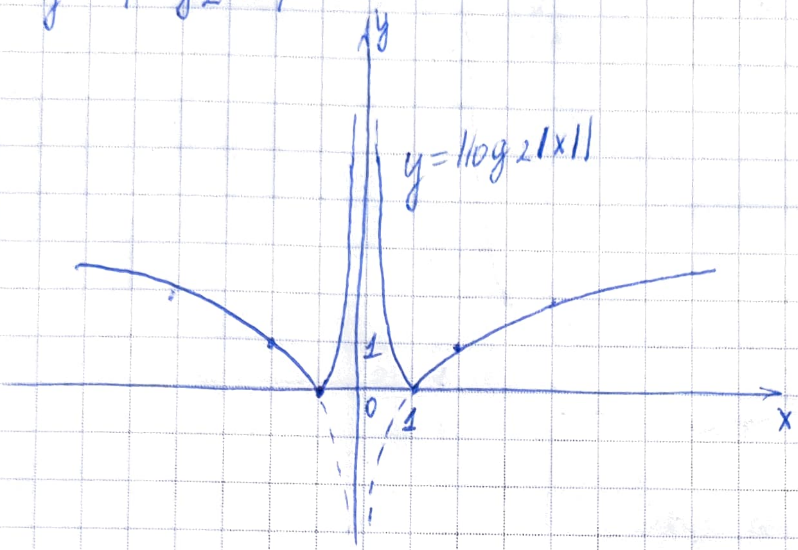
МАТ. АНАЛИЗ
(ПРАКТИКА)
1 КУРС

4) $y = |\log_2 |x||$



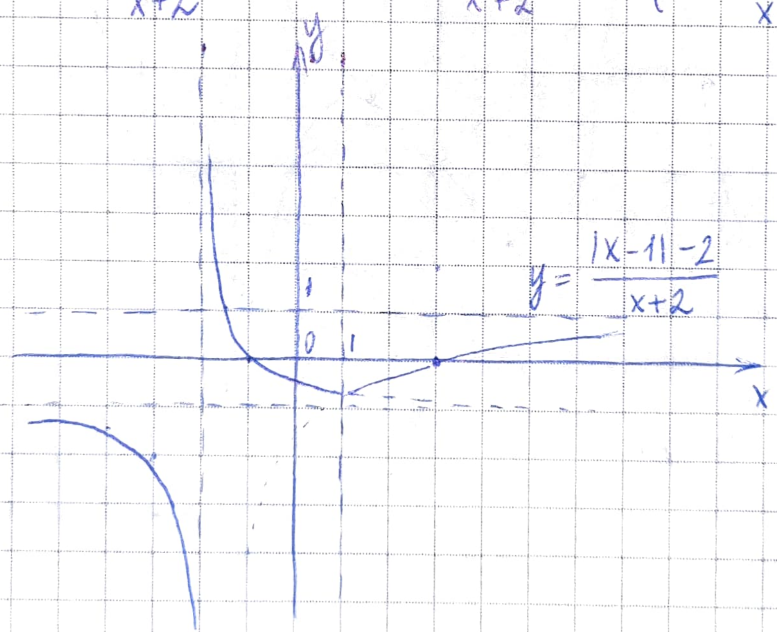
5) $y = \dfrac{|x-1|-2}{x+2}$

$x \geq 1$

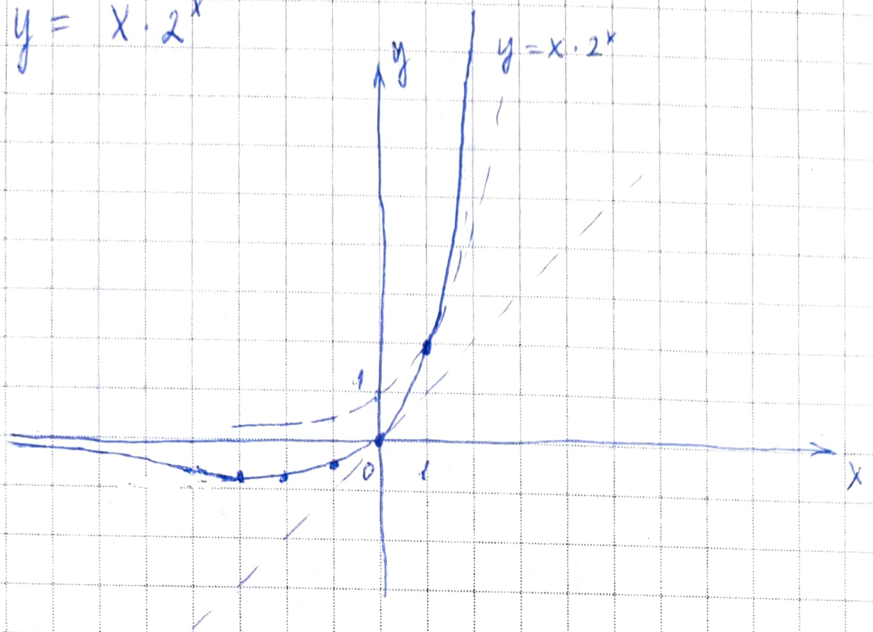
$\dfrac{x-3}{x+2} = 1 - \dfrac{5}{x+2}$

$x < 1$

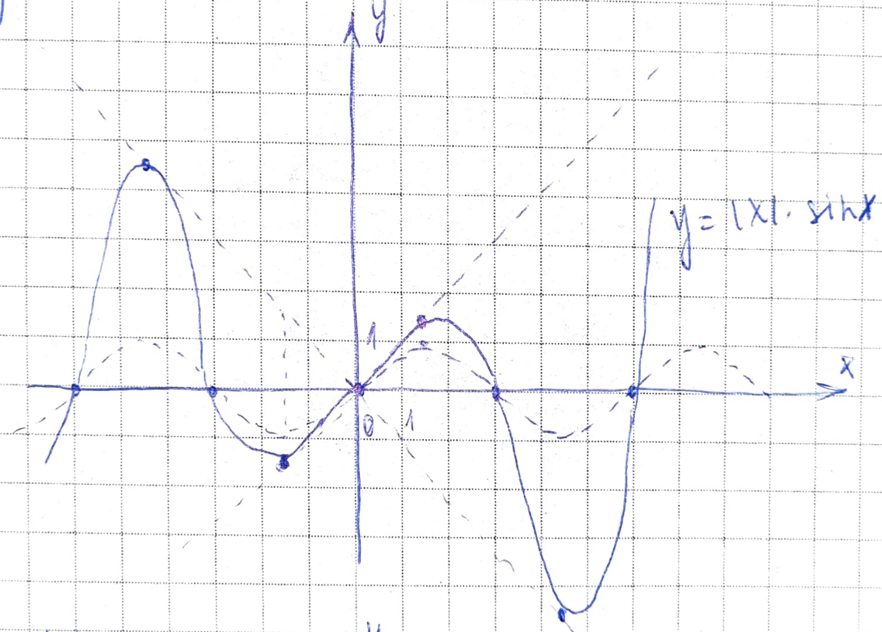
$\dfrac{-x-1}{x+2} = -\left(1 - \dfrac{1}{x+2}\right) = -1 + \dfrac{1}{x+2}$



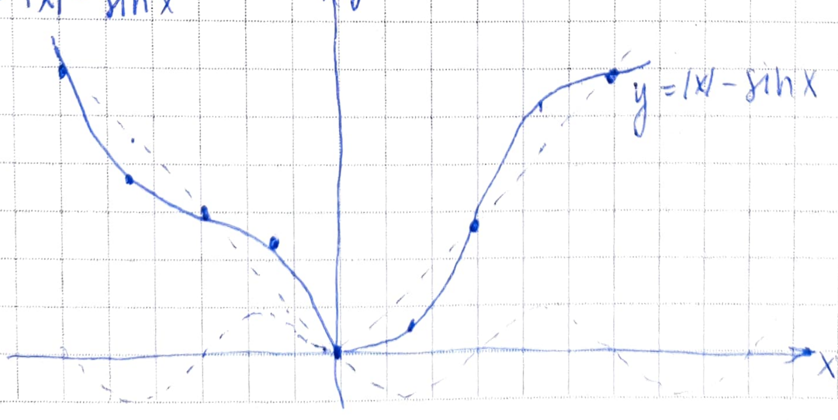
6) $y = x \cdot 2^x$

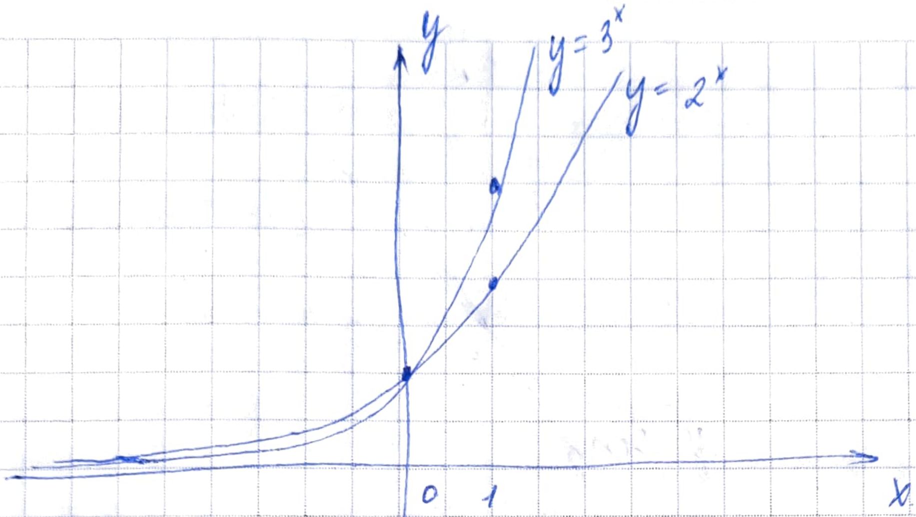


7) $y = |x| \cdot \sin x$

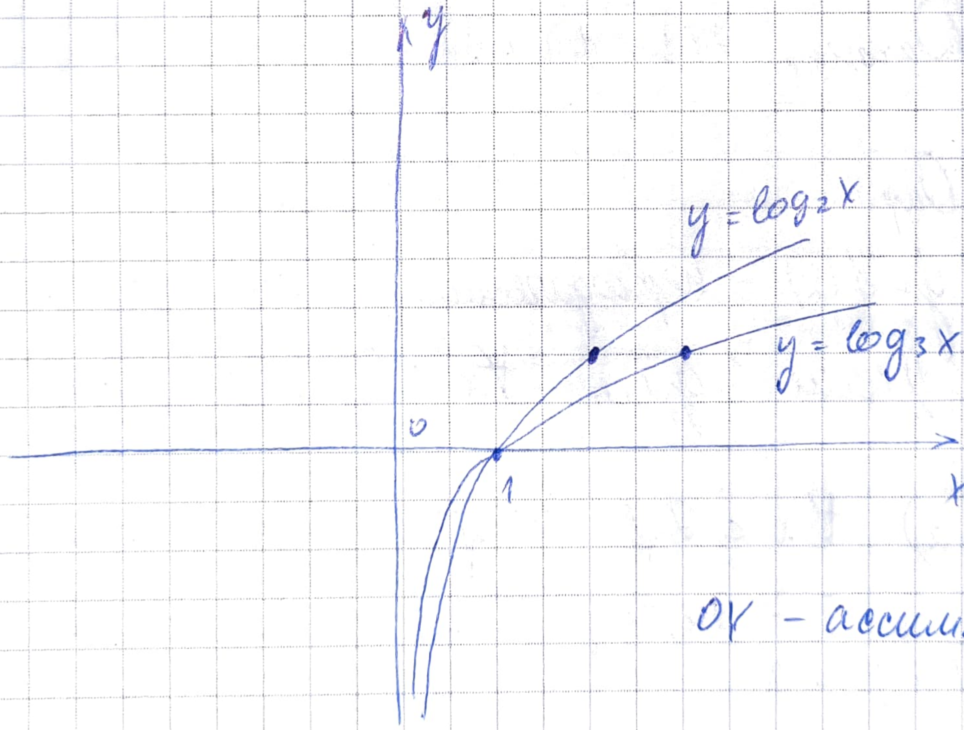


8) $y = |x| - \sin x$

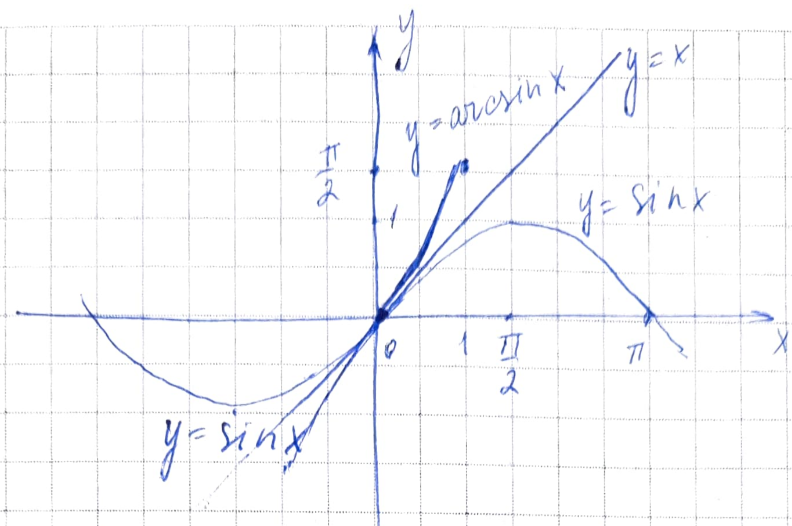




Ox – ассимпт.



Oy – ассимпт.



Триг. опр. – числ. ось расп. по кругу $O(0,0), 1$

$E_{\arcsin x} = [-\frac{\pi}{2}; \frac{\pi}{2}]$

$D_{\arcsin x} = [-1; 1]$

Опр.

$y=f(x)$ – периодическая с пер. $T>0$

1) если для $\forall x \in D_f \Rightarrow \begin{cases} x+T \in D_f \\ x-T \in D_f \end{cases}$

2) $\forall x \in D_f \Rightarrow f(x+T) = f(x) = f(x-T)$

T – период $\Rightarrow$ $n \cdot T$ – период, если $n, T \in \mathbb{N}$

Теорема

$y = const$

$T = \infty$

$f(x)$ – явл. периодической с пер. T

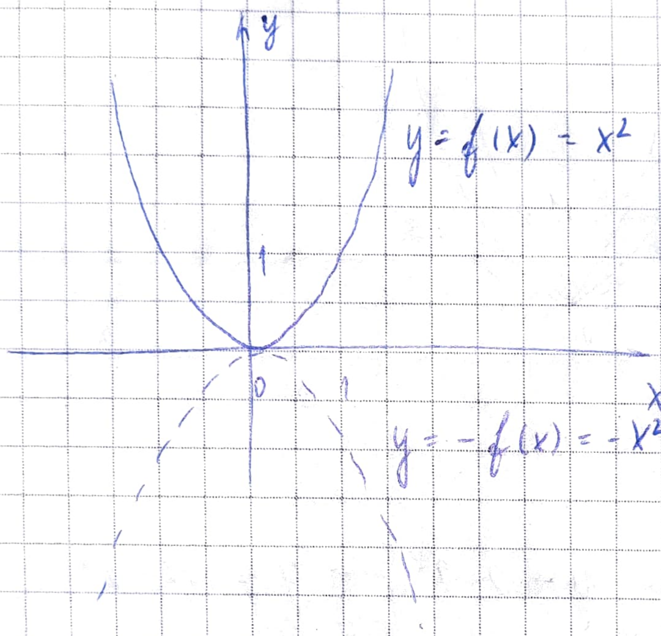
$\Leftrightarrow$ её граф. переходит в себя

при сдвиге вдоль OX на T вправо/влево

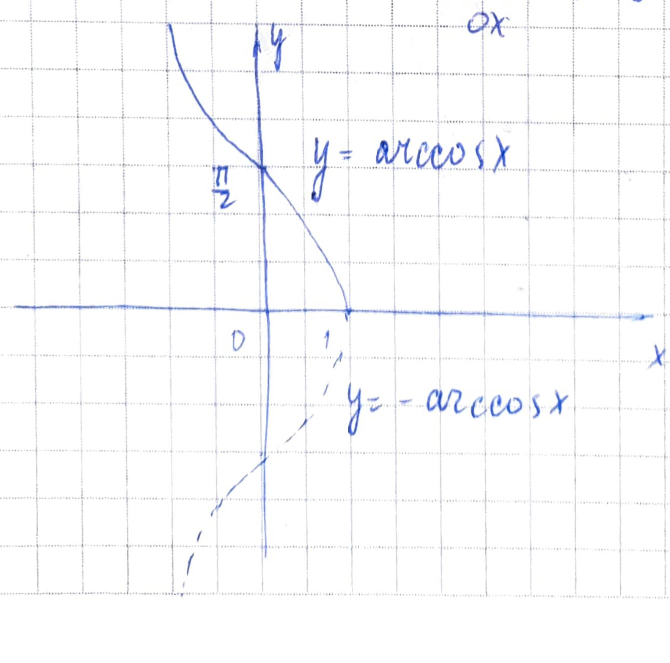
Предположим, что известен граф. ф-и
$y = f(x)$

1) $y = -f(x)$ - симметрия отн. оси OX

$$y = f(x) \xrightarrow[\text{OX}]{\text{сим. отн.}} y = -f(x)$$

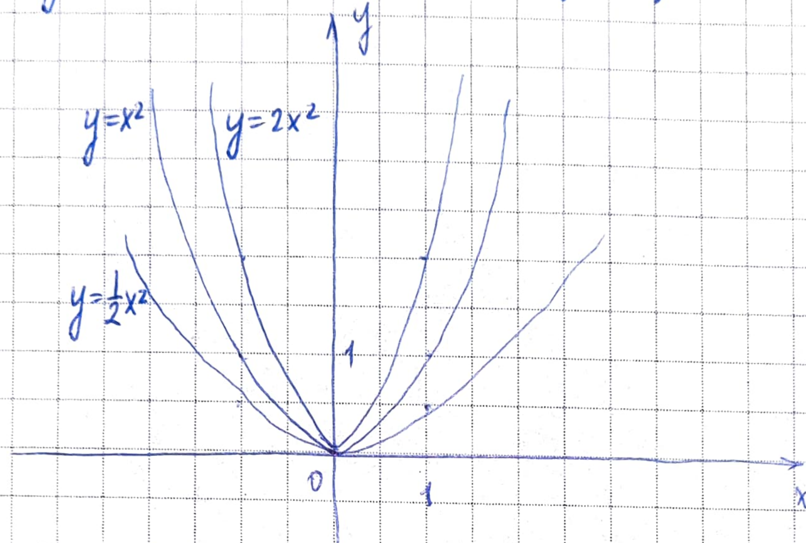


$$y = -\arccos x \xleftarrow[\text{OX}]{\text{сим. отн.}} y = \arccos x$$



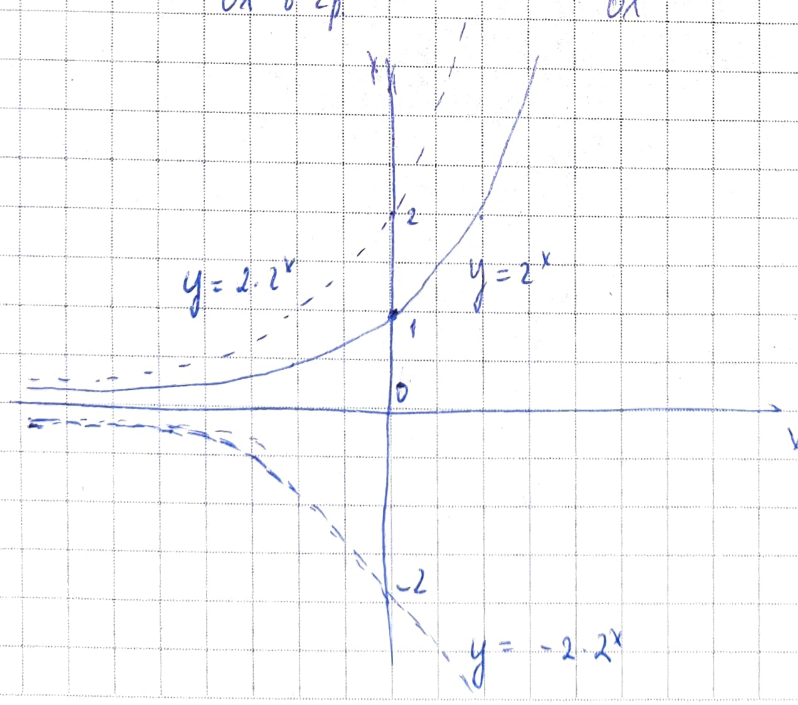
2) $y = k \cdot f(x)$ – растяж от Ox в k раз
$k>0$

$y = x^2$, $y = 2x^2$, $y = \frac{1}{2}x^2$



• $y = -2 \cdot 2^x$

$y = 2^x \xrightarrow[\text{Ox в 2р.}]{\text{раст. от}} y = 2 \cdot 2^x \xrightarrow[\text{Ox}]{\text{симм. отн.}} y = -2 \cdot 2^x$



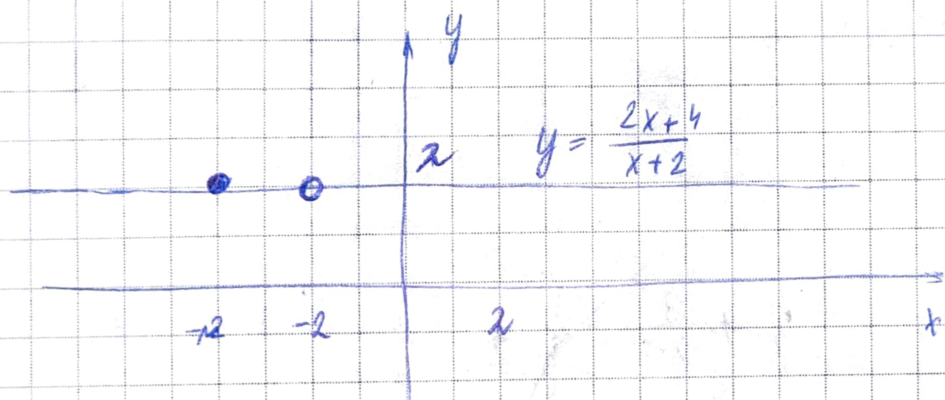
• $y = \frac{ax+b}{cx+d}$ — дробно-линейная ф-я,

если $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

$ad - bc \neq 0$

$ad \neq bc$

$\frac{a}{c} \neq \frac{b}{d}$



Рациональная ф-я — отн. 2ух многочленов

(Мн-н ст. n — вида $P_n(x) = a_n x^n + a_{n-1} x^{n-1} \dots a_1 x + a_0$)

$a_n \dots a_0 \in R$, $a_n \neq 0$

$R(x) = \frac{P_n(x)}{Q_m(x)}$

Замечание: 1) $R_1(x) + R_2(x) = R_0(x)$

2) $R_1(x) \cdot R_2(x) = R_*(x)$

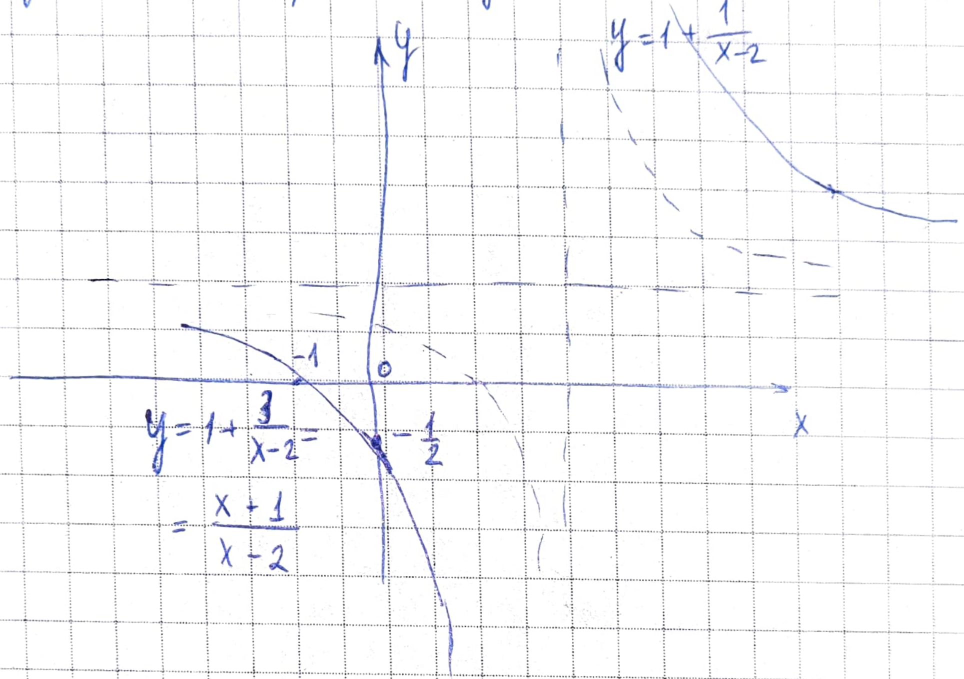
★ дробь зад. рац. ф-ю, правильная, если степ. числ. < степ. знамен.; иначе, если $\geq$ — неправильная

• $y = \frac{x+1}{x-2} = \frac{x-2+2+1}{x-2} = 1 + \frac{3}{x-2} =$

$= 3 \cdot \frac{1}{x-2} + 1$

$\frac{1}{x} \longrightarrow \frac{1}{x-2} \longrightarrow 3\,\frac{1}{x-2} \longrightarrow 1 + 3\,\frac{1}{x-2}$

сдвиг на 2 вдоль Ox | раст. от Ox в 3 р. | сдвиг на 1 вдоль OY



• $y = \frac{3x+1}{5x+2} = \frac{3}{5} \cdot \frac{x+\frac{1}{3}}{x+\frac{2}{5}} = \frac{3}{5} \cdot \left(1 - \frac{1}{15} \Big/ x+\frac{2}{5}\right) =$

$= \frac{3}{5} - \frac{1}{25} \cdot \frac{1}{x+\frac{2}{5}}$

$y = \frac{1}{x} \longrightarrow y = \frac{1}{x+\frac{2}{5}} \longrightarrow y = \frac{1}{25} \cdot \frac{1}{x+\frac{2}{5}} \longrightarrow y = -\frac{1}{25} \cdot \frac{1}{x+\frac{2}{5}} \longrightarrow$

сдвиг на $-\frac{2}{5}$ вдоль Ox | раст. от Ox с $k = \frac{1}{25}$ | ~~сим~~ сим. отн. Ox | сдвиг на $\frac{3}{5}$ вдоль OY

$y = -\frac{1}{25} \cdot \frac{1}{x+\frac{2}{5}} + \frac{3}{5}$

$$f(kx+b) = f\left(k\left(x+\frac{b}{k}\right)\right) \qquad k>0$$

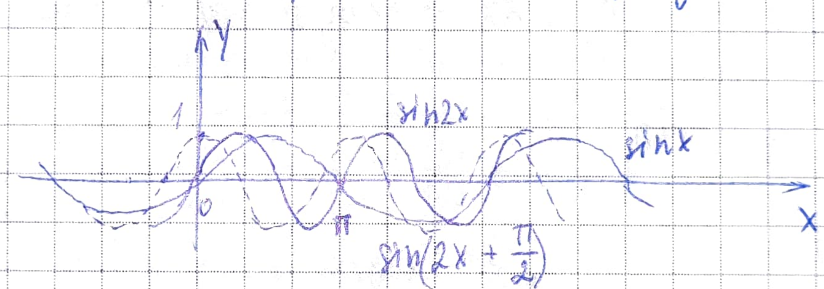
$y = f(x) \longrightarrow y = f(kx) \longrightarrow y = f\left(k\left(x+\frac{b}{k}\right)\right)$

сжат. в k раз к OY; сдвиг вдоль OX влево на b/k

$$y = \sin\left(2x+\frac{\pi}{2}\right)$$

$y = \sin x \rightarrow y = \sin 2x \rightarrow y = \sin\left(2\left(x+\frac{\pi}{4}\right)\right)$

сжат к OY в 2 р; сдвиг на $-\frac{\pi}{4}$ вдоль OX



$$y = \sin 3x + \cos 3x =$$

$y = a\cos x + b\sin x = \sqrt{a^2+b^2}\left(\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\cos\varphi_0}\cos x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\sin\varphi_0}\sin x\right)$

$= \sqrt{a^2+b^2}\,(\cos x\cos\varphi_0 + \sin x\sin\varphi_0) =$

$= \sqrt{a^2+b^2}\cdot\cos(x-\varphi_0)$

$y = \sqrt{1^2+1^2}\left(\frac{1}{\sqrt2}\cos 3x + \frac{1}{\sqrt2}\sin 3x\right) =$

$= \sqrt2\left(\cos\frac{\pi}{4}\cos 3x + \sin\frac{\pi}{4}\sin 3x\right) =$

$= \sqrt2\cos\left(3x-\frac{\pi}{4}\right) = \sqrt2\cos\left(3\left(x-\frac{\pi}{12}\right)\right)$

$y = \cos x \rightarrow \cos 3x \rightarrow \sqrt2\cos 3x \rightarrow \sqrt2\cos 3\left(x-\frac{\pi}{12}\right)$

$y = \cos^4 x - \sin^2 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= \cos 2x$

$y = f(kx + b) = f\left(-|k|\left(x + \frac{b}{k}\right)\right)$

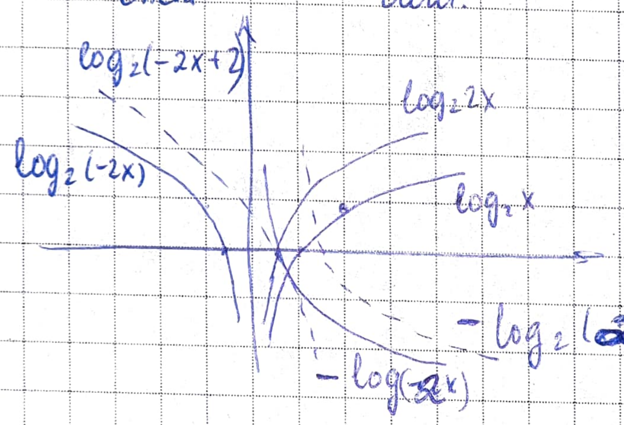
$k < 0$

$y = f(x) \longrightarrow y = f(|k| \cdot x) \longrightarrow y = f(-|k| \cdot x) \longrightarrow -|k|\left(x + \frac{b}{k}\right)$

к ОУ в $|k|$ — симм. отн. ОУ — сдвиг на $-\frac{b}{k}$ вдоль ОХ

$y = \log_2(-2x + 2) = \log_2(-2(x - 1))$

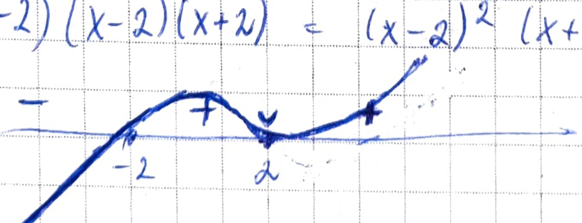
$\log_2 x \xrightarrow{\text{сжат.}} \log_2 2x \xrightarrow{\text{симм.}} \log_2(-2x) \xrightarrow{\text{сдвиг}} \log_2(-2(x-1))$



$y = (x-2)(x^2-4)$

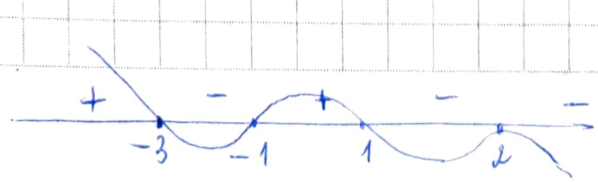
$P_n(x) = A_n (x-2)^3 (x+1)(x-4)^2 (x^2+x+7)(x^2+9)^5$

$y = (x-2)(x-2)(x+2) = (x-2)^2(x+2)$



$(1+x)(x^2+1)$

$y = (1 - x^4)(x+3)(x-2)^2 = (1 - x^4)(x+3)(x-2)^2 =$



Пределы

$\mathbb{N} \ni n \xrightarrow{f} f(n) = a_n \in \mathbb{R}$

$\lim\limits_{n\to\infty} a_n = a \iff \forall \varepsilon > 0 \ \exists N(\varepsilon) \ /$

$\forall n > N(\varepsilon) \ |a_n - a| < \varepsilon$

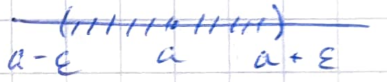
$\Downarrow$

$-\varepsilon < a_n - a < \varepsilon$

$\Downarrow$

$a - \varepsilon < a_n < a + \varepsilon$

$\Downarrow$



$U_\varepsilon(a) = (a - \varepsilon; a + \varepsilon)$

$\lim\limits_{n\to\infty} \frac{1}{n} = 0$

1) $\forall \varepsilon > 0$

2) $\left|\frac{1}{n} - 0\right| < \varepsilon$

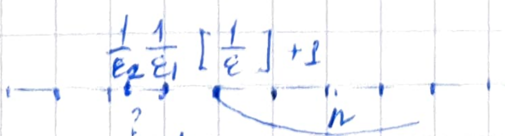
$\Updownarrow$

$\frac{1}{n} < \varepsilon$

$\Updownarrow$ все числа > 0

$n > \frac{1}{\varepsilon}$

3) $N(\varepsilon) = \left[\frac{1}{\varepsilon}\right] + 1$



$\Uparrow$

$\forall n > N(\varepsilon) = \left[\frac{1}{\varepsilon}\right] + 1$

Опр. $[x]$ – цел. часть числа – наиб. цел. число, кот. $\leq x$

$[2] = 2$

$[2,5] = 2$

$[-2,5] = -3$

BE DIFFERENT

Беск. большие посл.

<u>Опр.</u> $\lim\limits_{n\to\infty} a_n = +\infty$

$\{a_n = n\}_{n=1}^{\infty}$

$\Leftrightarrow \forall M > 0 \quad \exists N(M)$

$\forall n > N(M) \Rightarrow a_n > M$

<u>Опр.</u> $\lim\limits_{n\to\infty} a_n = -\infty$

$\{a_n = -n\}_{n=1}^{\infty}$

$\Leftrightarrow \forall M > 0 \quad \exists N(M)$

$\forall n > N(M) \Rightarrow a_n < -M$

<u>Опр.</u> $\lim\limits_{n\to\infty} a_n = \infty$

$\{a_n = (-1)^n \cdot n\}_{n=1}^{\infty}$

$\Leftrightarrow \lim\limits_{n\to\infty} |a_n| = +\infty$

Беск. малые посл.

<u>Опр.</u> $\lim\limits_{n\to\infty} a_n = 0$

$\{a_n\}$ - б.м. $\Rightarrow \left\{\frac{1}{a_n}\right\}$ б.б.

$\forall n, a_n \neq 0$

$\{b_n\}$ - б.б. $\Rightarrow \left\{\frac{1}{b_n}\right\}$ - б.м.

$$\begin{cases} x_0 = x(t_0) = 1 \\ y_0 = y(t_0) = 1 \end{cases}$$ $M_0(x_0, y_0) = M_0(1, 1)$

$y'_x(t_0) = y'_x(0) = 0$

M_0 — крит. т.

$y''_{xx} = \frac{-1}{2 \cdot 1 \cdot 1} = -\frac{1}{2}$

вып. вверх

при $t = t_0$ — лок. max

Метод мат. инд. (не прим. к конечн.)

$F(n)$ верно при $\forall n \in N$

1) $n = 1$ ($= n_0$) — база индукции

2) верно при $n = k$

3) верно при $n = k+1$?

8) $y(x) = \ln x$

$y' = \frac{1}{x} = x^{-1}$

....

$y^{(n)} = (-1)^{n-1} \cdot (n-1)!\, x^{-n}$

1) $\underbrace{y' = (-1)^0 \cdot 0!\, x^{-1}}_{F(1)} = x^{-1}$ — верно

2) верно при $n=k$ $\quad y^{(k)}=\underbrace{(-1)^{k-1}(k-1)!\,x^{-k}}_{F(k)}$

3) $y^{(k+1)}=\left(y^{(k)}\right)'=$

$=\left((-1)^{k-1}(k-1)!\,x^{-k}\right)'=$

$=(k-1)!\,(-1)^{k-1}\cdot(-k)\cdot x^{-(k+1)}=$

$=\underbrace{(-1)^{k}\cdot k!\,x^{-(k+1)}}_{F(k+1)}$

9) $y=\sin x$ $\quad y^{n}=(\sin x)^{(n)}$

$y'=(\sin x)'=\cos x=\sin\left(\frac{\pi}{2}+x\right)$

$y''=-\sin x=\sin\left(\frac{\pi}{2}\cdot 2+x\right)$

$y'''=-\cos x=\sin\left(\frac{3\pi}{2}+x\right)$

$y^{IV}=\sin x=\sin\left(\frac{4\pi}{2}+x\right)$

$y^{n}=\sin\left(\frac{\pi}{2}\cdot n+x\right)$ $\quad n\in\mathbb{N}$

1) $F(1)=\sin\left(\frac{\pi}{2}\cdot 1+x\right)=\cos x$

2) $F(k)=\sin\left(\frac{\pi}{2}\cdot k+x\right)$ – верно

3) $F(k+1)=\left(F(k)\right)'=\sin\left(\frac{\pi}{2}\cdot(k+1)+x\right)$

$F(k+1)'=\left(\sin\left(\frac{\pi}{2}\cdot(k+1)+x\right)\right)'=$

$=\sin\left(\frac{\pi}{2}(k+1)+x\right)=\left(\sin\left(\frac{\pi}{2}\cdot k+x\right)\right)'=\cos\left(\frac{\pi}{2}k+x\right)=\sin\left(\frac{\pi}{2}k+x+\frac{\pi}{2}\right)$

13) $z = \sqrt{x^2+y^2}$ $\qquad dz - ?$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

$$dz = \frac{x}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}}dy$$

13) $u = e^{s/t}$ $\qquad du - ?$

$$\frac{\partial u}{\partial s} = e^{s/t} \cdot \frac{1}{t}$$

$$\frac{\partial u}{\partial t} = e^{s/t} \cdot \left(-\frac{s}{t^2}\right)$$

$$du = \frac{e^{s/t}}{t}ds + \left(-\frac{e^{s/t} \cdot s}{t^2}\right)dt =$$

$$= \frac{1}{t} \cdot e^{s/t}ds - \frac{s}{t^2}e^{s/t}dt$$

14) $z = 2xy$ найти $dz(x_0, y_0)$, $(x_0, y_0) = (5, 4)$

$$\frac{\partial z}{\partial x} = 2y \qquad \frac{\partial z}{\partial x}(x_0, y_0) = \frac{\partial z}{\partial \cancel{x}}\Big|_{(x_0, y_0)} = 8$$

$$\frac{\partial z}{\partial y} = 2x \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{\partial z}{\partial y}\Big|_{(x_0, y_0)} = 10$$

$$dz(x_0, y_0) = dz\Big|_{(x_0, y_0)} = 8dx + 10dy$$

15) $f(x_0+\Delta x, y_0+\Delta y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)} \Delta x + \frac{\partial f}{\partial y}\Big|_{(x_0,y_0)} \Delta y$

1) $f(x,y)$ — ?

2) x — ? $\quad \Delta x$ — ?

y — ? $\quad \Delta y$ — ?

3) $\frac{\partial f}{\partial x}$ — ? $\quad \frac{\partial f}{\partial y}$ — ?

4) $f(x_0,y_0) \quad \frac{\partial f}{\partial x}(x_0,y_0) \quad \frac{\partial f}{\partial y}(x_0,y_0)$

5) $f(x,y)$ — ?

$\sqrt[4]{2{,}2^3 \cdot 1{,}9} \approx$ — ?

1) $f(x,y) = \sqrt[4]{x^3 \cdot y}$

2) $x = 2{,}2 \quad \Delta x = 0{,}2 \quad x - x_0$

$y = 1{,}9 \quad \Delta y = -0{,}1 \quad y - y_0$

$x_0 = 2$

$y_0 = 2$

3) $\frac{\partial f}{\partial x} = \frac{3}{4} \cdot x^{-1/4} \cdot y^{1/4} = \frac{3}{4}\sqrt[4]{\frac{y}{x}}$

$\frac{\partial f}{\partial y} = \frac{1}{4}\sqrt[4]{\frac{x^3}{y^3}}$

4) $f(x_0,y_0) = f(2,2) = \sqrt[4]{2^3 \cdot 2} = 2$

Любовь Михайловна Лутина

Мат. анализ (практика)

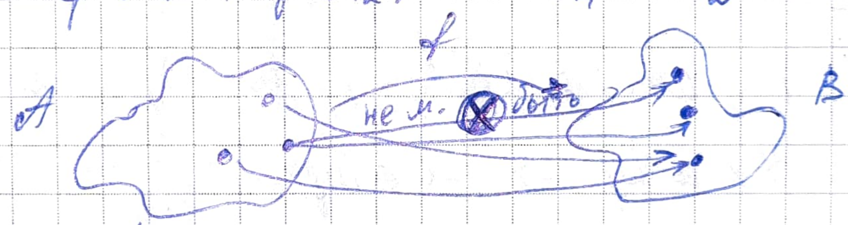
Не опаздывать, ксм

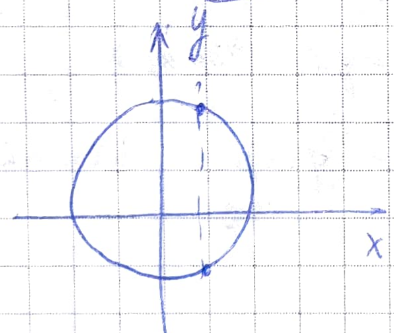
Эскизы графиков.

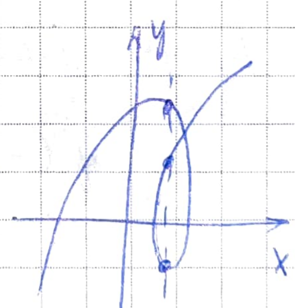
Преобразование графиков.

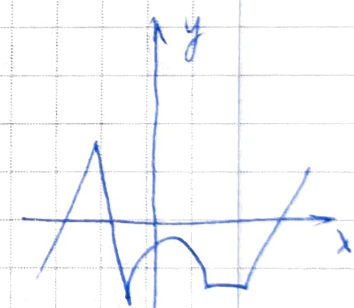
Опр. Функция – закон (правило), который числам одного числового мн-ва ставит в соответств. числа др. числ. мн-ва, причем

$$f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$$









$y=f(x)$ нет $x^2+y^2=R^2$ нет да $y=f(x)$

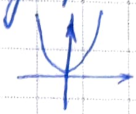
Область опр. ф-и — любое мн-во, кажд. т. которого зн. ф-ии можно найти (по закону, зад. ф-ию)

$y = x^2$ $\quad R$

$[-1; 2]$

$[0; 1]$

$[-1; 1]$



неогр.
чт.
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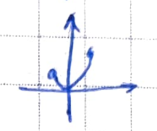
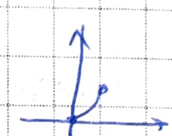


огр.
нечет.
не мон.



огр.
нечет.
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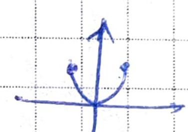


огр.
чет.
не мон

Опр.

Ф. $y=f(x)$ четная (нечет.) на D_f, если

1) $\forall x \in D_f \Rightarrow -x \in D_f$

2) $\forall x \in D_f \Rightarrow f(-x) = f(x)$ — четная

$f(-x) = -f(x)$ — нечетная

Теорема.

Ф. $y = f(x)$ явл. четной $\Leftrightarrow$ её график симметр. отн. оси OY

Теорема

Ф. $y = f(x)$ явл. нечетной $\Leftrightarrow$ её график центр. симметр. (переходит в себя при пов. на 180° отн. нач. коорд.)

Мат. анализ (5)

$y = 3^{\frac{x-1}{x+2}}$

$y = f(\varphi(x))$

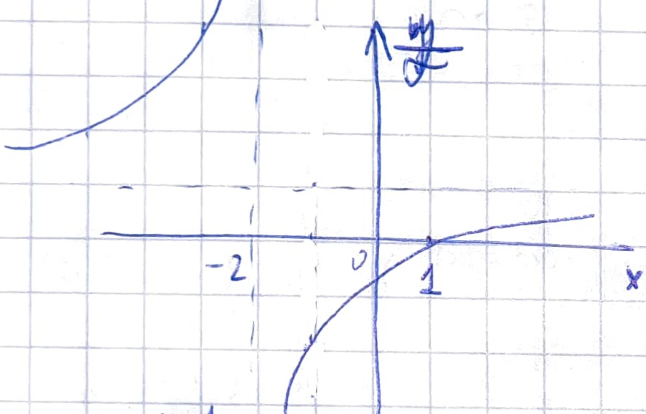
1) $z = \varphi(x)$

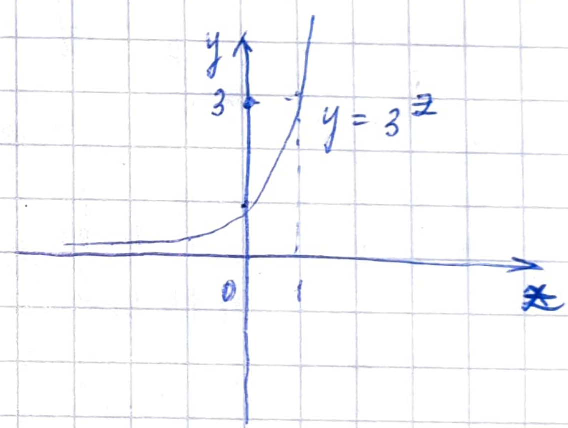
2) $y = f(z)$

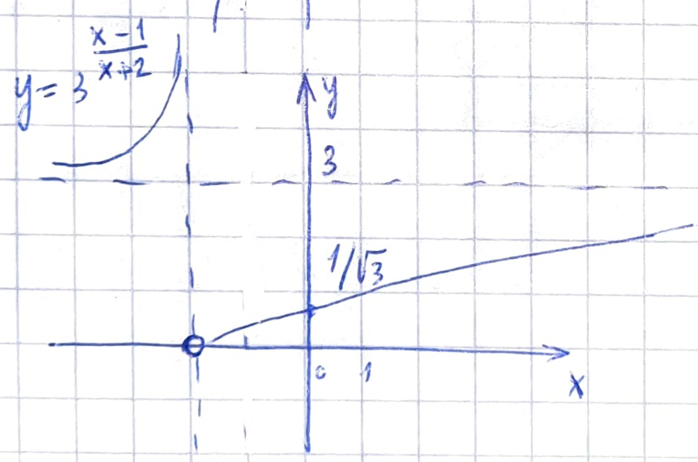
3) $y = f(\varphi(x))$

$z = \frac{x-1}{x+2}$ $\quad y = 3^z$

$z = 1 - \frac{3}{x+2}$







$x: (-\infty \nearrow -2)$

$(z: (1 \nearrow +\infty))$

$y: (3 \nearrow +\infty)$

$x: (-2 \nearrow +\infty)$

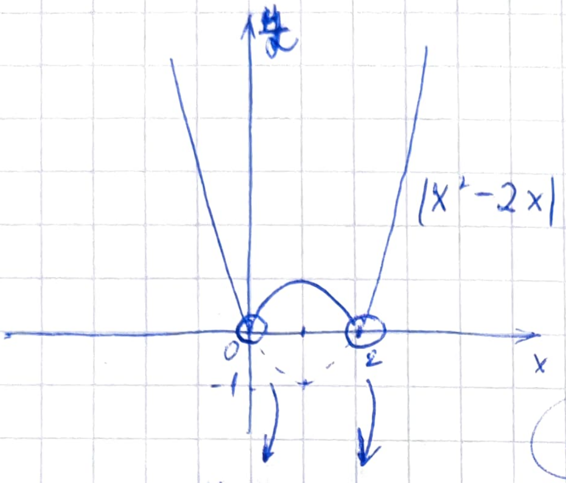
$(z: (-\infty \nearrow +1))$

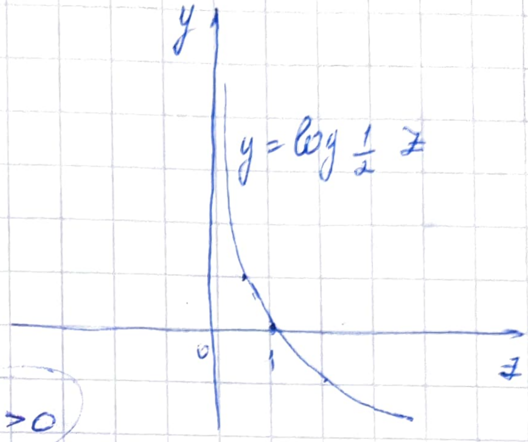
$y = (0 \nearrow 3)$

BE DIFFERENT

$y = \log_{1/2} |x^2 - 2x|$

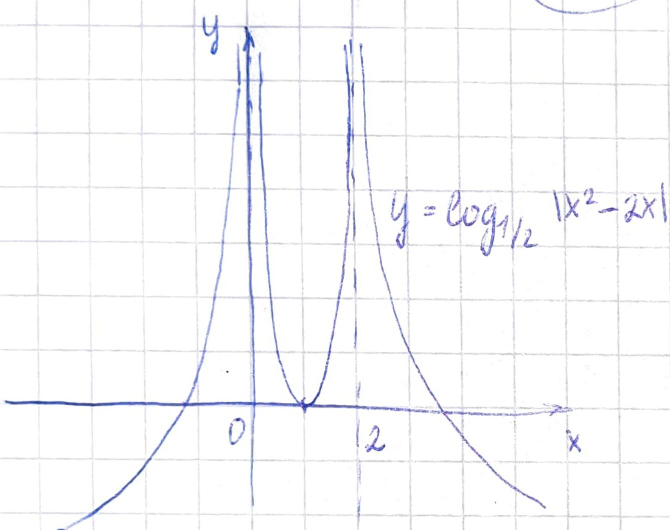
$\varphi(x) = |x^2 - 2x|$ $\qquad$ $y = \log_{\frac{1}{2}} z$





$z > 0$

О.О. : $z > 0$



1) $x : (-\infty \nearrow 0)$

$z : (+\infty \searrow 0)$

$y : (-\infty \nearrow +\infty)$

2) $x : (0 \nearrow 1]$

$z : (0 \nearrow 1]$

$y : (+\infty \searrow 0)$

4) $x : (2 \nearrow +\infty)$

$z : (0 \nearrow +\infty)$

$y : (+\infty \searrow -\infty)$

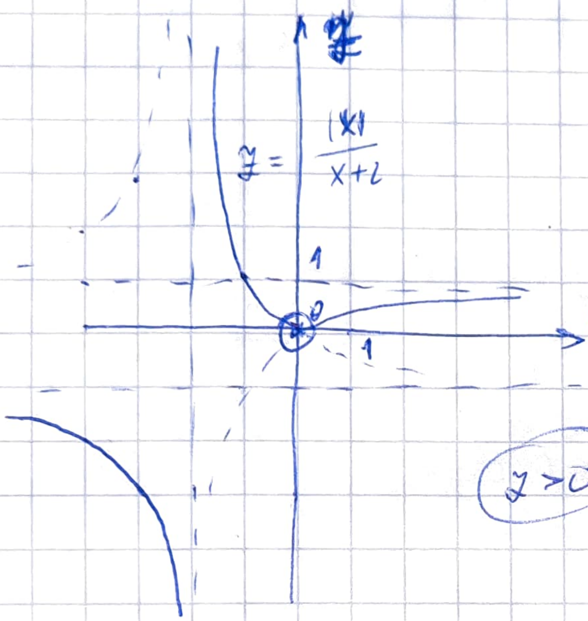
3) $x : [1 \nearrow 2)$

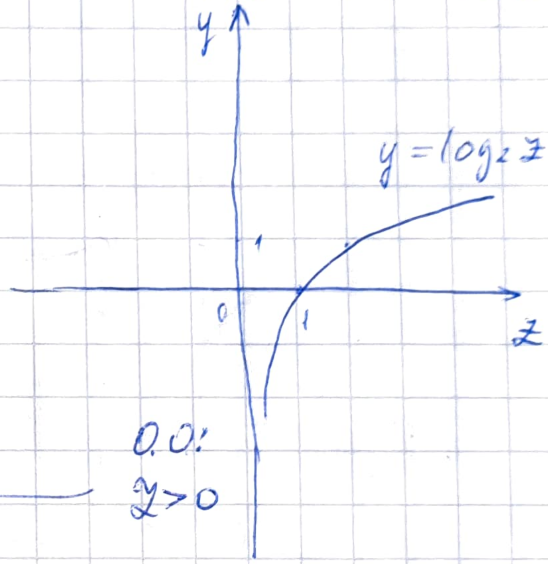
$z : [1 \searrow 0)$

$y : [0 \nearrow +\infty)$

$$y = \log_2 \frac{|x|}{x+2}$$

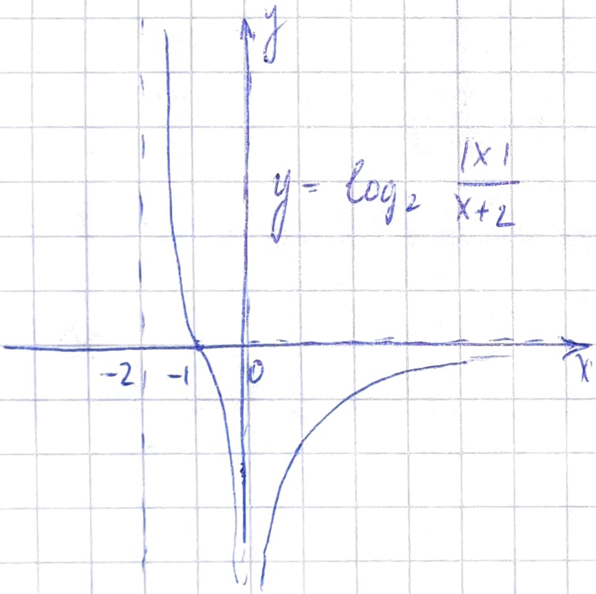
$$z = \frac{|x|}{x+2} = \begin{cases} x \geq 0: & z = \frac{x}{x+2} = 1 - \frac{2}{x+2} \\ x < 0: & z = -\frac{x}{x+2} = -1 + \frac{2}{x+2} \end{cases}$$





$z > 0$

О.О: $z > 0$



1) $x: (-2 \nearrow 0)$

$z: (+\infty \searrow 0)$

$y: (+\infty \searrow -\infty)$

2) $x: (0 \nearrow +\infty)$

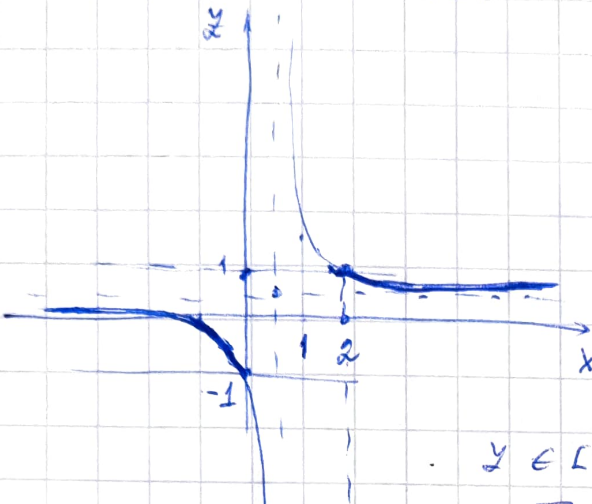
$z: (0 \nearrow 1)$

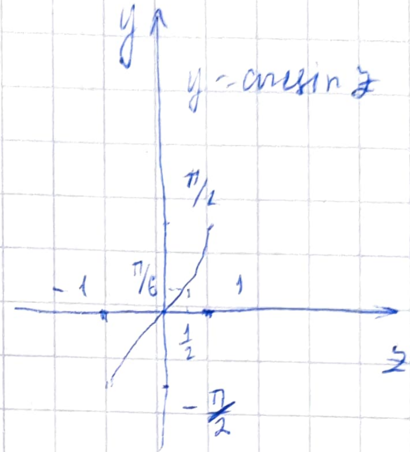
$y: (-\infty \nearrow 0)$

BE DIFFERENT

$$y = \arcsin \frac{x+1}{2x-1} \qquad y = \arcsin z$$

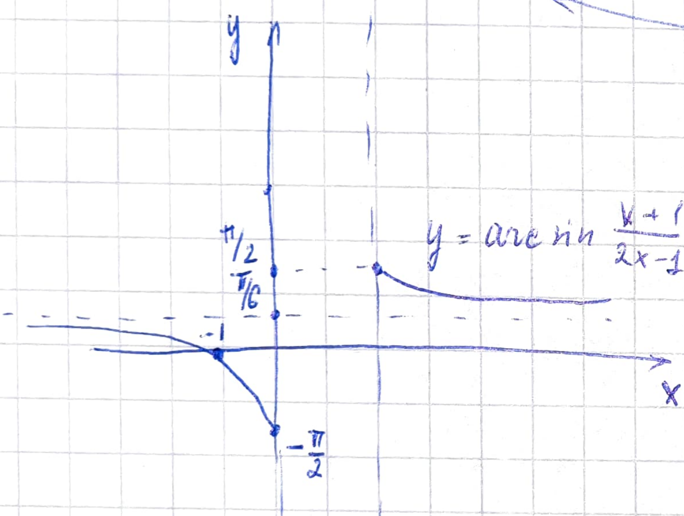
$$z = \frac{x+1}{2x-1} = \frac{1}{2}\left(\frac{x+1}{x-1/2}\right) = \frac{1}{2}\left(1 + \frac{3/2}{x-1/2}\right) = \frac{1}{2} + \frac{3}{4}\cdot\frac{1}{x-1/2}$$





$z \in [-1;1]$

О.О. $[-1;1]$



1) $x: (-\infty \uparrow 0]$

$z: (1/2 \downarrow -1]$

$y: (\pi/6 \downarrow -\frac{\pi}{2}]$

2) $x: [2 \uparrow +\infty)$

$z: [1 \downarrow \frac{1}{2})$

$y: [\frac{\pi}{2} \downarrow \frac{\pi}{6})$

$$\lim_{n\to\infty} (a_n + b_n)$$

$$\lim_{n\to\infty} = \frac{a_n}{b_n}$$

1) $$\lim_{n\to\infty} \frac{2n+1}{3n-2} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{2+\frac{1}{n}}{3-\frac{2}{n}} =$$

$$= \frac{\lim_{n\to\infty}\left(2+\frac{1}{n}\right)}{\lim_{n\to\infty}\left(3-\frac{2}{n}\right)} \overset{?}{=} \frac{\lim_{n\to\infty} 2 + \lim_{n\to\infty}\frac{1}{n}}{\lim_{n\to\infty} 3 - \lim_{n\to\infty}\frac{1}{n}} = \frac{2+0}{3-0} = \frac{2}{3}$$

$$\lim_{n\to\infty} \frac{2n+1}{3n-2} = \lim_{n\to\infty} \frac{2+\frac{1}{n}}{3-\frac{2}{n}} = \frac{2}{3}$$

2) $$\lim_{n\to\infty} \frac{3(n+1)^2}{2n^2} = \lim_{n\to\infty} \frac{3n^2+6n+3}{2n^2} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{3+\frac{6}{n}+3\cdot\frac{1}{n^2}}{2\cdot\frac{n^2}{n^2}} =$$

$$= \frac{3}{2}$$

$$\lim_{n\to\infty} \frac{(n+1)^3-(n-1)^3}{(n+1)^2+(n-1)^2} = \lim_{n\to\infty} \frac{n^3+3n^2+3n+1-n^3+3n^2-3n+1}{n^2+2n+1+n^2-2n+1} =$$

$$= \lim_{n\to\infty} \frac{6n^2+2}{2n^2+2} = \left[\frac{\infty}{\infty}\right] = \frac{6}{2} = 3$$

BE DIFFERENT

3) $$\lim_{n\to\infty} \frac{n^3-10n^2+1}{100n^2+2n+7} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{n-10+\frac{1}{n^2}}{1000+\left(2\frac{1}{n}+\frac{7}{n^2}\right)} = \infty$$

$$\lim_{n\to\infty} \frac{(n+1)^4-(n-1)^4}{(n+1)^4+(n-1)^4} \overset{\lim_{n\to\infty}}{=} \frac{n^4+4n^3+6n^2+4n+1-n^4+4n^3-6n^2+4n-1}{n^4+4n^3+6n^2+4n+1+n^4-4n^3+6n^2-4n+1} =$$

$$= \lim_{n\to\infty} \frac{8n^3+8n}{2n^4+12n^2+2} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{8+\frac{8}{n^2}}{2n+\frac{12}{n}+\frac{2}{n^3}} = 0$$

4) $$\lim_{n\to\infty} \frac{\sqrt[3]{n^3+2n-1}}{n+2} = \left[\frac{\infty}{\infty}\right] \overset{\lim_{n\to\infty}}{=} \frac{\sqrt[3]{n^3\left(1+\frac{2}{n^2}-\frac{1}{n^3}\right)}}{n+2} \overset{\lim_{n\to\infty}}{=} \frac{n\cdot\sqrt[3]{1+\frac{2}{n^2}-\frac{1}{n^3}}}{n+2} =$$

$$= \frac{\sqrt[3]{1+\frac{2}{n^2}-\frac{1}{n^3}}}{1+\frac{2}{n}} = \frac{1}{1} = 1$$

5) $$\lim_{n\to\infty} \frac{\sqrt[3]{n^2+n}}{n+1} = \lim_{n\to\infty} \frac{\sqrt[3]{n^3\left(\frac{1}{n}+\frac{1}{n^2}\right)}}{n+1} = \lim \frac{n\sqrt[3]{\left(\frac{1}{n}+\frac{1}{n^2}\right)}}{n+1} = \frac{\sqrt[3]{\frac{1}{n}+\frac{1}{n^2}}}{\frac{1}{n}+1} = 0$$

6) $$\lim_{n\to\infty} \frac{n!}{(n+1)!-n!} = \lim_{n\to\infty} \frac{n!}{(n+1)n!-n!} = \lim_{n\to\infty} \frac{1}{n+1-1} = 0$$

7) $$\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+3)!} = \lim_{n\to\infty} \frac{(n+2)(n+1)!+(n+1)!}{(n+3)(n+2)(n+1)!} = \frac{n+2+1}{(n+3)(n+2)} = 0$$

8) $$\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!} = \frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n-1)!} = \frac{n+2+1}{n+2-1} = 1$$

$$S_{\text{бүгл}} = a_1 \cdot \frac{1-q^n}{1-q}$$

9) $$\lim_{n\to\infty} \frac{1+\frac{1}{2}+\frac{1}{4}\dots\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}\dots\frac{1}{3^n}} = \lim_{n\to\infty} \frac{1\cdot\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-1/2}}{1\cdot\frac{1-(1/3)^{n+1}}{1-1/3}} = \lim_{n\to\infty}\frac{2}{3/2} = \frac{4}{3}$$

$$S_{\text{ар.}} = \frac{a_1+a_n}{2}\cdot n$$

10) $$\lim_{n\to\infty}\left(\frac{1}{n^2}(1+2+\dots+n)\right) = [0\cdot\infty] = \lim_{n\to\infty}\left(\frac{1}{n^2}\cdot\frac{1+n}{2}\cdot n\right) =$$

$$= \lim_{n\to\infty}\left(\frac{n+n^2}{2n^2}\right) = \frac{1}{2}$$

11) $$\lim_{n\to\infty}\left(\frac{1+2\dots+n}{n+2} - \frac{n}{2}\right) = [\,?-\infty\,] =$$

$$\lim_{n\to\infty}\left(\frac{\frac{1+n}{2}\cdot n}{n+2} - \frac{n}{2}\right) = \lim_{n\to\infty}\left(\frac{n+n^2}{2(n+2)} - \frac{n}{2}\right) = \lim_{n\to\infty}\frac{n^2+n-n^2-2n}{2(n+2)}$$

$$= \lim_{n\to\infty}\frac{-n}{2n+4} = -\frac{1}{2}$$

12) $$\lim_{n\to\infty}\frac{2^n-1}{2^n+1} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty}\frac{1-\frac{1}{2^n}}{1+\frac{1}{2^n}} = 1$$

13) $$\lim_{n\to\infty}\frac{2^{\frac{1}{n}}-1}{2^{\frac{1}{n}}+1} = \frac{0}{2} = 0$$

14) $$\lim_{n\to\infty}\left(\sqrt{2n+3}-\sqrt{2n+1}\right) = [(+\infty)-(\infty)] = [\infty-\infty] =$$

$$\lim_{n\to\infty}\frac{(\sqrt{2n+3}-\sqrt{2n+1})(\sqrt{2n+3}+\sqrt{2n+1})}{\sqrt{2n+3}+\sqrt{2n+1}} = \frac{2n+3-2n+1}{\sqrt{2n+3}+\sqrt{2n+1}} = \frac{2}{\infty} = 0$$

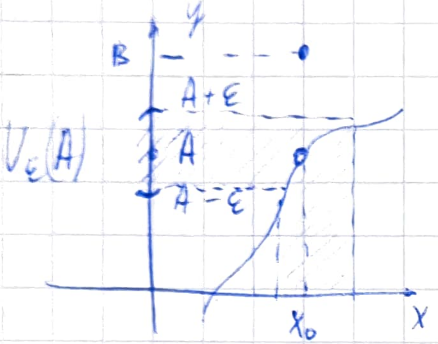
BE DIFFERENT

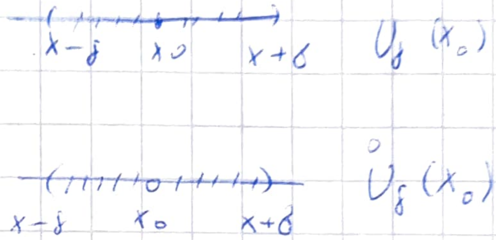
① $\lim\limits_{x\to x_0} f(x) = A$, если $\forall \varepsilon > 0$

$\exists\, \delta(x_0, \varepsilon) > 0 \mid \forall x \quad 0 < |x - x_0| < \delta$

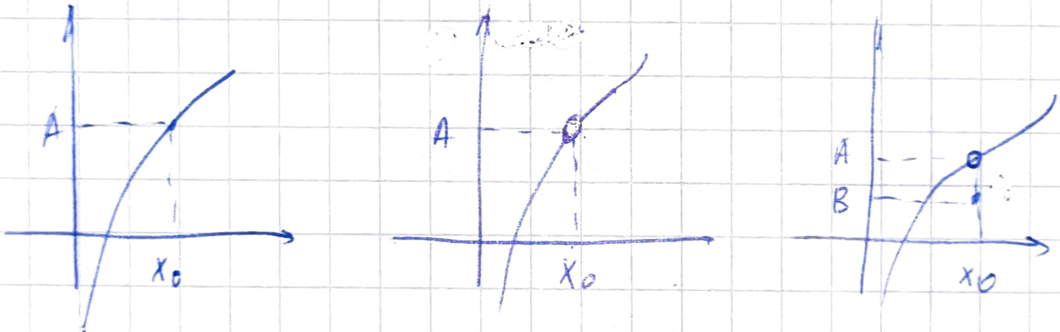
$|f(x) - A| < \varepsilon$

$A - \varepsilon < f(x) < A + \varepsilon$



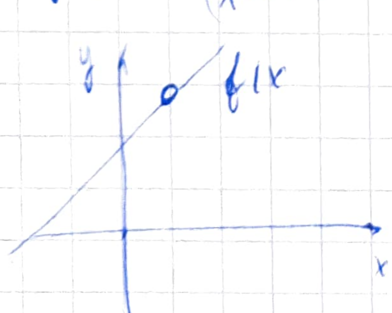


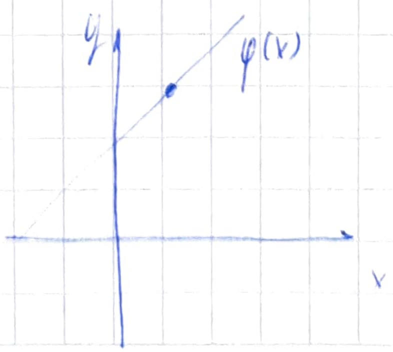
②



$\lim\limits_{x\to x_0} f(x) = f(x_0) = A \qquad \lim\limits_{x\to x_0} f(x) = A \qquad \lim\limits_{x\to x_0} f(x) = A \neq f(x_0) = B$

③ $f(x) = \dfrac{(x+2)(x-1)}{(x-1)}$ $\qquad \varphi(x) = x+2$





Мат. анал. 7 п

1) $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x^2-3x+2} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 2} \dfrac{(x-2)(x+3)}{(x-2)(x-1)} =$

$\lim\limits_{x\to 2} \dfrac{x+3}{x-1} = \dfrac{5}{1} = 5$

2) $\lim\limits_{x\to 2} \dfrac{x^3-12x+16}{x^2-4} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 2} \dfrac{(x-2)(x^2+2x-8)}{(x-2)(x+2)} =$

$\lim\limits_{x\to 2} \dfrac{x^2+2x-8}{x+2} = \dfrac{0}{4} = 0$

3) $\lim\limits_{x\to -2} \dfrac{x^3+3x^2+2x}{x^2-x-6} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -2} \dfrac{x(x+2)(x+1)}{(x+2)(x-3)} =$

$= \lim\limits_{x\to -2} \dfrac{x(x+1)}{x-3} = \dfrac{2}{-5} = -\dfrac{2}{5}$

4) $\lim\limits_{x\to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3}\right) = [\infty - \infty] = \lim\limits_{x\to 1} \left(\dfrac{1+x+x^2-3}{(1-x)(1+x+x^2)}\right) =$

$= \lim\limits_{x\to 1} \dfrac{(x-1)(x+2)}{-(x-1)(x^2+x+1)} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 1} \dfrac{-(x+2)}{x^2+x+1} = -1$

∞ ←

5) $\lim\limits_{x\to 2} \left(\dfrac{1}{x(x-2)^2} - \dfrac{1}{x^2-3x+2}\right) = \lim\limits_{x\to 2} \left(\dfrac{x-1 - x^2+2x}{x(x-2)^2(x-1)}\right) =$

$= \lim\limits_{x\to 2} \dfrac{-x^2+3x-1}{x(x-2)^2(x-1)} = \left[\dfrac{k}{0}\right] = \infty$

BE DIFFERENT

$x \to \infty \Leftrightarrow |x| \to +\infty \qquad x \to \pm\infty \begin{smallmatrix} \nearrow + \\ \searrow - \end{smallmatrix}$

6) $$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} =$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} (3x^2 + 3xh + h^2) =$$

$$= 3x^2$$

7) $$\lim_{x \to \pm\infty} \left(\sqrt{x^2+1} - x\right) = \left[\begin{array}{l} \lim\limits_{x \to +\infty} (\overset{\infty}{\sqrt{x^2+1}} - \overset{\infty}{x}) = [\infty - \infty] \\ \lim\limits_{x \to -\infty} (\underbrace{\sqrt{x^2+1}}_{\infty} \underbrace{-x}_{\infty}) = +\infty \end{array} \right.$$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)}{\sqrt{x^2+1} + x} = \lim_{x \to +\infty} \frac{x^2 + 1 - x^2}{\underbrace{\sqrt{x^2+1}}_{\infty} + \underbrace{x}_{\infty}} = 0$$

8) $$\lim_{x \to 0} \frac{\sqrt{1+x^2} - 1}{x} = \left[\frac{0}{0}\right] = \lim_{x \to 0} \frac{1 + x^2 - 1}{x\left(\sqrt{1+x^2} + 1\right)} =$$

$$\lim_{x \to 0} \frac{x}{\sqrt{1+x^2} + 1} = \frac{0}{2} = 0$$

9) $$\lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x - 5} = \left[\frac{0}{0}\right] = \lim_{x \to 5} \frac{x - 1 - 4}{\left(\sqrt{x-1} + 2\right)(x-5)} = \lim_{x \to 5} \frac{1}{\sqrt{x-1} + 2} =$$

$$= \frac{1}{4}$$

10) $$\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{x^2\left(\sqrt{x^2+16}+4\right)}{x^2\left(\sqrt{x^2+1}+1\right)} =$$

$$= \frac{\sqrt{x^2+16}+4}{\sqrt{x^2+1}+1} = \frac{8}{2} = 4$$

11) $$\lim_{x\to\pm\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+4}\right) = [\infty-\infty] =$$

$$= \lim_{x\to\pm\infty} \frac{x^2-2x-1 - x^2+7x-4}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+4}} = \frac{5x-5}{\sqrt{x^2\left(1-\frac{2}{x}-\frac{1}{x^2}\right)}+\sqrt{x^2\left(1-\frac{7}{x}+\frac{4}{x^2}\right)}} =$$

$$= \lim_{x\to\pm\infty} \frac{5x-5}{|x|\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{4}{x^2}}\right)} =$$

$$= \begin{cases} \lim\limits_{x\to+\infty} \dfrac{5x-5}{x\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{4}{x^2}}\right)} = \left[\dfrac{\infty}{\infty}\right] = \dfrac{5}{2} \\ \lim\limits_{x\to-\infty} \dfrac{5x-5}{-x\left(\sqrt{1-2/x+1/x^2}+\sqrt{1-7/x+4/x^2}\right)} = \left[\dfrac{\infty}{\infty}\right] = \dfrac{5}{-2} = -\dfrac{5}{2} \end{cases}$$

12) $$\lim_{x\to+\infty}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right) = [\infty-\infty] =$$

$$\cancel{\lim_{x\to+\infty}\left((x+1)^{2/3}-(x-1)^{2/3}\right) = \lim \frac{(x+1)^{4/3}-(x-1)^{4/3}}{(x+1)^{2/3}+(x-1)^{2/3}}}$$

$$= \lim_{x\to+\infty}\left(\sqrt[3]{x^2+2x+1}-\sqrt[3]{x^2-2x+1}\right) = \lim_{x\to+\infty}\left(\sqrt[3]{1+\frac{2}{x}+\frac{1}{x^2}}-\sqrt[3]{1-\frac{2}{x}+\frac{1}{x^2}}\right) =$$

$$\cancel{\lim_{x\to+\infty}\left((x+1)^{1/3}+(x-1)^{1/3}\right)\left((x+1)^{1/3}-(x-1)^{1/3}\right)}$$

$$= \lim_{x\to+\infty} \frac{(x+1)^{4/3}-(x-1)^{4/3}}{(x+1)^{2/3}+(x-1)^{2/3}} = \frac{\sqrt[3]{x^4+4x^3+6x^2+4x+1}-\sqrt[3]{x^4-4x^3+6x^2-4x+1}}{\sqrt[3]{x^2+2x+1}+\sqrt[3]{x^2-2x+1}}$$

$$= \frac{x\sqrt[3]{x+4+\frac{6}{x^2}+\frac{4}{x^3}+\frac{1}{x^4}} - x\sqrt[3]{x-4+\frac{6}{x^2}-\frac{4}{x^3}+\frac{1}{x^4}}}{x\sqrt[3]{x+\frac{2}{x}+\frac{1}{x^2}} \qquad x\sqrt[3]{x-\frac{2}{x}+\frac{1}{x^2}}} =$$

$$= 1 - 1 = 0$$

пусть $y = x^{\frac{1}{12}}$

13) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1} = \left[\frac{0}{0}\right] = \lim\limits_{y\to 1} \dfrac{y^4-1}{y^3-1} = \lim\limits_{y\to 1} \dfrac{(y-1)(y+1)(y^2+1)}{(y-1)(y^2+y+1)} =$

$\lim\limits_{y\to 1} \dfrac{(y+1)(y^2+1)}{y^2+y+1} = \dfrac{4}{3}$

пусть $y = (1+x)^{\frac{1}{6}}$ $\quad y \to 1$

14) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left[\frac{0}{0}\right] = \lim\limits_{y\to 1} \dfrac{y^3-1}{y^2-1} = \lim\limits_{y\to 1} \dfrac{(y-1)(y^2+y+1)}{(y-1)(y+1)} = \dfrac{3}{2}$

15) $\lim\limits_{x\to x_0} (f(x))^{h(x)}$

1. $\lim\limits_{x\to x_0} (\underset{\searrow A}{f(x)})^{h(x) \to B \neq 0} = A^B$

2. $\lim\limits_{x\to x_0} (\underset{\searrow A \neq 1}{f(x)})^{h(x) \to \infty} = \oplus$

3. $\lim\limits_{x\to x_0} (\underset{\searrow 1}{f(x)})^{h(x) \to \infty} = [1^\infty]$.

$\lim\limits_{x\to 0} \left(\dfrac{2x+2}{x+3}\right)^{1+x} = \left(\dfrac{2}{3}\right)^1 = \dfrac{2}{3}$

16) $\lim\limits_{x\to\infty} \left(\underset{\searrow \frac{1}{2}}{\dfrac{x+1}{2x+1}}\right)^{x^2 \to \infty} = 0$

17) $\lim\limits_{\substack{x\to x_0 \\ f(x)\to 1}} (f(x))^{h(x)} = \lim\limits_{x\to x_0} (1 + \underbrace{f(x)-1}_{\varphi(x) \to 0})^{h(x)} =$

$= \lim\limits_{\substack{x\to x_0 \\ \varphi(x)\to 0}} (1+\varphi(x))^{h(x)} = \lim\limits_{\substack{x\to x_0 \\ \varphi(x)\to 0}} \left(\underset{\searrow e}{(1+\varphi(x))^{\frac{1}{\varphi(x)}}}\right)^{\varphi(x) h(x) \to A} = e^A$

18) $$\lim_{x \to +\infty} \left(\frac{x-1}{x+1}\right)^x = [1^\infty] = \lim_{x \to +\infty} \left(1 + \underbrace{\frac{x-1}{x+1} - 1}_{\varphi(x)}\right)^x =$$

$$= \lim_{x \to +\infty} \left(1 + \frac{x-1-x-1}{x+1}\right)^x = \lim_{x \to +\infty} \left(1 + \frac{-2}{x+1}\right)^x =$$

$$= \lim_{x \to +\infty} \Big(\underbrace{\left(1 + \frac{-2}{x+1}\right)^{\frac{x+1}{-2}}}_{\to e}\Big)^{\overbrace{\frac{-2}{x+1} \cdot x}^{\to -2}} = e^{-2}$$

19) $$\lim_{x \to \infty} \left(\frac{x}{x+1}\right)^x = \lim_{x \to \infty} \left(1 + \frac{x}{x+1} - 1\right)^x = \lim_{x \to \infty} \Big(\underbrace{\left(1 + \frac{-1}{x+1}\right)^{\frac{x+1}{-1}}}_{\to e}\Big)^{\overbrace{\frac{-1}{x+1} \cdot x}^{\to -1}} = e^{-1}$$

20) $$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{\frac{x+1}{x}} = 1^1 = 1$$

21) $$\lim_{x \to \infty} \left(\frac{x+1}{x-2}\right)^{2x-1} = [1^\infty]$$

$$\lim_{x \to \infty} \left(1 + \frac{x+1}{x-2} - 1\right)^{2x-1} = \lim_{x \to \infty} \Big(\underbrace{\left(1 + \frac{3}{x-2}\right)^{\frac{x-2}{3}}}_{\to e}\Big)^{\overbrace{\frac{3(2x-1)}{x-2}}^{\to 6}} = e^6$$

22) $$\lim_{x \to \infty} \left(\frac{x^2+1}{x^2-1}\right)^{x^2+3x-2} = [1^\infty] = \lim_{x \to \infty} \Big(\underbrace{\left(1 + \frac{x^2+1-x^2+1}{x^2-1}\right)^{\frac{x^2-1}{2}}}_{\to e}\Big)^{\overbrace{\frac{2(x^2+3x-2)}{x^2-1}}^{\to 2}} = e^2$$

23) $$\lim_{x \to \infty} \left(\frac{x^2-2x+1}{x^2-4x+2}\right)^{x-3} = [1^\infty] = \lim_{x \to \infty} \left(1 + \frac{\cancel{x^2}-2x+1-\cancel{x^2}+4x-2}{x^2-4x+2}\right)^{x-3}$$

$$= \lim_{x \to \infty} \left(1 + \frac{2x-1}{x^2-4x+2}\right)^{x-3} = \lim_{x \to \infty} \Big(\underbrace{\left(1 + \frac{2x-1}{x^2-4x+2}\right)^{\frac{x^2-4x+2}{2x-1}}}_{\to e}\Big)^{\overbrace{\frac{(2x-1)(x-3)}{x^2-4x+2}}^{\to 2}} = e^2$$

24) $\lim_{x\to+\infty} \left(\ln(x+1) - \ln(x+2)\right) = [\infty - \infty] = \lim_{x\to+\infty} \left(\ln \frac{x+1}{x+2}\right) =$?

$\ln\left(\lim_{x\to+\infty} \frac{x+1}{x+2}\right) = \ln 1 = 0$

25) $\lim_{x\to 0} \left(\frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}}\right) = [\infty \cdot 0] = \lim_{x\to 0} \left(\frac{1}{2} \ln \left(1+\frac{1+x}{1-x}-1\right)^{\frac{1}{2}}\right) =$

$= \lim_{x\to 0} \left(\ln \left(1+\frac{2x}{1-x}\right)^{\frac{1}{2x}}\right) = \ln \left(\lim_{x\to 0} \left(\left(1+\frac{2x}{1-x}\right)^{\frac{1-x}{2x}}\right)^{\frac{2x}{1-x}\cdot\frac{1}{2x}}\right) =$

$= \ln e^1 = 1$

D:

~~241-245~~

~~247-251~~

~~191-196~~

~~198-200~~

~~202-210~~

~~213~~

Мат анал. 9

1) Производная ф-и, зад. неявно

$$\boxed{x+y=e^{x-y}} \qquad y'(x)-?$$

$$x+y(x)=e^{x-y(x)}$$

$$1+y'_x=e^{x-y(x)}(1-y'_x)$$

$$1+y'_x=e^{x-y}-e^{x-y(x)}\cdot y'_x$$

$$y'_x+y'_x\cdot e^{x-y}=e^{x-y}-1$$

$$y'_x=\frac{e^{x-y}-1}{e^{x-y}+1}$$

$$y'_x=\frac{x+y-1}{x+y+1}$$

2) $$x^3+4y^3-3yx^2=0 \qquad y'_x-?$$

$$x^3+4(y(x))^3-3y(x)\cdot x^2=0$$

$$3x^2+4\cdot 3(y(x))^2 y'_x-3y'_x\cdot x^2-3y(x)\cdot 2x=0$$

$$y'_x(12y^2-3x^2)=6xy-3x^2$$

$$y'_x=\frac{2xy-x^2}{4y^2-x^2}=\frac{x(2y-x)}{(2y-x)(2y+x)}=\frac{x}{2y+x}$$

$$(y(x))'=\frac{x}{2y(x)+x}$$

$$(y(x))''=\frac{1\cdot(2y+x)-x(2y'_x+1)}{(2y(x)+x)^2}$$

$$y''_{xx} = \frac{2y+x - x\cdot\left(\frac{2x}{2y+x}+1\right)}{(2y+x)^2}$$

$$y''_{xx} = \frac{(2y+x)(2y+x) - x\,(x+2y+x)}{(2y+x)^3}$$

$$y''_{xx} = \frac{2y^2+3xy+x^2-2x^2-2xy}{(2y+x)^3}$$

$$y''_{xx} = \frac{2y^2+xy-x^2}{(2y+x)^3}$$

3) $\ell: 2y = 1 + xy^3$

$$2y(x) = 1 + x\cdot(y(x))^3$$

$$2y'_x = 1\cdot y^3(x) + x\cdot 3y^2(x)\cdot y'_x$$

$$2y'_x - 3xy^2 y'_x = y^3$$

$$y'_x = \frac{y^3}{2-3xy^2}$$

$$y'_x = \frac{(y(x))^3}{2-3x(y(x))^2}$$

$$y''_{xx} = \frac{3y^2y'_x(2-3xy^2) - y^3(3y^2+3y\cdot 2y\cdot y'_x)}{(2-3xy^2)^2}$$

$$y''_{xx} = \frac{6y^2y'_x - 9xy^4y'_x - 3y^5 - 6y^5y'_x}{(2-3xy^2)^2}$$

$M(1,1) \in L$

$y_x'(M) = \frac{1}{2-3} = -1$ (убывает $y(x)$)

$y_{xx}''(M) = \frac{6\cdot 1\cdot(-1) - 9\cdot 1\cdot 1\cdot(-1) - 3\cdot 1 - 6\cdot 1\cdot(-1)}{(2-3)^2}$

$= \frac{-6+9-3+6}{1} = 6$ (выпукла вниз)

4) $x^2 - 1 + \cos xy = 0$

$y_x', y_{xx}'' - ?$

$x^2 - 1 + \cos(x\,(y(x))) = 0$

$2x - \sin(x\cdot y(x))(1\cdot y(x) + x\cdot y_x') = 0$

$2x - \sin xy\,(y + xy_x') = 0$

$2x - y\sin xy - x\cdot\sin xy\cdot y_x' = 0$

$y_x' = \frac{2x - y\sin xy}{x\sin xy} = \frac{2}{\sin xy} - \frac{y}{x}$

$y_{xx}'' = \left(\frac{2}{\sin xy}\right)_x' - \left(\frac{y(x)}{x}\right)_x'$

$= 2\cdot(-1)\cdot\frac{1}{(\sin xy)^2} - \frac{y_x'\cdot x - y}{x^2}$

$= \frac{-2}{\sin^2 xy} - \frac{x\cdot y_x' - y}{x^2}$

Производная ф-и, зад. параметрически

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad t \in M \subset R$$

$$y'_x = \frac{y'_t}{x'_t} = \varphi(t)$$

$$y''_{xx} = \frac{y''_{tt} x'_t - y'_t x''_{tt}}{(x'_t)^3}$$

5) $$\begin{cases} x = a\,(t - \sin t) \\ y = a\,(1 - \cos t) \end{cases}$$

$$x'_t = a\,(1 - \cos t)$$

$$y'_t = a\,(\sin t)$$

$$x''_{tt} = a \cdot \sin t$$

$$y''_{tt} = a \cos t$$

$$y'_x = \frac{y'_t}{x'_t} = \frac{a \sin t}{a\,(1-\cos t)} = \frac{\sin t}{1 - \cos t}$$

$$= \frac{2 \sin t/2 \cdot \cos t/2}{2 \sin^2 t/2} = ctg\, t/2$$

$$y''_{xx} = \frac{y''_{tt} x'_t - x''_{tt} y'_t}{(x'_t)^3} =$$

$$= \frac{a \cos t \cdot a\,(1 - \cos t) - a \sin t \cdot a \sin t}{(a\,(1 - \cos t))^3}$$

$$= \frac{a^2\cos t - a^2\cos^2 t - a^2\sin^2 t}{a^3(1-\cos t)^3}$$

$$= \frac{\cos t - 1}{a(1-\cos t)^3} = \frac{-1}{a(1-\cos t)^3}$$

$$\begin{cases} x_0 = a\cdot\pi & x = a(t-\sin t) \\ y_0 = 2a & y = a(t-\cos t) \end{cases}$$

$$y'(t_0) = y_x'(M_0) = ctg\frac{\pi}{2} = 0$$

M_0 - крит. т.

$$y''_{xx}(t_0) = y''_{xx}(M_0) = \frac{-1}{a\cdot(1+1)^2} = \frac{-1}{4a} < 0$$

б) $\begin{cases} x = x(t) = t^3 + 3t \\ y = y(t) = t\,\mathrm{arctg}\,t - \ln(\sqrt{1+t^2}) \end{cases}$

$$x_t' = 3t^2+3$$

$$y_t' = 1\cdot\mathrm{arctg}\,t + t\cdot\frac{1}{1+t^2} - \frac{1}{\sqrt{1+t^2}}\cdot\frac{1}{2\sqrt{1+t^2}}\cdot 2t = \mathrm{arctg}\,t + \frac{t}{1+t^2} +$$

$$\cancel{}\ \cancel{1+t^2} - \frac{t}{1+t^2} = \mathrm{arctg}\,t$$

$$y_x' = \frac{y_t'}{x_t'} = \frac{\mathrm{arctg}\,t}{3(t^2+1)}$$

$$y''_{xx} = \frac{y''_{tt}\cdot x_t' - y_t' x''_{tt}}{(x_t')^3} =$$

$$= \frac{\frac{1}{1+t^2} \cdot 3(t^2-1) - \operatorname{arctg} t \cdot (6t)}{(3t^2+3)^3} =$$

$$= \frac{t^2-1}{27(t^2+1)^4} - \frac{2t \cdot \operatorname{arctg} t}{9(t^2+1)^3}$$

7) $\begin{cases} x(t) = e^t(\cos t + \sin t) \\ y(t) = e^t(\cos t - \sin t) \end{cases}$ $t_0 = 0$

$\begin{cases} x'_t = 2e^t \cos t \\ y'_t = -2e^t \sin t \end{cases}$

$\begin{cases} x''_{tt} = 2e^t \cos t + 2e^t(-\sin t) = 2e^t(\cos t - \sin t) \\ y''_{tt} = -2e^t(\sin t + \cos t) \end{cases}$

$$y'_x = \frac{y'_t}{x'_t} = -\frac{2e^t \sin t}{2e^t \cos t} = -\operatorname{tg} t$$

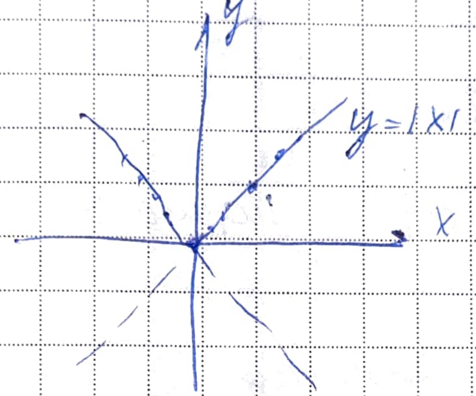
$$y''_{xx} = \frac{y''_{tt} x'_t - y'_t x''_{tt}}{(x'_t)^3} =$$

$$= \frac{-2e^t(\sin t + \cos t)\, 2e^t \cos t + 2e^t \sin t \cdot 2e^t(\cos t - \sin t)}{8 \cdot e^{3t} \cos^3 t}$$

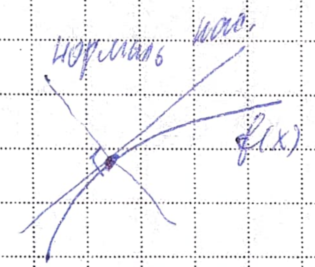
$$= \frac{-\sin t \cos t - \cos^2 t + \sin t \cos t - \sin^2 t}{2e^t \cos^3 t}$$

$$= -\frac{1}{2e^t \cdot \cos^3 t}$$

Касательная — предельное положение секущих, если оно $\exists$



нет кас. в 0



Ур кас. и норм. к кривой L, проход. через $M_0(x_0, y_0) \in L$

касат.: $y - y_0 = y'_x(M_0)(x - x_0)$

нормаль: $\cancel{x} - x_0 = -y'_x(M_0)(y - y_0)$

1) $y = \sin x$ $\qquad y' = \cos x$

$\begin{cases} x_0 = \pi/4 \\ y_0 = \sqrt{2}/2 \end{cases}$ $\qquad x_0 = \frac{\pi}{4}$, $y_0 = \sqrt{2}/2$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$$8y - 4\sqrt{2} = 4\sqrt{2}x - \pi\sqrt{2}$$

$$\boxed{4x - 4\sqrt{2}y - \pi + 4 = 0}$$ кас.

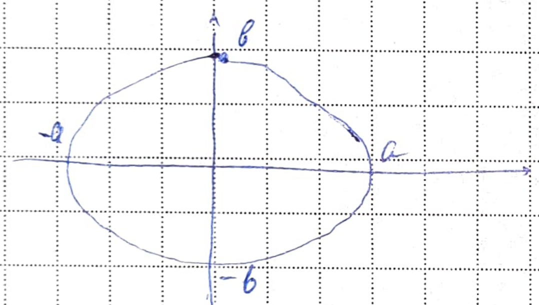
2) $$x - \frac{\pi}{4} = -\frac{\sqrt{2}}{2}\left(y - \frac{\sqrt{2}}{2}\right)$$

$$4x - \pi = -\sqrt{2}y + 2$$

$$\boxed{4x + \sqrt{2}y - \pi - 2 = 0}$$ нормаль

Ур кас. и норм. к эллипсу

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



в т. $M_0(x_0, y_0) \in$ эл.

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2x^2 + a^2(y(x))^2 = a^2b^2$$

$$2b^2x + a^2 \cdot y'(x) \cdot 2y(x) = 0$$

$$b^2x + a^2y \cdot y'_x = 0$$

$$y'_x = -\frac{b^2x}{a^2y} \quad y \neq 0$$

$$y - y_0 = -\frac{b^2x}{a^2y}(x - x_0)$$

$$-a^2y^2 + a^2yy_0 = +b^2x^2 - b^2xx_0$$

$$a^2y^2 + b^2x^2 = a^2yy_0 + b^2xx_0$$

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{yy_0}{b^2} + \frac{xx_0}{a^2}$$

$$\boxed{\frac{x x_0}{a^2} + \frac{y y_0}{b^2} = 1}$$ кас. элипса

кас. и норм. к кривой

$$\begin{cases} x = t\cos t \\ y = t\sin t \end{cases}$$ при $t_0 = \frac{\pi}{2}$

1) $x_0 = 0$

$y_0 = \frac{\pi}{2}$

2) y'_x:

$$\begin{cases} x'_t = \cos t - t\sin t \\ y'_t = \sin t + t\cos t \end{cases}$$

$$y'_x = \frac{\sin t + t\cos t}{\cos t - t\sin t}$$

$$y'_x(M_0) = y'_x(t_0) = \frac{1+0}{0 - \frac{\pi}{2}\cdot 1} = -\frac{2}{\pi}$$

кас: $y - \frac{\pi}{2} = -\frac{2}{\pi}(x-0)$

$2\pi y - \pi^2 = -4x$

$$\boxed{4x + 2\pi y - \pi^2 = 0}$$

нормаль $x - 0 = +\frac{2}{\pi}\left(y - \frac{\pi}{2}\right)$

$2\pi x = 4y - 4\pi^2$

$$\boxed{\pi x - 2y + 2\pi^2 = 0}$$

$$\operatorname{tg}\varphi = |\operatorname{tg}(\varphi_1 - \varphi_2)| =$$

$$= \left| \frac{\operatorname{tg}\varphi_1 - \operatorname{tg}\varphi_2}{1 + \operatorname{tg}\varphi_1 \cdot \operatorname{tg}\varphi_2} \right|$$

Нахождение n-ой производной по ф-ле Лейбница

$$(u(x) \cdot v(x))^{(n)} = \sum_{k=0}^{n} C_n^k \, u^{(n-k)}(x) \cdot v^{(k)}(x)$$

$$y(x) = e^{-x} \cdot \sin x$$

$$(e^{-x})^{(m)} = (-1)^m \, e^{-x}$$

$$(\sin x)^{(m)} = \sin\left(\frac{\pi}{2} \cdot m + x\right)$$

$$y^{n}(x) = \sum_{k=0}^{n} C_n^k \, (-1)^{n-k} e^{-x} \cdot \sin\left(\frac{\pi}{2} k + x\right)$$

$$y(x) = (x^2 - 3x + 1) \cdot e^x$$

$$y(x)^{(n)} = \sum_{k=0}^{n} C_n^k \, (x^2 - 3x + 1)^{(k)} (e^x)^{(n-k)} =$$

$$C_n^0 \, (x^2 - 3x + 1)^{(0)} \cdot (e^x)^{(n)} + C_n^1 \, (x^2 - 3x + 1)^{(1)} (e^x)^{(n-1)} +$$

$$C_n^2 \, (x^2 - 3x + 1)^{(2)} (e^x)^{(n-2)} \ldots =$$

1) $u = f(x, y, z)$

$$\frac{\partial f}{\partial x} = f'_{x}$$

$$\frac{\partial f}{\partial y} = f'_y$$

$$\frac{\partial f}{\partial z} = f'_z$$

2) $z = x^3 + 3x^2y - y^3$

$$\frac{\partial z}{\partial x} = 3x^2 + 3y \cdot 2x = 3x^2 + 6yx$$

$$\frac{\partial z}{\partial y} = 0 + 3x^2 \cdot 1 - 3y^2 = 3x^2 - 3y^2$$

3) $z = \frac{y}{x}$

$$\frac{\partial z}{\partial x} = \left(\frac{y}{x}\right)'_x = y\,(x^{-1})'_x = y\,(-1) \cdot x^{-2} = \frac{-y}{x^2}$$

$$\frac{\partial z}{\partial y} = \left(\frac{y}{x}\right)'_y = \frac{1}{x} \cdot (y)'_y = \frac{1}{x}$$

4) $z = \ln(x^2 + y^2)$

$$\frac{\partial z}{\partial x} = \frac{1}{x^2+y^2} \cdot 2x = \frac{2x}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2}$$

5) $z = \operatorname{arctg} \frac{y}{x}$

$$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(\frac{y}{x}\right)'_x = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{-y}{x^2} = \frac{-y}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(\frac{y}{x}\right)'_y = \frac{x}{x^2+y^2}$$

6) $z = \frac{xy}{x-y}$

$$\frac{\partial z}{\partial x} = \frac{(xy)'_x \cdot (x-y) - xy\,(x-y)'_x}{(x-y)^2} =$$

$$= \frac{y(x-y) - xy \cdot 1}{(x-y)^2} = \frac{xy - y^2 - xy}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(xy)'_y \cdot (x-y) - xy\,(x-y)'_y}{(x-y)^2} =$$

$$= \frac{x(x-y) - xy\,(-1)}{(x-y)^2} = \frac{x^2 - xy + xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

7) $z = \ln(\sqrt{x} + \sqrt{y})$

явл. решением $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = \frac{1}{2}$

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x}+\sqrt{y}} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{x}+\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}$$

$$\frac{\sqrt{x}}{2(\sqrt{x}+\sqrt{y})} + \frac{\sqrt{y}}{2(\sqrt{x}+\sqrt{y})} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2}$$

8) Д-ть, что $z = x \cdot \sin\frac{y}{x}$ – реш.

ур-е $x \cdot \frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

$\frac{\partial z}{\partial x} = \sin\frac{y}{x} + x \cdot \left(\sin\frac{y}{x}\right)'_x = \sin\frac{y}{x} + x\cos\frac{y}{x} \cdot \left(-\frac{y}{x^2}\right) =$

$= \sin\frac{y}{x} - \frac{y}{x}\cos\frac{y}{x}$

$\frac{\partial z}{\partial y} =$ ~~sin~~ $x \cdot \cos\frac{y}{x} \cdot \frac{1}{x} = \cos\frac{y}{x}$

~~$x \cdot \sin\frac{y}{x} + xy \cdot \cos\frac{y}{x} \cdot x \cdot \left(-\frac{y}{x^2}\right) +$~~

$x\sin\frac{y}{x} - y\cos\frac{y}{x} + y\cos\frac{y}{x} = x \cdot \sin\frac{y}{x} \equiv z$

9) $u = x^y$ Д-ть, что u – решение

$\frac{x}{y} \cdot \frac{\partial u}{\partial x} + \frac{1}{\ln x} \cdot \frac{\partial u}{\partial y} = 2u$

$\frac{\partial u}{\partial x} = y \cdot x^{y-1}$

$\frac{\partial u}{\partial y} = x^y \cdot \ln x$

$\frac{x}{y} \cdot y \cdot x^{y-1} + \frac{1}{\ln x} \cdot x^y \cdot \ln x = x^y + x^y = 2x^y \equiv 2u$

10) Д-ть $u = e^{\frac{x}{t^2}}$ – решение $2x \cdot \frac{\partial u}{\partial x} + t\frac{\partial u}{\partial t} = 0$

$$\frac{\partial u}{\partial x} = e^{\frac{x}{t^2}} \cdot 1 \cdot \frac{1}{t^2}$$

$$\frac{\partial u}{\partial t} = e^{\frac{x}{t^2}} \cdot (-2) \cdot \frac{x}{t^3}$$

$$2x \cdot \frac{1}{t^2} e^{\frac{x}{t^2}} + \cancel{t} \cdot \frac{-2x}{t^{\cancel{3}2}} \cdot e^{\frac{x}{t^2}} = 0 \quad \equiv 0$$

11) $f(x, y)$

главн. ч. приращ. ф.

$$\underbrace{f(x+\Delta x, y+\Delta y) - f(x,y)}_{\Delta f} = \overbrace{\frac{\partial f}{\partial x} \cdot \Delta x + \frac{\partial f}{\partial y} \cdot \Delta y}$$

$$+ \underbrace{o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)}_{\to 0}$$

$$\underbrace{df(x,y)}_{\text{диффер. ф.}} = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \qquad \begin{cases} \Delta x = dx \to 0 \\ \Delta y = dy \to 0 \end{cases}$$

12) Найти диф. ф-и $z = x^2 y$ dz – ?

$$\frac{\partial z}{\partial x} = 2xy$$

$$\frac{\partial z}{\partial y} = x^2$$

$$dz = 2xy\,dx + x^2 dy$$

$u = u(x, y, z)$

$\text{grad}\, u = \left\{ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right\}$ – градиент ф-и u

$\text{grad}\, u \Big|_{M_0(x_0, y_0, z_0)} = \left\{ \frac{\partial u}{\partial x}\Big|_{M_0}, \frac{\partial u}{\partial y}\Big|_{M_0}, \frac{\partial u}{\partial z}\Big|_{M_0} \right\}$

Производная по направлению $\bar{l}$

$\frac{\partial u}{\partial l} = \frac{\partial u}{\partial \bar{l}}$ Опр. lim...

$\bar{l} = \{a, b, c\}$

$\bar{e}_{\bar{l}} = \frac{\bar{l}}{|\bar{l}|} = \frac{1}{\sqrt{a^2+b^2+c^2}} \{a, b, c\}$

Теорема $\frac{\partial u}{\partial \bar{l}}\Big|_{M_0} = (\text{grad}\, u, \bar{e}_{\bar{l}})\Big|_{M_0}$

$u = x^2 + y^2 - 3xyz$

$M_0(1, 2, 3)$ $\qquad \frac{\partial u}{\partial l}\Big|_{M_0}$ – ?

$\bar{l} = \{2, 1, 2\}$

1) $\text{grad}\, u\Big|_{M_0} = \left\{ \frac{\partial u}{\partial x}; \frac{\partial u}{\partial y}; \frac{\partial u}{\partial z} \right\}_{M_0} =$

$(\bar{m}, \bar{n}) = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 z_2 = |\bar{m}| \cdot |\bar{n}| \cdot \cos\varphi$

$= \{2x - 3yz,\ 2y - 3xz,\ -3xy\}\Big|_{M_0(1,\ 2;\ 3)} =$

$= \{2 - 18,\ 4 - 9,\ -6\} = \{-16,\ -5,\ -6\}$

$\bar{e}_l - ?$

2) $|\bar{e}| = \sqrt{4 + 1 + 4} = 3$

$\bar{e}_l = \frac{1}{3}\{2; 1; 2\} = \{\frac{2}{3}; \frac{1}{3}; \frac{2}{3}\}$

3) $\frac{\partial u}{\partial l} = \frac{1}{3}(-16 \cdot 2 - 5 \cdot 1 - 6 \cdot 2) =$

$= \frac{1}{3}(-32 - 5 - 12) = -\frac{49}{3}$

$u = x^2 + y^2 + z^2 - 6x$

$\frac{\partial u}{\partial l}$ в т. $M_0(1, 0, 1)$ по напр. к т. $M_1(2; 1; 4)$

$\bar{e} = \overline{M_0 M_1} = \{2 - 1,\ 1 - 0,\ 4 - 1\} = \{1, 1, 3\}$

1) $\operatorname{grad} u = \{\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\} = \{2x - 6,\ 2y,\ 2z\}$

$\operatorname{grad} u\Big|_{M_0} = \{2 - 6,\ 0\ 2\} = \{-4; 0; 2\}$

2) $\bar{e}_l = \frac{1}{\sqrt{11}}\{1, 1, 3\}$

$|\bar{e}| = \sqrt{1 + 1 + 9} = \sqrt{11}$

3) $\frac{\partial u}{\partial \bar{e}}\Big|_{M_0} = (\operatorname{grad} u|_{M_0}, \bar{e}_{\bar{e}}) =$

$= \frac{1}{\sqrt{11}}(-4\cdot 1 + 0\cdot 1 + 2\cdot 3) = \frac{2}{\sqrt{11}}$

$\frac{\partial u}{\partial e}\Big|_{M_0} = (\operatorname{grad} u|_{M_0}, \bar{e}_{\bar{e}}) = |\operatorname{grad} u|_{M_0}| \cdot |\bar{e}_{\bar{e}}| \cdot \cos\varphi$

$|\bar{e}_{\bar{e}}| = 1$, $\cos\varphi \in [-1; 1]$

grad – напр. в пр-ве аргументов максимального роста ф-и

$z = \operatorname{arctg}\frac{y}{x}$

$\frac{\partial z}{\partial \bar{e}}\Big|_{M_0} - ?$ $M_0(1; -1)$ $M_1(3; 5)$

$\bar{e} = \overline{M_0 M_1} = \{2, 6\}$

1) $\frac{\partial z}{\partial x} = \frac{1}{1+(\frac{y}{x})^2}\cdot\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{1+(\frac{y}{x})^2}\cdot\left(\frac{x}{x^2}\right) = \frac{x}{x^2+y^2}$

$\operatorname{grad} z|_{M_0} = \{\frac{1}{2}, \frac{1}{2}\} = \frac{1}{2}\{1, 1\}$

2) $|\bar{e}| = 2\sqrt{10}$

$\bar{e}_{\bar{e}} = \frac{1}{2\sqrt{10}}\{2, 6\} = \frac{1}{\sqrt{10}}\{1, 3\}$

$* \ \bar{x} = \{x_1, x_2, x_3\} = (x_1, x_2, x_3)$

2) $\left.\frac{\partial z}{\partial l}\right|_{M_0} = \frac{1}{2} \cdot \frac{1}{\sqrt{10}} (1 \cdot 1 + 1 \cdot 3) = \frac{2}{\sqrt{10}}$

$ax + by + cz + d = 0$ $\quad a, b, c \neq 0$ одновр.

$a, b, c, \in \mathbb{R}$

– ур-ие п-ти в $\mathbb{R}^3$

Предп., что п-ть $S \subset \mathbb{R}^3$

задана $F(x, y, z) = 0$

Пусть $M_0 \in S$

Кас. п-ть $\underbrace{\left.\frac{\partial F}{\partial x}\right|_{M_0} (x - x_0) + \left.\frac{\partial F}{\partial y}\right|_{M_0} (y - y_0) + \left.\frac{\partial F}{\partial z}\right|_{M_0} (z - z_0)}_{\text{скал. произв. векторов}} = 0$

и $\begin{cases} \operatorname{grad} F|_{M_0} = \left\{ \left.\frac{\partial F}{\partial x}\right|_{M_0}, \left.\frac{\partial F}{\partial y}\right|_{M_0}, \left.\frac{\partial F}{\partial z}\right|_{M_0} \right\} \\ \bar{u} = \{x - x_0, y - y_0, z - z_0\} \end{cases}$

$\bar{p} = \{m, n, k\}$ – напр. вектор

$M_0 (x_0, y_0, z_0)$

$l: \frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{k}$

Нормаль к п-ти — перпендикуляр к п-ти через M_0

$$\frac{x-x_0}{\frac{\partial F}{\partial x}\Big|_{M_0}} = \frac{y-y_0}{\frac{\partial F}{\partial y}\Big|_{M_0}} = \frac{z-z_0}{\frac{\partial F}{\partial z}\Big|_{M_0}}$$

$S: \ z = x^2 + 2y^2 \quad M_0(1,1,3)$

$\Leftrightarrow \ F(x,y,z) = \underbrace{x^2 + 2y^2 - z}_{} = 0$

$\frac{\partial F}{\partial x} = 2x \qquad \frac{\partial F}{\partial x}\Big|_{M_0} = 2$

$\frac{\partial F}{\partial y} = 4y \qquad \frac{\partial F}{\partial y}\Big|_{M_0} = 4$

$\frac{\partial F}{\partial z} = -1 \qquad \frac{\partial F}{\partial z}\Big|_{M_0} = -1$

Кас. п-ть

$2(x-1) + 4(y-1) - 1(z-3) = 0$

$2x + 4y - z - 3 = 0$

Нормаль:

$$\frac{x-1}{2} = \frac{y-1}{4} = \frac{z-3}{-1}$$

$S: z^2 = xy \quad M_0(x_0, y_0, z_0) \in S$

$\Leftrightarrow \underbrace{xy - z^2}_{F(x,y,z)} = 0$

$\frac{\partial F}{\partial x} = y \qquad \left.\frac{\partial F}{\partial x}\right|_{M_0} = y_0$

$\frac{\partial F}{\partial y} = x \qquad \left.\frac{\partial F}{\partial y}\right|_{M_0} = x_0$

$\frac{\partial F}{\partial z} = -2z \qquad \left.\frac{\partial F}{\partial z}\right|_{M_0} = -2z_0$

Кас. пл-ть:

$y_0(x - x_0) + x_0(y - y_0) - 2z_0(z - z_0) = 0$

$y_0 x + x_0 y - 2z_0 z \underbrace{- x_0 y_0 - x_0 y_0 + 2z_0^2}_{=0} = 0$

т.к. $M_0 \in S$

$x_0 y + y_0 x - 2z_0 z = 0$

Нормаль.

$\frac{x - x_0}{y_0} = \frac{y - y_0}{x_0} = \frac{z - z_0}{-2z_0}$

$$S - \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$ — однополост. гиперболоид.

1) в т. $M_0(x_0, y_0, z_0) \in S$

2) в т. $M_1(a, b, c)$

$$S: \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 = 0 \qquad M_0(x_0, y_0, z_0) \in S$$

1) $$\left.\frac{\partial F}{\partial x}\right|_{M_0} = \left.\frac{2x}{a^2}\right|_{M_0} = \frac{2x_0}{a^2}$$

$$\left.\frac{\partial F}{\partial y}\right|_{M_0} = \left.\frac{2y}{b^2}\right|_{M_0} = \frac{2y_0}{b^2}$$

$$\left.\frac{\partial F}{\partial z}\right|_{M_0} = -\left.\frac{2z}{c^2}\right|_{M_0} = -\frac{2z_0}{c^2}$$

Кас. п-ть:

$$\frac{2x_0}{a^2}(x - x_0) + \frac{2y_0}{b^2}(y - y_0) - \frac{2z_0}{c^2}(z - z_0) = 0$$

$$2\frac{x_0}{a^2}x + \frac{2y_0}{b^2}\cdot y - \frac{2z_0}{c^2}\cdot z \underbrace{- \frac{2x_0^2}{a^2} - \frac{2y_0^2}{b^2} + \frac{2z_0^2}{c^2}}_{= -2} = 0 \Big| :2$$

$$\boxed{\frac{x_0}{a^2}x + \frac{y_0}{b^2}\cdot y - \frac{z_0}{c^2}\cdot z - 1 = 0}$$

Нормаль:

$$\frac{x - x_0}{\frac{2x_0}{a^2}} = \frac{y - y_0}{\frac{2y_0}{b^2}} = \frac{z - z_0}{-\frac{2z_0}{c^2}} \qquad \frac{x - x_0}{\frac{x_0}{a^2}} = \frac{y - y_0}{\frac{y_0}{b^2}} = \frac{z - z_0}{-\frac{z_0}{c^2}}$$

2) $\frac{x}{a} + \frac{y}{b} + \frac{z}{-c} = 1$ к. в т. $M_0(a, b, c)$

$z = f(x,y)$ $x = x(t)$ $y = y(t)$

$z = f(x(t), y(t))$ – ф-я одн. пер.

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$z = x^2 + xy + y^2$ $x = t^2$ $y = t$

1) $z = t^4 + t^3 + t^2$

$$\frac{dz}{dt} = 4t^3 + 3t^2 + 2t$$

2) $\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} =$

$= (2x + y) \cdot 2t + (x + 2y) \cdot 1 =$

$= (2t^2 + t) \cdot 2t + (t^2 + 2t) =$

$= 4t^3 + 3t^2 + 2t$

Правило Бернулли-Лопиталя

$$\lim_{x\to a}\frac{f(x)}{\varphi(x)}=\left[\frac{\infty}{\infty},\ \frac{0}{0}\right]=\lim_{x\to a}\frac{f'(x)}{\varphi'(x)}$$

если $\exists\, f', \varphi'$.

Не всегда работает:

1) $\lim\limits_{x\to+\infty}\dfrac{x-\sin x}{x+\sin x}=\left[\dfrac{\infty}{\infty}\right]=\lim\limits_{x\to+\infty}\dfrac{1-\cos x}{1+\cos x}$ – предела нет

$\parallel$

$$\lim_{x\to+\infty}\frac{\frac{x}{x}-\frac{\sin x}{x}\to 0}{\frac{x}{x}+\frac{\sin x}{x}\to 0}=1$$

2) $\lim\limits_{x\to1}\dfrac{x^5-1}{2x^3-x-1}=\left[\dfrac{0}{0}\right]=\lim\limits_{x\to1}\dfrac{5x^4}{6x^2-1}=1$

3) $\lim\limits_{x\to0}\dfrac{x-\operatorname{arctg}x}{x^3}=\left[\dfrac{0}{0}\right]=\lim\limits_{x\to0}\dfrac{1-\frac{1}{1+x^2}}{3x^2}=$

$=\lim\limits_{x\to0}\dfrac{\cancel{1}+x^2-\cancel{1}}{3x^2(1+x^2)}=\lim\limits_{x\to0}\dfrac{1}{3(1+x^2)}=\dfrac{1}{3}$

4) $\lim\limits_{x\to+\infty}\dfrac{\ln x}{x}=\left[\dfrac{\infty}{\infty}\right]=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{x}}{1}=0$

$\lim\limits_{x\to+\infty}\dfrac{\ln^2x}{x}\ (=0)=\lim\limits_{x\to+\infty}\dfrac{2\ln x\cdot\frac{1}{x}}{1}=\lim\limits_{x\to+\infty}\dfrac{2\ln x}{x}=$

$=\lim\limits_{x\to+\infty}\dfrac{2\cdot\frac{1}{x}}{1}=\lim\limits_{x\to+\infty}\dfrac{2}{x}=0$

$\lim\limits_{x\to+\infty}\dfrac{\ln x}{\sqrt{x}}\ (=0)=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}=\lim\limits_{x\to+\infty}\dfrac{2}{\sqrt{x}}=0$

$x \in \mathcal{D}_{odp}\, f$

$f^{-1}(f(x)) = x$

$$\boxed{\lim_{x\to+\infty} \frac{(\ln x)^{\alpha}}{x^{\beta}} = 0} \qquad \forall \beta, \alpha > 0$$

5) $$\lim_{x\to 0} \frac{\sin x - x\cos x}{\sin^3 x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\cancel{\cos x} - (\cancel{\cos x} - x\sin x)}{3\sin^2 x\cos x} =$$

$$= \frac{x}{3\frac{\sin x}{\ }\cos x} = \frac{1}{3}$$

$\downarrow$
1

6) $$\lim_{x\to 0_+} (x\cdot \ln x) = [0\cdot\infty] = \lim_{x\to 0_+} \frac{\ln x}{\frac{1}{x}} =$$

$$= \left[\frac{\infty}{\infty}\right] = \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x = 0$$

7) $[0^0]$, $[\infty^0]$, $[1^\infty]$

$$\lim_{x\to 0_+} x^x = \lim_{x\to 0_+} e^{\ln x^x} = \lim_{x\to 0_+} e^{x\cdot\ln x} =$$

$$= e^{\lim\limits_{x\to 0_+} x\cdot\ln x} = e^0 = 1$$

8) $$\lim_{x\to\frac{\pi}{2}} (\sin x)^{\operatorname{tg} x} = [1^\infty] = \lim_{x\to\frac{\pi}{2}} e^{\operatorname{tg} x\cdot\ln(\sin x)} =$$

$$\left[\Rightarrow \lim_{x\to\frac{\pi}{2}} (\operatorname{tg} x\cdot\ln(\sin x)) = [\infty\cdot 0] = \right.$$

$$\left. = \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\sin x}\cdot\cos x}{-\frac{1}{\sin^2 x}} = \lim_{x\to\frac{\pi}{2}} \frac{\sin x\cdot\cos x}{-1} = 0\right]$$

$$= e^0 = 1$$

9) $\lim\limits_{x\to+\infty}(3x^2+3^x)^{\frac{1}{x}}=[\infty^0]=\lim\limits_{x\to+\infty} e^{\ln(3x^2+3^x)^{\frac{1}{x}}}=$

$= e^{\lim\limits_{x\to+\infty}\frac{1}{x}\ln(3x^2+3^x)} =$

$\Big[=\lim\limits_{x\to+\infty}\frac{1}{x}\ln(3x^2+3^x)=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{3x^2+3^x}(6x+3^x\ln 3)}{1}$

$=\lim\limits_{x\to+\infty}\dfrac{6x+3^x\ln 3}{3x^2+3^x}=\lim\limits_{x\to+\infty}\dfrac{\frac{6x}{3^x}+\ln 3}{\frac{3x^2}{3^x}+1}=\ln 3\Big]$

$\frac{6x}{3^x}\to 0$, $\frac{3x^2}{3^x}\to 0$

$= e^{\ln 3}=3$

10) $\lim\limits_{x\to+\infty}\frac{x}{e^x}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to+\infty}\frac{1}{e^x}=0$

$$\boxed{\lim_{x\to+\infty}\frac{x^\alpha}{(e^x)^\beta}=0} \quad \forall\alpha,\beta>0$$

$\lim\limits_{x\to+\infty}\frac{x}{3^x}=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to+\infty}\frac{1}{3^x\cdot\ln 3}=0$

11) Таблица эквивалентности

при $x\to 0$

$\sin x\sim x$	$\ln(1+x)\sim x$
$\operatorname{tg} x\sim x$	$\ln(1-x)\sim -x$
$\operatorname{arctg} x\sim x$	$e^x\sim 1+x$
$\arcsin x\sim x$	$\cos x\sim 1-\frac{x^2}{2}$
	$(1+x)^\alpha\sim 1+\alpha x$

$\frac{\pi}{3,1}$ — ?

$\pi \sim 3$ $\quad \frac{3}{3,1} < 1$

$\pi \sim 3,14$ $\quad \frac{3,14}{3,1} > 1$

12) Формула Тейлора

$x \to 0$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + o(x^n)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + o(x^3)$$

$$\operatorname{tg} x = x + \frac{x^3}{3} + o(x^4)$$

$$\operatorname{arctg} x = x - \frac{x^3}{3} + o(x^4)$$

$$\arcsin x = x + \frac{x^3}{3!} + o(x^4)$$

$o(\varphi(x))$:

при $x \to a$

$$\lim_{x \to a} \frac{o(\varphi(x))}{\varphi(x)} = 0$$

$$\frac{o(\varphi(x))}{\varphi(x)} = o + \underbrace{\alpha(x)}_{\substack{\text{б.м.ф} \\ \text{при } x \to a}} \to 0$$

$$o(\varphi(x)) = \underbrace{\alpha(x)}_{\substack{\text{б.м.ф} \\ \text{при } x \to a}} \cdot \varphi(x)$$

$x \to 0$

13) $o(x^2) \cdot o(x^3) = \underbrace{\alpha(x)}_{\text{б.м.ф}} \cdot x^2 \cdot \underbrace{\beta(x)}_{\text{б.м.ф}} \cdot x^3 = \underbrace{(\alpha(x) \cdot \beta(x))}_{\text{б.м.ф}} x^5 = o(x^5)$

14) $o(x^2) \cdot x^3 = \underbrace{\alpha(x)}_{\text{б.м.ф.}} \cdot x^2 \cdot x^3 = o(x^5)$

15) $\frac{o(x^6)}{x^4} = \frac{\alpha(x) \cdot x^6}{x^4} = \alpha(x) \cdot x^2 = o(x^2)$

16) $o(x^2) + o(x^4) = o(x^2)$!

① $o(x^2) + o(x^4) = \alpha(x) \cdot x^2 + \beta(x) \cdot x^4 =$

$= x^2 \underbrace{(\alpha(x) + \beta(x) \cdot x^2)}_{\substack{\text{б.м.ф} \\ \text{при } x \to 0}} = o(x^2)$

② $o(x^2) + o(x^4) = \alpha(x) \cdot x^2 + \beta(x) \cdot x^4 =$

$x^4 (\underbrace{\alpha(x) \cdot \frac{1}{x^2}}_{\text{б.м.}} + \underbrace{\beta(x)}_{\to 0}) - ?$

17) $x^3 = o(x^2)$!

$$x^3 = \underset{\downarrow \atop 0}{x} \cdot x^2 = \alpha(x) \cdot x^2 = o(x^2)$$

18) Неверно!

$$\lim_{x\to 0} \frac{x-\sin x}{x^3} = \lim_{x\to 0} \frac{x-x}{x^3} = \lim_{x\to 0} \frac{0}{x^3} = \lim_{x\to 0} 0 = 0$$

Верно!

$$\lim_{x\to 0} \frac{x-\sin x}{x^3} = \lim_{x\to 0} \frac{x-\left(x-\frac{x^3}{3!}+o(x^4)\right)}{x^3} =$$

$$= \lim_{x\to 0} = \frac{\frac{x^3}{6}+o(x^4)}{x^3} = \lim_{x\to 0}\left(\frac{1}{6}+o(x)\right) = \frac{1}{6}$$

19) $$\lim_{x\to 0} \frac{\ln(1+x)-x}{x^2} = \lim_{x\to 0} \frac{x-\frac{x^2}{2}+o(x^2)-x}{x^2} =$$

$$= \lim_{x\to 0} \frac{-\frac{x^2}{2}+o(x^2)}{x^2} = \lim_{x\to 0}\left(-\frac{1}{2}+\underset{\downarrow}{o(1)}\right) = -\frac{1}{2}$$

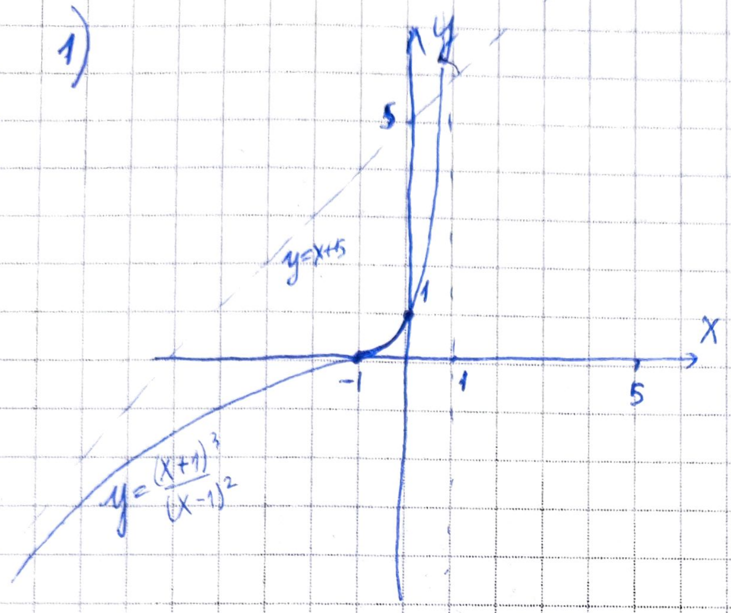
$\left(o(1) = \alpha(x)\cdot 1 = \alpha(x)\right)$

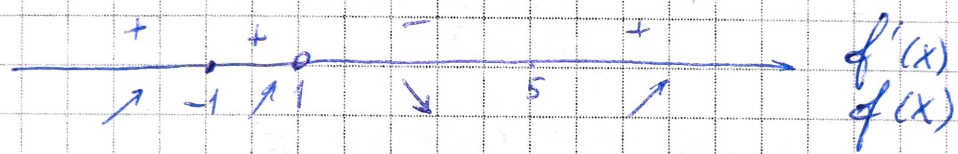
20) $$\lim_{x\to 0} \frac{e^x-1-x}{x^2} = \lim_{x\to 0} \frac{1+x+\frac{x^2}{2!}+o(x^2)-1-x}{x^2} =$$

$$= \lim_{x\to 0} \frac{\frac{x^2}{2!}+o(x^2)}{x^2} = \lim_{x\to 0}\left(\frac{1}{2}+o(1)\right) = \frac{1}{2}$$

Мат. ан. (14)

1)





$$y = \frac{(x+1)^3}{(x-1)^2}$$

1) $D_f = \{x \neq 1\}$

2) $\begin{cases} x = 0 \\ y = 1 \end{cases}$ $\quad \begin{cases} x = -1 \\ y = 0 \end{cases}$

3) $\lim\limits_{x \to 1_+} \frac{(x+1)^3}{(x-1)^2} = \left[\frac{8}{0_+} \right] = \infty$

$\lim\limits_{x \to 1_-} = \lim\limits_{x \to 1_+} = \infty$

4) $y' = \frac{3(x+1)^2(x-1)^2 - 2(x-1)(x+1)^3}{(x-1)^4} =$

$= \frac{(x+1)^2\left(3(x-1) - 2(x+1)\right)}{(x-1)^3} = \frac{(x+1)^2(x-5)}{(x-1)^3}$

$\begin{cases} x_{min} = 5 \\ y_{min} = \frac{27}{2} \end{cases}$

4) $y'' = \frac{\left(\left(2(x+1)(x-5) + (x+1)^2\right)(x-1)^3 - 3(x+1)^2(x-5)(x-1)^2\right)}{(x-1)^{6}}$

$= \frac{(x+1)\left((2x-10+x+1)(x-1) - 3(x+1)(x-5)\right)}{(x-1)^4}$

$= \frac{3(x+1)(x^2 - 4x + 3 - x^2 + 4x + 5)}{(x-1)^4}$

$= \frac{(x+1) \cdot 24}{(x-1)^4}$

5) при $x \to \pm\infty$ $\quad y_1 = k_\pm x + b_\pm$

$k_+ = \lim\limits_{x \to +\infty} \frac{f(x)}{x} = \lim\limits_{x \to +\infty} \frac{(x+1)^3}{x(x-1)^2} = 1$

$k_- = k_+ = 1$

$b_+ = \lim\limits_{x \to +\infty} (f(x) - k_+ \cdot x) =$

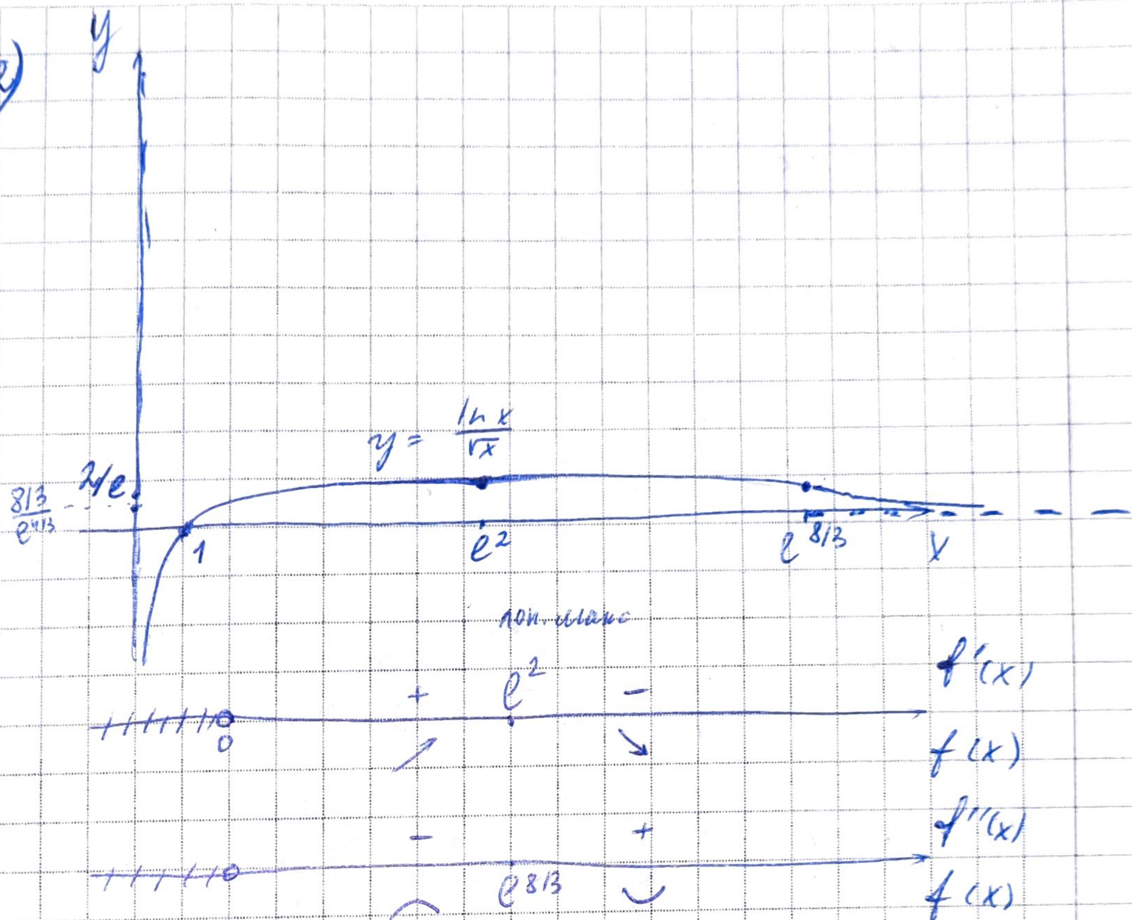
$= \lim\limits_{x \to +\infty} \left(\frac{(x+1)^3}{(x-1)^2} - x\right) =$

$= \lim\limits_{x \to +\infty} \frac{5x^2 + 2x + 1}{x^2 - 2x + 1} = 5$

$b_- = b_+ = 5$

$y = x + 5$ – ас. при $x \to \pm\infty$

2)



$$y = \frac{\ln x}{\sqrt{x}}$$

1) $D_f = \{x>0\}$

2) $\begin{cases} y=0 \\ \ln x = 0 \end{cases}$ $\quad \begin{cases} y=0 \\ x=1 \end{cases}$

3) $\lim\limits_{x \to 0_+} \frac{\ln x}{\sqrt{x}} = \left[\frac{-\infty}{0_+}\right] = -\infty$

4) $y' = \dfrac{\frac{1}{x}\cdot\sqrt{x} - \ln x \cdot \frac{1}{2\sqrt{x}}}{x} = \dfrac{2 - \ln x}{x \cdot 2\sqrt{x}}$

$\begin{cases} y'=0 \\ x = e^2 \end{cases}$ $\quad y = \frac{2}{e}$

$$y' = \frac{\frac{1}{x} \cdot \sqrt{x} - \ln x \frac{1}{2\sqrt{x}}}{x} =$$

$$= \frac{2 - \ln x}{x \cdot 2\sqrt{x}}$$

$$y'' = \frac{-\frac{1}{x} \cdot x \cdot 2\sqrt{x} - (2 - \ln x) \cdot 2 \cdot \frac{3}{2} \cdot x^{\frac{1}{2}}}{4x^3} =$$

$$= \frac{-2 - (2 - \ln x)^3}{4x^2 \sqrt{x}} = \frac{3\ln x - 8}{4x^2 \cdot \sqrt{x}}$$

$$\begin{cases} y'' = 0 \\ x = e^{8/3} \end{cases} \qquad y = \frac{\ln e^{8/3}}{\sqrt{e^{8/3}}} = \frac{8}{3 \cdot e^{4/3}}$$

5) $y_+ = k_+ x + b_+$

$$k_+ = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\ln x}{x^{3/2}} = \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{\frac{3}{2} x^{\frac{1}{2}}} = 0$$

$$b_+ = \lim_{x \to +\infty} (f(x) - x) = \lim_{x \to +\infty} \left(\frac{\ln x}{\sqrt{x}} - 0\right) = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \to +\infty} \frac{2}{\sqrt{x}} = 0$$

при $x \to +\infty$ $\quad y = 0$

№3. Диф. ф-и многих переменных

1) $f(x,y) = \frac{x(x-y)}{y^2} = xy^{-2}(x-y)$

$$\frac{\partial f}{\partial x} = y^{-2}(x-y) + xy^{-2} = \frac{2x-y}{y^2}$$

$$\frac{\partial f}{\partial y} = -2y^{-3}\cdot x(x-y) + xy^{-2} = -\frac{x(x-y)}{y^3} + \frac{x}{y^2} =$$

$$= \frac{-x^2+xy+xy}{y^3} = \frac{2xy-x^2}{y^3}$$

$$dz = \frac{2x-y}{y^2}\,dx + \frac{2xy-x^2}{y^3}\,dy$$

2) $f(x,y) = \sin\frac{x}{y}\cdot\cos\frac{y}{x}$

$$\frac{\partial f}{\partial x} = \cos\frac{x}{y}\cdot\frac{1}{y}\cdot\cos\frac{y}{x} + \sin\frac{x}{y}\cdot\sin\frac{y}{x}\cdot\frac{y}{x^2}$$

$$\frac{\partial f}{\partial y} = -\cos\frac{x}{y}\cdot\frac{x}{y^2}\cdot\cos\frac{y}{x} - \sin\frac{x}{y}\cdot\sin\frac{y}{x}\cdot\frac{1}{x}$$

$$dz = \left(\frac{1}{y}\cos\frac{x}{y}\cos\frac{y}{x} + \frac{y}{x^2}\sin\frac{x}{y}\cdot\sin\frac{y}{x}\right)dx$$
$$- \left(\frac{x}{y^2}\cos\frac{x}{y}\cos\frac{y}{x} + \frac{1}{x}\sin\frac{x}{y}\cdot\sin\frac{y}{x}\right)dy$$

$$\bar{l}_{\bar{c}} = \frac{\bar{c}}{|\bar{c}|} = \frac{1}{\sqrt{a^2+b^2+c^2}}\cdot\{a, b, c\}$$

№4) Производная по направлению

1) $f(x,y,z) = x^3 + 2xy^2 + 3yz^2$ $M_0(3,3,1)$

$l = \{2,2,1\}$

1. $\text{grad } u = \{2x^2 + 2y^2;\ 4xy + 3z^2;\ 6yz\}$

$\text{grad } u|_{M_0} = \{36, 39, 18\}$

2. $\bar{e}_{\bar{l}} = \frac{1}{\sqrt{4+4+1}} \cdot \{2, 2, 1\} = \frac{1}{3}\{2, 2, 1\}$

3. $\frac{\partial u}{\partial \bar{l}}\Big|_{M_0} = (\text{grad } u|_{M_0}, \bar{e}_{\bar{l}}) =$

$= \frac{1}{3}(36 \cdot 2 + 39 \cdot 2 + 18 \cdot 1) = 24 + 26 + 6 = 56$

2) $f(x,y) = x \cdot \sin(x+y)$ $M_0\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$l\{-5, 0\}$

1. $\text{grad } u \{\sin(x+y) + x \cdot \cos(x+y);\ x \cdot \cos(x+y)\}$

$\text{grad } u|_{M_0} = \{\sin\frac{\pi}{2} + \frac{\pi}{4} \cdot \cos\frac{\pi}{2};\ \frac{\pi}{4} \cdot \cos\frac{\pi}{2}\} =$

$= \{1;\ 0\}$

2. $\bar{e}_{\bar{l}} = \frac{1}{\sqrt{25+0}} \{-5; 0\}$

3. $\frac{\partial u}{\partial \bar{l}}\Big|_{M_0} = (\text{grad } u|_{M_0} \cdot \bar{e}_{\bar{l}}) =$

$= \frac{1}{5}(1 \cdot (-5) + 0 \cdot 0) = -1$

№5 1) $z = x^2 + y^2$ $M_0(1,1,2)$

$F(x,y,z) = x^2 + y^2 - z = 0$

$\frac{\partial F}{\partial x} = 2x$ $\left.\frac{\partial F}{\partial x}\right|_{M_0} = 2$

$\frac{\partial F}{\partial y} = 2y$ $\left.\frac{\partial F}{\partial y}\right|_{M_0} = 2$

$\frac{\partial F}{\partial z} = -1$ $\left.\frac{\partial F}{\partial z}\right|_{M_0} = -1$

Кас. п-ть: $2(x-1) + 2(y-1) - 1(z-2) = 0$

$2x + 2y - z - 2 = 0$

Нормаль: $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{-1}$

2) $z = 2x^2 - 4y^2$ $M_0(-2,1,4)$

$F(x,y,z) = 2x^2 - 4y^2 - z = 0$

$\frac{\partial F}{\partial x} = 4x$ $\left.\frac{\partial F}{\partial x}\right|_{M_0} = -8$

$\frac{\partial F}{\partial y} = -8y$ $\left.\frac{\partial F}{\partial y}\right|_{M_0} = -8$

$\frac{\partial F}{\partial z} = -1$ $\left.\frac{\partial F}{\partial z}\right|_{M_0} = -1$

Кас. п-ть: $-8(x+2) - 8(y-1) - 1(z-4) = 0$

$+8x + 8y + z + 4 = 0$

Нормаль: $\frac{x+2}{-8} = \frac{y-1}{-8} = \frac{z-4}{-1}$

№1 1) $\lim\limits_{x\to 0} \dfrac{1-\cos x^2}{x^2 \sin x^2} = \lim\limits_{x\to 0} \dfrac{\sin x^2 \cdot 2x}{2x\cdot \sin x^2 + x^2 \cdot \cos x^2 \cdot 2x} =$

$= \lim\limits_{x\to 0} \dfrac{\sin x^2}{\sin x^2 + x^2 \cdot \cos x^2} = \lim\limits_{x\to 0} \dfrac{\operatorname{tg} x^2}{\operatorname{tg} x^2 + x^2} =$

$\lim\limits_{x\to 0} \dfrac{\cos x^2 \cdot 2x}{\cos x^2 \cdot \cancel{2x} + 2x \cos x^2 - x^2 \cdot \sin x^2 \cdot \cancel{2x}} = \dfrac{1}{2}$

2) $\lim\limits_{x\to\infty} \left(x - 2x^2 \ln\left(1+\dfrac{1}{2x}\right)\right) =$

$\lim\limits_{x\to\infty} \left(\dfrac{1 - 2x \ln\left(1+\frac{1}{2x}\right)}{\frac{1}{x}}\right) =$

$= \lim\limits_{x\to\infty} \dfrac{\frac{1}{x} - 2\ln\left(1+\frac{1}{2x}\right)}{\frac{1}{x^2}} =$

$= \lim\limits_{x\to\infty} \dfrac{-\frac{1}{x^2} - 2\cdot\dfrac{1}{1+\frac{1}{2x}}}{-2\cdot\frac{1}{x^3}} = \lim\limits_{x\to\infty} \dfrac{2x^4 + x^3 + 4x^6}{4x^3 + 2x^2} =$

$= \lim\limits_{x\to\infty} \dfrac{4 + \frac{2}{x^2} + \frac{1}{x^3}}{\frac{4}{x^3} + \frac{2}{x^4}} = \infty$