# Seminar 9

a) $x \uparrow 1 \Rightarrow y \uparrow \hat{\beta}$

b) $x \uparrow 1\% \Rightarrow y \uparrow \hat{\beta}\%$

c) $x \uparrow 1\% \Rightarrow y \uparrow \frac{\hat{\beta}}{100}\%$

d) $x \uparrow 1 \Rightarrow y \uparrow \beta \cdot 100\%$

$(|\beta| < 0{,}1)$

**Q1**

$x \uparrow \quad y \uparrow$

d) $\log y_i = \beta_1 + \beta_2 x_i + u_i$

$\Delta \log \hat{y}_i$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = \hat{\beta}_2 \cdot \Delta x_i$$

else

$x \uparrow 1 \Rightarrow y \uparrow (e^{\beta_2} - 1) \cdot 100\%$

$$100 \cdot \hat{\beta}_2 = \frac{100 \cdot \Delta y_i / y_i}{\Delta x_i}$$

$$\Delta x_i = 1 \Rightarrow 100 \cdot \frac{\Delta y_i}{y_i} = 100 \cdot \hat{\beta}_2 \, \%$$

**Q2**

$$\widehat{\ln y_i} = \underset{(2)}{-10} + \underset{(0{,}01)}{0{,}07} \ln x_i + \underset{(0{,}01)}{0{,}07} \cdot z_i + \underset{(0{,}1)}{0{,}9} \cdot d_i, \quad R^2 = 0{,}95$$

$y_i$ – GDP of country (1000 \$)

$z_i$ – Dummy var. $\begin{cases} 1 & \text{law imp.} \\ 0 & \text{else} \end{cases}$

$d_i$ – Investment (1000 \$)

$x_i$ – Consumption (1000 \$)

- Cons. $\uparrow$ 1% $\Rightarrow$ GDP $\uparrow$ 0,07 % on average
- for countries with law implemented

GDP ↑ by 7% on average

$\cdot\ 100(e^{0,07}-1) = 7,25\%$

$x \uparrow 1 \Rightarrow y \uparrow 100(e^{\beta_2}-1)\% \approx y \uparrow 100 \cdot \beta_2 \%$

$e^{\beta_2} \approx 1+\beta_2 \qquad |\beta_2| < 0,1$

$$\log y_i = \beta_1 + \beta_2 x_i + u_i \qquad (*)$$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = \hat{\beta}_2 \cdot \Delta x_i$$

$$\frac{\Delta \hat{y}_i}{\hat{y}_i} = \frac{e^{\hat{\beta}_1 + \hat{\beta}_2(x_i+\Delta x_i)} - e^{\hat{\beta}_1 + \hat{\beta}_2 x_i}}{e^{\hat{\beta}_1 + \hat{\beta}_2 \cdot x_i}}$$

$$= e^{\hat{\beta}_2 \Delta x_i} - 1$$

$$\Delta x_i = 1 \qquad 100 \cdot \frac{\Delta \hat{y}_i}{\hat{y}_i} = (e^{\hat{\beta}_2} - 1) \cdot 100$$

$d_i \uparrow 1$ th.\$ $\Rightarrow y_i \uparrow 90\%$ on average

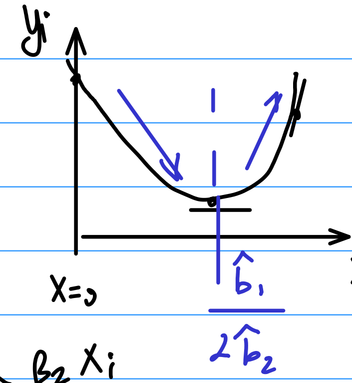
$d_i \uparrow 1$ th\$ $\Rightarrow y \uparrow (e^{0,5}-1) \cdot 100 =$ 145,96%

Q3 $\quad y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i \quad (\Delta x_i)$

$$\Delta \hat{y}_i = \hat{\beta}_1 \Delta x_i + \hat{\beta}_2 \Delta x_i^2 =$$

$$= \hat{\beta}_1 \Delta x_1 + 2\hat{\beta}_2 x_i \Delta x_i =$$

$$= \Delta x_i (\beta_1 + 2\beta_2 x_1)$$




$\Leftarrow \qquad x_i = 0$

$\Delta \hat{y}_i = \Delta x_i \cdot \beta_1$

$\Delta \hat{y}_i = \Delta x_i^2 \cdot \beta_2$

$\Delta x_i = 1 \qquad \Delta \hat{y}_i = \beta_1 + 2\beta_2 x_i$

e.g. for $x_i$ = 10 hours

$- \frac{\hat{\beta}_1}{2\hat{\beta}_2}$ – extr. of parabola

(?) effect of additional hour

b) $y_i = \beta_0 + \beta_1 \cdot x_{1i} + \beta_2 \cdot x_{2i} + \beta_3 \cdot \underline{x_{1i} x_{2i}} + \varepsilon_i$

$$\Delta \hat{y}_i = \hat{\beta}_1 \cdot \Delta x_{1i} + \hat{\beta}_2 \cdot \Delta x_{2i} + \hat{\beta}_3 \Delta x_{1i} x_{2i} =$$

$$= \hat{\beta}_1 \cdot \Delta x_{1i} + \hat{\beta}_2 \cdot \Delta x_{2i} +$$

$$+ \hat{\beta}_3 \cdot x_{2i} \cdot \Delta x_{1i} + \hat{\beta}_3 \cdot x_{1i} \cdot \Delta x_{2i}$$

$\Delta x_{1i}$ : $\quad \Delta \hat{y}_i = (\hat{\beta}_1 + \hat{\beta}_3 \cdot x_{2i}) \Delta x_{1i}$

$\Delta x_{2i} = 0$

$\Delta x_1 \uparrow 1 \Rightarrow \Delta y_i \uparrow \hat{\beta}_1 \qquad x_2 = 0$

for $x_2 = 5 \Rightarrow \ldots$

Q4

1) $$y_i = \alpha \cdot K_i^{\beta_1} \cdot L_i^{\beta_2} \cdot \varepsilon_i$$

$\varepsilon_i \sim N_i$

$\varepsilon_i \sim \log N_i$

$$\ln y_i = \ln \alpha + \beta_1 \ln K_i + \beta_2 \cdot \ln L_i + \ln \varepsilon_i$$

$$\Rightarrow \text{OLS}$$

2) $$y_i = \alpha \cdot K_i^{\beta_1} \cdot L_i^{\beta_2} + \varepsilon_i$$

$$\not\Rightarrow \text{OLS}$$

$$y_i = f(X'_i) + \varepsilon_i$$

$$RSS = \sum \hat{\varepsilon}_i^2 = \sum \left(y_i - f(x)\right)^2 \xrightarrow[\hat{\beta}_1, \dots, \hat{\beta}_k]{} \min$$

Non-linear Least Squares