Time Series

- Cross Sectional Data   n obs.   1 data per.
- Time Series   1 obs.   t data per.
- Panel Data   n obs.   t data per.

Problem 1

a) $HOUS_t = \beta_1 + \beta_2 \cdot DPI_t + u_t$

b) $HOUS_t = \beta_1 + \beta_2 \cdot DPI_t + \beta_3 DPI_{t-1} + u_t$

c) $HOUS_t = \beta_1 + \beta_2 \cdot DPI_t + \beta_3 HOUS_{t-1} + u_t$

Problem 2.   ADL(1, 0)

$$y_t = \beta_1 + \beta_2 \cdot x_t + \beta_3 y_{t-1} + u_t$$

$$L y_t = y_{t-1}$$

$$L^2 y_t = y_{t-2}$$

$$\vdots$$

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 L y_t + u_t$$

$$(1 - \beta_3 L) y_t = \beta_1 + \beta_2 \cdot x_t + u_t$$

$$y_t = \frac{\beta_1}{1-\beta_3 L} + \frac{\beta_2}{1-\beta_3 L} x_t + \frac{u_t}{1-\beta_3 L}$$

$$|\beta_3| < 1 \qquad \frac{1}{1-\beta_3 L} = 1 + \beta_3 L + \beta_3^2 L^2 + \dots$$

$$y_t = \underbrace{\frac{\beta_1}{1-\beta_3}}_{\beta_1^*} + \beta_2 x_t + \beta_2 \beta_3 \underbrace{L x_t} + \beta_2 \beta_3^2 \underbrace{L^2 x_t} + \dots + u_t^*$$

$$\rightarrow \quad y_t = \beta_1^* + \beta_2 x_t + \beta_2 \beta_3 x_{t-1} + \beta_2 \beta_3^2 x_{t-2} + \dots + u_t^*$$

ADL(p, q) ARDL

$$y_t = \beta_1 + \beta_2 \cdot x_t + \beta_3 y_{t-1} + u_t$$

$\hookrightarrow$ AR(1)

$$u_t = \varepsilon_t + \rho \varepsilon_{t-1}$$

$$y_{t-1} = \beta_1 + \beta_2 \cdot x_{t-1} + \beta_3 y_{t-2} + \varepsilon_{t-1} + \rho \varepsilon_{t-2}$$

SR and LR

$$y_t = \beta_1 + \beta_2 \cdot X_t + \beta_3 Y_{t-1} + u_t$$

$\uparrow$ short term marginal effect

$$\bar{y} = \beta_1 + \beta_2 \bar{x} + \beta_3 \bar{y}$$

$$(1 - \beta_3) \bar{y} = \beta_1 + \beta_2 \cdot \bar{x}$$

$$\bar{y} = \frac{\beta_1}{1 - \beta_3} + \frac{\beta_2}{1 - \beta_3} \cdot \bar{x}$$

Long-term marginal effect

$$\beta_2 \left(1 + \beta_3 + \beta_3^2 + \dots\right)$$

Problem 3. $\qquad ADL(0;1)$

$$\lg CAT_t = \beta_1 + \beta_2 \lg DPI_t + \beta_3 \lg DPI_{t-1} +$$
$$+ \beta_4 \lg PRCAT_t + \beta_5 \lg PRCAT_{t-1} + u_t$$

β_2 SR m.e. of $\lg DPI$

$$\overline{\lg CAT} = \beta_1 + \beta_2 \overline{\lg DPI} + \beta_3 \overline{\lg DPI} +$$
$$\beta_4 \cdot \overline{\lg PRCAT} + \beta_5 \overline{\lg PRCAT}$$

$\theta = \beta_2 + \beta_3$ LR m.e. of $\lg DPI$

$\phi = \beta_4 + \beta_5$

$$\lg CAT_t = \beta_1 + \beta_2 \lg DPI_t + (\theta - \beta_2) \lg DPI_{t-1} +$$
$$+ \beta_4 \lg PRCAT_t + (\phi - \beta_4) \lg PRCAT_{t-1} + u_t$$

$$\lg CAT_t = \beta_1 + \beta_2 \overbrace{(\lg DPI_t - \lg DPI_{t-1})}^{\Delta \lg DPI_t} +$$

$$\theta \lg DPI_{t-1} + \beta_4 (\lg PRCAT_t - \lg PRCAT_{t-1}) +$$

$$+ \phi \lg PRCAT_{t-1} + u_t$$

$$\lg CAT_t = \beta_1 + \beta_2 \Delta \lg DPI_t + \theta \lg DPI_{t-1}$$

$$+ \beta_4 \Delta \lg PRCAT_t + \phi \lg PRCAT_{t-1} + u_t$$

Koyck Distribution

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 \cdot X_{t-1} + \ldots + u_t$$

Problems when est. β_i

→ multicollinearity

→ large # of parameters is est. =>

=> d.of. - small

→ possible insignificance of coef.

before reg. with "long" lag

Problem 8. (UoL and ICEF Exam problem).

An econometrician having **quarterly data** for 12 years (plus current values 49 observations total) believes that current total consumption expenditure C_t is dependent not only on current value of disposable personal income Y_t and current price index P_t, but also on the last **two** years values of disposable personal income Y_{t-k}. She estimates using OLS the equation:

$$\hat{C}_t = \overset{\beta_0}{99} + \overset{\beta_1}{0.9}Y_t - \overset{\beta_2}{0.2}Y_{t-1} - 0.4Y_{t-2} - 0.2Y_{t-3} - 0.1Y_{t-4} + \left[0.04Y_{t-5} + 0.1Y_{t-6} + 0.4Y_{t-7} - \overset{\beta_9}{0.02}Y_{t-8} \right] + \overset{\beta_{10}}{0.4}P_t \quad R^2 = 0.99$$

$$(91)\ (0.32)\ (0.28)\ (0.29)\ (0.29)\ (0.28)\ (0.30)\ (0.33)\ (0.33)\ (0.30)\ (0.31)$$

(a) What econometric phenomena can be observed in the equation above? What econometric problems (if any) are likely connected with these phenomena?

(b) Explain how you would test the hypothesis that consumption is dependent on disposable income for the last year only against the alternative hypothesis that consumption is dependent on the last two years. Give details of the information you need for this test.

(c) A colleague suggests that you should use an infinite lag model instead of the model above in **(a)**. How would you estimate this model on the basis of Koyck distribution;

(d) How would you estimate the same model on the basis of Koyck transformation?

(b) $H_0: \beta_9 = \beta_8 = \beta_7 = \beta_6 = 0$

$$F = \frac{(RSS_R - RSS_{UR})/4}{RSS_{UR} / 49 - 11 - 8}$$