I Dummy Variables

P1 How to test

1) change in intercept
2) ... slope
3) ... both

▷ Categories vs Reference

$$y_i = \beta_1 + \beta_2 X_i + \beta_3 D_i + u_i \quad (+ \beta_4 \cdot X_i D_i)$$

where $D_i = \begin{cases} 1, & \text{if male} \\ 0 \end{cases}$

(!) interpretation for which group

$\hat{\beta}_1$ ... for females

(*) $y_i^F = \beta_1 + \beta_2 X_i + u_i$ $\quad N_F$

$+$

$y_i^M = (\beta_3 + \beta_1) + (\beta_2 + \beta_4) X_i + u_i$ $\quad N_m$

$=$

N

Dummy Variable Trap

$$D_i^M = \begin{cases} 1, & \text{if } M \\ 0 \end{cases}$$

$$y_i = \beta_1 \bar{1}_i + \beta_2 D_i^M + \beta_3 D_i^F + \ldots + \varepsilon_i \qquad D_i^F = 1 - D_i^M = \begin{cases} 1, & \text{if } F \\ 0 \end{cases}$$

$$\bar{1} = D^M + D^F$$

$$y_i = \beta_2 D_i^M + \beta_3 D_i^F + \ldots + \varepsilon_i$$

β_2^M – average y for M

β_3^F – average y for F

$$y_i = \beta_2 D_i^M + \beta_3 D_i^F + \beta_4 \cdot D_i^R + \beta_5 D_i^{NR} + \varepsilon_i$$

$$\bar{1} = D_i^M + D_i^F \qquad \bar{1} = D_i^R + D_i^{NR}$$

$$D_i^R = \underbrace{D_i^M + D_i^F}_{\bar{1}} - D_i^{NR}$$

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$Var(\hat{\beta}_i) = \frac{\sigma_\varepsilon^2}{TSS_i \cdot R_i^2}$$

$\Sigma(x_i - \bar{x})$ $\qquad x_i \mid x_{-i}$

1) change in intercept

2) ... slope

3) ... both

$$y_i = \beta_1 + \beta_2 X_i + \beta_3 \cdot D_i + \beta_4 D_i \cdot X_i + u_i$$

1) t - test for β_3

$$H_0 : \beta_3 = 0$$

$$t = \frac{\hat{\beta}_3}{se(\hat{\beta}_3)} \sim t_{N-4}$$

2) t - test for β_4

3) F - test for β_3 & β_4

$$H_0: \beta_3 = 0, \beta_4 = 0$$

$$F = \frac{(RSS_R - RSS_{UR})/2}{RSS_U / N-4} \sim F_{2, N-4}$$

Equivalent to Chow test stat

$$\parallel$$

$$F = \frac{(RSS_{pool} - (RSS_M + RSS_F))/2}{(RSS_M + RSS_F)/2 \cdot 2}$$

$$y_i^M = \beta_1^M + \beta_2^M X_i + u_i$$

$$y_i^F = \beta_1^F + \beta_2^F X_i + u_i$$

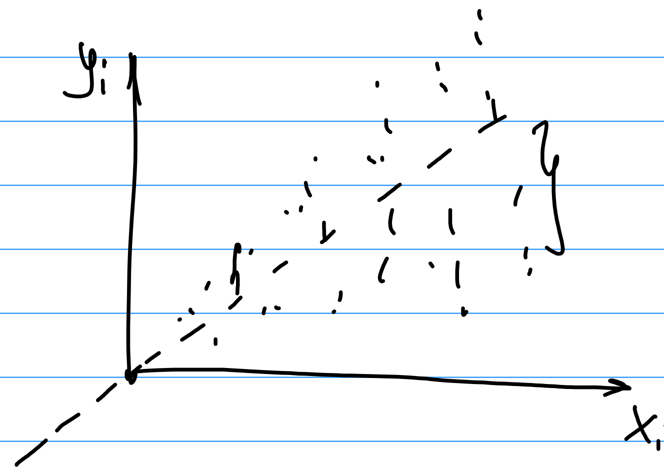
$$H_0: \beta_1^F = \beta_1^M, \beta_2^F = \beta_2^M$$

(II) Heteroscedasticity

(P2) (1) $y_i = \beta X_i + u_i \qquad u_i \sim N(0, \sigma_i^2)$

X_i - deterministic

a) $E(u_i) \neq$ const



b) Derive $V(\hat{\beta}_{OLS})$ & Explain how to obtain robust s.e.

$$Var(\hat{\beta}) = E((\hat{\beta} - E(\hat{\beta}))^2) =$$

$$E((\hat{\beta} - \beta)^2) = \left\{ \hat{\beta}_{OLS} = \frac{\sum X_i y_i}{\sum X_i^2} \right\} =$$

$$= E\left(\frac{(\sum X_i u_i)^2}{(\sum X_i^2)^2} \right) =$$

$$= \frac{E(\sum X_i^2 u_i^2 + \sum\sum X_i u_i X_j u_j)}{(\sum X_i^2)^2} =$$

$$= \frac{\sum X_i^2 E(u_i^2)}{(\sum X_i^2)^2} + \frac{\sum\sum X_i X_j \overset{0}{\overset{\shortparallel}{E(u_i u_j)}}}{(\sum X_i^2)^2} =$$

$$= \frac{\sum X_i \sigma_i^2}{\left(\sum X_i^2\right)^2}$$

For White s.e. :

$$\hat{Var}(\hat\beta) = \frac{\sum X_i \hat{u}_i^2}{\left(\sum X_i^2\right)^2}$$

c) $H_0: \beta = 1$ * Based on $\hat\beta_{OLS}$

$H_a: \beta > 1$

$$t = \frac{\hat\beta - 1}{se(\hat\beta)} \overset{H_0}{\sim} t_{N-1}$$

$$se(\hat\beta) = \sqrt{\frac{\sum X_i \hat{u}_i^2}{\left(\sum X_i^2\right)^2}}$$

if $\underbrace{t^{obs} > t^{crit}_{0{,}95,\ n-1}}$ $\Rightarrow$ H_0 is rejected

$\alpha = 5\%$

(P3) $y_i = \alpha \cdot X_i + \varepsilon_i$

$E(\varepsilon_i) = 0$

$\boxed{E(\varepsilon_i^2) = \sigma^2 X_i^2}$

$E(\varepsilon_i \varepsilon_j) = 0 \quad , i \neq j$

X_i - fixed

a) Derive WLS estimator $\hat{\alpha}_{WLS}$

b) Show consistency

a) WLS equation: weights $\sim \sqrt{Var(\varepsilon_i)}$

$$\frac{y_i}{X_i} = \alpha + \frac{\varepsilon_i}{X_i}$$

$$\hat{\alpha}_{OLS} = \frac{\Sigma\, y_i / x_i}{n}$$

b)

$$\hat{\alpha}_{WLS} = \frac{1}{N} \cdot \Sigma \frac{\alpha \cdot X_i + \varepsilon_i}{X_i} =$$

$$= \alpha + \frac{1}{N} \cdot \Sigma \frac{\varepsilon_i}{X_i}$$

1. $$E(\hat{\alpha}_{WLS}) = \alpha + \frac{1}{N} \cdot \Sigma \frac{E(\varepsilon_i)}{X_i}$$

2. $$Var(\hat{\alpha}_{WLS}) = E\left((\hat{\alpha} - \alpha)^2\right) =$$

$$= E\left(\frac{1}{N} \cdot \Sigma \frac{\varepsilon_i}{x_i}\right)^2 = \frac{1}{N^2} E\left(\Sigma \frac{\varepsilon_i}{X_i}\right)^2 =$$

$$= \frac{1}{N^2}\left(E\left(\Sigma \frac{\varepsilon_i^2}{X_i^2} + \Sigma\Sigma \frac{\varepsilon_i \varepsilon_j}{X_i X_j}\right)\right) = \{ E(\varepsilon_i \varepsilon_j) = 0 \}$$

$$= \frac{1}{N^2} \cdot \sum \frac{E(\varepsilon_i^2) = \sigma^2 \cdot \cancel{X_i^2}}{\cancel{X_i^2}} = \frac{\sigma^2}{N}$$

$$Var(\hat{\alpha}_{WLS}) = \frac{\sigma^2}{N} \xrightarrow[N \to \infty]{} 0$$