HSE and University of London Double Degree Programme in Data Science and Business Analytics

Elements of Econometrics, 2023-2024

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Class 4: Multiple linear regresion.

Problem 1

By definition $R^2_{adj} = \bar{R}^2 = 1 - \frac{RSS/(n-k)}{TSS/(n-1)}$. Compare it with the definition of conventional R^2, what is the difference? Comment on the meaning of $(n-k)$ and $(n-1)$ in this formula.

Problem 2

(a) Recall F-test for linear restrictions.

(b) Show that test for significance is actually F-test with naive restricted model (model on a constant term).

(c) Let the regression equation be

$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$

Number of observations equals to 20

Outline briefly how would you test:

(a) $\beta_2 = 0$

(b) $\beta_2 = 1$

(c) β_1, β_2 jointly insignificant

(d) $\beta_1 = \beta_2$

(e) $\beta_1 + \beta_2 = 1$

(f) $\beta_1 + \beta_2 = 1$ against $\beta_1 + \beta_2 < 1$

Problem 3

Let a regression equation be:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t;\ t = 1,2,\dots,T$$

What are normal equations of OLS for the multiple linear regression in covariance form? Use sample variance and sample covariance functions to represent normal equations of OLS **without taking derivatives**.

Derive formulas for regression coefficients in the case of two regressors?

$$b_2 = \frac{\text{Cov}(X_2,Y)\text{Var}(X_3) - \text{Cov}(X_3,Y)\text{Cov}(X_2,X_3)}{\text{Var}(X_2)\text{Var}(X_3) - [\text{Cov}(X_2,X_3)]^2}$$

$$b_3 = \frac{\text{Cov}(X_3,Y)\text{Var}(X_2) - \text{Cov}(X_2,Y)\text{Cov}(X_2,X_3)}{\text{Var}(X_2)\text{Var}(X_3) - [\text{Cov}(X_2,X_3)]^2}$$