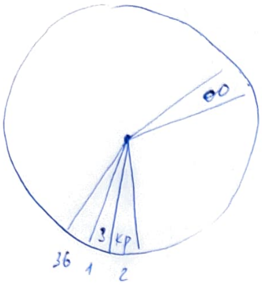
Лекция 1

Теория рисков



36+00

$+1 \cdot \frac{18}{37} - 1 \cdot \frac{19}{37} = -\frac{1}{37}$ (к) $\quad Var\, X_к \approx 1$

$35 \cdot \frac{1}{37} - 1 \cdot \frac{36}{37} = -\frac{1}{37}$ (13) $\quad Var\, X_N \approx 34$

- $X_1, \dots, X_n \longrightarrow E(F(X_1, \dots, X_n))$ ← risk

$C_+ - C_0 \qquad$ P&L

- $X \sim \frac{e^{-x^2/2}}{\sqrt{2\pi}} = p(x)$, $N(0,1)$

$L^2(\mathbb{R}, p(x))$ Гильб. пр-во функ.

← ск. пр.

$$(f, g) = \int_{\mathbb{R}} f(x) \cdot \bar{g}(x) \cdot p(x)\, dx$$

$$f \in L^2(\mathbb{R}, p) \iff \int |f|^2 p\, dx = \|f\|_2^2$$

сход-ся

$$y = \varphi(X), \quad E\varphi(X) = 0, \quad E|\varphi|^2 = \int_{\mathbb{R}} |\varphi(x)|^2 p(x)\, dx = \|\varphi(x)\|_2^2 < \infty$$

$1, x, x^2 \dots$ ортогонализация (полиномы Эрмита) Hermit

$\varphi_0(x) = 1$, $\varphi_1(x) = x$, $\varphi_2(x) = x^2 - 1$, $\varphi_3(x) = x^3 - 3x \dots$

орт. $\qquad$ орт.

$H_0(x) = 1 \quad H_1(x) = x \dots \qquad H_n(x) = x^n + \dots$

[illegible] ортогон. (но не ортонорм-ны) полиномы Вика (Wick)

$$e^{ax - a^2/2} = \sum_{n=0}^{\infty} \frac{a^n}{n!} :x^n: \quad \Leftarrow \text{пр. ф. пол. Вика}$$

полином n ст.

$\boxed{* \text{ B.S. } e^{wt - w^2/2} \ldots}$

$$e^{ax - a^2/2} = \left(1 + ax + \frac{a^2x^2}{2} + \frac{a^3}{3!}\cdot x^3 + \ldots\right)\cdot\left(1 - \frac{a^2}{2} + O(a^4)\right)$$

$$= 1 + ax + \frac{a^2}{2}\cdot x^2 - \frac{a^2}{2} + \frac{a^3x^3}{3!} - \frac{3a^4}{3!}$$

$:1: = 1$, $:x: = W_1(x) = x$, $W_2(x) = x^2 - 1$, $W_3(x) = x^3 - 3x$

пол. Эрмита:

$:x^n: \; W_n(x) = H_n(x)$

$$e^{ax - a^2/2} = \sum_{n=0}^{\infty} \frac{a^n}{n!}\cdot H_n(x)$$

Thm. полин. обр. ортог. систему. (не ортонорм.)

$E\, W_n(X)\cdot W_m(X) = n!\,\delta_{nm}$

$\|W_n(X)\|_2^2 = n!$

$$\frac{W_n(X)}{\sqrt{n!}} = \frac{:X^n:}{\sqrt{n!}}$$

ИW1 $H_{n+1}(x) = xH_n(x) - n\cdot H_{n-1}(x)$ ф-ла Родригеса

$$e^{ax - a^2/2} = \sum_{n=0}^{\infty} \frac{a^n}{n!} H_n(x)$$

$\partial/\partial x \Rightarrow$

$$(x-a)e^{ax-a^2/2} = \sum_{n=1}^{\infty} \frac{a^{n-1}}{(n-1)!} H_n(x)$$

HW1

a) $\partial/\partial x$; $\partial^2/\partial a \partial x$

$(X_1, \ldots, X_n)$ $\quad EX_i = 0$

$$X_1' = \frac{X_1}{\sqrt{\operatorname{Var} X}}, \ldots, X_n' = \frac{X_n}{\sqrt{\operatorname{Var} X_n}}$$

безразмерные ($\operatorname{Var} X = 1$)

$$\rho = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & & \rho_{2n} \\ & & \ddots & \vdots \\ & & \rho_{n,n-1} & 1 \end{pmatrix}$$

$\rho = \rho^T \quad \det \rho > 0$

$$\max_i \sum_{j: j \neq i} |\rho_{ij}| = \rho < 1$$

(опр. сил. стимулир.)

$\|B - I\|_{L^p} \le \rho < 1$

$A\bar{x}, \bar{x} \in \mathbb{R}^n$

$\|\bar{x}\|_p = \left(\sum_{i=1}^n |x_i|^p\right)^{1/p}$, $p = \infty$ $\quad \|\bar{x}\|_\infty = \max_i |x_i|$

$B = I + A \quad \|A\|_p \le \rho < 1$ $\quad$ м-ца ограничения

$\Rightarrow \det B > 0$

$\neq$. $p(\bar{x}) = \dfrac{1}{(2\pi)^{n/2}\sqrt{\det B}} \cdot e^{-1/2 (B^{-1}x,\, x)}$

$L^2(R^n,\, p(x))$, $L^2(\varphi(\bar{x}),\ E(\varphi(\bar{x}))=0)$

↳ пр-во ф-ий
↳ гаус. вел. обобщ.

Пусть φ – произв.

$$\frac{\|\varphi(x_1, \dots, x_n)\|_B}{\|\varphi(x_1, \dots, x_n)\|_I}$$

Th 1. $B \neq I$

$$\exists (y_n): \frac{\|y_n\|_B}{\|y_n\|_I} \xrightarrow[n\to\infty]{} \infty$$

$$\exists (\tilde{y}_n) \cdot \frac{\|\tilde{y}_n\|_B}{\|\tilde{y}_n\|_I} \longrightarrow 0$$

$B \neq I$

$\mathrm{Tr}\, B = \lambda_1 + \dots + \lambda_n = n \Rightarrow B = I$ (противоречие)

$\lambda_n \geq 0$

$\Rightarrow \exists\, (\lambda_{max} > 1),\ (\lambda_{min} < 1)$
$\lambda_+ \qquad \lambda_-$

$\xi_+ \in R^n$

$B\xi_+ = \lambda_+ \xi_+$

$\varphi(\bar{x}) = (\bar{X}, \xi_+)$ ← ск. пр.

$y_n = (X, \xi_+)^n$

Ǝ сопрач.
⇑
$x = B^{1/2} y$

$\|(\xi \cdot \bar{x})^n\|_2^2 = \int_{R^n} (\xi_+, \bar{x})^{2n}\, e^{-(B^{-1}x,\, x)/2}\, dx \Big/ \sqrt{\det B}\,(2\pi)^{d/2}$ ← собств. вектор

$$\ldots = \int_{\mathbb{R}^n} (\xi, B^{1/2} y)^{2n} e^{-y^2/2} / (2\pi)^{d/2} \, dy = \lambda_+^n \int_{\mathbb{R}^n} (\xi, y)^{2n} e^{-y^2/2} / (2\pi)^{d/2} \, dy$$

$$= \lambda_+^n \, \|\varphi(x)\|_2^2$$

↳ собств. вект. => глав. компоненты

Цель: не можем оценить cova => φ-я

$$\tilde{L}^2(\bar{X}) = \varphi_1(x_1) + \ldots + \varphi_n(x_n)$$

$$\tilde{L}^2(\mathbb{R}^n, B) = \varphi_1(x_1) + \ldots \varphi_n(x_n)$$

↳ пол. Эрмита

$$\varphi(\bar{X}) \in \tilde{L}^2$$

$$c_- \le \frac{\|\varphi(\bar{x})\|_B}{\|\varphi(\bar{x})\|_I} \le c_+$$

$$\varphi_i(X_i) = \sum_{k=1}^{\infty} c_{ik} :X_i^k:$$

$$(X_1, X_2) : \quad B = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

$$E :X_1^m: \cdot :X_2^n: = n! \, \delta_{mn} \rho^n$$

$$E \frac{:X_1^n:}{\sqrt{n!}} \cdot \frac{:X_2^m:}{\sqrt{m!}} = \delta_{mn} \rho^n$$

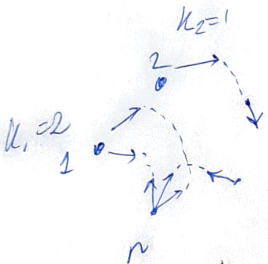
$$E\, e^{aX_1 - a^2/2} \cdot e^{bX_2 - b^2/2} = E \sum_{n,m=0}^{\infty} \frac{a^n}{n!} \frac{b^m}{m!} :X_1^n::X_2^m:$$

$$e^{-a^2/2 - b^2/2}\, E e^{aX_1 + bX_2} = e^{-(a^2+b^2)/2} \cdot e^{\sigma^2/2} = e^{ab\rho} = \sum_{n=0}^{\infty} \frac{(ab)^n}{n!} \rho^n$$

$$\sigma^2 = \operatorname{Var}(aX_1 + bX_2) = a^2 + b^2 + 2ab\rho$$

$$E :X_1^{k_1}::X_2^{k_2}: \ldots :X_n^{k_n}: = 0, \quad k_1 + \ldots + k_n = n \pmod 2)$$

$k_1 + \ldots k_n$ · even #



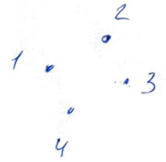
$n = 5 \quad \sum k = 8$

$\downarrow$ диаграмма Γ

$\ldots : \Gamma$

$$\sum_\Gamma \rho_{15}\rho_{15} \cdot \rho_{23}\rho_{45} = \sum_\Gamma \prod_n \rho_{ij}$$

$$E :X_1::X_2::X_3::X_4: = E X_1 X_2 \cdot X_3 \cdot X_4 =$$



$$= \rho_{12}\rho_{34} + \rho_{13}\rho_{24} + \rho_{23}\rho_{14}$$

(А при $\rho_{ij} = \rho \Rightarrow 3\sigma^4$)

$$E :X_1: \ldots :X_n:$$

$$= e^{-(a_1^2 + \ldots a_n^2)/n} \cdot E e^{aX_1 + \ldots + a_n X_n}$$

$$= e^{-(\sum a_i^2)/n} \cdot e^{\sigma^2/2} = e^{\sum_{i,j}}$$

$$e^{a_n X_n - a_n^2/2} = \sum \ldots$$

$$E(a_1 X_1 + \ldots a_n X_n)^2 = \sum a_i a_j E X_i X_j = \sigma^2$$

Th $E :X_1^{k_1}: \ldots :X_n^{k_n}: = \rho^N \prod_{i=1}^{n} (k_i!)^{3/2}$ ← corr. коррелятор

$$\rho = \max_i \sum_{j \neq i} |\rho_{ij}|$$

$k_1 + \ldots + k_n = 0 \mod(2)$

$k_1 + \ldots + k_n = 2N$

$A = [a_{ij}]$, $A = A^* \geq 0$

$A: \mathbb{R}^n \to \mathbb{R}^n$

$B = [b_{ij}] = B^* > 0 \quad \Longrightarrow c_{ij} \geq 0$

$C = [a_{ij} b_{ij}]$ Schur product

Hint: $X_1, \ldots, X_n \sim N(0, A)$

$Y_1, \ldots, Y_n \sim N(0, B)$

$X_1 Y_1, \ldots, X_n Y_n \quad A \cdot B$

Перколяция $\mathbb{Z}^2$ $\quad \varepsilon(x)$, $x \in \mathbb{Z}^2$

$$\varepsilon(x) = \begin{cases} 1 & p \\ 0 & q = 1-p \end{cases}$$

$A = \{x : \varepsilon(x) = 1\}$

при малых p малые кластеры (все моменты известны)

если $p \geq \tilde{p} \Rightarrow$ протекание

Лекция 2

Теория риска

$$\tilde{L}_2 = \{\underbrace{\varphi_1(x_1) + \dots + \varphi_n(x_n)}_{= f(\bar{x})}\}$$

$(x_1, \dots, x_n)$

$$B = \begin{bmatrix} 1 & & \rho_{1n} \\ & \ddots & \\ & & 1 \end{bmatrix}$$

$$\max_i \sum_{j:\, j \neq i} |\rho_{ij}| = \rho < 1$$

Th-1 $y \in \tilde{L}_2$

$$1 - \rho \le \frac{\|y\|_B^2}{\|y\|_I^2} \le 1 + \rho$$

...

$\|B - I\| = \rho$

Лемма Гершгорина

$A = [a_{ij}] = A^*$

$$R_{ii} = \sum_{j,\, j \neq i} |a_{ij}|$$

Δ_{ii}

$\{\lambda_i\} \in \bigcup_i \Delta_i$

$A = I + \rho$ $\quad \{\lambda_i\} \in [1 - \rho;\ 1 + \rho]$

$$0 < 1 - \rho \le \lambda_{max} \le 1 + \rho$$

$$y = \sum_{i=1}^n \varphi_i(x_i) = \sum_{i=1}^n \sum_{l=0}^\infty \frac{a_{il}}{l!} :x_i^l: = \sum_{l=0}^\infty \frac{1}{l!} \underbrace{\left(\sum_{i=1}^n a_{il} :x_i^l:\right)}_{Q_l(\bar{x})}$$

Thm. $E(:X_i^l: :X_j^m:) = \delta_{lm}\, l!\, \rho_{ij}^l$

$Q_l(\bar{x}) \perp Q_m(\bar{x}),\ l \neq m$

$$\|y\|_B^2 = \sum_{l=0}^\infty \frac{1}{(l!)^2} (Q_l(\bar{a}), Q_l(\bar{x}))_B \qquad \|y\|_I^2 = \sum \frac{1}{l!} (Q_l, Q_l)_I$$

$$\| (Q_l(\bar{x}), Q_l(\bar{x})) \|_B = \sum_{i,j=1}^{n} a_{il} a_{jl} \rho_{ij}^{l} C!$$

$$I + R = [\delta_{ij} + \rho_{ij}]$$

$$I + R^{l} = [\delta_{ij} + \dots + \rho_{ij}^{l}]$$

$$\sum_{j: j \neq i} |\rho_{ij}^{l}| \le \rho^{l-1} \cdot \rho = \rho^{l}$$

$$\lambda_{max}^{(l)} \le \rho^{2}$$

Оцениваем:

$$\lambda_{min} |x|^2 \le (Ax, x) \le \lambda_{max} |x|^2$$

$$\lambda_{max}(B) \le 1 + \rho^{l}$$

$$1 - \rho \le \frac{\| y \|_B^2}{\| y \|_I^2} \le 1 + \rho$$

↳ оценка из алгоритма

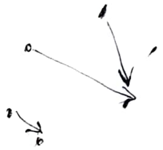
$$\lambda_{min}(B) \le \frac{\| y \|_B^2}{\| y \|_I^2} \le \lambda_{max}(B)$$

Спектральные оценки

Теория перколяций

Случайный граф

$A (N < n)$



n точек

$$\binom{n}{2} = \frac{n(n+1)}{2}$$

N – число действий $\Rightarrow \Gamma_N$

N – кол. пар ребер

$\Gamma_N = \cup_i C_i$

кл. содерж. точку

$E|C_i| - ? \Rightarrow E|C(x)| \le \frac{N}{n}$

→ кластеры

P (попасть в x ...)

Оценка сверху $\frac{N}{n} = \rho < 1$

$$\frac{|C(x)|}{n} \le \rho + \rho^2 + \dots < \frac{1}{1-\rho}$$

Books

Случ. размещения

Случ. графы

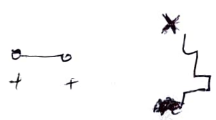
Thm Site percolation $\mathbb{Z}^2$

$$x \in \mathbb{Z}^2 : \quad \varepsilon(x) = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$$

$A_1 = \{x : \varepsilon(x) = 1\}$ — случ. мн-во

$x \sim x'$

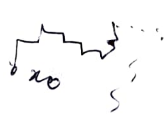
$\varepsilon(x) = \varepsilon(x') = 1$, $|x - x'| = 1$



* для групп полин. роста на реш. если и сущ. погр ∞, то только одна

$p \ll 1 \Rightarrow$ все компоненты конечные (p-as)

▷ Пусть $\exists\, x_0$:



$\gamma(t) \to \infty$

$\gamma(0) = x_0$

берем самонепересекающ.

$p^n = \{$ на γ, $|\gamma| = n$ стоят только $+1\}$

с $p > 0$ $\exists\, \gamma$: ↗ бесконеч. комп.

$\forall n \ \exists\, \gamma(0), \dots, \gamma(n)$
$\quad +1 \qquad +1$

$2{,}63 \le \alpha_2 \le 2{,}64$

$\frac{\ln \#\{\gamma_n\}}{n} \to \alpha_2$ (const)

$\#\{\gamma_n\} \le 4 \cdot 3^{n-1}$ ← оц. сверху

↑ самонепересекающийся

$4 \cdot 3^{n-1} p^n < \infty$ — P(найдется самонепер. путь n)

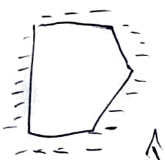
$p < 1/3$ $\left(\hat{p}_c \approx 0{,}59 \right)$

По лем. Бореля-Кантелли число конечно =>

Нач. с нек. n будут только противоп. события => прот. верх.

+ не прос. => $p < 1$ $\theta(p)$ конечна

если нет протек.



запирающий контур содерж. n точек

$\#(\gamma_{n}(m)) \le 8m\,7^m$

$P = \sum 8m\, q^m 7^m < \infty$

$q < 1/7$ если $p > 6/7$,

то + прос.

на бесконечность

если ~~[illegible]~~

$0{,}48 < p < 0{,}52$ не прос. ни "+", ни "−"

(в смысле "ч.")

Lecture 1. Hermite-Itô-Wick polynomials: Properties, Estimations, and Combinatorics

1 Introduction to Hermite-Itô-Wick polynomials.

In this section we define Wick polynomials and give some of their properties. They are sometimes also called Hermite-Itô-Wick polynomials. Let $X \sim N(0,1)$ on some probability space $(\Omega, \mathcal{F}, P)$. Let

$$L^2 = L^2\left(\mathbb{R}^1, p(x)\,\mathrm{d}x\right),$$

where $p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, be the space of all function φ such that $\mathrm{E}\,|\varphi(X)|^2 < \infty$ and $\mathrm{E}\,\varphi(X) = 0$. This is a Hilbert space with norm

$$\|\varphi\| = \sqrt{\mathrm{E}\,\varphi(X)^2} = \sqrt{\int_{\mathbb{R}} \varphi^2(x)p(x)\mathrm{d}x}.$$

Let $\mathcal{L}^2 = \mathcal{L}^2(\Omega, P)$ be the space of all random varibles of the form $Y = \varphi(X)$ for $\varphi \in L^2$. This is a Hilbert space with norm

$$\|Y\| = \sqrt{\mathrm{E}\,Y^2}.$$

We now introduce a natural basis for the space $\mathcal{L}^2$. This basis has been known under different names and notations; our presentation follows [1]. Let $\psi_X(a) = e^{aX - a^2/2}$. The Taylor expression of this function is given by

$$e^{aX-a^2/2} = \sum_{n=0}^{\infty} \frac{a^n}{n!} {:}X^n{:}, \tag{1}$$

Lecture 2. Wick polynomials in Risk Theory

Let 0 be the current time and let T be a certain time in the future (say 1 week or 1 month ahead). Assume that our portfolio has value v_0 at time 0 and v_T at the time T. Let $X_T = v_T - v_0$ be P&L (Profit and Loss). This is a random variable (on some probability space) whose distribution can be reconstructed statistically (at least in principle) if we have a "good model" for the evolution of $X_t, t \geq 0$.

One of the fundamental problems in financial mathematics is <u>risk measurement</u>. Mathematically, the risk of X_T is a real number depending on the law of X_T. Let us describe two popular risk measures.

(a) Standard deviation: $\mathrm{Var}(X_T) = \sigma^2$.

Consider the case of roulette. If we bet \$1 on "red" our profit is

$$X_1 = \begin{cases} +1 \text{ with probability } 18/38 \\ -1 \text{ with probability } 20/38 \end{cases}.$$

If we bet on a single number, say "13," our profit is

$$X_2 = \begin{cases} 35 \text{ with probability } 1/38 \\ -1 \text{ with probability } 37/38 \end{cases}.$$

It is clear that betting on a single number is "more risky" than betting on a color. The σ^2-measure gives

$$\mathrm{Var}(X_1) \approx 1 \ll \mathrm{Var}(X_2) \approx 33.$$

The standard deviation risk measure is especially natural in the case of static portfolios of linear instruments (equities, futures etc.). However, in the asymmetric situation, the variance is not an adequate measure of risk. If, say, $X_T \geq 0$ then the risk is zero, but $\sigma^2(X_T)$ can be very large.

(b) Value-at-Risk: $VaR_\lambda(X_T)$, $\lambda \in (0, 1)$

Value-st-Risk is the λ-quantile of the distribution of X_T with the opposite sign:

$$VaR_\lambda(X_T) = -q_\lambda(X_T),$$

where

$$q_\lambda(X_T) = \inf\left(x : P\left(X_T \leq x\right) > \lambda\right).$$

Lecture 3. Percolation of Random Fields on Lattices

In this lecture we will apply the estimation of joint moments of Wick polynomials of dependent Gaussian random variables to the percolation problem for (correlated) random fields. More details can be found in the review [3] and the monograph [2].

Let $F(x, w), x \in \mathbb{Z}^d, d \geq 2$ be a Gaussian homogeneous field on the lattice. Assume that $\mathrm{E}F(\cdot) = 0$ and $\mathrm{E}F(x, \cdot) F(y, \cdot) = B(x - y)$ are the first two correlation functions. We will use the normalization $\mathrm{Var}F(\cdot) = B(0) = 1$.

Assume that h is some level and let

$$A^+(h) = \{x : F(x, w) \geq h\} \text{ and } A^-(h) = \{x : F(x, w) < h\}.$$

We are interesting in the topological structure of 1-connected or $\sqrt{2}$-connected components of the random sets $A^{\pm}(h)$. Since this problem is very difficult even for i.i.d. random variables (classical site Bernoulli percolation) we will present results of the following nature: one can find such $h^+ = h^+(d, B(\cdot))$ that for $h > h^+$ the set $A^-(h)$ percolates to infinity and $A^+(h)$ contains only a bounded connected component and the volume of "typical" such component (for example, component containing fixed point, say $x = 0$) has exponential moments. In other terms, there exists an infinite 1-connected "ocean" where $\{x : F(x, \cdot) < h\}$ containing relatively small islands with $F \geq h$ (of course, each islands can contain lakes, where again $F(\cdot) < h$, etc.).

Let us prove this fact for the case when the random variables $F(x, w)$, $x \in \mathbb{Z}^d$ are independent, i.e., $B(z) = \delta_0(z)$. Let us put

$$p = P\{F(\cdot) \geq h\} = \int_h^\infty \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx,$$

$q = 1 - p$ and apply the classical contours method of Hummerslay for the Bernoulli field. The Gaussian nature of the i.i.d. random variables is irrelevant, we will work only with the probabilities $p, q > 0$.

Our first goal is to prove that for small p (i.e., large h) the set $A^+(h)$ does not percolate. Assume that A^+ percolates to ∞, then $P\{0 \in \text{infinite cluster of " + "}\} > 0$. This means that, for any $n \geq 1$, one can find a selfavoiding path of length n on $\mathbb{Z}^d$, starting at $x = 0$ and containing only "+." But the number of such paths $\leq 2d(2d-1)^n$, i.e., the probability that such a path exists (event B_n) is smaller than $c\,[p\,(2d-1)^n]$, where c is a constant. If $p < \frac{1}{2d-1}$ then $\sum_n P(B_n) < \infty$ and the Borel-Cantelli Lemma tells us that P-a.s. there is no percolation of $A^+(h)$ to infinity.

Lecture: Stoch. Frontier Model

$\ln y_i = y_i^* - u_i$

$\ln y_i^* = x_i \beta + v_i$ $\Leftrightarrow$ $\ln y_i = x_i \beta + \varepsilon_i$, $\varepsilon_i = v_i - u_i$.

$u_i \sim N^+(0, \sigma^2)$ $v_i \sim N(0, \sigma^2)$

(or truncated $N(\mu, \sigma^2)$)

Log-Likelihood function -
density fun. of composed error term

$E(u_i \mid \hat{\varepsilon}_i) = E(u_i \mid y_i - x_i \hat{\beta})$

PDF

$$E(u_i \mid \varepsilon_i) = \frac{\sigma_* \phi\left(\frac{\mu_{*i}}{\sigma_*}\right)}{\Phi\left(\frac{\mu_{*i}}{\sigma_*}\right)} + \mu_{*i}$$

CDF

Kumbakhar, Sun
R code
(Wang Econ. Letters)

$$\frac{\partial E(u \mid \varepsilon)}{\partial z_u} \neq \delta_u$$

closed-
skew normal
distr.

If z is dummy

$E(u \mid D=1) - E(u \mid D=0)$

Input and output endog. since economic desicions are made for both var. and depend on ec. beh. (prof. max, revenue max, ...)