The International College of Economics and Finance
Econometrics-2021-2022.
Home assignment 9. Nonstationary Time Series.
To be submitted by March, 13, 23:55

1. [30 marks] Investigating the series for stationarity.

1.1. [10 marks] Investigate the series for stationarity, and also their first and second differences.

It is assumed that u_t is stationary, initial state (e.g. x_0) is fixed, $E u_t = 0$, $\operatorname{var} u_t = \sigma_u^2$, and $E(u_s u_t) = 0$ if $s \neq t$.

a) $x_t = \alpha_0 + u_t + u_{t-1}$; $t = 1, 2, ..., T$.

b) $x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 t + u_t$; $|\alpha_1| < 1$; $t = 1, 2, ..., T$.

1.2. [10 marks] ☐ What is the Dickey-Fuller test to detect whether the time series is difference stationary and time stationary. Explain clearly what is difference stationarity and time stationarity.
☐ What is Augmented Dickey-Fuller tests to detect whether the time series is non-stationary. What is its difference from the test considered in (b)? What are their comparative advantages and disadvantages?

1.3. [10 marks] The researcher wants to test whether time series X_t is stationary using augmented Dickey-Fuller test without time trend including three additional lags: $X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + u_t$ where u_t is distributed independently of X_t with zero mean and constant variance. Derive Dickey-Fuller equation corresponding time series model under consideration. What is the null hypothesis? What is the decision rule?

2. [20 marks] A researcher has time series data for aggregate consumption, C, and aggregate disposable personal income, Y, for a certain country. She establishes that the logarithms of both series are I(1) (integrated of order one) and she correctly hypothesizes that the long-run relationship between them may be represented as

$$C_t = \lambda Y_t v_t \qquad (1)$$

where λ is a constant and v_t is a multiplicative disturbance term. It may be assumed that $\log v_t$ is normally distributed with zero mean and constant variance.

2.1. [10 marks] ☐ Transform the equation (1) into additive form. The researcher believes that $\log C_t$ and $\log Y_t$ are cointegrated. Explain what is meant when two time series are described as being cointegrated in the context of the problem under consideration. How should she demonstrate this?
☐ Investigate the properties of the growth rates of consumption and income. Prove that under assumption above the growth rates of consumption and income are stationary.
☐ Are the growth rates of these indicators equal? Is it correct to describe the growth rates of consumption and income as being cointegrated.

2.2. [10 marks] The researcher is also interested in the short-run dynamics of the relationship and correctly hypothesizes that they may be represented by ADL(1,2) process with two lags for income variable

$$\log C_t = \beta_1 + \beta_2 \log C_{t-1} + \beta_3 \log Y_t + \beta_4 \log Y_{t-1} + \beta_5 \log Y_{t-2} + \varepsilon_t \qquad (2)$$

where ε_t is identically and independently distributed and drawn from a normal distribution with zero mean.

☐ Derive long run relationship between the equilibrium values of $\log C_t$ and $\log Y_t$: $\overline{\log C_t}$ and $\overline{\log Y_t}$.

☐ Using long run relationship between $\log C_{t-1}$ and $\log Y_{t-1}$ as a possible cointegrating relationship derive error-correction type model for the model (2).

□ Describe the structure and dynamics of error correction model. Explain why fitting the error-correction model, rather than (2) directly, avoids a potentially important problem. Which assumptions are necessary to use error correction model? How these assumptions could be tested?

In answering the practical questions, no general theoretical/mathematical explanations are acceptable, nor any information on the topic that is not directly related to the question.

3. **[50 marks]** Use data file **ha09_dataNN.wf1.** The purpose of this exercise is to investigate the behavior of bitcoin prices P_t (in dollars), their differences DP_t and corresponding dynamics of the volume of trade VOL_t using daily data (140 observations) Full description of variables can be found in the file **HA09 Data Description.pdf**. The main focus should be on the problems associated with stationarity and nonstationarity of time series and the problems of cointegration.

3.1. [10 marks]. □ Construct the graphs of prices and volume of trade and their differences. Looking at these graphs what are your initial guesses about stationarity of the given time series? Explain.
□ Construct the graphs of correlograms of these time series. What additional information can we learn from them?

3.2. [10 marks]. □ Use Dickey-Fuller unit-root test to test four time series considered in 3.1 for stationarity. Try different manual and automatic options of Dickey-Fuller test explaining your choice. What are your conclusions? Explain why you use also augmented D-F test. Indicate appropiate Dickey-Fuller equations.

3.3. [10 marks]. □ Test series P_t, DP_t, V_t and DV_t for nonstationarity using the scaled estimator of the slope coefficient $T(\hat{\beta}_2 - 1)$. Use tests with and without trend at least once. Indicate in each case the null hypothesis and used critical values. Do the results of these tests coincide with your conclusions based on t-tests in 3.2?

□ Investigate the series P_t, DP_t, V_t and DV_t for nonstationarity using Dickey-Fuller F-test for the nonstationarity (consider versions of the test using 0 and 1 extra lags). In each case, indicate which processes it is possible to discriminate by the test used.
□ What are comparative advantages and disadvantages of three different ADF tests for nonstationarity?

3.4. [10 marks]. □ Test the time series P_t and V_t for cointegration, using regression with V_t as the dependent variable. Investigate also cointegration of V_t with prices $OPEN_t$, $CLOSE_t$, $HIGH_t$, LOW_t instead of P_t. Are time series DP_t and DV_t cointegrated? Explain.

3.5. [10 marks]. □ Develop and estimate error correction model for the time series P_t and V_t, using simple linear regression V_t on P_t as a cointegrating relationship. Explain its structure and behaviour.
□ Construct the graph of the behavior of the forecasted value of V_t (call it VF_t) based on the naive difference model $\Delta V_t = \beta_2 \Delta P_t + u_t$ in comparison with the actual volume of trade V_t, and then repeat this procedure using predictions PFZ_t based on the error correction model (ECM). Are there any advantages in ECM?

If you have any questions please ask at <u>Vladimir.tcherniak@gmail.com</u>