

Panel data

↓	①	Cross - sections	x_i
	②	Time series	x_t
	③	Panel data	x_{it}

Advantages:

1. More observations :

$$\begin{array}{ccc} n & \cdot & T \\ | & & | \\ \# \text{ obj} & & \# \text{ Time periods} \end{array} \rightarrow \text{balanced panel}$$

2. Account for time dynamics

3. Fight with endogeneity

$$y_{it} = \mu_i + \beta x_{it} + \epsilon_{it}$$

Pooled Regression

$$y_{it} = \beta_0 + \sum_{j=1}^k \beta_j x_{jit} + \epsilon_{it} \quad [+ \mu_i = 0]$$

Fixed Effect

$$y_{it} = \beta_0 + \mu_i + \sum \beta_j x_{jit} + \epsilon_{it}$$

$$\exists j \quad \text{cov}(\mu_i, x_{jit}) \neq 0$$

1) LSDV (Dummy Variable)

2) FD (First Difference)

3) Within - method

① LSDV

$$y_{it} = \sum \beta_j x_{jit} + \sum_{i=1}^n \alpha_i D_i + \epsilon_{it}$$

$$D_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

Test: FE vs Pooled

F-test $H_0: \alpha_2 = \dots = \alpha_n$

② FD (first difference)

$$y_{it} = \beta_0 + \sum \beta_j x_{jit} + \mu_i + \varepsilon_{it} \quad (*)$$

(*) - lag (*)

$$\underbrace{y_{it} - y_{it-1}}_{\Delta y_{it}} = \sum \beta_j \underbrace{(x_{jit} - x_{jit-1})}_{\Delta x_{it}} + \underbrace{\varepsilon_{it} - \varepsilon_{it-1}}$$

③ Within - transformation

$$y_{it} - \bar{y}_i = \beta_0 + \sum \beta_j x_{jit} + \mu_i + \varepsilon_{it} \\ - \left(\beta_0 + \sum \beta_j \bar{x}_{ji} + \mu_i + \bar{\varepsilon}_i \right)$$

$$y_{it} - \bar{y}_i = \sum \beta_j (x_{jit} - \bar{x}_{ji}) + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

Random Effects

$$y_{it} = \beta_0 + \sum \beta_j x_{jit} + \underbrace{\mu_i}_{u_{it}} + \epsilon_{it}$$

Assume:

$$E(\mu_i) = 0$$

$$\text{cov}(\mu_i, x_{jit}) = 0$$

$\forall j$

Estimate using GLS

Test: RE vs Pooled

Breusch-Pagan (LM) test

$$H_0: \sigma^2_{\mu_i} = 0 \quad \forall i \Rightarrow \text{objects are homogeneous}$$

FE vs RE

True	Pooled	RE	FE
$\mu_i = 0$	BLUE	not efficient	not efficient
$\text{cov}(\mu_i, X_{it}) = 0$	not efficient	BLUE	not efficient
$\text{cov}(\mu_i, X_{it}) \neq 0$	biased + incons.	biased + incons.	BLUE

Hausman test

H_0 : RE consistent

H_0 : RE inconsistent \Rightarrow FE

$$(\hat{\beta}_{FE} - \hat{\beta}_{RE})^T (\hat{V}(\hat{\beta}_{FE}) - \hat{V}(\hat{\beta}_{RE})) (\hat{\beta}_{FE} - \hat{\beta}_{RE}) \sim \chi^2_k$$