***Question* 1.** **(a)** What is meant by the term 'spurious regression'? Explain how you would determine whether or not an estimated equation is a 'spurious regression'?

(b) Explain what you understand by difference-stationary and trend-stationary time series.

(c) Consider a time series process

$$\ln Y_t = \alpha + \beta t + u_t;\ t = 1, 2, \ldots, T$$

Examine the order of integration of $\ln Y_t$.

***Question* 2.** **(a)** Explain what is Dickey-Fuller test.

(b) Explain what is Augumented Dickey-Fuller test.

Q1. c) $E(\ln y_t) = \alpha + \beta t \Rightarrow$ non-stationary

$E(\Delta \ln y_t) = E(\Delta u_t + \beta t - \beta t - \beta) = -\beta$

$Var(\Delta \ln y_t) = Var(\Delta u_t) = 2\sigma_u^2$

$Cov(\Delta \ln y_t, \Delta \ln y_{t-s}) =$

$Cov(\Delta u_t, \Delta u_{t-s}) =$

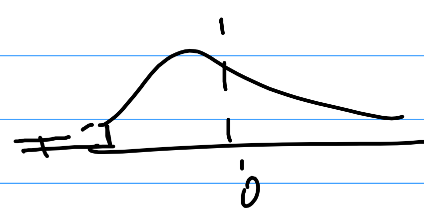
$$Cov(u_t - u_{t-1}, u_{t-s} - u_{t-s-1}) = \begin{cases} s=1, & -\sigma_u^2 \\ s>1, & 0 \end{cases}$$

Q2. a) $\rightarrow y_t = \beta y_{t-1} + \varepsilon_t \qquad H_0: \beta = 1$

$\rightarrow \Delta y_t = (\beta - 1) y_{t-1} + \varepsilon_t$

↳ $H_0: \beta - 1 = 0$

$H_a: \beta - 1 < 0$



$t^{DF}_{obs} < DF^{crit}_{NC} \Rightarrow$ stationary

$t^{DF} = \dfrac{\beta - 1}{se(\beta)} \sim DF_{NC}$

b) ADF test of order p:

$$y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 t + \beta_4 y_{t-2} + \dots + \beta_{p+2} y_{t-p} + \varepsilon_t$$

for order p = 2

$$\Delta y_t = \beta_1 + \underbrace{(\beta_2 + \beta_3 - 1)}_{= \gamma_1} y_{t-1} - \beta_3 \Delta y_t + \varepsilon_t$$

$$H_0: \gamma_1 = 0$$

$$\beta_2 + \beta_3 - 1 = 0$$

$$t_{obs}^{DF} = \frac{\gamma_1}{se(\gamma_1)} \sim DF_{TC}$$

...

NC: RW (I(1)) vs Stationary

C: RW with drift (around lin. trend) I(1) vs Stationary (shifted by const.)

CT: DS (I(2)) vs TS

RW with trend (around quad. trend) vs stat. process + lin. trend

Question 3. Consider a linear regression model:

$$y_t = \alpha + \beta x_t + \varepsilon_t,\ t = 1, \ldots, T,$$

where the zero mean error ε_t exhibits autocorrelation of an unknown form. We assume our processes are covariance stationary and exhibit weak dependence. The regressor and error may be assumed to be independent.

(a) Explain what it means to say that $\{\varepsilon_t\}_{t=1}^{T}$ is covariance stationary. Provide an intuitive discussion of the requirements and indicate why these requirements are desirable. **(3 marks)**

(b) Recognising that the errors exhibit autocorrelation, discuss how we can conduct statistical inference on β using the OLS estimator. Specifically, discuss how you can test the hypothesis $H_0 : \beta = 0.7$ against the alternative $H_1 : \beta < 0.7$ using the OLS estimator. **(5 marks)**

a) E, Cov, Var do not dep. on time

ε_t - stationary $\Leftrightarrow$ ε_t - i.i.d

if $E(\varepsilon_t) \neq$ const => inconsistent / biased

$Var(\varepsilon_t) \neq$ const => heteroscedasticity

$Cov(\varepsilon_t, \varepsilon_{t-s}) \neq$ const => unbiased / consistent
$\neq 0$ inefficiency

b) $t = \dfrac{\hat{\beta} - 0{,}7}{se_{HAC}(\hat{\beta})} \sim t_{n-2}$

10. This question is based on "Capital Accumulation and Growth: A New Look at the Empirical Evidence", by Bond et al. (*Journal of Applied Econometrics*, 2010). In this article, the authors are interested in a regression model for the (logarithm of) output-per-capita y_t in a given country and time period t similar to:

0 inst < 1 end. var.

struct. form

$$y_t = \alpha + \rho y_{t-1} + \beta x_t + \gamma t + \varepsilon_t, \quad |\rho| < 1, \tag{10.1}$$

underindetified

where x_t is (the logarithm of) investment-per output. Imagine that investment rates are also affected by current output-per-capita, so that

2 inst. > 1 end. var.

$$x_t = \varphi_0 + \varphi_1 y_t + u_t. \tag{10.2}$$

over identified

Equations (10.1) and (10.2) then form a simultaneous equation model. Both errors ε_t and u_t have zero mean.

instr ≥ # end. var. in RHS

For (a) and (b), we start by assuming that ε_t and u_t are not serially correlated.

(a) **(5 marks)** Obtain the reduced form equations for y_t and x_t (Note: the variables t and y_{t-1} are exogenous). Are equations (10.1) and (10.2) identified? Discuss.

$$y_t = \frac{\alpha + \beta\varphi_0}{1-\beta\varphi_1} + \frac{\rho}{1-\beta\varphi_1} y_{t-1} + \frac{\gamma}{1-\beta\varphi_1} t + \left[\frac{\beta u_t + \varepsilon_t}{1-\beta\varphi_1}\right]$$

$$x_t = \frac{\varphi_0 + \varphi_1\alpha}{1-\beta\varphi_1} + \frac{\rho\varphi_1}{1-\beta\varphi_1} y_{t-1} + \frac{\varphi_1\gamma}{1-\beta\varphi_1} t + \left[\frac{\varphi_1\varepsilon_t + u_t}{1-\beta\varphi_1}\right]$$