Autocorrelation

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$Cov(u_t, u_s) \neq 0 \qquad t \neq s$$

First-order AC:

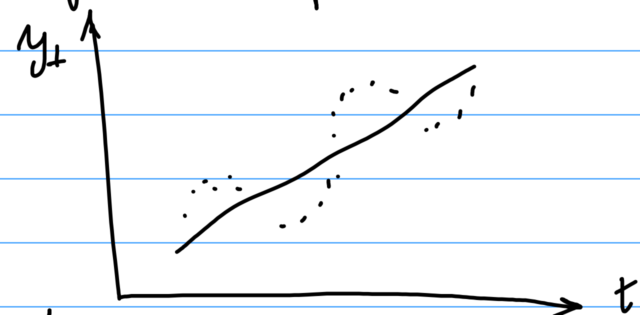
$$u_t = \rho u_{t-1} + \varepsilon_t$$

satisfies GM cond:

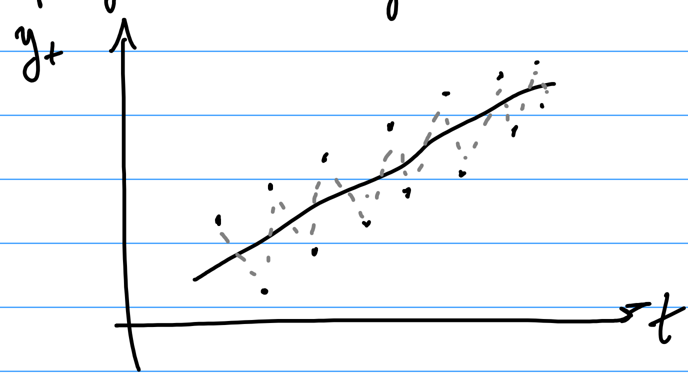
- $E(\varepsilon_t) = 0$
- $E(\varepsilon_t^2) = \sigma_\varepsilon^2$
- $E(\varepsilon_t, \varepsilon_s) = 0, \; t \neq s$

ρ - 1st order autocor. coef.

if $\rho > 0$ pos. AC



if $\rho < 0$ neg. AC

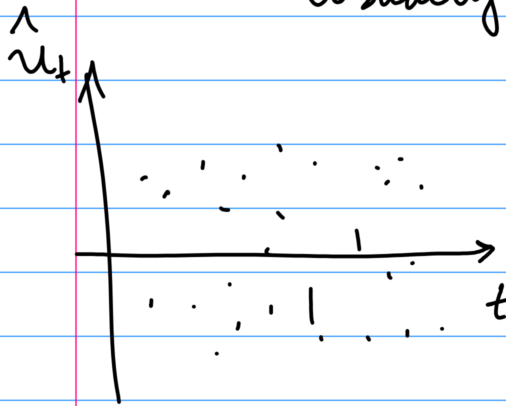


Second-order AC:

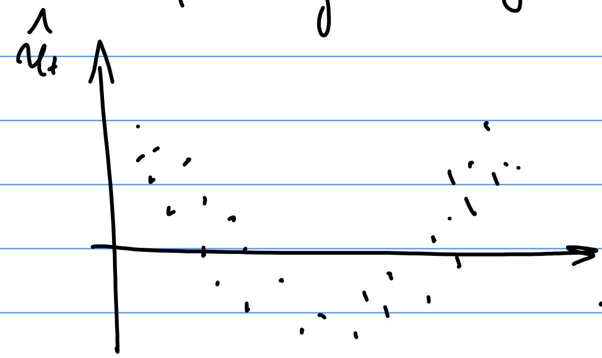
$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$$

Higher-order AC (seasonal AC)

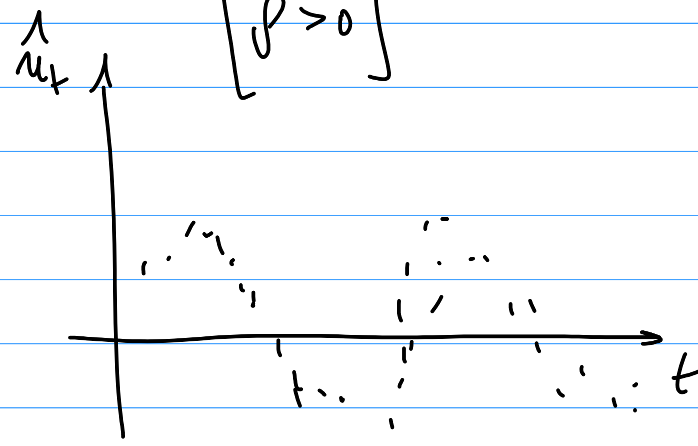
usually for quaterly | montly data



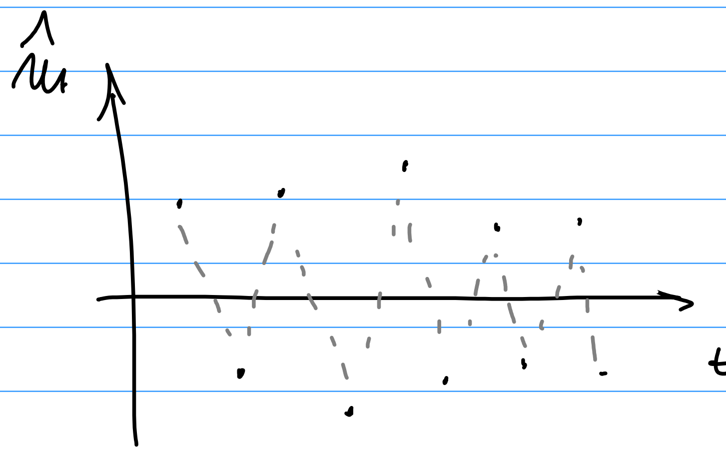
$\rho = 0$



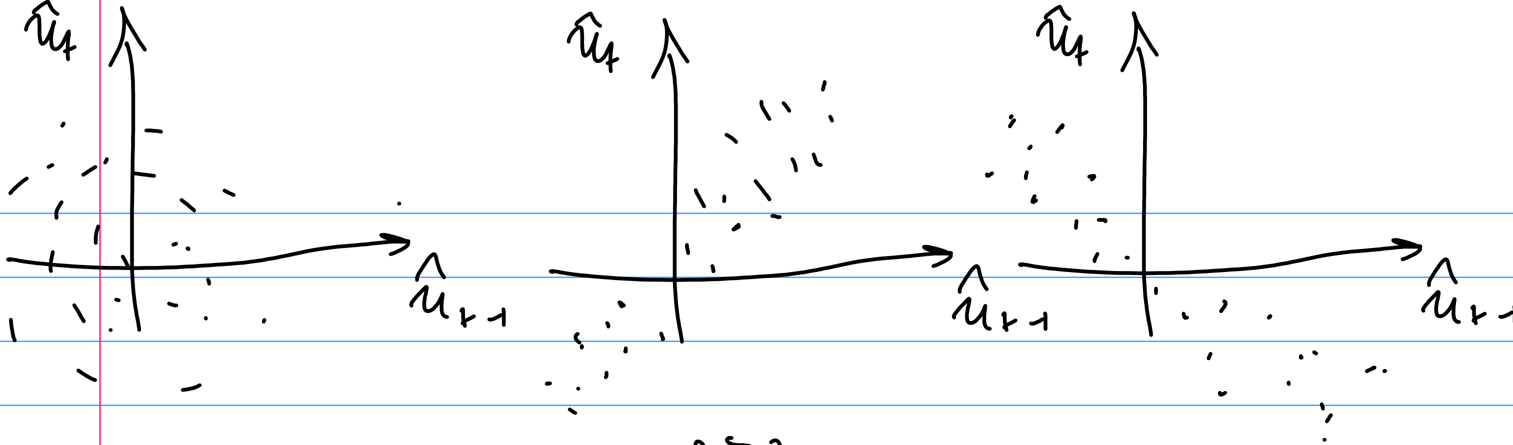
$[\rho > 0]$



When dataset is small
it's hard to
dist. between
$\rho < 0$ and $\rho = 0$



$\rho < 0$



$\rho = 0$ $\qquad$ $\rho > 0$ $\qquad$ $\rho < 0$

$$\hat{u}_t = \rho \, \hat{u}_{t-1} + \varepsilon_t$$

True AC (vs) Autocorr. caused by misspecification

$u_t = \rho u_{t-1} + \varepsilon_t$

① AC caused by omitted variable

True: $y_t = \beta_1 + \beta_2 X_{1t} + \beta_3 X_{2t} + u_t$

Est: $y_t = \beta_1 + \beta_2 X_{1t} + u_t^*$

$u_t^* = \beta_3 X_{2t} + u_t$

If X_{2t} autocorr. variable

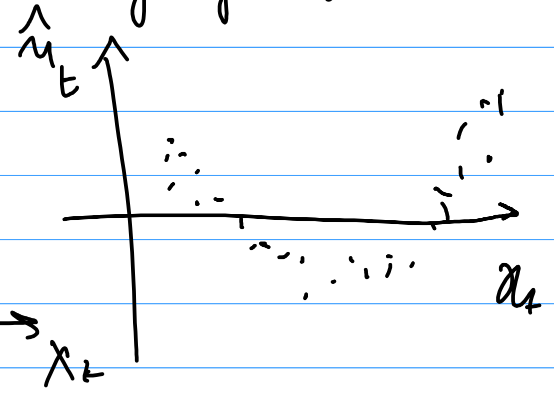
u_t is relatively small

$\Rightarrow u_t^*$ - autocorrelation

② AC caused by wrong functional form

True: $\ln y_t = \beta_1 + \beta_2 \ln X_t + u_t$

Est: $y_t = \beta_1 + \beta_2 X_t + u_t^*$



Consequences of AC
(no lagged dep. var.)

1) True AC => no bias of regr. coeff.

2) Pos. AC => s.e. underestimated

t-stats are inflated

3) test are invalid

DW test

Assumptions: 1) for 1^{st} order AC

2) for models with intercept

3) without lagged dep. variable

↳ Durbin h-test

$$DW = \frac{\sum(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2}$$

$$\frac{cov(\varepsilon_t, \varepsilon_{t-1})}{var(\varepsilon_t)}$$

$$DW = \frac{\sum(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2} = \frac{\sum \hat{\varepsilon}_t^2}{\sum \hat{\varepsilon}_t^2} - 2\frac{\sum \hat{\varepsilon}_t \hat{\varepsilon}_{t-1}}{\sum \hat{\varepsilon}_t^2} + \frac{\sum \hat{\varepsilon}_{t-1}^2}{\sum \hat{\varepsilon}_t^2}$$

$$\xrightarrow{P} 1 - 2\rho + 1 = 2\cdot(1-\rho)$$

$\rho = 1$ $\quad DW = 0$

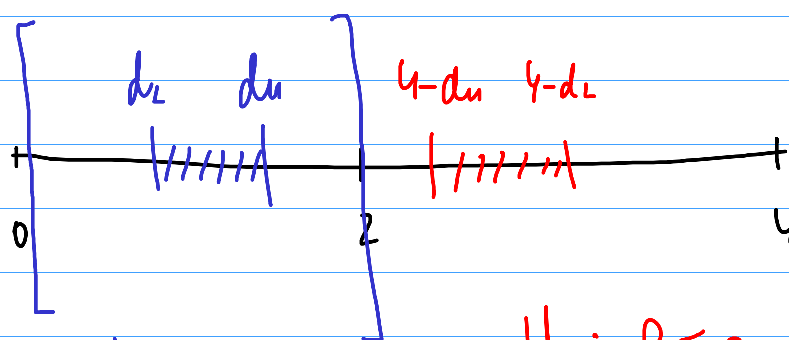
$\rho = 0$ $\quad DW = 2$

$\rho = -1$ $\quad DW = 4$

$-1 < \rho < 1$ $\quad 0 < DW < 4$

$H_0: \rho = 0$ $\quad H_a: \rho > 0 \quad (\rho < 0)$

$$DW = \frac{\sum (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2}$$



$H_a: \rho > 0$ $\quad H_a: \rho < 0$

Alt: $DW^* = 4 - DW$

$DW > d_u$ $\quad H_0$ isn't rej $\Rightarrow$ no AC

$DW < d_L$ $\quad H_0$ is rej $\Rightarrow$ pos. AC

$d_L < DW < d_u$ $\quad$ can't rej / not rej H_0

Durbin h - test

* for test lagged explained variable

$$h = \hat{\rho}\sqrt{\frac{n}{1 - n\,Var(\hat{\beta}_{y_{t-1}})}} \sim N(0,1)$$

$$DW \longrightarrow 2(1-\rho)$$

$$\hat{\rho} = 1 - 0{,}5 \cdot DW$$ Const. est

$\left\{ \hat{u}_t = \rho \hat{u}_{t-1} + e_t \right.$ ρ biased down.

$$h = (1 - 0{,}5\,DW)\sqrt{\frac{n}{1 - n\,s^2_{\hat{\beta}_{y_{t-1}}}}}$$

Limitations:

1) Only 1 lagged dep. variable
2) Only for 1st order AC
3) when there's neg. value under sq. ~~root~~ => can't calculate