Autocorrelation

Remedies:

1) Specification:

True: $y_t = \alpha + \beta X_t + \underbrace{\gamma Z_t + \varepsilon_t}_{''}$

Est: $y_t = \alpha + \beta X_t + u_t$

$u_t = \gamma Z_t + \varepsilon_t$

if z_t – AR process $\Downarrow$ Autocorr. of errors

else $\downarrow$ NOT

2) AR(1) transform (Special GLS)

Cohrain - Orcutt procedure

$$y_t = \beta_1 + \beta_2 X_t + u_t \quad \Big| \cdot L \cdot \rho$$

Assumption: $u_t = \rho u_{t-1} + \varepsilon_t$

$$y_t = \beta_1 + \beta_2 X_t + \rho u_{t-1} + \varepsilon_t$$

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$$\rho y_{t-1} = \beta_1 \cdot \rho + \beta_2 \cdot \rho X_{t-1} + \rho u_{t-1}$$

$=$

$$\underbrace{y_t - \rho y_{t-1}}_{} = \beta_1 (1-\rho) + \beta_2 (\underbrace{X_t - \rho \cdot X_{t-1}}_{}) + \varepsilon_t$$

$$y^*_t = \beta^*_1 + \beta_2 X^*_t + \varepsilon_t$$

s.t.: $\hat{\rho} \nearrow \hat{u}_t = \rho \hat{u}_{t-1} + v_t$

$\searrow DW \longrightarrow 2(1-\rho)$

$$\hat{\rho} = 1 - \frac{DW}{2}$$

Price - Winsten correction

for 1^{st} observation

$$y^*_t = \beta_1 + \beta_2 X^*_t + \varepsilon_t$$

$$y_1 = \beta_1 + \beta_2 X_1 + u_1 \quad \Big| \cdot \sqrt{1-\rho^2}$$

$$\sigma_u^2 = Var(\rho u_{t-1} + \varepsilon_t) = \rho^2 \sigma_u^2 + \sigma_\varepsilon^2 + 2\overset{0}{Cov(\rho u_{t-1}, \varepsilon_t)}$$

$$\sigma_u^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2} \qquad (1-\rho^2)\,\sigma_u^2 = \sigma_\varepsilon^2 \Rightarrow$$

3) MA(1) Transform:

$$\hat\beta = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y \leftarrow \quad \text{GLS}$$

$$\Omega = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \sigma_\varepsilon^2 \qquad \text{if } \Omega = \hat\Omega \Rightarrow \text{FGLS}$$

$$\Downarrow$$

$$\hat\beta = (X'X)^{-1}X'y \Rightarrow \text{OLS}$$

$$\Omega = \begin{pmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{pmatrix} \sigma_\varepsilon^2$$

$$\Downarrow$$

WLS

AR(1) in error term $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$

$$\Omega = \frac{\sigma_\varepsilon^2}{1-\rho^2}\begin{pmatrix} 1 & \rho & \cdots & \rho^{n-1} \\ \rho & \ddots & & \vdots \\ \vdots & & \ddots & \rho \\ \rho^{n-1} & \cdots & & 1 \end{pmatrix} \qquad cov(\rho_t, \rho_{t-k}) = \frac{\rho^k}{1-\rho^2}$$

MA(1)

$$\varepsilon_t = u_t + \rho u_{t-1}$$

$$\Omega = \sigma_\varepsilon^2 \begin{pmatrix} 1 & r & 0 & \cdots & 0 \\ r & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & r \\ 0 & \cdots & 0 & r & 1 \end{pmatrix}$$

$$Var(\varepsilon_t) = (1+\rho^2)\sigma_u^2$$

4) Adding Lag of y_t

$$y_t = \beta_1 + \beta_2 X_t + \beta_3 \cdot y_{t-1} + \underbrace{u_t}_{\rho u_{t-1} + \varepsilon_t}$$

Common Factor Test:

From CO transform:

$$y_t = \rho y_{t-1} + \beta_1(1-\rho) + \beta_2(X_t - \rho \cdot X_{t-1}) + \varepsilon_t$$

$$y_t = \overset{\lambda_1}{\beta_1(1-\rho)} + \overset{\lambda_2}{\beta_2} X_t - \overset{\lambda_3}{\rho\beta_2} \cdot X_{t-1} + \overset{\lambda_4}{\rho} y_{t-1} + \varepsilon_t$$

↳ ARDL(1, 1) with restrictions:

$$\lambda_3 = -\lambda_2 \cdot \lambda_4$$

(non-linear restriction

$$\Rightarrow \quad n \cdot \log \frac{RSS_R}{RSS_u} \sim \chi^2_1$$

5) ARDL(p, q)