Time Series

- Cross Sectional Data — N obs — 1 time period
- → Time Series — 1 obs — T time period
- Panel Data — N obs — T time period

Problem 1

a) $HOUS_t = \beta_1 + \beta_2 DPI_t + u_t$

b) $HOUS_t = \beta_1 + \beta_2 \cdot DPI_t + \beta_3 \cdot DPI_{t-1} + u_t$ — ADL(0,1)

c) $HOUS_t = \beta_1 + \beta_2 \cdot DPI_t + \beta_3 \cdot HOUS_{t-1} + u_t$ — ADL(1,0)

ADL(p,q)

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ARDL(p,q)

Problem 2

ARL(1,0)

$u_t \sim N(0, \sigma_u^2)$

$$y_t = \beta_1 + \beta_2 \cdot x_t + \beta_3 y_{t-1} + u_t$$

$$|\beta_3| < 1$$

$$L y_t = y_{t-1}$$

$$L^2 y_t = y_{t-2}$$

$$y_t = \beta_1 + \beta_2 \cdot x_t + \beta_3 \cdot L y_t + u_t$$

$$(1 - \beta_3 L) y_t = \beta_1 + \beta_2 \cdot x_t + u_t$$

$$y_t = \frac{\beta_1}{1-\beta_3 L} + \frac{\beta_2}{1-\beta_3 L} x_t + \frac{u_t}{1-\beta_3 L}$$

$$\beta_1 + \beta_3 L \beta_1 + \beta_3^2 L^2 \beta_1 \dots$$

$$= \beta_1 \to \beta_3 \beta_1 \dots$$

$$\frac{1}{1-\beta_3 L} = \left(1 + \beta_3 L + \beta_3^2 L^2 + \beta_3^3 L^3 + \dots\right)$$

$$q = \beta_3 L$$

$$y_t = \frac{\beta_1}{1-\beta_3} = \beta_1^* + \beta_2 \cdot x_t + \beta_2 \cdot \beta_3 L x_t +$$

$$+ \beta_2 \beta_3^2 L^2 x_t + \dots + u_t^*$$

$$y_t = \beta_1^* + \beta_2 x_t + \beta_2 \beta_3 x_{t-1} + \beta_2 \cdot \beta_3^2 x_{t-2} + \dots + u_t^*$$

ADL (0, ∞)

Problems :

ADL(1,0)

$$y_t = \beta_1 + \beta_2 \cdot x_t + \beta_3 y_{t-1} + u_t$$

↳ AR(1)

$$y_{t-1} = \beta_1 + \beta_2 \cdot x_{t-1} + \beta_3 \cdot y_{t-2} + \varepsilon_{t-1} + \rho \varepsilon_{t-2}$$

$$u_t = \varepsilon_t + \rho \cdot \varepsilon_{t-1}$$

$$\sim N(0,1)$$

Autocorrelation of error term

Interpretation

Short-term marg. effect:

$$y_t = \beta_1 + \beta_2 \cdot x_t + \beta_3 y_{t-1} + u_t$$

Long-term marg. effect

$$\bar{y} = \beta_1 + \beta_2 \cdot \bar{x} + \beta_3 \bar{y}$$

$$(1 - \beta_3)\bar{y} = \beta_1 + \beta_2 \cdot \bar{x}$$

$$\bar{y} = \frac{\beta_1}{1-\beta_3} + \frac{\beta_2}{1-\beta_3} \bar{x}$$

$$\frac{\beta_2}{1-\beta_3} = \beta_2 + \beta_2\beta_3 + \beta_2\beta_3^2 + \dots$$

Problem 4 $\quad CAT_t = \log(cat_t)$, etc.

a)
$$CAT_t = \beta_1 + \beta_2 \cdot DPI_t + \beta_3 DPI_{t-1} + \beta_4 \cdot PR_t + \beta_5 PR_{t-1} + u_t$$

SR m.e. of $DPI = \beta_2$

$$\overline{CAT} = \beta_1 + (\beta_2 + \beta_3)\overline{DPI} + (\beta_4 + \beta_5)\overline{PR}$$
$$\theta \qquad\qquad \phi$$

LR m.e. of $DPI = \beta_2 + \beta_3$

b)
$$CAT_t = \beta_1 + \beta_2 \cdot DPI_t + (\theta - \beta_2) DPI_{t-1} + \beta_4 \cdot PR_t + (\phi - \beta_4) PR_{t-1} + u_t$$

ΔDPI_t — first difference

$$CAT_t = \beta_1 + \beta_2 (DPI_t - DPI_{t-1}) + \theta DPI_{t-1} + \beta_4 (PR_t - PR_{t-1}) + \phi PR_{t-1} + u_t$$

"ΔPR_t

$$CAT = \beta_1 + \beta_2 \Delta DPI_t + \theta DPI_{t-1} +$$

$$+ \beta_4 \Delta PR_t + \phi PR_{t-1} + u_t$$