Question 1 (30 points) A student runs logarithmic regressions of the total expenditures of USA citizens on beer B (billions of dollars) on price index PB for the period 1996-2020 (100% corresponds to the level of the year 2020).

$$\ln \hat{B}_t = 1.56 + 0.21 \ln PB_t \qquad R^2 = 0.82 \qquad \text{(1-OLS)}$$
$$(0.09)\ (0.02) \qquad d = 0.76$$
$$\ln \hat{B}_t = 5.18 - 0.28 \ln PB_t \qquad R^2 = 0.92 \qquad \text{(2-AR(1))}$$
$$(4.84)\ (0.20) \qquad d = 2.61$$

1) Sources of endogeneity:

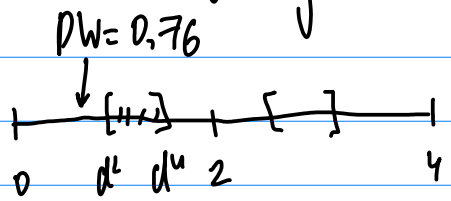
- OVB
- Simultaneity

(1) Interpretation is incorrect

AC?

$d = 0,76$ $\quad H_0: \rho = 0$

$H_a: \rho > 0$



$d = 0,76 < d^L = 1,05$

1%, 25

=> H_0 is rejected => AC

□ What is the autoregressive transformation AR(1)? Show mathematically how autoregressive transformation works. Was this transformation successful (equation 2)?

$E(u_t) = 0, \quad E(u_t^2) = \sigma_u^2$

(1) $\ln B_t = \beta_1 + \beta_2 P_t + u_t \quad | \cdot \rho$

$u_t = \rho u_{t-1} + \varepsilon_t$

$E\,\varepsilon_t = 0; \quad E\,\varepsilon_t^2 = \sigma_\varepsilon^2$

$s \neq t \quad E(\varepsilon_t \varepsilon_s) = 0$

(2) $\ln B_t = \beta_1 + \beta_2 P_t + \rho u_{t-1} + \varepsilon_t \quad | - \rho \cdot L(1)$

$\ln b_t - \rho \ln B_{t-1} = \beta_1(1-\rho) + \beta_2(P_t - \rho P_{t-1}) + \varepsilon_t$

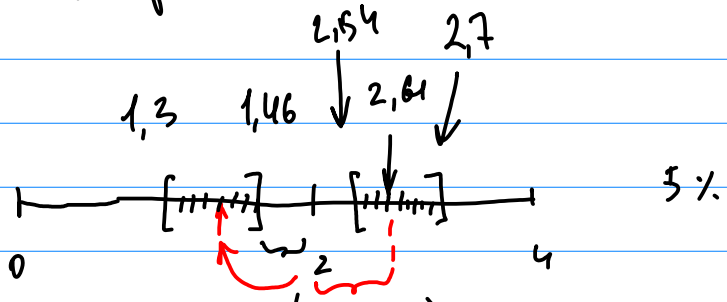
How-to: until conv.

1) $\hat{\rho} = 1 - \frac{DW}{2}$ $\quad [DW \xrightarrow{d} 2\rho - 2]$

2) Transform

$H_0: \rho = 0$

$H_a: \rho < 0$



$(2 - 1{,}46) + 2 = 2{,}54$

$2 - (2{,}61 - 2) = 1{,}39$

(b) (18 points) □ The teacher advised the student to include into equation more variables: $\ln DPI_t$ – logarithm of the gross disposable personal income (billions of dollars) and $TIME_t$ (which is equal 1 in 1996):

$$\ln \hat{B}_t = 7.23 - 0.37 \ln PB_t + 0.55 \ln DPI_t + 0.049 TIME_t \qquad R^2 = 0.93$$
$$(1.74)\ (0.11) \qquad (0.22) \qquad (0.012) \qquad d = 1.26 \qquad \text{(3-OLS)}$$

Give interpretation to the coefficients of equation (3) and compare them with those of (1) and (2). Was the teacher's advice helpful?

1) $-0{,}37$ elasticity Cons on ?

$+0{,}55$ elasticity cons on DPI

↑ Time 1 year $\Rightarrow$ $\hat{\beta}_{Time} \cdot 100 = 4{,}9$ %

DW: 4 parems, 25 obs, 5%.

$1{,}1 < 1{,}26 < 1{,}66$

□ Additionally the student decided to do Breusch-Godfrey test for the first order autocorrelation getting value of the test statistic equal to 3.3088. Explain what is Breusch-Godfrey test and help the student interpret the test results.

aux: $\ln B_t = \beta_1 + \beta_2 \ln P_t + \beta_3 \cdot \ln D_t + \beta_4 T_t + \beta_5 \hat{u}_{t-1} + \varepsilon_t$

$\hat{u}_{t-1}$ ↑ (3)

$H_0: \rho = 0$

$H_a: \rho \neq 0$

$T \cdot R^2 = 3{,}31 \sim \chi^2_1$

$\chi^2_{1,5\%} = 3{,}84 \Rightarrow$ no AC

□ A student friend said that beer consumption can be explained mainly by the habit of drinking beer. So she tries one more equation with the lagged variable

$$\ln \hat{B}_t = 0.62 - 0.08 \ln PB_t + 0.90 \ln B_{t-1} \qquad R^2 = 0.90$$
$$(0.51)\ (0.10) \qquad (0.07) \qquad d = 2.18 \qquad \text{(4-OLS)}$$

Was friend's advice helpful?

$$h = \hat{\rho} \cdot \sqrt{\frac{n}{1 - n \cdot Var(\hat{\beta}_{y_{t-1}})}} =$$

$$= \left(1 - \frac{2.18}{2}\right) \cdot \sqrt{\frac{24}{1 - 24 \cdot (0,07)^2}} = -0,461$$

$$|h^{obs}| < |-1,96| \quad \Rightarrow \quad \text{no AC}$$

Question 2. (30 points) The student of the course 'Elements of Econometrics' decided to investigate the influence of income DPI_t and relative prices on home electronics $PREL_{it}$ on the expenditures on home electronics EL_{it}. Different types of electronics (TV, computers, phones etc. – 7 items in total), considered in 1977-1921 (45 years) form a panel. The model under investigation is

$$\log(EL_{it}) = \beta_1 + \beta_2 \log(DPI_t) + \beta_3 \log(PREL_{it}) + u_{it};$$

The student runs three alternative approaches to the evaluation of this model: 1) pooled OLS regression, 2) fixed effects panel regression model, and 3) random effects panel regression model. The results of the estimation are shown below

$$\log(EL)_{it} = \underset{(2.47)}{4.28} + \underset{(0.24)}{0.63}\log(DPI_t) - \underset{(0.15)}{0.83}\log(PREL_{it}) + e_{it} \quad R^2 = 0.11 \quad DW = 0.02 \quad RSS = 514.9; \quad \text{(1 - Pooled)}$$

$$\log(EL)_{it} = \underset{(0.46)}{3.81} + \underset{(0.05)}{0.62}\log(DPI_t) - \underset{(0.05)}{0.92}\log(PREL_{it}) + e_{it} \quad R^2 = 0.98 \quad DW = 1.05 \quad RSS = 12.5; \quad \text{(2 - Fixed)}$$

$$\log(EL)_{it} = \underset{(0.73)}{3.18} + \underset{(0.04)}{0.65}\log(DPI_t) - \underset{(0.06)}{0.89}\log(PREL_{it}) + e_{it} \quad R^2 = 0.73 \quad DW = 1.04 \quad RSS = 12.7; \quad \text{(3 - Random)}$$

(you can assume that the LSDV approach is used when evaluating the fixed effects model)

(a) (12 marks) □ The student is worried that none of estimated equations (1)-(3) contained information about unobserved heterogeneity which, as he remembers from the lecture, is one of the main features of panel data. Explain to the student the differences between the three methods, stating the reasons for the absence of unobserved heterogeneity term in the results.

□ Help the student interpret the coefficients of the regression (2). The printout of regression (2) contains data on fixed effects (see table to the right). Explain to the student what are fixed effects and how this information can be used in the interpretation of the regression results.

EL Type	Fixed Effect
1	1.228209
2	-1.886449
3	2.008554
4	0.636180
5	-1.041255
6	-0.207966
7	-0.737273

(b) (18 marks) □ Help the student to choose between models (1-pooled) and (2-fixed). Do appropriate test stating null hypothesis and giving explanation to the paremeters used, and make a conclusion.

(2) $H_0: \beta_{D_1} = \ldots = \beta_{D_7} = 0$

$$F = \frac{(RSS_{POOL} - RSS_{FE}) / (7-1)}{RSS_{FE} / (7.45 - 9)} = 2049.8$$

$$F_{1\%,\,6,\,300} = 2.186$$