Testing for Autocorrelation

$$y = X\beta + \varepsilon$$

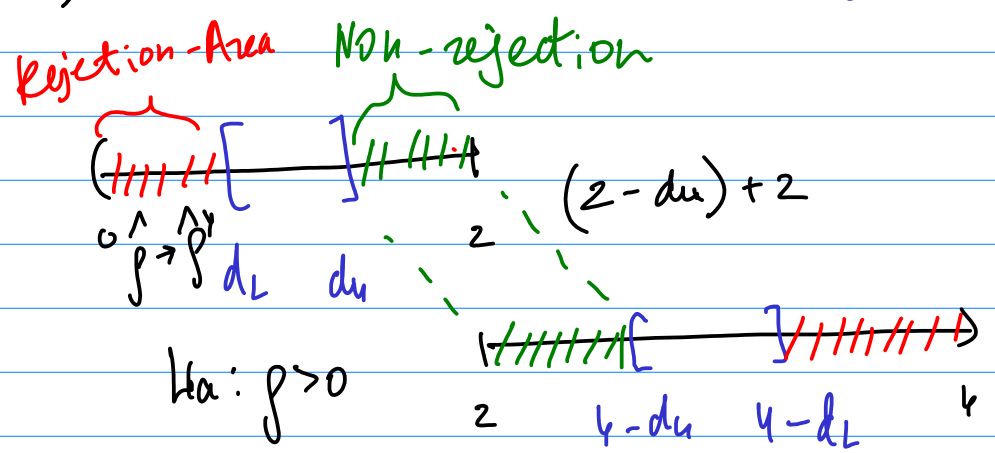
$$\hookrightarrow \varepsilon_t = \rho\,\varepsilon_{t-1} + u_t$$

(I) Durbin Watson:

Disadvantages: 1) Only for 1st order AC

$$DW = \frac{\sum(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum \hat{\varepsilon}_t^2} \longrightarrow 2\cdot(1-\rho) \in (0;4)$$

2) Inconclusive Areas $[d_L ; d_u]$



$H_a: \rho > 0$

$H_a: \rho < 0$

3) Only for models with const ($k \geq 2$, if k - est par.)

4) Only for models w/o y_t:

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

$$\varepsilon_t = \underbrace{\rho\,\varepsilon_{t-1} + u_t}$$

WN

if we add lag y_t

we underestimate $\hat{\rho}$

$\Rightarrow$ $\hat{\rho}$ biased towards 0

DW biased towards 2

* Hence, if $DW \in [0, d_L)$ $\Rightarrow$ H_0 is definitely rejected, else

(II) h - Durbin test

$H_0: \rho = 0$

1) Only for 1st Autocorrelation

2) Only for 1 lag of y_t

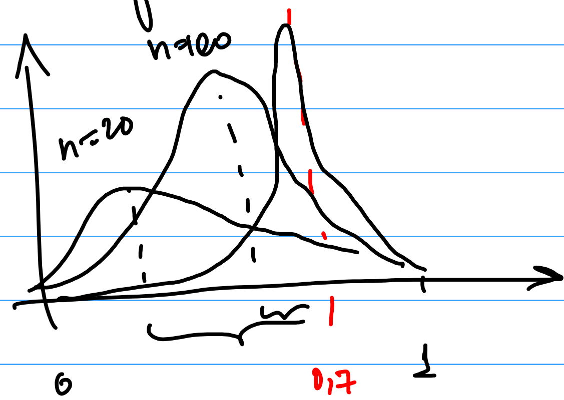
$$h = \underset{\underset{1 - DW \cdot 0{,}5}{\parallel}}{\hat{\rho}} \sqrt{\frac{n}{1 - n \cdot \hat{Var}(\hat{\beta}_{y_{t-1}})}} \overset{H_0}{\sim} N(0; 1)$$

3) Sometimes can't be calculated because of $\sqrt{x}$, $x<0$

$$DW \longrightarrow 2(1 - \rho)$$

$$\hat{\rho} = 1 - \frac{DW}{2} \longrightarrow \rho$$

$$\hat{\varepsilon}_t = \rho\, \hat{\varepsilon}_{t-1} + u_t$$



→ for small n

$\hat{\rho}$ biased towards 0

B6 :

H_0: no AC of q

$$\hat{\varepsilon}_t = \alpha_0 + \beta_1 \cdot X_{1t} + \beta_2 \cdot X_{2t} + \dots$$

$$+ \rho_1 \underbrace{\hat{\varepsilon}_{t-1} + \dots + \rho_q \hat{\varepsilon}_{t-q}} + u_t$$

↳ R^2_{aux}

$$LM = n \cdot R^2_{aux} \sim \chi^2_q$$