Panel Data

y_i — cross-sections

y_t — TS

y_{it} — Panel Data

$\mu_i = 0$ Pooled OLS $\quad y_{it} = \alpha + \beta X_{it} + \varepsilon_{it}$

$cov(\mu_i, X) \neq 0$ Fixed Effect $\quad y_{it} = \cancel{\alpha} + \beta X_{it} + \mu_i + \varepsilon_{it}$

↳ LSDV, FD, Within-Est.

μ_i: unobserved heterogeneity of i

$cov(\mu_i, X) = 0$ Random Effect $\quad y_{it} = \underbrace{\alpha + \beta X_{it}}_{OLS} + \underbrace{\mu_i + \varepsilon_{it}}_{v_{it}}$

↳ GLS

* $E(\mu_i | X) = E(\mu_i) = 0$

	FE	RE	OLS	All
$\mu_i = 0$	not eff	not eff.	V	cons., unbiased
$cov(\mu_i, X) = 0$	not eff.	V	not eff.	cons., unbiased
$cov(\mu_i, X) \neq 0$	V	biased & inconsistent		

FE

① LSDV

$$y_{it} = \beta X_{it} + \mu_1 D_i^{(1)} + \dots + \mu_N \cdot D_i^{(N)} + \varepsilon_{it}$$

I Pooled OLS vs FE :

(FE) $$y_{it}^{UR} = \beta X_{it} + \alpha + \mu_2 D_i^{(2)} + \dots + \mu_N D_i^{(N)} + \varepsilon_{it}$$

$H_0 : \mu_2 = \dots = \mu_N = 0 \rightarrow$ F-test

(Pooled) $$y_{it}^{R} = \beta X_{it} + \alpha + \varepsilon_{it}$$

II Pooled OLS vs RE:

Breusch - Pagan Test :

$$\nu_{it} = \mu_i + \varepsilon_{it}$$

$$Var(\nu_{it}) = \sigma_\mu^2 + \sigma_\varepsilon^2$$

$H_0 : \sigma_\mu^2 = 0$

$$BP = \frac{NT}{2 \cdot (T-1)} \cdot \left(\frac{\sum_i \left(\sum_t \hat{\varepsilon}_{it} \right)^2}{\sum_i \sum_t \hat{\varepsilon}_{it}^2} - 1 \right)^2 \overset{H_0}{\sim} \chi_1$$

III RE vs FE

Hausman Test

H_0: RE – consistent, $cov(\mu_i, X) = 0$

H_a: RE – inconsistent, $cov(\mu_i, X) \neq 0 \Rightarrow$ FE

$$(\hat{\beta}_{RE} - \hat{\beta}_{FE})^T \left(\hat{Var}(\hat{\beta}_{RE}) - \hat{Var}(\hat{\beta}_{FE}) \right)^{-1} (\hat{\beta}_{RE} - \hat{\beta}_{FE}) \sim \chi^2_k$$

2 FD

$t = 1, 2$

$$\begin{array}{r} y_{i2} = \alpha + \beta X_{i2} + \varepsilon_{i2} + \mu_i \\ - \quad y_{i1} = \alpha + \beta X_{i1} + \varepsilon_{i1} + \mu_i \\ \hline \Delta y_i = \beta \Delta X_i + \Delta \varepsilon_i \end{array}$$

$$\hat{\beta}_{FD} = \frac{\sum \Delta X_i \Delta y_i}{\sum (\Delta X_i)^2}$$

$t = 2 \Rightarrow$ FD identical to LSDV

RE using FGLS

Ω - cov. matrix of error

$$\operatorname{cov}(\nu_{it}, \nu_{jp}) = \begin{cases} i=j,\ t=p & \operatorname{Var}(\nu_{it}) = \sigma_\mu^2 + \sigma_\varepsilon^2 \\ i=j,\ t\neq p & \operatorname{cov}(\mu_i + \varepsilon_{it}, \mu_i + \varepsilon_{ip}) \end{cases}$$

$\nu_{it} = \mu_i + \varepsilon_{it}$

$= \sigma_\mu^2$

$$\Omega = \begin{pmatrix} \sigma_\varepsilon^2 + \sigma_\mu^2 & \sigma_\mu^2 & 0 & & 0 & \\ \sigma_\mu^2 & \sigma_\varepsilon^2 + \sigma_\mu^2 & & & & \\ & & \sigma_\varepsilon^2 + \sigma_\mu^2 & \sigma_\mu^2 & & \\ & 0 & \sigma_\mu^2 & \sigma_\varepsilon^2 + \sigma_\mu^2 & 0 & \\ & & & & \sigma_\varepsilon^2 + \sigma_\mu^2 & \sigma_\mu^2 \\ & 0 & & 0 & \sigma_\mu^2 & \sigma_\varepsilon^2 + \sigma_\mu^2 \end{pmatrix}$$

$$\hat{\beta}_{GLS} = (X' \hat{\Omega}^{-1} X)^{-1} X' \hat{\Omega} y$$

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'y$$

③ Within Estimates

$$y_{i1} = \alpha + \beta x_{i1} + \varepsilon_{i1} + \mu_i$$

$$\vdots$$

$$y_{iT} = \alpha + \beta x_{iT} + \varepsilon_{iT} + \mu_i$$

$$\bar{y}_i = \alpha + \beta \bar{x}_i + \bar{\varepsilon}_i + \mu_i$$

$$\underbrace{(y_{it} - \bar{y})}_{= \tilde{y}_{it}} = \beta \underbrace{(x_{it} - \bar{x})}_{= \tilde{x}_{it}} + (\varepsilon_{it} - \bar{\varepsilon})$$

$$\hat{\beta}_{\text{within}} = \frac{\sum_i \sum_t \tilde{x}_{it} \tilde{y}_{it}}{\sum_i \sum_t (\tilde{x}_{it})^2}$$