Autocorrelation

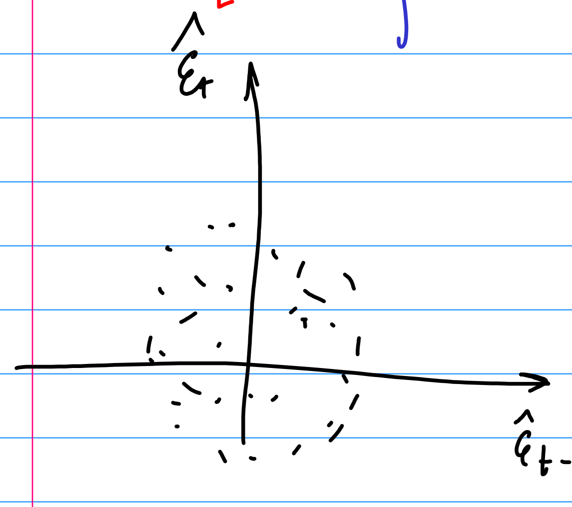
$$y_t = \alpha + x_t + \varepsilon_t$$

$$cov(\varepsilon_i, \varepsilon_j) \neq 0$$

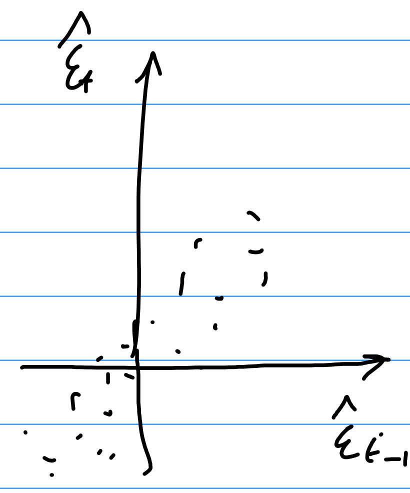
Autocorrelation of order 1:

$$[\varepsilon_t = \rho \cdot \varepsilon_{t-1} + u_t] \qquad u_t - WN$$

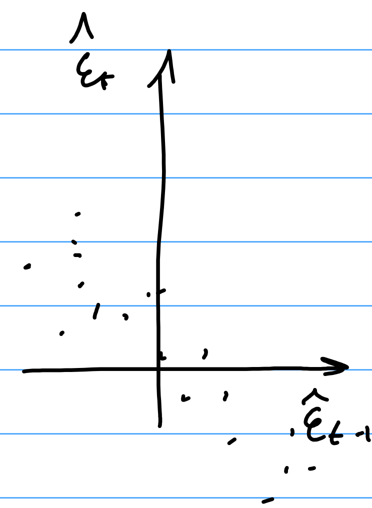
①



$\rho = 0$

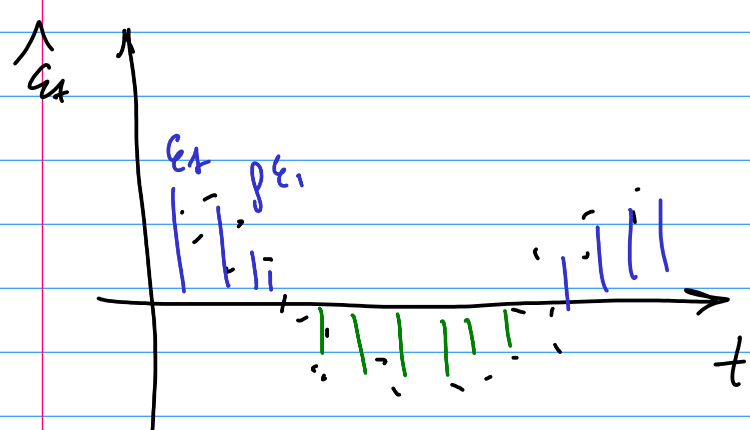


$\rho > 0$

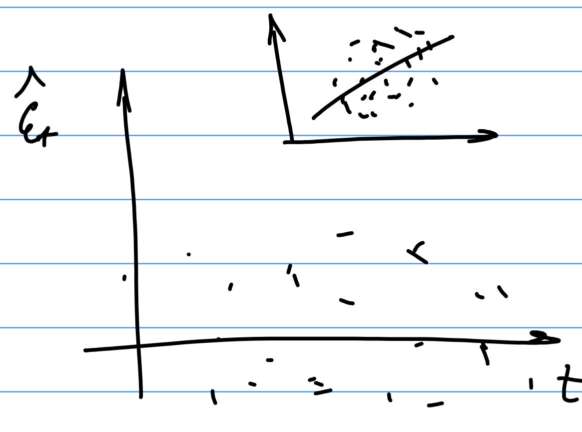


$\rho < 0$

②

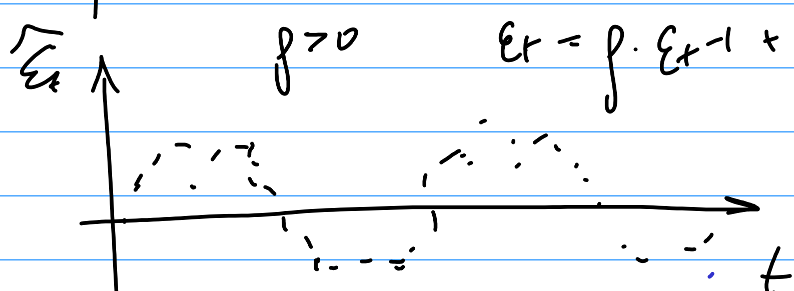


$\rho > 0$

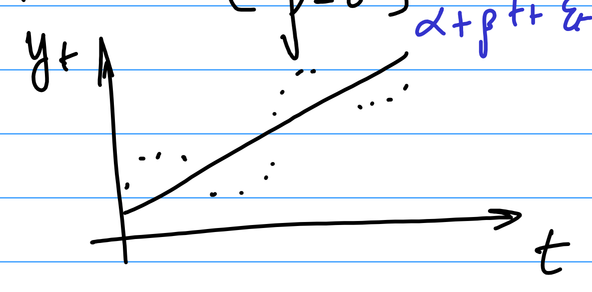


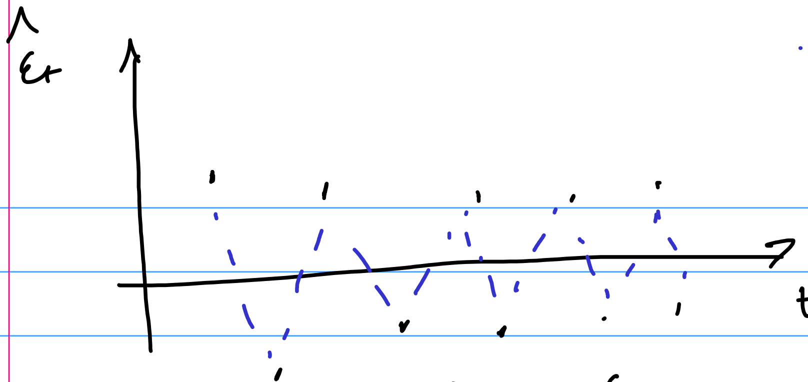
$$\varepsilon_t = \rho \cdot \varepsilon_{t-1} + u_t$$

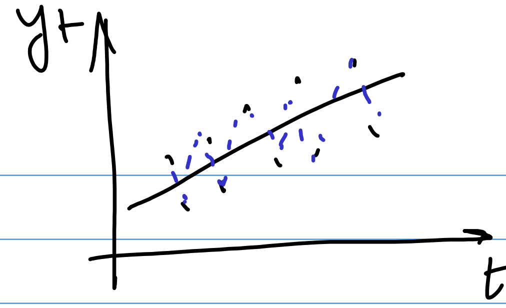
$[\rho = 0]$



$\rho > 0$







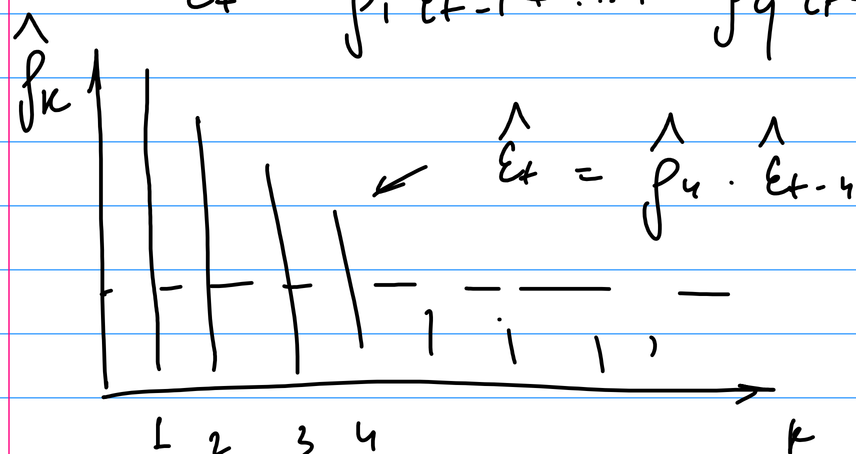
$\rho < 0 \qquad \varepsilon_t = \rho \cdot \varepsilon_{t-1} + u_t$

Autocorrelation of order q:

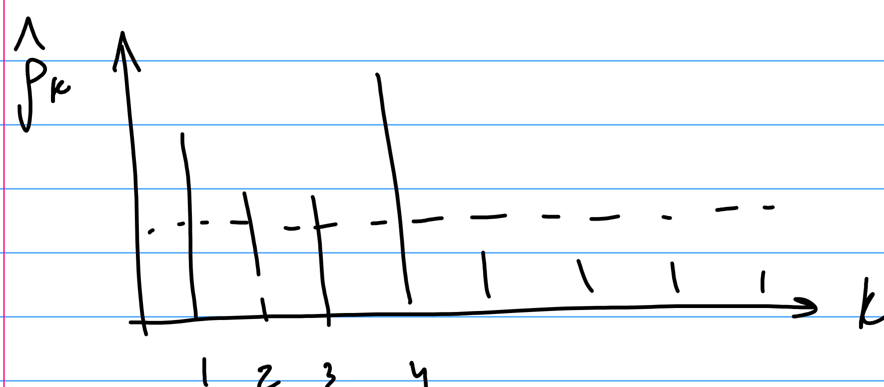
$$y_t = \alpha + \beta X_t + \varepsilon_t$$

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \dots + \rho_q \varepsilon_{t-q} + u_t$$

(3)



ACF



Seasonal autocorrelation of errors

Types of autocorrelation:

1) True autocorrelation:

$$y_t = f(X_t) + \varepsilon_t$$

↳ correctly specified!

$$\boxed{\varepsilon_t = \rho\,\varepsilon_{t-1} + u_t}$$

$$E(\varepsilon'\varepsilon) = \begin{pmatrix} E(\varepsilon_1\varepsilon_1) & E(\varepsilon_1\varepsilon_2) & \cdots \\ \vdots & \ddots & \\ & & E(\varepsilon_T\varepsilon_T) \end{pmatrix} =$$

$$= \begin{pmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon^2 & \rho^2\sigma_\varepsilon^2 & \cdots \\ \rho\sigma_\varepsilon^2 & \ddots & & \\ \rho^2\sigma_\varepsilon^2 & & \ddots & \\ \vdots & & & \sigma_\varepsilon^2 \end{pmatrix} = \Omega$$

$\hat{\Omega}$ ↳ weighting matrix for GLS

$$\mathrm{Cov}(\varepsilon_t, \varepsilon_{t-1}) =$$

$$\mathrm{Cov}(\rho\varepsilon_{t-1} + u_t, \varepsilon_{t-1}) = \rho\sigma_\varepsilon^2$$

2) False Autocorrelation

- Omitted Variable

TRUE: $y_t = \alpha + \beta_1 X_{1t} + \beta_2 X_{2t} + \varepsilon_t$, $\quad \varepsilon_t$ - WN

EST: $\boxed{y_t = \alpha + \beta_1 X_{1t} + v_t}$

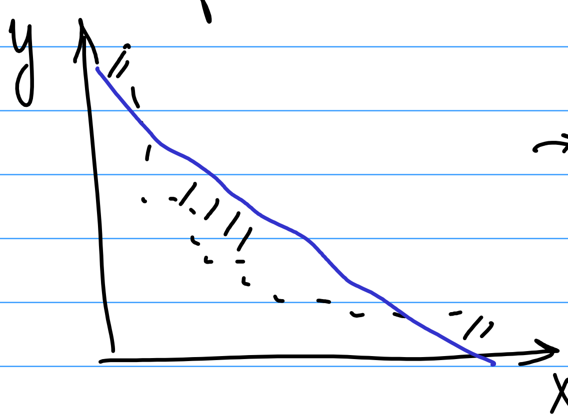
$$v_t = \beta_2 X_{2t} + \underbrace{\varepsilon_t}_{WN}$$

$$\downarrow$$

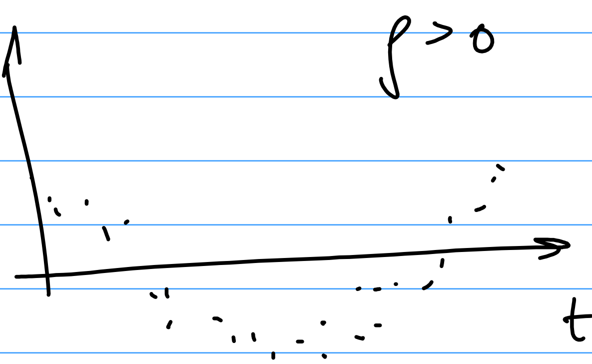
$$\rho \cdot X_{2t-1} + w_t$$

IF $\quad cov(X_{2t}, X_{2t-1}) \neq 0 \Rightarrow cov(v_t, v_{t-1}) = 0$

- Misspecification : Incorrect functional form:



$\rightarrow$



Consequences of Autocorrelation

1) True autocorrelation => biased and incons. estimator of cov matrix of regressor coefs.

$\Downarrow$

$se(\hat{\beta})$ - biased and incons.

2) If y_{t-1} is in RHS => endogeneity

Testing Autocorrelation

DW test: $H_0: \rho = 0$

$H_a: \rho > 0$ or $\rho < 0$

$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$

1) Autocorr. of 1st
2) Const in a model
3) No y_{t-1} in RHS

$$DW = \frac{\sum_{t=2}^{T} (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum_{t=1}^{T} \hat{\varepsilon}_t^2}$$

$$DW = \frac{\sum_{t=2}^{T} \hat{\varepsilon}_t^2}{\sum_{t=1}^{T} \hat{\varepsilon}_t^2} - \frac{2\sum_{t=2}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-1} / T}{\sum_{t=1}^{T} \hat{\varepsilon}_t^2 / T} + \frac{\sum_{t=2}^{T} \hat{\varepsilon}_{t-1}^2}{\sum_{t=1}^{T} \hat{\varepsilon}_t^2}$$

$$\xrightarrow{T\to\infty} \quad 1 - 2\cdot\frac{E(\varepsilon_1\varepsilon_2)}{\underbrace{E(\varepsilon_1^2)}} + 1$$

$$E(\varepsilon_1\varepsilon_2) - E(\varepsilon_1)E(\varepsilon_2)$$

$$\underset{E(\varepsilon_1)=0}{=} 2 - 2\cdot\frac{Cov(\varepsilon_1,\varepsilon_2)}{Var(\varepsilon_1)}$$

$$Var(\varepsilon_1) = E(\varepsilon_1^2) - \underbrace{E^2(\varepsilon_1)}_{=0}$$

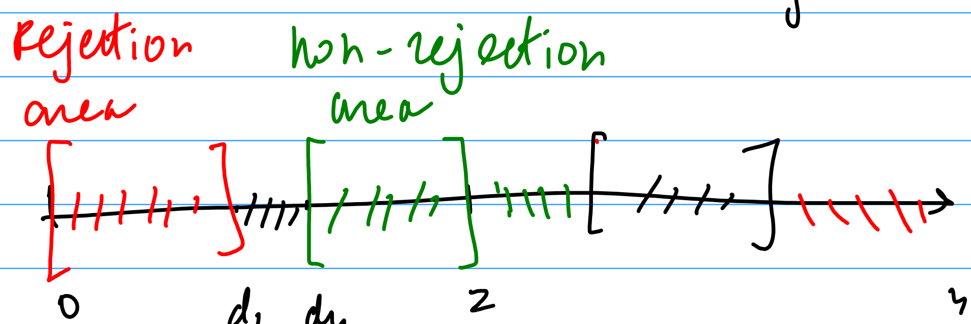
$$= 2 - 2\rho$$

$$DW \xrightarrow{T\to\infty} 2 - 2\rho$$

$\rho = -1$ $\qquad DW = 4$

$\rho = 0 \longleftrightarrow DW = 2$

$\rho = 1$ $\qquad DW = 0$



$\rho > 0$

$d_U < DW < 2 \Rightarrow H_0$ not rej

$d_L < DW < d_U \Rightarrow$ no statistical inference

$0 < DW < d_L \Rightarrow H_a: \rho > 0$