Seminar 9.

a) $X \uparrow 1 \Rightarrow y \uparrow \hat{\beta}_1$

b) $X \uparrow 1\% \Rightarrow y \uparrow \hat{\beta}_1\%$

c) $X \uparrow 1\% \Rightarrow y \uparrow \frac{\hat{\beta}_1}{100}$

$X \uparrow \frac{1}{100} \Rightarrow y \uparrow \frac{\hat{\beta}}{100}$

d) $X \uparrow 1 \Rightarrow y \uparrow \beta 100\%$

d) $\Delta \log(\hat{y}_i) = \Delta(\hat{\beta}_0 + \hat{\beta}_1 \cdot X_i)$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = \hat{\beta}_1 \Delta X_i$$

$$100 \cdot \hat{\beta}_1 = \frac{\Delta \hat{y}_i / \hat{y}_i \cdot 100}{\Delta X_i}$$

$\Delta X_1 = 1 \Rightarrow \frac{\Delta \hat{y}_i}{\hat{y}_i} \cdot 100\% \uparrow 100 \cdot \hat{\beta}_1 \%$

Q2

$|\hat{\beta}_2| < 0{,}1$ approx "$100 \cdot \hat{\beta}_2 \%$"

$d \uparrow 0 \to 1 \qquad \Delta d_i = 1$

$|\hat{\beta}_2| \geq 0{,}1$
$\Delta X_i \geq 1$

"$100 (e^{\hat{\beta}_2} - 1) \%$" exact formula

$$|\hat{\beta}_2| < 0{,}1 : \quad e^{\hat{\beta}_2} \approx 1 + \hat{\beta}_2$$

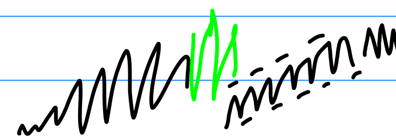
a)

$$\Delta \log(\hat{y}_i) = \Delta(\hat{\beta}_0 + \hat{\beta}_1 \cdot x_i)$$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = \hat{\beta}_1 \Delta x_i$$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = \frac{e^{(\hat{\beta}_0 + \hat{\beta}_1 \cdot (\Delta x_i + x_i))} - e^{(\hat{\beta}_0 + \hat{\beta}_1 \cdot x_i)}}{e^{(\hat{\beta}_0 + \hat{\beta}_1 \cdot x_i)}}$$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = e^{\hat{\beta}_1 \cdot \Delta x_i} - 1$$



$\sqrt[2]{x}$ $BC(2)$ $\frac{x^\lambda - 1}{\lambda}$

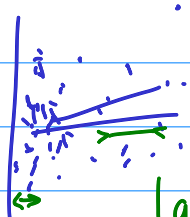
TBATS

Box-Cox

$\Delta x_i = 1 \Rightarrow \frac{\Delta \hat{y}_i}{\hat{y}_i} = e^{\hat{\beta}_1} - 1$

$\lambda \to 1$

$\lambda \to 0$



$$100 \frac{\Delta \hat{y}_i}{\hat{y}_i} \% = (e^{\hat{\beta}_1} - 1) \cdot 100 \%$$

$lg(x)$ $BC(\lambda)$ $0{,}7$

$$y_i = \alpha \cdot K^{\beta_1} \cdot L^{\beta_2}$$

Ex

$$\widehat{\ln y_i} = \underset{(2)}{-10} + \underset{(0{,}01)}{0{,}07} \ln x_i + \underset{(0{,}1)}{0{,}9}\, d_i + \underset{(0{,}01)}{0{,}07}\, z_i$$

$\overline{\ln y_i} = -10$ $x_i = 1$ v $d_i = 0$ v $z_i = 0$

$\bar{y}_i = \exp(-10)$

y_i – GDP of country (thsnd. \$).

x_i – consumption (thsnd. \$)

sign.: [if cons ↑ 1 % => GDP ↑ 0,07 % on average other par. being]

z_i – Investment (thsnd. \$)

sign.: if z_i ↑ 1 th \$ => GDP ↑ $0{,}07 \cdot 100 = 7\%$ $(e^{0{,}07} - 1) \cdot 100 = 7{,}25\%$

d_i – $\begin{cases} 1 & \text{law implemented} \\ 0 & \text{no law} \end{cases}$

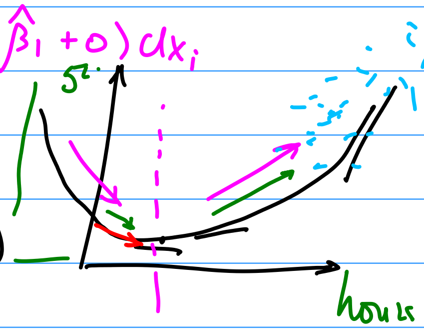
$100(e^{0,9}-1)\% = \boxed{146\%}$

in counties where law implemented

$100 \cdot 0,9 = 90\%$

$X_i = 0$: $\bar{y}_i - \hat{\beta}_0$

$dy = (\hat{\beta}_1 + 0)\, dx_i$



Q3

a) $y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{1i}^2 + \varepsilon_i$

$$d\hat{y}_i = d(\hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{1i}^2)$$

$$d\hat{y}_i = \hat{\beta}_1\, dx_{1i} + 2\hat{\beta}_2 X_{1i} \cdot dx_{1i}$$

$\hat{\beta}_2 > 0$

$$\left[d\hat{y}_i = (\hat{\beta}_1 + 2\hat{\beta}_2 x_{1i})\, dx_{1i} \right]$$

$-\dfrac{\hat{\beta}_1}{2\hat{\beta}_2}$

$\Delta x_i^2 \uparrow 1 \Rightarrow y_i \uparrow \hat{\beta}_2$ $\quad dx_{1i} = 1$ $\quad d\hat{y}_i = (\hat{\beta}_1 + 2\hat{\beta}_2 x_{1i})$

* an additional hour $\uparrow$ grade given that $x_i = 4h$

b) $\underset{\text{grade}}{y_i} = \beta_0 + \beta_1 \underset{\text{hours}}{X_{1i}} + \beta_2 \underset{\text{grade}(t-1)}{X_{2i}} + \beta_3 X_{1i} X_{2i} + \varepsilon_i$

$$d\hat{y}_i = d(\hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 \cdot X_{2i} + \hat{\beta}_3 X_{1i} X_{2i})$$

$$d\hat{y}_i = \hat{\beta}_1 \cdot dx_{1i} + \hat{\beta}_2\, dx_{2i} + \hat{\beta}_3 X_{1i}\, dx_{2i} + \hat{\beta}_3 \cdot x_{2i} \cdot dx_{1i}$$

$$d\hat{y}_i = \underbrace{(\hat{\beta}_1 + \hat{\beta}_3 X_{2i})}\, dx_{1i} + (\hat{\beta}_2 + \hat{\beta}_3 X_{1i})\, dX_{2i}$$

(*) $\text{grade}_{t-1} = 7$

(*) $\hat{\beta}_1 / \hat{\beta}_2 \quad x_{2i} = 0 / X_{1i} = 0$