VAR

(P1) Cointegrated TS:

1) Same order of integration $I(d)$, $d \geq 1$

↳ ADF test

2) $\exists$ lin. comb. of these TS that is stationary

$y_t, X_t, Z_t - I(1)$ $\quad y_t = \alpha_1 + \alpha_2 X_t + \alpha_3 Z_t + u_t$

$u_t = -\alpha_1 + y_t - \alpha_2 X_t - \alpha_3 Z_t$

↳ ADF test

(P2) ECM

$$\Delta y_t = \underbrace{\theta_1 + \gamma_2 \cdot \Delta X_t}_{\text{SR dynamics}} + \pi \cdot u_{t-1} + \varepsilon_t$$

$$u_{t-1} = [y_{t-1} - \alpha - \beta X_{t-1}]$$

$[y_{t-1} - \alpha - \beta X_{t-1}]$ — LR dynamics

π - adjustment rate

$y_t = \alpha + \beta X_t + \varepsilon_t$

$X_t, y_t - I(1)$

↳ spurious regression

$[\Delta y_t = \theta_1 + \gamma_2 \Delta X_t + \varepsilon_t$

↳ only SR dynamics

Granger Causality

$$y_t = \alpha_1 + \alpha_2 y_{t-1} + \dots + \alpha_m y_{t-m} + \beta_1 x_{t-1} + \dots + \beta_m x_{t-m} + \varepsilon_t$$

$H_0: \beta_1 = \dots = \beta_m = 0$

x does not Granger cause y

$$x_t = \alpha_1 + \alpha_2 y_{t-1} + \dots + \alpha_m y_{t-m} + \beta_1 x_{t-1} + \dots + \beta_m x_{t-m} + \varepsilon_t$$

$H_0: \alpha_1 = \dots = \alpha_m = 0$

y does not cause x

VAR(p)

VAR(1):
$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} I \begin{pmatrix} u_t \\ \varepsilon_t \end{pmatrix}$$

VAR(5)
$$\begin{cases} y_t = \phi_{11} y_{t-1} + \varphi_{12} x_{t-1} + \varphi_{13} \cdot y_{t-2} \\ \qquad + \phi_{14} x_{t-2} + \dots + \psi_1 + u_t \\ x_t = \dots \end{cases}$$

goodness-of-fit vs complexity

$$AIC = -2 \log L(\hat{\theta}) + 2p$$

$p \uparrow \Rightarrow \log L \uparrow \qquad p \uparrow \Rightarrow 2p \uparrow$

$\Rightarrow -2 \log L \downarrow$

$$SC = BIC = -2(\log L(\hat{\theta}) + \log(T) \cdot p$$

(P14) LR elasticity:

$$\frac{\alpha_4 + \alpha_5}{1 - \alpha_2 - \alpha_3}$$

$$\tilde{y} = \frac{\alpha_1}{1 - \alpha_2 - \alpha_3} + \frac{\alpha_4 + \alpha_5}{1 - \alpha_2 - \alpha_3} \tilde{X}$$

↳ LR equilibrium / Coint. relationship