ФИНАНСЫ

(1)

$$(NPV) = -1600 + \frac{10000}{1,1} + \frac{(-10000)}{1,21} = -773$$

$$(IRR:) -1600 + \frac{10000}{1+IRR} + \frac{-10000}{1+IRR} = 0$$

$$\Rightarrow IRR_1 = 1/4 \quad IRR_2 = 4$$
$$=25\% \qquad\qquad =400\%$$

$$\hookrightarrow [-1600(1+IRR) + 10000](1+0,1) - 10000 = 0$$

реин. ст.

$$NPV = \sum \frac{CF_t}{(1+k)^t} \quad vs \quad IRR: \; I_0 = \sum \frac{CF_t}{(1+IRR)^t}$$

(2)

$$\begin{cases} \sigma^2(R_p) \to min \\ a+b=1 \end{cases}$$

$$\sigma^2(R_p) = a^2\sigma^2(X) + (1-a)^2\sigma(Y) + 2a(1+a)\, r_{xy}\, \sigma(X)\sigma(Y) \to min$$

$a \in [0;1]$

$$\frac{\partial \sigma(R_p)}{\partial a} = 0 \Rightarrow a^* = \frac{\sigma_y^2 - r_{xy}\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2 - 2r_{xy}\sigma_x\sigma_y}$$

+

1. PV Relations

- Balance Sheet:

Assets	Liabilities
Cash	Equity
Capital	Debt
Intangibles	

- Income Statement:

Sources of funds = Uses of funds

$$\Delta S + \Delta B + NI = I + D + T + C$$

- The Perpetuity

$$PV = \frac{C}{r} \quad , \ C - \text{const cashflow}$$

$$PV = \frac{C}{r-g} \quad , \ g - \text{growth rate}$$

$$\triangleright \ PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \dots$$

$$\frac{1+r}{1+g} \times PV = \frac{C}{1+g} + \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \dots$$

$$\left[\frac{1+r}{1+g} - 1\right] \times PV = \frac{C}{1+g}$$

$$PV = \frac{C}{r-g} \ , \quad r > g$$

- The Annuity

$$PV = \frac{C}{1+r} + \dots + \frac{C}{(1+r)^T}$$

$$r \times PV = C - \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r} - \frac{C}{r} \cdot \frac{1}{(1+r)^T}$$

$$PV = C \times \frac{1}{r} \times \underbrace{\left[1 - \frac{1}{(1+r)^T}\right]}_{ADF(r,T)}$$

- Inflation

$$r_{real} = \frac{1 + r_{nom}}{1+\pi} - 1 \approx r_{nom} - \pi$$

2. Fixed - Income Securities

- zero-coupon bonds or STRIPS

$$P_0 = \frac{F}{(1+r)^T}$$

$$P_0 = \frac{F}{(1+R_1)\dots(1+R_T)} \quad \dots = \quad \frac{F}{(1+r_{0,T})^T}$$

$\uparrow$ Today's T-Year Spot Rate

$$P_{0,1} = \frac{F}{1+R_1} \rightarrow r_{0,1}$$

$$P_{0,2} = \frac{F}{(1+R_1)(1+R_2)} \rightarrow r_{0,2}$$

...

$\{P_{0,1}, \dots, P_{0,T}\} \rightarrow \{r_{0,1}, \dots, r_{0,T}\}$ ← Term Structure of Int. Rates

↳ cont. Info about future int rates

$$\frac{P_{0,1}}{P_{0,2}} = \frac{F}{1+R_1} : \frac{F}{(1+R_1)(1+R_2)} = (1+R_2)$$

↳ One-Year forward rates

$$\frac{P_{0,t-1}}{P_{0,t}} = 1 + f_t = \frac{(1+r_{0,t})^t}{(1+r_{0,t-1})^{t-1}} \qquad \left(\text{e.g. } f_4 = \frac{(1+r_{0,4})^4}{(1+r_{0,3})^3} - 1\right)$$

- forw. int. rates are today's rates for trans. between two future dates (t_1 and t_2, e.g.)
- note: future spot rates can be different from current corresp. forw. rates
- short sales - fin. transaction in which an ind. can sell a security (that he doesn't own) by borrowing the sec. from another party, selling it and receiving the proceeds, and then buying back the sec. and returning it to the original owner at a later date (possibly w/ cap. gain or loss)

- Coupon Bonds

$$P_0 = \frac{C}{1+R_1} + \frac{C}{(1+R_1)(1+R_2)} + \dots + \frac{C+F}{(1+R_1)\dots(1+R_T)}$$

$$P_0 = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \dots + \frac{C+F}{(1+y)^T}$$

↑ summarized fut. spot rates

y = YTM

- liq. pref. model

$$E_0[R_k] = f_k - \text{liquidity premium} \qquad (E_0[R_k] < f_k)$$

↑ exp. fut. spot ↑ current forw

- In General: $p = C \cdot P_{0,1} + C\, P_{0,2} + \dots + (C+F)\, P_{0,T}$

↑ if not => arbitrage opportunities

e.g. if > => short coupon bond
buy C disc. bond of all mat. up to T
and F disc. bond of mat. T
=> no risk, pos. profits = arbitrage

- Macaulay Duration

$$D_m = \sum^T k \cdot w_k \quad , \quad \sum^T w_k = 1$$

↑ weighted average term to mat.

$$w_k = \frac{c_k/(1+y)^k}{P} = \frac{PV(c_k)}{P}$$

$$p = \sum^T \frac{c}{(1+y)^k}$$

sens. of bond prices to y changes

$$\frac{\partial p}{\partial y} = -\frac{1}{1+y} \sum^T k \cdot \frac{c_k}{(1+y)^k}$$

$$\frac{1}{p} \cdot \frac{\partial p}{\partial y} = -\frac{D_m}{1+y} = -D_m^*$$

↑ mod. dur.

$$\text{Annual } D_m = \sum^T \frac{k \cdot w_k}{q}$$

$$\text{Annual } D_m^* = \frac{An.\, D_m}{1 + y/q}$$

- Convexity - sens. of dur. to y changes

$$\frac{1}{p} \cdot \frac{\partial^2 p}{\partial y^2} = V_m$$

- $$P(y') \approx P(y) + \frac{\partial p}{\partial y}(y)(y'-y) + \frac{\partial^2 p}{\partial y^2}(y) \cdot \frac{(y'-y)^2}{2} = P(y)\left[1 - D_m^*(y'-y) + \frac{1}{2} V_m (y'-y)^2\right]$$

- Decomp. of corp. Bond yields

Promised YTM - yield if default doesn't occur

Expected YTM - prob. -weighted average of all pos. yeilds

Def. premium = diff. between prom. y and exp. y

Risk premium = diff. between exp. y on a risky bond and the y on a risk - free bond of similar maturity and coupon rate

3. Equities

The Primary Market (Underwriting)

- Venture cap. ;
- IPO
- SEO (secondary / seasoned)
- stock insurance

Secondary market (Resale Market)

- Organized exchanges (NYSE, AMEX, NASDAQ)
- Brokers, ECN's
- OTC : Nasdaq.

- Common Stock valuation

$$P_t = \sum^{\infty} \frac{E_t [D_{t+k}]}{(1+r_{t+k})^k} = \sum^{\infty} \frac{D}{(1+r)^k} = \frac{D}{r}$$

if $E_t [D_{t+k}] = D$, $r_{t+k} = r$

- Gordon growth model

$$P_t \sum^{\infty} \frac{E_t [D_{t+k}]}{(1+r_{t+k})^k} = \sum^{\infty} \frac{D (1+g)^{k-1}}{(1+r)^k} = \frac{D}{r-g} \quad , \quad r>g$$

$$P_t = \frac{D}{r-g}$$

$$\left[r = \frac{D}{P_t} + g = \frac{P_0 (1+g)}{P_t} + g \right]$$

- Payout ration : div / earnings = DPS/EPS
- Retained earnings : earn − div
- Plowback Ratio b : ret. earn / tot. earn.
- Book value BV : cum. retained earn.
- Ret. on Book eq. ROE : earn./BV

e.g. $D_1 = p \times EPS$

$g = b \times ROE$

$P_0 = \frac{D_1}{r-g} = \frac{EPS_1}{r}$

$$NPV_1 = -0{,}4 \cdot 8{,}33 + \frac{0{,}25 \cdot 3{,}33}{0{,}15} = 2{,}22$$

ret. rate

0,15 ← r = cost of cap

$NPV_2 = 2{,}22 \cdot \underset{\uparrow g}{11}$

$$P_0 = \frac{EPS_1}{r} + PVGO$$

⇑

$$PVGO = \frac{NPV_1}{r-g}$$

↑ pr. val. of growth op. ← dif. bet. growth / no growth

4. Forward / Futures Contracts

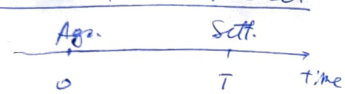
- Types of derivatives

Forwards / Future – a contr. to exchange an asset in the future at a spec. price and time

Options – gives the holder the right to buy (call option) or sell (put option) an asset at a spec. price

Swaps – an agreement to exchange a series of CF at a spec. prices and times

Forward Contract



- the price fixed now for fut. exchange is forw. price
- the buyer of the underlying is said to be "long" the forw.

↳ tom. if the value of the asset beyond P then buyer is going to make money

spot prices ↗ ⇒ "long" profit, "short" lose

Example:

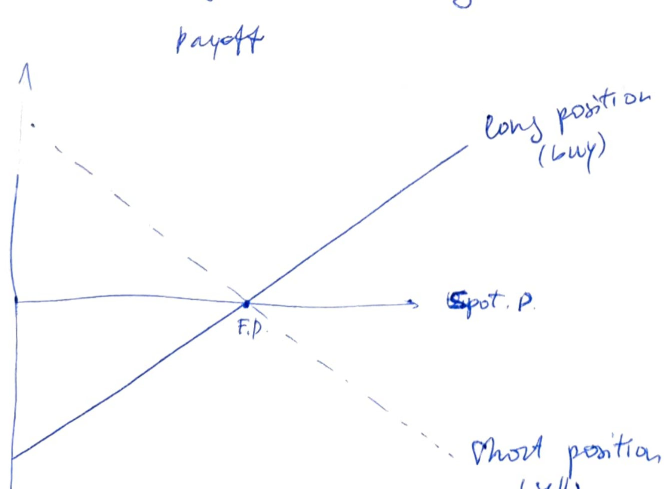
Current P = 160

3-month forw. contr. for 1000 tons at 165

long side will buy 1000 tons from short side at 165 in 3 m.

Def.: futures – exchange-traded, stand., forward-like contract that is marked to market daily

Guaranteed by the clearing house ⇒ no counter-party risk



Valuation of Forwards and Futures

S_t – spot at t

$F_{t,T} \approx H_{t,T}$

Date	Forward contract	Outright Asset Purchase
0	Pay \$0 for contr. with forw price \$$F_{0,T}$	Borrow \$$S_0$ Pay \$$S_0$ for Asset
T	pay \$ $F_{0,T}$ own asset	Pay back \$$S_0(1+r)^T$, cum. storage cost Deduce cum "conv. yeild" own asset
Cost	\$ $F_{0,T}$	\$ $S_0(1+r)^T$ + net storage costs

$$F_{0,T} \approx H_{0,T} = (1+r_f)^T S_0 + FV_T$$

$$\frac{F_{0,T}}{(1+r)^T} \approx \frac{H_{0,T}}{(1+r)^T} = S_0 + PV_0$$

e.g. gold $\quad F_{t,H} \approx H_{t,T} = (1+r_f)^{(T-t)} S_t$

↳ easy to store

no div

gas $\quad F_{t,H} \approx H_{t,T} = (1+r_f+c-y)^{(T-t)} S_t$

cost of storage, convenience yeild

% per period

financials $\quad F_{t,H} \approx H_{t,T} = (1+r_f-d)^{(T-t)} S_t$

ann. div. yeild

e.g. $\quad$ F. Price $= $ Spot Price $\left(1 + \text{6-mont int. rate} - y\right)^{6/12}$

$0{,}73 = 0{,}77\left(1 + 0{,}034 - y\right)^{6/12}$ $\Rightarrow y = 14{,}18$

$$y = 1 + r_f - (F/S_0)^{12/4}$$

5. Options

S_t - stock price

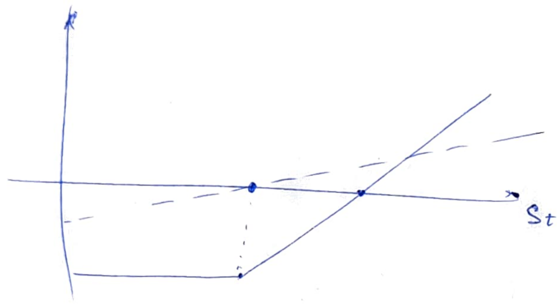
C_t - call price

P_t - put price

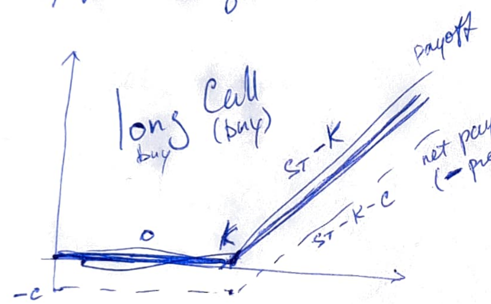
K - strike price

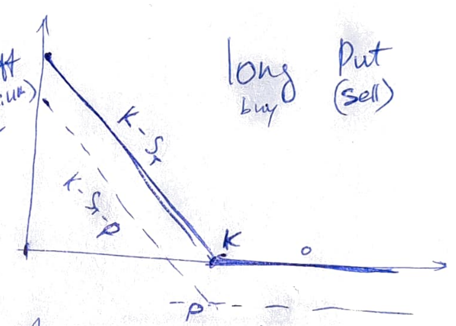
payoff at T $\begin{cases} C_T = Max\,[0;\ S_T - K] \\ P_T = Max\,[0;\ K - S_T] \end{cases}$

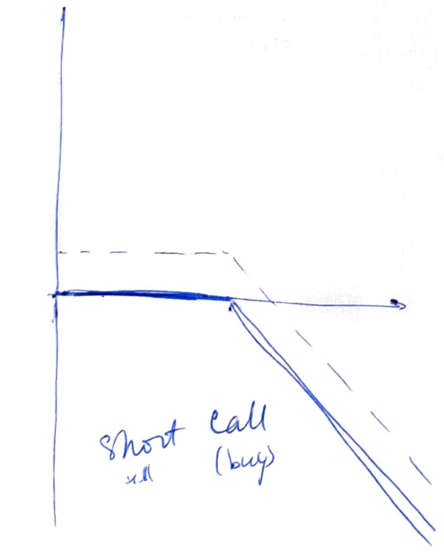
Call option vs stock return

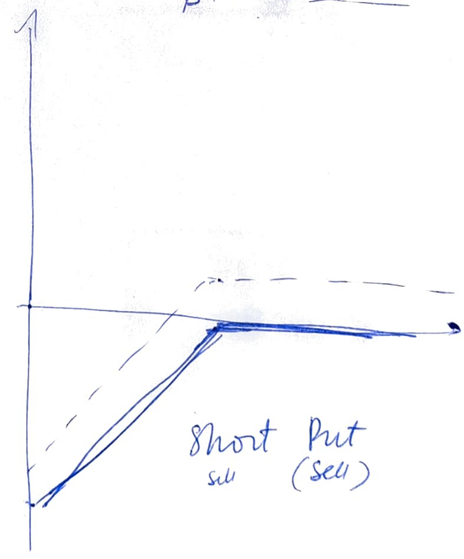


payoff diagrams

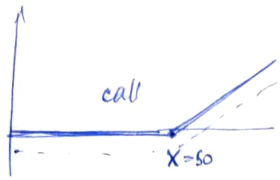


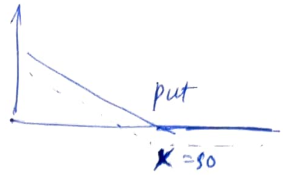


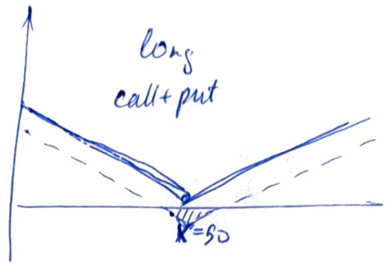




e.g.

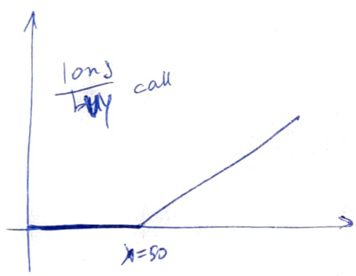


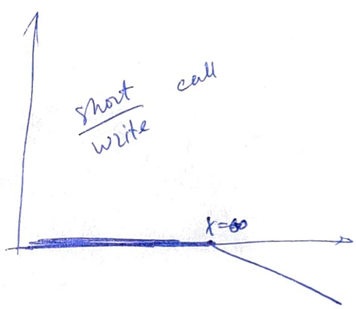


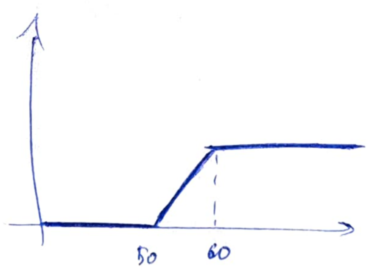


=> bets on volatility

e.g. $Call_1 - Call_2$







=> 

get rid of unbound fin. gain to fin positions

$V = E + D$

Eq. $\propto$ Call $\quad Max[0, V-B] = E$

Debt $\propto$ Put $\quad Max[V, B] \equiv D$

ann. implied volatility

VIX index

Valuation of Options

- Binomial Option-Pricing Model

$$C_1 = Max\ [S_1 - K, 0]$$

$$S_1 \begin{cases} \xrightarrow{p} uS_0 \\ \xrightarrow{1-p} dS_0 \end{cases} \qquad u > d \qquad C_1 \begin{cases} \xrightarrow{p} Max\ [uS_0 - K, 0] = C_u \\ \xrightarrow{1-p} Max\ [dS_0 - K, 0] = C_d \end{cases}$$

$$C_0 = f(S_0, K, u, d, p, r)$$

- $$V_0 = \underset{\substack{\uparrow \\ \text{shares of stock}}}{S_0 \Delta} + \underset{\substack{\uparrow\ \$B \\ \text{riskless bonds}}}{B} \qquad V_1 \begin{cases} \xrightarrow{p} uS_0\Delta + \overset{(1+r^*)}{r}B \\ \xrightarrow{1-p} dS_0\Delta + rB \end{cases}$$

Δ^*, B^* s.t. $\Rightarrow C_u, C_d$

$$V_1 \begin{cases} \xrightarrow{p} C_u \\ \xrightarrow{1-p} C_d \end{cases} \Rightarrow \qquad \Delta^* = \frac{(C_u - C_d)}{S_0(u-d)} \qquad B^* = \frac{(uC_d - dC_u)}{r(u-d)} \qquad d < r < u$$

$$\Rightarrow \quad C_0 = V_0 = S_0\Delta^* + B^* = \frac{1}{r}\left(\overbrace{\frac{r-d}{u-d}}^{p^*} C_u + \frac{u-r}{u-d} C_d\right)$$

if violated $\Rightarrow$ arbitrage

$C_0 > V_0 \Rightarrow$	Sell C_0	Buy V_0	$C_0 - V_0 > 0$	now
	Owe $C_1 = V_1$			tomorrow
$C_0 < V_0 \Rightarrow$	Buy C_0	Sell V_0	$V_0 - C_0 > 0$	now
	Owe $V_1 = C_1$			tomorrow

multi-period $\Rightarrow$ bin-tree

$$C_0 = \frac{1}{r^n} \sum \binom{n}{k} p^{*k} (1-p^*)^{n-k}\ Max\ [0, u^k d^{n-k} S_0 - K]$$

$$p^* = \frac{r-d}{u-d}$$

$\Rightarrow$ cont version $\Rightarrow$ Black-Scholes/Merton

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} = rC$$

$$\text{s.t.:} \begin{cases} C(S,T) = Max\ [S-K, 0] \\ C(0,t) = 0 \end{cases}$$

6. Risk and Return

$$R_{it} = \frac{D_{it} + P_{it} - P_{it-1}}{P_{it-1}} = \frac{D_{it} + P_{it}}{P_{it-1}} + 1$$

Return

Excess Return $= R_{it} - r_f$

Risk Premium $= E[R_{it}] - r_f$

· Anomalies:

size Ef.

Jan. Ef.

Value Ef

Momentum

Accruals & Issuances

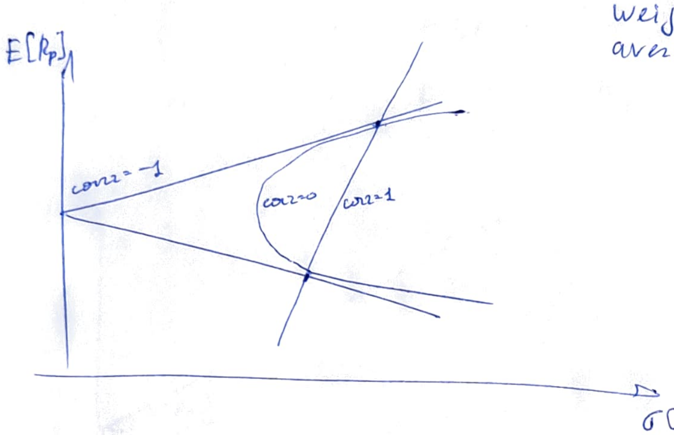
7. Portfolio Theory

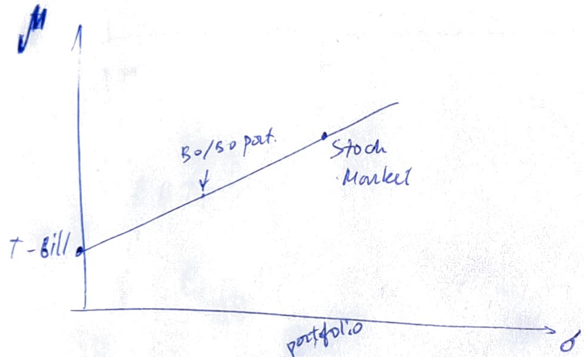
$$w_i = \frac{N_i P_i}{N_1 P_1 + \dots + N_n P_n} \qquad \sum w_i = 1$$

$$R_p = w_1 R_1 + \dots + w_n R_n$$

$$E[R_p] = w_1 \mu_1 + \dots + w_n \mu_n = \mu_p$$

weighted average

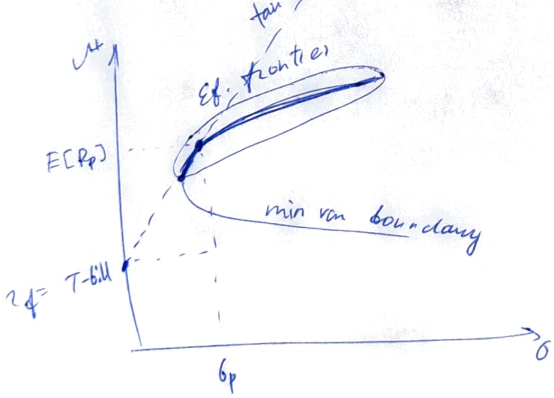




$$E[R_p] = w_a \mu_a + w_b \mu_b$$

$$Var[R_p] = w_a^2 \mu_a^2 + w_b^2 \mu_b^2 + 2 w_a w_b \sigma_a \sigma_b \rho_{ab}$$

$Var[R_p]$ = Average cov ; if $w_i = \frac{1}{n}$



Sharpe Ratio $= \dfrac{E[R_p] - r_f}{\sigma_p}$

8. CAPM & APT

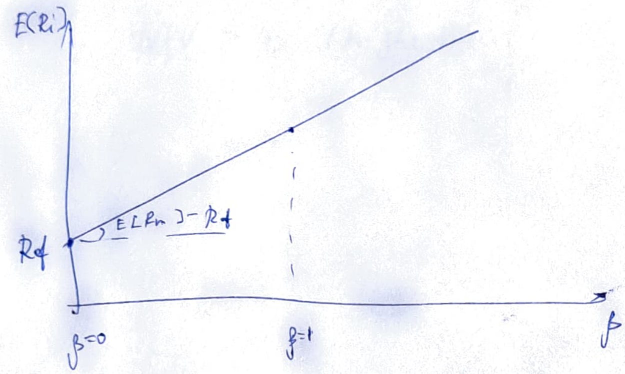
CML: $E[R_p] = R_f + \underbrace{\frac{\sigma_p}{\sigma_m}}_{\to \beta_p} (E[R_m] - R_f)$

$$E[R_i] = R_f + \beta_i (E[R_m] - R_f)$$

$\beta_i = 1 \Rightarrow E[R_i] = E[R_m]$

$\beta_i = 0 \Rightarrow E[R_i] = R_f$

$\beta_i < 0 \Rightarrow E[R_i] < R_f$



APT

$$R_i - R_f = \alpha_i + \beta_{i1} F_1 + \dots + \beta_{ik} F_k + \varepsilon$$

F_k = factor k excess return

$$E[R_i] - R_f = \beta_{i1} F_1 + \dots + \beta_{ik} F_k$$

9 Capital Budgeting

$$NPV = CF_0 + \frac{CF_1}{1+r_1} + \dots + \frac{CF_T}{(1+r_T)^T}$$

$$\bar{R}_{project} = R_f + \beta_{project}(\bar{R}_m - R_f)$$

$$IRR: \quad I_0 = \frac{CF_1}{1+IRR} + \dots + \frac{CF_t}{(1+IRR)^T}$$

accept if $IRR > IRR^*$ (highest)

$NPV > 0$ (highest)

Рипинской (10)

Bonds & Derivatives

derivatives → forwards (OTC) ; futures (бирж., стандарт.)

lot ≈ 100 акций

→ long (об-во купить)

→ short (об-во продать)

→ SWAP → FX-SWAP } вал. SWAP

→ cross-currency SWAP XCS | кросс-вал. SWAP

→ IRS

fix t_0 leg¹ leg² spot rate

leg¹ t_1 в t_0 fixed leg² swap rate

FX - SWAP

t_0 t_0 t_1

realised PL (прибл.)

стоим. вал. в стоим. инстр.

★ SPV

CIP Condition - covered int. parity:

London Inter Bank Offered Rate

$$1\ year: \quad (1 + R^{Rus}_{EUR})^1 = \frac{F^{(1y)}}{S}(1 + R^{Rus}_{USD})^1$$

$$1\ year: \quad (1 + R^{Rus}_{EUR})^1 = \frac{F^{(1y)}}{S}(1 + R^{LIBOR}_{USD})^1 \times (1 + R_{Basis\ SWAP})$$

1. International bond (NNI) — ① валюта
2. Euro bond (INA) ↔ ② страна эмиссии
3. National bond (NNN) — ③ эмитент

④ "LIBOR-OIS spread"

euro.dol. USA

overnight indexed swap

$$dS = S\mu dt + \sigma S dW$$

$$\frac{dS}{S} = \mu dt + \sigma dW \longrightarrow$$

Heston: $\sigma \sim$ CIR

$$\frac{dS}{S} = \mu(S,t)\, dt + \sigma(S,t)\, dW$$

$\Downarrow$

BS:

risk-neutrality

μ - risk-free rate

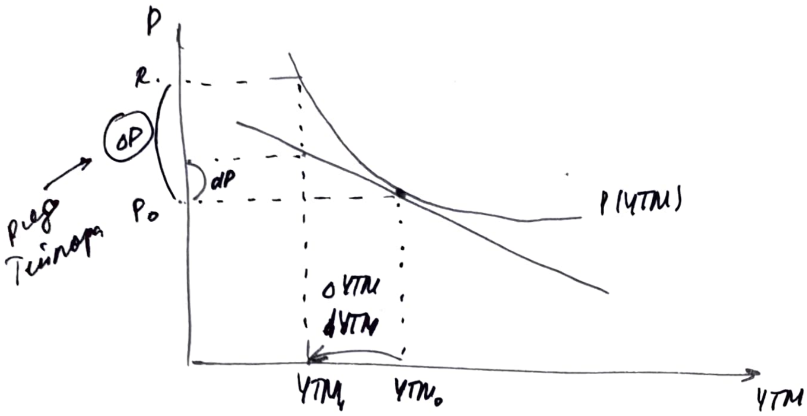
вне зависимости от предпочтений агентов

e.g. $r_f = 2\% = E(R_S)$

Forward

Финансы (10)

$$P = \sum_{i=1}^{n} \frac{CF_i}{(1+YTM)^i} \qquad P = \sum_{i=1}^{n} \frac{CF}{(1+r_i)^i}$$



$$\frac{\partial P}{\partial YTM} = \frac{dP}{dYTM}$$

$dYTM \to 0$

$$\frac{dP}{dYTM} = P'_{YTM} = \sum \frac{-i \cdot CF_i}{(1+YTM)^{i+1}} = P\left(-\sum i \cdot \frac{CF_i/(1+YTM)^i}{P\cdot(1+YTM)}\right) = -\frac{P}{(1+YTM)} \cdot D \Rightarrow \boxed{-P \cdot D' \cdot dYTM = dP}$$

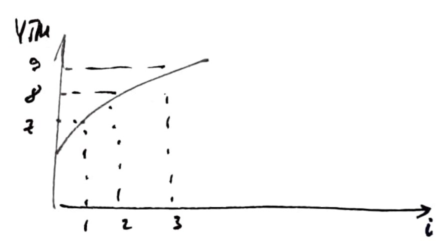
$$W_i = \frac{CF_i \cdot df_i}{\sum CF_i \cdot df_i}$$

$$D = \sum_{i=1}^{n} i \cdot \frac{CF_i/(1+r_i)^i}{\underbrace{\sum_{i=1}^{n} CF_i/(1+r_i)^i}_{P}} \leftarrow \sum i \cdot w_i$$

$$\frac{D}{1+YTM} = D'$$

дюрация Маколи

модиф. дюрация



i	CF_i	$CF_i \cdot df_i$	W_i	$W_i \cdot i$
1	10	9,35	9,1	0,091
2	10	8,57	8,3	0,166
3	110	84,94	82,6	2,478
	130	102,86	100%	2,735

рын. цена форв.

ост. до погаш.

$F_0 = S_0 \cdot e^{(r-q)t}$

q — div. yield

$PL: f_0 = (F_0 - K)e^{-rt}$

финанс. результат

Margin call – требование пополнения марж. счета клиента (брокера), вызванный отр. переоценкой торг. стратегии в размере, превышающем мин. размер марж. счета

(ex)	FP = 43 $	FP = 39 $
Action now	1. Занять 40$ под 5% на 3м. 2. Покупаем акцию 3. Закл. Forw. с ценой 43 $ Short	1. Продать акцию (Short) 40$ 2. Инв. 40$ под 5% на 3 м. 3. Закл. Forw. с ценой 39 (Long)
Action 3 m.	1. Исполняем Forw. (прод. за 43$) 2. Возвращаем кредит $40 \cdot e^{0{,}05 \cdot 0{,}25}$	1. Исполнить Forw. (купить за 39) 2. Закрыть Short 3. Получим от инв. $40 \cdot e^{0{,}05 \cdot 0{,}25}$
	Profit = 43 − 40,5 = 2,5	Profit = 40,5 − 39 = 1,5

S = 40

r = 5%

M = 3 mon

non-div. p. stock

$FV = PV \cdot e^{rt}$

справ. цена

$\Rightarrow F_0 = S_0 \cdot e^{rt}$

	Stock Payoff	Option Payoff	Extra payoff from holding stock insted of option
Option exercised	S	$S - K$	K
Option expires unexercised	S	0	S

Когда $S \gg K$, то цена опциона $\longrightarrow S - \text{Pay-off} \cdot e^{-rt}$

stock-price, strike, risk-free rate

$$C(\underset{-}{K};\ \underset{+}{T};\ \underset{+}{\sigma};\ \underset{+}{r_f};\ \underset{+}{S})$$

call-option-price

до погашения

ст. откл. доходности

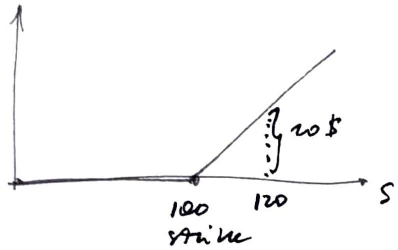
$$\sigma_z^2 = \beta^2 \cdot \sigma_m^2 + \sigma_e^2$$

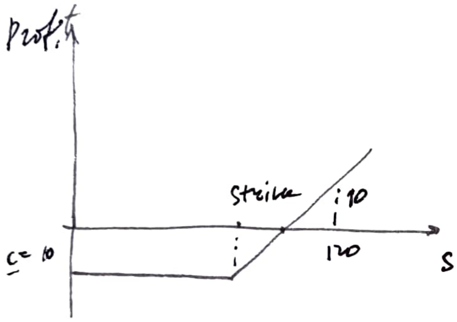
Options

Опцион (+call / -put) – право купить / продать баз. актив по цене strike в опр. дату (евр.)
до опр. даты (амер.)

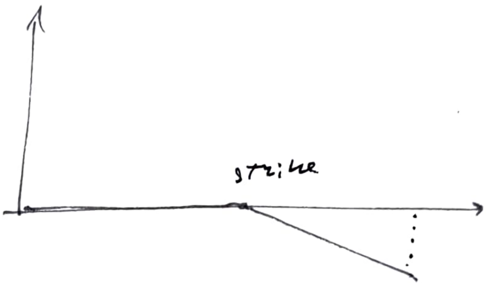
Short – обяз-во

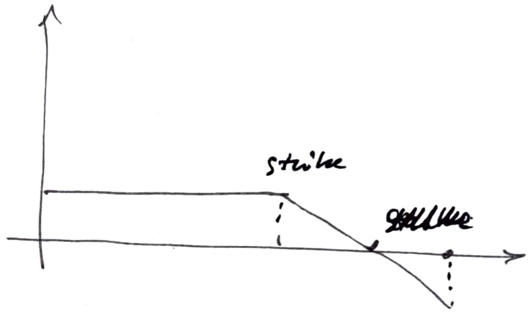
Long-call



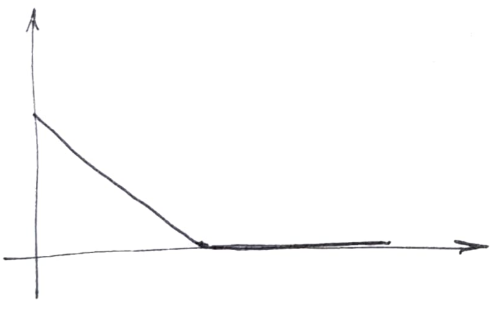


Short-call



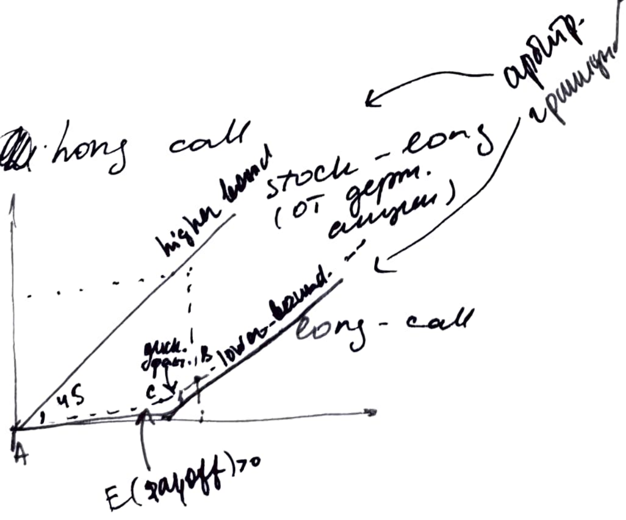


Long-put



Short-put

Long call



Option Chain
UL = 'LKOD'
Maturity = 2M
K ∈ [...]
TYPE ∈ [P; K]

1) Daily closing prices of com. stocks

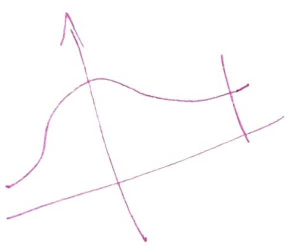
3-month Gov Treasury Bills

↳ Investigation period

5 years (July 1995 - June 2000)

Estimation period

1 year (July 2000 - June 2001)



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2) APT : factor analysis

- FX rate if entering a foreign m.

Macro :

- 0 = Mean ; not GDP
  ⇒ GDP growth

  changes in n. G. r. of GDP

- changes in default risk premium
  (spread between yields of AAA and Baa bonds)

- changes in the shape of the term structure

- changes in exp. inflation
- changes in price level

utility

XIII. Portfolio Theory

Portfolio – combination of securities

$w = \{w_1, w_2, \dots, w_n\}$ – weights : $\boxed{\sum_{i=1}^{n} w_i = 1}$ $n \in \mathbb{N}$

$$w_i = \frac{N_i P_i}{\sum_{j=1}^{n} N_j P_j}$$

e.g. $N = -200$ means 200 short-sold shares $N \in \mathbb{R}$

* dollar-neutral $\sum w_i = 0$
* long positions (e.g. mut. fund) $\sum w_i > 0$
* short positions $\sum w_i < 0$

GOAL: not predicting but estim. underlying par. (stable over time)

"130–30" : long $\leq 130\%$

short $\geq -30\%$

Mean-variance analysis:

$R_p = w_1 R_1 + \dots + w_n R_n$

$E[R_p] = w_1 \mu_1 + \dots + w_n \mu_n = \mu_p$ ← Weighted Average

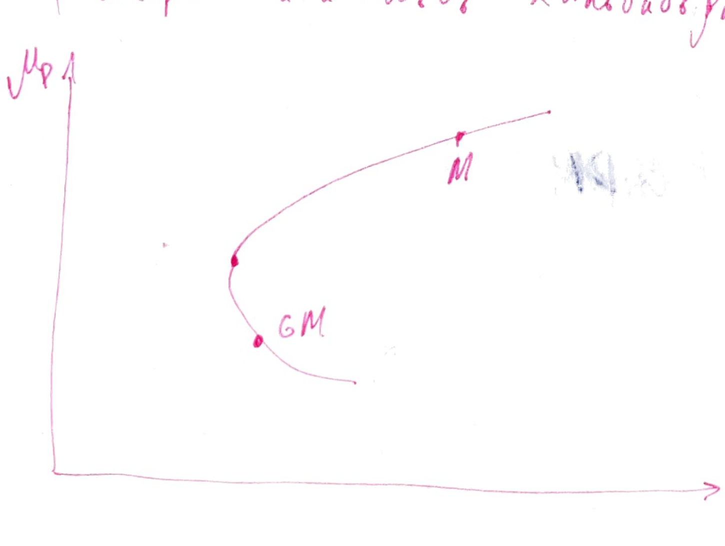
$Var[R_p] = E[(R_p - \mu_p)^2] = E[(w_1(R_1 - \mu_1) + \dots + w_n(R_n - \mu_n))^2]$

$E[w_i w_j (R_i - \mu_i)(R_j - \mu_j)] = w_i w_j Cov(R_i, R_j) = w_i w_j \cdot \underbrace{\sigma_i \sigma_j \cdot \rho_{ij}}_{\sigma_{ij}}$

e.g. $R_p = w_a R_a + w_b R_b$ $(w_a + w_b = 1)$

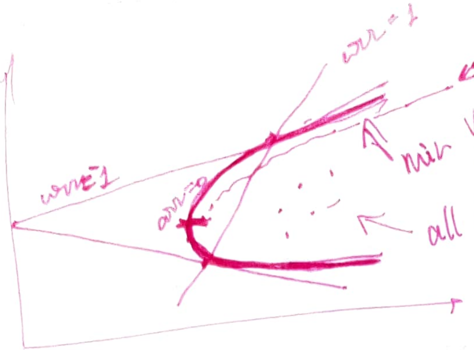
$E(R_p) = w_a \mu_a + w_b \mu_b$

$Var(R_p) = w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2 w_a w_b \sigma_a \sigma_b \rho_{ab}$



$$\hat{\sigma}_{ij} = \frac{1}{T} \sum_{t=1}^{T} (R_{it} - \hat{\mu}_i)(R_{jt} - \hat{\mu}_j)$$

Trade-off
* lots of data
  ⇒ built in non-stat.
* less data
  ⇒ error introduced by not using enough data



depends on our preferences

The General Case:

$E[R_p] = w_1 \mu_1 + \dots + w_n \mu_n$

$Var[R_p] = \sum w_i^2 \sigma_i^2 + \sum w_i w_j \, cov[R_i, R_j]$

Special case (equally weighted portfolio $w_i = \frac{1}{n}$)

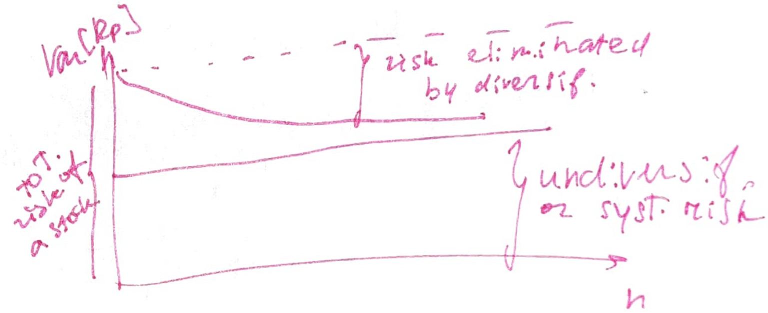
$Var[R_p] = \frac{1}{n^2} \sum \sigma_i^2 + \frac{1}{n^2} \sum cov[R_i, R_j]$

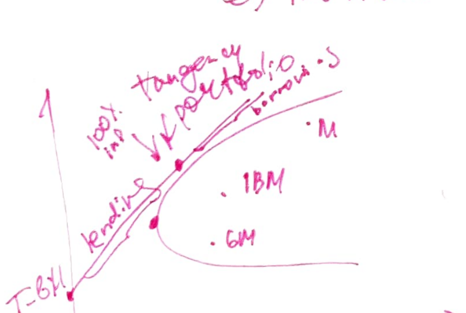
$= \frac{1}{n} \overline{Var} + \frac{n-1}{n} \overline{cov} \overset{\text{cl}}{\approx} \overline{cov}$

Systematic Risks:

- credit
- liquidity
- volatility
- Business Cycle
- Value / Growth

=> motivation for lin. factors





more securities => slope ↑

Alpha

measure of a mutual fund's risk-adjusted performance

→ tan portfolio maximizes the fund's alpha

CAPM Model

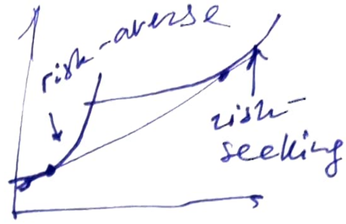
The tan portfolio:

Sharp ratio : measure of a portfolio's risk-return trade-off

$\hookrightarrow \beta = \frac{E[R_p] - r_f}{\sigma_p}$ ← p risk premium ← volatility

tan portfolio has the highest possible sharp ratio

key points



1. Diversification reduces risks
2. Cor. are more important than var.
3. Investors should try to hold P on the ef. frontier
4. With a riskless asset all inv should hold the tan port.
   (max trade-off between risk & exp. return)

M - market portfolio - tan P

↳ S&P 500, Russel 2000

The CAPM

- Ef. P are comp. of M and T-Bills
- $E[R_p] = R_f + \frac{\sigma_p}{\sigma_m}(E(R_m) - R_f)$

  ratio of p risk to mark. risk

  $\approx 15\%$, $\approx 7\%$

  ↳ required rate of return / cost of cap. for ef port.

for any asset : $\beta_i = \frac{Cov[R_i, R_m]}{Var[R_m]}$ ← β is the correct measure of risk (not sigma) except for ef. portfolios

measure sensitivity of stock to mark movement

Sharpe-Linter CAPM:

SML: $E[R_i] = R_f + \beta_i (E[R_m] - R_f)$

from equilibrium

CML: $E(R_p) = R_f + \frac{\sigma_p}{\sigma_m}(E(R_m) - R_f)$

rate of return you should have, given certain β_p

SML: $E(R_p) = R_f + \beta_i (E(R_m) - R_f)$

? Really α or is it due to other factors

$$\boxed{\alpha = E[R_i] - \{R_f + \beta_i(E(R_m) - R_f)\}}$$

linear
$$\begin{cases} R_p = w_1 R_1 + \dots + w_n R_n \\ \frac{Cov(R_p, R_m)}{Var(R_m)} = w_1 \frac{Cov(R_1, R_m)}{Var(R_m)} + \dots + w_n \frac{Cov(R_n, R_m)}{Var(R_m)} \\ \beta_p = \beta_1 w_1 + \dots + \beta_n \cdot w_n \end{cases}$$

Measure dev. from $E(R_p)$

α - difference (+/-?) how far away we are from efficiency ~~$R_p - E(R_p)$~~

$R_p - E(R_p)$ → return - what you should have with given β

→ how far away you are from efficiency depends upon how much risk is unsystematic

Emp. properties of Stock Returns (USA)

1) r slightly positive

2) Return on more risky assets > return on less risky assets

3) Returns on risky assets can be highly cor. to each other

4) Returns on risky assets are serially uncorrelated

* Size effect (premium): small stocks do better than large)

January effect

Price-to-book : value premium

low-momentum stocks (smaller)

high-momentum stocks (larger)

XII. Risk & Return

STOCK RETURN

- How volatile are stock returns?
- Are returns predictable?
- How does volatility change over time?
- What types of stocks have the highest returns?

"EF." MARKETS

- Random, unpredictable
- Quick react. to news of prices
- Investors can't earn abnormal, risk-adj returns

Return $R_{it} = \dfrac{D_{it} + P_{it} - P_{it-1}}{P_{it-1}} = \dfrac{D_{it} + P_{it}}{P_{it-1}} - 1$

Exp. Ret. $E(R_{it})$

Excess Ret. $R_{it} - r_f$

Risk Pr. $E(R_{it}) - r_f$

Mean $\mu_i = E(R_{it})$ $\quad \hat{\mu}_i = \frac{1}{T} \sum R_{it}$

Var $\sigma_i^2 = E[(R_{it} - \mu_i)^2]$ $\quad \hat{\sigma}_i^2 = \frac{1}{T-1} \sum (R_{it} - \hat{\mu}_i)^2$

Median : 50% percentile (1/2 pr. that $R_t <$ Median)

Skewness : symmetric dist.? (−) big losses are more likely
(+) big gains are more likely

Correlation : how closely variables move together?

$Cov(R_{it}, R_{jt}) = E((R_{it} - \mu_i)(R_{jt} - \mu_j))$

$Corr(R_{it}, R_{jt}) = \dfrac{Cov(R_{it}, R_{jt})}{\sigma_i \sigma_j}$

I.) PV

$$V_0 = CF_0 + \sum \frac{CF_i}{(1+r)^i}$$

NPV > 0 ! take , NPV < 0 reject

ex. $-230000 + \sum_{i=1}^{3} \frac{90000}{(1{,}04)^i} = 19{,}758$ (>0)

The Perpetuity

I $PV = \frac{c}{1+r} + \ldots = \sum_{i=1}^{\infty} \frac{c}{(1+r)^i}$

$$PV \cdot (1+r) = c_0 + \sum_{i=1}^{\infty} \frac{c_i}{(1+r)^i}$$

$$pv \cdot (1+r) = \frac{c_0}{1 - \frac{1}{1+r}}$$

$$PV \cdot r = c_0 \quad \Rightarrow \quad \boxed{PV = \frac{c}{r}}$$

II $PV = \frac{c}{1+r} + \frac{c(1+g)}{(1+r)^2} + \frac{c(1+g)^2}{(1+r)^3} + \ldots$

$$\frac{1+r}{1+g} PV = \frac{c}{1+g} + \frac{c}{1+r} + \frac{c(1+g)}{(1+r)^2} + \ldots$$

$$\frac{1+r}{1+g} PV = \frac{c}{1 - \frac{1+g}{1+r}} \quad \Rightarrow \quad PV = \frac{c}{r-g}, \quad r > g$$

The Annuity

$$PV = \frac{c}{1+r} + \ldots \frac{c}{(1+r)^T}$$

$$\boxed{PV = \frac{c}{r} - \frac{c}{r} \cdot \frac{1}{(1+r)^T}}$$

⊖ perp ⊕ perp at (T+1)

$$ADF = \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] \rightarrow \times C = PV$$

↳ annuity discount factor

$$r_{EAR} = \left(1 + \frac{r}{n}\right)^{n} - 1$$

↳ effective annual rate

$$W^{R}_{t+k} = \frac{W_{t+k}}{(1+\pi)^{k}}$$

$$r^{R} = \boxed{\frac{1+r^{n}}{1+\pi} - 1} \approx r^{n} - \pi$$

II Fixed-Income Securities

$$P_0 = \frac{F}{(1+r)^T}, \quad \text{if } r = \text{const}$$

$$P_0 = \frac{F}{\prod_{i=1}^{T}(1+r_i)} \quad \text{if } r \neq \text{const}$$

one-year spot rates

$$= \frac{F}{(1+r_{0,T})^T}$$

Today's T-Year Spot Rate

Ex.

maturity : 5
price : 0,797

$\Rightarrow \quad 0{,}797 = \frac{1}{(1+r_{0,5})^5} \Rightarrow r = 4{,}64\%$

$$\frac{P_{0,1}}{P_{0,2}} = \frac{F}{(1+R_1)} : \frac{F}{(1+R_1)(1+R_2)} = 1 + R_2$$

↓ one-year forward rates

$$\frac{P_{0,t-1}}{P_{0,t}} = 1 + f_t = \frac{(1+r_{0,t})^t}{(1+r_{0,t-1})^{t-1}}$$

→ forward int rates – today's rates for transactions between t_1 and t_2

$$P_0 = \sum_{i=1}^{T} \frac{C}{(1+y)^t} + \frac{F}{(1+y)^T}, \quad y - \text{YTM}$$

Measures of Int.-Rate Risk

Duration – sensitivity of price ↔ yield

Weighted average term to maturity:

$D_m = \sum k \cdot w_k$, where $\sum w_k = 1$

$\boxed{\text{weighted average}}$

longer duration → more sensitive

$$w_k = \frac{PV(C_k)}{P} = \frac{\frac{C_k}{(1+y)^k}}{P}$$

$$P = \sum \frac{C_k}{(1+y)^k}$$

$$\frac{\partial P}{\partial y} = \frac{-1}{1+y} \underbrace{\sum k \frac{C_k}{(1+y)^k}}_{D_m \cdot P} \Rightarrow \underbrace{\frac{1}{P}\frac{\partial P}{\partial y}}_{\text{sensitivity}} = -\frac{D_m}{1+y}$$

Ex

$q \cdot t$	c_h	$c_h / (1+y/q)^{t}$	$\frac{t \times PV}{2P}$	$\frac{t \cdot (t+1) \cdot PV}{4P}$
t	CF_t	$PV(CF_t)$	$t \cdot PV(CF_t)$	
1	3,5	3,4	3,4	
⋮	3,5	2,85	19,92	
8	103,5	81,70	653,83	
		Σ 103	Σ 738	

$$D = \frac{738{,}28}{103{,}5} = \frac{\sum t \cdot PV(CF_t)}{\sum PV(CF_t)}$$ – dur.

$$D^* = \frac{D}{1+y} = \frac{7{,}13}{1{,}03} = 6{,}92$$ – mod. dur.

$$\Delta = D^* \cdot P = 6{,}92 \cdot 103{,}5 = 716$$ – price risk at $y = 0{,}03$

* Annual $D_h = \sum h \cdot w_h / q$

Annual $D_h^* = \frac{D_m}{1 + y/q}$

$$\frac{\partial^2 P}{\partial y^2} = \frac{1}{(1+y)^2} \sum h(h+1) \frac{c_h}{(1+y)^h}$$

$$\frac{1}{P} \frac{\partial^2 P}{\partial y^2} = V_h$$ – convexity

sensitivity of duration

* 2nd ord. appr.

$$P(y') \approx P(y) + \frac{\partial P}{\partial y}(y) \cdot (y'-y) + \frac{\partial^2 P}{\partial y^2}(y) \frac{(y'-y)^2}{2}$$

$$= P(y) \left[1 - D_m^*(y'-y) + \frac{1}{2} V_m (y'-y) \right]$$

$$\boxed{P = \sum P_j}$$

$$D_m^*(P) = -\frac{1}{P} \frac{\partial P}{\partial y} = \sum_j \frac{P_j}{P} \cdot D_m^*$$

$$V_m^*(P) = -\frac{1}{P} \frac{\partial^2 P}{\partial y^2} = \sum_j \frac{P_j}{P} V_{m,j}^*$$