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**DYNAMIC PROCESSES FOR SCARCE RESOURCES**

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**2017**

# Abstract

We take a differential game approach to study the optimal choices of managerial firms concerning efforts in process innovation while operating under resource scarcity. Game theory is a theory of conflict and cooperation between rational decision makers. We consider process innovation that reduces marginal cost. We demonstrate how changes in different parameters alter the incentive for cost-reduction innovation, using Pontryagin’s maximum principle. Alongside with this, we are making an overview of several economic models operating on a market for scarce resources or dealing with innovative processes.

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# Introduction

Game theory is a theory of conflict and cooperation between rational decision makers. It relies heavily on mathematical models and has proven useful to address problems of conflict and cooperation in economics and management science, as well as other areas of social science. Differential games are dynamic game models used to study systems that evolve in continuous time and where the system dynamics can be described by differential equations.

Modern game theory has evolved enormously since its inception in the 1920s. The branch of game theory known as dynamic games descended from the pioneering work on differential games by Isaacs, Pontryagin and his school, and from seminal papers on stochastic games by Shapley. Since then dynamic game theory has had significant impact in such diverse disciplines as applied mathematics, economics, systems theory, engineering, operations research, biology, ecology and the environmental sciences. More importantly game theory is proven to be very effective, considering obtaining oligopolistic market outcome.

Differential game theory conceptualizes problems of this kind by assuming that time evolves continuously and the system dynamics can be described by a set of ordinary differential equations. And since many problems are characterized by multiple interdependent decision makers, acting independently or cooperatively, continuous-time dynamic optimization models of these agents are regarded as differential games, being perfect in application to economics and management science.

We are going to examine continuous-time dynamic optimization of continuous-time oligopolistic market models, which operate under scarcity, along with numerous types of dynamic games which can represent that market and different types of optimization that can be applied to obtain optimal result. We will also make a review on different innovative practices and most importantly different models of resource economies will be assessed.

To begin with, scarcity is the fundamental economic problem, which states that society has not enough resources to fulfill all human wants and needs. Today natural common resources face increasing environmental risks as reported in numerous scientific studies. The most common prediction is a widespread reduction in the renewability of the stock of natural resources. Declining fish stocks, a decrease in global water availability, an overall decline in crop yields and a decrease in growth rates of tropical forests are just a few examples.

The outline of the paper is as follows. Section 1 investigates different types of dynamic games, which can be used to represent needed model, and dynamic optimization with special attention paid to Pontryagin’s maximum principle. Section 2 presents a number of models with innovative processes, showing which strategies can each producer follow while running a firm under circumstances requiring developments of some kind. Several questions regarding resource economy will be shown in Section 3. Finally, Section 4 demonstrates a dynamic duopoly in a market of scarce resource with cost-reduction innovation being invested by both firms.

## Related literature

Differential games have been applied to a variety of different economic models. They have been successfully used in economics, finance, optimization, and stochastic control. Dockner, Jergensen, Van Long and Sorger (2000) considered its application to management science. Nowak and Szajowski (2004) especially investigate dynamic games and risk-sensitive Nash equilibria in games of resource extraction.

Stochastic differential game approach to study the optimal choices of managerial firms concerning efforts in product and process innovation and in particular queueing models and patenting were reviewed by Keller (2007). Cellini and Lambertini (2007) investigate the timing of adoption of product and process innovation by using a differential game in which firms may invest in both activities, as well as Lambertini and Mantovani (2005) who investigated simultaneous innovation activities in duopoly. That question was also raised by Utterback and Abernathy (1975) who assessed a dynamic model of process and product innovation. Dynamic optimization and differential games were reviewed by Itaya (1999): in his work he presented several packages for obtaining necessary conditions of constrained static optimization problems, solving differential equations and boundary-value problems. Yeltekin, Cai and Judd (2015) presented a method is based on recursive game theory and provided an iterative scheme, which delivers an accurate approximation to the true equilibrium value correspondence of a dynamic game for dynamic oligopoly with endogenous capacity. They stated that in some approaches, firms cannot change their capacity state throughout the game and then the game would reduce to a repeated game (rather than a dynamic one). In others, firms can invest in higher capacity but cannot divest. This restriction has the advantage of transforming an infinite horizon dynamic game into a finite state game that can be solved by backward induction.

Fesselmeyer and Santugini (2012) consider the effect of environmental risk on the extraction of a common resource and also focus on the tragedy of commons. Regev, Gutierrez, Schreiber and Zilberman (1998) assess biological foundations of renewable resource exploitation. Eso, Nocke and White (2007) show competitive market for scarce resources, yielding perhaps an unexpected result that an increase in the quantity of available capacity can result in a reduction in the total quantity of output.

A big class of linear-quadratic differential games has been widely reviewed by Ling (2010), who assess its application to lottery model for rationing public resources. While Amir (2000) provided its application in resource economics, industrial organization, macroeconomics, market games, and experimental and empirical economics based on the stochastic games paradigm.

My work also connects to the large theoretical literature on dynamic oligopolies. Kirman and Sobel (1974) reviewed dynamic oligopoly with inventories, also assessing differential game with stochastic demand application. Goettler and Gordon (2008) provided a model for durable goods oligopoly with endogenous innovation, using Markov-Perfect Nash Equilibria for obtaining results and later in their other work (2012) showed effects of product innovation and competition in dynamic oligopolistic market. They provided a dynamic model in which firms endogenously determine steady-state innovation rate and used it to evaluate the effect on innovation of three measures of competition: entry costs, product substitutability, and innovation spillovers. Exogenous improvements in the outside good generate industry-wide depreciation shocks. Benoit and Krishna (1987) studied a dynamic model of duopoly in which firms choose both prices and quantities, investigating how flexible capacities change market outcome. Chintagunta and Rao (1996) formulated a differential game model for dynamic pricing in a duopolistic market, introducing the question of brand choice and brand preferabilities. Their results indicate that, given equal marginal costs for brands, in steady-state equilibrium, the brand with the higher preference level charges a higher price.

In addition, there was an amount of research made in the area of innovative processes. Some relevant results were yielded by Cai and Rajan (2004) in their research on investment in innovations and collusion, and by Levidow, Zaccaria and Maia (2014) making a review on innovative practices in water irrigation. Ho, Savin and Terwiesch (2002) discuss new product diffusion under supply constraint, using Bass diffusion model as a well-known parametric approach to estimating new product demand trajectory over time. This paper generalizes the Bass model in the presence of a supply constraint. As a result of supply constraint, potential customers who are not able to obtain the new product join the waiting queue, generating backorders and potentially reversing their adoption decision, resulting in lost sales.

Finally, a big area of literature refers to the resource scarcity and food and water scarcity in particular. Biological aspect of water scarcity problem was assessed by Rijsberman (2006). Wutich and Brewis (2014) made a research on anthropology of resource insecurity, showing sociological aspect of the problem of water and food shortage. Lopes, Pereira and Fontes (2014) studied the dynamic optimization of an irrigation problem, characterizing the optimal solution by applying the necessary conditions of optimality in the form of the maximum principle. Podimata and Yannopoulos (2015) presented a paper examining the potential for water conflict when water consumption for irrigation takes place, using game theory as a platform that provides predictions about strategies of irrigation followed by stakeholders.

# Dynamic differential games

This section deals with the theory and applications of dynamic differential games. The theory of dynamic games evolved from static game theory which originated in the 1920s with seminal works of John von Neumann and made great progress in the 1950s with the papers by John Nash. Central concepts in game theory have been influenced by one-person decision theory (utility theory).

In a dynamic differential game decision maker can act independently or cooperatively in the pursuit of their own best interests. In the area of differential games, cooperative theory is far less developed than non-cooperative theory and almost all applications in economics and management science are in the non-cooperative setup. Differential games belong to a subclass of dynamic games called state space games. In a state space game, the modeler introduces a set of state variables to describe the state of a dynamic system at any particular instant during play. The hypothesis is that the payoff-relevant influence of past events is adequately summarized in the state variables. To illustrate, the state vector may consist of the current capital stocks of N oligopolistic firms and these stocks can be influenced by the firms through the choices of their individual investment rates. If a state space game is cast as a differential game, the assumption is that time evolves continuously and that the evolution over time of the state variables can be modelled by a set of differential equations. Discrete time models involve the assumption that no decisions are made between the time instants that define the periods.

Application of differential games includes a number of studies in the following areas of economics and management science: capital accumulation and investments, R&D and technological innovations, Cournot oligopoly, pricing and advertising decisions in marketing, natural resource extraction, pollution control, etc.

## Dynamic game equilibria

In this section we now turn to notable types of game equilibria, which can be obtained in dynamic games. To begin with, there are two forms of solutions to the same dynamic problem: the open-loop solution and the closed-loop, or feedback, solution.

An open-loop strategy is a strategy which is conditioned on current time only, that is, a minimal amount of information. Like any other type of strategy, an open-loop strategy is fixed at the start of the game. The open-loop assumption means that the players leave all information except time out of consideration, or that they must choose open-loop strategies since they cannot observe anything other than their own actions and time. The use of open-loop strategies has been criticized for being static in nature, not allowing for genuine strategic interaction between the players during the play of the game. In problems in the economics of renewable resources and the environment, the commitment that lies in open-loop strategies can be seen as a reflection of far-sightedness and concern for the conservation of resources and the environment.

Further, if the planning horizon is short, an open-loop strategy could be employed as a representation of a rigid strategy for short-term operational or tactical planning.

An alternative way of thinking about open-loop strategies is as Markovian strategies where at each stage players use only constant functions of the current state. In the systems theory and macroeconomics literature, Markovian strategies are usually referred to as feedback, or no-memory, strategies.

There are several important properties of open-loop equilibria. In deterministic Markov one-person dynamic optimization, there always exists an optimal open-loop strategy. This fact is certainly intuitive, as is its failure in the presence of chance moves or stochastic transitions.

Open-loop equilibria are typically much simpler to analyze than Markovian equilibria. In particular, the usually difficult question of existence of pure-strategy equilibrium is most often straightforward in the open-loop case.

The use of Markovian strategies is a natural choice in the setup of state space games where the history of the game till time t is summarized in the value of the state vector at time. The choice of Markovian strategies is also motivated by their simplicity: players react only to factors which are payoff-relevant and constitute an intertemporal link in the game (namely, the state variables). The Nash equilibrium in a game played with Markovian strategies is called a Markovian Nash equilibrium, or feedback Nash equilibrium, and play an important role in optimal control theory.

Yet another relevant type of game equilibria is subgame-perfect equilibrium (SPE) which is used largely in the repeated game case, where the key to finding the equilibrium value correspondence involves defining a recursive operator that maps future SPE payoffs into SPE current payoffs. This type of equilibria was observed by Yeltekin, Cai and Judd [10]. SPE is a stronger equilibrium notion, than Nash. In games with a non-trivial dynamic structure, Nash equilibrium is too permissive. Consider a Nash equilibrium of an infinitely repeated game with perfect monitoring. Associated with each history that cannot occur in equilibrium is a subgame. The notion of a Nash equilibrium imposes no optimality conditions in these subgames, opening the door to violations of sequential rationality. Subgame- perfection strengthens Nash equilibrium by imposing the sequential rationality requirement that behavior should be optimal in all circumstances, both those that arise in equilibrium (as required by Nash equilibrium) and those that arise out of equilibrium.

Last type of equilibria, worth mentioning is risk-sensitive Nash equilibrium concept in static non-cooperative games, which can be also used in two-stage stochastic games of resource extraction. In games of resource extraction randomness is connected with a transition probability function. A risk-sensitive Nash equilibrium exists if and only if the noncooperative game has a Nash equilibrium.

## Dynamic optimization

Continuous-time dynamic optimization models play a vital role in analyzing time-dependent activities in an economic system. Each agent, or player, has its own objective to be maximized. There are several well-known approaches to solving dynamic problems: backward induction, the Bellman equation, the Lagrangian method and the Hamiltonian method. All of them must satisfy the principle of dynamic optimization. We will concentrate on Hamilton-Jacobi-Bellman equation and on Pontryagin’s maximum principle.

The Hamilton-Jacobi-Bellman (HJB) equation lies at the heart of the dynamic programming approach to optimal control problems. This approach is based on the important principles of embedding and recursion. To explain these two principles, formally, we denote that problem by . The principle of embedding says that we should solve not only the given problem but rather the entire family of problems . Here, is the problem that starts at time *t* in initial state *x* and can be stated as follows:

subject to

The principle of embedding alone does not help us in any way: it tells us to solve infinitely many problems instead of a single one. However, if combined with the principle of recursion it leads to the powerful HJB equation. Recursion means that we start at the 'smallest' problems of the entire family and work our way backwards to the 'largest' problems.

We now present an intuitive argument for the fact that the optimal value function *V* satisfies the partial differential equation, which is called the Hamilton-Jacobi-Bellman equation:

,

where is the scalar product of two n-dimensional vectors.

Pontryagin's maximum principle is often employed to solve dynamic optimization problems. The principle states that the solutions must satisfy boundary-value differential equations. It is a first order condition for smooth problems, comparable to the condition that the gradient vector of a function must vanish at a local maximum of *g*. The latter condition is also satisfied at local minima and other critical points. Only if we have some additional information on the global curvature properties of *g* (like concavity) can we infer from the condition *g(x) =* 0 that *x* is indeed a maximum. The situation is similar in the case of an optimal control problem. Even if a certain control path *u* satisfies the maximum principle it need not be an optimal path. If the problem has certain curvature properties, however, then any control path which satisfies the maximum principle is an optimal path.

The function *H* is called the (current-value) Hamiltonian function and plays a prominent role in Pontryagin's maximum principle. The variable is called the (current-value) costate variable associated with the state variable *x*, or the (current-value) adjoint variable.

Maximized Hamiltonian function :

Considering some optimal control problem and defining the Hamiltonian function *H* and the maximized Hamiltonian function *H\*,* assume that the state space *X* is a convex set and that the scrap value function S is continuously differentiate and concave. Let *u* be a feasible control path with corresponding state trajectory *x*. If there exists an absolutely continuous function such that the maximum condition ; the adjoint equation and the transversality condition are satisfied, and such that the function *x* is concave and continuously differentiable, then *u* is an optimal path.

## Linear-Quadratic Game of natural resources extracting

The class of linear-quadratic differential games is the workhorse model in this field and has been widely applied in macroeconomics, international pollution control and natural resources extracting. That class allows the dynamic system to be reduced to a pair of differential equations, and hence the standard phase-diagram method can be applied to study the equilibria of the model.

Consider a simple linear-quadratic model with two players, one state variable and one control variable for each player in a game of resource extraction shown in Ling’s [11] research paper. The game is given by the pair of optimal control problems:

s. t.

For instance, one can view this as an international pollution control problem, if we assume as the natural decay rate, and all other parameters are positive. Thus *x(t)* is the nonnegative stock of pollution at time t, *u(t)* is the abatement activity for country *i*, and the goal of each country *i* is to minimize the present discounted cost due to the pollution damage and the abatement activity.

The linear quadratic structure yields an important outcome that cannot be obtained in general. Specifically, the costate variable is less than the shadow value of the initial state, seeing as the strategic effect is positive under the stated assumptions on the parameters *k* and *l*. Hence the costate variable overestimates the increase in minimum costs due to an increase in the initial state. This result has an important policy implication. For instance, in the resource extracting models, if the policy maker somehow wants to protect the stock of resource by regulating the extraction rates of the firms, he would probably overestimate the value of the stock if he interprets the costate variable as the shadow value in the conventional way. Therefore, the policy maker will over-regulate the extraction rates and hence reduce the welfare of the economy.

# Innovation processes

The problem in this section concerns the area of research and development. R&D activities are aimed at developing new technologies, production processes, or products. The game theoretic approach to R&D views innovations as being developed not in isolation, but in competitive environments. Sometimes one thinks of the R&D activities of competing firms as a race of being the first to reach a technological breakthrough. Dynamic game models of R&D often rest on the following three assumptions. First, no firm knows in advance exactly how much it must spend to develop the innovation. Second, there are several possible paths to the successful development of the innovation. Third, R&D activities are costly but lead to the accumulation of know-how, which influences positively the probability of winning the race.

## Innovation patterns and stages of development

Basically, according to Utterback and Abernathy [19] one firm can choose different patterns of innovation: performance maximizing, sales maximizing, or cost minimizing. Thus, a firm at one time may attempt to be the first to introduce technically advanced products (performance-maximizing), or to watch others innovate but be prepared to quickly adapt and introduce new product variations and features (sales-maximizing), or to enter the market later in the product life cycle with simpler and less expensive versions (cost-minimizing).

In their research Utterback and Abernathy [19] describe three different stages of process development which are referred as uncoordinated, segmental and systemic.

Stage I: uncoordinated process, product performance-maximizing strategy: A firm with a performance-maximizing strategy might be expected to emphasize unique products and product performance, often in the anticipation that a new capability will expand customer requirements.

Stage II: segmental process, sales-maximizing strategy: As an industry and its product group mature, price competition becomes more intense. Therefore, production systems, designed increasingly for efficiency, become mechanistic and rigid. Tasks become more specialized and are subjected to more formal operating controls. Innovations leading to better product performance might be expected to be less likely, unless performance improvement is easy for the customer to evaluate and compare. As obvious improvements are introduced it becomes increasingly difficult to better past performance, users develop loyalties and preferences.

Stage III: systemic process, cost-minimizing strategy: As a process becomes more highly developed and integrated and as investment in it becomes large, selective improvement of process elements becomes increasingly more difficult. The process becomes so well integrated that changes become very costly, because even a minor change may require changes in other elements of the process and in the product design. Process redesign typically comes more slowly at this stage, but it may be spurred either by the development of new technology or by a sudden or cumulative shift in the requirements of the market. If changes are resisted as process technology and the market continue to evolve, then the stage is set for either economic decay or a revolutionary as opposed to evolutionary change. As the product life cycle evolves product variety tends to be reduced and the product becomes standardized.

There are some important statements can be made about these three stages of development.

First, most innovations introduced by firms in Stage I will be original, while in Stage III most will be adopted (from material suppliers, equipment suppliers, by license, imitation, etc.)

Second, innovations introduced in Stage I will require little perceived change in process technology. Innovations introduced in Stage II will require the greatest degree of perceived change in process technology, while those in Stage III will result in only incremental and/or adopted process changes.

Third, costs of innovations introduced by firms in Stage I will be relatively greater than those for Stage II and will be lowest for firms in Stage III.

Fourth, most firms in Stage I will be relatively small while most firms in Stages II and III will be relatively large.

## Stochastic game of R&D competition

In game theory, a stochastic game, a dynamic game with probabilistic transitions played by one or more players, was introduced by Lloyd Shapley in the early 1950s. It has been widely applied to the timing of product and device innovations. Uncertainty is also inherent in the process of technological innovation: R&D expenditures will be engaged in an unforeseeable environment and possibly lead to innovations after a random time interval. Uncertainties generally affect the fundamentals of the standard differential game problem: discounted profit functional, differential state equations of the system, initial states.

There are some special features of economic applications of stochastic games: predictive power of models and simplicity. A key advantage of this approach is that it not limited to open-loop behavior.

Let’s have look on an interesting model introduced by Keller [4], which can be useful in our research later. In this game N firms have competing R&D projects. The dynamic game supposes that: no firm knows in advance the amount of R&D that must be invested, R&D activities are costly but contribute to higher accumulation of common know-how and then have positive externalities, one successful innovation may be achieved by using different paths. Resources in R&D positively influence the probability of successful innovations. Once a firm has won the competition, it acquires a monopolistic position.

The time to complete a project is a random variable, whose probability distribution . The random variables are stochastically independent, since knowledge is supposed to have no spillover between firms. , where the date of an innovation. Let be the rate of R&D effort. The rate of the distribution is assumed to be proportional to the effort

where is the survival probability and is the hazard rate.

We assume that and the game is played over a fixed and finite horizon T. The costs of R&D efforts are quadratic in the investment rate. This stochastic game belongs to the class of piecewise deterministic games. The objective functional of firm *i* is its expected discounted profit:

This expression consists of three terms, which weights are the probabilities. The first reflects the fact that firm *i*'s value of net benefits (in present value terms) is equal to if this firm succeeds in becoming the innovator. The second term shows that firm *i*'s value of net benefits (in present value terms) is equal to if this firm loses the innovation race. The third term represents the present value of the cost of R&D effort at time *t*. All R&D activities on the project are assumed to terminate at the instant of time where a firm makes the innovation. All three monetary components are weighted by their corresponding probabilities.

In the R&D game the players would condition their strategies on *y(t),* the state variable, which represents the aggregate stock of know-how. A single state equation is given by:

We look for a Nash equilibrium with open-loop effort strategies. To obtain the needed result we use Pontryagin’s maximum principle, which is also discussed in Section 1.4. Defining the present-value Hamiltonians, we have:

in which are present-value costate variables. Assuming that the equilibrium effort rates are strictly positive, the necessary conditions for Hamiltonian maximization provide the candidate strategies

where we note that a candidate effort rate depends only on the costate, and not on the state *y* itself. The costates must satisfy the differential equations:

Assuming, that these candidates act symmetrically (all firms use identical effort strategies), each candidate have a following strategy:

in which

## Stochastic game of patent race

In this section we present a market model introduced by Keller [4], where N incumbents and one entrant aim to invent a new product, and patent their innovations. In this model as well as in section 1.2 and 2.1 stochastic relationship is assumed between R&D investments and the time at which an innovation will occur.

The probability that the firm i succeeds at or before the date t is

The conditional probability that the firm i will succeed in the instant, given that it has not already succeeded is

Finally, let us suppose the following constant non-discounted profits: the incumbent i’s profit before any innovation occurs, the incumbent i’s profit if he winsthe patent race, the incumbent i’s profit if he loses the patent race to the entrant, the incumbent i’s profit if he loses the patent race to another incumbent,and the profit of the entrant when he wins the patent race.

Integrating (16) and (17), we have the system for the R&D subgame

Best response functions

The equation (18) is the incumbent i’s BRF with upward sloping19, given the R&D investments of his opponents. Similarly, (19) is the entrant’s BRF with upward sloping, given the R&D investments of his opponents.

The expected pre-innovation profit only occurs in the incumbent’s BRF. As a consequence to the shift, we observe with [Gayle, 2001] that the R&D spending equilibrium of theincumbent will be less than that the one of the entrant.

## Game with collusions

In a dynamic differential game decision maker can act independently or cooperatively in the pursuit of their own best interests. In the area of differential games, cooperative theory is far less developed than non-cooperative theory and almost all applications in economics and management science are in the non-cooperative setup.

Strategies based on action history have been extensively used in dynamic games of tacit collusion where the players use threats of punishment as a means to sustain collusive behavior. Collusion to secure monopolistic profits is in practice almost universally illegal, but history has seen a remarkably high number of such arrangements (concerning prices, product specifications, advertising, output limitations, market shares, and exclusive selling areas). The general structure of tacit collusion problems is as follows. It would be better if all players stick to their collusive strategies than if all players played non-cooperatively, but for each player it is better unilaterally to maximize his payoff (i.e., to cheat on the collusive outcome) if all the other players stick to their collusive strategies. Since actions are observable, the players will know whether cheating has taken place.

Cai and Rajan [16] consider a two-stage model in which two firms first invest in R&D to reduce their marginal production costs, and then either compete or collude in the output market. When they collude, they bargain over a cartel agreement to divide the collusive profit. If bargaining breaks down, they revert to duopolistic competition. For both a location model and a linear demand model, we show that firms invest more in R&D in the first stage under collusion than under competition. We demonstrate via example that social welfare may be greater under collusion than under competition in the location model.

In the first stage, the two firms simultaneously and non-cooperatively make cost-reduction investments. Investments determine marginal costs, which become publicly known at the end of the first stage.

To simplify matters and focus on the more interesting cases, we make several assumptions.

The first is that marginal production costs of both firms are constant in quantities. Firm i’s marginal cost depends on its investment , with (investment reduces marginal cost) and (diminishing returns of investment).

A second assumption is that firms have the same cost-reduction technology, so for the same I. By investing , firm i incurs investment cost of , where and . Since c(I) is strictly decreasing and hence invertible, we can think of firms as directly choosing marginal costs with the associated investment costs , where and . We assume that F satisfies the Inada conditions: , and , for some ∈ (0,R). To ensure concave profit functions, we further require F to be sufficiently convex, so that . for all c.

We now study the case in which the two firms choose investments non-cooperatively in the first stage, but collude in the output market in the second stage. Once investments are made and the cost structure is known, the monopoly profit is higher than the sum of the two firms’ profits if they compete against each other. Hence the firms have incentives to collude on the monopoly outcome. Whether collusion is incentive compatible, however, will depend on how the firms divide the net surplus from collusion. If they do not reach a collusive agreement, they will compete against each other. Thus no firm should get less than its competition profit in the collusion outcome. We use the Nash Bargaining Solution as the outcome of the surplus division, with the competitive outcome as the disagreement point.

Under collusion, if a firm reduces its marginal cost, it has two effects. First, it raises the monopoly profit. Second, by shifting its own disagreement point, it captures a larger share of the monopoly profit. This leads to an over-investment in cost reduction.

In conclusion, compared with oligopolistic competition, collusion in the output market can provide additional incentives (via the “strategic bargaining effect”) for firms to invest in cost-reduction. When this effect is strong, it is possible that overall social welfare is greater under collusion. This analysis provides another reason that one needs to be cautious in applying the conventional wisdom of competition policies to certain industries that may have strong strategic bargaining effects. We conjecture that such industries are likely to have the following features: (i) innovation investments are important and hard to monitor; (ii) marginal investment costs do not increase very fast; (iii) demand is not very elastic.

# Resource economy

In this chapter we present a number of models in resources and environmental economics in which economic agents (firms or countries) exploit natural resources or the environment in an intertemporal context, taking into account the strategic behavior of other agents. Another natural source of strategic dynamic competition is also due to technological progress. Here, technological progress is modelled as process R&D, with firms expending resources to lower their unit costs. These papers a priori postulate differential games with stochastic duration corresponding to the occurrence of a success in an R&D project that would lead to a patent.

Recent economic literature on growth posits that given non-ending technological progress, food production will continue to outpace demand for several centuries, ignoring natural resource limita- tions with optimistic views about the role of tech- nology in surmounting resource scarcity and environmental degradation

## Stochastic game of R&D and resource extraction

In this section we investigate a game combining the problem of determining optimal R&D efforts with the extraction of an exhaustible resource as described in Dockner, Jergensen, Van Long and Sorger [1].

In section 1.2 we have already introduced a stochastic game where the date of successful completion of the innovation by one of the oligopolists is a random variable with a probability distribution that is known to depend on the oligopolists' R&D efforts. In this modification, an importer of a nonrenewable resource seeks to develop a new technology the output of which can be substituted for imports of the nonrenewable resource. But in this innovation game we look for a Markovian Nash equilibrium. We can interpret the state variables as the firms' respective stocks of know-how, having been acquired through their R&D efforts.

The differential game is played between a country which exports a nonrenewable resource and an importing country which seeks to invent a substitute technology in order to become less dependent of the imports of the resource. The time of the importing country's innovation is uncertain but can be affected by the country's R&D efforts. A key issue in the problem is the fact that the resource exporter must take into account the incentives of the importing country to try to develop a backstop technology which can be substituted for the depletable imported resource. The resource extractor is assumed to be a monopolist supplier of the resource. The strategic interdependence between the two countries lies in the fact that the producer's extraction policy has an impact on the importer's innovation efforts, and vice versa.

Denote the resource-extracting country by *P* and the importing country (or a homogeneous group of importing countries) by *C*. If *C* did not have the possibility of inventing a substitute technology, an optimal depletion policy for *P* would follow the Hoteling rule of exhaustible resource extraction: marginal revenues should grow over time at a rate equal to the interest rate until the stock is depleted. The Hoteling rule is discussed later in Section 3.

Countries *P* and *C* seek to maximize their respective national welfare functions. The unit extraction cost of country P is constant and equals c > 0. Country C inverse demand function for the resource does not change over time and is denoted by p(q). Denote the extraction rate , where denotes a fixed upper bound on *P*'s extraction rate; the remaining stock *s* evolves according to the differential equation

Country *C* attempts to invent a technology that would be a perfect substitute for the exhaustible resource. The unit production cost to be incurred by the substitute technology is constant, equal to *b*. We assume that *b ≤ c*, which means should the backstop technology be invented, there will be no demand at all from country *C* for the resource. Denote by *u(t)* the R&D effort rate of country *C*.

The objective functional of country *P*, is expected profit

where we recall that country P receives zero profit for t > x (since demand vanishes). Assume that the instantaneous profit function n is strictly concave for . The objective functional of country C is expected welfare, given by

in which

is consumers' surplus (if imports are q) in country C. The function / in (10.30) represents country C's cost of R&D efforts. The latter assumption has the implication that zero R&D effort is suboptimal and for the strategy u we can confine our interest to interior solutions.

This differential game belongs to the class of piecewise deterministic games.. The game has two modes only: mode 0 is active before country C has made the innovation and mode 1 becomes active if C succeeds in making the innovation. Thus, as in the model of the previous section, there can be at most one switch of mode. The switching time is the random variable r and the probability distribution of r is given by F (cf. equation (10.28)). The game has only one state variable s, the dynamics of which are given by (10.27).

We look for a stationary Markov perfect Nash equilibrium. We need to construct a pair of value functions that are bounded and continuously differentiable, and that satisfy the HJB equations. The strategy pair (Q(h, s), U(h, s)) is a stationary Markov perfect Nash equilibrium (provided, of course, that value functions can be found that satisfy the HJB equations).

We have to resort to a qualitative analysis and wish to make this analysis in the payoff space, that is, in the (VP,VC) plane.

The following properties can be established.

(ii) The Vp = 0 isocline is upward sloping for 0 < Vp < 7tm/r and becomes vertical for Vp = 7tm/r. If Vp goes to zero, the isocline approaches the Vc axis.

(iii) The Vf = 0 isocline is upward sloping for V^\_ < Vc < Vc and goes to zero if Vc approaches V^\_. The isocline becomes horizontal at Vc = Vc.

Hence, both value functions are strictly increasing functions of the remaining stock. It can also be shown that the value functions are bounded and continuously differentiable for s e [0, oo). The equilibrium strategies are constructed by inserting the value functions so identified into (10.38) and (10.39). Finally, it can be shown that there exists a finite instant of time after which either country C has made the innovation (and some stock of resource is still remaining) or the resource has been completely used up.

## Games with nonrenewable resources

We begin with a simple model of exploitation of a common-property, nonrenewable resource such as an oil field. We compare the benchmark cooperative solution with a noncooperative open-loop Nash equilibrium and a Markov perfect Nash equilibrium. In the open-loop version of the game, we show that the nature of the solution depends on how we restrict the set of feasible strategies available to each agent. This is an important issue from the modelling point of view, especially in the context of common property resources, because this context dramatically highlights the interdependence among agents, not only in terms of payoffs but also in terms of what one agent can do given the actions of others.

Let x(t) and ct(t) denote respectively the stock of the resource and player f s rate of extraction at time t. We assume that ct[t) > 0 and that, if x(t) = 0, then the only feasible rate of extraction is ct(t) = 0. The transition equation is assumed to be

Each player / has a utility function wfe), defined for all ct > 0, which is strictly concave and increasing (we allow the case where w(0) = — oo). Utility is discounted at a constant rate r > 0, and player /'s objective function is

Given that all players have the same utility function, it seems natural to assume that, when they cooperate, the objective is to maximize the sum of the integrals of discounted payoffs. Let c(t) denote the rate of extraction of the representative player. The resulting optimal control problem consists in maximizing subject to

The Hamiltonian function ) from which we obtain the conditions u'(c(t)) -

Since the Hamiltonian is concave in (c, x), any feasible path {c\*(-), \*\*(•)» ^\*(-)} that satisfies conditions (12.3) and (12.4) is optimal if it also satisfies the transversality condition , (12.5) where x(t) is any feasible path.

It is easy to show that, along an optimal path, extraction is positive for all t and V(0 is positive and rises at the rate of interest: Condition (12.6) says that marginal utility rises at the rate r. This is known as the Hotelling rule. (Clearly, any path that does not exhaust the stock will be dominated by a feasible path that exhausts the stock, giving rise to more consumption over some time interval.)

The noncooperative case: We now turn to the noncooperative solution. An interesting question is whether the cooperative solution described above can be achieved as a Nash equilibrium of a noncooperative game. Thus, as we will see later, under the common access regime there may exist an incentive for each player to exhaust the resource stock in finite time, as everyone tries to grab a bigger share of the total stock. Whether this occurs or not depends on how severely we restrict the choice set of each player.

Theorem 12.2 Let 0 < r\ < 1 and assume that weakly feasible open-loop replies are permitted. Then each player has an incentive to deviate from the cooperative extraction path if and only if 1 — rj > l/N.

The intuition behind theorem 12.2 is as follows. Because 0 < r\ < 1, zero consumption does not cause utility to fall to minus infinity. Therefore, it may pay for a player (and hence, by symmetry, for all players) to deviate from the cooperative extraction path and exhaust the stock at some finite date Tt. In doing so, the player would capture that part of the stock that his opponents plan to consume after Tt. This would improve his payoff provided that the gains he would get from this 'theft' exceed the loss caused by having nothing to consume after Tt when marginal utility of consumption would be high. Clearly the gains are greater, the greater the number of players that are 'robbed'. This is why the cooperative solution can be supported as an open-loop Nash equilibrium only if N < 1/(1 - v)4

In view of theorem 12.2 the following question arises: given that 0 < r\ < 1 and N > 1/(1 — if), is there an open-loop Nash equilibrium that implies exhaustion of the stock at some finite time T > 0? The answer is no. Owing to limitation of space, we do not offer a formal argument here, but only sketch a proof. First, if a player / chooses a finite Th then player / s best reply must involve 7} < Tt. It follows that in any open-loop Nash equilibrium all players must plan to exhaust the stock at the same finite time, say T. Consider two cases: (a) the consumption path continuously falls to zero (i.e., limt^Tc(t) = 0), and (b) there is a jump discontinuity at T. Clearly (a) violates the Hotelling rule because marginal utility has a constant elasticity 77. As for (b), since each player's planning horizon (as distinct from planned exhaustion date) is infinite, and since each player i knows that his opponents stop extracting at time T, he can improve his payoff by rescheduling his consumption stream so that his consumption is always positive, without fear of being 'robbed' and without changing his total lifetime accumulated consumption.

Notice that with r\ G (0, 1) we have to assume D > 0 (which means that N is sufficiently small) to obtain a symmetric Markov perfect Nash equilibrium. If D < 0, then no such equilibrium exists. As theorem 12.2 and the ensuing discussion show, the condition D > 0 also ensures that the cooperative equilibrium is supported as an open-loop Nash equilibrium when players are allowed to choose weakly feasible open-loop strategies. Comparing the two equilibria, we see that the Markov perfect Nash equilibrium implies a faster extraction rate and a lower value Vt(x) for all players. The intuition behind this is as follows. In the Markov perfect equilibrium each player knows that, if he tries to be more conservationist, this will encourage other firms to extract more. Therefore there is little incentive to be conservationist. By contrast, in the open-loop equilibrium, given the other players' extraction paths, player f s effort to be conservationist does not induce the others to extract more.

After a thorough discussion, we consider some variations of the basic model of common-property nonrenewable resources: the so-called doomsday problem, and the problem where utility depends directly on both the stock of the resource and the flow of consumption.

### Doomsday problem

In this section we consider some variations of the basic model presented in the previous section. First, we look at the case where each player can choose his termination date, that is, the date at which, from his point of view, the game ends. This is sometimes called the doomsday problem.

This type of problem is sometimes referred to as the doomsday problem. This name suggests the gloomy picture of an individual or a community that has to decide on a time beyond which nothing matters any more. A slightly different interpretation is the case of a firm that dissolves itself when its business activities are declared completed.

In the preceding section, we assumed that the time horizon is infinite so that, if the resource stock is exhausted at some finite time T, then after time T each player's utility flow is w(0) (which may be positive or negative), and each player's total payoff is

We now consider a different formulation. More specifically, we assume that each player / can choose a terminal date Tt such that his utility flow stops at Tt and anything that happens after Tt does not count as far as he is concerned. His total payoff is therefore

Upon reflection, the result that the output of the mining firm, under the assumptions just stated, should be at the point of minimum average cost, rather than at the point where marginal cost equals price, is not surprising. To see this one should note the distinction between a resource-extracting firm and a 'normal' firm (i.e., the type of static firm considered in any first-year textbook in microeconomics). For a resource-extracting firm, one more unit of extraction (i.e., output) today means one less unit of extraction at some time in the future. This is not the case with the normal firm. The reader is invited to prove that if r > 0 (and p and F are constant) then the extraction c(t) falls over time, approaching cM as t approaches T.

### Stock-dependent utility

In many resource extraction problems, the utility function of each player depends not only on his extraction rate, but also on the remaining stock. For example, the stock may be a good proxy for the recreational value of the resource (think of a sand beach), or it may have a direct effect on a firm's profit (for example, the cost of harvesting fish may depend on the stock of fish).

The following example is a special case of a class of problems involving the long-term decline in effectiveness of a pesticide. Let x(t) denote the effectiveness of a pesticide and af(t) the rate of application of the pesticide by farmer i. Insects tend to develop resistance to the pesticide over time. To capture this phenomenon, we postulate that x(t) = —b Ylu=\ aiif) ^ x(i) > 0 and x(t) = 0 when x(t) = 0, where b is a positive parameter. We distinguish the nominal dose a((t) from the effective dose a((t)x(t) and we assume that farmer f s profit rate at time t is 7rz(7) = [aj(f)x(t)]a, where 0 < a < 1. The cooperative problem is to maximize

It is clear that the rate of decline in effectiveness of the pesticide in this Markov perfect Nash equilibrium is greater than the rate obtained in the cooperative solution. This result corresponds to similar results obtained in most models of noncooperative exploitation of a stock.

## Games with renewable resources

Renewable resources such as fish stocks and forests are considered next, at first in the standard format of simultaneous choice of Markovian strategies.

Let x(t) denote the stock of fish at time t, and ct(t) country z's rate of harvest of this renewable resource. The state variable x(t) evolves according to the differential equation where g(-) is the natural growth function which is assumed to satisfy the properties for some x > 0.

This implies that g(x) reaches a maximum at x > 0 and that there exists a stock level xM > 0 such that g(x) is negative for all x > xM. We call x the maximum sustainable yield stock level. The payoff to country is

where w(-) is a strictly concave and increasing function and r > 0 is the rate of discount.5 In what follows, we assume that g'(0) > r. This assumption means that the consumers are not too impatient and, as will be seen shortly, it ensures that the cooperative solution implies convergence of the stock to a strictly positive steady state level.

It is easy to show that the cooperative solution implies that the stock converges to a steady state level x\* that is the unique solution of the equation

Now consider the case where the countries play a noncooperative game. We assume u(c) = B + (I — rj)~lcl~v, where B is a constant (possibly equal to 0) and rj > 1 so that u(0) = — oo. With this utility function, no country has an incentive to drive the stock to zero in finite time, because to do so would result in a payoff integral of minus infinity. It follows from this fact and from the fact that the stock does not appear in the integral that the cooperative solution can be supported by a symmetric open-loop Nash equilibrium. However, we have pointed out that these equilibria are in general not subgame perfect. Let us therefore find a Markov perfect Nash equilibrium for this game.

This is to be expected. Everyone tries to capture more, because each one knows that if he captures less, this will tend to increase x in the future, which will lead others to capture more because the harvesting strategy has the property that ct>\x) > 0.

We now consider a modified fishery model by assuming that the utility function is linear (that is, u(c) = c) and that no country can harvest more than a maximum amount cm. This constraint may, for example, reflect the limited quantity of vessels. The natural growth function g(x) is assumed to have all the properties listed in the fishery model of the preceding subsection. In particular, it is assumed that there exists a value x\* > 0 such that g'(x\*) = r. Assume that N = 2 and that 2cm > g(x) (here x is the maximum sustainable yield), so that if both countries harvest at the maximum rate then the resource stock will be exhausted in finite time. The interpretation of this assumption is as follows. If one player, say player 1, chooses to harvest at the maximum rate cm, then no matter what player 2 does, the stock of fish will keep on declining to zero in finite time. In other words, player 2 cannot ensure a steady state with a positive stock when player 1 is greedy. Notice that, if player 1 plays this strategy, then player 2 faces a very simple control problem and, given assumption 12.1, it is easy to show that player 2's optimal exploitation is to set c2(t) = cm until the resource is exhausted.

### Common-Pool Resources Extraction game

Beginning with Gordon (1954) and Scott (1955), the common-pool resource problem attracted a lot of attention by economists. A classical common-pool resources extraction model is given by

s.t.

x(t) is the stock of the resource at time t, with the initial reserve ,

Suppose that the steady state of OLNE lies in the region that F’>0, the steady state comparative statics with respect to the discount rate are

They show that an increase in the discount rate reduces the symmetric steady state extraction rate and the stock of the resource, which is counterintuitive at first glance. For instance, how can the stock fall if the extraction rate has also fallen? An increase in the discount rate increases the symmetric steady state extraction rate and therefore reduces the stock of the resource

In Amir [12]: This is one of the areas of economics that has witnessed a high level of research activity involving stochastic/dynamic games as the key methodological approach. The seminal paper of Levhari and Mirman considers two agents noncooperatively exploiting a natural resource. Using standard induction, Levhari and Mirman showed for this “Great Fish War” that (i) for every finite horizon with end-period T, there is a unique Markovian equilibrium with linear consumption strategies and logarithmic value functions (ii) the limits of these strategies as constitute a Markov-stationary equilibrium of the infinite-horizon game (iii) a tragedy of the commons prevails in both cases, in that the given equilibria are not Pareto-optimal and lead to overconsumption of the resource stock (relative to a Pareto-optimal path), and (iv) the equilibrium resource stock converges to a unique globally stable steady-state level. Specifically, agents coordinate on cooperative extraction paths secured by the threat of reversion to the Markov-stationary strategies in case of defection.

### Tragedy of commons

We study in [5] the effect of environmental risk on the extraction of a common resource.

It is the purpose of this paper to consider whether the tragedy of the commons is reduced or exacerbated in the presence of environmental risk.

For state of the environment st, the resource available at the beginning of period t is determined by

We focus on two effects of environmental risk consistent with the scientific evidence exposed above: renewability and quality. Focusing on a completely exogenous event, the change in behavior is solely due to reducing exposure to risk (instead of altering it).

Using a dynamic and non-cooperative game in which an environmental event impacts both the renewability (the future quantity) and the quality of the resource, we show that the anticipation of such an event has an ambiguous effect on present extraction and the tragedy of the commons. On the one hand, a risk of a reduction in the renewability induces the agents to extract less in the present. On the other hand, a risk of a deterioration in the quality of the resource induces the agents to extract more in the present. In particular, when environmental risk induces conservation (when the risk of less renewability is more important than the risk of quality deterioration), there is a larger decrease in present harvesting under social planning than in the non-cooperative game, and the tragedy of the commons is worsened.

In view of our results, the reason for a change in behavior due to environmental risk is solely motivated by a reduction in the exposure to risk (and not manipulation of likelihood of risk as discussed above). The effect of uncertainty is shown to be ambiguous as well, and can lead to conservation. We show that when environmental risk induces conservation (i.e., when the risk of less renewability is more important than the risk of quality deterioration), the presence of the risk leads to a stronger decrease in present harvesting under social planning than in the non-cooperative game. Although agents choose to harvest less, they do not internalize the risk that too much extraction creates for others, and, thus, decrease their own extraction too little. The social planner, on the other hand, internalizes this effect and decreases harvesting more. This disparity in conservation leads to a worsening of the tragedy of the commons.

### Biological model of resource extraction

The model leads to global implications on the relationship between economic growth and the ability of modern societies to maintain the environment at a sustainable level.

A physiologically based population dynamics model [6] of a renewable resource is used as the basis to develop a model of human harvesting. The model incorporates developing technology and the effects of market forces on the sustainability of common property resources. The bases of the model are analogies between the economics of resource harvesting and allocation by firms and adapted organisms in nature.

Where M – mass of trophic level, D – maximal rate per unit mass, h(s) – proportion of demand acquired, v(D) – respiration cost rate per unit mass as a function of potential extraction rate, – conversion efficiency, α – unit of extraction, с – per unit mass cost spent on adaptedness to meet expected environmental hazard. Dh(c) – per unit capital harvest rate

Where x – resource, y – product, D – harvesting, g(x) – renewal rate, – wastage rate, , c – rate of consumption

Dh(s) – per capita harvest rate, yDh(s) – harvest by all firms, vDy – costs of production, Cy – Consumption.

In other words, maximizing the present value of benefits obtained from the consumption stream:

where b is an instantneous discount rate, U(C) – concave utility function, – discounting factor, – discount rate.

– value multipliers or costate variables

Necessary conditions: , ;



As equations are defined only for , we restrict attention to [0,∞] x [0,,)] in the (x, ) phase space.

– extinction level, – unexploited carrying capacity, – regeneration rate equals discount rate, – competitive solution, – optimal steady state resource level,

Specifically, the paper makes the following points:

(1) it shows how economic and ecological theories may be unified;

(2) it punctuates the importance of time frame in the two systems (evolutionary versus market);

(3) it shows, contrary to prevailing economic wisdom, how technological progress may be detrimental to resource preservation;

The technology parameter (a) determines what proportion of the resource can be exploited. In general, if a is sufficiently low compared to the maximum biomass regeneration rate of the resource, the resource will survive any size predator population and demand rate. This result is in sharp contrast to the conventional view that technological improvements are the driving force behind increased income per capita and supports population growth. As discussed above, increasing the technology of harvesting (a) can also drive a resource under public control to extinction, but this also depends on the discount rate d. If the parameter determines what proportion of the resource can be exploited, than its increase leads to reduction of Xc and increases the likelihood of driving Xe

(4) it shows how the anticipated effects of high discount rates on resource use can be catastrophic when synergized by progress in harvesting technology;

The decrease in the population of human firms is mitigated by the effect of increases in the discount rate that raise the individual’s rate of resource exploitation. The lower bound xd on the societal solution suggests that if the discount rate (d) is sufficiently small, technological improvements should not drive a publicly regulated resource to extinction. Reduces the societal steady state solution Xs, but the level of Xc is unaffected. The decrease in the number of firms increases discount rate that raises the individual's rate of resource exploitation.

(5) it suggests that increases in efficiency of utilization of the harvest encourages higher levels of resource exploitation; and

Higher implies lower wastage and for competitive solution may drive the resource population quickly to extinction. Increasing u does not affect xd, which is the lower bound on the societal solution. The competitive solution moves to the left, so that reducing waste has a similar effect to that of improving technology. In a competitive framework, the resource level may be driven closer to extinction. Despite a higher u, their a is typically low, hence regardless of their perception of environmental uncertainty, efficient utilization of resources may have little impact on the renewable resource.

(6) Increase of v reduces /| 1 shadow costs and the payoffs consequently reduces the optimal population density.

Also competitive solution moves to the right, but Xd is not affected. An increase of n reduces l1 and the payoff and consequently reduces the optimal population density. Note also that the competitive solution xc moves to the right and away from extinction level (Fig. 2e), but xd is not affected. As n approaches u, xs approaches xu and marginal gains fall (i.e. it is no longer cost effective to harvest) and suggests that the societal level xs increases with n. From a policy point of view, one may interpret an increase in n as a tax levied on harvesting capacity, suggesting effective policy implications for preserving resources.

(7) it shows the effects of environmental degradation on consumer and resource dynamics.

The resource base for x is often eroded by human activities including those used to harvest the resource x. Decreasing the resource base parameter (M0) has the obvious effect of shifting x; to the left and l:1 downward, implying lower competitive and societal solutions. In addition, the payoff and optimal population density increase (Fig. 2f). Clearly, there are synergistic effects of reduced environmental carrying capacity and factors enhancing over exploitation increase the likelihood of resource extinction.

To sum up, with increasing technological process rate, or wastage rate, or resource base increasing the amount of resource produced, which leads the resource closer to extinction. On the contrary, with increasing discount rate and maintenance costs happens the opposite – the amount of resource produced decreases, which leads the resource away from extinction point.

Competitive markets have failed to provide an appropriate mechanism for pricing resources with free access (Gordon, 1954; Hilborn et al., 1995), and consequently, over exploitation of resources has been a common practice in forestry an fisheries because the cost of renewable resources is largely neglected by harvesters.

The golden rule of economically balanced growth (that maximizes the steady state per-capita consumption) stipulates that the rate of saving associated capital accumulation is obtained when marginal productivity of capital is equated to population growth and capital depreciation rates. However, our model shows that as wastage is reduced, the optimal steady state level of the resource is reduced, again contradicting common wisdom. This effect increases with increasing discount rate, and is similar to the synergistic effect between technology and discount rate. This occurs because the payoff for the firms increases following better technology and lower wastage. Increases in the cost to firms detract from growth, countering gains from decreases in wastage, and thus leading to higher resource levels. A way of increasing cost is the use of Pigouvian taxes on firm capacity (D) suggesting interesting policy implications. Of course, restricting harvests may be efficient but often hard to enforce as harvesters find ways to circumvent regulations.

Our model points to the need to simultaneously control technology and the discount rate. It is obviously impossible, nor is it desirable to regulate the advance of technology, hence the major option left is to reduce the discount rate below the market equilibrium and also to regulate the harvest. Regulation of harvest can be accomplished via a Pigouvian tax on the capacity of the firm which in our model drives down firm numbers, but leaves open the question of increasing size of the remaining firms. Clearly, ecology and economics are at a crossroads of conflict: the alternatives are sustainable renewable resource management based on sound biology, or will over exploitation and mutual annihilation result as we scramble for ever decreasing resources.

## Competition for scarce resources

In particular, we characterize certain benchmarks and solve for the unique equilibrium of the second-period subgame (Cournot competition with capacity constraints).

Denote the total available capacity by K, and the capacities of the firms, determined in the first-period auction (or through efficient Coasian bargaining), by k, where

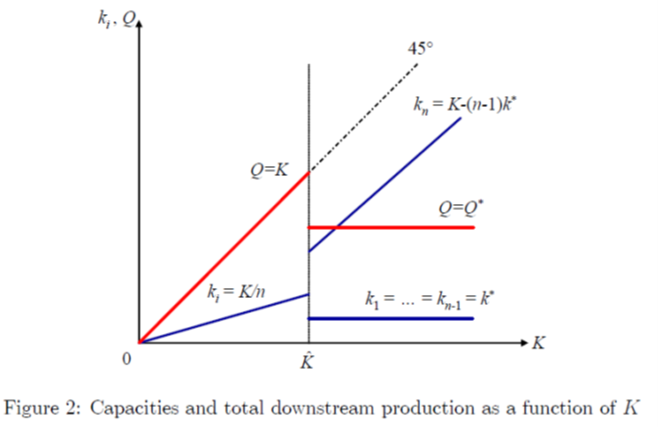
Denote the inverse demand function in the downstream market by P(Q), where Q is the total production. We assume that P is twice differentiable, and that both P(Q) and P’(Q)Q are strictly decreasing for all Q > 0. Firm i’s cost of producing q ≤ k units is c(q), while its cost of producing more than k units is infinity. We assume that c is twice differentiable, strictly increasing, and strictly convex. Finally, we assume that producing a limited amount of the good is socially desirable: P(Q) – c’(Q) is positive for Q = 0, and negative as Q → ∞.

We can write the profit of firm i in the downstream market for quantity q ≤ k and total output from firms other than i, , as

We show that the eﬃcient allocation of production capacity can turn a competitive industry and downstream market into an imperfectly competitive one. Even though downstream firms have symmetric production technologies, the downstream industry structure will be symmetric only if capacity is suﬃciently scarce. Otherwise it will be asymmetric, with one large “fat” capacity-hoarding firm and a fringe of smaller “lean and fit” firms, so that Tobin’s Q varies inversely with firm size. As demand or input quantity varies, the industry may switch between symmetric and asymmetric phases, generating predictions for firm size and costs across the business cycle. Surprisingly, an increase in available capacity resulting in such a switch can cause a reduction in total output and consumer surplus.

Starting at the threshold with symmetric firms (which is the socially most efficient point), an expansion in demand generates a rise in the price of output, but the industry remains symmetric. By contrast, a contraction in demand causes all firms but one to shrink their output, while the remaining firm absorbs the excess capacity.

In summary, the efficient capacity allocation is either asymmetric with exactly one firm receiving excess capacity and the others all receiving k\*, or symmetric with all capacities binding in the downstream market. Note that the asymmetric outcome can arise as the solution only when it is feasible, that is, when K ≥ Q\* ≡ (n − 1)k\* + r((n − 1)k\*). If K < Q∗ then we know the efficient capacity allocation is symmetric, k = K/n for all i.



Total production is lower than the Cournot output K\*. However, the total surplus falls discretely as K exceeds K\*. This is because the firms’ total profit is continuous at K = K\* by equation (8), but the consumer surplus falls discontinuously together with the total output in the downstream market. The policy consequence is that a social planner should restrict the quantity sold in the capacity auction to K\* whenever K exceeds K\*.

In particular, we show that a small slump in demand may lead to a relatively large drop in the downstream production, and that the industry becomes more asymmetrical and concentrated during a contraction than it is during a demand-driven expansion

More surprisingly, it shows that trying to increase input availability can easily be a misguided policy measure in such markets– even though firms face binding capacity (or input) constraints, an increase in input availability will lead to a reduction in output if it leads to a change in industry structure. Rather than encourage entry into the upstream market, it might be preferable to change the method by which the input is allocated.

### Auction

Instead, the assumption reflects the fact that in practice, when inputs are allocated among producers (either via an auction or bargaining among the firms) the consumers of the final good are not present, and their interests are not represented.

In the context of our model, a Vickrey-Clarke-Groves (VCG) auction with bids that are contingent on the entire allocation constitutes an efficient auction. This mechanism works as follows. Participants are requested to submit their monetary valuations for all possible allocations of the goods. The auctioneer chooses the allocation that maximizes the sum of the buyers’ reported valuations. Then, each buyer pays the difference between the other buyers’ total valuation in the hypothetical case that the goods were allocated efficiently among them (excluding him) and in the allocation actually selected by the auctioneer. The rules induce all participants to submit their valuations for every allocation honestly, and the outcome of the auction is efficient

There are other auction forms–simpler and more widely used in practice–that also yield an efficient capacity allocation in the context of our model. In particular, the uniform-price share auction (first analyzed by Wilson (1979)) is one such mechanism.

Dynamic auctions, where each unit of capacity is auctioned off separately over time, may be good candidates for such mechanisms. Suppose, for example, that the total capacity to be sold is divided into small units. At each point in time, one “unit” of capacity is sold at a second-price auction, with no discounting between periods. At first glance, it may seem surprising that this mechanism does not yield the same “efficient” result as the ones considered in Section 4.1. In fact, if K is sufficiently large and there are constant returns to scale, then the dynamic auction proposed above is socially more desirable than those auctions. The brief intuition for this result is the following. Under constant returns to scale, an “efficient” capacity auction would allocate all available capacity to one firm. In order to get the same result in the dynamic auction, one firm would have to outbid all the others for each capacity unit, and pay the marginal profit of the first capacity unit every time. Since the marginal profit of capacity is decreasing, this is unprofitable for the large firm and monopoly cannot be sustained. It is an open question whether a dynamic auction would do socially better than the mechanisms studied in Section 4.1 under decreasing returns to scale.

### Lotteries

Since these lottery formats are common at the state and federal levels, examining their optimal pricing strategies may have great policy implications.

Allocating resources through lotteries has primarily been viewed as fair because each participant has the same chance of being drawn. A common assumption in these literatures is that the prices of entry and awarded units of the good are zero, despite the fact that empirical observation reveals that agencies usually charge positive prices in administering lotteries. An individual’s decision to enter the lottery is based on the expected utility theorem, and she will choose to enter if and only if the expected payoff is nonnegative.

This chapter deals with a class of lotteries in which the allocation rule is uniform and payments are anonymous. That is, each entrant has the same chance of being awarded a unit, and the payments only depend on the realization of the lottery and not on any personal information about the participants. This class of lotteries is referred to as regular lotteries

From the above definition, it is easy to show that the expected payments for individuals in all regular lotteries are the same. As discriminatory lottery mechanisms can be constructed, the regular lottery provides a benchmark from which to evaluate these alternative mechanisms.

Lemma 3.1. For any regular lottery, if an individual with private valuation v enters the lottery, then all individuals with private values greater than v also enter the lottery.

Once the number of entrants is known, the expected revenue (R) and the expected consumer surplus (CS) can be defined as

The optimal lottery price   will depend upon the pricing arrangement chosen by the agency. If the lottery format is known, then the unique optimal (uniform) price may be determined. To illustrate, consider two widely used uniform-price lotteries: the user-pay (UP) lottery and all-pay (AP) lottery. The UP lottery is characterized by a tariff P that is incurred solely by the Q entrants who are drawn, while the AP lottery is characterized by a nonrefundable tariff T that is paid by all entrants. Equation indicates that optimal prices of AP lotteries increase as the available quantity of the good increases. While this may appear counterintuitive at first glance, there is a straightforward explanation. Namely, when the available quantity of the good increases, more individuals are willing to enter the lottery in AP lottery, and the agency may be able to increase the price to screen the lower value participants to maintain efficiency.

However, different lottery formats may have significant differences with respect to Pareto implications or the wealth distribution. For example, unsuccessful participants in AP lottery will suffer losses, and hence are worse off, while no one gets hurt in UP lottery.

# Model formulation and computation

The basic assumptions for our dynamic model are:

We have several growing for firms which leads us to a market structure known as oligopoly. These firms could have grown faster and better if they were not facing supply constraint. All firms produce only one good, which is a scarce resource itself. Firms can enter and exit freely. Collusions do not take place in the interest of simplicity.

The resource is allocated between firms by the auction run by government, which determines the amount of a good that each firm can produce. That means the constraint is basically the sum of all of the firms capacities or total available capacity.

These firms could have grown faster and better if they were not facing supply constraint. That fact makes them look for a way to change market outcome by altering different parameters, which leads us to the second research question.

To sum up, in our model we have n growing firms, that produce one product under supply constraint and can exit market freely. Government allocate scarce resource between firms and prohibits collusions.

“Which decisions each producer has to make while running a firm producing under resource scarcity?”. Each firm can use following strategies in response to changes.

We came up with six basic strategies:

Increase capacity, Increase resource base, Lower consumption, Lower maintenance costs, Increase technological "progress rate", Sell excess capacity or exit

1. Increase capacity by reallocation (buying additional capacity from other firms ) If firm is willing to increase its capacities, it can buy that additional amount from other firms.

2. Invest in increasing of resource base

3. Lower consumption C by altering discount rate or wastage rate : Consumption can be altered to increase capacity available for future production. Though it can lower current profits. Demand restrictions to increase capacity available for future production. Strategy is used by social planner, basically government, to regulate markets with extremely scarce resources (like water), the shortage of which can lead to social insecurities.

4. Lower maintenance costs :To lower the costs of production of each unit of a good a firm has to look for new technologies which will allow them to replace a natural resource with a cheaper and easier to produce alternative, or to alter exchange rates of the Production Cost function. Obviously, the less the costs, the better a firm can grow.

5. Increase technological

a. Investments in increasing of , if

b. Investments in technology application, if

A firm can invest in its own innovations or to copy the innovations that were made by its rivals. The first strategy leads to a process known as learning-by-doing. This influences a system positively, but innovations cannot keep improving the system at same high rates, so innovation rate depends positively on the amount of investments, but is a concave function. The second strategy in dynamics leads us to a situation, where after a successful innovation of a firm all the other firms will adapt the technology. That leads all firms of the system to have the same innovation rate. After that a leader of a previous period has to decide whether to continue investing in innovations, which will lead to its even bigger growth or to stop there.

6. Sell excess capacity or exit

Additionally some words on collusions and cartelization: Increasing powers to alter the market price; Cooperative investments; Altering allocation; Strategies lead to a different market outcome closer to monopolistic, but are omitted for simplicity.

Which model represents decision-making in the market for scarce resources? It is differential game model.

Since we have n players with several strategies concerning price, quantity of production and amount of investments, we can create a system of differential equations that set the model.

When several agents are involved in a model and they act independently or cooperatively, continuous-time dynamic optimization models of these agents are regarded as differential games. Each agent, or player, has its own objective to be maximized. I have special packages of commands in Wolfram Mathematica, which allows me to obtain approximate solutions of differential games and demonstrate the contribution of each economic agent.

To solve a problem of dynamic optimization approximately we can apply the commands from Wolfram Mathematica. They can help us deal with dynamic maximization, obtaining Pareto optimal solutions or open-loop Nash equlibrium solutions.

Thus, we came up with an economic model, which represents decision making in the market for scarce resource by finding optimal solution maximizing present value under influence of costates

To begin with we are considering differential duopoly game over continuous time t∈[0;∞), with a-la Cournot competition. All firms sell completely homogeneous product, which is made out of scarce resource. At any instant firms choose the quantity level and the investment level in process innovation. Process innovation is formalized as a reduction in the unit cost of production, while expansion of resource base affects the amount of resource available for producing.

The demand system operates as follows:

p\_i (t)=a-b[q\_i (t)+s∑\_(j≠i)▒〖q\_j (t) 〗]

Where a, b are positive parameters, q is the amount of a good produced by a firm i. Since i,j={1,2} and the goods are perfect substitutes, therefore, s(t)=1, we have:

p\_i (t)=a-b[q\_i (t)+q\_j (t)]

Instantaneous profits are given by

π\_i (t)=[ a-b(q\_i (t)+q\_j (t))]\*q\_i (t)-c\_i (t)\*x\_i (t)-β〖[k\_i (t)]〗^2

Where c is the cost of producing q from x units of resource by a firm i and q\_i (t)=θx\_i (t), θ is wastage rate, k indicates the effort made by firm i to reduce the cost of production and β is a positive parameter, which is a reverse measure of process innovation since investing in any type of R&D implies decreasing returns to innovative activity.

We assume that i aims at maximizing the discounted profit flow: П\_i (t)=∫\_0^∞▒〖π\_i (t) 〗 e^(-rt) dt, where

(dc\_i (t))/dt=c ̇\_i=c\_i (t)[-k\_i (t)-〖δk〗\_j (t)+η]

Where δ∈[0;1] measures the positive technological spillover that firm i receives from the process innovative activity of the rival, while η∈[0;1] is the depreciation rate, assumed to be costant over time and common to all the firms.

Rate of change of i's marginal cost over time is linear: c ̇\_i/(c\_i (t) )=-k\_i (t)-〖δk〗\_j (t)+η

The instantaneous cost of investing in process innovation is C\_i (k\_i (t))=β〖[k\_i (t)]〗^2

Next, we have to verify that the candidates constitute an equilibrium by, for instance, assessing concavity of the maximized Hamiltonians. This is trivial in our case because the Hamiltonians are linear in the state y and the candidate strategies do not depend on y, which allows us to conclude that the maximized Hamiltonians are concave in the state variable y.

The corresponding Hamiltonian function writes:

The initial condition is c(0) = c0 ∈ (0;a] .Firm i’s first order conditions (FOCs) on controls are:

Lemma 2 The open-loop Nash equilibrium of the game with firms investing only in process innovation is subgame (or Markov) perfect.

The first derivative in (49) shows that the optimal R&D effort for process innovation is non-monotone in the level of the marginal cost, with R&D efforts initially increasing as c(t) departs from c0 (provided c0 ∈ (a/2, a)), and then decreasing as soon as c(t) goes below a/2. The remaining two properties in (49) are intuitive, since (i) the higher is the substitutability between products, the lower are gross profits at any instant; as a consequence, internal funds for process R&D shrink; and (ii), steady state marginal cost is positively related to substitutability, which replicates the property we have already highlighted in the previous subsection concerning product innovation. As for (i), also in this case a higher level of substitutability decreases the available funds for financing process innovation R&D.

We have considered horizontal product innovation that reduces product substitutability, and process innovation reducing marginal production cost. The solution concept we have adopted is the closed-loop Nash equilibrium.

[7]

# Discussion and conclusion

To sum up, we’ve been able to create a model of a market for scarce resources, which can be set by a system of differential equations, which helps with computing of the optimal solution for each firm and shows the effects of different parameters, that determine that model.

Further researches in that field will allow me to solve dynamic optimization problem for firms in not only in dynamic deterministic, but also stochastic market which operate under resource scarcity. Than I will examine, how government can alter markets outcome, is total social surplus maximized and whether collusion and cartelization will change it. Finally, I am going to apply that theory to the market of water, which differs from any scarce resource market by social weight and more controlling government policies.

Food and water shortages are two of the greatest challenges facing humans in the coming century. Our goal in this essay is to advance identifying, theorizing, and testing a broader anthropology of resource insecurity.

Water is definitely physically scarce in densely populated arid areas, Central and West Asia, and North Africa, with projected availabilities of less than 1000 m3/capita/ year. [23]

Agriculture is responsible for approximately 70% of water withdrawals, but 90% of the water consumption (FAO, 2014). This account of water abstraction for satisfying agricultural needs goes up to 95% in developing countries. [25]

Often investment in technological improvements has incurred higher water prices, however, without gaining the full potential benefits through water efficiency. Farmers can better use technological systems already installed, adopt extra technologies, enhance their skills in soil and water management, tailor cropping patterns to lower water demand and usage, reduce agrochemical inputs, etc. [22]

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