Autocorrelation

Remedies:

1) Specification:

True: $y_t = \beta_1 + \beta_2 X_t + \underbrace{\beta_3 Z_t + \varepsilon_t}_{\text{"}} \sim WN$

Est: $y_t = \beta_1 + \beta_2 X_t + u_t$

$+ \gamma \cdot t +$ $\qquad Z_t$ - AR process

$\Downarrow$

$y_t = \alpha + \beta X_t + \varepsilon_t$ $\qquad$ autocorrelation in u_t

$\Delta y_t = \alpha + \beta \Delta X_t + \varepsilon_t$

2) AR(1) transform

Cohrain - Orcutt

(1) $y_t = \beta_1 + \beta_2 X_t + u_t \quad | \cdot L\rho$

Assume: $u_t = \rho u_{t-1} + \varepsilon_t \sim WN$

(2) $y_t = \beta_1 + \beta_2 X_t + \rho \underbrace{\cancel{u_{t-1}}} + \varepsilon_t$

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$L \cdot \rho \cdot (1)$ $\quad \rho y_{t-1} = \beta_1 \cdot \rho + \beta_2 \cdot \rho X_{t-1} + \cancel{\rho} \cdot u_{t-1}$

$=$

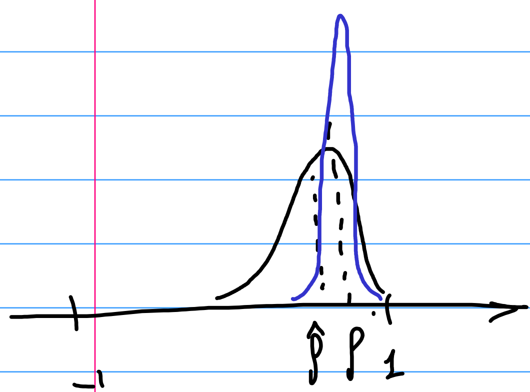
$$\underbrace{y_t - \rho y_{t-1}}_{y^*} = \beta_1 (1-\rho) + \beta_2 \underbrace{(X_t - \rho X_{t-1})}_{X^*} + \varepsilon_t$$

$$y_t^* = \beta_1^* + \beta_2 X_t^* + \varepsilon_t$$

$\hat{\rho} \nearrow$ (1) $\hat{u}_t = \rho \hat{u}_{t-1} + \varepsilon_t$ $\qquad \hat{\rho} = \dfrac{\sum \hat{u}_{t-1}\hat{u}_t}{\sum \hat{u}_{t-1}^2}$

$\hat{\rho} \searrow$ (2) DW $\longrightarrow 2(1-\rho)$

$$\hat{\rho} = 1 - \frac{DW}{2}$$



Price – Winsten Correction

(for 1st observation)

$t = [2;T] \qquad y_t^* = \beta_1^* + \beta_2 X_t^* + \varepsilon_t \qquad \sigma_\varepsilon^2$

$t = 1 \qquad y_1 = \beta_1 + \beta_2 X_1 + u_1 \ \Big| \cdot\sqrt{1-\rho^2} \qquad \sigma_u^2$

$$\sigma_u^2 = Var(u_t) = var(\rho u_{t-1} + \varepsilon_t) =$$

$$= \rho^2 \sigma_u^2 + \sigma_\varepsilon^2 + 2\,\overset{=0}{Cov(\rho u_{t-1}, \varepsilon_t)}$$

$$\sigma_u^2 = \rho^2 \sigma_u^2 + \sigma_\varepsilon^2 \qquad \left[(1-\rho^2)\cdot \sigma_u^2 = \sigma_\varepsilon^2 \right]$$

3) MA(1) transform - GLS

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$$

Ω - covariation matrix of errors

$$\Omega = \sigma_\varepsilon^2 \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} Var(\varepsilon_1) & Cov(\varepsilon_1, \varepsilon_2) & \cdots \\ cov(\varepsilon_2, \varepsilon_1) & Var(\varepsilon_2) & \cdots \\ \vdots & \vdots & \end{pmatrix}$$

$\Downarrow$

$$\hat{\beta} = (X'X)^{-1}X'y$$

$$\Omega = \sigma_\varepsilon^2 \begin{pmatrix} X_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & X_n \end{pmatrix}$$

$\Downarrow$

WLS

$\hat{\Omega}$

FGLS

$AR(1): \quad u_t = \rho u_t + \varepsilon_t$

$$\Omega = \frac{\sigma_\varepsilon^2}{1-\rho^2} \begin{pmatrix} 1 & \rho^1 & \cdots & \rho^{n-1} \\ \rho^1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho^1 \\ \rho^{n-1} & \cdots & \rho^1 & 1 \end{pmatrix}$$

$$Var(u_t) = \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

$$Cov(u_t, u_{t-k}) = Var(u_t) \cdot \rho^k$$

$MA(1): \quad u_t = \varepsilon_t + \rho \varepsilon_{t-1}$

$$\Omega = \sigma_u^2 \begin{pmatrix} 1 & 2 & 0 & \cdots & 0 \\ 2 & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 2 \\ 0 & \cdots & 0 & 2 & 1 \end{pmatrix}$$

$$Cov(u_t, u_{t-2}) = Cov(\varepsilon_t + \rho\varepsilon_{t-1}; \varepsilon_{t-2} + \rho\varepsilon_{t-3})$$
$$= 0$$

$$y_t = \overset{\lambda_1}{\rho} y_{t-1} + \overset{\lambda_2}{\beta_1 (1-\rho)} + \overset{\lambda_3}{\beta_2} X_t - \overset{\lambda_4}{\beta_2 \rho} X_{t-1} + \varepsilon_t$$

ARDL(1,1) with restriction

$$\lambda_4 = -\lambda_1 \cdot \lambda_3$$

Common Factor Test

$$n \cdot \log \frac{RSS_R}{RSS_{UR}} \sim \chi^2_1$$