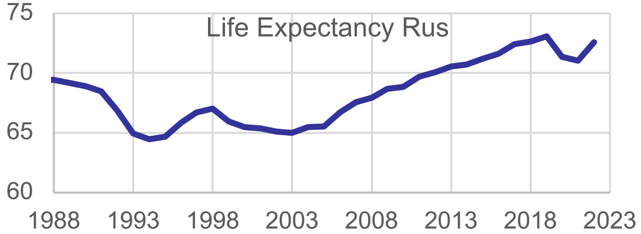
Elements of Econometrics.
Lecture 26.
Revision Time Series 3.

FCS, 2022-2023

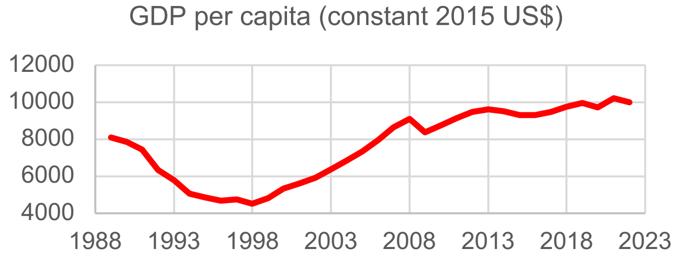
Example: the regression of Life Expectancy on the Real GDP Per Capita in Russia, 1988-2022



Dependent Variable: LIFE_EXP
Method: Least Squares
Date: 03/11/23 Time: 10:27
Sample (adjusted): 18 51
Included observations: 34 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	58.84583	0.953172	61.73683	0.0000
GDP_PC	0.001236	0.000121	10.21293	0.0000

R-squared	0.765230	Mean dependent var	68.29374
Adjusted R-squared	0.757894	S.D. dependent var	2.721508
S.E. of regression	1.339098	Akaike info criterion	3.478893
Sum squared resid	57.38190	Schwarz criterion	3.568678
Log likelihood	-57.14117	Hannan-Quinn criter.	3.509512
F-statistic	104.3039	Durbin-Watson stat	0.350227
Prob(F-statistic)	0.000000		



Dependent Variable: LIFE_EXP
Method: Least Squares
Date: 03/11/23 Time: 14:08
Sample: 19 51
Included observations: 33

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	13.13820	5.175146	2.538711	0.0166
GDP_PC	0.000371	0.000117	3.163232	0.0036
LIFE_EXP(-1)	0.767223	0.086434	8.876423	0.0000

R-squared	0.935161	Mean dependent var	68.26713
Adjusted R-squared	0.930838	S.D. dependent var	2.759211
S.E. of regression	0.725635	Akaike info criterion	2.282968
Sum squared resid	15.79637	Schwarz criterion	2.419014
Log likelihood	-34.66897	Hannan-Quinn criter.	2.328743
F-statistic	216.3416	Durbin-Watson stat	1.134956
Prob(F-statistic)	0.000000		

Potential problems with the model:

- The static regression (left) can be **spurious** if the series are **non-stationary**;
- The DW statistics (left) reflects severe autocorrelation, likely due to incorrect model specification;
- The AR(1;0) model (right) is better but still can be spurious if the series are non-stationary, and there is residuals autocorrelation as may be confirmed by Breusch-Godfrey test

Time series processes to be tested for stationarity:

$X_t = \beta_2 X_{t-1} + \varepsilon_t$ – autoregressive process of the order 1, AR(1)

$X_t = \beta_1 + \beta_2 X_{t-1} + \varepsilon_t$ – AR(1) with a constant

$X_t = X_{t-1} + \varepsilon_t$ - random walk

$X_t = \beta_1 + X_{t-1} + \varepsilon_t$ - random walk with drift (β_1 is the drift)

$X_t = \beta_1 + \beta_2 t + \varepsilon_t$ – deterministic trend

$X_t = \beta_1 + \beta_2 X_{t-1} + \ldots + \beta_{m+1} X_{t-m} + \varepsilon_t$ - *autoregressive process of the order m, AR(m)*

$X_t = \beta_1 + \varepsilon_t + \beta_2 \varepsilon_{t-1} + \ldots + \beta_{k+1} \varepsilon_{t-k}$ – moving average of the order k, MA(k)

$X_t = \beta_1 + \alpha_1 X_{t-1} + \ldots + \alpha_m X_{t-m} + \varepsilon_t + \beta_2 \varepsilon_{t-1} + \ldots + \beta_{k+1} \varepsilon_{t-k}$ - ARMA(m,k)

AR(1) process with $|\beta_2|<1$

Conditions for weak stationarity:

1. **The population mean of the distribution is independent of time.**

$$X_t = \beta_2 X_{t-1} + \varepsilon_t \qquad -1 < \beta_2 < 1$$

$$X_t = \beta_2^t X_0 + \beta_2^{t-1}\varepsilon_1 + \ldots + \beta_2^2 \varepsilon_{t-2} + \beta_2 \varepsilon_{t-1} + \varepsilon_t \qquad E(X_t) = \beta_2^t X_0 \to 0$$

2. **The variance of the distribution is independent of time.**

$$\operatorname{var}(X_t) = \operatorname{var}\left(\beta_2^t X_0 + \beta_2^{t-1}\varepsilon_1 + \ldots + \beta_2^2 \varepsilon_{t-2} + \beta_2 \varepsilon_{t-1} + \varepsilon_t\right)$$

$$= \left(\beta_2^{2(t-1)} + \ldots + \beta_2^4 + \beta_2^2 + 1\right)\sigma_\varepsilon^2 = \left(\frac{1-\beta_2^{2t}}{1-\beta_2^2}\right)\sigma_\varepsilon^2 \to \left(\frac{1}{1-\beta_2^2}\right)\sigma_\varepsilon^2$$

3. **The covariance between its values depends only on the distance in time.**

$$\operatorname{cov}(X_t, X_{t+s}) = \operatorname{cov}(X_t, \beta_2^s X_t) + \operatorname{cov}\left(X_t, \left[\beta_2^{s-1}\varepsilon_{t+1} + \ldots + \beta_2^2 \varepsilon_{t+s-2} + \beta_2 \varepsilon_{t+s-1} + \varepsilon_{t+s}\right]\right)$$

$$= \beta_2^s \operatorname{var}(X_t) - \textit{does not asymptotically depend on } t.$$

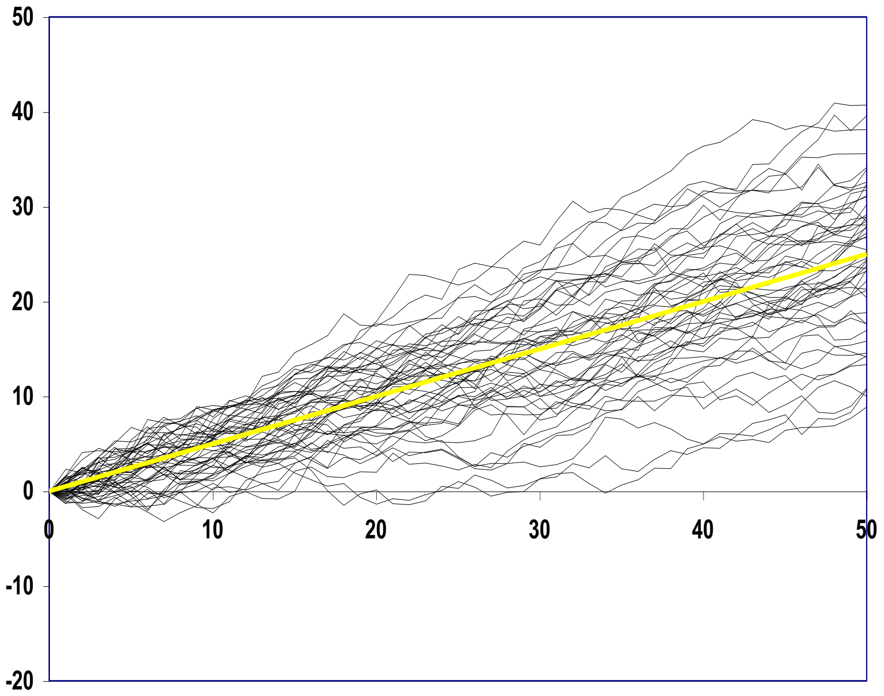
All 3 conditions hold. As $\beta_2^s \to 0$ if $s \to \infty$, the process is weakly persistent. The variance and covariance are unaffected by adding intercept β_1.

NONSTATIONARY PROCESSES: RANDOM WALK WITHOUT AND WITH DRIFT

Random walk $$X_t = X_{t-1} + \varepsilon_t$$

$$X_t = X_0 + \varepsilon_1 + ... + \varepsilon_{t-1} + \varepsilon_t$$

$$E(X_t) = X_0 + E(\varepsilon_1) + ... + E(\varepsilon_n) = X_0$$



Random walk with drift $$X_t = \beta_1 + X_{t-1} + \varepsilon_t$$

$$X_t = \beta_1 t + X_0 + \varepsilon_1 + ... + \varepsilon_{t-1} + \varepsilon_t$$

$$E(X_t) = X_0 + \beta_1 t$$

$$\sigma_{X_t}^2 = \mathrm{var}\left(X_0 + \varepsilon_1 + ... + \varepsilon_{t-1} + \varepsilon_t\right) = \mathrm{var}\left(\varepsilon_1 + ... + \varepsilon_{t-1} + \varepsilon_t\right) = \sigma_\varepsilon^2 + ... + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 = t\sigma_\varepsilon^2$$

The population variance of X_t is directly proportional to *t*, its distribution spreads out as *t* increases. The process is nonstationary.

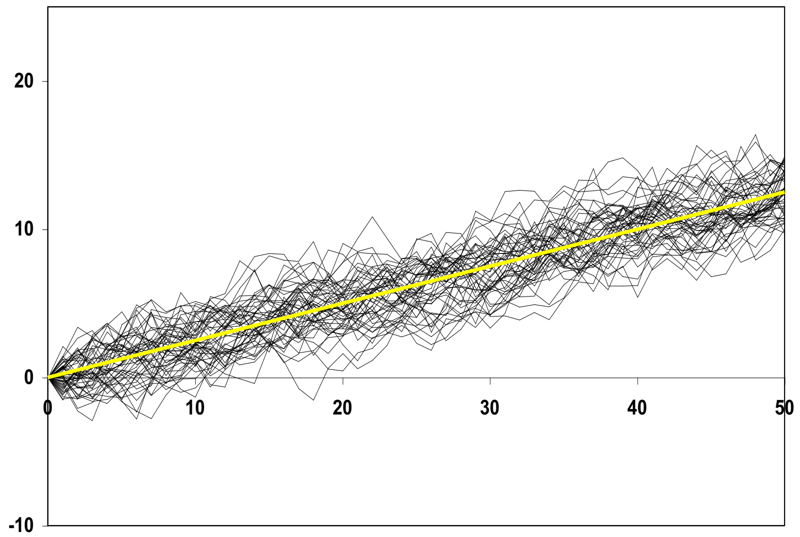
NONSTATIONARY PROCESSES: DETERMINISTIC TIME TREND

$$X_t = \beta_1 + \beta_2 t + \varepsilon_t$$

$$E(X_t) = \beta_1 + \beta_2 t \qquad \sigma_{X_t}^2 = \sigma_\varepsilon^2$$

The expected value changes in time, while the population variance does not. The process is nonstationary.

Time trend



More facts on Stationarity

Necessary condition for stationarity of autoregressive process of the order m (AR(m)) $X_t=\beta_1+\beta_2 X_{t-1}+...+\beta_{m+1}X_{t-m}+\varepsilon_t$ *is*

$$\sum_{i=2}^{m+1}\beta_i < 1.$$

Sufficient condition for stationarity of AR(m) process is $\sum_{i=2}^{m+1}|\beta_i| < 1.$

Any MA(k) *series* $X_t = \beta_1 + \varepsilon_t + \beta_2 \varepsilon_{t-1} + ... + \beta_{k+1} \varepsilon_{t-k}$ (moving averages of the order k) is stationary.

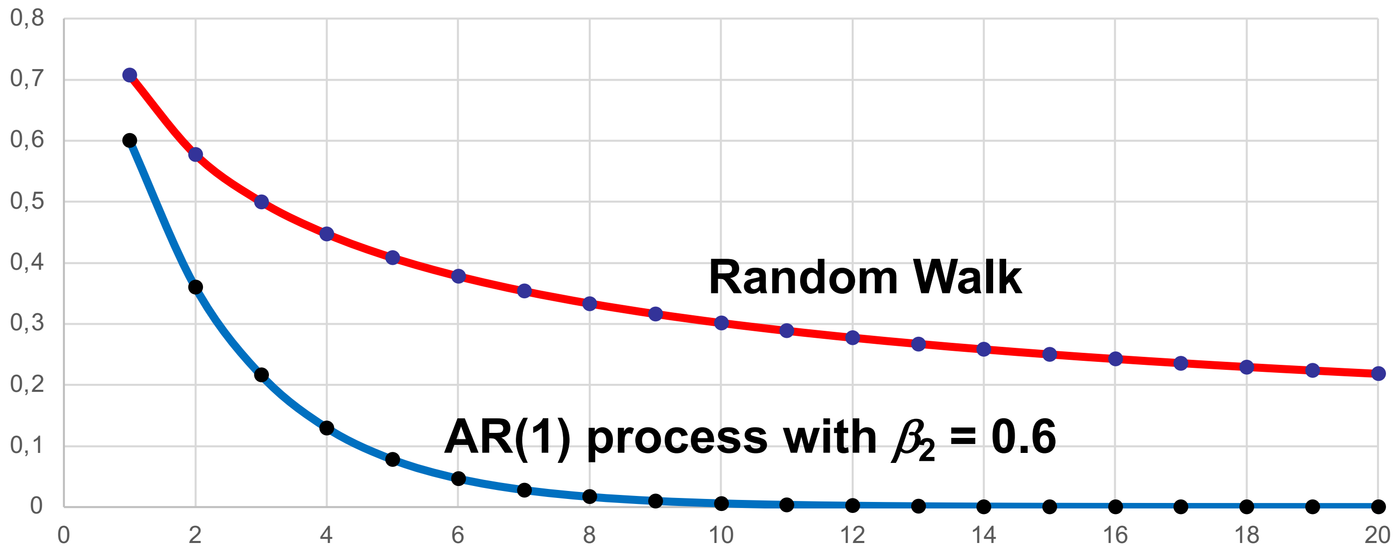
Stationarity of the ARMA(m,k) model

$X_t = \beta_1 + \alpha_1 X_{t-1} + ... + \alpha_m X_{t-m} + \varepsilon_t + \beta_2 \varepsilon_{t-1} + ... + \beta_{k+1} \varepsilon_{t-k}$ depends on its AR part only.

DETECTING NONSTATIONARITY: CORRELOGRAM

Autocorrelation function

$$\rho_k = \frac{\mathrm{E}((X_t - \mu_X)(X_{t+k} - \mu_X))}{\sqrt{\mathrm{E}((X_t - \mu_X)^2)\mathrm{E}((X_{t+k} - \mu_X)^2)}} \quad \text{for } k = 1, \ldots$$



AR(1) process with β_2 = 0.6

$$X_{t+i} = \beta_2 X_{t+i-1} + \varepsilon_{t+i}$$

$$\rho_k = \beta_2^k = 0.6^k$$

Random Walk

$$X_{t+i} = X_{t+i-1} + \varepsilon_{t+i}$$

$$\rho_k = \sqrt{t/(t+k)}$$

$$\rho_k = \sqrt{1/(1+k)} \quad (if \;\; t = 1)$$

Correlograms of an AR(1) process with β_2 = 0.6 and of the Random Walk for k=1,…,20.

FORMAL TESTS FOR NONSTATIONARITY

The model $Y_t = \beta_1 + \beta_2 Y_{t-1} + \delta t + \varepsilon_t$

Transformed: $Y_t - Y_{t-1} = \beta_1 + (\beta_2 - 1)Y_{t-1} + \delta t + \varepsilon_t$

Alternatives $\beta_2 = 1$ **or** $-1 < \beta_2 < 1$

$\delta = 0$ **or** $\delta \neq 0$

Case (a): Stationary AR(1)	$\beta_1 = *$	$\lvert\beta_2\rvert < 1$	$\delta = 0$
Case (b): Random Walk	$\beta_1 = 0$	$\beta_2 = 1$	$\delta = 0$
Case (c): Random Walk with Drift	$\beta_1 \neq 0$	$\beta_2 = 1$	$\delta = 0$
Case (d): Stationary AR(1) around Trend	$\beta_1 = *$	$\lvert\beta_2\rvert < 1$	$\delta \neq 0$
Case (e): Random Walk around Trend	$\beta_1 = *$	$\beta_2 = 1$	$\delta \neq 0$

There are five cases. β_1 = * means β_1 is unrestricted.
Cases (a) and (b) do not include time trend, cases (c) and (d) include trend.

TESTS OF NONSTATIONARITY

Augmented Dickey–Fuller tests

General autoregressive process

Necessary condition for stationarity: $\sum_{i=2}^{p+1} \beta_i < 1.$

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \ldots + \beta_{p+1} Y_{t-p} + \delta t + \varepsilon_t$$

$$Y_t - Y_{t-1} = \Delta Y_t = \beta_1 + \left(\beta_2^* - 1\right) Y_{t-1} + \beta_3^* \Delta Y_{t-1} + \ldots + \beta_{p+1}^* \Delta Y_{t-p} + \delta t + \varepsilon_t$$

$$\boldsymbol{\beta_2^* = \beta_2 + \ldots + \beta_{p+1}}$$

Under the null hypothesis of non-explosive nonstationarity, the test statistics $T(b_2^* - 1)$, *t*, and *F* asymptotically have the distributions with the critical values available in /Dougherty/.

In practice, the *t* test is particularly popular and is generally known as the augmented Dickey–Fuller (ADF) test.

The number of lags can be set by the user or chosen with an Information Criteria, Schwarz (SIC) or Akaike (AIC),

Time Series Data: Order of Integration

1. Weakly dependent (stationary) time series are integrated of order zero (= I(0))
2. If a time series has to be differenced one time in order to obtain a weakly dependent series, it is called integrated of order one (= I(1))
3. If a time series has to be differenced *m* times in order to obtain a weakly dependent series, it is called integrated of order *m* (= I(m))

Examples for I(1) processes

$$y_t = y_{t-1} + e_t \;\Rightarrow\; \Delta y_t = y_t - y_{t-1} = e_t \leftarrow$$

After differencing, the resulting series are weakly dependent (because e_t is weakly dependent).

$$\Delta \log(y_t) \approx (y_t - y_{t-1})/y_{t-1}$$

Differencing of the log function provides the growth rates (approx.) which are often weakly dependent (stationary).

Differencing is often a way to achieve weak dependence of economic time series with clear interpretation of the indicators.

ADF TESTS OF NONSTATIONARITY, AN EXAMPLE:

REAL GDP PER CAPITA IN RUSIA AND ITS FIRT DIFFERENCE

Null Hypothesis: GDP_PC has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=8)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.724361	0.8665
Test critical values:	1% level	-2.636901	
	5% level	-1.951332	
	10% level	-1.610747	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GDP_PC)
Method: Least Squares
Date: 03/18/23 Time: 20:02
Sample: 19 51
Included observations: 33

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GDP_PC(-1)	0.007015	0.009684	0.724361	0.4741

R-squared	-0.001662	Mean dependent var	57.44915
Adjusted R-squared	-0.001662	S.D. dependent var	433.8014
S.E. of regression	434.1618	Akaike info criterion	15.01455
Sum squared resid	6031887.	Schwarz criterion	15.05989
Log likelihood	-246.7400	Hannan-Quinn criter.	15.02980
Durbin-Watson stat	0.960181		

Null Hypothesis: DGDP_PC has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=8)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.110847	0.0029
Test critical values:	1% level	-2.639210	
	5% level	-1.951687	
	10% level	-1.610579	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DGDP_PC)
Method: Least Squares
Date: 03/18/23 Time: 20:01
Sample (adjusted): 20 51
Included observations: 32 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DGDP_PC(-1)	-0.474187	0.152430	-3.110847	0.0040

R-squared	0.237898	Mean dependent var	1.307313
Adjusted R-squared	0.237898	S.D. dependent var	430.7110
S.E. of regression	376.0040	Akaike info criterion	14.72783
Sum squared resid	4382749.	Schwarz criterion	14.77363
Log likelihood	-234.6452	Hannan-Quinn criter.	14.74301
Durbin-Watson stat	2.202234		

The GDP_PC variable is non-stationary, while its first difference DGDP_PC is stationary. Hence, the series is I(1) (integrated of the order 1).

COINTEGRATION

The order of linear combination of series $X_1, \ldots, X_n$ is usually equal to the maximum order of integration among the series.

Two or more series (with the same order of integration k≥1) are called **cointegrated** if there exists their **stationary** linear combination.

$$Y_t = \beta_1 + \beta_2 X_{2t} + \ldots + \beta_k X_{kt} + u_t \qquad u_t = Y_t - \beta_1 - \beta_2 X_{2t} - \ldots - \beta_k X_{kt}$$

$$e_t = Y_t - b_1 - b_2 X_{2t} - \ldots - b_k X_{kt} - test\ for\ unit\ root$$

If there exists linear relationship between variables Y_t, X_{2t}, …, X_{kt}, the disturbance term u_t is measuring the deviation between the terms in the model. The test is indirect since done with e_i which are more close to stationary than u_i. Hence the critical values are lower.

Asymptotic Critical Values of the Dickey-Fuller Statistic for a Cointegrating Relationship with Two Variables

Regression equation contains:	5%	1%
Constant, but no trend	–3.34	–3.90
Constant and trend	–3.78	–4.32

In the model

$$LIFE\_EXP_t = \lambda_0 + \lambda_1 LIFE\_EXP_{t-1} + \lambda_2 GDP\_PC_t + \varepsilon_t$$

for Russia (see above) the Dickey-Fuller statistics is -3.43<-3.34, and hence we reject the H_0 of non-stationarity of the error term at 5% level.
We have found the cointegrating relationship.

Fitting Models with Nonstationary Time Series: Detrending

In the models with variables which include time trends, removal of the trends, or detrending, allows to avoid getting spurious regressions.

Detrending of each variable in the model is equivalent to including the time trend as an explanatory variable.

Economic indicators often behave not as series including time trends, but as random walks.

If you detrend a series which is a random walk with a drift, then its variance increases proportionally to time, the series does not become stationary, and hence the problem of spurious regressions is not resolved.

Fitting Models with Nonstationary Time Series: Differencing

If having random walk time series, differencing is a procedure which can be applied for making it stationary:

subtracting $Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1}$ from $Y_t = \beta_1 + \beta_2 X_t + u_t$, we get $\Delta Y_t = \beta_2 \Delta X_t + \Delta u_t$

The series ΔY_t and ΔX_t are stationary, and the coefficient β_2 can be estimated from this model.

The new disturbance term Δu_t is subject to autocorrelation, and appropriate remedial measures should be applied. Only in the case of severe autocorrelation of u_t in the initial model (ρ is close to 1) differencing helps to reduce autocorrelation.

If Y_t and X_t are unrelated I(1) processes, absence of their relationship will be revealed in the differenced model, so the problem of spurious regressions will be resolved.

Shortcomings of the differenced model:

- constant disappears
- only short-run relationships can be investigated since in the long-run equilibrium $\Delta Y = \Delta X = 0$

Differencing: the Model for Life Expectancy and Real GDP per capita, Russia, 1989-2022

The variables $LIFE\_EXP_t$ and GDP_PC_t are I(1). Then we apply the differencing:

subtracting $LIFE\_EXP_{t-1} = \beta_1 + \beta_2 GDP\_PC_{t-1} + u_{t-1}$ from $LIFE\_EXP_t = \beta_1 + \beta_2 GDP\_PC_t + u_t$,
we get the model $\Delta LIFE\_EXP_t = \beta_2 \Delta GDP\_PC_t + \Delta u_t$ with stationary variables, and we can estimate β_2 from it:

Dependent Variable: DLIFE_EXP
Method: Least Squares
Date: 03/18/23 Time: 18:43
Sample: 19 51
Included observations: 33

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DGDP_PC	0.000706	0.000306	2.306366	0.0277

R-squared	0.128044	Mean dependent var	0.103888
Adjusted R-squared	0.128044	S.D. dependent var	0.811508
S.E. of regression	0.757775	Akaike info criterion	2.312974
Sum squared resid	18.37514	Schwarz criterion	2.358323
Log likelihood	-37.16408	Hannan-Quinn criter.	2.328233
Durbin-Watson stat	1.335653		

Error Correction Model

$$Y_t^* = \alpha_1 + \alpha_2 X_t$$

$$\Delta Y_t = \lambda\left(Y_{t-1}^* - Y_{t-1}\right) + \delta\Delta X_t + u_t = \lambda\left(\alpha_1 + \alpha_2 X_{t-1} - Y_{t-1}\right) + \delta\left(X_t - X_{t-1}\right) + u_t =$$
$$= \lambda\alpha_1 + \delta X_t + \left(\lambda\alpha_2 - \delta\right) X_{t-1} - \lambda Y_{t-1} + u_t$$

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \beta_3 X_t + \beta_4 X_{t-1} + u_t$$

$$\beta_1 = \lambda\alpha_1 \qquad \beta_3 = \delta$$

$$\beta_2 = 1 - \lambda \qquad \beta_4 = \lambda\alpha_2 - \delta$$

Y* is a desirable (appropriate) unobserved value of Y. In the short run, $\Delta Y_t = Y_t - Y_{t-1}$, is determined by two components: closing the discrepancy between its previous "appropriate" and actual values, $Y^*_{t-1} - Y_{t-1}$, and a straightforward response to ΔX_t. This is ADL(1,1) model.

Fitting Models with Nonstationary Time Series: Error Correction Model.

ADL(1,1) model $$Y_t = \beta_1 + \beta_2 Y_{t-1} + \beta_3 X_t + \beta_4 X_{t-1} + \varepsilon_t$$

In equilibrium $$\bar{Y} = \beta_1 + \beta_2 \bar{Y} + \beta_3 \bar{X} + \beta_4 \bar{X}.$$

Hence $$\bar{Y} = \frac{\beta_1}{1-\beta_2} + \frac{\beta_3 + \beta_4}{1-\beta_2} \bar{X}$$

Cointegrating relationship $$Y_t = \frac{\beta_1}{1-\beta_2} + \frac{\beta_3 + \beta_4}{1-\beta_2} X_t$$

Cointegrating relationship describes the long-run effects. We will construct a model combining short-run and long-run dynamics.

Fitting Models with Nonstationary Time Series: Error Correction Model.

ADL(1,1) model $$Y_t = \beta_1 + \beta_2 Y_{t-1} + \beta_3 X_t + \beta_4 X_{t-1} + \varepsilon_t$$

Cointegrating relationship $$Y_t = \frac{\beta_1}{1-\beta_2} + \frac{\beta_3 + \beta_4}{1-\beta_2} X_t$$

$$\begin{aligned}
Y_t - Y_{t-1} &= \beta_1 + (\beta_2 - 1)Y_{t-1} + \beta_3 X_t + \beta_4 X_{t-1} + \varepsilon_t \\
&= \beta_1 + (\beta_2 - 1)Y_{t-1} + \beta_3 X_t - \beta_3 X_{t-1} + \beta_3 X_{t-1} + \beta_4 X_{t-1} + \varepsilon_t \\
&= (\beta_2 - 1)\left(Y_{t-1} - \frac{\beta_1}{1-\beta_2} - \frac{\beta_3 + \beta_4}{1-\beta_2} X_{t-1} \right) + \beta_3 (X_t - X_{t-1}) + \varepsilon_t
\end{aligned}$$

$$\Delta Y_t = (\beta_2 - 1)\left(Y_{t-1} - \frac{\beta_1}{1-\beta_2} - \frac{\beta_3 + \beta_4}{1-\beta_2} X_{t-1} \right) + \beta_3 \Delta X_t + \varepsilon_t .$$

The ADL(1,1) relationship may be rewritten to incorporate the cointegrating relationship by subtracting Y_{t-1} from both sides, subtracting $\beta_3 X_{t-1}$ from the right side and adding it back again, and rearranging. The Error Correction Model is obtained.

Fitting Models with Nonstationary Time Series: Error Correction Model.

$$\Delta Y_t = (\beta_2 - 1)\left(Y_{t-1} - \frac{\beta_1}{1-\beta_2} - \frac{\beta_3 + \beta_4}{1-\beta_2} X_{t-1} \right) + \beta_3 \Delta X_t + \varepsilon_t .$$

The Error Correction Model states that the change in *Y* in any period will be governed by the change in *X* and the discrepancy between Y_{t-1} and the value predicted by the cointegrating relationship.

If *Y* and *X* are *I(1)* and the cointegrating relationship found, then ΔY_t, ΔX_t, and the error correction term are *I(0).*

Problem of estimation: the coefficients for calculating the error correction term are not known. Solution: Engle-Granger two step procedure.

Error Correction Model Estimation: Engle-Granger Two Step Procedure.

$$\Delta Y_t = (\beta_2 - 1)\left(Y_{t-1} - \frac{\beta_1}{1-\beta_2} - \frac{\beta_3 + \beta_4}{1-\beta_2} X_{t-1} \right) + \beta_3 \Delta X_t + \varepsilon_t .$$

If *Y* and *X* are *I(1)*, ΔY_t, ΔX_t to be used in further modeling.

Step 1: the cointegrating relationship is estimated, its residuals tested for stationarity. If the hypothesis of nonstationarity is rejected, go to Step 2.

Step 2: estimate the Error Correction Model using the residuals for cointegrating relationship.

Engle and Granger: asymptotically, in the cointegrating term the estimates of β*s* can be used instead of true values, and hence the residuals from the cointegrating regression can be further used.

Granger Causality

Granger test for causality (1969): regress current value of *X* on the past values of *X* and *Y*, and the current value of *Y* on the past values of *X* and *Y*:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + ... + \alpha_m Y_{t-m} + \beta_1 X_{t-1} + ... + \beta_m X_{t-m} + \varepsilon_t \quad (1)$$

$$X_t = \alpha_0 + \alpha_1 X_{t-1} + ... + \alpha_m X_{t-m} + \beta_1 Y_{t-1} + ... + \beta_m Y_{t-m} + \varepsilon_t \quad (2)$$

Test statistic: F-statistic for H_0: $\beta_1=\beta_2=\ldots=\beta_m=0$. If in (1) *Ho* is rejected, then X Granger causes Y.

If in (2) *Ho* is rejected, then Y Granger causes X.

Independence, or Unidirectional or bilateral Granger causality is possible.

EViews: select the group of series, indicate “Granger Causality” option in “View” menu. Then EViews runs bivariate regressions for all possible pairs of series in the group selected. The reported F-statistics are the Wald test statistics for H_0 for each equation. For the variables $LIFE\_EXP_t$ and GDP_PC_t (Russia, 1989-2022) the result of Granger causality test is:

Pairwise Granger Causality Tests
Date: 03/18/23 Time: 20:59
Sample: 19 51
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
LIFE_EXP does not Granger Cause GDP_PC	32	0.18243	0.8343
GDP_PC does not Granger Cause LIFE_EXP		3.75582	0.0364

Vector Autoregression (VAR)

Vector autoregression (VAR) is used for systems of interrelated time series. In the VAR model every endogenous variable in the system is a function of the lagged values of all endogenous variables in the system, and of the values of exogenous variables.

VAR model:

$$Y_t = A_1 Y_{t-1} + \ldots + A_m Y_{t-m} + BX_t + \varepsilon_t$$

where Y is a vector of endogenous variables, X is a vector of exogenous variables, A and B are matrices of coefficients to be estimated, and ε is a vector of innovations (uncorrelated with their own lagged values and with all of the right-hand side variables).

For the model relating the variables $LIFE\_EXP_t$ and GDP_PC_t for Russia, 1989-2022, the result of VAR model application is:

Vector Autoregression Estimates
Date: 03/18/23 Time: 21:06
Sample (adjusted): 20 51
Included observations: 32 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LIFE_EXP	GDP_PC
LIFE_EXP(-1)	1.191847	59.61501
	(0.18766)	(107.230)
	[6.35118]	[0.55595]
LIFE_EXP(-2)	-0.387664	-61.44602
	(0.18001)	(102.858)
	[-2.15361]	[-0.59739]
GDP_PC(-1)	0.000631	1.470569
	(0.00031)	(0.17871)
	[2.01752]	[8.22866]
GDP_PC(-2)	-0.000367	-0.502011
	(0.00034)	(0.19560)
	[-1.07220]	[-2.56649]
C	11.41542	392.8918
	(5.91044)	(3377.31)
	[1.93140]	[0.11633]