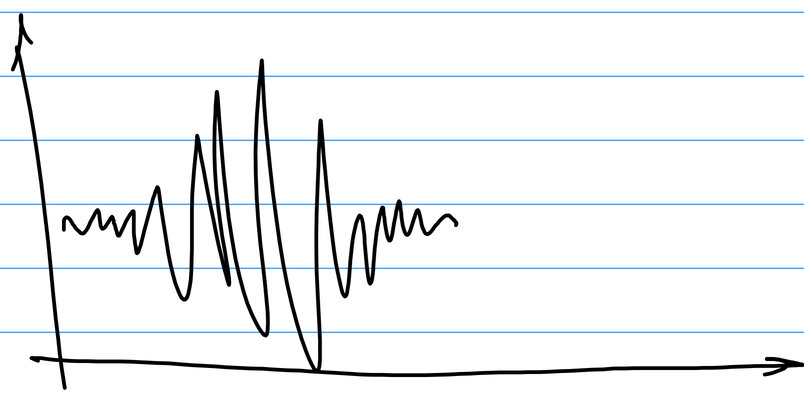
Question 4.

Let us consider the relationship between the natural logarithm of GDP, GDP_t, and the lagged long-term interest rates $rate_t$, and $rate_{t-1}$.

(a) (10 points) □ Assume that GDP_t and $rate_t$ are difference stationary, or they are integrated of order 1: $GDP_t \in I(1)$, $rate_t \in I(1)$. What does it mean? How to test that a time series is $I(1)$?



□ What is cointegration? How to test whether time series GDP_t and $rate_t$ are cointegrated.

- Same Order of integration
- $\exists$ lin. comb. which is stationary (z_t)

$$z_t = a_0 + a_1 \cdot GDP_t + a_2 r_t$$

$$GDP_t = \frac{z_t}{a_1} - \frac{a_2}{a_1} \cdot r_t - \frac{a_0}{a_1}$$

$I(1)$ $\quad$ $I(0)$ $\quad$ $I(1)$

$$GDP_t = b_0 \quad b_1 \quad r_t + \varepsilon_t$$

$b_0 = a_0/a_1$, $\quad b_1 = a_2/a_1$, $\quad \varepsilon_t = z_t/a_1$

ADF test for $\hat{\varepsilon}_t$

(b) (10 points) □ If GDP_t and $rate_t$ are not co-integrated, what are the properties of an OLS estimator in the regression of GDP_t on $rate_t$?

Spurious regression: $\alpha_2 = 0$

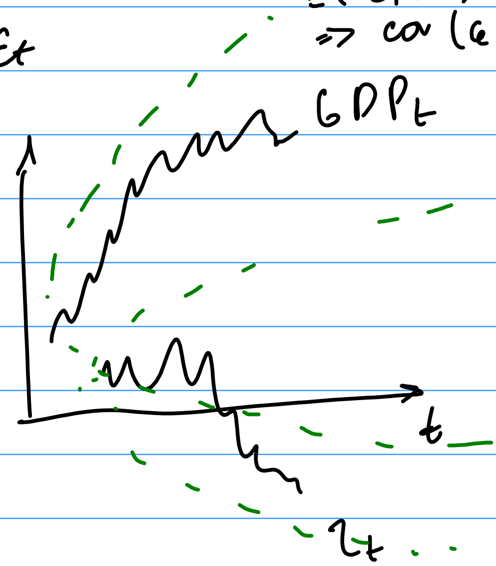
$E(\varepsilon_i | X) = 0$

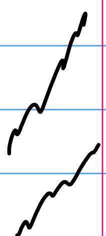
$\Rightarrow cov(\varepsilon_i, X) = 0$

- $GDP_t = \alpha_1 + \alpha_2 r_t + \varepsilon_t$

$I(1) = \alpha_1 + 0 + I(1)$

$\Rightarrow$ ε_t violates GMT

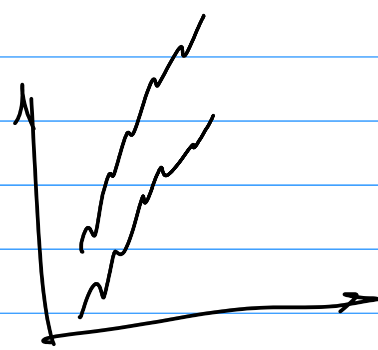




(*) TS: $GDP_t = \alpha_1 + \alpha_2 r_t + \varepsilon_t$

Why spurious? $f(t) = \alpha_1 + 0 + \varepsilon_t$

$\alpha_2 = 0$



α_2 biased

$\Rightarrow H_0: \alpha_2 = 0$

will be rejected (falsly) with higher prob.

□ Discuss the implications of the existence of cointegration between GDP_t and $rate_t$ on the short-run and long-run relationship of two variables.

$GDP - GDP^* \mid K, L$

Koyck Transf. ↗ PAM ↘ AEM

Co'int. relationship = LR relationship

$u_t \mid \pi_t - \pi_t^e$

(c) (20 points) Consider the following regression model, where ε_t has mean zero and is uncorrelated with GDP_{t-1}, GDP_{t-2}, $rate_t$, and $rate_{t-1}$, where $GDP_t \in I(1)$, $rate_t \in I(1)$ and are cointegrated:

ARDL(2, 1)

$$GDP_t = \alpha + \beta_1 GDP_{t-1} + \beta_2 GDP_{t-2} + \gamma_1 rate_t + \gamma_2 rate_{t-1} + \varepsilon_t \quad (1)$$

□ Derive the Error Correction Model (ECM) representation of the equation, and discuss the long-run equilibrium relation for GDP_t and $rate_t$.

$a_0 + a_1 \cdot GDP_{t-1} + a_2 rate_{t-1}$

ECM: $\Delta GDP_t \mid \underset{\uparrow \text{SR}}{\Delta rate_t}, \; \overset{\text{LR}\nearrow}{z_{t-1}}$

$z_{t-1} \updownarrow \hat{\varepsilon}_t$ – residuals from $GDP \mid rate$

ΔGDP_t =

$$GDP_t - GDP_{t-1} = \alpha + (\beta_1 - 1) GDP_{t-1} + \beta_2 GDP_{t-2} + \gamma_1 r_t + \gamma_2 r_{t-1} + \varepsilon_t$$

$$\Delta GDP_t = \alpha + \gamma_1 \Delta r_t + (\gamma_2 + \gamma_1) r_{t-1} + (\beta_1 - 1 + \beta_2) \cdot GDP_{t-1} + - \beta_2 \Delta GDP_{t-1} + \varepsilon_t$$

$(GDP_{t-1} - GDP_{t-2})$

SR

$$\Delta GDP_t = -\beta_2 \Delta GDP_{t-1} + \gamma_1 \Delta r_t +$$

$$\underbrace{(\beta_1+\beta_2-1)}_{=\,\lambda} \cdot \Big(\underbrace{GDP_{t-1} - \frac{\alpha}{\beta_1+\beta_2-1} - \frac{\gamma_1+\gamma_2}{\beta_1+\beta_2-1}\cdot z_{t-1}}_{=\, z_t / a_1}\Big) + \varepsilon_t$$

speed of adjustment