Adaptive Expectations Model

(*) $y_i = \alpha + \beta X_i^* + \varepsilon_i$

Adaptive exp. hypothesis: $\underbrace{(1-\tilde{\lambda}) = \lambda}$ ← speed of adjustment

$$\underset{\Delta X_i^*}{\underbrace{X_i^* - X_{i-1}^*}} = \lambda (X_i - X_{i-1}^*)$$

$$X_i^* = \lambda X_i + (1-\lambda) X_{i-1}^* = \qquad (1)$$

$$\left\{ \begin{array}{l} \text{plug in} \\ \text{lagged (1)} \end{array} \right\} = \lambda (X_i + (1-\lambda) X_{i-1} + (1-\lambda)^2 X_{i-2} + \dots)$$

$$= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j}$$

Plug in (1) to (*)

$$y_i = \alpha + \beta (\lambda X_i + (1-\lambda) X_{i-1}^*) + \varepsilon_i$$

$$= \alpha + \beta \sum_{j=0}^{\infty} w_j X_{i-j}$$

$$w_j = \lambda (1-\lambda)^j$$

ADL - form:

$$y_i = \alpha_0 + \beta_0 X_i + (1-\lambda) y_{i-1} + v_i$$

$\alpha_0 = \alpha \lambda \qquad \beta_0 = \beta \lambda \qquad v_i = \varepsilon_i - \lambda \varepsilon_{i-1}$

SR effect: $\beta_0 = \beta\lambda$

LR effect: β

$$\bar{y} = \alpha\lambda + \beta\lambda\bar{x} + (1-\lambda)\bar{y}$$

$$\lambda\bar{y} = \lambda\alpha + \lambda\beta\bar{x} \Rightarrow \text{LR effect } \beta$$

$$\bar{y} = \alpha + \beta\bar{x}$$

(AEM) $\leftrightarrow x_t^*$

$$y_t = \alpha + \beta x_t^e + \varepsilon_t$$

$$\Delta x_t^e = \lambda(x_t - x_{t-1}^e) \qquad 0 < \lambda \leq 1$$

revision of expectation

λ - speed of adjustment

$$x_t^e = x_{t-1}^e + \lambda x_t - \lambda x_{t-1}^e =$$

$$= (1-\lambda)x_{t-1}^e + \lambda x_t$$

$\lambda \approx 1 \Rightarrow$ revision fast

$\lambda \approx 0 \Rightarrow$ revision slow

(P6) $y_t = \beta_1 + \beta_2 x^e_{t+1} + u_t$ (1) New indexes: $x^e_t \to x^e_{t+1}$

$x^e_{t+1} - x^e_t = \lambda (x_t - x^e_t)$ (2)

$$y_t = \beta_1 + \beta_2 \lambda x_t + \beta_2 \lambda (1-\lambda) x_{t-1} + \dots$$

$$+ \beta_2 \lambda (1-\lambda)^{s-1} x_{t-s+1} + \beta_2 (1-\lambda)^s x^e_{t-s+1} + u_t$$

(P7) from (2) express $x^e_{t+1} \to (1)$

(3) $y_t = \beta_1 + \beta_2 (\lambda x_t + (1-\lambda) x^e_t) + u$

$$\dots = \beta_1 + \beta_2 \lambda x_t + \beta_2 \lambda (1-\lambda) x_{t-1} + \dots$$

$$+ \beta_2 \lambda (1-\lambda)^{s-1} x_{t-s+1} + \boxed{\beta_2 (1-\lambda)^s x^e_{t-s+1}} + u_t$$

Same using Lag operator:

$$x^e_{t+1} = \lambda x_t + (1-\lambda) x^e_t$$

$$\lambda L x^e_{t+1} = \lambda x_t$$

$$x^e_{t+1} = (\lambda L)^{-1} \lambda x_t \qquad \{ \lambda = 1 - \tilde{\lambda} \}$$

$$x^e_{t+1} = (1 - \tilde{\lambda} L)^{-1} (1 - \tilde{\lambda}) x_t$$

$$= (1 - \tilde{\lambda}) \sum_{k=0}^{\infty} \tilde{\lambda}^k x_{t-k}$$

$$= \sum_{k=0}^{\infty} \beta_k x_{t-k} \qquad \beta_k = (1-\tilde{\lambda}) \tilde{\lambda}^k = \lambda (1-\lambda)^k$$

$\Rightarrow$ plug in (1)

(P8) Estimation using Koyck distribution

Approximation (3)

$$y_t = \beta_1 + \beta_2 \quad \lambda X_t + (1-\lambda) X_t^e) + u_t$$

$$\ldots = \beta_1 + \beta_2 \lambda X_t + \beta_2 \lambda (1-\lambda) X_{t-1} + \ldots$$

$$+ \beta_2 \lambda (1-\lambda)^{s-1} X_{t-s+1} + u_t$$

Estimation : fix λ => no non-linearity

$$y_t = \beta_1 + \beta_2 (\underbrace{\lambda X_t + \ldots}_{= X^{(\lambda)}})$$

Hence use OLS

Calc. RSS

=> min RSS

(P11) Est. using Koyck transform:
(with Koyck distribution)

(3) $y_t = \beta_1 + \beta_2 (\lambda x_t + (1-\lambda) x_t^e) + u_t$

$= \beta_1 + \beta_2 \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_{t-j} + u_t$

KT: $(3) - \text{lag}(3) \cdot (1-\lambda)$

$(1-\lambda) \cdot \Big| \; y_{t-1} = \beta_1 + \beta_2 \lambda \sum (1-\lambda)^j x_{t-j-1} + u_{t-1}$

$y_t - (1-\lambda) y_{t-1} = \lambda \beta_1 + \beta_2 \lambda x_t + u_t - (1-\lambda) u_{t-1}$

$\Rightarrow \quad y_t = (1-\lambda) y_{t-1} + \lambda \beta_1 + \beta_2 \lambda x_t + u_t - (1-\lambda) u_{t-1}$

$\Rightarrow$ ADL(1,0)

$1 - \hat{\lambda} = 0{,}85$

$\hat{\lambda} = 0{,}15$ $\Rightarrow$ rather slow revision

P12

Est. using Koyck transform:
(without Koyck distribution)

$$\begin{cases} y_t = \beta_1 + \beta_2 \underbrace{X_{t+1}^{e}} + u_t & (*) \\ X_{t+1}^{e} - X_t^{e} = \lambda (X_t - X_t^{e}) & (**) \end{cases}$$

KT: lag $(*)$

$$y_{t-1} = \beta_1 + \beta_2 X_t^{e} + u_{t-1}$$

$$\beta_2 X_t^{e} = -\beta_1 + y_{t-1} - u_{t-1}$$

$$y_t = \beta_1 + \beta_2 X_t^{e} + \beta_2 (\lambda (X_t - X_t^{e})) + u_t = \ldots$$

$$\ldots = \lambda \beta_1 + \beta_2 \lambda X_t + (1-\lambda) y_{t-1} + u_t - (1-\lambda) u_{t-1}$$

(ECM)

Error Correction Model

(1) $y^*_t = \alpha + \beta X_t + \varepsilon_t$

ECM hypothesis:

(2) $$\underbrace{y_t - y_{t-1}}_{\Delta y_t} = (1-\gamma)\underbrace{(y^*_t - y^*_{t-1})}_{\Delta y^*_i \text{ - change potential value}} + (1-\lambda)\underbrace{(y^*_{t-1} - y_{t-1})}_{\text{previous disequilibrium}}$$

$0 < \gamma < 1, \quad 0 < \lambda < 1$

$\gamma = \lambda \Rightarrow$ PAM

Plugging (2) to (1)

$$y_t - y_{t-1} = (1-\gamma)\left(\beta(X_t - X_{t-1}) + \varepsilon_t - \varepsilon_{t-1}\right) + (1-\lambda)\left(\alpha + \beta X_{t-1} + \varepsilon_{t-1} - y_{t-1}\right)$$

$$y_t = \alpha_0 + \beta_0 X_t + \beta_1 X_{t-1} + \lambda y_{t-1} + v_i$$

$\alpha_0 = (1-\lambda)\alpha \quad \beta_0 = (1-\gamma)\beta \quad \beta_1 = (\gamma-\lambda)\beta$

$$v_i = (1-\gamma)\varepsilon_t + (\gamma-\lambda)\varepsilon_{t-1}$$

SR: $(1-\gamma)\beta$

LR: $\dfrac{\beta_0 + \beta_1}{1-\lambda} = \dfrac{\beta - \gamma\beta + \gamma\beta - \lambda\beta}{1-\lambda} = \beta$