Panel Data

- Cross - Sections x_i
- Time series x_t

- Panel Data x_{it}

Advantages : 1) More data
2) Can observe dynamic changes
3) Can fight endogeneity

$n \times T$ - Balanced panel

↑ ↑

obj # time periods

$$y_{it} = \beta \cdot x_{it} + \mu_i + \varepsilon_{it}$$

μ_i - unobserved regions' specifics

↳ unobserved heterogeneity

Pooled Regression

$$y_{it} = \beta_0 + \beta_1 x_{it} + \varepsilon_{it} \quad [+ \mu_i]$$

$$\mu_i = 0$$

Fixed Effects

$$y_{it} = \mu_i + \beta X_{it} + \varepsilon_{it}$$

$$\mathrm{Cov}(\mu_i, X_{it}) \neq 0$$

1) Dummy Variables
   LSDV
2) First Difference
   FD
3) Within - group method

1 LSDV:

$$y_{it} = \sum_{j=1}^{k} \beta_j x_{jit} + \sum_{i=1}^{n} \alpha_i \cdot D_i + \varepsilon_{it}$$

$$D_1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \qquad i = 1$$

2 FD:

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j x_{jit} + \alpha_i + \varepsilon_{it} \qquad (*)$$

Lag $(*)$

$$y_{it-1} = \beta_0 + \sum_{j=1}^{k} \beta_j x_{jit-1} + \alpha_i + \varepsilon_{it-1}$$

$(*)$ − Lag $(*)$

$$\Delta y_{it} = \sum \beta_j \Delta x_{jit} + \underbrace{\varepsilon_{it} - \varepsilon_{it-1}}_{}$$

d.of. FD = # d.of. LSDV

③ Within - Group Method

$$y_{it} - \bar{y}_i = \beta_0 + \sum_{j=1}^{k} \beta_j X_{jit} + \alpha_i + \varepsilon_{it}$$
$$- \left(\beta_0 + \sum \beta_j \bar{X}_{ji} + \alpha_i + \bar{\varepsilon}_i\right)$$

$$y_{it} - \bar{y}_i = \sum \beta_j (X_{jit} - \bar{X}_{ji}) + \varepsilon_{it} - \bar{\varepsilon}_i$$

Test: FE vs Pooled

F - test:

$$\alpha_2 = \alpha_3 = \dots = \alpha_n = 0$$

Random Effects

$$y_{it} = \beta_0 + \sum \beta_j x_{jit} + \left[\underbrace{\mu_i + \varepsilon_{it}}_{v_{it}}\right]$$

Assume:

$\forall j$ $\quad cov(\mu_i, x_{jit}) = 0$

$E(\mu_i) = 0$

Estimated with GLS

Test: RE vs Pooled

Breusch - Pagan Test

$H_0: \sigma^2_{\mu_i} = 0 \quad \forall i$ (Objects are homogenous)

FE vs RE

True:	Pooled	RE	FE
$\alpha_i = 0$	unbiased (U) consistent (C) efficient (E)	ineff.	ineff.
$Cov(\alpha_i, X_{it}) = 0$	ineff.	unbiased (U) consistent (C) efficient (E)	ineff.
$Cov(\alpha_i, X_{it}) \neq 0$	biased incons.	biased incons.	unbiased (U) consistent (C) efficient (E)

Hausman test:

H_0: RE consistent

H_a: RE incons. $\Rightarrow$ FE

$$(\hat{\beta}_{FE} - \hat{\beta}_{RE})' \left(\hat{V}(\hat{\beta}_{FE}) - \hat{V}(\hat{\beta}_{RE}) \right) (\hat{\beta}_{FE} - \hat{\beta}_{RE})$$

$$\sim \chi^2_k$$