The International College of Economics and Finance
Econometrics - 2020. Mid-term exam, October 22

Part 1. (30 minutes). In each of 12 multiple choice tests indicate the correct answer.

1. In the Simple Linear Regression Model $Y_i = \beta_1 + \beta_2 X_i + u_i$ the population covariance of the OLS estimators of the intercept and slope coefficient $cov(b_1,b_2)$ is

1) positive if the mean value of X is positive;
2) negative if the mean value of X is positive;
3) may be positive or negative if the mean value of X is positive;
4) may be positive or negative if the mean value of X is negative;
5) does not depend on the mean value of X.

$\sigma^2_{\hat{\beta}} = \sigma^2_u (X'X)^{-1}$

$\Rightarrow Cov(\hat{\beta}_0, \hat{\beta}_1) = = \frac{-\bar{X}\sigma^2_u}{\sum(X_i - \bar{X})^2}$

2. For the Model $Y_i = \beta_2 X_i + u$ (Model A assumptions satisfied, $i=1,\dots,n$) the estimator $b_2 = \frac{((Y_1 + Y_2 + Y_3)/3) - \bar{Y}}{((X_1 + X_2 + X_3)/3) - \bar{X}}$ is:

~~1)~~ non-linear estimator of β_2; 2) unbiased estimator of β_2; ~~3)~~ efficient estimator of β_2;
~~4)~~ biased estimator of β_2; ~~5)~~ not an estimator of β_2.

3. In a simple regression with an intercept $\hat{Y} = b_1 + b_2 X$, the estimated slope coefficient b_2 is equal to zero. Then the determination coefficient R^2 is

1) Equal to one;
2) Not equal to 1 or 0;
3) In some situations can be negative;
4) Equal to zero
5) Can not be calculated for the model due to violation of assumptions.

4. If a new observation and a new explanatory variable are added in the Linear Regression Model, then for OLS-estimation, the following is true:

1) The Residual Sum of Squares (*RSS*) decreases; the Determination Coefficient R^2 may increase, decrease or stay the same;
2) The Residual Sum of Squares (*RSS*) increases; the Determination Coefficient R^2 may increase, decrease or stay the same;
3) Both the Determination Coefficient R^2 and the Residual Sum of Squares (*RSS*) may increase, decrease or stay the same;
4) The Determination Coefficient R^2 increases; the Residual Sum of Squares (*RSS*) may increase, decrease or stay the same;
5) The Determination Coefficient R^2 decreases; the Residual Sum of Squares (*RSS*) may increase, decrease or stay the same;

HW: TSS, ESS, RSS, R^2 $\Rightarrow$ ESS, R^2

is a new obs. is added

$$\bar{y}' = \frac{n}{n+1}\bar{y} + \frac{1}{n+1} y_{n+1}$$

$$TSS' = \sum (y_i - \bar{y}')^2 =$$

$$= \sum_{i=1}^{n+1} ((y_i - \bar{y}) + (\bar{y} - \bar{y}'))^2 =$$

$$= TSS + 2\underbrace{\sum (y_i - \bar{y})}_{=0} \cdot (\bar{y} - \bar{y}') +$$

$$+ n(\bar{y} - \bar{y}')^2 + (y_{n+1} - \bar{y}')^2$$

$$= TSS + \frac{n}{n+1}(y_{n+1} - \bar{y})^2$$

$$\Rightarrow TSS' \geq TSS$$

$$RSS'_{n+1} = \sum_{i=1}^{n+1} (Y_i - \hat{Y}_i)^2 =$$

$$= \underbrace{\sum_{i=1} (Y_i - \hat{Y}_i)^2}_{RSS'_n} + (Y_{n+1} - \hat{Y}_{n+1})^2 \geq$$

$$\geq \quad RSS'_n \geq RSS_n$$

by prop. of OLS

$\hat{\beta}$ min RSS

5. Using a sample of 570 observations, the following earnings function has been estimated: $\log(EARN) = -0.41 + 0.22ASVABC + 0.34\log(S)$ ($EARN$ – hourly earnings in Rubles, $ASVABC$ – abilities indicator, S – length of studies). For this earnings function, it can be said that if S increases for one per cent, then hourly earnings increase (on average, others equal) approximately for

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0.34 Rubles; 2) 0.34%; 3) 34%; 4) 0.0034%; 5) 34 Rubles.

6. For the Model $Y_i = \beta_1 + \beta_2 X_i + u$ (Model A assumptions satisfied) the estimator $b_2 = \frac{\sum (Y_i - \bar{Y})}{\sum (X_i - \bar{X})}$ is, generally speaking:

$\Leftarrow \quad \Sigma(X_i - \bar{X}) = 0$

1) biased and inconsistent estimator of β_2;
2) biased but consistent estimator of β_2;
3) unbiased but inefficient estimator of β_2;
4) equal to the OLS estimator;
5) can not be calculated;

7. Introduction of two linear restrictions on parameters in a regression model, estimated using OLS

1) results in minor increase of the sum of squared errors if at least one of the restrictions is not valid;
2) results in significant increase of the sum of squared errors if at least one of the restrictions is valid;
3) results in minor increase of the sum of squared errors if both restrictions are valid;
4) results in significant increase of the sum of squared errors only if both restrictions are valid;
5) all the above is incorrect.

8. Let $Y^* = \lambda_1 + \lambda_2 Y$. In a simple regression model, b^*_1 is the OLS estimator of the intercept in the regression of Y^* on X, and b_1 is the OLS estimator of the intercept in the regression of Y on X. Then the following is correct:

1) $b^*_1 = \lambda_1 + \lambda_2 b_1$; 2) $b^*_1 = b_1$; 3) $b^*_1 = b_1 * \lambda_2$; 4) $b^*_1 = \lambda_1 + b_1$; 5) none of the above.

9. There are (1) and (2) versions of the Multiple Regression Model:

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + u \quad (1)$$

$$Y - X_4 = \beta_1 + \beta_2 (X_2 + X_3) + u \quad (2)$$

The Model (2) is the Model (1) with the following restrictions:

1) $\beta_3=0; \beta_4=0;$ 2) $\beta_2=\beta_3; \beta_4=1;$ 3) $\beta_{2+}\beta_3=1; \beta_4=1;$ 4) $\beta_2=\beta_3;\ \beta_4=0;$

5) The Model (2) is not a restricted version of the Model (1).

10. A student regressed *Y* and *log(Y)* on *X*, with the intercept (regressions 1 and 2 respectively). Then he did Zarembka scaling *Y*=Y/geometric mean of Y*, and regressed *Y** and *log(Y*)* on *X* (regressions 3 and 4). Then the following is correct:

1) All the coefficients' estimates, including the intercept, are the same in regression 3 as in regression 1;
2) All the coefficients' estimates, including the intercept, are the same in regression 4 as in regression 2;
3) All the coefficients' estimates, except the intercept, are the same in regression 3 as in regression 1;
4) All the coefficients' estimates, except the intercept, are the same in regression 4 as in regression 2;
5) None of the above.

11. The population variance of prediction error σ_{PE}^2 $= \sigma_u^2 \cdot \left(1 + \frac{1}{n} + \frac{(X_{n+1} - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right)$

1) is always greater than the population variance of disturbance term σ_u^2;
2) is always less than the population variance of disturbance term σ_u^2;
3) can be greater, less then or equal to the population variance of disturbance term σ_u^2;
4) is always equal to the population variance of disturbance term σ_u^2;
5) is not related to the population variance of disturbance term σ_u^2.

12. Root mean squared error of prediction $\sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h}$ is always

1) Greater or equal to the Mean absolute error of prediction $\sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| / h$
2) Less or equal to the Mean absolute error of prediction;
3) May be greater or less than the Mean absolute error of prediction;
4) Equal to the Mean absolute error of prediction;
5) Can not be compared with the Mean absolute error of prediction since it has different dimensity.

RMSE $\geq$ MAE

RMSE = MAE $\Rightarrow$ all errors are of the same magnitude