

# VAR

(P1)

Cointegrated TS:

① the order of integration  $I(d)$ ,  $d \geq 1$   
↳ ADF test

②  $\exists$  lin. com that is stationary

$$y_t = \alpha_1 + \alpha_2 x_t + \alpha_3 z_t + u_t$$

$$u_t = y_t - \alpha_1 - \alpha_2 x_t - \alpha_3 z_t$$

↳ ADF stationary

(P2)

ECM

$$y_t = \alpha_1 + \alpha_2 x_t + \varepsilon_t$$

$$\Delta y_t = \beta_1 + \beta_2 \Delta x_t + \underbrace{\pi \cdot u_{t-1}}_{\text{LR dynamics}} + \varepsilon_t$$

LR dynamics

$$[y_{t-1} - \alpha_1 - \alpha_2 x_{t-1}]$$

$$y_t, x_t \sim I(1) \quad (*)$$

↳ spurious regression

$$\Delta y_t = \beta_1 + \beta_2 \Delta x_t + \varepsilon_t$$

↳ only SR dynamics

Aux. reg.:  $y_t | x_t \Rightarrow L(\hat{u}_t)$

$\pi$  - adjustment rate

②

# Granger Causality

$$y_t = \alpha_0 + \alpha_1 \cdot y_{t-1} + \dots + \alpha_m y_{t-m} + \beta_1 \cdot x_{t-1} + \dots + \beta_m x_{t-m} + \epsilon_t$$

$$H_0: \beta_1 = \dots = \beta_m = 0$$

X does not Granger cause Y

$$x_t = \alpha_0 + \alpha_1 \cdot y_{t-1} + \dots + \alpha_m y_{t-m} + \beta_1 \cdot x_{t-1} + \dots + \beta_m x_{t-m} + \epsilon_t$$

$$H_0: \alpha_1 = \dots = \alpha_m = 0$$

③

VAR(p) :

VAR(5) for  $y_t, x_t$

$$\text{VAR}(1): \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix} \quad \begin{matrix} \rightarrow 2 \times 11 \\ \text{vecs} \end{matrix}$$

Akaike

$$AIC = \underbrace{-2 \log L(\hat{\theta})}_{\text{Goodness of fit}} + \underbrace{2p}_{\text{complexity}}$$

$$p \uparrow \Rightarrow \log L \uparrow \Rightarrow -\log L \downarrow$$

$$p \uparrow \Rightarrow 2p \uparrow$$

$$AIC = \ln \frac{RSS}{T} + \frac{2p}{T}$$

$$BIC = -2 \log L(\hat{\theta}) + \log(T) \cdot p$$

(P14)

LR elasticity (Analyse LR dynamics)

$$\frac{\alpha_4 + \alpha_5}{1 - \alpha_2 - \alpha_3}$$

$$1 - \alpha_2 - \alpha_3$$

LR equilibrium:

$$\tilde{y} = \frac{\alpha_1}{1 - \alpha_2 - \alpha_3} + \frac{\alpha_4 + \alpha_5}{1 - \alpha_2 - \alpha_3} \cdot \tilde{x}$$