MLE and Binary Choice Model

Problem 1: p - prob. of event

n - # obs

m - # of event occurence

$$L(\theta \mid T_1, \ldots, T_n) = f(\theta \mid T_1) \cdot \ldots \cdot f(\theta \mid T_n)$$

$$\ln L(\theta \mid T_1, \ldots, T_n) = \sum f(\theta \mid T_i)$$

$$L(p \mid n, m) = \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m}$$

$$\ell(p \mid n, m) = \log \frac{n!}{m!(n-m)!} + m \log p + (n-m) \log (1-p)$$

$$\frac{d\ell}{dp} = 0 + \frac{m}{p} - \frac{n-m}{1-p} = 0$$

$$m(1-p) = (n-m)p$$

$$\hat{p} = \frac{m}{n}$$

$$\frac{d^2 \ell}{dp^2} = -\frac{m}{p^2} - \frac{(n-m)}{(1-p)^2} < 0$$

$\hat{p}$ - max: min

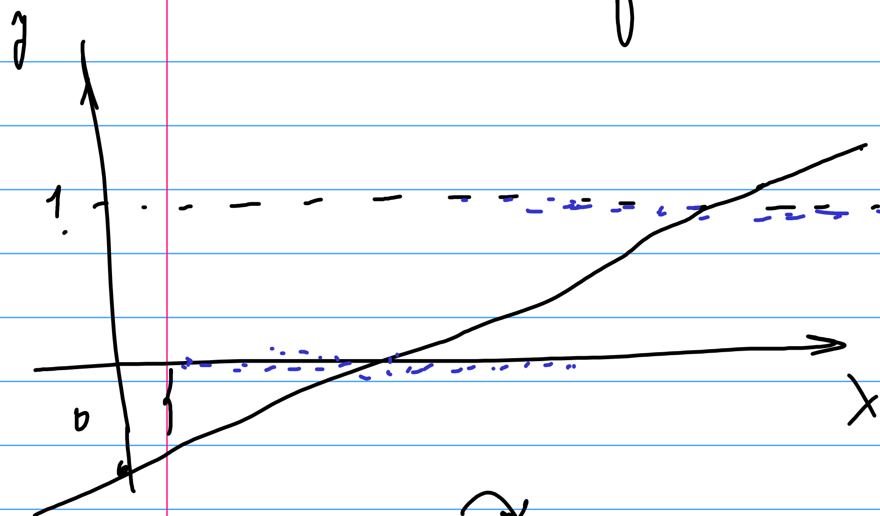
Binary Choice Models

$$y_i = \begin{cases} 0 & \text{else} \\ 1, & \text{if event occurred} \end{cases}$$

Linear Probability Model

$$p_i = P(y_i = 1) = \beta_1 + \beta_2 x_i$$

$$\hat{\beta}_1 = -0.3 \qquad \hat{\beta}_2 = 0.1$$



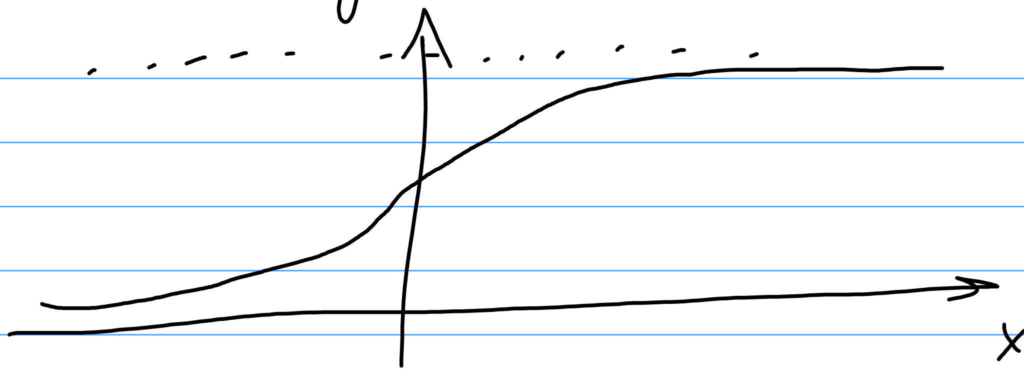
$1 \to 2$

$100 \to 101$

Disadvantages:

1. Prob. can be $p_i < 0$ and $p_i > 1$
2. Not realistic assumption that p changes linearly (w.r.t. X)
3. Heteroscedasticity (+ depends β)

Logit Model



$$P(y_i=1) = F(z_i) = \frac{1}{1+e^{-z_i}}$$

$$z_i = \beta_1 + \beta_2 x_i$$

$$\hat{\beta}_1 = -9 \qquad \hat{\beta}_2 = 0{,}5$$

Interpretation:

a) $X_i = 15$ $\quad P(y_i=1) = \frac{1}{1+e^{-(-9+0{,}5\cdot 15)}} =$

$$= 0{,}18$$

b) marginal effect

$$\frac{dP(y_i=1)}{dx} = \frac{e^{-(\beta_1+\beta_2 x)}}{\left(1+e^{-(\beta_1+\beta_2 x)}\right)^2}\cdot \beta_2$$

$$\frac{d\hat{p}(y_i=1)}{dx} = \frac{e^{-(-9+0,5\cdot 15)}}{\left(1+e^{-(-9+0,5\cdot 15)}\right)^2} \cdot 0,5 = 0,07$$

for $x_i = 100$

$$\frac{d\hat{p}(y_i=1)}{dx} = 8 \cdot 10^{-19}$$

In practice:

a) marginal effect for average (for $\bar{x}$)

b) average margin effect

$$= \frac{\text{m.e. for } x_i}{n}$$

if x – binary variable

$$\hat{p}(y_i=1) = \frac{1}{1+e^{-(-2+2\cdot x)}}$$

$$\hat{p}(x=1) - \hat{p}(x=0) = \frac{1}{1+e^{0}} - \frac{1}{1+e^{2}} = 0,38$$

Likelihood Ratio test

(to compare R and UR)

$$LR = -2(\ln L_R - \ln L_{UR}) \sim \chi^2_q$$

$$\left\{ = \ln\left(\frac{L_{UR}}{L_R}\right)^2 \right\}$$

q – # numb. of rest.

- Pseudo-R^2 (Mc Fadden R^2)

$$\text{Pseudo-}R^2 = 1 - \frac{\ln L}{\ln L_0} \in [0, 1]$$

$\hookrightarrow$ likelihood in naive (on a const)

- ratio of correctly predicted outcomes
- correlation between outcomes and predicted probabilities

$y_i = 1$ if hired

	Model 1	Model 2	Model 3
X	–	0,49	0,49
Gender	–	–	0,15
Black	–	–	– 0,32
Const	– 0,32	– 1,02	– 99
ln L	– 68	– 62	– 61

$$\text{Pseudo-}R^2 = 1 - \frac{\ln L}{\ln L_0}$$

a) Pseudo $R^2 = 0$ Model 1

Pseudo $R^2 = 1 - 62/68 = 0{,}05$ Model 2

Pseudo $R^2 = 1 - 61/68 = 0{,}1$ Model 3

b) $LR = -2(\ln L_R - \ln L_{UR})$

$= -2(-68 + 61) = 14 \sim \chi^2_3$

$\alpha = 5\%$ $\chi^2_{crit} = 7{,}81$

c) $LR = -2(-62 + 61) = 2 \sim \chi^2_2$

$\alpha = 5\%$ $\chi^2_{crit} = 5{,}99$

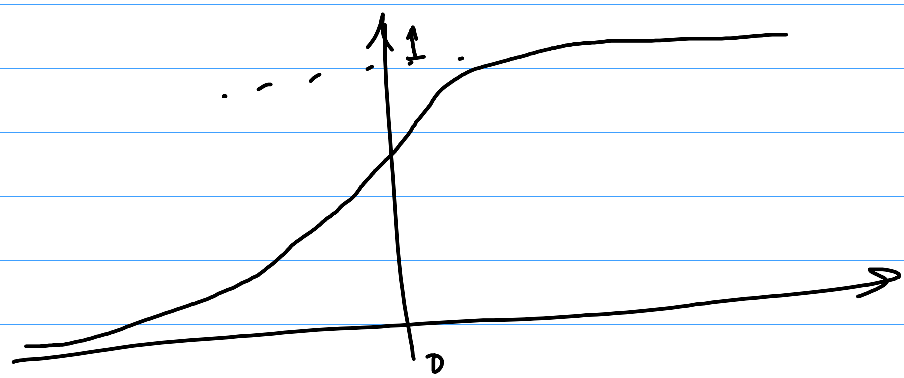
d) $\bar{x} = 1{,}4$

$$p(y_i = 1) = \frac{1}{1 + e^{-z_i}}$$

$$\frac{d\hat{p}}{dx} = \frac{e^{-(-1{,}02 + 0{,}49 \cdot 1{,}4)}}{\left(1 + e^{-(-1{,}02 + 0{,}49 \cdot 1{,}4)}\right)^2} \cdot 0{,}49 = 0{,}12$$

Probit Model

$P(y_i = 1) = \Phi(z_i)$, $z_i = \beta_1 + \beta_2 x_{2i} + \dots + \beta_k x_{ki}$



$$\frac{dP(y_i = 1)}{dx_j} = \Phi'(z_i) \cdot \beta_j =$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{(\beta_1 + \beta_2 x_{2i} + \dots + \beta_k \cdot x_{ki})^2}{2}} \cdot \beta_j$$