HSE and University of London Double Degree Programme in Data Science and Business Analytics

Elements of Econometrics, 2023-2024

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Class 4: Multiple linear regresion.

Problem 1

By definition $R^2_{adj} = 1 - \frac{RSS/(n-k)}{TSS/(n-1)}$.

Compare it with the definition of conventional R^2, what is the difference? Comment on the meaning of $(n-k)$ and $(n-1)$ in the formula.

Problem 2

(a) Recall F-test for linear restrictions.

(b) Show that test for significance is actually F-test with naive restricted model (model on a constant term).

(c) Let the regression equation be

$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$

Number of observations equals to 20

Outline briefly how would you test:

(a) $\beta_2 = 0$

(b) $\beta_2 = 1$

(c) β_1, β_2 jointly insignificant

(d) $\beta_1 = \beta_2$

(e) $\beta_1 + \beta_2 = 1$

(f) $\beta_1 + \beta_2 = 1$ against $\beta_1 + \beta_2 < 1$

Problem 3

Let a regression equation be:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t : t = 1, 2, \ldots, T$$

What are normal equations of OLS for the multiple linear regression in covariance form? Use sample variance and sample covariance functions to represent normal equations of OLS without taking derivatives.

Show that formulas for regression coefficients in the case of two regressors are the following

$$b_2 = \frac{\operatorname{Cov}(X_2, Y)\operatorname{Var}(X_3) - \operatorname{Cov}(X_3, Y)\operatorname{Cov}(X_2, X_3)}{\operatorname{Var}(X_2)\operatorname{Var}(X_3) - [\operatorname{Cov}(X_2, X_3)]^2}$$

$$b_3 = \frac{\operatorname{Cov}(X_3, Y)\operatorname{Var}(X_2) - \operatorname{Cov}(X_2, Y)\operatorname{Cov}(X_2, X_3)}{\operatorname{Var}(X_2)\operatorname{Var}(X_3) - [\operatorname{Cov}(X_2, X_3)]^2}$$