$$\Delta \hat{P}_t = 160.58 - 0.02 P_{t-1} \qquad R^2 = 0.01, \tag{1}$$
$$\quad (134.00) \quad (0.014)$$

$$\Delta \hat{DP}_t = -0.97 DP_{t-1} \qquad R^2 = 0.487 \tag{2}$$
$$\quad (0.075)$$

$$\Delta \hat{VOL}_t = 1.48 \cdot 10^8 - 0.144 VOL_{t-1} - 0.224 \Delta (VOL_{t-1}) + 91320.24t \qquad R^2 = 0.14 \tag{3}$$
$$(871445.3) \quad (0.045) \quad (0.073) \quad (871445.3)$$

$$\Delta \hat{VOL}_t = 1.55 \cdot 10^8 - 0.143 VOL_{t-1} - 0.224 \Delta (VOL_{t-1}) \qquad R^2 = 0.14 \tag{4}$$
$$(65210866) \quad (0.044) \quad (0.073)$$

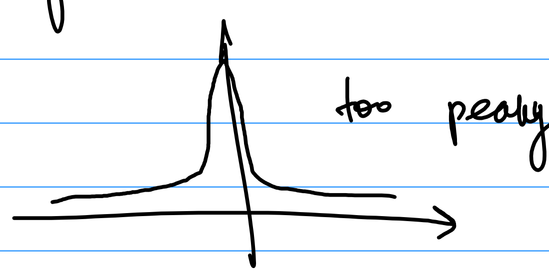
(b) Now for equations (1-4) carry out the Dickey-Fuller test using the scaled estimator of the slope coefficient $T(\hat{\beta}_2 - 1)$ to test series P_t, DP_t and VOL_t for nonstationarity where β_2 is the slope coefficient of the autoregression $Y_t = \beta_2 Y_{t-1} + u_t$. Indicate in each case the null hypothesis and used critical values. Do the results of these tests coincide with your conclusions based on t-tests? Comment the meaning of $T(\hat{\beta}_2 - 1)$ statistic and explain why the difference $(\hat{\beta}_2 - 1)$ should be multiplied by T rather than by $\sqrt{T}$.

$$Y_t = \beta_2 Y_{t-1} + u_t$$

if $|\beta_2| < 1$ $\qquad \sqrt{T}(\hat{\beta}_2 - \beta_2) \sim N(0,1)$

$\uparrow$ # of obs

if $\beta_2 = 1$ $\qquad \sqrt{T}(\hat{\beta}_2 - 1) \nsim N(0,1)$



$$T(\hat{\beta}_2 - 1) \sim DF_{sc.dist}$$

$$\Delta P_t = \beta_1 + (\beta_2 - 1) P_{t-1} + u_t$$

$$DF_{sc,\,obs} = T(\beta_2 - 1) = 183 \cdot (-0{,}02) = -3{,}66$$

$DF_{3c,\, crit} = -13{,}72$

$-3{,}66 > -13{,}72$ P_t – non-stationary

(c) The student obtained for equation (1) $\Delta P_t = \beta_1 + (\beta_2 - 1)P_{t-1} + u_t$, the value of F-statistics for testing simultaneously two restrictions $\beta_1 = 0, \beta_2 = 1$: $F = 1.00$. How to use this information to conduct Dickey-Fuller F-test for the nonstationarity of the P_t? Following this approach describe how you can investigate also eqations (2-4) to test the series DP_t and VOL_t for the nonstationarity using Dickey-Fuller F-test: for each case indicate theoretical equation, restriction(s), and the rule for choosing the critical value of ADF F-statistic from the appropriate table. Indicate also in each case the random processes which the test allows to discriminate. What are comparative advantages and disadvantages of three different ADF tests for nonstationarity?

$P_t = \beta_1 + \beta_2 P_{t-1} + u_t$

$\Delta P_t = \beta_1 + (\beta_2 - 1)P_{t-1} + u_t$

$H_0: \beta_1 = 0, \ \beta_2 - 1 = 0$

$F_{obs} = 1$

$F_{crit} = 4{,}67$

$1 < 4{,}67 \Rightarrow$ P_t – non-stationary

Cointegration

$I(0)$: $MA(1)$, $AR(1)$

$I(1)$: ΔX_t – stationary

$I(2)$: $\Delta(\Delta X_t)$ – stationary

Problem 2: $X_t = \alpha_0 + \alpha_1 t + u_t$

$$Z_t = X_t - \alpha_1 t = \alpha_0 + u_t$$

$$E(Z_t) = \alpha_0$$

$$Var(Z_t) = \sigma_u^2$$

$$Cov(Z_t, Z_{t-s}) = 0 \qquad \forall s \geq 1$$

Problem 3: $\ln y_t = \alpha + \beta t + u_t$

$$E(\ln y_t) = \alpha + \beta t$$

$$\Delta \ln y_t = \alpha + \beta t + u_t - \alpha - \beta(t-1) - u_{t-1} =$$

$$= \beta + \underbrace{u_t - u_{t-1}}_{\varepsilon_t}$$

$$E(\Delta \ln y_t) = \beta$$

$$Var(\Delta \ln y_t) = 2 \cdot \sigma_u^2$$

$$Cov(\Delta \ln y_t, \Delta \ln y_{t-s}) = \begin{cases} s = 1 & \sigma_u^2 \\ s \geq 1 & 0 \end{cases}$$

Problem 4. What time series are called cointegrated?

1) All of the same order of integration

2) $\exists$ lin. comb of these t.s. s.t.

it is stationary

$\Leftrightarrow$ res. of lin. reg. $X_i \mid X_{-i}$

is stationary

$$a_0 + a_1 X_t + a_2 y_t + a_3 z_t = e_t$$

X_t, y_t, z_t – $I(1)$

e_t – stationary

if fix $a_2 = 1$:

$$y_t = -a_0 - a_1 X_t - a_3 z_t + \underbrace{e_t}_{\text{stationary}}$$