APL(p, q)

$$y_t = \beta_0 + \gamma_1 y_{t-1} + \dots + \gamma_p y_{t-p} +$$
$$\alpha_0 x_t + \alpha_1 x_{t-1} + \dots + \alpha_q x_{t-q} + \varepsilon_t$$

APL(1, 0) $\longrightarrow$ ADL(0, ∞)

Problems:
1) NC
2) large # est. params
3) possible inefficiency

(1) Geometrically Distributed (Koyck's) Lag

(2) Polynomially Distributed (Almond's) Lag

Koyck's Lag Model $(0 < \lambda < 1)$

DL-form:

$$y_t = \alpha + \beta(1-\lambda)\sum_{j=0}^{\infty} \lambda^j X_{t-j} + \varepsilon_t$$

AR-form:

$$y_t = \overbrace{\alpha(1-\lambda)}^{\alpha_0} + \overbrace{\beta(1-\lambda)}^{\beta_0} X_t + \lambda y_{t-1} + \varepsilon_t - \lambda\varepsilon_{t-1}$$

Short-term effect: $\beta(1-\lambda)$

Long-term effect: $\beta_0 / 1-\lambda = \beta$

$$\bar{y} = \alpha_0 + \beta_0 \bar{X} + \lambda \bar{y}$$

Almond's Lag Model

$$y_t = \alpha + \sum_{j=0}^{q} \beta_j X_{t-j} + \varepsilon_t \qquad (1)$$

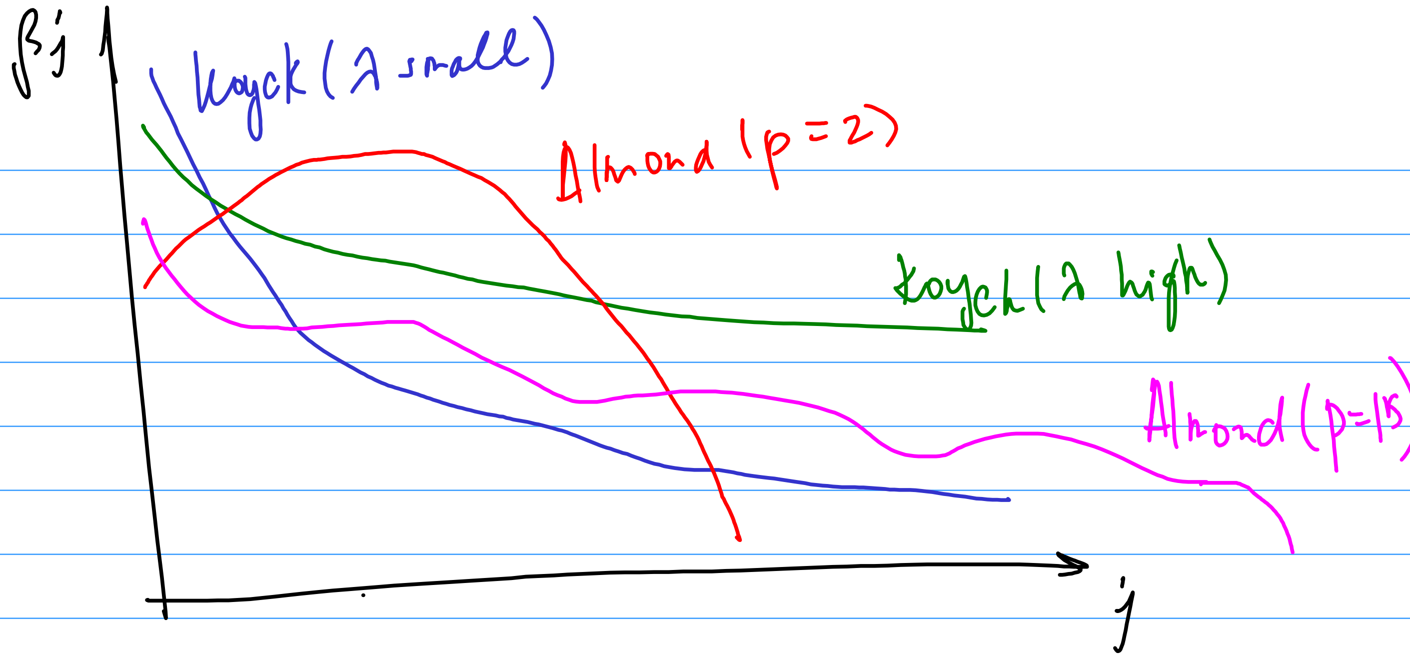
$$\beta_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2 + \dots + \gamma_p j^p = \sum_{k=0}^{p} \gamma_k j^k$$

$$y_t = \alpha + \sum_{k=0}^{p} \gamma_k z_{tk} + \varepsilon_t$$

$$z_{tk} = \sum_{j=0}^{q} j^k X_{t-j}$$

Short-term effect: $\beta_0 = \gamma_0$

Long-term effect: $\beta = \sum \beta_j = \sum_{k=0}^{p} \gamma_k \cdot \sum_{j=0}^{q} j^k$



β_j
Koyck (λ small)
Almond (p = 2)
Koyck (λ high)
Almond (p=15)
j

Problem 8. (UoL and ICEF Exam problem).

An econometrician having **quarterly data** for 12 years (plus current values 49 observations total) believes that current total consumption expenditure C_t is dependent not only on current value of disposable personal income Y_t and current price index P_t, but also on the last **two** years values of disposable personal income Y_{t-k}. She estimates using OLS the equation:

$$\hat{C}_t = 99 + 0.9Y_t - 0.2Y_{t-1} - 0.4Y_{t-2} - 0.2Y_{t-3} - 0.1Y_{t-4} + 0.04Y_{t-5} + 0.1Y_{t-6} + 0.4Y_{t-7} - 0.02Y_{t-8} + 0.4P_t \quad R^2 = 0.99$$
$$(91)\ (0.32)\ (0.28)\ (0.29)\ (0.29)\ (0.28)\ (0.30)\ (0.33)\ (0.33)\ (0.30)\ (0.31)$$

(a) What econometric phenomena can be observed in the equation above? What econometric problems (if any) are likely connected with these phenomena?

(b) Explain how you would test the hypothesis that consumption is dependent on disposable income for the last year only against the alternative hypothesis that consumption is dependent on the last two years. Give details of the information you need for this test.

(c) A colleague suggests that you should use an infinite lag model instead of the model above in **(a)**. How would you estimate this model on the basis of Koyck distribution;

(d) How would you estimate the same model on the basis of Koyck transformation?

(c) $$C_t = \beta_0 + \underbrace{\beta \sum_{j=0}^{\infty} \lambda^j y_{t-j}} + \gamma P_t + \varepsilon_t \quad (*)$$

$|\lambda| < 1$

I Non-linear estimation procedure

- fix λ
- calculate $\sum_{j=0}^{\infty / q} \lambda^j y_{t-j}$
- Estimate using OLS : β_0, β, γ
- calculate RSS

Repeat changing λ to min RSS

(d) II Est. based on Koyck's transformation

$$(1) \quad C_t = \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^j Y_{t-j} + \gamma P_t + \varepsilon_t$$

Lagged (1) first order (+) multiply by λ

$$(2) \quad \lambda C_{t-1} = \lambda \beta_0 + \beta \sum_{j=0}^{\infty} \lambda^{j+1} Y_{t-j-1} + \lambda \gamma P_{t-1} + \lambda \varepsilon_{t-1}$$

Subtract (1) - (2):

$$C_t = \overset{\alpha_1}{\overset{\shortparallel}{\lambda}} C_{t-1} - (1 \overset{\alpha_2}{\overset{\shortparallel}{-}} \lambda) \beta_0 + \beta \cdot Y_t + \gamma P_t$$
$$- \lambda \gamma P_{t-1} + \varepsilon_t - \lambda \varepsilon_{t-1}$$

ADL(1, 1) with restrictions

Can't use OLS

=> Non-linear est. technique

(change λ min RSS)

Economic Models that Generate Geom. Lag Behaviour

1) Partial Adjustment Model
2) Adaptive Expectation Model

PAM

$$y_t^* = \alpha + \beta \cdot x_t + \varepsilon_t$$

$$y_t \neq y_t^*$$

$0 < \lambda < 1$

$$\underset{\substack{\parallel \\ y_t - y_{t-1}}}{\Delta y_t} = \lambda \left(y_t^* - y_{t-1} \right)$$

PAM

$$\begin{cases} y_t^* = \alpha + \beta x_t + \varepsilon_t \\ y_t - y_{t-1} = \lambda \left(y_t^* - y_{t-1} \right) \end{cases}$$

$$y_t - y_{t-1} = \lambda \alpha + \lambda \beta x_t + \lambda \varepsilon_t - \lambda y_{t-1}$$

$$y_t = \alpha_0 + \underset{\substack{\parallel \\ \lambda\beta}}{\beta_1} x_t + \underset{\substack{\parallel \\ (1-\lambda)}}{\beta_2} y_{t-1} + u_t$$

ADL(1,0)

$$\Delta y_t = y_t - y_{t-1} = \lambda (y_t^* - y_{t-1})$$

$$y_t = \lambda y_t^* + (1-\lambda) y_{t-1}$$

↳ weighted sum

λ – adjustment rate

$\lambda \approx 0 \Rightarrow$ slow adjustment

$\lambda \approx 1 \Rightarrow$ fast adjustment

$$y_t = \lambda \alpha + \lambda \beta x_t + (1-\lambda) y_{t-1} + \lambda \varepsilon_t$$

Short-term effect $\dfrac{\partial y_t}{\partial x_t} = \lambda \beta$

Long-term effect: $\dfrac{\partial y_t^*}{\partial x_t} = \beta$

$$\bar{y} = \lambda \alpha + \lambda \beta \bar{x} + (1-\lambda) \bar{y}$$

$$\lambda \beta / \lambda = \beta$$