Autocorrelation

$$y_i = \beta_1 + \beta_2 x_i + u_i$$

$$u_i = \rho u_{i-1} + \varepsilon_i$$

$\Rightarrow$ inefficiency

$$y_i = \beta_1 + \beta_2 x_i + \beta_3 y_{i-1} + u_i$$

$$u_i = \rho u_{i-1} + \varepsilon_i$$

$\Rightarrow$ inconsistency

Autocorrelation

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

AC $\Rightarrow$ $cov(u_i, u_j) \neq 0$ $(i \neq j)$

First-order autocorrelation

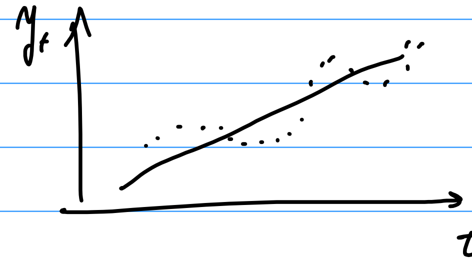
$$u_t = \rho u_{t-1} + \varepsilon_t$$

ε_t sat. GM:

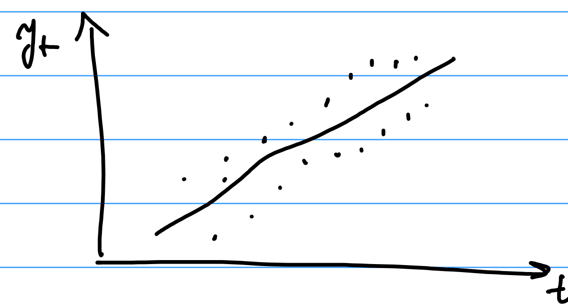
$E(\varepsilon_t) = 0$
$E(\varepsilon_t) = \sigma^2_\varepsilon$
$E(\varepsilon_i, \varepsilon_j) = 0$ $i \neq j$

ρ – 1st order autocorr. coefficient

$\rho > 0$ – pos. autocor.



$\rho < 0$ – neg. autocor.



$-1 < \rho < 1$

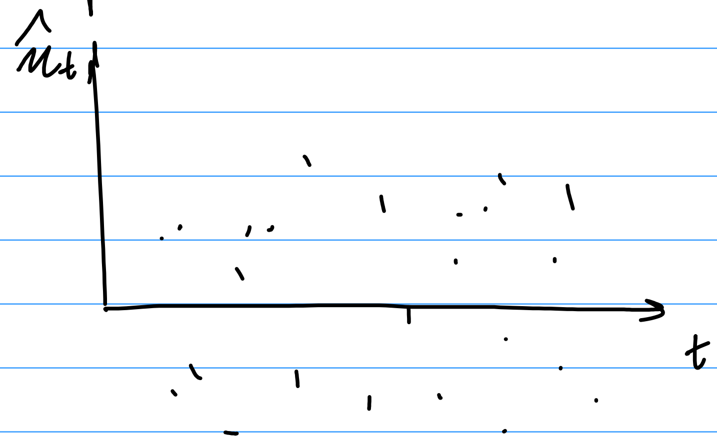
Second-order AC:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$$

Higher-order AC: often in case of quarter / monthly data

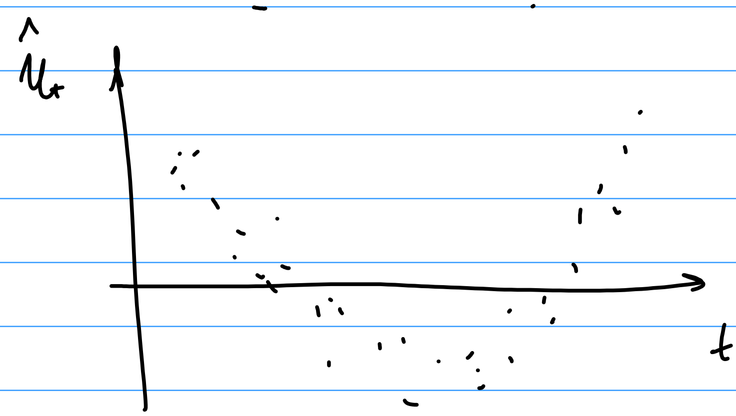
Detecting Autocorrelation

1) Graphical Method



no AC

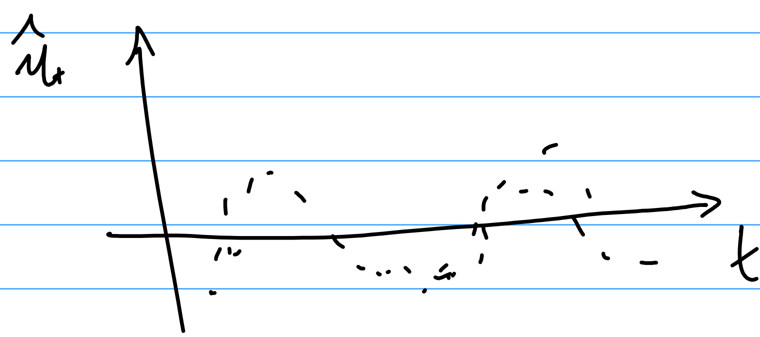
$(\rho = 0)$

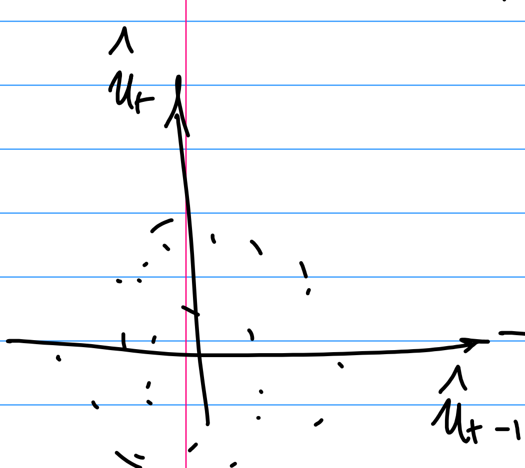


Pos. AC

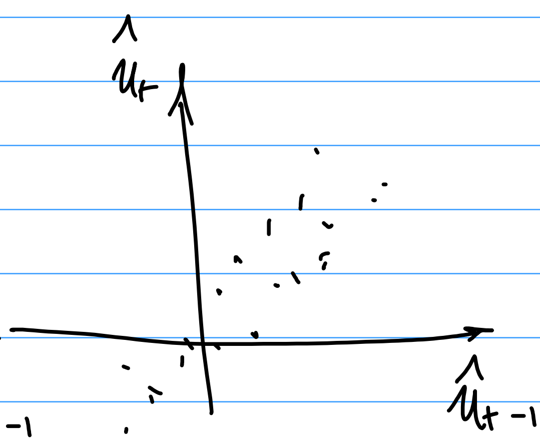
$\rho > 0$

$\{u_t = \rho u_{t-1} + \varepsilon_t\}$

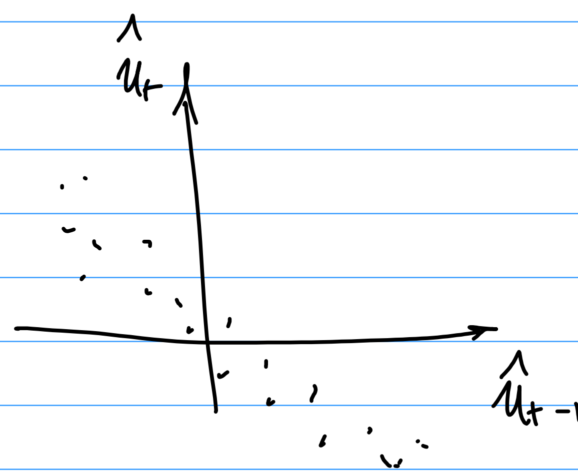




$\rho = 0$



$\rho > 0$



$\rho < 1$

True AC vs AC caused by misspecification

1) AC because of omitted variable

True $Y_i = \beta_1 + \beta_2 X_{1t} + \beta_3 X_{2t} + u_t$

Est $Y_i = \beta_1 + \beta_2 X_{1t} + u_t^*$

$u_t^* = \beta_3 X_{2t} + u_t$

X_{2t} - autocorrelated variable

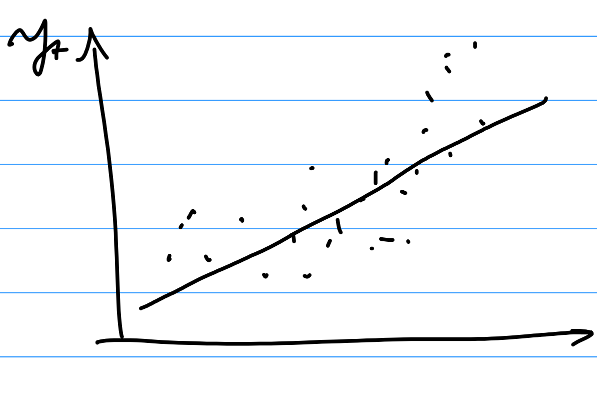
$\Rightarrow$ u_t are relatively small

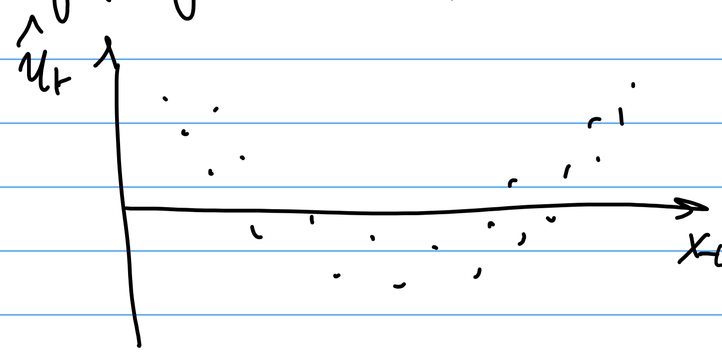
u_t^* will be autocorrelated

2) Wrong functional form

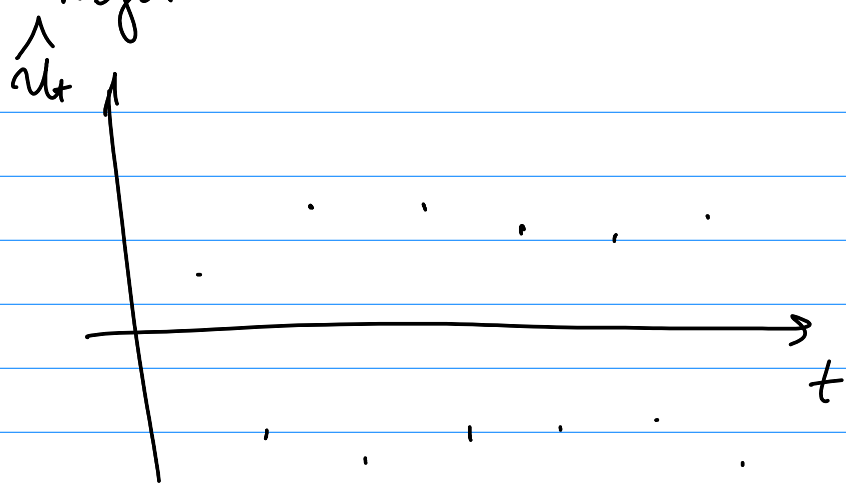
True : $\ln Y_t = \beta_1 + \beta_2 \ln X_t + \varepsilon_t$

Est: $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t^*$





Negative AC:



When small dataset can seem like normally distributed error term

DW test

Autocorrelation consequences:

1. True AC doesn't lead to bias of coefs.
2. Pos. AC => s.e. are underest.
3. test are inappropriate.

DW test

$$DW = \frac{\sum(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2} \sim DW \text{ distribution}$$

H_0: $\rho = 0$

H_a: $\rho > 0$ ($\rho < 0$)

Assumptions:

1) Only for 1st AC
2) For model with const. term
3) No lags of dep. variable in the model

$$DW = \frac{\sum(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2} = \frac{\sum(\hat{\varepsilon}_t^2 - 2\hat{\varepsilon}_t\hat{\varepsilon}_{t-1} + \hat{\varepsilon}_{t-1}^2)}{\sum \hat{\varepsilon}_t^2} =$$

$$= \frac{\sum \hat{\varepsilon}_t^2}{\sum \hat{\varepsilon}_t^2} - \frac{2\sum \hat{\varepsilon}_t \hat{\varepsilon}_{t-1}}{\sum \hat{\varepsilon}_t^2} + \frac{\sum \hat{\varepsilon}_{t-1}^2}{\sum \hat{\varepsilon}_t^2} \xrightarrow{\rho}$$

$$= 1 - 2\rho + 1 = 2(1-\rho)$$

$$\frac{cov(\varepsilon_t, \varepsilon_{t-1})}{Var(\varepsilon_t)}$$

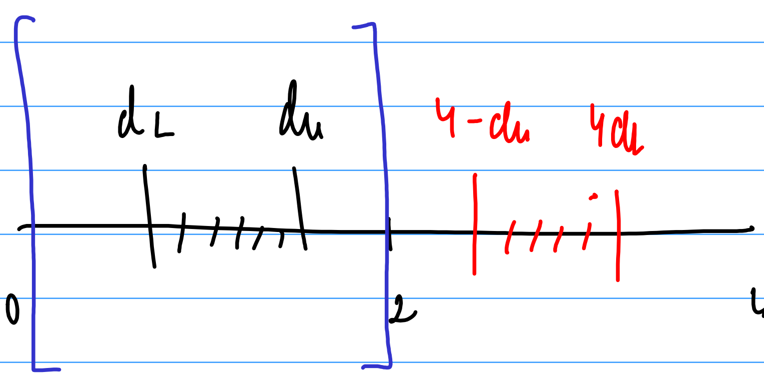
$-1 < \rho < 1$

$\rho = 0 \qquad DW = 2$

$\rho = 1 \qquad DW = 0$

$\rho = -1 \qquad DW = 4$

$0 < DW < 4$



$k = \#$ of est. par

$H_a: \rho > 0$ $\qquad$ $H_a: \rho < 0$ $\qquad$ / $DW^* = 4 - DW$

$DW > d_u \Rightarrow H_0$ is not rej $\Rightarrow$ no AC

$DW < d_L \Rightarrow H_0$ is rej $\Rightarrow$ pos. AC

$d_L < DW < d_u \Rightarrow$ uncertainty

Shortcomings of DW test

1) Uncertainty zone

2) Only for 1st order AC

3) Only for mod. with const

4) Only for mod. without lagged dep. variable

Advantages of DW test:

1) Works well on small samples

Durbin h-test

$$h = \hat{\rho}\sqrt{\frac{n}{1 - n\,Var(\hat{\beta}_{y_{t-1}})}} \sim N(0;1)$$

$$DW \approx 2 - 2\hat{\rho}$$

$$\hat{\rho} = 1 - 0{,}5 \cdot DW$$ | consisent

$$\left\{ \hat{u}_t = \rho \hat{u}_{t-1} + \varepsilon_t \right.$$

biased ↓ for small samples

$$h = (1 - 0{,}5\,DW)\sqrt{\frac{n}{1 - n\,s^2_{\hat{\beta}_{y_{t-1}}}}}$$