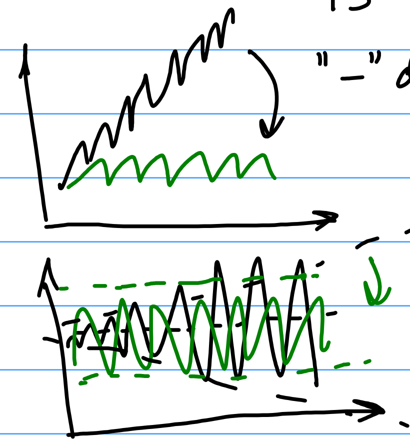
Non-stationary TS

(P1) Weakly-Stationary

1) $E(X_t) = const$

2) $Var(X_t) = const$

3) $cov(X_t, X_{t+s}) = f(s)$



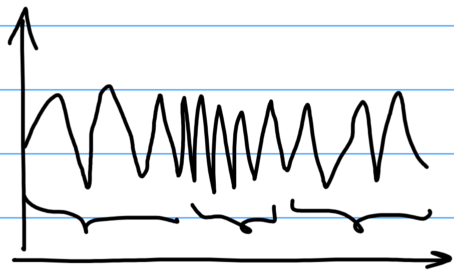
TS-process

"–" det. trend

log

Box-Cox Transf.

$$y_t^{(\lambda)} = \begin{cases} \frac{y^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \ln y & \lambda = 0 \end{cases}$$



Garch or SV

(P2) Stationary Processes:

MA(q) $\forall q$ – always stationary

AR(p), ARMA(p,q) equations

has all roots $|\lambda| \neq 1$ => has stationary solutions

(P3) $y_t = 1{,}4\, y_{t-1} + \varepsilon_t$

$y_t = \beta y_{t-1} + \varepsilon_t$, $0 < \beta < 1$, ε_t – WN

$E(y_0) = 0$

$Var(y_0) = \frac{1}{1-\beta^2} \cdot \sigma_\varepsilon^2$

$$E(y_t) = E(\beta^t y_0 + \beta^{t-1}\varepsilon_1 + \dots + \varepsilon_t) =$$

$$= \beta^t \underset{\overset{\shortparallel}{0}}{E(y_0)} = 0$$

$$Var(y_t) = \beta^{2t} Var(y_0) + \beta^{2t-2}\sigma_\varepsilon^2 + \dots + \sigma_\varepsilon^2 =$$

$$= \beta^{2t} \cdot \frac{1}{1-\beta^2} \cdot \sigma_\varepsilon^2 + \frac{1-\beta^{2t}}{1-\beta^2} \cdot \sigma_\varepsilon^2 =$$

$$= \frac{1}{1-\beta^2} \cdot \sigma_\varepsilon^2 \qquad \frac{b_0}{1-q} = \sum_{q=0}^{t} \beta^q \cdot \sigma_\varepsilon^2$$

$$Cov(y_t, y_{t+s}) = Cov(y_t, \beta^s y_t + \beta^{s-1}\varepsilon_{t+1} + \dots + \varepsilon_{t+s})$$

$$= \beta^s Var(y_t) + 0$$

④ For sample large enough

y_t – weakly persistent / dependent

$cov(y_t, y_{t+s}) \to 0, \quad s \to \infty$

(P4) Non - stationary :

$y_0 = 0$

Random Walk : $y_t = y_{t-1} + \varepsilon_t$

↳ DS - process

$E(y_t) = E(\varepsilon_0) = 0$

DS:

$\Delta y_t = \varepsilon_t$

$Var(y_t) = t \cdot \sigma_\varepsilon^2$

$\forall s > 0 \quad Cov(y_t, y_{t+s}) = t \cdot \sigma_\varepsilon^2$

$\forall s > 0 \quad Cov(y_t, y_{t-s}) = (t-s)\sigma_\varepsilon^2$

TS - process : $y_t - \hat{\alpha} - \hat{\beta} t \Rightarrow$ stat.

$y_t = \underbrace{\alpha + \beta t}_{} + \underbrace{\varepsilon_t}_{}$

$\Delta y_t = y_t - y_{t-1} = \beta + \varepsilon_t - \varepsilon_{t-1}$

(P5) Spurious Regressions:

TS - process :