Эконометрика

Семинары

1.1. $x_i = \begin{cases} 0 \\ 1 \end{cases}, \quad i = \overline{1, 1055}$ $\qquad \hat{p} = 0{,}54$

x_i	0	1
	$1-p$	p

$$\hat{p} = \frac{\sum x_i}{1055} = 0{,}54$$

$$E(x_i) = 0(1-p) + 1 \cdot p = p$$

$$Var(x_i) = \left(0^2(1-p) + 1^2 \cdot p\right) - p^2 = p - p^2$$

$$E(\hat{p}) = E\left(\frac{x_1 + \ldots + x_n}{1055}\right) = \frac{\sum E(x_i)}{1055} = p$$

$$Var(\hat{p}) = Var\left(\frac{x_1 + \ldots + x_n}{1055}\right) = \frac{1}{1055^2} \cdot 1055(p - p^2) = \frac{p-p^2}{1055}$$

$Cov = 0$

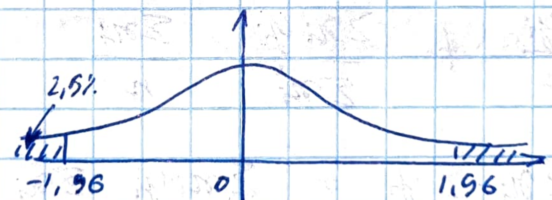
$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

Пусть $H_0: \; p = 0{,}5$ верна

$$E(\hat{p}) = 1/2$$

$$Var(\hat{p}) = 1/1055 \cdot 1/4$$

при $\alpha = 0{,}05$:



$$P(-2 < N(0;1) < 2) \approx 95\%$$

$$Z_{obs.} = \frac{\hat{p} - E(\hat{p})}{Var(\hat{p})} = \frac{0{,}54 - 0{,}5}{\sqrt{0{,}00024}} \approx 2{,}58$$

$Z_{cr} = 1{,}96$

$Z_{obs} > Z_{cr} \Rightarrow H_0$ – отв.

qnorm(0,975)
qt()
qf()

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p - p^2}{1055}}}$$

$$p\left(-1{,}96 < \frac{\hat{p} - p}{\sqrt{\frac{p - p^2}{1055}}} < 1{,}96\right) = 95\%$$

$$\hat{p} - 1{,}96\sqrt{\frac{p - p^2}{1055}} < p < \hat{p} + 1{,}96\sqrt{\frac{p - p^2}{1055}}$$

$\Downarrow$ ~~аппрокс~~

аппроксимация по LLN ($\hat{p} \approx p$)

DRY = do not repeat yourself

$$\hat{p} - 1{,}96\sqrt{\frac{\hat{p} - \hat{p}^2}{1055}} < p < \hat{p} + 1{,}96\sqrt{\dots}$$

2.1. Метод МНК (OLS)

corr - парная линейная величина (если corr = 0 связь м.б. но не линейн.)

$$y_i = \beta x_i + u_i$$

$$\hat{y}_i = \hat{\beta} x_i$$

$$\min \sum (y_i - \hat{y}_i)^2 = \sum (y_i - \hat{\beta} x_i)^2 \longrightarrow \min_{\hat{\beta}}$$

$$\partial Q / \partial \hat{\beta} = \sum(-2x_i(y_i - \hat{\beta} x_i)) = 0$$

$$\hat{\beta} = \frac{2\sum x_i y_i}{2\sum x_i^2} = \frac{\sum x_i y_i}{\sum x_i^2}$$

Проходит ли $\hat{y}_i = \hat{\beta} x_i$ через $(\bar{x}, \bar{y})$ - центр массы?

$$\frac{\sum y_i}{n} \neq \frac{\sum x_i y_i}{\sum x_i^2} \cdot \frac{\sum x_i}{n}$$

$$\sum x_i^2 \cdot \sum y_i \neq \sum x_i y_i \cdot \sum x_i$$

2.2. OLS $y_i = \beta_0 + \beta_1 x_i + u_i$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{\beta}_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} \qquad \bar{x} = \frac{\Sigma x}{n}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x}$$

$n = 20$ $\Sigma x_i = 20$ $\Sigma x_i^2 = 400$ $\Sigma x_i y_i = 500$

$\Sigma y_i = 40$ $\Sigma y_i^2 = 900$

$$\hat{\beta}_1 = \frac{\Sigma x_i y_i - \cancel{\Sigma x_i \bar{y}} - \Sigma \bar{x} y_i + \cancel{\Sigma \bar{x} \bar{y}}}{\Sigma x_i^2 - 2\Sigma x_i \bar{x} + n\bar{x}^2} =$$

$$= \frac{\Sigma x_i y_i - n \cdot \bar{x} \cdot \bar{y}}{\Sigma x_i^2 - n\bar{x}^2} = \frac{500 - 20 \cdot 40/20}{400 - 20 \cdot 20/20} = \frac{23}{19}$$

$$\hat{\beta}_0 = \frac{40}{20} - \frac{23}{19} \cdot \frac{20}{20} = \frac{15}{19}$$

3.1. $\mathrm{Var}(y_i | X) = \mathrm{Var}(u_i | X) = \sigma^2$

$$\mathrm{Var}(y_i(x_i - \bar{x}) | X) = (x_i - \bar{x})^2 \cdot \mathrm{Var}(y_i | X) = (x_i - \bar{x})^2 \cdot \sigma^2$$

- $\mathrm{Var}(R_1 + R_2 + R_3) = \mathrm{Var}_1 + \mathrm{Var}_2 + \mathrm{Var}_3 + 2\mathrm{cov}_{12} + 2\mathrm{cov}_{23} + 2\mathrm{cov}_{13}$

4.1. Дифференциалы

$f(x) = x^3$ $df = 3x^2 \cdot dx$

$f(b_1, b_2) = b_1^2 + 2b_2 \cdot b_1$. $df = (2b_1 + 2b_2)\, db_1 + 2b_1\, db_2$

$df = \Sigma \frac{\partial f}{\partial b_k} db_k$ ($= 0$ в т. opt, т.к. $df = \Sigma 0 \cdot db_k$)

$$d\begin{pmatrix} b_1 & b_2^2 \\ b_1 + b_2 & b_2 \cdot b_1 \end{pmatrix} = \begin{pmatrix} db_1 & 2b_2\, db_2 \\ db_1 + db_2 & b_2\, db_1 + b_1\, db_2 \end{pmatrix}$$

Св-ва: 1) $dA = 0_{m\times n}$, где $A_{m\times n}$ – м-ца констант

2) $d(RS) = (dR) \cdot S + R(dS)$, где R, S – м-цы, $\exists R \cdot S$

3) $d(R+S) = dR + dS$

4.2. $\underbrace{Y}_{} = \underbrace{\beta_1 \cdot 1 + \beta_2 \cdot X + \ldots + \beta_5 \cdot d}_{} + u$

$Y_{n\times 1} \qquad X_{n\times 5}$

$\hat{y}_{n\times 1} = \hat{\beta}_1 1 + \ldots + \hat{\beta}_5 \cdot d = X_{n\times 5} \cdot \hat{\beta}_{5\times 1}$

$\hat{u} = y - \hat{y}$ – ошибки прогнозов

$$\min_{\hat{\beta}} \sum \hat{u}_i^2 = \hat{u}^T \cdot \hat{u}$$

FOC: $dQ = \underbrace{d\hat{u}^T_{1\times n} \cdot \hat{u}_{n\times 1}}_{1\times 1} + \underbrace{\hat{u}^T_{1\times n} \cdot d\hat{u}_{n\times 1}}_{1\times 1} = 2\sum u_k d_{u_k} = 2\hat{u}^T d\hat{u} =$

$= 2(y-\hat{y})^T d(y-\hat{y}) = 2(y - X\cdot\hat{\beta})^T d(y - X\hat{\beta})$

$= 2(y - X\hat{\beta})^T (\underset{0}{\overset{\shortparallel}{dy}} - \underset{0}{\overset{\shortparallel}{dX}}\hat{\beta} - X \cdot d\hat{\beta})$

$= \underbrace{2(y - X\hat{\beta})^T_{1\times n} \cdot (-X)_{n\times 5} \cdot d\hat{\beta}_{5\times 1}}_{1\times 1} = 0$

$2(y - X\hat{\beta})^T \cdot (-X) = 0 \quad | :(-2)$

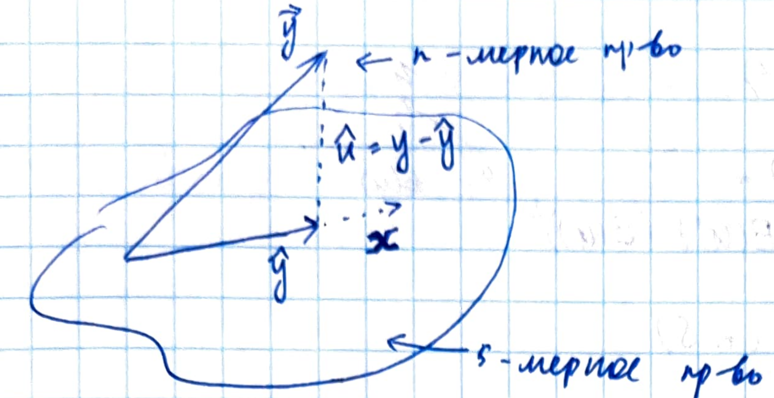
$(y^T - \hat{\beta}^T \cdot X^T) \cdot X = 0$

$(A+B)^T = A^T + B^T$ $\qquad y^T X - \hat{\beta}^T \cdot X^T \cdot X = 0$

$(AB)^T = B^T A^T$ $\qquad (y^T X)^T = (\hat{\beta}^T X^T X)^T$

$X^T y = X^T X \hat{\beta}$

$Ax = b \Rightarrow x = A^{-1} b$ $\qquad \hat{\beta} = (X^T X)^{-1} \cdot X^T y$



$\sum \hat{u}_i^2 = |\hat{u}|^2 \to \min$

$\hat{u} \perp 1 \Rightarrow (\hat{u}_1, \dots, \hat{u}_n)\begin{pmatrix}1\\ \vdots \\ 1\end{pmatrix} = 0$

$\hat{u} \perp x \Rightarrow (\hat{u}_1, \dots, \hat{u}_n)\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix} = 0$

…

$\hat{u} \perp d \Rightarrow$ -//-

$\Rightarrow \hat{u}^T \cdot X = 0_{1\times 5}$

$$\underset{1\times n}{(\hat{u}_1, \dots, \hat{u}_n)} \cdot \underset{n\times 5}{\begin{pmatrix}1 & x_1 & \dots & d_1\\ \vdots & & & \\ 1 & x_n & \dots & d_n\end{pmatrix}} = \underset{1\times 5}{(0, \dots, 0)}$$

5.1. Dummy-переменные

Season ← z_i, v_i, l_i, o_i, где $z_i = \begin{cases} 1, & \text{если season = зима} \\ 0, & \text{иначе}\end{cases}$

$y_i = \beta_1 + \beta_2 x_i + \beta_k \cdot \underbrace{\text{season}}_{?}$

$\hat{y}_i = 60 - 3p_i - 20 z_i + 10 l_i + 5 o_i$ ← "разница между осенью и весной"

$z_i = 1 \qquad \hat{y}_i = 40 - 3p_i$

$v_i = 1 \qquad \hat{y} = 60 - 3p_i$

$l_i = 1 \qquad \hat{y}_i = 70 - 3p_i$

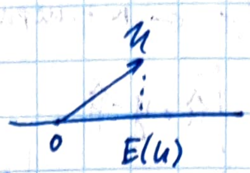
$o_i = 1 \qquad \hat{y}_i = 65 - 3p_i$

↳ $\hat{y}_i = 40 - 3p_i + 20 v_i + 30 l_i + 25 o_i$

data1 ←
data %>%
mutate(...)
filter(...)

\[... \]

6.1. $\sigma^2(u) = E(u^2)$



$$E(u - E(u)^2) = E(u^2) - (E(u))^2$$

$$\langle R, S \rangle = E(R \cdot S)$$

$$Cov(u, x) = E(ux) - E(u) \cdot E(x)$$

Св-во (сим.):

1) R, S независимые $\to Cov(f(R), h(S)) = 0, \ \forall f, h$

$\Downarrow$

2) $E(R|S) = const \to Cov(R, f(S)) = 0, \ \forall f$

$\Downarrow$

3) $Cov(R, S) = 0$

6.2. Гетероскедастичность

y_i	x_i
2	1
5	4
3	9

$$Var(u_i | X) = \sigma^2 \cdot x_i$$

$$y_i = \beta \cdot x_i + u_i$$

$$\hat{\beta}_{OLS} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{49}{98}$$ ← была бы эф., если бы $Var(u_i|X) = \sigma^2$ (по ТГМ)

$\tilde{y}_i$	$\tilde{x}_i$
2	1
2,5	2
1	3

где $\tilde{y}_i = \frac{y_i}{\sqrt{x_i}} = \beta \frac{x_i}{\sqrt{x_i}} + \frac{u_i}{\sqrt{x_i}}$

$\Rightarrow Var(\tilde{u}_i | \tilde{x}_i) = \sigma^2 \Rightarrow$ эф. МНК-оц

$$\hat{\beta}_{alt} = \frac{\sum \tilde{x}_i \tilde{y}_i}{\sum \tilde{x}_i^2} = \frac{10}{14}$$

6.3. $y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot z_i + u_i$

Пусть $Var(u_i) = \delta_i^2$

$$Var(u) = \begin{pmatrix} \delta_1^2 & & \overset{cov(u_1, u_n)}{0} \\ & \ddots & \\ 0 & & \delta_n^2 \end{pmatrix} = \delta^2 \cdot I$$

$(X'X)' = X'X$

$$Var(\hat{\beta}) = Var((X'X)^{-1} X'y) =$$

$$= (X'X)^{-1} X' \cdot Var(u) \cdot ((X'X)^{-1} X')' =$$

$((X'X)^{-1})' = (X'X)^{-1}$

$$= (X'X)^{-1} X' \cdot Var(u) \cdot X (X'X)^{-1}$$

$(X^{-1})' = (X')^{-1}$

В реальности $Var(u)$ не изв. => исп. $\hat{Var}(u)$

6.3.1. Гомоскедастичность

$$\hat{Var}(u) = \begin{pmatrix} \hat{\delta}^2 & & 0 \\ & \ddots & \\ 0 & & \hat{\delta}^2 \end{pmatrix} = \hat{\delta}^2 \cdot I = \frac{RSS}{n-k} \cdot I$$

число набл.

где $\hat{\delta}^2 = \frac{RSS}{n-k}$ – знач. оценка для δ^2

$$Var(\hat{\beta}) = \hat{\delta}^2 \cdot (X'X)^{-1}$$

6.3.2. Гетероскедастичность

$$\hat{Var}(\hat{\beta}) = (X' \cdot X)^{-1} X' \cdot \hat{Var}(u) \cdot X \cdot (X' \cdot X)^{-1}$$

предп. гомоск. $\hat{Var}(u) = \frac{RSS}{n-k} \cdot I$

предп. гетероск. HC0 – $\hat{Var}(u) = \begin{pmatrix} \hat{u}_1^2 & & 0 \\ & \ddots & \\ 0 & & \hat{u}_n^2 \end{pmatrix}$

HC1 (STATA)
HC2
HC3 (R)
HC4

A^{-1}

$AA^{-1} = A^{-1}A = \mathbb{1}$

$\det A^{-1} = \frac{1}{\det A}$

$(AB)^{-1} = B^{-1}A^{-1}$

$(A^{T})^{-1} = (A^{-1})^{T}$

$(kA)^{-1} = k^{-1}\cdot A^{-1}$

$E^{-1} = E$

$Ax = b \Rightarrow x = A^{-1}\cdot b$

8.4. $y_i = \beta x_i + u_i$

y	x
2	1
8	2
8	3

$$X'X = (1\;2\;3)\begin{pmatrix}1\\2\\3\end{pmatrix} = 14$$

$$(X'X)^{-1} = \frac{1}{14}$$

$$\hat\beta = (X'X)^{-1}\cdot X'\cdot y = \frac{1}{14}(1\;2\;3)\begin{pmatrix}2\\8\\8\end{pmatrix} = 3$$

$$\hat y_i = 3x_i \Rightarrow \hat y = \begin{pmatrix}3\\6\\9\end{pmatrix}$$

$$(y-\hat y) = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$$

$$RSS = (y-\hat y)^{T}\cdot(y-\hat y) = 6$$

$$\widehat{Var}_{HOMO}(u) = \frac{RSS}{n-k}\cdot\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = \frac{6}{3-1}I = 3\cdot I$$

$$\widehat{Var}(\hat\beta) = 3\cdot(X'X)^{-1} = 3\cdot 1/14 = 3/14$$

$$\hookrightarrow t = \frac{\hat\beta}{se(\hat\beta)} = \frac{3-0}{\sqrt{3/14}}$$

HC0: $\widehat{Var}(u) = \begin{pmatrix}\hat u_1^2 & & 0\\ & \ddots & \\ 0 & & \hat u_n^2\end{pmatrix}$

HC1: $\widehat{Var}(u) = \frac{n}{n-k}\begin{pmatrix}\hat u_1^2 & & 0\\ & \ddots & \\ 0 & & \hat u_n^2\end{pmatrix}$,

где $\widehat{Var}(u_{jj}) = \frac{n}{n-k}\cdot\hat u_j^2$

HC3: $\widehat{Var}_{jj}(u) = \frac{\hat u_j^2}{(1-H_{jj})^2}$, где $H = X(X'X)^{-1}X'$

$$\widehat{Var}_{HC0}(u) = \begin{pmatrix}1&0&0\\0&4&0\\0&0&1\end{pmatrix}$$

$$\widehat{Var}(\hat\beta) = \frac{1}{14}\cdot(1\;2\;3)\cdot\begin{pmatrix}1&0&0\\0&4&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\frac{1}{14} = \frac{26}{196}$$

$$\widehat{Var}_{HC1}(u) = \frac{3}{2}\cdot\begin{pmatrix}1&0&0\\0&4&0\\0&0&1\end{pmatrix}$$

$$\widehat{Var}(\hat\beta) = 3/2\cdot 26/196 = 39/196$$

$$H = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \frac{1}{14} \cdot (1\ 2\ 3) = \frac{1}{14} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$$\widehat{Var}_{HC3}(u) = \begin{pmatrix} \frac{1}{(1-1/14)^2} & 0 & 0 \\ 0 & \frac{4}{(1-4/14)^2} & 0 \\ 0 & 0 & \frac{1}{(1-9/14)^2} \end{pmatrix} = \begin{pmatrix} (14/13)^2 & 0 & 0 \\ 0 & (14/10)^2 & 0 \\ 0 & 0 & (14/5)^2 \end{pmatrix}$$

$\Rightarrow \widehat{Var}(\hat\beta) = \ldots$

A^T

$A'' = A$

$(AB)' = B'A'$

$(\lambda A)' = \lambda A'$

$\det A = \det A'$

7.1. $SER = S_{\hat u} = \sqrt{\frac{\sum \hat u_i^2}{n-k}} = \sqrt{\frac{RSS}{n-k}}$

$R^2 = ESS/TSS = 1 - RSS/TSS = [corr(X,Y)]^2 = 1 - \frac{\hat\sigma^2}{\hat\sigma_y^2}$

$\hookrightarrow \bar R^2 = 1 - \frac{n-1}{n-k} \cdot \frac{RSS}{TSS} = 1 - \frac{S_{\hat u}^2}{S_y^2}$

$\leq R^2$

$$corr(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

$$scorr(X,Y) = \frac{\sum(x_i - \bar x)(y_i - \bar y)}{\sqrt{\sum(x_i-\bar x)^2 \cdot \sum(y_i - \bar y)^2}}$$

Найти наилучш. лин. аппроксимацию Y через X ($\min E(\varepsilon_i^2)$)?

7.2 $Y = \alpha_1 + \alpha_2 \cdot X + u$

$corr(X,Y) = sign(\alpha_2) \cdot \sqrt{\alpha_2 \beta_2}$

$X = \beta_1 + \beta_2 \cdot Y + w$

$E(Y) = 7 \quad E(X) = 8 \quad Var\left(\begin{pmatrix} X \\ Y \end{pmatrix}\right) = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$

7.2.1. $\hat Y = \alpha_1 + \alpha_2 \hat X$

$E[(y - \alpha_1 - \alpha_2 x)^2] = E(y^2) + \alpha_1^2 + \alpha_2^2 E(x^2) - 2\alpha_1 E(y) - 2\alpha_2 E(XY) + 2\alpha_1\alpha_2 E(X)$

$\partial E/\partial\alpha_1 \Rightarrow \alpha_1 = -\alpha_2 E(X) + E(y) = -8\alpha_2 + 7 = 0$

$\partial E/\partial\alpha_2 \Rightarrow \alpha_2 \cdot \underbrace{E(X^2)}_{5+64} = \underbrace{E(XY)}_{3+56} - \alpha_1 \cdot \underbrace{E(X)}_{8}$

$\Rightarrow \hat Y = 2{,}2 + 0{,}6X$

7.2.2. $\hat{X} = \beta_1 + \beta_2 Y$

$$E\left[(X - \beta_1 - \beta_2 Y)^2\right] \to \min_{\beta_1, \beta_2}$$

$$E((\ )^2) = Var(X - \beta_1 - \beta_2 Y) + \underbrace{(E(X - \beta_1 - \beta_2 Y))^2}_{=0}$$

$$= Var(X) + \beta_2^2 Var(Y) - 2\beta_2 Cov(X,Y) =$$

$$= 5 + \beta_2^2 \cdot 2 - 2 \cdot 3 \cdot \beta_2 \to \min_{\beta_2}$$

$$\Rightarrow \beta_2 = \frac{3}{2} = \frac{Cov(X,Y)}{Var(Y)}$$

$$corr(X,Y) = \sqrt{3/5 \cdot 3/2} = \sqrt{0{,}9}$$

8.1. Предположения:

8.1.1. $Var(u_i | X) = \sigma^2$ – гомоск.

$\hat{\beta}$ – сост.

ТГМ применима $\Rightarrow$ $\hat{\beta}$ – эф-на

$$t = \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \xrightarrow{d} N(0;1), \quad u_i \sim N, \quad u_i \sim t_{n-k}$$

8.2.2. $Var(u_i | X) = f(x_i)$ – гетероск.

$\hat{\beta}$ – сост.

ТГМ не прим. $\Rightarrow$ $\hat{\beta}$ – не эф-на

$$t = \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \not\to N(0;1)$$

$\Rightarrow$ исп. $se_{HC}(\hat{\beta}_k)$

$\hat{\Sigma} \nearrow SER^2 \cdot I$, $\hat{\Sigma} \searrow HC$

$$\hat{Var}_{HC}(\hat{\beta}|X) = (X'X)^{-1} \cdot X' \cdot \hat{\Sigma} \cdot X (X'X)^{-1}$$

8.2. $y_i^{UR} = \beta_1 + \beta_2 x_i + \beta_3 z_i + \beta_4 w_i + u_i$

$H_0: \beta_3 = 0 \vee \beta_4 = 0$

$y_i^R = \beta_1 + \beta_2 x_i + u_i$

число огр. в H_0

$$F = \frac{(R^2_{UR} - R^2_R)/q}{(1 - R^2_{UR})/n - k_{UR}} = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/n - k_{UR}}$$

только при гомоск-ти

число коэф. в y_i

8.3. Тест Вальда

$H_0: \beta_3 = 0$ $\qquad \dfrac{\hat\beta_3 - 0}{se(\hat\beta_3)} \xrightarrow{d} N(0;1)$, $\sim t_{n-k}$

$$W = (\hat\beta_3 - 0) \cdot (\hat{Var}(\hat\beta_3|X)) \cdot (\hat\beta_3 - 0) \xrightarrow{d} \chi^2_q$$

$H_0: \beta_3 = 0 \vee \beta_4 = 0$

$$W = (\hat\beta_3 \ \hat\beta_4) \cdot (\hat{Var}\begin{pmatrix}\hat\beta_3 \\ \hat\beta_4\end{pmatrix}|X)^{-1} \begin{pmatrix}\hat\beta_3 \\ \hat\beta_4\end{pmatrix} \xrightarrow{d} \chi^2_q$$

9.1. $y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot w_i + \beta_4 \cdot z_i + u_i$

$n = 24$; $RSS = 15$; $\sum(y_i - \bar y - w_i + \bar w)^2 = 20$

$H_0: \beta_2 + \beta_3 + \beta_4 = 1 \vee \beta_2 = 0 \vee \beta_3 = 1$

Без НС-коррекции:

$$\underbrace{y_i - w_i}_{k_i} = \beta_1 + u_i \Rightarrow \hat\beta_1 = \frac{\sum k_i}{n}$$

$$RSS_R = \sum(k_i - \hat k_i)^2 = \sum(y_i - w_i - \hat\beta_1)^2 =$$

$$= \sum\left(y_i - w_i - \frac{\sum(y_i - w_i)}{n}\right)^2 = \sum(y_i - \bar y - w_i + \bar w)^2 = 20$$

$RSS_{UR} = 15$

$$F = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/n-k} = \frac{(20-15)/3}{15/(24-4)} = 2{,}22$$

↳ при верной H_0 $\quad F \sim F_{q,\,n-k}\ (\sim F_{3;\,24-4})$

$\alpha = 5 \quad F_{cr} = 3{,}098 \Rightarrow H_0$ не отв.

9.2. $M1: \; y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot w_i + u_i \quad ; \forall i$

$M2: \; y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot w_i + \beta_4 \cdot d_i + u_i \quad ; \; d_i = \begin{cases} 1, & i = 13 \\ 0, & i \neq 13 \end{cases}$

$M3: \; y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 w_i + u_i \quad ; \forall i \neq 13$

$TSS_1 = TSS_2$, т.к. зависит от числа набл. $n_1 = n_2$

$RSS_2 \leq RSS_1$
$R_2^2 \geq R_1^2$, т.к. зависит от числа пер. $k_2 > k_1$

$TSS_3 > TSS_2$
$R_2^2 > R_3^2$ $\quad \Leftrightarrow$ остаток в 13 набл. $(=0)$
$RSS_2 = RSS_3$

10.1. $y_i = \beta_1 + \beta_2 \cdot x_i + u_i$

$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot x_i^*$

$x_i^* = x_i + v_i$ – зашумленная пер.

$E(v_i) = 0$

$E(u_i | X) = 0 \qquad Var(u_i | X) = 0$

$v_i = \begin{pmatrix} x_i \\ x_i^* \\ y_i \end{pmatrix}$ i.i.d. $\Leftrightarrow \begin{pmatrix} x_i \\ x_i^* \\ y_i \end{pmatrix} \not\leftrightarrow \begin{pmatrix} x_j \\ x_j^* \\ y_j \end{pmatrix}$

v_i нез. с. x_i, u_i

10.2. LLN: если w_i - i.i.d

то $\underset{n\to\infty}{plim}\ \bar{w}_i = E(w_i)$

10.2.1. $x_i \sim N(7;10)$ - i.i.d.

- $\underset{n\to\infty}{plim}\ \bar{x}_n = 7$
- $\underset{n\to\infty}{plim} \frac{\sum x_i^2}{n} = \underset{n\to\infty}{plim} \frac{\sum z_i}{n} = E(z_i) = E(x_i^2) =$

$= Var(x_i) + (E(x_i))^2 = 59$

- $\underset{n\to\infty}{plim} \frac{\sum (x_i - \bar{x})^2}{n} = \underset{n\to\infty}{plim} \frac{\sum x_i}{n} + \underset{n\to\infty}{plim} \frac{n\bar{x}^2 - 2n\bar{x}^2}{n}$

$= 59 - \underset{n\to\infty}{plim}\ \bar{x}^2 = 59 - plim\ \bar{x} \cdot plim\ \bar{x} = 59 - 7 \cdot 7 = 10$

- $\underset{n\to\infty}{plim} \frac{\sum (x_i - \bar{x})^2}{n-1} = \underset{n\to\infty}{plim} \frac{\sum (x_i - \bar{x})^2}{n} \cdot \underset{n\to\infty}{plim} \frac{n}{n-1} = 10 \cdot 1 = 10$

10.2.2. $x_i \sim N(7;10)$ - i.i.d.

$y_i \sim N(-2;5)$ - i.i.d.

$cov(x_i; y_i) = -3$

- $\underset{n\to\infty}{plim} \frac{\sum x_i y_i}{n} = E(x_i y_i) = Cov(x_i y_i) + E(x_i) E(y_i)$

$= -3 + 7 \cdot (-2) = -17$

- $\underset{n\to\infty}{plim}\ \bar{x} \cdot \bar{y} = 7 \cdot (-2) = -14$
- $cov(x_i, y_i) = \underset{n\to\infty}{plim} \frac{\sum (x_i - \bar{x}) y_i}{n} =$

$= \underset{n\to\infty}{plim} \frac{\sum x_i y_i}{n} - \underset{n\to\infty}{plim} \frac{\sum \bar{x} y_i}{n} = -17 - (-14) = -3$

- $cov(x_i, y_i) = \underset{n\to\infty}{plim} \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} =$

$= plim \frac{\sum x_i y_i}{n} + plim\ \bar{x}\bar{y} - 2\, plim\ \bar{x}\bar{y} = -3$

10.2.3. $x_i, y_i, \begin{pmatrix} x_i \\ y_i \end{pmatrix}$ – i.i.d.

$$\underset{n\to\infty}{\mathrm{plim}} \frac{\sum x_i}{n} = E(x_i)$$

$$\underset{n\to\infty}{\mathrm{plim}} \frac{\sum x_i y_i}{n} = E(x_i y_i)$$

$$\underset{n\to\infty}{\mathrm{plim}} \frac{\sum x_i^2}{n} = E(x_i^2)$$

$$\underset{n\to\infty}{\mathrm{plim}} \frac{\sum (x_i - \bar{x}_i)^2}{n} = Var(x_i)$$

$$\underset{n\to\infty}{\mathrm{plim}} \frac{\sum (x_i - \bar{x})(y_i - \bar{x})}{n} = Cov(x_i, y_i)$$

10.3. $y_i = \beta_1 + \beta_2 \cdot x_i + u_i$

$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot x_i^*$,

где $x_i^* = x_i + v_i$

$Var(x_i) = 10$
$Var(v_i) = 5$
$Var(u_i) = 8$

$\beta_2 = 2$

10.3.1. $$\hat{\beta}_2 = \underset{n\to\infty}{\mathrm{plim}} \frac{\frac{\sum (x_i^* - \bar{x}^*)(y_i - \bar{y})}{n}}{\frac{\sum (x_i^* - \bar{x}^*)^2}{n}} = \frac{Cov(x_i^*, y_i)}{Var(x_i^*)} =$$

$$= \frac{Cov(x_1 + v_1, \beta_1 + \beta_2 \cdot x_i + u_i)}{Var(x_1 + v_1)} = \frac{Cov(x_1, \beta_2 x_1)}{Var(x_1) + Var(v_1)} =$$

$$= \frac{\beta_2 \cdot 10}{10 + 5} = \frac{2}{3}\beta_2$$

10.3.2. $y_i = \beta_1 + \beta_2 \cdot x_i + u_i$

$x_i^* = x_i + v_i$ – изм. себя

$\tilde{x}_i = x_i + w_i$ – изм. соседа $\quad E(w_i) = 0$

$$\underset{n\to\infty}{\mathrm{plim}} \hat{\beta}_2^{МНК} = 2/3\,\beta_2$$

$$\underset{n\to\infty}{\mathrm{plim}} \hat{\beta}_2^{МН} = \mathrm{plim} \frac{\sum (y_i - \bar{y})(\tilde{x}_i - \bar{\tilde{x}}_i)}{\sum (x_i^* - \bar{x}^*)(\tilde{x}_i - \bar{\tilde{x}}_i)} = \frac{Cov(y_i, \tilde{x}_i)}{Cov(x_i^*, \tilde{x}_i)} =$$

$$= \frac{Cov(\beta_1 + \beta_2 \cdot x_1 + u_1, x_1 + w_1)}{Cov(x_1 + v_1, x_1 + w_1)} = \frac{\beta_2 Var(x_1)}{Var(x_1)} = \beta_2$$

11.1. $y_i = \beta_1 \cdot x_i + \beta_2 \cdot w_i + u_i$

$\hat{y}_i = \hat{\beta}_1 \cdot x_i$

$E(x_i) = E(w_i) = E(u_i) = 0$

$$Var\left(\begin{pmatrix} x_i \\ w_i \\ u_i \end{pmatrix}\right) = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 9 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\hat{y}_i = \hat{\beta}_1 \cdot x_i$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} \Rightarrow plim\left(\frac{\sum x_i y_i}{\sum x_i^2}\right) = \frac{E(x_i y_i)}{E(x_i^2)} = \frac{4\beta_1 + (-1)\cdot\beta_2}{4}$$

Пусть $E(z_i) = 0$

$$Var\left(\begin{pmatrix} x_i \\ w_i \\ u_i \\ z_i \end{pmatrix}\right) = \begin{pmatrix} 4 & -1 & 0 & 2 \\ -1 & 9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 16 \end{pmatrix}$$

<u>2-шаговый МНК</u>

1) X на Z : $\hat{x}_i = \hat{\gamma} \cdot z_i \Rightarrow plim\, \hat{\gamma}$ - ?

2) Y на $\hat{X}$ $\Rightarrow plim\, \hat{\beta}_1 = \beta_1$; $\hat{y}_i = \hat{\beta}_1 \cdot \hat{x}_i$

$$\underset{n\to\infty}{plim}\, \hat{\gamma} = plim \frac{\sum z_i x_i}{\sum z_i^2} = \frac{E(z_i x_i)}{E(z_i^2)} = \frac{2+0}{16} = \frac{1}{8}$$

$$\underset{n\to\infty}{plim}\, \hat{\beta}_1 = plim \frac{\sum \hat{x}_i y_i}{\sum(\hat{x}_i^2)} = plim \frac{1/8 \sum z_i y_i}{1/64 \sum z_i^2} = 8 \frac{E(z_i, y_i)}{E(z_i^2)} =$$

$$= 8 \cdot \frac{Cov(z_i, y_i)}{16} = \frac{1}{2} \cdot Cov(z_i, \beta_1 \cdot x_1 + \beta_2 \cdot w_1 + u_1) = \frac{1}{2} \cdot 2 \cdot \beta_1 = \beta_1$$

Пр. : з/п певца

тест на чтение во 2 классе

спос. к пению

$$y_i = \beta_1 \cdot x_i + \beta_2 \cdot \cancel{a}_i + u_i$$

w_i

месяц рождения