Home Assignment #1.

Problem 1

$$X_1 = \Delta_0 S_0 \left(\frac{S_1 - S_0}{S_0} - r \right) r + (1+r) X_0$$

Then

$$X_1(H)(1+r)X_0 = \Delta_0 S_0[u(1+r)];$$

$$X_1(T)(1+r)X_0 = \Delta_0 S_0[d(1+r)].$$

We assume that following condition holds $d < 1+r < u$, then $S_0[u(1+r)] > 0$ and $S_0[d(1+r)] < 0$.

So

$$X_1(H) > (1+r)X_0 \Rightarrow X_1(T) < (1+r)X_0;$$

$$X_1(T) > (1+r)X_0 \Rightarrow X_1(H) < (1+r)X_0.$$

And if $X_0 = 0$

$$X_1(H) > 0 \Rightarrow X_1(T) < 0;$$

$$X_1(T) > 0 \Rightarrow X_1(H) < 0.$$

Problem 2

$$X_1(u) = \Delta_0 * 8 + \Gamma_0 * 3 - 5/4(4\Delta_0 + 1.20\Gamma_0) = 3\Delta_0 + 1.5\Gamma_0$$

$$X_1(d) = \Delta_0 * 2 - 5/4(4\Delta_0 + 1.20\Gamma_0) = -3\Delta_0 - 1.5\Gamma_0 = -X_1(u)$$

So if $Pr(X_1 > 0) > 0 \Rightarrow Pr(X_1 < 0) > 0$ and there is no arbitrage if the time-zero price of the option is 1.20.

Problem 3

$$V_0 = \frac{1}{1+r} \left(\frac{1+r-d}{u-d} S_1(H) + \frac{u-1-r}{u-d} S_1(T) \right) = \frac{S_0}{1+r} \left(\frac{1+r-d}{u-d} u + \frac{u-1-r}{u-d} d \right) = S_0$$