DLM

ARDL$(0,\infty)$ → Geometrically → PAM, AEM, ECM

DL(∞) → Polynomially

① PAM :

$$y_t^* = \alpha + \beta X_t + \varepsilon_t$$

y_t^* – unobserved (equlium) level of Y

PA hypothesis: $y_t - y_{t-1} = (1-\lambda)(y_t^* - y_{t-1})$

$0 < \lambda < 1$

$$y_t = (1-\lambda)\, y_t^* + \lambda\, y_{t-1}$$

$\lambda \approx 0$ fast adjustment

$\lambda \approx 1$ slow adjustment

② Adaptive Expectation Model (AEM)

$$\left[y_t = \alpha + \beta X_t^* + \varepsilon_t \right]$$

X_t^* - unobserved (expected) value of X

AE hypothesis : $X_t^* - X_{t-1}^* = (1-\lambda)(X_t - X_{t-1}^*)$

$0 < \lambda < 1$

$\lambda \approx 0$ fast revision

$\lambda \approx 1$ slow revision

$$X_t^* = (1-\lambda) X_t + \lambda X_{t-1}^* =$$

$$= (1-\lambda) X_t + \lambda \left((1-\lambda) X_{t-1} + \lambda X_{t-2}^* \right)$$

$$= (1-\lambda)\left(X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \dots\right)$$

$$= (1-\lambda) \sum_{j=0}^{\infty} \lambda^j X_{t-j}$$

ARDL$(0,\infty)$ $y_t = \alpha + \beta \sum_j w_j X_{t-j} + \varepsilon_t$ $\quad w_j = (1-\lambda)\lambda^j$

in lag form: SR: $\beta(1-\lambda)$ LR: $\beta \sum w_j = \beta(1-\lambda)\sum \lambda^j = \beta$

ARDL$(1,0)$ $y_t = \alpha_0 + \beta_0 X_t + \lambda y_{t-1} + v_t$

$\alpha_0 = \alpha(1-\lambda)$ $\qquad$ SR: $\beta_0 = \beta(1-\lambda)$

$\beta_0 = \beta(1-\lambda)$ $\qquad$ LR: $\tilde{y} = \alpha_0 + \beta_0 \tilde{X} + \lambda \tilde{y}$

$v_t = \varepsilon_t - \lambda \varepsilon_{t-1}$ $\qquad \Rightarrow \beta_0 / 1-\lambda = \beta$

Alternative notations:

$$y_t = \alpha + \beta x_t^e + \epsilon_t$$

↳ * sometimes denoted as x_{t+1}^e

x_t^e - unobserved expectation $E(x_{t+1} | I_t)$

AEH: $\Delta x_t^e = (1-\lambda)(x_t - x_{t-1}^e)$

↑ revision of expectations

$$x_t - x_{t-1}^e = x_t - E(x_t | I_{t-1})$$

↳ forecast error

Solution using lag operator $L x_t = x_{t-1}$

$L^k x_t = x_{t-k}$

$$x_t^e = \lambda x_{t-1}^e + (1-\lambda) x_t$$

$$x_t^e - \lambda x_{t-1}^e = (1-\lambda) x_t$$

$$(1-\lambda L) x_t^e = (1-\lambda) x_t$$

$$x_t^e = (1-\lambda L)^{-1} (1-\lambda) x_t$$

↓

$$\frac{1}{1-\lambda L}$$

$$X_t^e = (1-\lambda)\sum \lambda^j L^j X_t$$

$$= (1-\lambda)\sum \lambda^j X_{t-k}$$

$$y_t = \alpha + \beta(1-\lambda) \cdot \sum \lambda^j X_{t-j} + \varepsilon_t$$

$$= \alpha + \beta \cdot B(L)\, X_t + \varepsilon_t$$

where $B(L) = (1-\lambda) \cdot (1-\lambda)^{-1}$

Estimation:

① ARDL$(0,\infty)$: $y_t = \alpha + \beta \cdot (1-\lambda) X_t + \beta(1-\lambda) \cdot \lambda X_{t-1} + \beta(1-\lambda)\lambda^2 X_{t-2} + \dots + u_t$

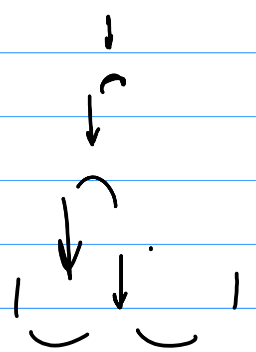
↳ Approximately ARDL$(0,s)$

$$y_t = \alpha + \beta(1-\lambda) X_t + \dots$$
$$\dots \beta(1-\lambda)\lambda^s X_{t-s} + u_t$$

Algorithm:

1) fix $\hat{\lambda}$

2) fit $y \mid \big[\underbrace{(1-\lambda)X_t + \dots + (1-\lambda)\lambda^s X_{t-s}}_{1 \text{ regressor}}\big]$



$0 < \lambda < 1$

3) find $\hat{\lambda}, \hat{\beta}$, RSS

4) change $\hat{\lambda}$

5) repeat 1 - 4 until RSS min

(2) ARDL(1,0) ;

$$y_t = (1-\lambda)\alpha + (1-\lambda)\beta X_t + \lambda y_{t-1} + \underbrace{u_t - \lambda u_{t-1}}$$

λ - revision rate

$\lambda = 0{,}85$ slow adjustment

$$\beta = \frac{\beta_0}{1-\lambda} = 0{,}2$$

(3) Error Correction Model

$$\left[y_t^* = \alpha + \beta \cdot X_t + \varepsilon_t \right]$$

$\hookrightarrow$ unobserved (equilibrium) value y $f(X_t)$

EC Hypothesis: $y_t - y_{t-1} = (1-\gamma)\cdot(y_t^* - y_{t-1}^*)$

$+ (1-\lambda)\cdot(y_{t-1}^* - y_{t-1})$

Δy_t^* – change in equilibrium value of y

$\parallel$

$\beta \, \Delta X_t + \Delta \varepsilon_t$

$y_{t-1}^* - y_{t-1}$ – previous disequilibrium

IF $\gamma = \lambda \Rightarrow$ PAM

$$y_t - y_{t-1} = (1-\gamma)(\beta(X_t - X_{t-1}) + \varepsilon_t - \varepsilon_{t-1})$$

$$+ (1-\lambda)(\alpha + \beta X_{t-1} + \varepsilon_{t-1} - y_{t-1})$$

ARDL(1,1): $y_t = \alpha_0 + \beta_0 X_t + \beta_1 X_{t-1} + \lambda y_{t-1} + v_t$

$\alpha_0 = (1-\lambda)\alpha$

$\beta_0 = (1-\gamma)\beta$

$\beta_1 = (\gamma - \lambda)\beta$

$v_t = (1-\gamma)\varepsilon_t + (\gamma-\lambda)\varepsilon_{t-1}$

SR: $\beta_0 = (1-\gamma)\beta$

LR: $\frac{\beta_0 + \beta_1}{1-\lambda} = \frac{(1-\lambda)\beta}{1-\lambda} = \beta$