Autocorrelation

Remedies :

1) Specification

true: $y_t = \alpha + \beta X_t + \gamma \underbrace{Z_t + \varepsilon_t}_{=}$ WN

Est : $y_t = \alpha + \beta X_t + u_t$

$u_t \swarrow Z_t$ $\qquad$ $u_t \searrow$ else

Z_t - AR process $\Downarrow$ ε_t - autocorr.

else $\downarrow$ Not

2) AR(1) Transform (special case of GLS)

Cochrain - Orcutt

$$\frac{\sigma_u^2}{\Sigma(X_i-\bar{X})^2}$$

$$y_t = \beta_1 + \beta_2 X_t + u_t \quad \Big| \cdot L\rho$$

Assume: $u_t = \rho u_{t-1} + \varepsilon_t$ ←

$$y_t = \beta_1 + \beta_2 X_t + \cancel{\rho u_{t-1}} + \varepsilon_t$$

$$\rho y_{t-1} = \rho\beta_1 + \rho\beta_2 X_{t-1} + \cancel{\rho u_{t-1}}$$

$$y_t - \rho y_{t-1} = \beta_1(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + \varepsilon_t$$

$$y_t^* = \beta_1^* + \beta_2 X_t^* + \varepsilon_t \sim WN$$

$$\beta = (X'X)^{-1}X'y$$

$$\begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \end{bmatrix}\begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \quad \hat{\rho}$$

$$\mathrm{Var}(\hat{\beta}_2) = \sigma^2 (X'X)^{-1} = \frac{\sigma_\varepsilon^2}{\Sigma(x_i^* - \bar{x}^*)^2}$$

$$\hat{u}_t = \rho \hat{u}_{t-1} + v_t \qquad \hat{\rho} = \frac{\Sigma \hat{u}_t \cdot \hat{u}_{t-1}}{\Sigma \hat{u}_{t-1}^2}$$

$$DW \longrightarrow 2(1-\rho)$$

$$\hat{\rho} = 1 - \frac{DW}{2}$$

Price-Winsten Correction: (for 1st obs)

$$y_t^* = \beta_1^* + \beta_2 x_t^* + \varepsilon_t \qquad \mathrm{Var}(\varepsilon_t) = \sigma_\varepsilon^2$$

$$\sqrt{1-\rho^2}\cdot \Big| \; y_1 = \beta_1 + \beta_2 X_1 + u_1 \qquad \mathrm{Var}(u_t) = \sigma_u^2 \;\; \text{LH}$$

$$= \mathrm{Var}(\rho u_{t-1} + \varepsilon_t) =$$

RHS

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad = \underline{\rho^2 \sigma_u^2 + \sigma_\varepsilon^2} + 2 \cdot 0$$

$$\Rightarrow \sigma_u^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2}$$

$$\sigma_\varepsilon^2 = (1-\rho^2)\sigma_u^2$$

③ MA(1) → GLS

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1} \cdot X'\Omega^{-1}y$$

$$\Omega = \sigma_\varepsilon^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$\Downarrow$

$$\hat{\beta} = (X'X)^{-1} \cdot X'y$$

$$\Omega = \sigma_\varepsilon^2 \begin{pmatrix} X_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \\ 0 & \cdots & 0 & X_n \end{pmatrix}$$

$\Downarrow$

WLS

AR(1) : $\varepsilon_t = \rho\,\varepsilon_{t-1} + u_t$

$$\Omega = \frac{\sigma_\varepsilon^2}{1-\rho^2} \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \rho \\ \rho^{n-1} & \cdots & & \rho & 1 \end{pmatrix}$$

$\downarrow$

$$\mathrm{cov}(\varepsilon_t, \varepsilon_{t-k}) = \frac{\rho^k}{1-\rho^2}$$

MA(1) : $\varepsilon_t = u_t + \rho\, u_{t-1}$

$$\Omega = \sigma_\varepsilon^2 \begin{pmatrix} 1 & 2 & 0 & \cdots & 0 \\ 2 & \ddots & \ddots & & 0 \\ 0 & \ddots & \ddots & \ddots & 2 \\ 0 & \cdots & 0 & 2 & 1 \end{pmatrix}$$

$$\overset{\lambda_1}{} \quad \overset{\lambda_2}{} \quad \overset{\lambda_3}{} \quad \overset{\lambda_4}{}$$

$$y_t = \rho y_{t-1} + \beta_1 (1-\rho) + \beta_2 X_t - \beta_2 \rho X_{t-1}) + \varepsilon_t$$

ARDL (1, 1) with restrictions

$$\lambda_4 = - \lambda_1 \cdot \lambda_3$$

non-linear restriction

Common Factor test

$$n \cdot \log \left(\frac{RSS_R}{RSS_{UR}} \right) \sim \chi^2_1$$