

Testing Stationarity

Dickey-Fuller Test (t -test)

$$y_t = y_{t-1} + \underline{\varepsilon_t} \Rightarrow \Delta y = \underline{\varepsilon_t}$$

$$(i) \quad \Delta y_t = \underbrace{\rho}_{=} y_{t-1} + \varepsilon_t$$
$$y_t - y_{t-1}$$

$$H_0: \rho = 0$$

y - non-stationary
RW

$$H_a: \rho < 0$$

y - stationary

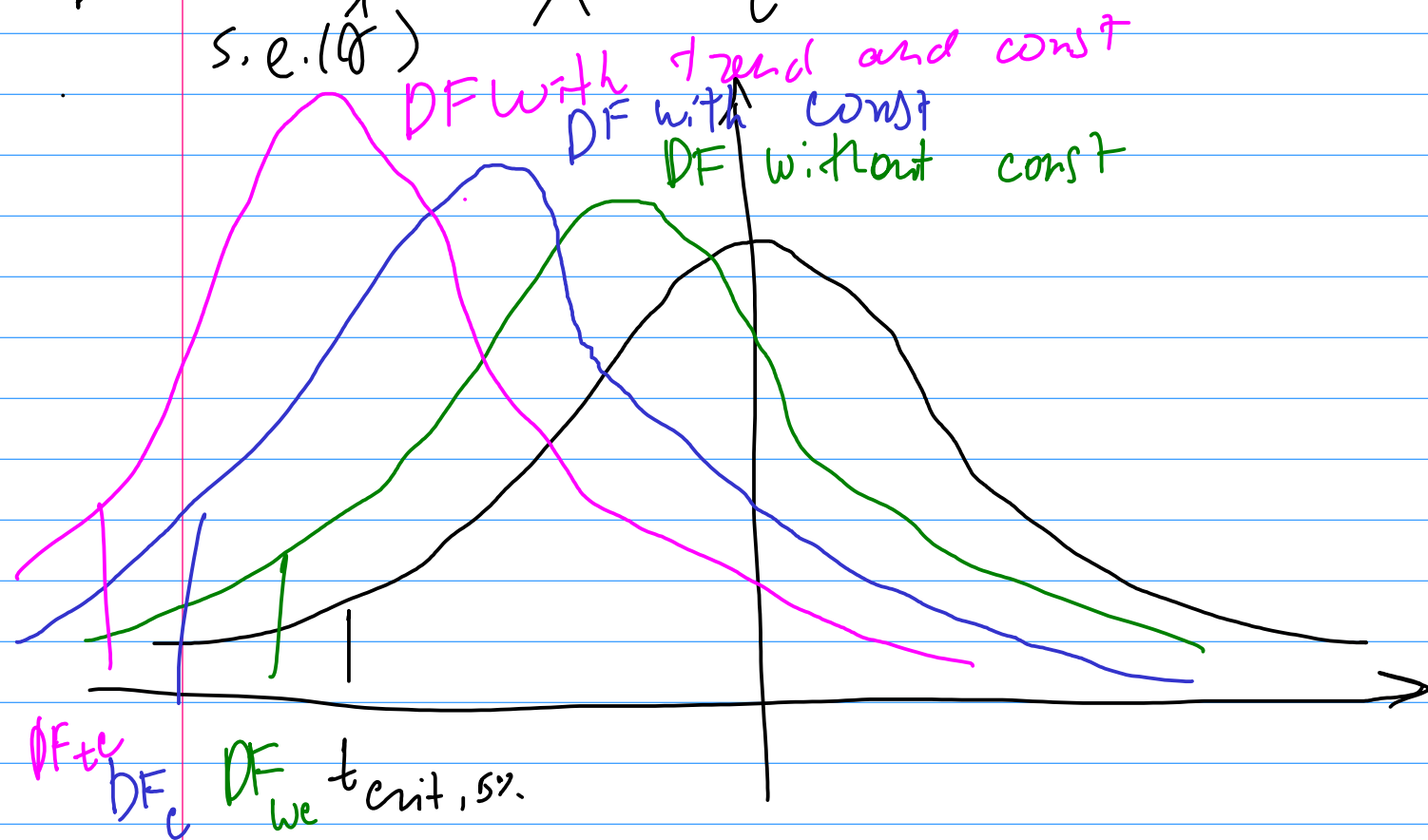
$$(ii) \quad \Delta y_t = \mu + \rho y_{t-1} + \varepsilon_t$$

y - RW with drift

$$(iii) \quad \Delta y_t = \mu + \rho y_{t-1} + \beta t + \varepsilon_t$$

y - trend Stationary TS

$$DF = \frac{\hat{\sigma}^2}{s.e.(\hat{\sigma}^2)} \quad \times \quad t$$



Augmented DF-test

$$AR(p): y_t = \mu + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + \varepsilon_t$$

$$\Delta y_t = \mu + \beta y_{t-1} + \alpha_1 \Delta y_{t-1} + \dots + \alpha_{p-1} \Delta y_{t-(p-1)} + \varepsilon_t$$

$$y_t = \alpha + \beta t + \varepsilon_t$$

$$y_t - \hat{\alpha} - \hat{\beta} t = \hat{\varepsilon}_t$$

Problem 10

$$(1) \quad \hat{\Delta P_t} = 160.58 - 0.02P_{t-1} \quad R^2 = 0.01, \\ (134.00) \quad (0.014)$$

$$(2) \quad \hat{\Delta DP_t} = -0.97DP_{t-1} \quad R^2 = 0.487 \\ (0.075)$$

$$(3) \quad \hat{\Delta VOL_t} = 1.48 \cdot 10^8 - 0.144VOL_{t-1} - 0.224\Delta(VOL_{t-1}) + 91320.24t \quad R^2 = 0.14 \\ (871445.3) \quad (0.045) \quad (0.073) \quad (871445.3)$$

$$(4) \quad \hat{\Delta VOL_t} = 1.55 \cdot 10^8 - 0.143VOL_{t-1} - 0.224\Delta(VOL_{t-1}) \quad R^2 = 0.14 \\ (65210866) \quad (0.044) \quad (0.073)$$

$$(1) \quad P_t = \beta_1 + \beta_2 P_{t-1} + u_t$$

$$P_t - P_{t-1} = \beta_1 + (\beta_2 - 1) P_{t-1} + u_t$$

$$\Delta P_t$$

$$H_0: \beta_2 - 1 = 0$$

$$DF = \frac{-0.02}{0.014} = -1.43 > -2.88$$

$\Rightarrow P_t$ - non-stationary

$$(2) \quad H_0: \beta_2 - 1 = 0$$

$$DF = \frac{-0.97}{0.075} = -12.93 < -2.88 (5\%)$$

$$< -3.48 (1\%)$$

$\Rightarrow DP_t$ - stationary

$$(3) \quad y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \theta t + u_t$$

$$\Delta y_t = \beta_1 + (\beta_2 - 1) y_{t-1} + \beta_3 y_{t-2} + \theta t + u_t$$

$$\Delta y_t = \beta_1 + (\beta_2 + \beta_3 - 1) y_{t-1} - \beta_3 \Delta y_{t-1} + \theta t + u_t$$

$$H_0: \beta_2 + \beta_3 - 1 = 0$$

$$DF = \frac{-0,144}{0,045} = -3,2 > -3,44$$

↳ VOL_t - non-stationary