29)\ (0.29)\ (0.28)\ (0.30)\ (0.33)\ (0.33)\ (0.30)\ (0.31)$$

(a) What econometric phenomena can be observed in the equation above? What econometric problems (if any) are likely connected with these phenomena?

(b) Explain how you would test the hypothesis that consumption is dependent on disposable income for the last year only against the alternative hypothesis that consumption is dependent on the last two years. Give details of the information you need for this test.

(c) A colleague suggests that you should use an infinite lag model instead of the model above in **(a)**. How would you estimate this model on the basis of Koyck distribution;

(d) How would you estimate the same model on the basis of Koyck transformation?

(c) $C_t = \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^j y_{t-j} + \gamma P_t + u_t$

$|\lambda| < 1$

I Estimation (Non-linear approach)

- fix λ
- calculate $\sum_{j=0}^{q} \lambda^j y_{t-j}$
- Estimate β_0, β, γ using OLS
- calculate RSS

$\Rightarrow$ Repeat changing λ to min RSS

d) II Estimation using Koyck's transformation

(1) $$C_t = \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^j Y_{t-j} + \gamma P_t + u_t$$

Lagging (1) by one period

+ multiplying by λ

(2) $$\lambda C_{t-1} = \lambda \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^{j+1} Y_{t-j-1} + \lambda \gamma P_{t-1} + \lambda u_{t-1}$$

Subtr.: (1) − (2)

$$C_t = \lambda C_{t-1} + (1-\lambda)\beta_0 + \beta Y_t + \gamma P_t +$$
$$+ \lambda \gamma P_{t-1} + u_t + \lambda u_{t-1}$$

ADL(1,1) with restrictions

can't use OLS

⇒ Non-linear estimation technique (changing λ to min RSS)

$\lambda \in (0,1)$

Economic Model that Generate Geometric Lag Behaviour

1) Partial Adjustment Model
2) Adaptive Expectations Models

PAM

$$y_t^* = \alpha + \beta x_t + \varepsilon_t$$

$$y_t \neq y_t^*$$

$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

$0 < \lambda < 1$

$$\underset{\substack{\shortparallel \\ y_t - y_{t-1}}}{\Delta y_t} = \lambda (y_t^* - y_{t-1})$$

$$\left\{ y_t = \lambda y_t^* + (1-\lambda) y_{t-1} \right\}$$

λ – speed of adjustment

y_t – weighted sum y_t^* and y_{t-1}

$\lambda \approx 1$ fast adjustments

$\lambda \approx 0$ slow adjustments

$$\begin{cases} y_t^* = \alpha + \beta x_t + \varepsilon_t \\ y_t - y_{t-1} = \lambda (y_t^* - y_{t-1}) \end{cases}$$

$$y_t - y_{t-1} = \lambda \alpha + \lambda \beta x_t + \lambda \varepsilon_t - \lambda y_{t-1}$$

$$y_t = \lambda \alpha + \lambda \beta x_t + (1-\lambda) y_{t-1} + \lambda \varepsilon_t$$

$\hookrightarrow$ ADL(1,0) with restr.

$$\frac{\partial y_t}{\partial x_t} = \lambda \beta \quad \text{— short-term effect}$$

$$\wedge$$

$$\frac{\partial y_t^*}{\partial x_t} = \beta \quad \text{— long-term effect}$$

$$\bar{y} = \lambda \alpha + \lambda \beta \bar{x} + (1-\lambda) \bar{y}$$

$$\left\{ \bar{y} = \frac{\lambda \alpha}{\lambda} + \frac{\lambda \beta}{\lambda} \bar{x} \right\}$$

Example

$$y_t = \lambda\alpha + \lambda\beta x_t + \underset{\substack{\parallel \\ 0,89}}{(1-\lambda)} y_{t-1} + \lambda\varepsilon_t$$

$1-\lambda = 0,89 \Rightarrow \lambda = 0,11$

Short-term effect $\lambda\beta = 0,013$

Long-term effect $\beta = \dfrac{\lambda\beta}{\lambda} = \dfrac{0,013}{0,11} =$

$= 0,12$