Problem 1

Consider this model, the equation

$$C_t = \alpha + \beta Y_t + \varepsilon_t,$$

where C_t is real consumption per capita; Y_t - real income per capita; ε_t - random error, is complemented by the ratio

$$Y_t = C_t + I_t,$$

where I_t is real investment per capita.

This leads to the system of equations

$\gamma \cdot I_t \ ; \ \gamma = 0$

$$\begin{cases} C_t = \alpha + \beta Y_t + \varepsilon_t \\ Y_t = C_t + I_t \end{cases}$$

Q1. St.
$$\begin{cases} C_t = \alpha + \beta Y_t + \varepsilon_t \\ Y_t = C_t + I_t \end{cases}$$

Red.
$$\begin{cases} C_t = \dfrac{\alpha}{1-\beta} + \dfrac{\beta}{1-\beta} I_t + \dfrac{1}{1-\beta} \varepsilon_t \\ Y_t = \dfrac{\alpha}{1-\beta} + \dfrac{1}{1-\beta} I_t + \dfrac{1}{1-\beta} \varepsilon_t \end{cases}$$

$$\mathrm{Cov}(Y_t, \varepsilon_t) = \frac{1}{1-\beta} \cdot \mathrm{Cov}(\varepsilon_t, \varepsilon_t) = \frac{\sigma_\varepsilon^2}{1-\beta} \neq 0$$

Q2

$$\begin{cases} Q_t = a_0 + a_1 P_t + a_2 Y_t + u_t \\ Q_t = b_0 + b_1 P_t + b_2 R_t + v_t \end{cases}$$

i	Q_t	P_t	1	Y_t	R_t	
1	1	a_1	a_0	a_2	0	$\to$ I.
2	1	b_1	b_0	0	b_2	$\to$ I

① $(b_2)_{1\times 1}$ $\quad 1 = g-1 = 1$

$g = 2$

$(a_2)_{1\times 1}$ $\quad 1 = 2-1$

$\text{rank}(A) \geq g-1$

$$\begin{cases} Q_t = a_0 + a_1 P_t + u_t \\ Q_t = b_0 + b_1 P_t + b_2 R_t + b_3 S_t + v_t \end{cases}$$

1) OVER I.

2) rank$(A) = 0$

$\Rightarrow$ NOT I.

i	Q_t	P_t	1	R_t	S_t
1	1	a_1	a_0	0	0
2	1	b_1	b_0	b_2	b_3

$\left((b_2, b_3) \right)$ rank$(A) = 1 = g - 1 = 1$

Q3. St.:

$$\begin{cases} Q_t = a_0 + a_1 P_t + a_2 Y_t + u_t \\ Q_t = b_0 + b_1 P_t + b_2 R_t + v_t \end{cases}$$

Red.:

$$\begin{cases} Q_t = \pi_{11} + \pi_{21} Y_t + \pi_{31} R_t + w_{t1} \\ P_t = \pi_{12} + \pi_{22} Y_t + \pi_{32} R_t + w_{t2} \end{cases}$$

St:

$$Q_t - a_1 P_t = a_0 + a_2 Y_t + u_t$$

$$Q_t - b_1 P_t = b_0 + b_2 R_t + v_t$$

$$(Q_t, P_t)\begin{pmatrix} 1 & 1 \\ -a_1 & -b_1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -a_1 & -b_1 \end{pmatrix}^{-1} =$$

$$= \left((1 \quad y_t \quad R_t)\begin{pmatrix} a_0 & b_0 \\ a_2 & 0 \\ 0 & b_2 \end{pmatrix} + (u_t, v_t) \right)\begin{pmatrix} 1 & 1 \\ -a_1 & -b_1 \end{pmatrix}^{-1}$$

$\pi_{11} = \dots$

$\dots$

$\pi_{32} = \dots$

$\hat{\pi}_{11}, \dots, \hat{\pi}_{32} \Rightarrow \hat{a}_0, \dots, \hat{b}_2$

$$(Q_t, P_t) = (1, y_t, R_t)\,\Pi + (u_t, V_t)\cdot\tilde{\Pi}$$

$$\Pi = \underbrace{\begin{pmatrix} a_0 & b_0 \\ a_2 & 0 \\ 0 & b_2 \end{pmatrix}}_{3\times 2} \cdot \frac{1}{a_1 - b_1} \cdot \underbrace{\begin{pmatrix} -b_1 & -1 \\ a_1 & 1 \end{pmatrix}}_{2\times 2} = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \\ \pi_{31} & \pi_{32} \end{pmatrix}$$

Gen.

$$\begin{cases} \gamma_{11} y_{t1} + \dots + \gamma_{g1} Y_{tg} = \beta_{11} X_{t1} + \dots + \beta_{k1} X_{tk} + u_{t1} \\ \vdots \\ \gamma_{1g} y_{t1} + \dots + \gamma_{gg} Y_{tg} = \beta_{1g} X_{t1} + \dots + \beta_{kg} X_{tk} + u_{tg} \end{cases}$$

x_t : exogenous + predetermined

Q_t - endog. ; Q_{t-1}

St: $y_t \Gamma = x_t B + u_t \quad | \cdot (\Gamma^{-1})$

Red. $y_t = x_t B \Gamma^{-1} + u_t \Gamma^{-1}$

$$A = \begin{bmatrix} \Gamma \\ B \end{bmatrix}$$

$$\Phi_i a_i = 0 \qquad i = \overline{1, g}$$

$$\Phi_i \quad r_i \times (g+K)$$

Rank: $\quad \operatorname{rank}(\Phi_i A) = g-1$

Order: $\quad k_i \geq g-1$

$$A = \begin{pmatrix} 1 & 1 \\ -a_1 & -b_1 \\ a_0 & b_0 \\ a_2 & 0 \\ 0 & b_2 \end{pmatrix} \qquad \Phi_2 = (0,0,0,1,0)$$

$$\Phi_1 a_1 = 0 \quad \Rightarrow \quad \Phi_1 = (0,0,0,0,1)$$

$$\Phi_1 A_1 = (0,0,0,0,1) \begin{pmatrix} 1 \\ -b_1 \\ b_0 \\ 0 \\ b_2 \end{pmatrix} = b_2$$

$$\operatorname{rank}(\Phi_1 A_1) = \operatorname{rank}(\Phi_1 A) = 1$$

$$A = \begin{pmatrix} 1 & 1 \\ -a_1 & -b_1 \\ a_0 & b_0 \\ 0 & b_1 \\ 0 & b_2 \end{pmatrix}$$

$$\Phi_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Phi_1 A = \begin{pmatrix} 0 & b_1 \\ 0 & b_2 \end{pmatrix}$$

2 $\operatorname{rank}(\Phi_2 A) = 0$

$\Rightarrow$ not id.

$\Rightarrow$ system is part. identified

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$\operatorname{Rank}(\Phi_1 A) = 1$

$\Rightarrow$ id.

$k_1 = 2 > g - 1 = 1$