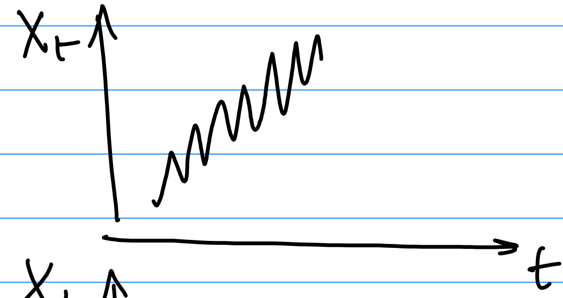
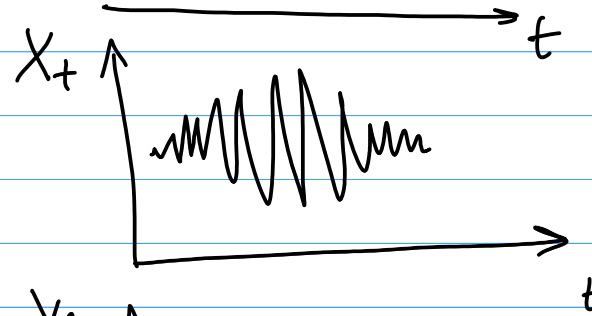
Not - Stationary Time Series

Weak (Covariance) Stationary

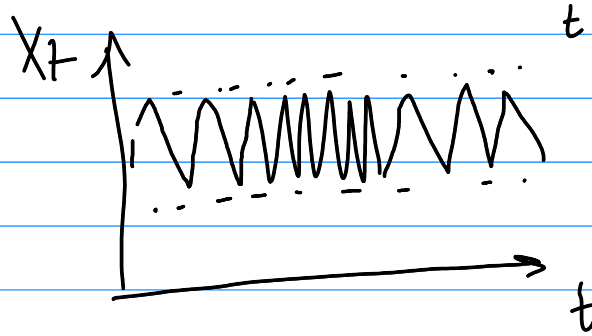
1. $E(X_t) = const$



2. $Var(X_t) = const$



3. $Cov(X_t, X_{t+s}) = f(s)$



Stationary Processes

1. AR(1) in finite samples

$$X_t = \beta_1 \cdot X_{t-1} + \varepsilon_t$$

$$0 < \beta_1 < 1$$

2. MA(1), MA(2)

$$X_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

$$X_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2}$$

Non-Stationary Processes

1. Trend Stationary

$$X_t = \alpha + \beta t + \varepsilon_t$$

2. RW

$$X_t = X_{t-1} + \varepsilon_t$$

RW with drift

$$X_t = \alpha + X_{t-1} + \varepsilon_t$$

Problem 3. $x_t = \alpha + \beta t + \varepsilon_t$

1) $E(X_t) = \alpha + \beta t \Rightarrow X_t$ - non-stationary

Problem 4. $X_t = X_{t-1} + \varepsilon_t \qquad X_{t-1} = X_{t-2} + \varepsilon_{t-1}$

$X_t = X_{t-2} + \varepsilon_{t-1} + \varepsilon_t = \ldots = X_0 + \varepsilon_1 + \ldots + \varepsilon_t$

1) $E(X_t) = E(X_{t-1}) + E(\varepsilon_t) = \ldots$

$\ldots = E(X_0) + 0 = E(X_0)$

if X_0 r.v. with $E(X_0) = \mu$

2) $Var(X_t) = \sigma^2_{X_0} + t \cdot \sigma^2_\varepsilon$

$\Rightarrow X_t$ non. stationary

Problem 5. $X_t = \beta_2 X_{t-1} + \varepsilon_t$ finite sample

(+) X_0 r.v. with $E(X_0) = 0$

$$Var(X_0) = \frac{1}{1-\beta_2^2} \sigma_\varepsilon^2$$

$$X_t = \beta_2(\beta_2 X_{t-2} + \varepsilon_{t-1}) + \varepsilon_t = \dots$$

$$\dots = \beta_2^t X_0 + \beta_2^{t-1} \varepsilon_1 + \dots + \beta_2 \varepsilon_{t-1} + \varepsilon_t$$

1) $E(X_t) = \beta_2^t E(X_0)$

if $E(X_0) = 0$ $\Rightarrow$ 1^{st} prop. isn't violated

2) $Var(X_t) = \beta_2^{2t} \sigma_{X_0}^2 + \beta_2^{2t-2} \sigma_\varepsilon^2 + \dots + \beta_2^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2$

$$= \beta_2^{2t} \sigma_{X_0}^2 + \frac{1-\beta_2^{2t}}{1-\beta_2^2} \sigma_\varepsilon^2$$

$$Var(X_0) = \frac{1}{1-\beta_2^2} \sigma_\varepsilon^2$$

$$Var(X_t) = \beta_2^{2t} \cdot \frac{1}{1-\beta_2^2} \sigma_\varepsilon^2 + \frac{1-\beta_2^{2t}}{1-\beta_2^2} \sigma_\varepsilon^2 =$$

$$\frac{1}{1-\beta_2^2} \sigma_\varepsilon^2$$

3)

$$X_t = \beta_2 X_{t-1} + \varepsilon_t$$

$$X_{t+s} = \beta_2^s X_t + \beta_2^{s-1} \varepsilon_{t+1} + \dots + \varepsilon_{t+s}$$

$$Cov(X_t, X_{t+s}) = \beta_2^s Var(X_t) + Cov(X_t ; \beta_2^{s-1} \cdot \varepsilon_{t+1} + \dots + \varepsilon_{t+s})$$

$$= \beta_2^s Var(X_t) + 0$$

Hence X_t is stationary

Problem 7 $\quad X_t = \varepsilon_t + \alpha_1 \cdot \varepsilon_{t-1}$

1) $E(X_t) = 0$

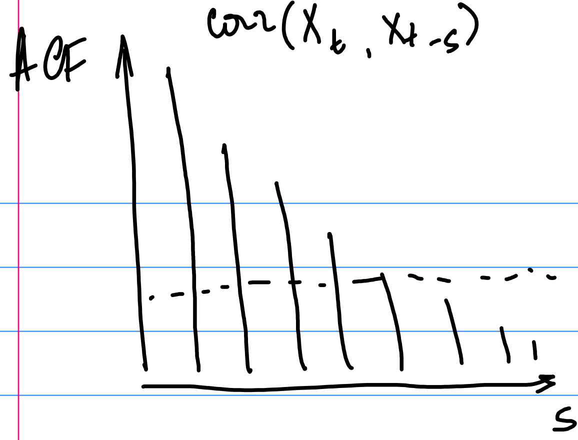
2) $Var(X_t) = \sigma_\varepsilon^2 + \alpha_1^2 \cdot \sigma_\varepsilon^2$

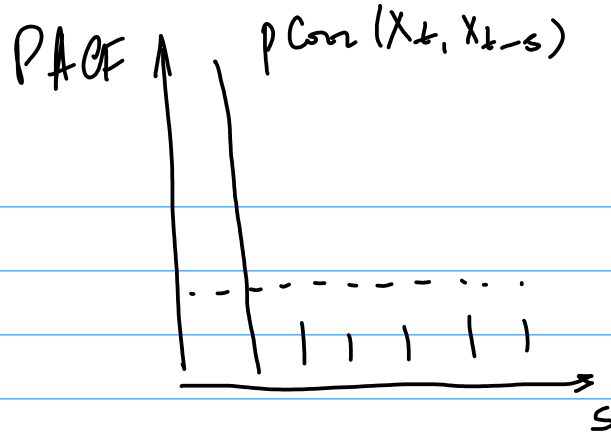
3)
$$Cov(X_t, X_{t+s}) = \begin{cases} s=1: & Cov(\varepsilon_t + \alpha_1 \varepsilon_{t-1} ; \varepsilon_{t+1} + \alpha_1 \cdot \varepsilon_t) = \alpha_1 \cdot \sigma_\varepsilon^2 \\ s>1: & 0 \end{cases}$$

$$X_t, X_{t-1}, X_{t-2}$$

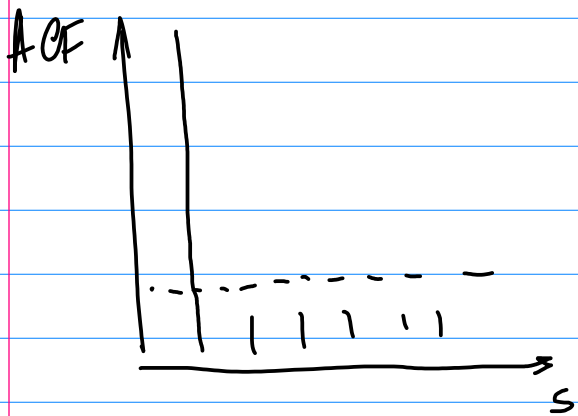
$$X_t, \underbrace{X_{t-1}}_{fix}, X_{t-2}$$

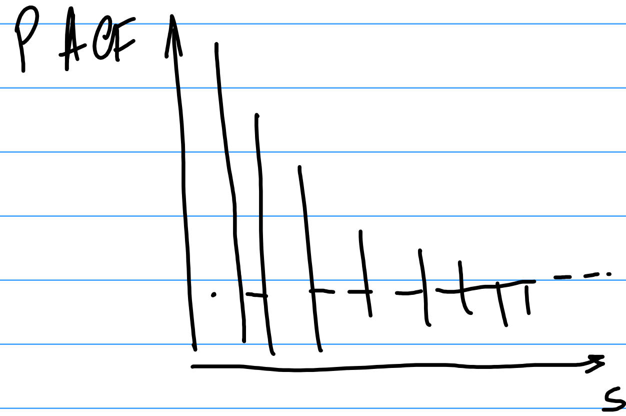
AR:



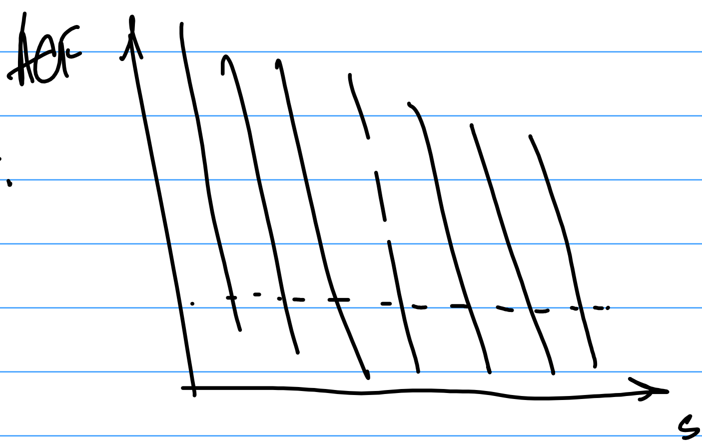


MA:





non-stat.



a) $X_t = \varepsilon_t + \alpha_1 \cdot \varepsilon_{t-1}$

1) $E(X_t) = 0$

2) $Var(X_t) = \sigma_\varepsilon^2 + \alpha_1^2 \cdot \sigma_\varepsilon^2$

3) $$Cov(X_t, X_{t+s}) = \begin{cases} s=1: & \alpha_1 \cdot \sigma_\varepsilon^2 \\ s>1: & 0 \end{cases}$$

$$Corr(X_t, X_{t+s}) = \begin{cases} s=1, & \dfrac{\alpha_1 \cdot \sigma_\varepsilon^2}{(1+\alpha_1^2)\sigma_\varepsilon^2} = \dfrac{\alpha_1}{1+\alpha_1^2} \\ s>1, & 0 \end{cases}$$

b) $AR(1)$: $X_t = \beta_2 X_{t-1} + \varepsilon_t$

$$Corr(X_t, X_{t+s}) = \rho^s = \frac{\beta_2^s \sigma_\varepsilon^2 / 1-\beta_2^2}{\sigma_\varepsilon^2 / 1-\beta_2^2} = \beta_2^s$$