

**Model exam 2.****SECTION A**

Answer **ALL** questions from this section (questions 1-5).

**Question 1L** Explain what you understand by a dummy variable. Under what circumstances would you use dummy variables in econometric analysis?

**Question 1R** Discuss how dummy variables can be used to test

- (a) change in intercept;
- (b) change in slope;
- (c) changes in both intercept and slope.

**Question 2.** Explain what you understand by omitted variable bias using regression model without intercept.

**Question 3.**

Consider the simple linear regression model

$$Y_i = \beta X_i + u_i, \quad i = 1, \dots, n.$$

We assume that the errors  $\{u_i\}_{i=1}^n$  are independent normal random variables with zero mean. The regressor  $\{X_i\}_{i=1}^n$  is non-stochastic (fixed under repeated sampling). You suspect that the errors exhibit heteroskedasticity.

(a) Explain what we mean by the concept of heteroskedasticity. Enhance your answer with the help of a graphical illustration.

(b) Derive the variance of the OLS estimator for  $\hat{\beta}$  in the presence of heteroskedasticity and explain how a robust (White) standard error for  $\hat{\beta}$  can be obtained.

(c) Suppose you want to test  $H_0 : \beta = 1$  against  $H_1 : \beta > 1$ . Discuss how you would conduct this test based on the OLS estimator, recognizing the presence of heteroskedasticity. Please provide a detailed answer.

**Question 4** Consider a model

$$y_t = \alpha x_t + u_t; \quad t = 1, 2, \dots, T$$

where  $E(u_t) = 0$ ,  $E(u_t^2) = \sigma^2 x_t^2$  and  $E(u_s u_t) = 0$  if  $s \neq t$  for all  $s, t = 1, 2, \dots, T$ .  $x$ 's are fixed.

(a) Derive the weighted least squares (WLS) estimator  $\hat{\alpha}$ , of  $\alpha$  and also derive the variance of  $\hat{\alpha}$ .

(b) Is WLS estimator of  $\alpha$  consistent? Explain.

**Question 5.**

(a) Define the term 'multicollinearity' and provide a real life example where this problem is likely to occur.

(b) Examine whether the following statements are true or false. Give an explanation.

(i) In multiple regression, multicollinearity implies that the least squares estimators of the coefficients are biased and standard errors invalid.

(ii) If the coefficient estimates in an equation have high standard errors, this is evidence of high multicollinearity.

## SECTION B

**Question 6.** To investigate the relationship between the price of wine and consumption of wine, an economist runs the following regression on a sample of 32 individuals for one week in 2013:

$$\log(\hat{wine}) = 4.2514 - 0.8323 \log(price), \quad n = 32, \quad R^2 = 0.89.$$

(0.8911) (0.0031)

*wine* denotes the amount of wine consumed per week in millilitres (a medium glass contains 175ml) and *price* denotes the average price of a selection of wines during the week in GBP (£). The numbers in parentheses are the standard errors.

(a) Discuss what would happen to the parameter estimate of the slope coefficient if we had measured the amount of wine consumed per week in number of medium glasses instead of millilitres. Explain your answer.

(b) You are asked to test the hypothesis that the demand for wine has an elasticity equal to  $-1$  against a two-sided alternative, using a 5% level of significance. Clearly stating any assumptions you may need, carry out this test.

(c) Construct a 95% confidence interval for the price elasticity of demand and discuss how this interval can be used to carry out the test in (b).

(d) A famous TV chef suggests in a talk show that the demand for wine is less elastic (i.e. less negative) for people who have eaten at a restaurant during the week, arguing that eating in a restaurant encourages people to drink wine regardless of the price. To test this theory, the economist defines a dummy variable  $D_i$  that takes the value 1 if individual  $i$  ate at a restaurant during the week, and 0 otherwise. She obtains the following regression result:

$$\log(\hat{wine}) = 4.2133 - 0.8218 \log(price) + 0.0889 D \times \log(price), \quad n = 32, \quad R^2 = 0.92.$$

(0.8911) (0.0028) (0.0011)

(i) How does this regression help in assessing the TV chef's claim?

(ii) Conduct a test that may offer support for the TV chef's claim. Clearly specify the null and the alternative hypothesis. Explain clearly why your test is effective in answering the question of interest.

**Question 7.** Consider the model:

$$y_t = ax_t + u_t, \quad t = 1, \dots, T$$

where  $E(u_t) = 0$ ,  $E(u_t^2) = \sigma^2$ ,  $E(u_t u_{t'}) = 0$  for  $t \neq t'$ .

Suppose that the parameter  $a$  changes at a certain point  $s$  the sample, i.e.

$$a = a^*, \quad t = 1, \dots, s$$

$$a = a^{**}, \quad t = s+1, \dots, T$$

(a) Explain how you would estimate  $a^*$  and  $a^{**}$ :

(i) not using dummy variables;

(ii) using dummy variables.

(b) Suppose an econometrician ignores the change in  $a$ , and assumes that  $a$  is constant throughout the sample. Derive an expression for the bias of the OLS estimate of  $a$  as an estimate of  $a^*$ .

(c) Given data on  $x_t$  and  $y_t$  explain how you would test your model for a change in the slope coefficient at time  $s$  (i) using a t-test;

(ii) using F-test for restriction (what is the restriction?).

(d) An alternative to the test in (c) is a Chow test. Explain how you would apply a Chow test to this model and how the results would compare with the test you described in (c).