(c) (20 points) Consider the following regression model, where ε_t has mean zero and is uncorrelated with GDP_{t-1}, GDP_{t-2}, $rate_t$, and $rate_{t-1}$, where $GDP_t \in I(1)$, $rate_t \in I(1)$ and are cointegrated:

ADL(2,1)

$$GDP_t = \alpha + \beta_1 GDP_{t-1} + \beta_2 GDP_{t-2} + \gamma_1 rate_t + \gamma_2 rate_{t-1} + \varepsilon_t \quad (1)$$

□ Derive the Error Correction Model (ECM) representation of the equation, and discuss the long-run equilibrium relation for GDP_t and $rate_t$.

If GDP_t, r_t are cointegrated

$\exists$ stat. lin. comb. $z_t = \alpha_0 + \alpha_1 \cdot GDP_t + \alpha_2 \cdot r_t$

$\rightarrow GDP_t = \beta_0 + \beta_1 \cdot r_t + \underbrace{\frac{z_t}{\alpha_1}}_{= \varepsilon_t}$

Coint. relationship = LR relationship

$$\Delta GDP_t = \gamma_0 + \underbrace{\gamma_1 \cdot \Delta rate_t}_{\text{SR dynamics}} + v_t$$

Gener. case

in ECM:

$$\Delta GDP_t = \gamma_0 + \overbrace{\gamma_1 \Delta rate_t}^{SR} + \underbrace{\gamma_2 \, \varepsilon_{t-1}}_{LR} + v_t$$

$$\varepsilon_{t-1} = (GDP_{t-1} - \beta_0 - \beta_1 r_{t-1})$$

From (1): ΔGDP_t

$$\Delta GDP_t = GDP_t - GDP_{t-1} =$$

$$\alpha + (\beta_1 - 1) GDP_{t-1} + \beta_2 GDP_{t-2} + \gamma_1 z_t + \gamma_2 z_{t-1} + \varepsilon_t$$

$$\Delta GDP_t = \alpha + (\beta_1 - 1) GDP_{t-1} + \beta_2 \cdot GDP_{t-1} - \beta_2 \Delta GDP_{t-1} + (\gamma_1 z_t - \gamma_1 z_{t-1}) + \gamma_1 \cdot z_{t-1} + \gamma_2 z_{t-1} + \varepsilon_t$$

$$\Delta GDP_t = -\beta_2 \Delta GDP_{t-1} + \gamma_1 \Delta z_t + \underbrace{(\beta_1 + \beta_2 - 1)}_{\lambda} \left(GDP_{t-1} - \frac{\gamma_1 + \gamma_2}{\beta_1 + \beta_2 - 1} z_{t-1} - \frac{\alpha}{\beta_1 + \beta_2 - 1} \right) + \varepsilon_t$$

speed of adjustment

$\varepsilon_{t-1} \Rightarrow \hat{\varepsilon}_{t-1}$

□ Let $x_t = x_{t-1} + \varepsilon_t$, where ε_t is i.i.d. with mean zero and variance σ^2 for $t = 1, \ldots, T$. Is the time series x_t stationary?

X_0 – fixed / or with known dist.

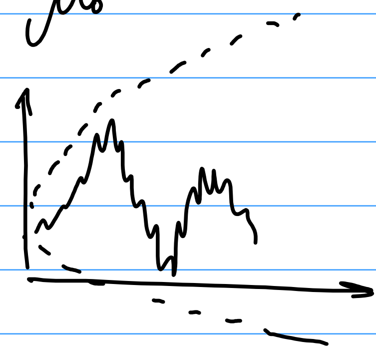
- RW
- RW with drift
- RW with trend

$I(1)$ $\quad X_t = X_{t-1} + \varepsilon_t$ – RW

$$E X_t = E X_{t-1} + E \varepsilon_t =$$

$$= E X_{t-2} + E(\varepsilon_t + \varepsilon_{t-1}) =$$

$$= \mu_0 + E\left(\sum_{s=1}^{t} \varepsilon_s\right) = \mu_0$$



$$Var(X_t) = Var(X_{t-1} + \varepsilon_t) =$$

$$Var(X_0) + Var\left(\sum_{s=1}^{t} \varepsilon_s\right) =$$

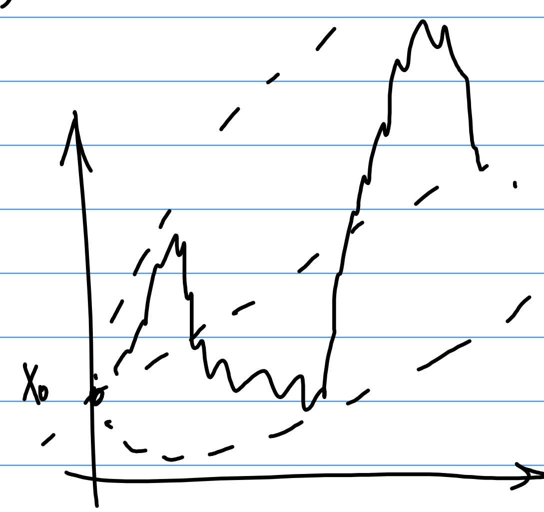
$$= Var(X_0) + \sigma^2 t$$

$t > s$ $\quad Cov(X_t, X_s) = \sigma^2 (t-s)$

$I(1)$ RW with drift:

$$X_t = \alpha + X_{t-1} + \varepsilon_t$$

$$E(X_t) = \alpha \cdot t + E X_0$$



$I(2)$ RW with trend:

$$X_t = \alpha + \beta t + X_{t-1} + \varepsilon_t$$

$$E(X_t) = \alpha t + \beta t^2 + E X_0$$