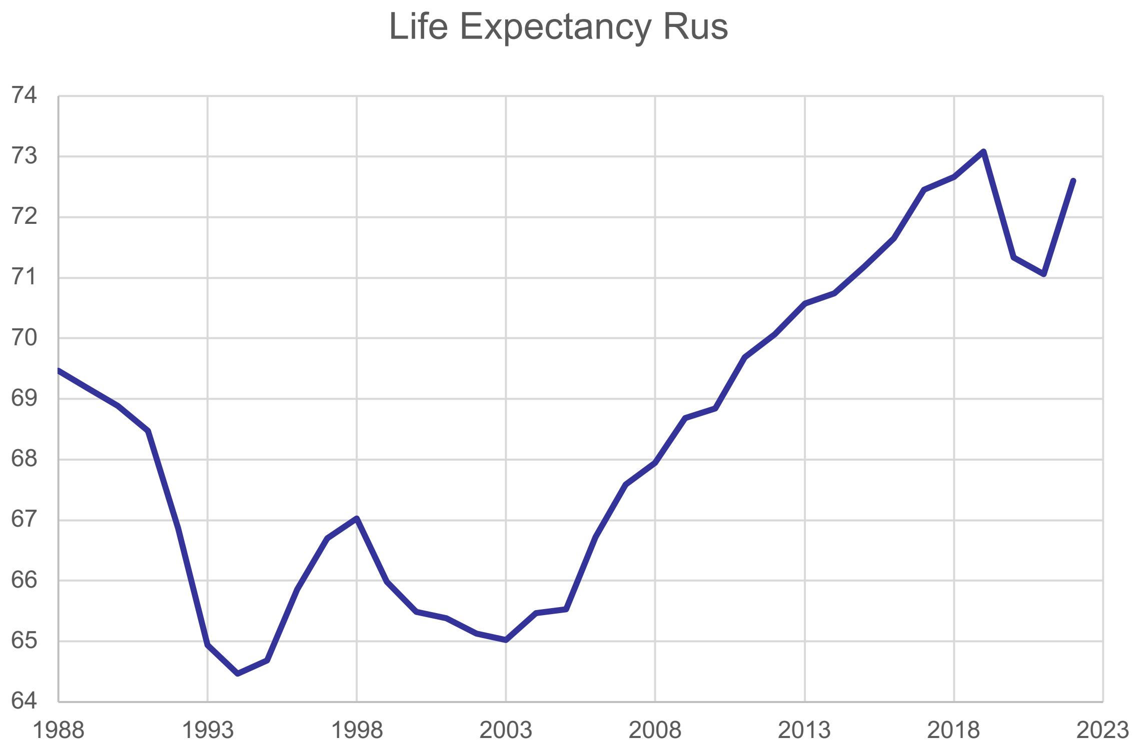
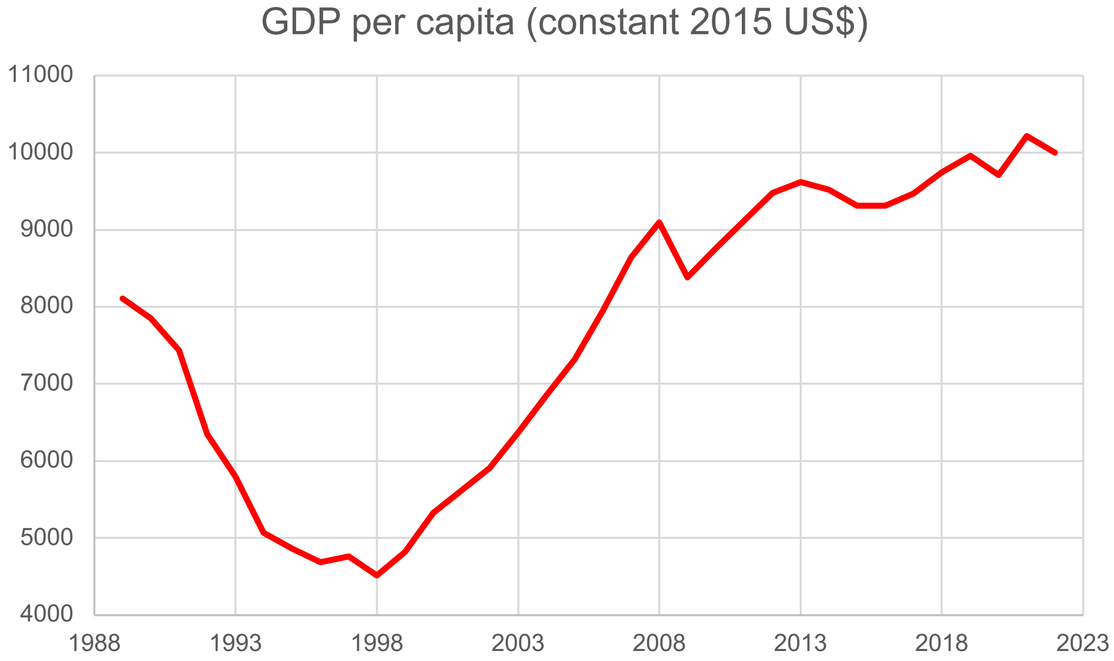
Elements of Econometrics.
Lecture 25.
Revision Time Series 1.

FCS, 2022-2023

Time Series Data: Life Expectancy (years) and Real GDP per capita in Russia, 1988-2022.





Are the series related? What is the dynamic model of their relationship?

AUTOCORRELATED DISTURBANCE TERM

Violated Assumption C.6 (Gauss-Markov 3 condition)
"The values of the disturbance term have independent distributions:
u_t is distributed independently of $u_{t'}$ for $t' \neq t$ "

Reasons: **disturbance term combines the influence of all factors not included in the model directly, and some of them may be autocorrelated in the case of time series data.**

Consequences: **in general, the regression coefficients remain unbiased, but OLS is inefficient. Standard errors estimated wrongly, t-tests invalid. If lagged dependent variable is a regressor, the OLS estimates are biased and inconsistent.**

Detection:

- **Breusch–Godfrey LM test (autocorrelation of order p; large samples);**
- **Durbin-Watson d-test (AR(1) type autocorrelation, finite samples, fixed values of explanatory variables, intercept, critical values depend on X's);**
- **Durbin h-test (model with lagged dependent variable as a regressor; large samples);**
- **F-tests and t-tests (large samples only).**

Remedial Measures: **special type of GLS, AR transformation.**

TESTS FOR AUTOCORRELATION

Breusch–Godfrey test (first or higher order autocorrelation)

$$Y_t = \beta_1 + \sum_{j=2}^{k} \beta_j X_{jt} + u_t \qquad \hat{u}_t = \gamma_1 + \sum_{j=2}^{k} \gamma_j X_{jt} + \sum_{s=1}^{q} \rho_s \hat{u}_{t-s}$$

First-order autoregressive autocorrelation: AR(1) $\quad u_t = \rho u_{t-1} + \varepsilon_t$

p's order autoregressive autocorrelation: AR(p) $\quad u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \ldots + \rho_p u_{t-p} + \varepsilon_t$

Valid also for MA(*q*) autocorrelation $\quad u_t = \lambda_0 \varepsilon_t + \lambda_1 \varepsilon_{t-1} + \lambda_2 \varepsilon_{t-2} + \ldots + \lambda_q \varepsilon_{t-q}$

For the Lagrange multiplier version of the test, the test statistic is nR^2 distributed as $\chi^2(q)$

The *F* test version compares *RSS* for the residuals regression with *RSS* for the same specification without the residual terms.

The test is valid only asymptotically.

TESTS FOR AUTOCORRELATION

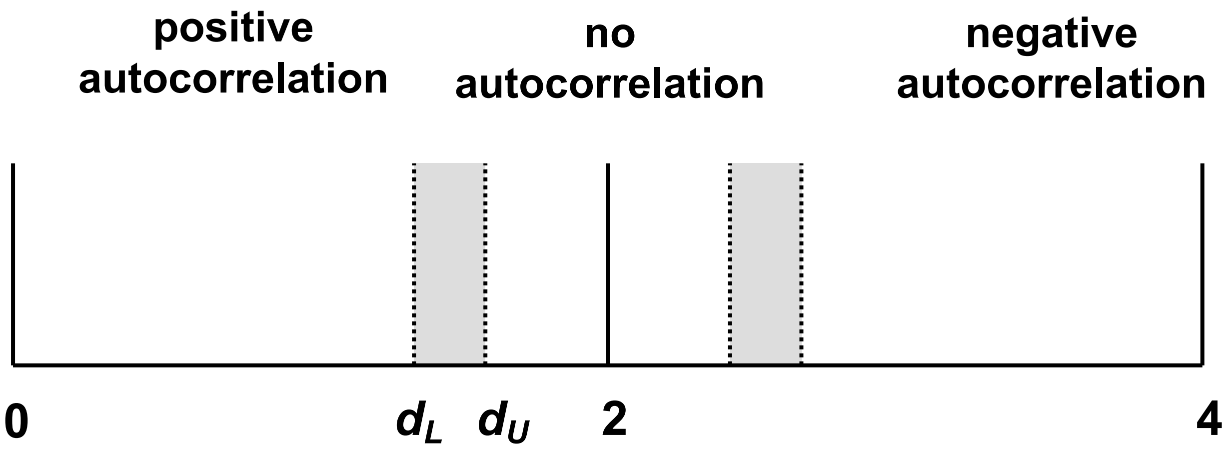
Durbin–Watson test

$$d = \frac{\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{T} \hat{u}_t^2}$$

In large samples $d \to 2 - 2\rho$

The Durbin–Watson test for AR(1) autocorrelation based on the Durbin–Watson *d* statistic calculated from the residuals.

In large samples *d* tends to 2 – 2ρ, where ρ is the parameter in the AR(1) relationship $u_t = \rho u_{t-1} + \varepsilon_t$.



The Durbin-Watson test is valid only when all the explanatory variables are deterministic.

The test is restricted by AR(1) autocorrelation.

The model should include the constant.

But the test is applicable for finite samples and is provided by standard packages.

TESTS FOR AUTOCORRELATION

Durbin's *h* test

$$h = \hat{\rho}\sqrt{\frac{n}{1 - ns^2_{\hat{\beta}_{Y(-1)}}}}$$

$$\boldsymbol{d \rightarrow 2 - 2\rho} \qquad \hat{\rho} = 1 - 0.5d$$

The *h* test is appropriate for the detection of AR(1) autocorrelation where the use of the lagged dependent variable as a regressor made the Durbin–Watson test inapplicable

Durbin *h* statistics has asymptotically standardized normal distribution under the H_0 hypothesis of no autocorrelation of u_t.

Correcting for serial correlation with strictly exogenous regressors

Under the assumption of AR(1) errors, one can transform the model so that it satisfies all Gauss-Markov assumptions. For this model, OLS is BLUE.

$$y_t = \beta_0 + \beta_1 x_t + u_t \leftarrow$$ Simple case of regression with only one explanatory variable. The general case works analogusly.

$$\rho y_{t-1} = \rho\beta_0 + \rho\beta_1 x_{t-1} + \rho u_{t-1} \leftarrow$$ Lag and multiply by ρ

$$\Rightarrow \quad y_t - \rho y_{t-1} = \beta_0(1-\rho) + \beta_1(x_t - \rho x_{t-1}) + u_t - \rho u_{t-1}$$

$$u_t = \rho u_{t-1} + e_t \Leftrightarrow u_t - \rho u_{t-1} = e_t \leftarrow$$ The transformed error satisfies the GM-assumptions.

Problem: The AR(1)-coefficient is not known and has to be estimated. The non-linear estimation should be applied, or a Cochrane-Orcutt iterative procedure.

ELIMINATING AR(1) AUTOCORRELATION. AUTOREGRESSIVE TRANSFORMATION WITH TWO REGRESSORS

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \qquad u_t = \rho u_{t-1} + \varepsilon_t$$

$$\rho Y_{t-1} = \beta_1 \rho + \beta_2 \rho X_{2t-1} + \beta_3 \rho X_{3t-1} + \rho u_{t-1}$$

$$Y_t - \rho Y_{t-1} = \beta_1 (1-\rho) + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + u_t - \rho u_{t-1}$$

$$Y_t = \beta_1 (1-\rho) + \rho Y_{t-1} + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + \varepsilon_t$$

$-$ *two nonlinear restrictions.*

Doing the AR(1) transformation, we get rid of the autocorrelation in the disturbance term. Only the innovation term ε_t remains. But the revised specification involves a nonlinear restriction, hence non-linear estimation technique is needed.

EViews:
Y=C(1)*(1-C(2))+C(2)*Y(-1)+C(3)*X2-C(2)*C(3)*X2(-1)+C(4)*X3-C(2)*C(4)*X3(-1)

Option in the EViews: add AR(1) to the list of explanatory variables in the initial regression. If the second order autocorrelation available, add AR(1) and AR(2); higher orders dealt respectively.

COCHRANE–ORCUTT ITERATIVE PROCEDURE

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$\rho Y_{t-1} = \beta_1 \rho + \beta_2 \rho X_{t-1} + \rho u_{t-1}$$

$$Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2 X_t - \beta_2 \rho X_{t-1} + u_t - \rho u_{t-1}$$

$$\widetilde{Y}_t = \beta_1' + \beta_2 \widetilde{X}_t + \varepsilon_t$$

$$\widetilde{Y}_t = Y_t - \rho Y_{t-1}$$
$$\widetilde{X}_t = X_t - \rho X_{t-1}$$
$$\beta_1' = \beta_1(1-\rho)$$

1. Regress Y_t on X_t using OLS
2. Calculate $\hat{u}_t = Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_t$ and regress $\hat{u}_t$ on $\hat{u}_{t-1}$ to obtain an estimate of ρ.
3. Calculate $\widetilde{Y}_t$ and $\widetilde{X}_t$ and regress $\widetilde{Y}_t$ on $\widetilde{X}_t$ to obtain revised estimates $\hat{\beta}_1$ and $\hat{\beta}_2$. Return to (2) and continue until convergence.

CO procedure allows to apply linear OLS for iterative estimation of non-linear model. After steps 1-3, we keep alternating between Step 2 and Step 3 until convergence is obtained. The first observation is lost if the CO is applied, but this can be compensated by adding the transformed first model observation (Prais-Winsten)

Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors (Newey-West)

In the model with autocorrelated disturbance term, standard errors are calculated incorrectly, and t-tests invalid.

Newey and West (1987) have proposed an estimator that is consistent in the presence of both heteroscedasticity and autocorrelation of unknown form.

To use the Newey-West method in the EViews, select the Options tab in the Equation Estimation. Check the box labeled Heteroscedasticity Consistent Covariance and press the Newey-West button. The formula used is:

$$\sigma^2_{\hat{\beta}_2} = E\left\{\left(\hat{\beta}_2 - E(\hat{\beta}_2)\right)^2\right\} = E\left\{\left(\hat{\beta}_2 - \beta_2\right)^2\right\} = E\left\{\left(\sum_{i=1}^{n} a_i u_i\right)^2\right\} = E\left\{\sum_{i=1}^{n} a_i^2 u_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} a_i a_j u_i u_j\right\} = \sum_{i=1}^{n} a_i^2 E\left(u_i^2\right) + \sum_{i=1}^{n}\sum_{j\neq i} a_i a_j E\left(u_i u_j\right) =$$

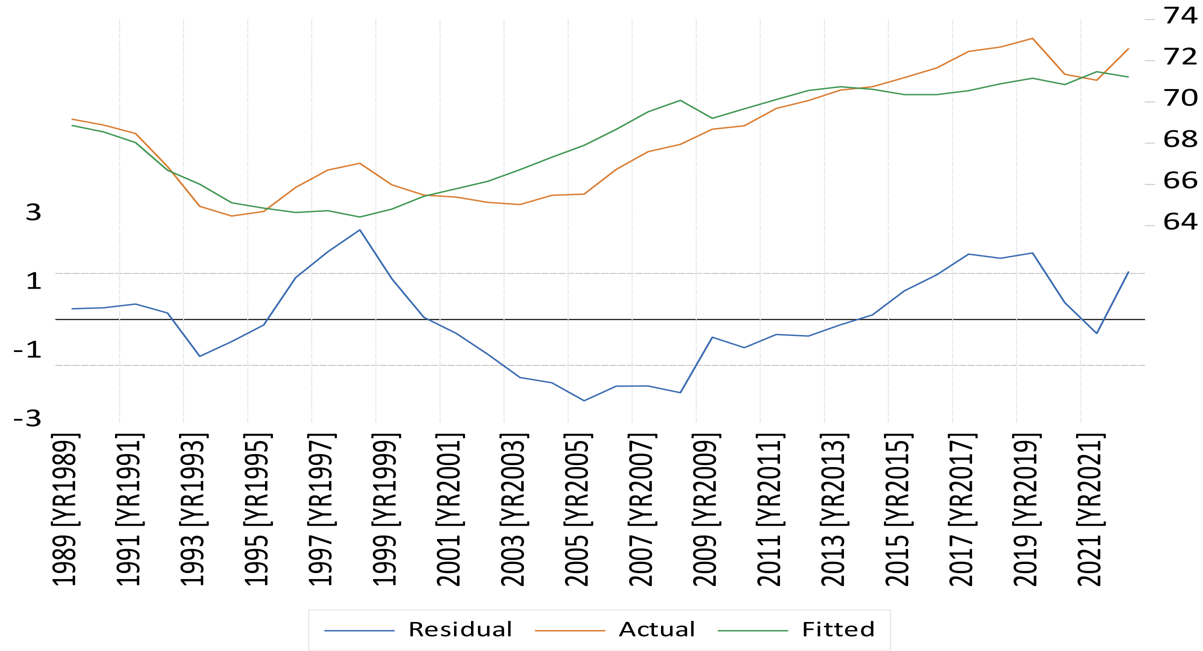
$$= \sum_{i=1}^{n} a_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} a_i a_j \sigma_{ij} = \frac{\sum_{i=1}^{n} x_i^{\ 2} \sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} x_i x_j \sigma_{ij}}{\left(\sum_{j=1}^{n} x_j^{\ 2}\right)^2}$$

Example: the regression of Life Expectancy on the Real GDP Per Capita in Russia, 1988-2022

Dependent Variable: LIFE_EXP
Method: Least Squares
Date: 03/11/23 Time: 10:27
Sample (adjusted): 18 51
Included observations: 34 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	58.84583	0.953172	61.73683	0.0000
GDP_PC	0.001236	0.000121	10.21293	0.0000

R-squared	0.765230	Mean dependent var	68.29374
Adjusted R-squared	0.757894	S.D. dependent var	2.721508
S.E. of regression	1.339098	Akaike info criterion	3.478893
Sum squared resid	57.38190	Schwarz criterion	3.568678
Log likelihood	-57.14117	Hannan-Quinn criter.	3.509512
F-statistic	104.3039	Durbin-Watson stat	0.350227
Prob(F-statistic)	0.000000		



Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 1 lag

F-statistic	65.57961	Prob. F(1,31)	0.0000
Obs*R-squared	23.08672	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/11/23 Time: 13:16
Sample: 18 51
Included observations: 34
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.299425	0.549905	-0.544503	0.5900
GDP_PC	4.37E-05	6.99E-05	0.625231	0.5364
RESID(-1)	0.841300	0.103888	8.098124	0.0000

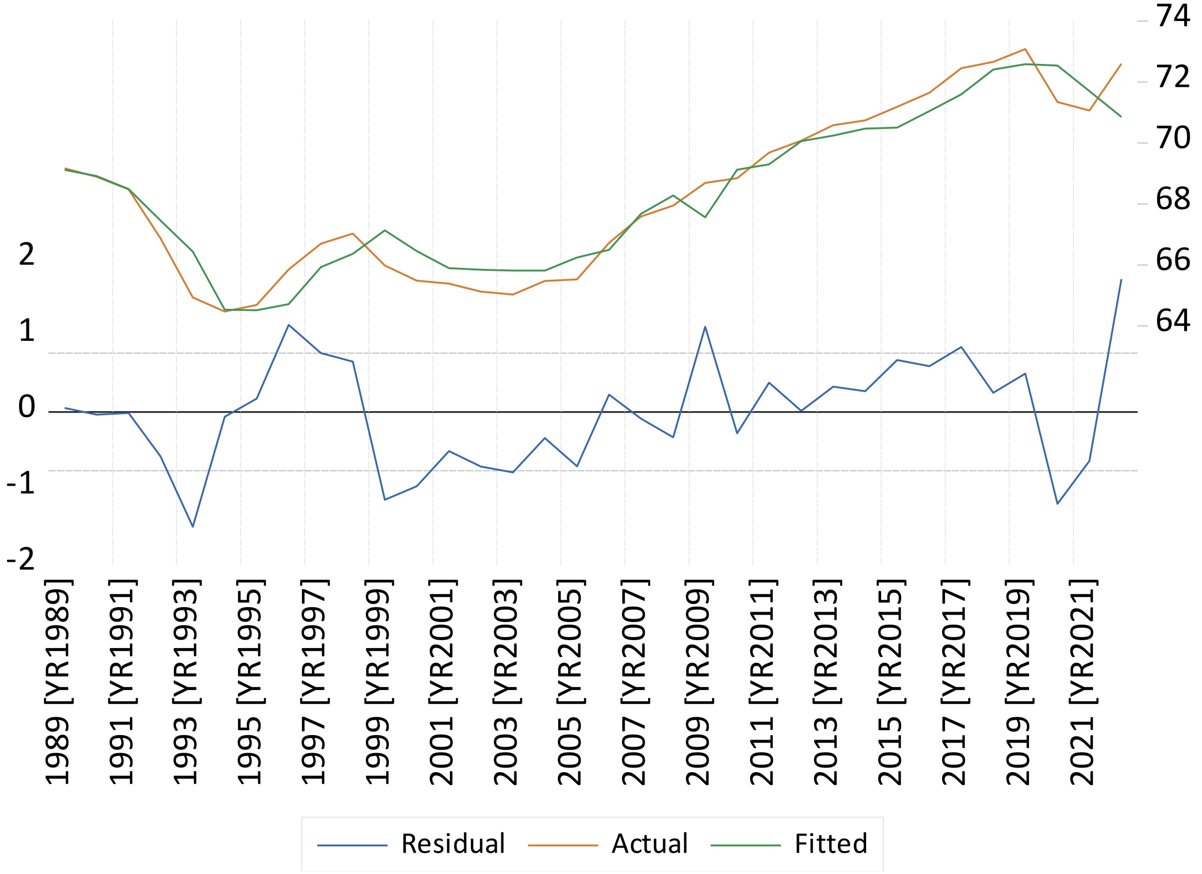
R-squared	0.679021	Mean dependent var	1.16E-14
Adjusted R-squared	0.658313	S.D. dependent var	1.318653
S.E. of regression	0.770805	Akaike info criterion	2.401336
Sum squared resid	18.41837	Schwarz criterion	2.536014
Log likelihood	-37.82270	Hannan-Quinn criter.	2.447265
F-statistic	32.78981	Durbin-Watson stat	1.448629
Prob(F-statistic)	0.000000		

Example: the regression of Life Expectancy on the Real GDP Per Capita in Russia, 1988-2022

Dependent Variable: LIFE_EXP
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/11/23 Time: 13:22
Sample: 18 51
Included observations: 34
Convergence achieved after 5 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	61.50916	2.374200	25.90732	0.0000
GDP_PC	0.000933	0.000317	2.938277	0.0063
AR(1)	0.862645	0.098805	8.730812	0.0000
SIGMASQ	0.525422	0.148333	3.542169	0.0013

R-squared	0.926911	Mean dependent var	68.29374
Adjusted R-squared	0.919602	S.D. dependent var	2.721508
S.E. of regression	0.771672	Akaike info criterion	2.469712
Sum squared resid	17.86435	Schwarz criterion	2.649283
Log likelihood	-37.98510	Hannan-Quinn criter.	2.530951
F-statistic	126.8189	Durbin-Watson stat	1.324195
Prob(F-statistic)	0.000000		



AR(1) as a Special Case of ADL(1,1) Model

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Restricted model (transformed AR(1)):

$$Y_t = \beta_1(1-\rho) + \rho Y_{t-1} + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + \varepsilon_t$$

Unrestricted ADL(1,1) model

$$Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 X_{2t} + \lambda_3 X_{2t-1} + \lambda_4 X_{3t} + \lambda_5 X_{3t-1} + \varepsilon_t$$

Restrictions embodied in the AR(1) process

$$\lambda_3 = -\lambda_1 \lambda_2 \qquad \lambda_5 = -\lambda_1 \lambda_4$$

The AR(1) model can be considered as a special (restricted) case of more general (unrestricted) ADL(1,1) model.

COMMON FACTOR TEST

Test statistic: $$n \log \frac{SSR_R}{SSR_U}$$

Restricted model SSR_R

$$Y_t = \beta_1(1-\rho) + \rho Y_{t-1} + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + \varepsilon_t$$

Unrestricted model SSR_U

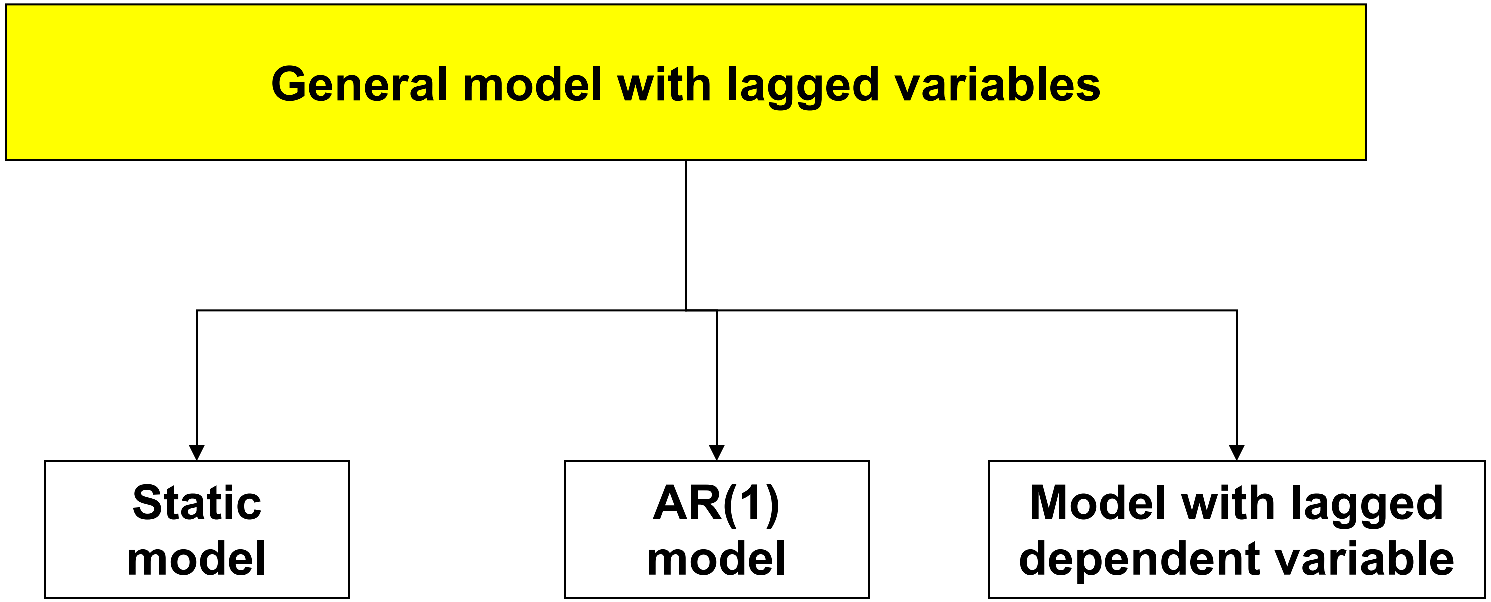
$$Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 X_{2t} + \lambda_3 X_{2t-1} + \lambda_4 X_{3t} + \lambda_5 X_{3t-1} + \varepsilon_t$$

Restrictions embodied in the AR(1) process

$$\lambda_3 = -\lambda_1 \lambda_2 \qquad \lambda_5 = -\lambda_1 \lambda_4$$

Under the null hypothesis that the restrictions are valid, the test statistic has a χ^2 (chi- squared) distribution with degrees of freedom equal to the number of restrictions. It is a large-sample test.

DYNAMIC MODEL SPECIFICATION



$$\lambda_1 = \lambda_3 = \lambda_5 = 0$$

$$\lambda_3 = -\lambda_1 \lambda_2$$

$$\lambda_5 = -\lambda_1 \lambda_4$$

$$\lambda_3 = \lambda_5 = 0$$

$$Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 X_{2t} + \lambda_3 X_{2t-1} + \lambda_4 X_{3t} + \lambda_5 X_{3t-1} + \varepsilon_t$$

General-to-specific approach should be used. We start with a model sufficiently general, and then simplify it if possible.

Example: the regression of Life Expectancy on the Real GDP Per Capita in Russia, 1988-2022

Dependent Variable: LIFE_EXP
Method: Least Squares
Date: 03/11/23 Time: 10:52
Sample (adjusted): 19 51
Included observations: 33 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.91886	5.902947	1.849731	0.0746
GDP_PC	0.000593	0.000302	1.964882	0.0591
GDP_PC(-1)	-0.000278	0.000349	-0.798010	0.4314
LIFE_EXP(-1)	0.805886	0.099548	8.095480	0.0000

R-squared	0.936554	Mean dependent var	68.26713
Adjusted R-squared	0.929991	S.D. dependent var	2.759211
S.E. of regression	0.730067	Akaike info criterion	2.321852
Sum squared resid	15.45694	Schwarz criterion	2.503247
Log likelihood	-34.31056	Hannan-Quinn criter.	2.382886
F-statistic	142.6940	Durbin-Watson stat	1.255686
Prob(F-statistic)	0.000000		

Dependent Variable: LIFE_EXP
Method: Least Squares
Date: 03/11/23 Time: 14:08
Sample: 19 51
Included observations: 33

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	13.13820	5.175146	2.538711	0.0166
GDP_PC	0.000371	0.000117	3.163232	0.0036
LIFE_EXP(-1)	0.767223	0.086434	8.876423	0.0000

R-squared	0.935161	Mean dependent var	68.26713
Adjusted R-squared	0.930838	S.D. dependent var	2.759211
S.E. of regression	0.725635	Akaike info criterion	2.282968
Sum squared resid	15.79637	Schwarz criterion	2.419014
Log likelihood	-34.66897	Hannan-Quinn criter.	2.328743
F-statistic	216.3416	Durbin-Watson stat	1.134956
Prob(F-statistic)	0.000000		

COMMON FACTOR TEST: EXAMPLE

Restricted model (AR(1))

$$LIFE\_EXP_t = \beta_1 + \beta_2 GDP\_PC_t + u_t \qquad u_t = \rho u_{t-1} + \varepsilon_t$$

$$LIFE\_EXP_t = \beta_1(1-\rho) + \rho LIFE\_EXP_{t-1} + \beta_2 GDP\_PC_t - \beta_2 \rho GDP\_PC_{t-1} + \varepsilon_t$$

Unrestricted model (ADL(1,1)

$$LIFE\_EXP_t = \lambda_0 + \lambda_1 LIFE\_EXP_{t-1} + \lambda_2 GDP\_PC_t + \lambda_3 GDP\_PC_{t-1} + \lambda_4 LGPRHOUS_t + \varepsilon_t$$

We compare the initial regression with AR(1) term (*SSR_R*=17.86) with the estimated with OLS the ADL(1,1) model with no restrictions on the parameters (*SSR_U*=15.46).

$$n \log\left(\frac{SSR_R}{SSR_U}\right) = 33 \log\left(\frac{17.86}{15.46}\right) = 38.12 \qquad \chi^2_{\text{crit}} = 6.63 \quad (1,\ 0.1\%)$$

We reject the restriction. We should choose more general model instead of assuming that the disturbance term is subject to an AR(1) process.

Then we test if the lagged regressor *GDP_PC* is needed in the ADL(1,1) model, using *t*-test. We reject H_0 and get the ADL(1,0) model.

$$LIFE\_EXP_t = \lambda_0 + \lambda_1 LIFE\_EXP_{t-1} + \lambda_2 GDP\_PC_t + \varepsilon_t$$