Autocorrelation

$$y_t = \alpha + \beta X_t + \varepsilon_t$$

1st order autocorrelation

$$\text{cov}(\varepsilon_i, \varepsilon_j) \neq 0$$

$$\varepsilon_t = \rho \cdot \varepsilon_{t-1} + u_t$$

$$\text{cov}(\varepsilon_t, \varepsilon_{t-s}) = \text{cov}(\rho \varepsilon_{t-1} + u_t, \varepsilon_{t-s}) = \text{cov}(\rho^2 \varepsilon_{t-2} + \rho u_t + u_{t-1}, \varepsilon_{t-s}) = \rho^s$$

$$E(u_t) = 0$$

$$E(u_t^2) = \sigma_u^2$$

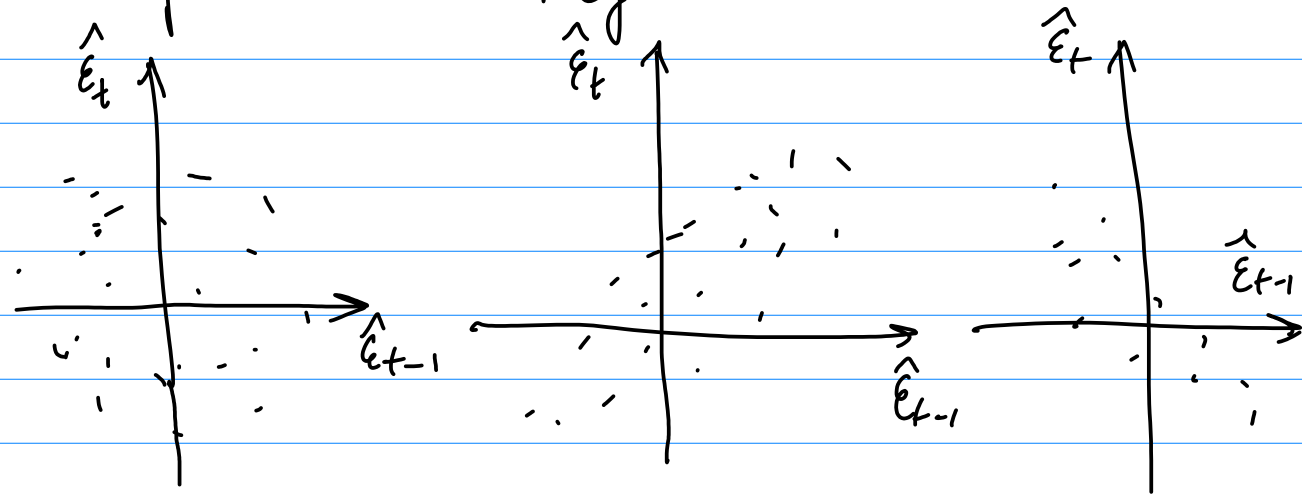
$$E(u_i, u_j) = 0 \quad \forall i \neq j$$

$$\varepsilon'\varepsilon = \begin{pmatrix} \sigma_\varepsilon^2 & \rho & \rho^2 & \cdots & \\ \rho & \ddots & & & \\ \rho^2 & & \ddots & & \rho \\ \vdots & & & \rho & \sigma_\varepsilon^2 \end{pmatrix}$$

$\rho > 0 \Rightarrow$ positive autocorrelation

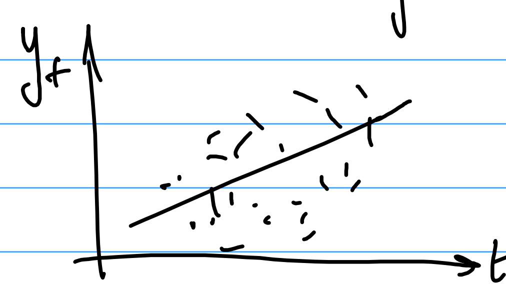
$\rho < 0 \Rightarrow$ negative autocorrelation

(1)



$\rho = 0$ $\quad\quad$ $\rho > 0$ $\quad\quad$ $\rho < 0$

$\rho = 0$



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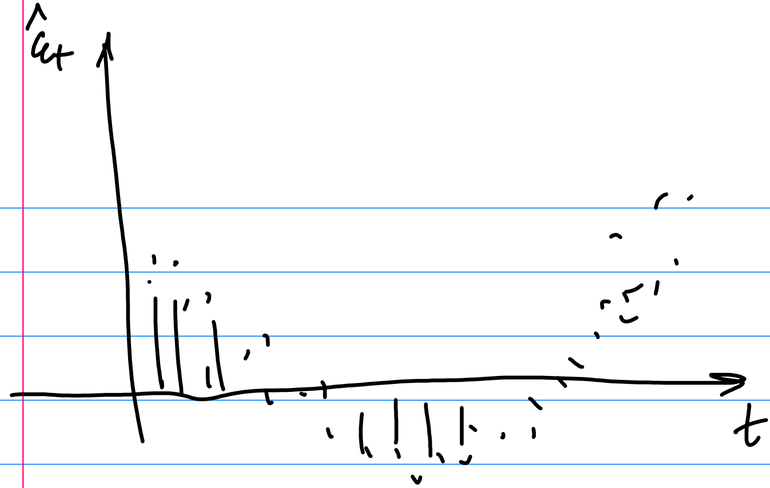
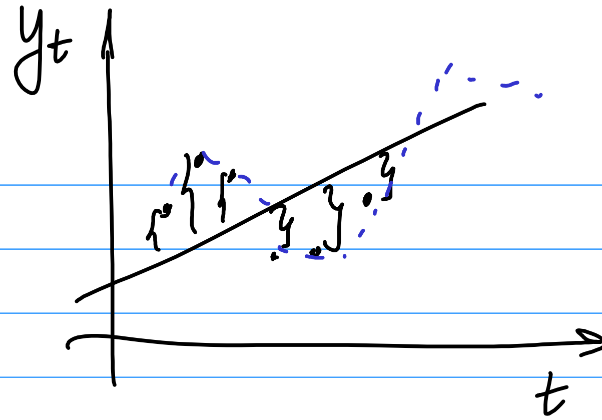
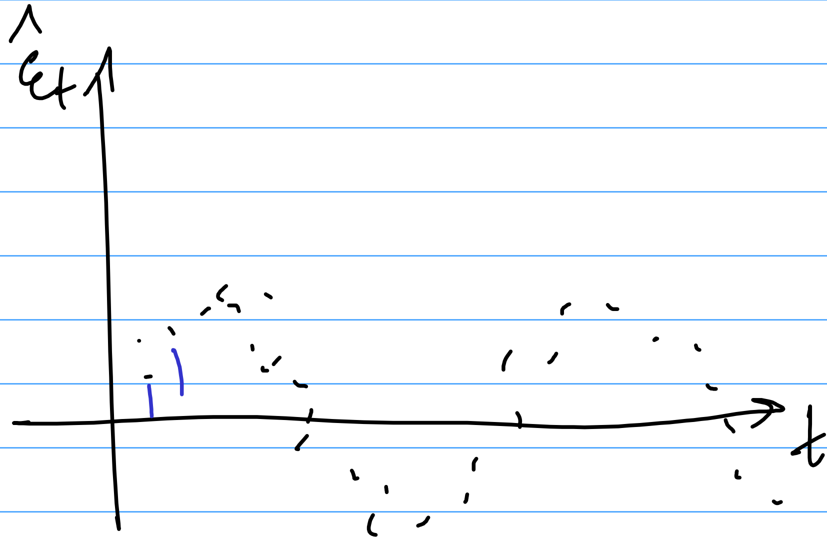


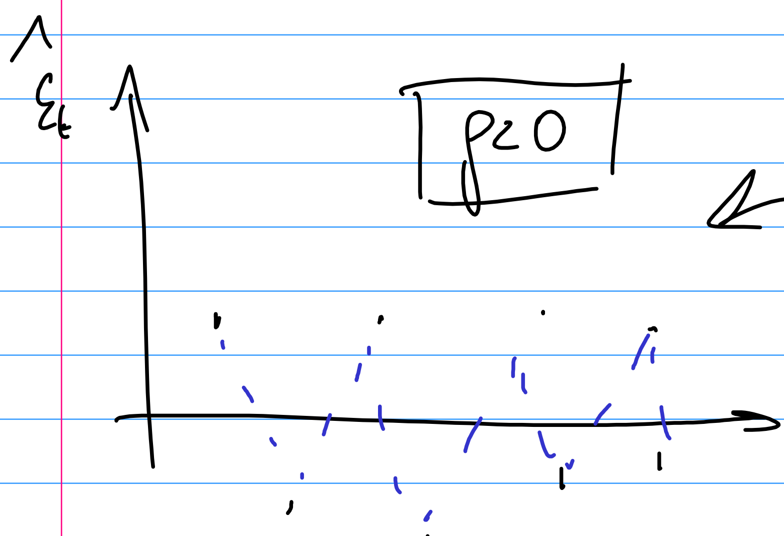


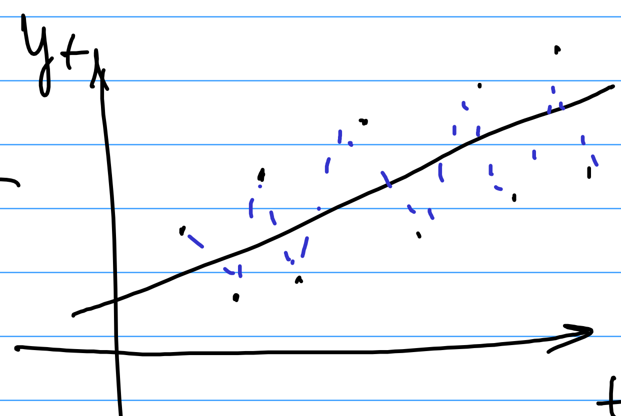


$\boxed{\rho > 0}$

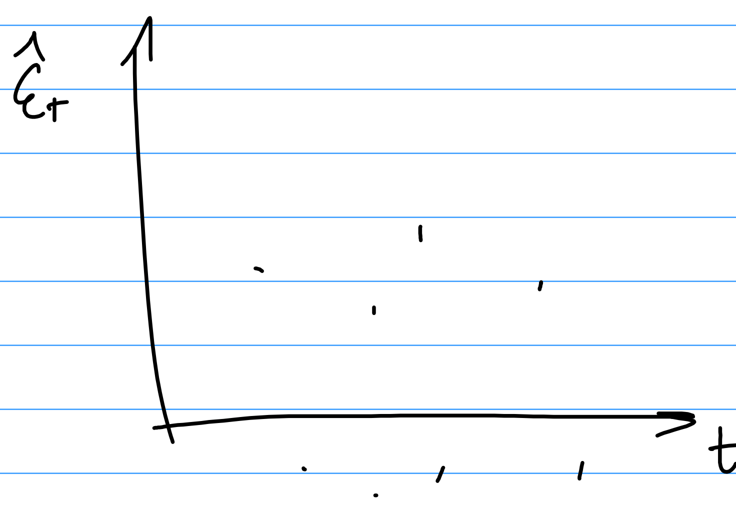
$$\varepsilon_t = \rho \cdot \underbrace{\varepsilon_{t-1}}_{<0} + u_t$$

$\boxed{\rho < 0}$





Small sample:

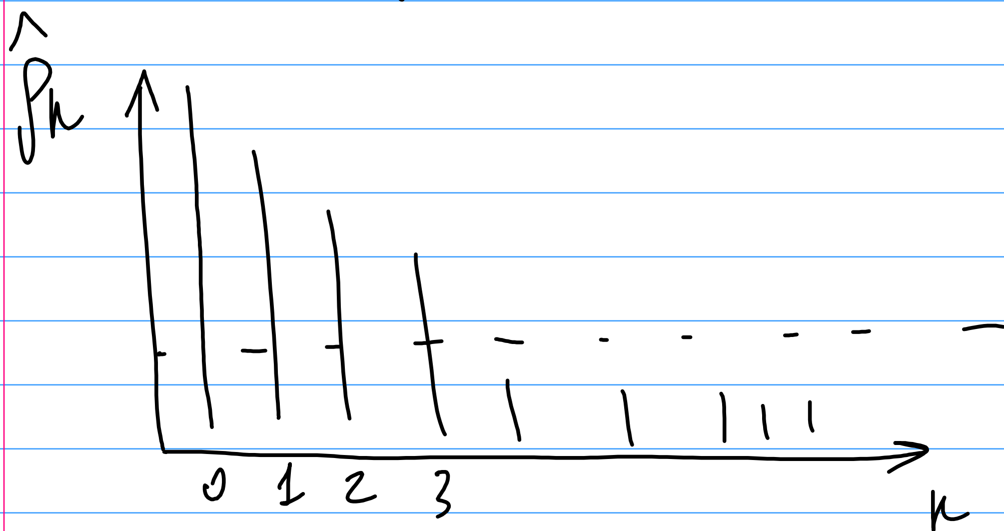


Autocorrelation of order q

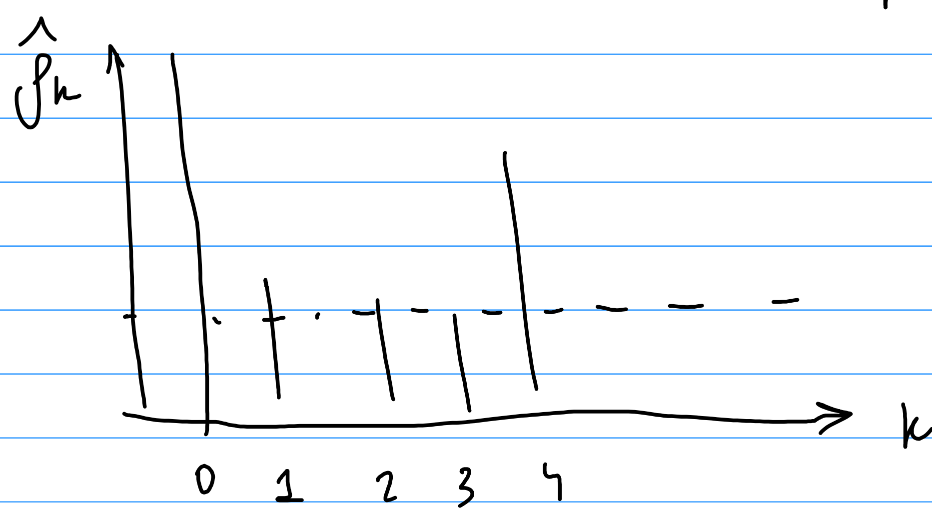
$$y_t = \alpha + \beta x_t + \varepsilon_t$$

$$\varepsilon_t = \rho_1 \cdot \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \dots + \rho_q \varepsilon_{t-q} + u_t$$

u_t – W.N.



Autocorrelation function



Seasonal autocorrelation

Causes of autocorrelation:

1) True autocorrelation:

If the model is correctly specified $\Rightarrow \varepsilon_t = \rho\,\varepsilon_{t-1} + u_t$

2) False autocorrelation:

- Omitted Variable

TRUE: $y_t = \alpha + \beta_1 X_{1t} + \underbrace{\beta_2 X_{2t} + \varepsilon_t}_{\parallel} \qquad \varepsilon_t - WN$

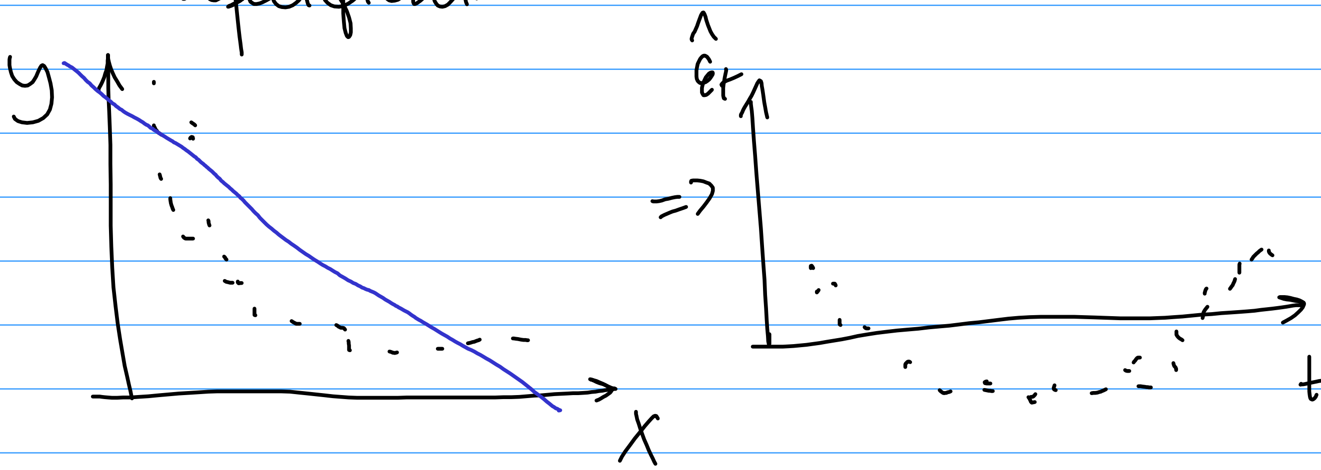
EST: $y_t = \alpha + \beta X_{1t} + \nu_t$

$$\nu_t = \beta_2 X_{2t} + \varepsilon_t$$

IF $cov(X_{2,t}, X_{2,t-1}) \neq 0$

$\Rightarrow cov(\nu_t, \nu_{t-1}) \neq 0$

- Misspecification



$$\ln y_t = \alpha + \beta \ln x_t + \varepsilon_t$$

TRUE: $y_t = \alpha' \cdot x_t^{\beta} \cdot \exp(\varepsilon_t)$

EST: $y_t = \alpha'' + \beta'' x_t + v_t$

$$\alpha' \cdot x_t^{\beta} \cdot \underbrace{\exp(\varepsilon_t)} - \alpha'' - \beta'' x_t = v_t$$

Autocorrelation consequences:

1) True autocorrelation => Biased and inconsistent estimator of covariance matrix of coefficient estimates

$\Downarrow$

$\hat{se}(\hat{\beta})$ - inconsistent & biased

2) If y_{t-1} is in RHS => endogeneity

DW test:

1) Autocorrelation of 1st order

2) Model with constant term

3) y_{t-1} is not in RHS

$H_0: \rho = 0$ vs $H_a: \rho > 0$ $\quad \varepsilon_t - \rho \varepsilon_{t-1} + u_t$

$$DW = \frac{\sum_{t=2}^{T} (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum_{t=1}^{T} \hat{\varepsilon}_t^2}$$

$$DW = \frac{\sum_{t=2}^{T} \hat{\varepsilon}_t^2}{\sum_{t=1}^{T} \hat{\varepsilon}_t^2} - 2\,\frac{\sum \hat{\varepsilon}_t \hat{\varepsilon}_{t-1} / T}{\sum \hat{\varepsilon}_t^2 / T} + \frac{\sum \hat{\varepsilon}_{t-1}^2}{\sum \hat{\varepsilon}_t^2}$$

$$\xrightarrow[T \to \infty]{} 1 - 2\,\frac{E(\varepsilon_t \varepsilon_{t-1})}{E(\varepsilon_t^2)} + 1$$

$$\underset{E(\varepsilon_t)=0}{=} 1 - 2\,\frac{\mathrm{Cov}(\varepsilon_t, \varepsilon_{t-1})}{\mathrm{Var}(\varepsilon_t)} + 1$$

$$\frac{\mathrm{Cov}(\varepsilon_t, \varepsilon_{t-1})}{\mathrm{Var}(\varepsilon_t)} = \rho$$

$$= 2 - 2\rho$$

$$DW \to 2 - 2\rho$$

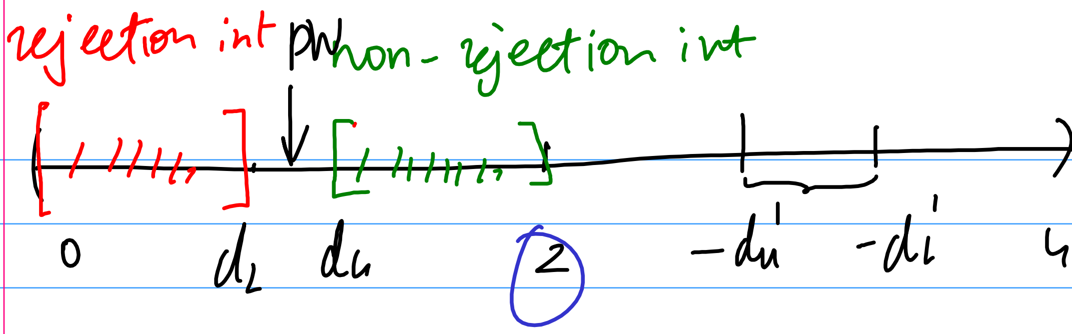
$\rho = -1$ $\quad DW = 4$

$[\rho = 0$ $\quad DW = 2]$

$\rho = 1$ $\quad DW = 0$

$$\rho \in (-1; 1)$$

$$DW \in (0; 4)$$



$H_a: \rho > 0$

$d_L < DW < d_u \Rightarrow$ no statistical inference is done

$2 < DW < d_u \Rightarrow H_0: \rho = 0$

$0 < DW < d_L \Rightarrow H_a: \rho > 0$