Breusch - Godfrey Test

$$y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + u_t, \quad t = \overline{1,T}$$

AR(p) - errors:

$$u_t = \rho_1 u_{t-1} + \dots + \rho_p u_{t-p} + \varepsilon_t$$

Est. aux. regression:

$$\hat{u}_t = \alpha_0 + \alpha_1 X_{1t} + \alpha_2 X_{2t} + \rho_1 \hat{u}_{t-1} + \dots + \rho_p \hat{u}_{t-p} + \varepsilon_t$$

$\hookrightarrow R^2_{aux}$

$$BG = n \cdot R^2_{aux} \sim \chi^2_p$$

$$n = T - p$$

Advantages:

1. No limitations for model structure
2. Exact critical values (unlike DW test)
3. Always applicable (unlike h-Durbin test)
4. Possible to test AC of higher orders

5. Applicable to MA - form of error term

$$u_t = \varepsilon_t + \mu_1 \varepsilon_{t-1} + \dots + \mu_q \varepsilon_{t-q}$$

E.g. MA(1):

$$u_t = \varepsilon_t + \rho \varepsilon_{t-1}$$

$$u_{t-1} = \varepsilon_{t-1} + \rho \varepsilon_{t-2}$$

Disadvantage: 1. Asymptotic test (for large sample only)

Fighting Autocorrelation

1) Correcting Specification
2) AR(1) - transform
   (Cochrain - Orcutt transform)
   ↳ GLS transform
   (generalized least squares)
3) MA(1) - transform
4) Using lagged dep. variable
5) More complex ADL(p, q)

Cochrain - Orcutt Transform

(1) $y_t = \beta_1 + \beta_2 X_t + u_t$

AR(1) form of error:

$$u_t = \rho u_{t-1} + \varepsilon_t$$

(2) $y_t = \beta_1 + \beta_2 X_t + \rho u_{t-1} + \varepsilon_t$

Lag (1) . ρ:

(3) $\rho y_{t-1} = \rho\beta_1 + \rho\beta_2 X_{t-1} + \rho u_{t-1}$

Subtract: (2) − (3)

$$\underbrace{y_t - \rho y_{t-1}}_{y_t^*} = \beta_1(1-\rho) + \beta_2 \underbrace{(X_t - \rho X_{t-1})}_{X_t^*} + \varepsilon_t$$

$\hat{\rho}$ can be est. from DW $\to 2(1-\rho)$

Est $y_t^* \mid X_t^* \Rightarrow \hat{\rho}_{new} \Rightarrow$

reest y_t^*, X_t^* new

Iterate until $\hat{\rho}$ converges

Price-Winston correction:

1st obs. is added with weight

$(x_t, y_t) \cdot \sqrt{1-\rho^2}$

MA(1) - transform:

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'y$$

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1}y$$

Ω - cov. matrix of errors

$\hat{\Omega}$ - est. (unbiased) of Ω

$$\hat{\beta}_{FGLS} = (X'\hat{\Omega}^{-1}X)^{-1} X'\hat{\Omega}^{-1}y$$

No AC:

$$\Omega = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \sigma_\varepsilon^2$$

MA(1) : $u_t = \varepsilon_t + \rho \varepsilon_{t-1}$

$$\Omega = \begin{pmatrix} 1 & \rho & 0 & \cdots & 0 \\ \rho & \ddots & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & \rho \\ 0 & \cdots & & \rho & 1 \end{pmatrix} \sigma_\varepsilon^2$$

$\hat{\Omega}$ using $\hat{\rho}$

$$u_t = \phi u_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t$$

ADL:

From CO transform:

$$y_t - \rho y_{t-1} = \beta_1(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + \varepsilon_t$$

ADL(1, 1) with non-lin. restrict.

$$\rightarrow y_t = \underbrace{\beta_1(1-\rho)}_{=\gamma_1} + \underbrace{\rho}_{=\gamma_2} y_{t-1} + \underbrace{\beta_2}_{=\gamma_3} X_t - \underbrace{\beta_2\rho}_{=\gamma_4} X_{t-1} + \varepsilon_t$$

$$\gamma_4 = -\gamma_2 \cdot \gamma_3$$

$$LF = n \log\left(RSS_R / RSS_{UR}\right) \sim \chi^2_1$$

Alternative: Wald test

$$\gamma_4 = -\rho \cdot \beta_3$$