Test for Autocorrelation

(I) Durbin-Watson Test:

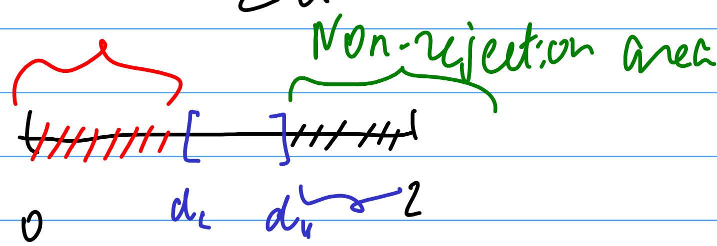
Disadvantages: 1) only for 1st order AC

2) inconclusive areas $[d_L; d_u]$

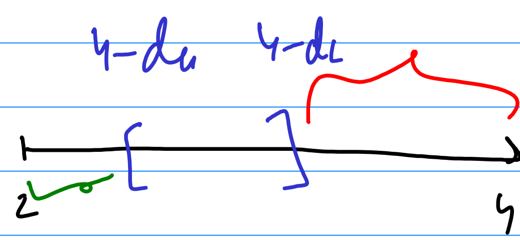
3) only for models with const term $(k>1)$

4) only for models without lag of y
=> DW will be biased towards 2 => h-Durbin

$$DW = \frac{\sum(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2} \longrightarrow 2(1-\rho) \in (0,4)$$



$H_a: \rho > 0$



$H_a: \rho < 0$

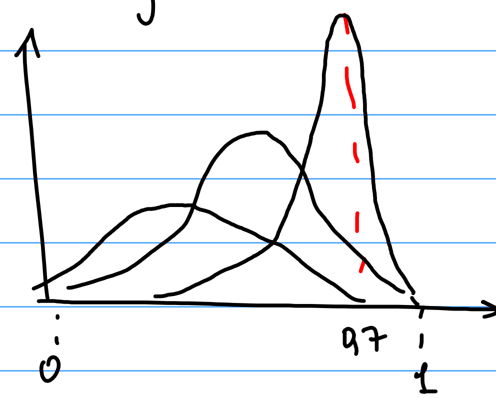
Advantages:

1) Works well for small samples

2) $DW \to 2(1-\rho)$

$$\hat{\rho} = 1 - \frac{DW}{2} \to \rho$$

Note: $\hat{\varepsilon}_t = \rho \hat{\varepsilon}_{t-1} + u_t$



(I) h-Durbin: $H_0: \rho = 0$

$$h = \hat{\rho} \cdot \sqrt{\frac{n}{1 - n \cdot \hat{Var}(\hat{\beta}_{y_{t-1}})}} \overset{H_0}{\sim} N(0;1)$$

$$\hat{\rho} = 1 - \frac{DW}{2}$$

Disadvantages:

1) Only for 1st order AC
2) Only for 1 lag of y_t
3) May be impossible to calculate, because of $\sqrt{x}$, $x<0$

Remedies for Autocorrelation

1) Specification

$\hookrightarrow$ false autocorrelation because of omitted variables

$$y_t = \alpha + \beta X_t + \underbrace{\varepsilon_t}_{= \rho \varepsilon_{t-1} + u_t}$$

2) AR - transformation $\longleftrightarrow$ Cochrain-Orchutt Procedure

$\hookrightarrow$ GLS

3) MA - transformation

$\hookrightarrow$ GLS

4) Adding lag of y_t

5) Using more complex ARDL(p,q)

Test Breusch-Godfrey :

H_0 : no Autocorrelation of order q

$$\hat{\varepsilon}_t = \alpha_0 + \alpha_1 \cdot X_t + \alpha_2 Z_t + \hat{\rho}_1 \underbrace{\hat{\varepsilon}_{t-1} + \dots + \hat{\rho}_q}_{} \hat{\varepsilon}_{t-q}$$

$q=1$ t-test $\rho_1 = 0$ F-test: $\rho_1 = \dots = \rho_q = 0$

$\hookrightarrow R^2_{aux}$

$$BG = n \cdot R^2_{aux} \sim \chi^2_q$$

$\hookrightarrow$ LM test

1) works with any model
   - w/o const
   - with any # of lags of y
2) asymptotic (only works with big samples)