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HW-07

Task 1.

$y_i = x_i'\beta + \nu_i - u_i,$

where

$\nu_i \sim N(0, \sigma_\nu^2),$

$u_i \sim N^+(0, \sigma^2(z)),$

$\sigma_i^2 = \sigma^2(z_i) = \exp(\delta + \gamma z_i)$

Analytical expression for inefficiency measures:

(i)

Let $b_1 = \delta, b_2 = \gamma$

Then $\ln \sigma^2 = \delta + \gamma z_i = b'z$ and $\sigma = \exp(1/2 * b'z)$

Therefore

$E(u|z) = \int_0^\infty u \frac{2}{\sqrt{2\pi}\sigma} \exp(-\frac{u^2}{2\sigma^2}) du = \sqrt{\frac{2}{\pi}} * \sigma = \sqrt{\frac{2}{\pi}} * \exp(1/2 * b'z)$

(ii)

$E(\exp(-u)|z) = \int_0^\infty \exp(-u) \frac{2}{\sqrt{2\pi}\sigma} \exp(-\frac{u^2}{2\sigma^2}) du = 2 \exp(\frac{\sigma^2(b'z)}{2})(1 - \Phi(\sigma(b'z)))$

(iii)

Marginal effects:

(a)

$\frac{\partial}{\partial z_i} E(u_i|z_i) = \frac{\partial}{\partial z_i} \sqrt{\frac{2}{\pi}} * \sigma(z'b) = \sqrt{\frac{2}{\pi}} \frac{\partial}{\partial z_i} \exp(1/2 * b'z) = \sqrt{\frac{2}{\pi}} \exp(1/2 * b'z) * 1/2 b_2 =$

$= \frac{b_2}{\sqrt{2\pi}} \exp(1/2 * b'z) = \frac{\gamma \sigma(b'z)}{\sqrt{2\pi}}$

(b)

$\frac{\partial}{\partial z_i} \exp(-E(u_i|z_i)) = \frac{\partial}{\partial z_i} \exp(-\sqrt{\frac{2}{\pi}} * \sigma(z'b)) =$

$= \exp(-\sqrt{\frac{2}{\pi}} * \sigma(z'b)) * (-\sqrt{\frac{2}{\pi}} \exp(1/2 * b'z) * 1/2 * b_2) =$

$= -\frac{\gamma\sigma(b'z)}{\sqrt{2\pi}} \exp(-\sqrt{\frac{2}{\pi}} * \sigma(z'b))$

$\frac{\partial}{\partial z_i} E(u_i|z_i)$ and $\frac{\partial}{\partial z_i} \exp(-E(u_i|z_i))$ has opposite signs.

(c)

$\frac{\partial}{\partial z_i}(E(\exp(-u_i)|z_i)) = \frac{\partial}{\partial z_i} 2\exp(\frac{\sigma^2(b'z)}{2})(1-\Phi(\sigma(b'z))) = 2*\exp(\frac{\sigma^2(b'z)}{2})*\sigma(b'z)* \exp(1/2*b'z)*1/2*b_2*(1-\Phi(\sigma(b'z))) + 2*\exp(\frac{\sigma^2(b'z)}{2})*(-\phi(\sigma(b'z))*\exp(1/2*b'z)* 1/2*b_2) = \gamma\sigma(b'z)\exp(\frac{\sigma^2(b'z)}{2})(\sigma(b'z)*(1-\Phi(\sigma(b'z))) - \phi(\sigma(b'z)))$

$\frac{\partial}{\partial z_i}(E(\exp(-u_i)|z_i))$ and $\frac{\partial}{\partial z_i} \exp(-E(u_i|z_i))$ has same signs,

because $\frac{\phi(\sigma(x))}{1-\Phi(\sigma(x))} > \sigma(x) => \frac{\phi(\sigma(b'z))}{1-\Phi(\sigma(b'z))} > \sigma(b'z) =>$
$(\sigma(b'z)*(1-\Phi(\sigma(b'z))) - \phi(\sigma(b'z))) < 0.$

Task 2.

First, we estimate quadratic regression:

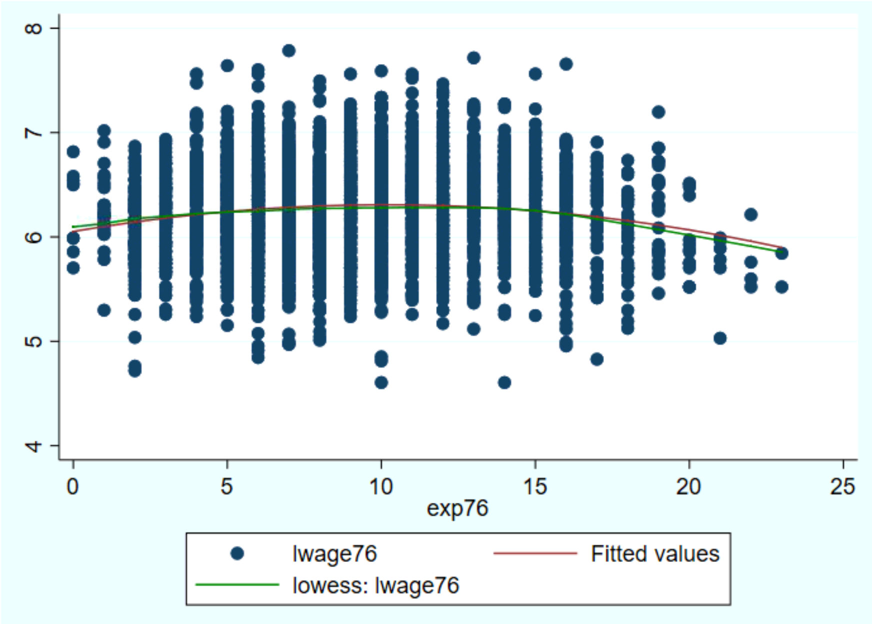
$lwage_i = const + exp_i + exp_i^2 + e_i$

Source	SS	df	MS			
				Number of obs	=	3,010
				F(2, 3007)	=	22.34
Model	8.67672483	2	4.33836241	Prob > F	=	0.0000
Residual	583.964921	3,007	.194201836	R-squared	=	0.0146
				Adj R-squared	=	0.0140
Total	592.641646	3,009	.196956346	Root MSE	=	.44068

lwage76	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
exp76	.0504443	.0076365	6.61	0.000	.035471	.0654176
exp762	-.002485	.0003738	-6.65	0.000	-.0032179	-.0017522
_cons	6.052608	.0352271	171.82	0.000	5.983536	6.121679

Second, we run locally weighted scatterplot smoothing (LOWESS) with bandwidth 0.8.

Plotting both predicted values on a same plot, where red line represents polynomial fit estimated with OLS and a green line represents the smoothed curve by non-parametric method, we can see, that the predictions differ insignificantly. That means that second-order polynomial is a reasonable model, as the shape of a curve is exactly the same as a non-parametric model would suggest. However, LOWESS fit shows, that a linear fit would not be a suitable model here.



STATA code:

```
reg lwage76 exp76 exp762
predict preg
lowess lwage76 exp76, generate (pnonp)
twoway (scatter lwage76 exp76) (line preg exp76, sort)
(line pnonp exp76, sort lcolor(green))
```