Revision Seminar 1

Model Exam 1
SECTION A

Answer **ALL** questions from this section (questions **1-5**). Each question in this section bears 8 marks

The formulations of the tasks are given as they were published in the texts of the UoL exams

Question 1. Consider a model $Y_i = \alpha + \beta X_i + u_i;\ \ i = 1,...,6$, where $E(u_i) = 0$; $E(u_i^2) = \sigma^2$ and $E(u_i u_j) = 0$ if $i \neq j$. The observations of X_i's are $X_1 = 1,\ X_1 = 2,\ X_1 = 3,\ X_1 = 4,\ X_1 = 5,\ X_1 = 6$.

The OLS estimator of β is $\hat{\beta}$ and $V(\hat{\beta}) = \dfrac{\sigma^2}{17.5}$. An alternative estimator of β is $\tilde{\beta} = \dfrac{1}{8}[Y_6 + Y_5 - Y_2 - Y_1]$.

Compare the sampling variance of $\tilde{\beta}$ with that of $\hat{\beta}$.

Question 2. Let the regression equation be $y_t = \beta x_t + u_t;\ \ t = 1, 2,...,T$, where $E(u_t) = 0$; $E(u_t^2) = \sigma^2$ and $E(u_s u_t) = 0$ if $s \neq t$.Obtain the ordinary least squares estimator of β. Show that the OLS estimator of β is linear and unbiased.

Question 3. Unbiasedness is not the only criterion by which to judge a statistical estimator. What other criteria are used and why are they used? Explain fully.

Question 4. Show that the function $Y = \beta_1 X^{\beta_2}$ has constant elasticity. Given data on *X* and *Y* explain how you would attempt to estimate the elasticity.

Question 5. Show that $\text{var}(Y_t) = \text{var}(\hat{Y}_t) + \text{var}(\hat{u}_t)$ for the simple linear model $Y_t = \alpha + \beta X_t + u_t$ where b_0 and b_1 are ordinary least squares estimates of β_0 and β_1, and $\hat{u}_t = Y_t - b_0 - b_1 X_t$. Explain how this expression is related to the properties of R^2, the coefficient of determination.

SECTION B

Question 6. We are interested in investigating the factors governing the precision of regression coefficients. Consider the model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$$

with OLS parameter estimates $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$. Under the Gauss—Markov assumptions, we have

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma_\varepsilon^2}{\sum_{i=1}^{n}(X_{2i} - \overline{X}_2)^2} \times \frac{1}{1 - r_{X_2 X_3}^2},$$

where σ_e^2 is the variance of ε and $r_{X_2 X_3}$ is the sample correlation between X_2 and X_3.

(a) Provide four factors that help with obtaining more precise parameter estimates for, say, $\hat{\beta}_2$. (4 marks)

(b) Assume that the true value of $\beta_3 = 0$, so that the above model includes an irrelevant variable. Discuss the effect of including this irrelevant variable on the unbiasedness and precision of $\hat{\beta}_2$. (4 marks)

(c) In light of your answer to (a) and (b), discuss the concept of near multicollinearity. What consequences does its presence have when considering single and joint significance testing of our slope parameters? (4 marks)

Question 7.

Consider the OLS estimator for β in the linear regression model:

$$Y_i = \beta X_i + \varepsilon_i, \ i = 1, ..., n,$$

where $\{(Y_i, X_i)\}_{i=1}^n$ form an i.i.d. sample from a population and the errors are drawn from an <u>unknown</u> distribution with mean zero and variance σ^2. You are told that X_i and ε_i are <u>uncorrelated</u> (not necessarily independent therefore!).

(a) (4 marks) Discuss the importance of convergence in probability.

(b) (4 marks) Discuss the importance of convergence in distribution.

Question 8.

Consider the human capital earnings function given by:

$$earnings_i = \beta_0 + \beta_1 educ_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i, \ i = 1, ..., n,$$

where *earnings* denotes the hourly earnings of an individual and *educ* and *exper* denote the years of schooling and experience, respectively. We assume we have obtained a random sample $\{(earnings_i, educ_i, exper_i)\}_{i=1}^n$ from the population. The errors $\{u_i\}_{i=1}^n$ are i.i.d. normal random variables with zero mean and variance σ^2. We assume independence between the errors and regressors (i.e., we ignore the usual ability bias problem).

(a) (2 marks) Discuss the rationale for including both *exper* and $exper^2$ in this model. In your answer be explicit about the expected signs for β_2 and β_3.

(b) (2 marks) Discuss <u>briefly</u> how would you test that *exper* has a significant effect on *earnings*?

(c) (4 marks) Provide a clear interpretation of the parameter β_1. You are told that you can obtain the estimator for β_1 by running the regression:

$$earnings_i^* = \beta_1 educ_i^* + e_i, \ i = 1, ..., n,$$

where $earnings_i^*$ and $educ_i^*$ are obtained from running a regression of *earnings* (and *educ*) on an intercept, *exper* and $exper^2$. Explain this statement.