

Breuch - Godfrey Test

$$y_t = \beta_1 + \beta_2 X_{1t} + \beta_3 X_{2t} + u_t \quad t=1, \dots, T$$

$$u_t = \rho_1 u_{t-1} + \dots + \rho_p u_{t-p} + \varepsilon_t$$

estimate aux. regression

$$\hat{u}_t = \alpha_1 + \alpha_2 X_{1t} + \alpha_3 X_{2t} + \rho_1 \hat{u}_{t-1} + \dots + \rho_p \hat{u}_{t-p} + \varepsilon_t$$

$$\rightarrow R^2_{aux}$$

$$BG = n R^2_{aux} \sim \chi^2_p$$

$$n = T - p$$

- Advantages:
1. No limitation on the structure
 2. No uncertainty zones (unlike DW)
 3. Always applicable (unlike h-Durbin when neg. values under root)
 4. Test any order of AC.

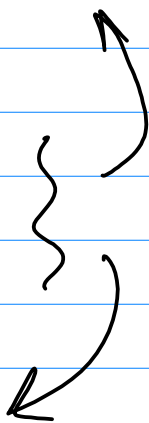
5. Also applicable for
MA - process errors

$$u_t = \varepsilon_t + \mu_1 \varepsilon_{t-1} + \dots + \mu_q \varepsilon_{t-q}$$

Disadvantage

1. Large samples
are required

Fight Autocorrelation

- 1) Correct Specification
 - 2) $\left\{ \begin{array}{l} \text{Cochran-Orcutt transformation} \\ \text{AR}(1) - \text{transform} \\ \text{(GLS)} \end{array} \right.$
generalized least squares
 - 3) $\text{MA}(1) - \text{transform}$
 - 4) Add lagged exp. variable
 - 5) More complex ADL (p.9)
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CO-transformation

$$(0) \quad y_t = \beta_1 + \beta_2 x_t + u_t \quad \nearrow$$

$$u_t = \rho u_{t-1} + \varepsilon_t \quad (*)$$

$$(1) \quad y_t = \beta_1 + \beta_2 x_t + \rho u_{t-1} + \varepsilon_t$$

↓ lagged (0) $\hat{\rho}$ from DW-test

$$(2) \quad \rho y_{t-1} = \rho \beta_1 + \rho \beta_2 x_{t-1} + \rho u_{t-1}$$

subtract (1) - (2)

$$\underbrace{y_t - \rho y_{t-1}}_{y_t^*} = \beta(1-\rho) + \beta_2 \underbrace{(x_t - \rho x_{t-1})}_{x_t^*} + \varepsilon_t \quad (3)$$

} given $\hat{\rho}$ from previous step
est: OLS $y_t^* | x_t^* \Rightarrow \hat{\rho}_{\text{new}}$

↑ Iterate until $\hat{\beta}$ converges

Price-Winsten correction:

add 1st obs $\cdot \sqrt{1 - \hat{\rho}^2}$

3) MA(1)

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

$$u_t = \varepsilon_t + \rho \varepsilon_{t-1}$$

$$u_{t-1} = \varepsilon_{t-1} + \rho \varepsilon_{t-2}$$

GLS - method

Ω - correlation matrix
of errors

$$\hat{\beta}_{GLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y$$

$$\hat{\beta}_{OLS} = (X' X)^{-1} X' y$$

$\hat{\Omega}$ given some assumptions.
 $\hat{V}(\hat{\beta}_{GLS}) = (X' \hat{\Omega} X)^{-1}$

If no AC:

$$\hat{\Omega}_{TNT} = \begin{bmatrix} 1 & & & 0 \\ & \ddots & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix} \sigma_{\varepsilon}^2 = I \cdot \sigma_{\varepsilon}^2$$

If MA(1)

$$\hat{\Omega} = \begin{bmatrix} 1 & \rho & 0 & \\ \rho & 1 & \ddots & \\ 0 & 0 & \ddots & \\ & & & 1 \end{bmatrix} \sigma_{\varepsilon}^2$$

If AR(1)

$$\text{cov}(u_t, u_{t-k}) = \rho^k \cdot \frac{\sigma_{\varepsilon}^2}{1-\rho^2}$$

$$\hat{\Omega} = \frac{\sigma_{\varepsilon}^2}{1-\rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{n-1} \\ \rho & 1 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

4) Adding lagged exp. variable

How to test $AR(1)$ vs $ADL(1,0)$

CD-form:

$$y_t - \rho y_{t-1} = \beta(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + \epsilon_t \quad (3)$$

$$(1) \quad y_t = \underbrace{\rho_0}_{\rho(1-\rho)} + \underbrace{\rho_1}_{\rho} y_{t-1} + \underbrace{\rho_2}_{\beta_2} X_t - \underbrace{\rho_3}_{\rho \beta_2} X_{t-1} + \epsilon_t$$

↖ $ADL(1,1)$ with non-lin. restr.

$$\rho_3 = -\rho_1 \cdot \rho_2$$

Test $AR(1)$ form by
Common Factor test

$$n \log \left(\frac{RSS_R}{RSS_{UR}} \right) \sim \chi^2_L$$

Test Wald for $\theta_3 = -\theta_1 \cdot \theta_2$