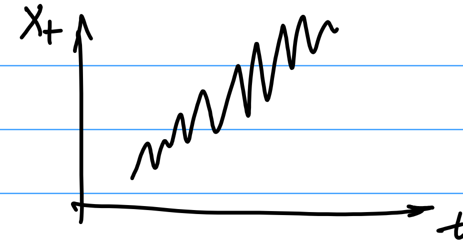
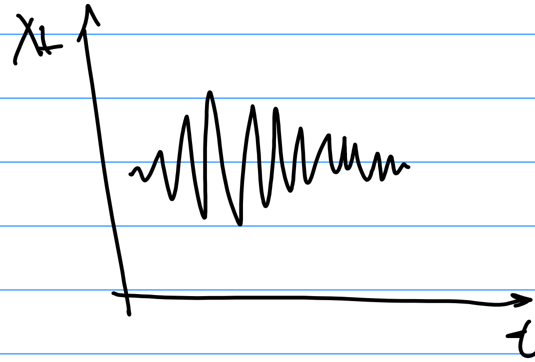
Non-Stationary TS

Weak Stationarity (Covariance)

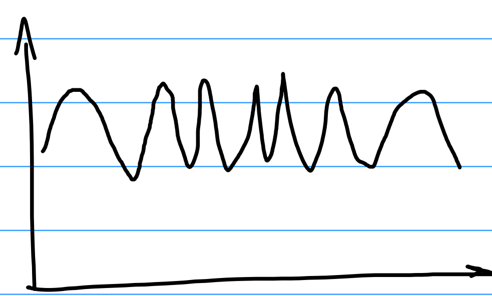
1. $E(X_t) = \text{const}$



2. $Var(X_t) = \text{const}$



3. $Cov(X_t, X_{t-s}) = f(s)$



Stationary Processes

- AR(1) for finite samples

$$X_t = \beta_1 X_{t-1} + \varepsilon_t,$$

$0 < \beta_1 < 1$

- MA(1)

$$X_t = \varepsilon_t + \alpha_2 \varepsilon_{t-1}$$

MA(2)

$$X_t = \varepsilon_t + \alpha_2 \varepsilon_{t-1} + \alpha_3 \varepsilon_{t-2}$$

Non-Stationary

- Time trend

$$X_t = \alpha + \beta t + \varepsilon_t$$

- RW

$$X_t = X_{t-1} + \varepsilon_t$$

- RW with drift

$$X_t = \alpha + X_{t-1} + \varepsilon_t$$

Problem 1:

$X_t = \alpha + \beta t + \varepsilon_t$ non-stationary unless $\beta = 0$

$E(X_t) = \alpha + \beta t$

Problem 2:

$X_t = X_{t-1} + \varepsilon_t$

1) $E(X_t) = E(X_{t-1}) + E(\varepsilon_t) =$

$= E(X_{t-2} + \varepsilon_{t-1}) =$

$= E(X_0 + \varepsilon_{t-1} + \varepsilon_{t-2} + \dots) = E(X_0)$

if X_0 is r.v. $\Rightarrow$

$E(X_t) = \mu$

2) $var(X_t) = var(X_0 + \varepsilon_t + \varepsilon_{t-1} \dots) =$

$\sigma^2_{X_0} + t \cdot \sigma^2_{\varepsilon_t}$

Problem 3:

AR(1) with $0<\beta_1<1$ is stationary

for finite samples

X_0 – r.v. with $E(X_0)=0$ $Var(X_0)=\frac{1}{1-\beta_2^2}\cdot \sigma_\varepsilon^2$

1) $X_t = \beta_1 \cdot X_{t-1} + \varepsilon_t =$

$= \beta_1^2 \cdot X_{t-2} + \beta_1 \varepsilon_{t-1} + \varepsilon_t = \dots$

$= \beta_1^t X_0 + \beta_1^{t-1}\varepsilon_1 + \beta_1^{t-2}\varepsilon_2 + \dots + \varepsilon_t$

$E(X_t) = \beta_1^t \cdot E(X_0)$

$\Rightarrow$ 1 prop. not violated if $E(X_0)=0$

2 $X_t = \beta_1^t X_0 + \beta_1^{t-1}\varepsilon_1 + \beta_1^{t-2}\varepsilon_2 + \dots + \varepsilon_t$

$\sigma_{X_t}^2 = \beta_1^{2t}\sigma_{X_0}^2 + \beta_1^{2t-2}\sigma_\varepsilon^2 + \dots + \sigma_\varepsilon^2$

$= \beta_1^{2t}\cdot\sigma_{X_0}^2 + \frac{1-\beta_1^{2t}}{1-\beta_1^2}\sigma_\varepsilon^2 = \dots$

$\left\{ \sigma_{X_0}^2 = \frac{1}{1-\beta_1^2}\cdot\sigma_\varepsilon^2 \right\}$

$\dots = \beta_1^{2t}\cdot\frac{1}{1-\beta_1^2}\cdot\sigma_\varepsilon^2 + \frac{1-\beta_1^{2t}}{1-\beta_1^2}\sigma_\varepsilon^2 = \frac{1}{1-\beta_2^2}\sigma_\varepsilon^2$

3) $X_t = \beta_1 X_{t-1} + \varepsilon_t$

$X_{t+s} = \beta_1 X_{t+s-1} + \varepsilon_{t+s} =$

$= \beta_1^s X_t + \beta_1^{s-1} \cdot \varepsilon_{t+1} + \ldots + \varepsilon_{t+s}$

$\mathrm{Cov}(X_t, X_{t+s}) = \beta_1^s \,\mathrm{Var}(X_t) +$

$+ \mathrm{Cov}(X_t, \beta_1^{s-1} \cdot \varepsilon_{t+1} + \ldots + \varepsilon_{t+s}) =$

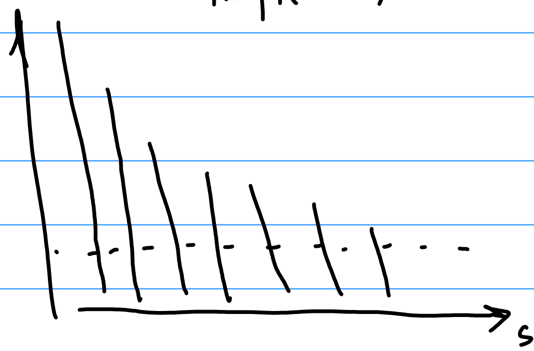
$\beta_1^s \, \sigma^2_{X_t}$ $\Rightarrow$ doesn't dep. on t

X_t, X_{t-1}, X_{t-2}

$Corr(X_t, X_{t-s})$ $\qquad$ $pCorr(X_t, X_{t-s})$

AR:

ACF



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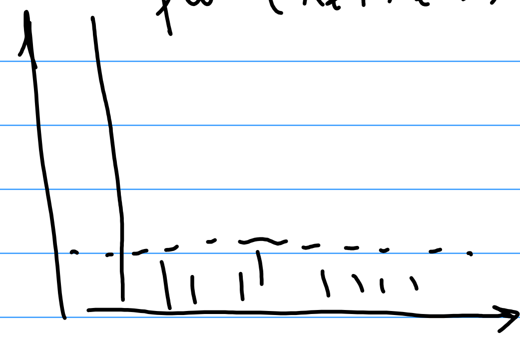

MA:

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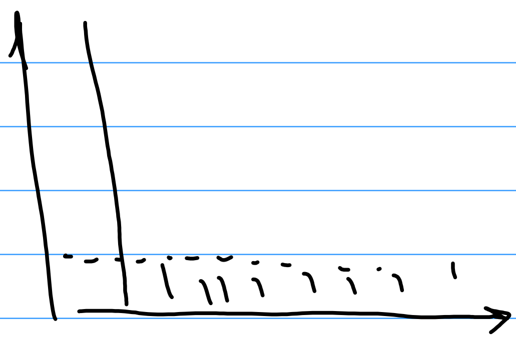

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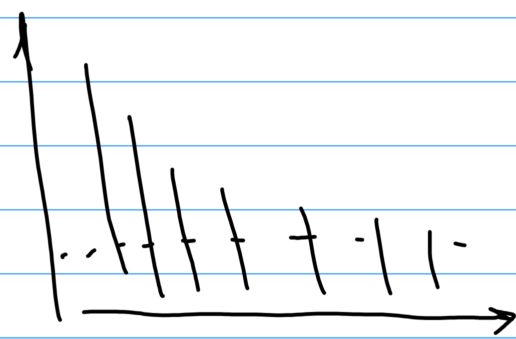
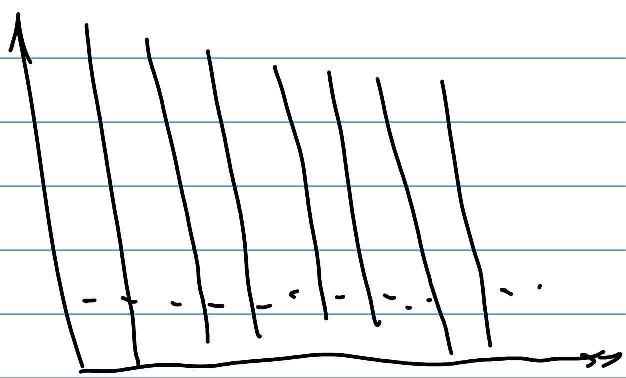


non-Stat.

ACF



Problem 4: $X_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1}$

1) $E(X_t) = 0$

2) $Var(X_t) = \sigma_\varepsilon^2 + \alpha_1^2 \cdot \sigma_\varepsilon^2$

3) $Cov(\varepsilon_t + \alpha_1 \varepsilon_{t-1}, \varepsilon_{t-1} + \alpha_1 \varepsilon_{t-2}) = \alpha_1 \sigma_\varepsilon^2, \quad s = 1$

$Cov(X_t, X_{t-s}) = 0, \quad s > 1$