Elements of Econometrics 2022-2023
Class 22. Dickey-Fuller Test

Problem 1. Explain what is unit root test.

Problem 2. Derive Dickey-Fuller equation.

Problem 3. How to take into account the presence of intercept? What is the difference in tests, what are their relative advantages and disadvantages?

Problem 4. What is Augmented dickey-Fuller test? How to include in the test more lagged variables? What is the reason for that?

Problem 5. How to choose between different numbers of lags?

Problem 6. How to take into account the presence of trend? What is the difference in tests, what are their relative advantages and disadvantages?

Problem 7. How to cope with nonstationarity? Detrending and differencing.

Problems from Exams

Problem 8. Describe briefly a unit root test to detect a non-stationary process. What is meant by augmented Dickey-Fuller test?

Problem 9. The researcher wants to test whether time series X_t is stationary using augmented Dickey-Fuller test including two additional lags: $X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + u_t$ where u_t is distributed independently of X_t with zero mean and constant variance.
  (i)   without time trend;
  (ii)   including time trend;

(a) Derive Dickey-Fuller equation corresponding time series model under consideration in case **(i)**. What is the null hypothesis? What is the decision rule?

(b)   What is Dickey-Fuller equation corresponding time series model in case **(ii)**. If the null hypothesis is rejected how to test the presence of time trend? If time trend is detected how to get rid of it?

Problem 10. Using daily data for the period from 2017-04-01 to 2017-09-30 (183 observations), a student tries to investigate the behavior of bitcoin prices P_t (in dollars), their differences DP_t and corresponding dynamics of the volume of trade VOL_t (see the corresponding graphs below). For time trend $t = 0$ at $4/01/2017$.
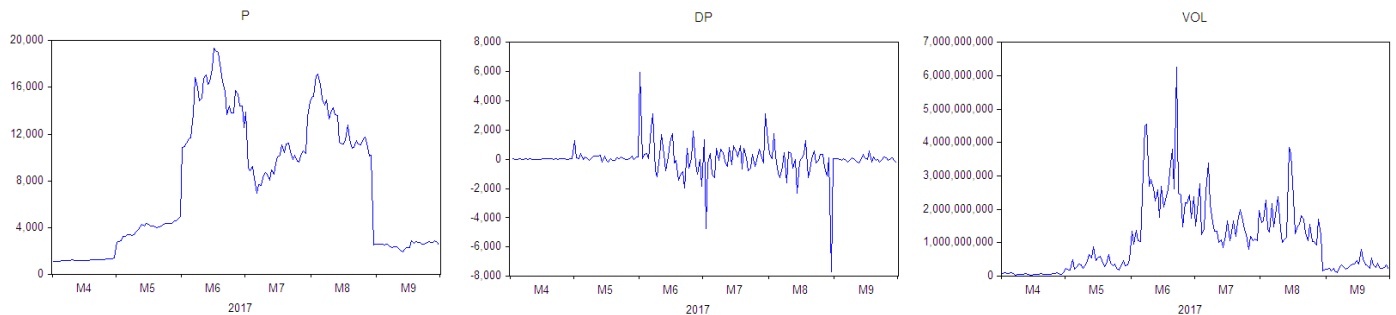
Suspecting that some of these time series might be non-stationary she runs some Dickey-Fuller equations to test the series for stationarity:

$$\hat{\Delta P_t} = 160.58 - 0.02 P_{t-1} \quad R^2 = 0.01$$
$$(134.00) \quad (0.014)$$
(1)

$$\hat{\Delta DP_t} = -0.97 DP_{t-1} \quad R^2 = 0.487$$
$$(0.075)$$
(2)

$$\hat{\Delta VOL_t} = 1.48 \cdot 10^8 - 0.144 VOL_{t-1} - 0.224\Delta(VOL_{t-1}) + 91320.24t \quad R^2 = 0.14$$
$$(871445.3) \quad (0.045) \quad (0.073) \quad (871445.3)$$
(3)

$$\hat{\Delta VOL_t} = 1.55 \cdot 10^8 - 0.143 VOL_{t-1} - 0.224\Delta(VOL_{t-1}) \quad R^2 = 0.14$$
$$(65210866) \quad (0.044) \quad (0.073)$$
(4)

(a) Looking at the graphs what are your initial guesses about stationarity of the given time series? Describe briefly how to use Dickey-Fuller equations to test time series under consideration for stationarity. For equations (1) - (4) perform the (Augmented) Dickey-Fuller t-test, each time clearly indicating the null hypothesis, the test statistic and critical values. Simultaneously explain how to find the critical values in the presence of a time trend? How to test the presense of time trend in time series? Compare the results of equations (3) and (4). Comment on the results.

(b) Now for equations (1-4) carry out the Dickey-Fuller test using the scaled estimator of the slope coefficient $T(\hat{\beta}_2 - 1)$ to test series P_t, DP_t and VOL_t for nonstationarity where β_2 is the slope coefficient of the autoregression $Y_t = \beta_2 Y_{t-1} + u_t$. Indicate in each case the null hypothesis and used critical values. Do the results of these tests coincide with your conclusions based on t-tests? Comment the meaning of $T(\hat{\beta}_2 - 1)$ statistic and explain why the difference $(\hat{\beta}_2 - 1)$ should be multiplied by T rather than by $\sqrt{T}$.

(c) The student obtained for equation (1) $\Delta P_t = \beta_1 + (\beta_2 - 1)P_{t-1} + u_t$, the value of F-statistics for testing simultaneously two restrictions $\beta_1 = 0, \beta_2 = 1$: $F = 1.00$. How to use this information to conduct Dickey-Fuller F-test for the nonstationarity of the P_t? Following this approach describe how you can investigate also eqations (2-4) to test the series DP_t and VOL_t for the nonstationarity using Dickey-Fuller F-test: for each case indicate theoretical equation, restriction(s), and the rule for choosing the critical value of ADF F-statistic from the appropriate table. Indicate also in each case the random processes which the test allows to discriminate. What are comparative advantages and disadvantages of three different ADF tests for nonstationarity?