VAR

(P1) Cointegrated TS:

- Same order of $I(d)$, $d \geq 1$

  ↳ ADF test

- $\exists$ stationary lin. comb.

$$y_t = \alpha + \beta X_t + \gamma \cdot Z_t + u_t$$

$$u_t = y_t - \alpha - \beta X_t - \gamma \cdot Z_t$$

↳ ADF-test

ECM

$$\Delta y_t = \psi_1 + \psi_2 \delta X_t + \pi \cdot \underbrace{u_{t-1}}_{[y_{t-1} - \alpha - \beta X_{t-1}]} + \varepsilon_t$$

LR equilibrium

$y_t | X_t \Rightarrow L(\hat{u}_t)$

$$y_t = \alpha + \beta X_t + \varepsilon_t$$

$y_t, X_t - I(1)$

spurious regression

$$\Delta y_t = \gamma_1 + \gamma_2 \Delta X_t + \varepsilon_t$$

↳ only SR effect

no Long-term equilibrium

Granger Causality

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_m \cdot y_{t-m} +$$
$$+ \beta_1 \cdot X_{t-1} + \dots + \beta_m \cdot X_{t-m} + \varepsilon_t$$

F-test $\quad H_0: \beta_1 = \dots = \beta_m = 0$

X does not Granger cause Y

$$X_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_m \cdot y_{t-m} +$$
$$+ \beta_1 \cdot X_{t-1} + \dots + \beta_m \cdot X_{t-m} + \varepsilon_t$$

$H_0: \alpha_1 = \dots = \alpha_m = 0$

Y does not Granger cause Y

VAR(1) $\qquad$ VAR(5) $\qquad \left. 5\, L(X_t) + 5\, L(Y_t) \right\} \times 2$

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

closeness-of-fit

\# of regressors

$$AIC = -2\log L(\hat{\theta}) + 2p$$

$\uparrow p \ \uparrow \log L \qquad \uparrow p \ \uparrow 2p$

$-\log L \downarrow$

$$BIC = -2\log L(\hat{\theta}) + \log(T)\,p$$

P14

$$y_t = \alpha_1 + \alpha_2 \cdot y_{t-1} + \alpha_3 y_{t-2} + \alpha_4 \cdot X_t + $$
$$+ \alpha_5 X_{t-1} + u_t$$

$$\tilde{y} = \alpha_1 + \alpha_2 \tilde{y} + \alpha_3 \tilde{y} + \alpha_4 \tilde{X} + \alpha_5 \tilde{X}$$

$\Rightarrow$ LR elasticity $\dfrac{\alpha_4 + \alpha_5}{1 - \alpha_2 - \alpha_3}$

LR equilibrium

$$\tilde{y} = \frac{\alpha_1}{1-\alpha_2-\alpha_3} + \frac{\alpha_4+\alpha_5}{1-\alpha_2-\alpha_3}\tilde{X}$$