Panel Data

Panel Data:

y_{it} ↗ i - object
↘ t - time

- balanced / unbalanced

Cross-Sections: y_i
t - fixed

Time Series: y_t
i - fixed

$i = \overline{1, N}$

Pooled (OLS) $\quad y_{it} = \alpha + \beta X_{it} + \varepsilon_{it}$

Fixed Effect $\quad y_{it} = \cancel{\alpha} + \mu_i + \beta x_{it} + \varepsilon_{it}$

↳ LSDV, FD, Within-Est. $\quad$ unobserved specifics of i

Random Effect $\quad y_{it} = \alpha + \beta X_{it} + \underbrace{\mu_i + \varepsilon_{it}}_{v_i}$

↳ GLS $\quad E(\mu_i | X) = E(\mu_i) = 0$

	FE	RE	Pool	All
$\mu_i = 0$	not efficient		V	unbiased and cons.
$cov(\mu_i, X_j) = 0$	not eff.	V	not eff.	unbiased and cons
$cov(\mu_i, X_j) \neq 0$	V	biased and incons.		

① FE vs OLS

LSDV: $y_{it} = \alpha + \sum_{i=2}^{N} \mu_i \cdot D_i + \beta X_{it} + \varepsilon_{it}$

$H_0: \mu_2 = \dots = \mu_N = 0$

using F-test

② RE vs OLS

Breusch - Pagan test

$H_0: \sigma_{\mu_i} = 0 \Rightarrow$ obs are homogeneous

$$\frac{n \cdot T}{2(T-1)} \cdot \left(\frac{\sum_{i=1}^{n} \left(\sum_{t=1}^{T} e_{it} \right)^2}{\sum_{i=1}^{n} \sum_{t=1}^{T} e_{it}^2} - 1 \right)^2 \overset{H_0}{\sim} \chi_1$$

③ RE vs FE

H_0: RE - consistent, $\operatorname{cov}(\mu_i, X_j) = 0$

H_a: RE - inconsistent, $\operatorname{cov}(\mu_i, X_j) \neq 0$

$\Rightarrow$ FE - consistent

$$(\hat{\beta}_{FE} - \hat{\beta}_{RE})' \left(\hat{V}(\hat{\beta}_{FE}) - \hat{V}(\hat{\beta}_{RE}) \right)^{-1} (\hat{\beta}_{FE} - \hat{\beta}_{RE}) \overset{H_0}{\sim} \chi_k^2$$

FE Estimation

① LSDV

$$y_{it} = \mu_1 D_i^{(1)} + \dots + \mu_N \cdot D_i^{(N)} + \beta X_{it} + \varepsilon_{it}$$

$$y_{it} = \alpha + \mu_2 D_i^{(2)} + \dots + \mu_N \cdot D_i^{(N)} + \beta X_{it} + \varepsilon_{it}$$

$-(N-1)$ d.o.f.

② FD

$t = 1, 2$

$$
\begin{aligned}
& y_{i2} = \beta X_{i2} + \cancel{\mu_i} + \varepsilon_{i2} \\
- \; & y_{i1} = \beta X_{i1} + \cancel{\mu_i} + \varepsilon_{i1} \\
\hline
& \Delta y_i = \beta \Delta X_i + u_i
\end{aligned}
$$

$$\hat{\beta}_{FD} = \frac{\sum \Delta X_i \Delta y_i}{\sum (\Delta X_i)^2}$$

if $T=2$ $\Rightarrow$ identical to LSDV

③ Within - transformation

$$
\begin{aligned}
+ \; & y_{i1} = \beta X_{i1} + \mu_i + \varepsilon_{i1} \\
& \dots \\
+ \; & y_{iT} = \beta X_{iT} + \mu_i + \varepsilon_{iT} \\
\hline
\end{aligned}
\qquad : T
$$

$$\bar{y}_i = \beta \bar{x}_i + \mu_i + \bar{\varepsilon}_i$$

$$(y_{it} - \bar{y}_i) = \beta(x_{it} - \bar{x}_i) + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

Random effect using FGLS

$$v_{it} = \varepsilon_{it} + \mu_i$$

s.t.: $\text{var}(\varepsilon_{it}) = \sigma_\varepsilon^2$

$$\text{var}(\mu_i) = \sigma_\mu^2$$

$$\text{Cov}(v_{it}, v_{jp}) = \begin{cases} i \neq j, \ t = p \\ i = j, \ t \neq p \end{cases}$$

$\text{Var}(v_{it}) = \sigma_\mu^2 + \sigma_\varepsilon^2$

$\text{Cov}(\varepsilon_{it} + \mu_i, \varepsilon_{jt} + \mu_i) = \sigma_\mu^2$

$$\Omega = \begin{pmatrix} \sigma_\mu^2 + \sigma_\varepsilon^2 & \sigma_\mu^2 & 0 & & 0 & \\ \sigma_\mu^2 & \sigma_\mu^2 + \sigma_\varepsilon^2 & & & & \\ & & \sigma_\mu^2 + \sigma_\varepsilon^2 & \sigma_\mu^2 & & \\ & 0 & \sigma_\mu^2 & \sigma_\mu^2 + \sigma_\varepsilon^2 & 0 & \\ & & & & \sigma_\mu^2 + \sigma_\varepsilon^2 & \sigma_\mu^2 \\ & 0 & 0 & & \sigma_\mu^2 & \sigma_\mu^2 + \sigma_\varepsilon^2 \end{pmatrix}$$

$$\hat{\beta}_{RE} = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}y \qquad \hat{\beta}_{OLS} = (X'X)^{-1}X'y$$