Testing for Non-stationary

DF

$$X_t = \beta X_{t-1} + \varepsilon_t$$
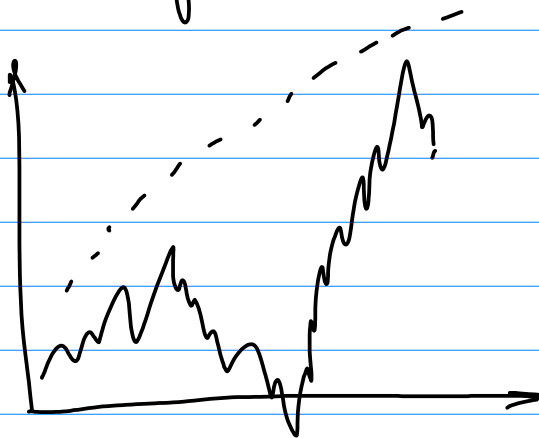
$$H_0 : \beta = 1$$

$$X_t - X_{t-1} = \beta X_{t-1} - X_{t-1} + \varepsilon_t$$
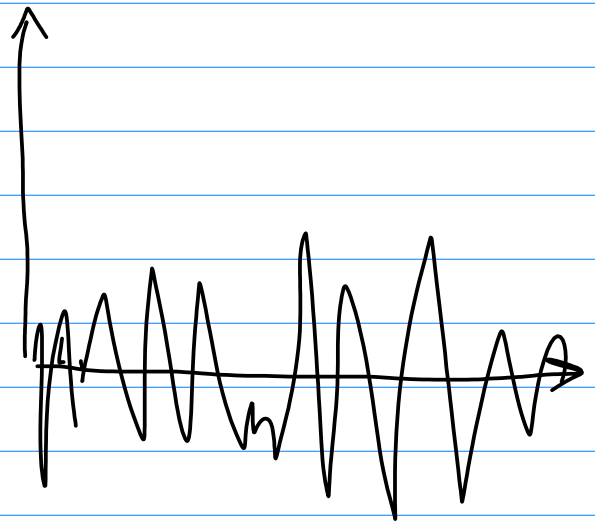
$$\Delta X_t = \underset{\gamma}{(\beta - 1)} X_{t-1} + \varepsilon_t$$

$$H_0 : \gamma = \beta - 1 = 0 \qquad\qquad H_a : \gamma < 0$$

$H_0 : RW$

$H_a :$ stationary with mean 0

$$t = \frac{\hat{\gamma} - 0}{se(\hat{\gamma})} \sim DF_{NC}$$

DF^c DF^{NC} t

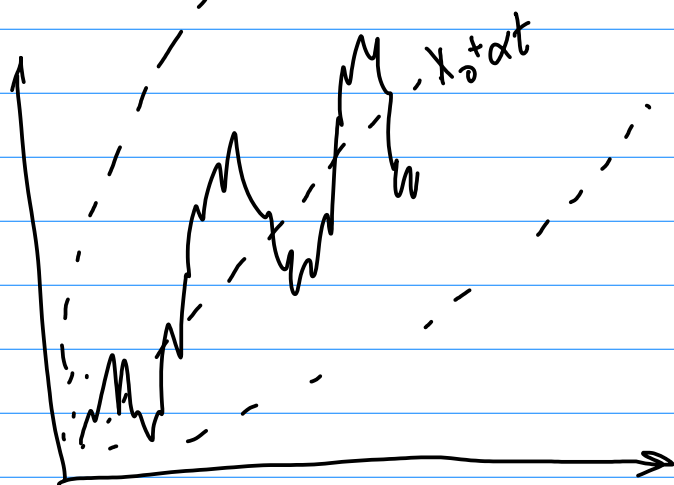$DF_{a, 5\%}$ $DF_{c, 5\%}$ $DF_{NC, 5\%}$ $t^c_{5\%}$
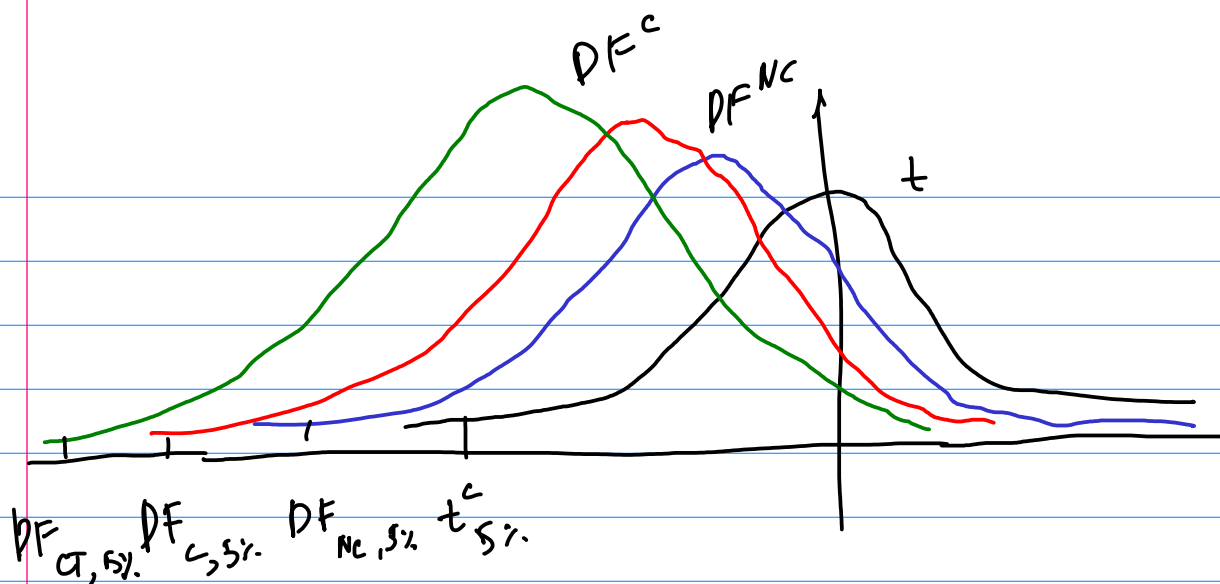
DF with const

$$\left[\Delta X_t = \alpha + \gamma X_{t-1} + \varepsilon_t \right] \qquad H_0 : \gamma = 0$$

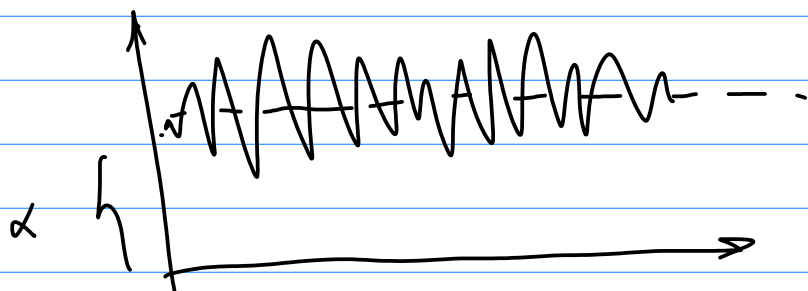$$X_t = \alpha + (\gamma+1) X_{t-1} + \varepsilon_t$$

if H_0 is true $\qquad X_t = \alpha + X_{t-1} + \varepsilon_t$

$X_0 + \alpha t$

$$X_t = X_0 + \alpha t + \sum z_t$$

z_t – stationary

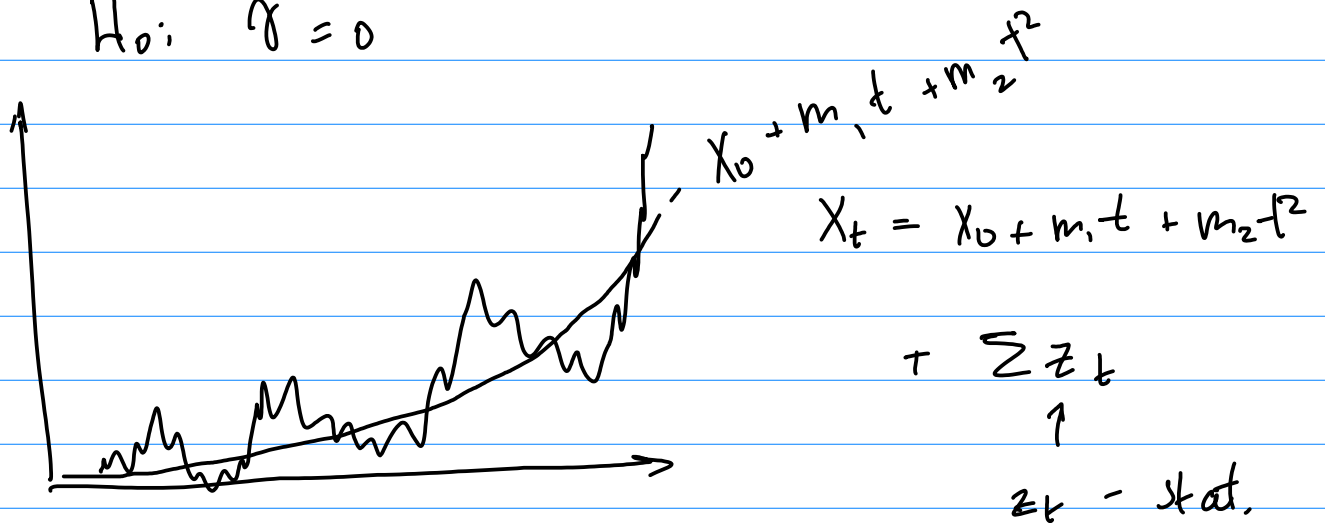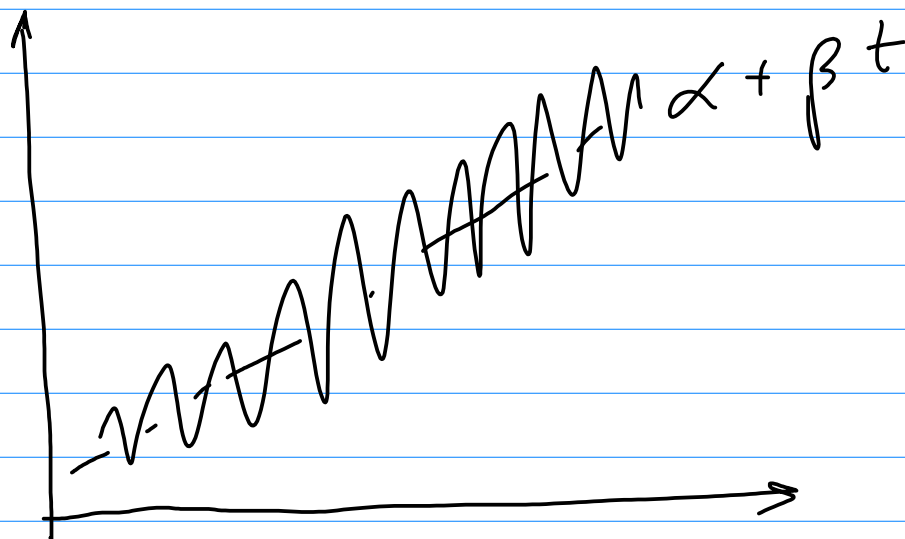$H_0 :$ RW with drift

$H_a :$ stat.

α

with mean α

DF with const and trend

$$\Delta X_t = \alpha + \beta t + \gamma X_{t-1} + \varepsilon_t$$

$$H_0: \gamma = 0$$

$$X_0 + m_1 t + m_2 t^2$$

$$X_t = X_0 + m_1 t + m_2 t^2$$

$$+ \sum Z_t$$

$$\uparrow$$

$$z_t - \text{stat.}$$

$$H_0 : RW \text{ with trend} \implies DS - \text{process}$$

$$\alpha + \beta t$$

$$H_a : TS - \text{process}$$

$$X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \varepsilon_t$$

$$\Delta X_t = \beta_0 + (\beta_1 - 1) X_{t-1} +$$

$$+ \beta_2 X_{t-2} + \beta_3 X_{t-2} + \underbrace{\beta_3 X_{t-3} - \beta_3 X_{t-2}}_{= -\beta_3 \Delta X_{t-2}} + \varepsilon_t$$

$$= \gamma$$

$$\Delta X_t = \beta_0 + \underbrace{(\beta_1 + \beta_2 + \beta_3 - 1)}_{= \gamma} X_{t-1}$$

$$- (\beta_2 + \beta_3) \Delta X_{t-1} - \beta_3 \Delta X_{t-2} + \varepsilon_t$$

$$H_0: \quad \beta_1 + \beta_2 + \beta_3 - 1 = 0 = \gamma$$