

**Elements of Econometrics. 2022-2023.**  
**Class 22. Cointegration**

**Problem 1.** What is the order of integration of the time series? What series are called integrated  $I(0)$ ? What series are called integrated  $I(1)$  and so on? What type of time series are MA(1), AR(1) and random walk ?

**Problem 2.** Explain what do you understand by difference-stationary and trend-stationary time series. Why is it important to know whether a variable is difference- or trend-stationary?

**Problem 3.** Consider a time series process

$$\ln Y_t = \alpha + \beta t + u_t; \quad t = 1, 2, \dots, T$$

Examine the order of integration of  $\ln Y_t$ .

**Problem 4.** What time series are called cointegrated?

**Problem 5.** Consider two time series variables  $Y_t$  and  $X_t$  both are integrated of order one. Explain how you will test that  $Y_t$  and  $X_t$  are cointegrated.

**Problem 6. (Computer practice)** Using file EXPEND.WF1, investigate whether time series COSM, DPI and PRELCOSM are cointegrated

**Problem 7.** Explain the meaning of spurious regression. Why is it important to know whether the time series used in the regression model are stationary? Consider two variables  $Y_t$  and  $X_t$ , where

$$Y_t = \alpha + Y_{t-1} + \varepsilon_t$$
$$X_t = \beta + X_{t-1} + \nu_t.$$

$\varepsilon_t$  and  $\nu_t$  are unrelated white noise processes. A researcher regresses  $Y_t$  on  $X_t$  and tests the significance of the slope coefficient. Discuss in detail the result he will get.

**Problem 8.** How can you make sure that some regression is spurious? Give an example and describe your actions.

**Problem 9. (Computer practice)** How to construct and use models in differences? What are their comparative advantages and disadvantages?

**Problem 10.** Briefly explain the following: Error correction model.

**Problem 11.** How to use **error correction model in practice**? What are their comparative advantages and disadvantages?

**Problem 12.** Explain what is Granger Causality. How to test Granger Causality? What are possible results of this test?

**Problem 13.** Explain what is Vector Autoregression??

## **Problems from real exams**

### **Problem 14. (ICEF Exam)**

Consider an  $ADL(2, 1)$  model:

$$Y_t = \alpha_1 + \alpha_2 Y_{t-1} + \alpha_3 Y_{t-2} + \alpha_4 X_t + \alpha_5 X_{t-1} + u_t; \quad t = 1, 2, \dots, T. \quad (1)$$

where both  $Y_t$  and  $X_t$  are  $I(1)$ ,  $u_t$  is the disturbance term where  $E(u_t) = 0$ ;  $E(u_t^2) = \sigma^2$  and  $E(u_s u_t) = 0$  for any  $s \neq t$ . We will assume that all variables are in logarithmic form so, for example  $\alpha_4$  can be interpreted as a short term  $X$ -elasticity of  $Y$ .

- (a) Explore long run dynamics of this model and find the long term  $X$ -elasticity of  $Y$ . How would you estimate a cointegrating relationship between  $Y_t$  and  $X_t$ ?
- (b) Transform the model (1) into the one relating to the first differences of  $X$  and  $Y$ . What are advantages and disadvantages of this model?
- (c) Supposing that the cointegrating relationship between  $X$  and  $Y$  had been found, express the model (1) as an error correction type model, and describe briefly its dynamics.