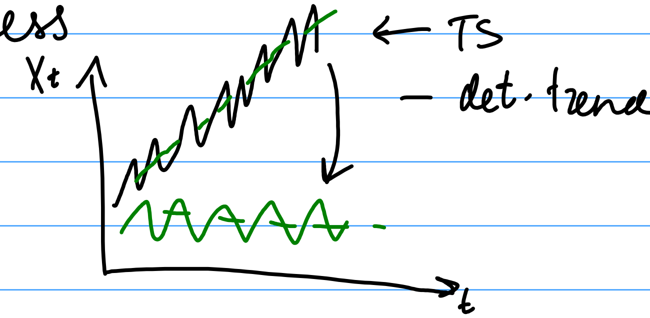
Non-stationarity

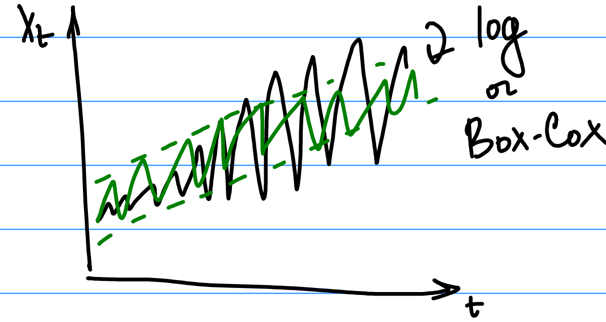
P1 **Weakly-stationary process**

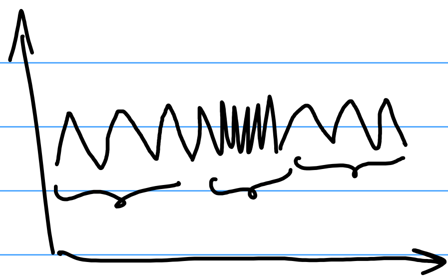
1) $E(X_t) = const$

2) $Var(X_t) = const$

3) $Cov(X_t, X_{t+k}) = \gamma_k = f(k)$







$$y_t^{(\lambda)} = \begin{cases} \dfrac{y^\lambda - 1}{\lambda}, & \lambda \neq 0 \\ \ln y, & \lambda = 0 \end{cases}$$

P2 **Stationary Processes:**

MA(1), MA(q), MA(∞) – all stationary

AR(1), $0 < \beta_1 < 1$

AR(p) or ARMA(p,q) equations has roots $|\lambda| < 1$ $\Leftrightarrow$ has stationary solutions

Non-stationary Processes:

Random Walk: $X_t = X_{t-1} + \varepsilon_t$

↳ Difference-stationary: $\Delta X_t = \varepsilon_t$

$\varepsilon_0 = 0$

$X_0 = 0$

$$E(X_t) = E(\varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_0)$$

$$= E(\varepsilon_0) = \varepsilon_0 = 0$$

$$Var(X_t) = t \cdot \sigma_\varepsilon^2 \Rightarrow$$

not stat.

$$Cov(X_t, X_{t-s}) = (t-s)\sigma_\varepsilon^2$$

$\Rightarrow$ not stat.

Trend - Stationary:

$$X_t = \underbrace{\alpha + \beta t}_{\text{lin. trend}} + \underbrace{\varepsilon_t}_{WN}$$

$X_t - \hat{\alpha} - \hat{\beta} t$

- stationary

1) $E(X_t) = \alpha + \beta t \Rightarrow$ non-stationary

2) $Var(X_t) = \sigma^2$

3) $Cov(X_t, X_{t-s}) = 0 \quad \forall s \neq 0$

$$\Delta X_t = \alpha + \beta t - X_{t-1} + \varepsilon_t$$

$$= \alpha + \beta t - \alpha - \beta(t-1) + \varepsilon_t - \varepsilon_{t-1}$$

$$= -\beta + \underbrace{\varepsilon_t - \varepsilon_{t-1}}$$

P3 $AR(1)$, $0 < \beta < 1$ — is stationary?

$$\begin{cases} E(X_0) = 0 \\ \sigma^2_{X_0} = \dfrac{1}{1-\beta^2} \cdot \sigma^2_\varepsilon \end{cases}$$

$$X_t = \beta X_{t-1} + \varepsilon_t$$

$$X_t = \beta^t X_0 + \beta^{t-1}\varepsilon_1 + \dots + \varepsilon_t$$

$$E(X_t) = \beta^t E(X_0) + \beta^{t-1} \cdot E(\varepsilon_1) + \dots + E(\varepsilon_t) =$$

$$= \beta^t \cdot E(X_0) = 0$$

$$Var(X_t) = \beta^{2t} \cdot Var(X_0) + \underbrace{\beta^{2t-2} \cdot \sigma^2_\varepsilon + \dots + \sigma^2_\varepsilon}$$

$$= \beta^{2t} \cdot \frac{1}{1-\beta^2} \cdot \sigma^2_\varepsilon + \frac{1-\beta^{2t}}{1-\beta^2} \cdot \sigma^2_\varepsilon =$$

$$= \frac{1}{1-\beta^2} \cdot \sigma^2_\varepsilon$$

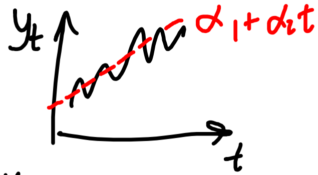
$$Cov(X_t, X_{t+s}) = \beta^s \cdot Var(X_t) + 0$$

$$X_{t+s} = \beta^s X_t + \beta^{s-1} \cdot \varepsilon_{t+1} + \dots + \varepsilon_{t+s}$$

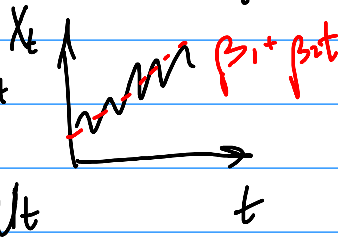
(*) $0 < \beta < 1$ + $s \to \infty$ $\Rightarrow$ $cov(X_t, X_{t+s}) \to 0$

(P4)

TS - DGP:



$$\begin{cases} y_t = \alpha_1 + \alpha_2 t + \varepsilon_t \\ X_t = \beta_1 + \beta_2 t + u_t \end{cases}$$



Est:

$$y_t = \gamma_1 + \gamma_2 X_t + v_t$$

$H_0: \gamma_2 = 0$

True $\gamma_2 = 0$

omitted variable t $\nearrow \alpha_2 \neq 0$, $\searrow \rho_{X_t, t} \neq 0$

$\Rightarrow$ $\hat{\gamma}_2$ - biased

$\Rightarrow$ probably won't be able to not reject true H_0

DS - DGP:

$$\begin{cases} y_t = y_{t-1} + \varepsilon_t \\ X_t = X_{t-1} + u_t \end{cases}$$

Est.:

$$y_t = \gamma_1 + \gamma_2 X_t + v_t$$

$H_0: \gamma_2 = 0$

Under True $\gamma_2 = 0$

$$\underbrace{y_t}_{RW} = \gamma_1 + \underbrace{v_t}_{RW}$$

$\Rightarrow$ GMT ass. are violated

$\Rightarrow$ $se(\hat{\gamma}_2)$, t-tests not valid

TS is weakly dependent

↳ $cov(X_t, X_{t+s}) \to 0, \quad s \to \infty$