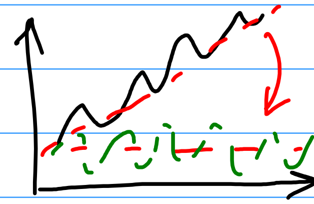
Non-stationary TS

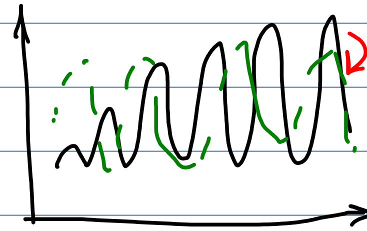
P1. Weak-stationary

1) $E(X_t) = \text{const}$

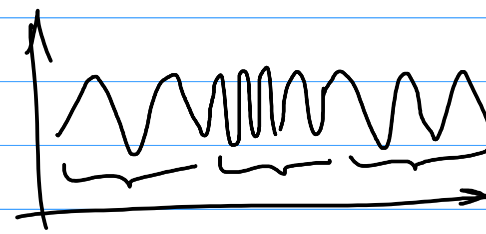
2) $Var(X_t) = \text{const}$

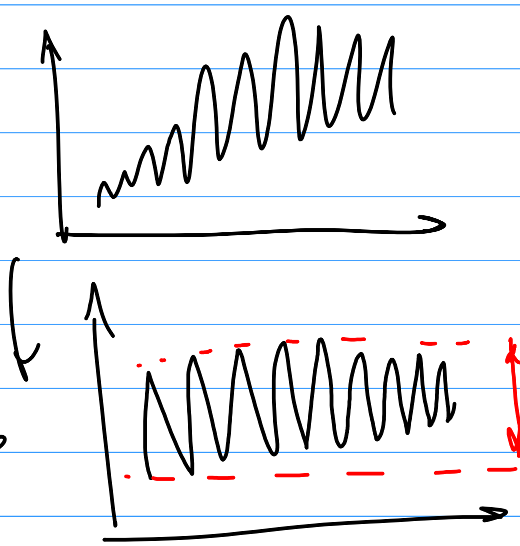
3) $Cov(X_t, X_{t+s}) = f(s)$





log





$\lambda = 0$

num. search for $\hat{\lambda}$

B.C. transformation

$$y = \begin{cases} \dfrac{y^\lambda - 1}{\lambda}, & \lambda \neq 0 \\ \ln y, & \lambda = 0 \end{cases}$$

P2. 1) Stationary series:

$y_t = 1{,}4\, y_{t-1} + \varepsilon_t$ MA(∞)

ARMA with AR part

with roots $|\lambda| < 1$

=> ARMA will have stationary solutions

2) Non-Stationary

Trend-Stationary: $X_t = \alpha + \beta t + \varepsilon_t$

$E(X_t) \neq$ const

$X_0 = 0$ $X_1 = X_0 + \varepsilon_1$

Random Walk: $X_t = X_{t-1} + \varepsilon_t$ $X_0 = 0 + \varepsilon$

$X_t = \varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_1 + 0$

$Var(X_t) = t\sigma^2 \neq$ const

$Cov(X_t, X_{t-s}) =$

$Cov(\varepsilon_t + \dots \varepsilon_1, \varepsilon_{t-s} + \dots + \varepsilon_1) =$

$(t-s)\cdot\sigma^2 \neq$ const

P3. AR(1), $0 < \beta_1 < 1$ $X_t = \beta_1 X_{t-1} + \varepsilon_t$

$E(X_0) = 0$ — is stationary

$$\rightarrow \sigma^2_{X_0} = \frac{1}{1-\beta_1^2} \cdot \sigma^2_\varepsilon$$

1) $E(X_t) = E(\beta_1^t X_0 + \beta_1^{t-1} \varepsilon_1 + \dots + \beta_1 \varepsilon_{t-1} + \varepsilon_t) = 0$

2) $Var(X_t) = \beta_1^{2t} Var(X_0) + \beta_1^{2t-2} \sigma^2_\varepsilon + \dots + \sigma^2_\varepsilon =$

$$= \frac{\beta_1^{2t}}{1-\beta_1^2} \cdot \sigma^2_\varepsilon + \frac{1-\beta_1^{2t-2}}{1-\beta_1^2} \cdot \sigma^2_\varepsilon$$

$$= \frac{1}{1-\beta_1^2} \cdot \sigma^2_\varepsilon$$

3) $Cov(X_t, X_{t+s}) = Cov(X_t, \varepsilon_{t+s} +$

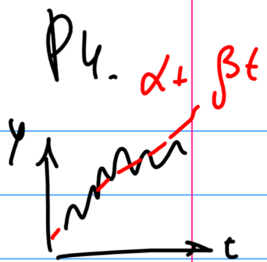
$\beta_1 \varepsilon_{t+s} + \dots + \beta_1^{s-1} \cdot \varepsilon_{t+1} + \beta_1^s \cdot X_t)$

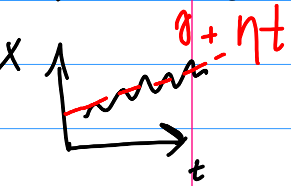
$= \beta_1^s \cdot Var(X_t)$

Weakly dependent time series:

$corr(X_t, X_{t+h}) \rightarrow 0$, $h \rightarrow \infty$

P4.





TS:

$$y_t = \alpha + \beta t + \varepsilon_t$$
$$x_t = \gamma + \eta t + u_t$$

DGP

EST:

$$y_t = c_1 + c_2 x_t + v_t$$

true: $c_2 = 0$

est.: c_2 biased

because t is omitted

DS:

$$y_t = \alpha + y_{t-1} + \varepsilon_t$$
$$x_t = \gamma + x_{t-1} + \varepsilon_t$$

EST:

$$(*)\ y_t = c_1 + c_2 x_t + v_t$$

true: $c_2 = 0$

given that:

H_0: $c_2 = 0$

$$y_t = c_1 + v_t$$

RW

$v_t = y_{t-1} + \varepsilon_t$

$= y_t - c_1$

$\Rightarrow$ v_t in $(*)$

RW

$\Rightarrow$ GMT ass. are violated

$$t = \frac{\hat{c}_2}{se(\hat{c}_2)} \overset{H_0}{\not\sim} t$$