OVB

$X_{1,2}$ - determ.; $E(\varepsilon_i)=0$; $E(\varepsilon_i\varepsilon_j)=0$
$E(\varepsilon_i^2)=\sigma^2$; $i \neq j$

Q2

True model: $y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$

Est. model: $y_i = \beta_1 X_{1i} + v_i$

$$\hat{\beta}_1 = \frac{\sum X_{1i} y_i}{\sum X_{1i}^2} = \frac{\sum X_{1i}(\beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i)}{\sum X_{1i}^2} =$$

$$= \beta_1 + \beta_2 \frac{\sum X_{1i} \cdot X_{2i}}{\sum X_{1i}^2} + \frac{\sum X_{1i}\varepsilon_i}{\sum X_{1i}^2}.$$

$$E(\hat{\beta}_1) = \beta_1 + \beta_2 \cdot \frac{\sum X_{1i} X_{2i}}{\sum X_{1i}^2} + \frac{\sum X_{1i} \cdot \overset{=0}{E(\varepsilon_i)}}{\sum X_{1i}^2}$$

$$= \beta_1 + \underbrace{\beta_2 \frac{\sum X_{1i} \cdot X_{2i}}{\sum X_{1i}^2}}_{\text{bias}}$$

No bias if $\beta_2 = 0$

or if $\sum X_{1i} \cdot X_{2i} = 0$ i.e. $X_1 \perp X_2$ orthogonal

Q5 Perfect MC $X_i = \sum_{j \neq i} \lambda_j X_j$

MC $cor(X_i, X_j) > \rho^*$

Q6 b) $H_0: \beta_{price} = -1$

$H_a: \beta_{price} \neq -1$

$$t = \frac{\hat{\beta} - (-1)}{se(\hat{\beta})} = \frac{-0{,}83 + 1}{0{,}0031} = 53{,}93 \overset{H_0}{\sim} t_{n-k=}$$

$=32-2=30$

for $\alpha = 5\%$ $t^{crit}_{30,\,0{,}975} = 2{,}042$

$|t^{obs}| > t^{crit} \Rightarrow H_0$ is rejected

c) $\{\hat{\beta} \pm t_{1-\alpha/2} \cdot se(\hat{\beta})\} =$

$= \{-0{,}83 \pm 2{,}042 \cdot 0{,}0031\} =$

$= \{-0{,}839\,;\ -0{,}826\}$ — two-sided 95% CI

to test $H_0: \beta = -1$

$-1 \notin CI \Rightarrow H_0$ is rejected

d) i) if $D_i = 1$

$\log w_i = \beta_1 + \beta_2 \log p_i + \beta_3 \log p_i \cdot D_i =$

$= 4{,}2 + (-0{,}82 + 0{,}09) \log p_i$

$$= 4.2 - 0{,}73 \log p_i$$

if $D_i = 0$

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ii) $H_0: \beta_3 = 0$

$H_a: \beta_3 > 0$

$$t = \frac{\hat{\beta}_3 - 0}{se(\hat{\beta}_3)} = \frac{0{,}09}{0{,}001} = 90 \overset{H_0}{\sim} t_{32-3}$$

$$t^{crit}_{0{,}05,\,29} = 1{,}699$$

$t^{obs} > t^{crit}$ H_0 is rejected

Q7 $D_t = \begin{cases} 1 & t = \overline{s+1;T} \\ 0 & t = \overline{1;s} \end{cases}$

$$y_t = aX_t + \lambda(D_t X_t) + u_t$$

$\hat{a}$ – est. for a^*

$\hat{a} + \hat{\lambda}$ – est for a^{**}

b) True : $y_t = aX_t + \lambda D_t X_t + u_t$

Est : $y_t = aX_t + \varepsilon_t$

$$\hat{a} = \frac{\Sigma X_t y_t}{\Sigma X_t^2} = \frac{\Sigma X_t(aX_t + \lambda D_t X_t + u_t)}{\Sigma X_t^2} =$$

$$= a + \lambda \frac{\Sigma D_t X_t^2}{\Sigma X_t^2} + \frac{\Sigma X_t u_t}{\Sigma X_t^2} =$$

$$= a + \lambda \frac{\sum_{t=s+1}^{T} x_t^2}{\sum_{t=1}^{T} x_t^2} + \frac{\sum x_t u_t}{\sum x_t^2}$$

$$E(\hat{a}) = a + \underbrace{\lambda \frac{\sum_{t=s+1}^{T} x_t^2}{\sum_{t=1}^{T} x_t^2}}_{\text{bias}}$$

$\hat{a}$ as est a^* is biased

and sign depends on λ

c) $y_t = a x_t + \lambda D_t x_t + u_t$

i) t-test : $H_0: \lambda = 0$

ii) F-test : $H_0: \lambda = 0$

UR: $y_t = a x_t + \lambda D_t x_t + u_t \Rightarrow RSS^{UR}$

R: $y_t = a x_t + u_t \Rightarrow RSS^{R}$

$$F = \frac{(RSS_R - RSS_{UR}) / \overset{=1}{q}}{RSS_{UR} / T - \underset{2}{\underset{\shortparallel}{k}}} \overset{H_0}{\sim} F_{1, T-2}$$

d) $t = \overline{1, s}$ $\quad y_t^1 = a_t^1 + u_t^1 \Rightarrow RSS_1$

$t = \overline{s+1, T}$ $\quad y_t^2 = a_t^2 + u_t^2 \Rightarrow RSS_2$

$$F = \frac{(RSS_R - RSS_{UR})/1}{RSS_{UR}/T-2} = \frac{\left(RSS_{pool} - (RSS_1 + RSS_2)\right)/1}{(RSS_1 + RSS_2)/T-2}$$

$$\underset{H_0}{\sim} F_{1,T-2}$$