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Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange

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Abstract. This paper examines the importance of the regular patterns in the behavior of electricity prices, and its implications for the purposes of derivative pricing. We analyze the Nordic Power Exchange's spot, futures, and forward prices. We conclude that the seasonal systematic pattern throughout the year, in particular, is of crucial importance in explaining the shape of the futures/forward curve. Moreover, in the context of the one and two factor models analyzed in this paper, a simple sinusoidal function is adequate in order capture the seasonal pattern of the futures and forward curve directly implied by the seasonal behavior of spot electricity prices.

Keywords: energy derivatives, electricity futures, seasonal effects.

JEL classification: G13

A growing number of countries worldwide, including the US, have recently undertaken restructuring processes in their electric power sectors. Although the speed and scope of the reforms varies across countries, such liberalization processes have been based on opening the electricity systems to competition wherever it was considered to be feasible, notably generation and retailing activities. The deregulating processes have been accompanied by the introduction of competitive wholesale electricity markets, and power derivative contracts, both OTC and exchange-traded, providing a variety of contract provisions to meet the needs of the electricity market participants. In the US, electricity futures and options contracts have been listed in recent years by the Chicago Board of Trade, the New York Mercantile Exchange, and the Minneapolis Grain Exchange.

Electricity may be considered as a *flow commodity* strongly characterized by its very limited storability and transportability. Both limits to the possibilities of "carrying" electricity across time and space turn out to be crucial in explaining the behavior of electricity spot and derivative prices as compared to other commodities. In other words, arbitrage across time and space, which is based on storability and transportation, is seriously limited, if not completely eliminated, in electricity markets. If the links across time and space provided by arbitrage break down, we would expect spot prices to be highly dependent on temporal and local supply and demand conditions. The limits of the arbitrage are also expected to affect decisively the relationship between spot and derivative prices.

The non-storability of electricity makes electricity delivered at different times and on different dates to be perceived by users as distinct commodities. In other words, prices are strongly dependent on the electricity needs (demand) and their determinants in every precise moment (this is to say, business activity, temporal weather conditions, and the like). Distinguishing between *on-peak* and *off-peak* electricity prices, or among prices corresponding to different time periods, such as seasons, is indeed important in power markets (such distinctions determine, for instance, derivative contractual terms). The non-storability of electricity is also likely to affect derivative pricing significantly, notably influencing on the shape of the forward curve and its behavior.

Transportation constrains for electricity come in the form of capacity limits in the transmission lines and transportation losses, which can make impossible or uneconomical the transmission of electricity among certain regions. These limitations make electricity contracts and prices highly local, i.e. strongly dependent on the local determinants of supply and demand (such as characteristics of the local generation plants, and local climate and weather conditions together with their derived uses of electricity).[1]

The number of papers addressing the specific valuation problems of electricity derivative contracts is still scarce. Several papers have pointed out some general characteristics of the power price behavior that should be considered, in their opinion, for the purposes of electricity derivative pricing. In particular, some have argued that a model for electricity prices should incorporate a form of time-varying volatility, and the possibility of jumps in prices (see Kaminski (1997), Eydeland and Geman (1998), and Deng (2000)). Others, on the contrary, have stressed the importance of the periodic seasonal behavior of electricity prices, and its reversion to mean (possibly non stationary) levels (Pilipovic (1998)). Nevertheless, the much needed empirical work is on its early stages. Only very limited and tentative work has been published to date (see Kaminski (1997), and particularly Pilipovic (1998)), and the bulk of the research on modeling electricity spot and derivative prices remains to be done. Probably this is due to the fact that competitive electricity markets and exchange-traded derivatives are relatively new and, consequently, long histories of liquid spot and exchange-traded derivatives prices do not exist.

This paper is mainly concerned with the importance of the predictable component in the behavior of electricity spot prices and its implications for derivative securities valuation. The value of a derivative security is the risk-adjusted discounted expected value of its future electricity-contingent payoffs. To the extent that these payoffs strongly depend on the future level of the electricity price and this has a clear predictable component, this predictability should be taken into account when making the expectations and consequently translated somehow to derivative prices. Of course, this would be the case only if such predictable component is a genuine feature of the underlying price behavior, as it is expected to be the case for electricity, and if it is sufficiently important. Indeed, genuine systematic behavior of electricity prices through time can be explained, for instance, by changes in demand following business activity, and the periodic behavior of consumption arising from the seasonal (average) evolution of temperatures.

We try to circumvent the indicated problems on the lack of historical time series for electricity prices by using data from one of the oldest spot and futures electricity markets in the world, the Nordic Power Exchange, Nord Pool ASA. The Nord Pool is a non-mandatory market for electric power contracting that has its roots in the liberalization of the electricity sectors in the Nordic European countries started in Norway with the Electricity Act of 1991. Thus, it has a history of spot and derivative contracting, with accompanying increasing trading volume, which can be traced back to the early nineties.[2]

The balance of the paper is organized as follows. In Section 1 we review the basic features of electricity contracting via the Nord Pool in both the physical and the financial markets. In particular, we explain the so-called *system price* underlying the derivative contracts. In Section 2 we provide a thorough analysis of the behavior of the system price in the Nord Pool. We stress the importance of several systematic patterns. We also provide evidence about a seasonal periodic pattern in the term structure of futures prices. In Section 3 we propose several one and two factor models for electricity spot prices, in light of the results of the previous section. We take into account the systematic effects by the inclusion of a deterministic component in the assumed models for the underlying spot price. All the models considered allow for closed form solutions for forward and futures prices. Section 4 presents and discusses the results of the estimation of the one factor processes for the underlying spot price, using electricity system price data from the Nord Pool. In Sections 5 and 6 we analyze actual futures and forward prices in the Nord Pool by comparing them with model prices. Section 7 concludes the paper with some final remarks.

1. Nord Pool—The Nordic Power Exchange

During the nineties, the Nordic European countries started a gradual deregulating process in their respective electricity sectors based on opening them as much as possible to competition. In the resulting Nordic electricity system, the transmission network is owned and operated by a number of independent transmission system operators, whose activity is subject to regulation and control by public authorities. This guarantees a non-discriminatory access to the grid to all market participants in the new electricity market.[3] The new Nordic wholesale electricity market combines both over the counter bilateral contracting and trading via the Nordic Power Exchange, Nord Pool ASA.

Established in January 1993, and first covering only the Norwegian market, the Nord Pool is currently a non-mandatory common multinational market which also includes Sweden since January 1996, Finland since June 15, 1998, and the western part of Denmark (Jutland and Funen) since July 1, 1999.[4] Basically, Nord Pool organizes two markets, a "physical market" (*Elspot*) and a "financial market" (*Eltermin* and *Eloption*), and also provides with clearing services.[5]

Elspot is a "spot" market where day-ahead electric power contracts are traded for physical delivery for each one of the 24 hours during the following day. Every contract in

Elspot refers to a load, in megawatt-hours (MWh, 1 MWh equals 1,000 kWh), during a given hour, and a price per MWh. A price, called the *System Price*, is fixed separately for each hour for the next day, based on the balance between aggregate supply and demand for all participants in the whole market area (the so-called Nordic Power Exchange Area), without considering capacity limits ("bottlenecks") in the grid among countries. A system price can thus be defined as the market clearing price at which market participants trade electricity for the entire exchange area when no transmission constrains apply. It is also used as reference price in settlements at the Nord Pool's financial market.

The respective national system operators have established different methods for handling bottlenecks situations (i.e. situations when the required electricity flow between any two given areas exceeds the capacity limits of the transmission lines), depending on the specific involved areas. Bottlenecks between any two countries as well as internal bottlenecks in Norway are all managed using the pricing mechanism in the spot market, implying price adjustments for the involved areas. In essence, when the required power flow between two or more areas exceeds the capacity limits two or more zonal prices are calculated besides the system price.[6] Internal bottlenecks in the other countries are managed directly by the national grid operators, and the cost of the regulation is financed through tariffs for power transmission.

An electric power system must be continuously balanced. In order to handle any unpredictable differences between the planned and the real exchange during delivery, once the Elspot market is closed, the national system operators have additionally set up *regulating* or *balance markets* from which the required upward or downward regulation is obtained on short notice.[7]

The Nord Pool's financial market, also known as the Eltermin and Eloption, allows trading in financial contracts such as forward and futures with delivery periods up to three years in advance. Since September 1995, none of these contracts entails physical delivery, they are all settled in cash against the system price in the spot market. They refer to a base load of 1 MW during every hour for a given *delivery period* of one day, one week, one block (four weeks), one season, and one year that may be available for trading depending on the type of contract. European and Asian style option contracts are also available for trading since October 29, 1999.

During the trading period analyzed in this paper, futures contracts with delivery periods of one season were available for trading up to three years in advance. There were three season contracts during a given year: Season 1 (with delivery period including weeks 1 to 16 of any given year), Season 2 (weeks 17 to 40), and Season 3 (weeks 41 to 52/53). They were available for trading until the beginning of the second previous season. Then, the season contracts were split into 3 to 6 block contracts.[8] Each block contract has a delivery period of 4 weeks (5 weeks for the case of the last block for a year with 53 weeks), and each one is available for trading until the beginning of the delivery period of the previous block. Then, they are split into weekly contracts with delivery period of one week each, which stop trading before the beginning of their respective delivery periods. Additionally, futures contracts with a delivery period of one day are also available for trading several days in advance until the day before they are due for delivery.

Annual and seasonal forward contracts are traded in the following way. Annual forward contracts with delivery period corresponding to the entire calendar year are available for trading till two trading days before the beginning of the delivery period. Seasonal contracts in turn are available for trading until the beginning of their respective delivery periods. There are three seasonal forward contracts for each annual period, namely: Winter 1 (with delivery period Jan. 1–Apr. 30), Summer (May 1–Sept. 30), and Winter 2 (Oct. 1–Dec. 31).

Exercising a European option results in opening a position in the underlying forward contract. Only options with underlying season and year forward contracts had been listed to date. Asian contracts, on the contrary, are settled in cash at the end of each settlement period against the arithmetic average system price in Elspot during the settlement period. There can be more than one settlement period for each Asian option, depending on the reference underlying contract. Only Asian options on block futures contract with settlement period corresponding to the delivery period of the reference futures contract have been available for trading to date.

Through its clearing function, currently conducted by a separate business area called the Nordic Electricity Clearing (NEC), Nord Pool guarantees settlement and delivery of all trades made at the market, by entering into the contracts as a legal conterparty for both the buyer and the seller. NEC also offers clearing services of standardized OTC bilateral financial contracts registered in the market for that purpose.

2. System Price Description

The Elspot data set was obtained from the Nord Pool's FTP server files, and consisted of twenty-four time series, one for each hour during a day, of daily (seven days a week) system prices, in Norwegian kroner (NOK) per MWh with two decimal points, for the seven year period starting January 1, 1993 and ending December 31, 1999.[9] The twenty-four price time series turn out to be highly correlated pair-wise. The linear correlation coefficients between any two hourly series during the sample period lie all above 0.94 (with a mean value of 0.98), and are always above 0.98 for any two consecutive hours.

The Nord Pool uses the arithmetic average of all hourly prices for a given day as the reference price in the cash-settlement calculations at expiration for the Eltermin derivative contracts. In accordance to this practice, we generated a new time series for this underlying variable by calculating the arithmetic average of the 24 available data for each day. We will refer to this average price as *the* system price or the spot price from now onwards. Table 1 reports the descriptive statistics for the average price and other related time series, as indicated therein, and Figure 1 plots them for the complete sample period.[10]

A first casual look to the average price series in Figure 1(a) reveals a quite erratic behavior of the system price. With a mean value of 142.57 NOK, it reached maximum and minimum values of 423.38 and 14.80, respectively, during the sample period. The highest price was reached during cold days around the Olympic Winter Games held in Lillehammer at the end of February 1994, and the second highest one during the very dry and cold year 1996. Nonstationarity tests have been conducted for both the price and

Table 1. Descriptive Statistics for the Daily System Price and Other Related Time Series (1993–1999)

Series	Autocorrelation Coefficient of Lag 1	2	3	4	5	7	14	21	28	35	ADF
					Panel A: All Seasons						
P_t	0.987	0.974	0.965	0.956	0.950	0.944	0.905	0.865	0.830	0.795	−2.901
$P_t - P_{t-1}$	0.007	−0.137	−0.033	−0.102	−0.117	0.315	0.323	0.288	0.268	0.264	
$\ln P_t$	0.982	0.962	0.949	0.939	0.933	0.931	0.874	0.830	0.791	0.740	−3.052
$\ln P_t - \ln P_{t-1}$	0.060	−0.184	−0.090	−0.112	−0.176	0.410	0.376	0.339	0.305	0.310	

Series	Number of Observations	Mean	Median	Minimum	Maximum	Standard Deviation	Skewness	Kurtosis
				Panel A: All Seasons				
P_t	2,556	142.569	132.070	14.800	423.380	66.370	0.755	3.540
$P_t - P_{t-1}$	2,555	0.006	−0.490	−96.430	95.350	10.603	0.668	16.034
$\ln P_t$	2,556	4.838	4.883	2.694	6.048	0.529	−0.909	4.487
$\ln P_t - \ln P_{t-1}$	2,555	0.000	−0.004	−0.718	0.701	0.099	0.461	14.397
				Panel B: Cold Seasons				
P_t	1,485	157.138	144.080	33.440	423.380	54.215	0.857	3.841
$P_t - P_{t-1}$	1,484	−0.189	−0.640	−96.430	95.350	10.205	0.643	25.155
$\ln P_t$	1,485	4.999	4.970	3.510	6.048	0.345	−0.252	3.812
$\ln P_t - \ln P_{t-1}$	1,484	−0.001	−0.005	−0.672	0.545	0.067	−0.293	23.651

Note: This table displays descriptive statistics for the daily spot system price (denoted P_t) and other related series as indicated in the first column. The sample period is from January 1, 1993 to December 31, 1999. Panel A displays the results for the whole set of data, panel B for cold seasons, and panel C for warm seasons. Warm seasons includes observations for any date t from May 1 to September 30, both inclusive. Under the heading ADF, the last column shows the value of the augmented Dickey–Fuller test t-statistic for the null hypothesis of a unit root, using twenty one lags in the relevant model, i.e. the statistic is the usual t-ratio for the ϕ coefficient in the model (see, e.g. Greene (1993: 19.5)): $\Delta y_t = \mu + \phi y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + u_t$ with $p = 21$, and $\Delta y_t \equiv y_t - y_{t-1}$, where y_t stands for P_t and $\ln P_t$, alternatively. MacKinnon critical values are: −3.4360 at the 1% level, −2.8632 at the 5%, and −2.5677 at the 10%.

Table 1. Descriptive Statistics for the Daily System Price and Other Related Time Series (1993–1999) *(Continued)*

Series	Number of Observations	Mean	Median	Minimum	Maximum	Standard Deviation	Skewness	Kurtosis
Panel C: Warm Seasons								
P_t	1,071	122.367	105.940	14.800	370.030	75.756	1.177	3.913
$P_t - P_{t-1}$	1,071	0.276	−0.290	−44.870	64.510	11.130	0.681	6.867
$\ln P_t$	1,071	4.615	4.663	2.694	5.914	0.646	−0.381	3.039
$\ln P_t - \ln P_{t-1}$	1,071	0.002	−0.002	−0.718	0.701	0.132	0.489	8.903

Note: This table displays descriptive statistics for the daily spot system price (denoted P_t) and other related series as indicated in the first column. The sample period is from January 1, 1993 to December 31, 1999. Panel A displays the results for the whole set of data, panel B for cold seasons, and panel C for warm seasons. Warm seasons includes observations for any date t from May 1 to September 30, both inclusive. Under the heading ADF, the last column shows the value of the augmented Dickey–Fuller test t-statistic for the null hypothesis of a unit root, using twenty one lags in the relevant model, i.e. the statistic is the usual t-ratio for the ϕ coefficient in the model (see, e.g. Greene (1993: 19.5)): $\Delta y_t = \mu + \phi y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + u_t$ with $p = 21$, and $\Delta y_t \equiv y_t - y_{t-1}$, where y_t stands for P_t and $\ln P_t$, alternatively. MacKinnon critical values are: −3.4360 at the 1% level, −2.8632 at the 5%, and −2.5677 at the 10%.

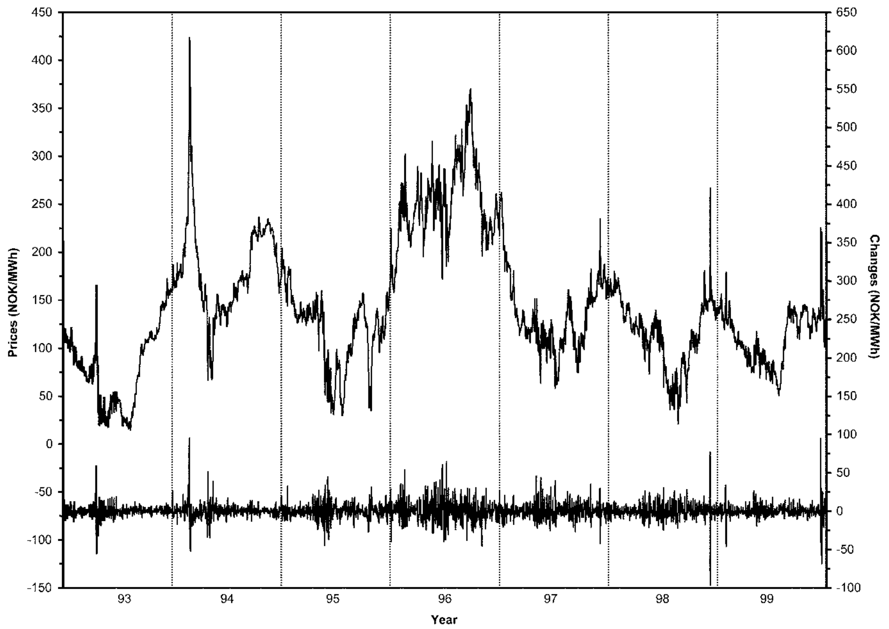

Figure 1 (a). Daily System Price Time Series (1993–1999). The figure plots the daily system price series (above) as well as its daily changes (below).

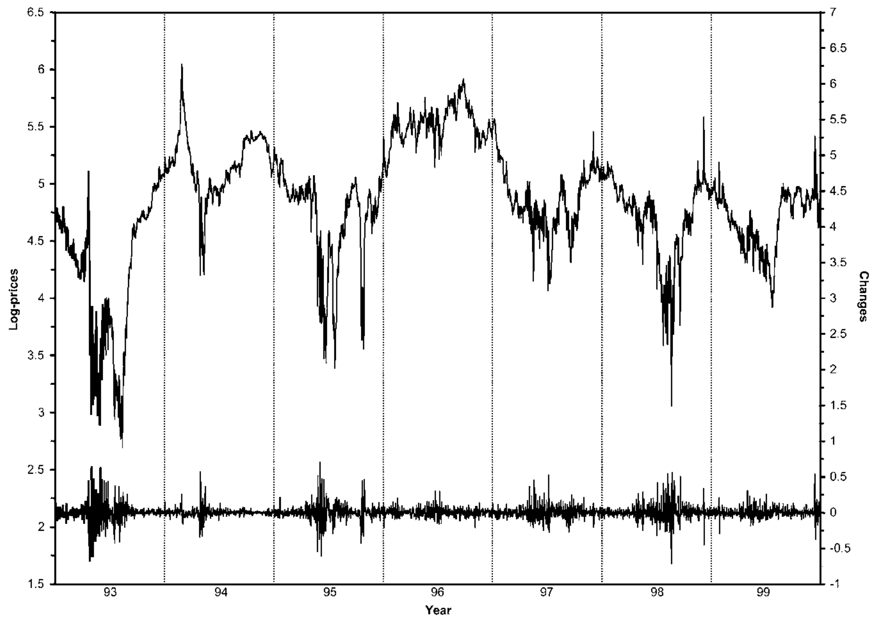

Figure 1 (b). The same daily system price series (1993–1999) is used as that of Figure 1(a). The figure plots the natural log of the daily system price series (above) together with its daily changes (below).

the log-price series using the augmented Dickey–Fuller *t*-test for a unit root (see Table 1). The presence of a unit root is rejected in both cases at the 5% significance level.[11]

Electricity prices are highly volatile, as measured by standard volatility measures. The standard deviation of the daily changes in log-prices is 0.099, which translates into an annualized volatility of 189%.[12] A significant difference exists, however, between cold and warm seasons.[13] Standard deviations of log-price changes show that warm seasons are twice as volatile as cold seasons (0.067 or 128% annualized, and 0.132 or 252% annualized, respectively). This result should be interpreted with care since the log transformation is expected to increase comparatively the volatility during periods with consistently lower prices and, consequently, to contribute to the significative difference in the standard deviations of changes in the log-price (which can be roughly interpreted as relative or percentage changes in prices) between warm and cold seasons. Note that warm seasons had a daily mean price about 22% lower than the mean for cold seasons during the sample period (see Table 1).[14]

The standard deviation of the price itself is 54.2 for cold seasons and 75.8 for warm seasons, indicating a significantly higher stability of the mean price for cold seasons, as compared to warm seasons, during the sample period. Changes in prices, in turn, show very similar standard deviations for both seasons.

Extreme electricity prices are relatively frequent in Elspot. This fact is reflected in the sample kurtosis coefficients. The kurtosis estimate for the whole sample period is 3.5, which is significantly different from three (the kurtosis for a normal distribution) under the null hypothesis of normality. This means that extremely low and high prices have a higher probability of occurrence than that dictated by a normal distribution with the same variance. The kurtosis estimates are similar for cold and warm seasons (3.8 and 3.9, respectively). The positive sign of the skewness estimates for the price series reveals that high extreme values for the price are in fact more probable than low extreme values. The log-price series shows a higher leptokurtosis than the price series (the kurtosis estimate is 4.5).

An interesting related question is whether such extreme values are a result of jumps, i.e. abnormally large variations, in prices. A look to the sample kurtosis coefficients for both changes in prices and changes in log-prices (or percentage changes in prices) reveals that large daily variations are indeed relatively frequent in electricity prices, this being particularly severe during cold seasons. Both complete series show sample kurtosis, 16 and 14.4 respectively, far in excess of 3, indicating leptokurtic distributions for both absolute and percentage changes in prices.

The excess kurtosis is extremely large during cold seasons, 25.15 and 23.65 respectively for the changes in prices and in the log-prices. The excess kurtosis is more than four and a half times higher in cold seasons than in warm seasons for (absolute) changes in prices, and it is about two and a half times higher for changes in log-prices (relative changes in prices). In addition, the highest daily price positive and negative changes took place at winter time (around 95 NOK of both signs, or nine times the standard deviation).[15] All these facts probably have to do with the shape of the supply curve (also called the *supply stack*), which exacerbates the jumps in prices as a result of jumps in demand during periods of higher demand, when additional generation units brought on-line correspond to more inefficient ones.[16] Many of the larger jumps in prices are due to temporary shocks in demand (very frequently linked to sudden and pronounced short term changes in temperatures) and prices rapidly return to previous levels, causing spikes in the behavior of spot prices.[17] The largest changes in log-prices, in turn, occurred during warm seasons (around 0.71 with both signs).

The degree of asymmetry of the distribution of daily changes in both prices and log-prices in not as large as the leptokurtosis, although all the skewness estimates are statistically significative under the null hypothesis of normality.

All these facts notwithstanding, the system price displays some signs of predictability. First, note that the first-differences in the level of both the price and the log-price series are significatively positively autocorrelated at several lags multiple of seven (see Table 1). This means that the increments of the price (log-price) from one day to the next helps to predict significantly the increment of the price (log-price) for the same couple of consecutive days up to several weeks into the future.[18] This has to do with the fact that demand for electricity varies following a somehow noticeable regular pattern within the week.

Demand for electricity power also varies regularly within the day influencing price levels. Intra-day and intra-week regular patterns in the level of prices are mainly determined by business activity, and they might change along the year following changes in the main uses of power across seasons. Figure 2(a) displays the average hourly pattern within the day across days of the week. There is a clear difference in both shape and mean level between weekdays on the one hand and weekends on the other.[19] As it was expected, a division of the weekdays into working days (i.e. non-holiday weekdays) on the one hand and non-working days (holidays) on the other reveals that weekdays holidays show an hourly pattern shape and mean level very similar to weekends (see Figure 2(b)).[20]

With the exception of year 1996, it can be noticed a seasonal pattern in the level of prices along the year, by a casual inspection of Figure 1 (see also the differences across seasons in Figure 2). Table 1, Panels B and C, shows that the system price had mean, median, maximum and minimum values during cold seasons all above the corresponding ones during warm seasons, being the mean price for cold seasons about 35 NOK (a 28%) higher than the one for warm seasons. This regular pattern has mainly to do with seasonal changes in climate along the year which strongly determine heating needs, and also with day-light length which influences lighting needs.[21] Weather conditions, however, are responsible for significant departures from the usual patterns described above, as it happened to be the case in 1996.

One question that arises immediately is how market participants in Eltermin perceive that seasonal pattern and its importance, and whether they eventually incorporate it in their derivatives valuation process. Figure 3 plots the evolution of the term structure of futures prices for almost two complete years of fortnightly observations, from January 1998 through November 1999. Each observation consists of the complete term structure of futures prices, i.e. the complete set of futures (closing) prices, corresponding to every listed futures contract on that date. Figure 3(a) displays the curves altogether on a single two-dimensional graph, without explicitly indicating the observation date (shorter curves correspond to later observation dates). Figure 3(b) shows the evolution across time of the term structure of futures prices.[22]

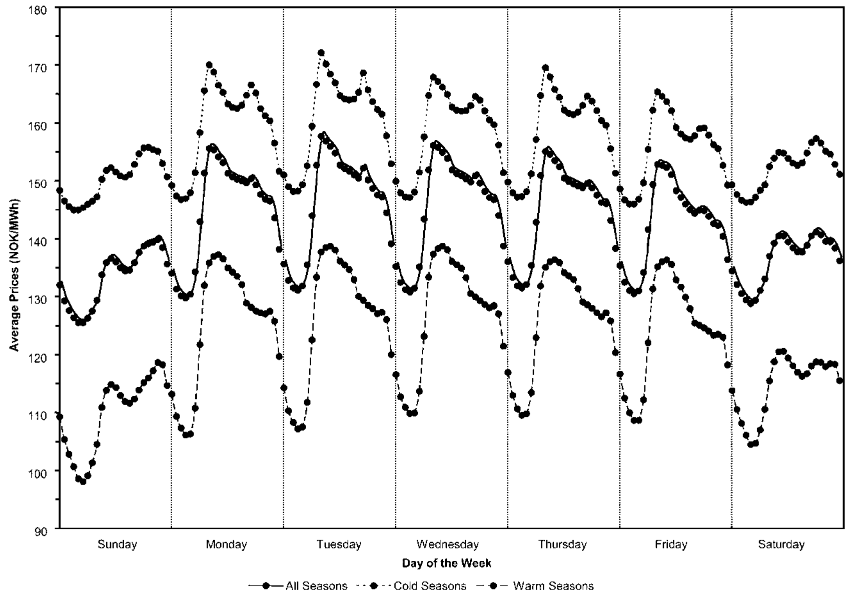

Figure 2(a). Hourly Average Patterns (1993–1999). The figure displays the hourly mean prices throughout the week across seasons. A warm season includes the months from May through September.

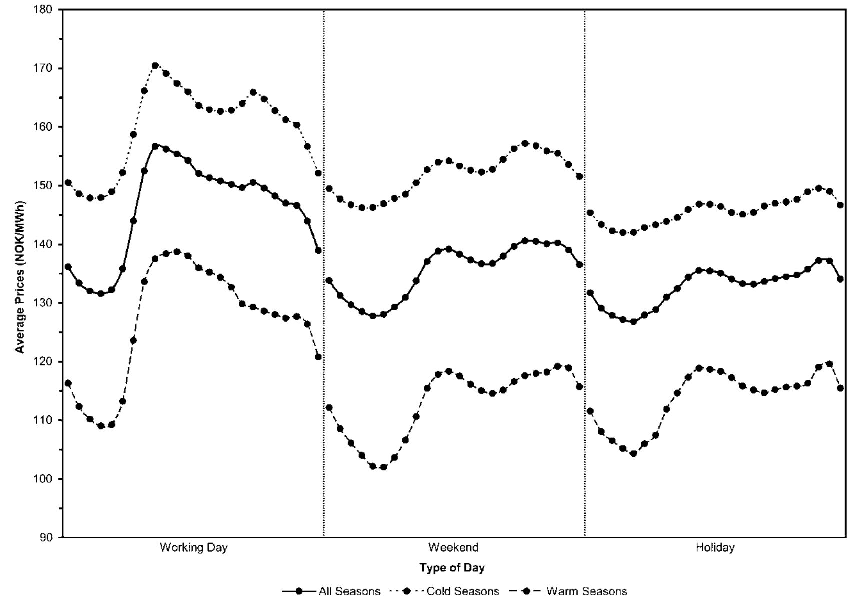

Figure 2(b). Hourly Average Patterns (1993–1999). The figure displays the hourly mean prices separately for working days, weekends, and holidays (only official holidays in Norway have been considered) across seasons. A warm season includes the months from May through September.

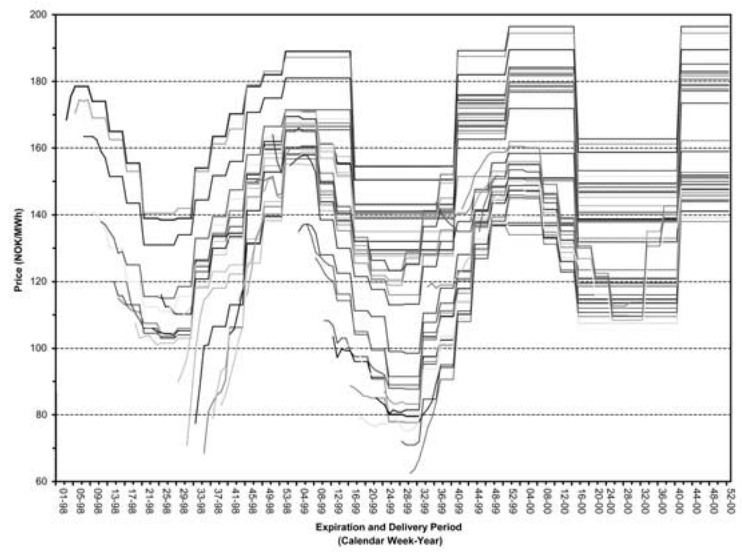

Figure 3(a). Term Structure of Futures Prices in the Nord Pool (1998–1999). This figure plots the term structures of futures (closing) prices from January 1998 to November 1999, using fortnightly observations. (Observation dates are the first trading day of every two weeks, starting on Monday, January 5, 1998.) Only listed futures contracts have been considered (regardless of whether they were traded or not on the observation dates). For any curve, every turning point indicates approximately a different price corresponding to a subsequent maturity, and the length of the associated flat section represents the length of the delivery period as measured in weeks.

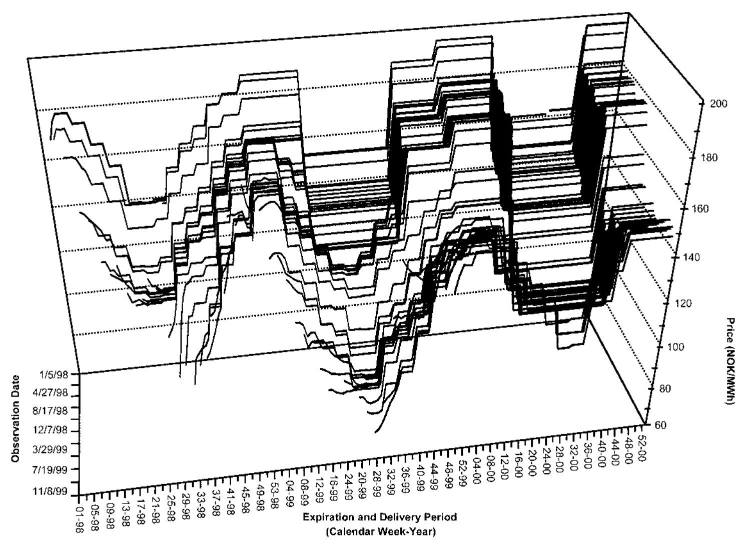

Figure 3(b). The same futures curves as those of Figure 3(a) are used. This plot allows to observe the evolution of the futures curves along time (January 1998–November 1999).

An immediate conclusion arises from a casual inspection of the figure: the seasonal component in the evolution of the system price is incorporated by market participants in their valuation processes and, as a matter of fact, it constitutes a prominent explanation for the shape of the futures curve for electricity contracts in the Nord Pool. The futures curve's shape displays one peak and one valley per year, in total accordance with the behavior of the system price.

3. Models

In this section we discuss the models for the dynamics of the spot system price and their implications for the valuation of derivative securities. We describe the behavior of the spot price in terms of two types of components. The first one is a totally predictable deterministic component that accounts for regularities in the evolution of prices, such as a deterministic trend and any genuine periodic behavior. The second component is stochastic and will be assumed to follow a particular continuous time diffusion process. The final models considered differ with respect to the stochastic process assumed for the spot price, the number of stochastic factors considered, and the way the deterministic component is incorporated into the model.[23]

For simplicity, in what follows we assume that interest rates are constant. In this setting forward and futures prices are equal. This assumption is less accurate for the longest maturity contracts. The extension of the models considered, in order to allow for stochastic interest rates (which implies the inclusion of one or two additional factors), would considerably complicate the discussion and obscure the main points of the analysis.

3.1. One Factor Model Based on the Spot Price

We start by expressing the stochastic process followed by the spot price, represent by P_t with $t \in [0, \infty)$, as the sum of two components. The first one is considered to be totally predictable, and is represented by a known deterministic function of time, $F = f(t)$. The second one is a diffusion stochastic process X_t. That is,

$$P_t = f(t) + X_t. \tag{1}$$

In particular, we assume that X_t follows a stochastic process of the form:

$$dX_t = -\kappa X_t\, dt + \sigma\, dZ \tag{2}$$

where $\kappa > 0$, $X(0) = x_0$, and dZ represents an increment to a standard Brownian motion Z_t. Therefore, X_t follows a stationary mean-reverting process, or Ornstein–Uhlenbeck process, with a zero long-run mean and a speed of adjustment κ. Recalling that $X_t = P_t - f(t)$, and assuming that the function $F = f(t)$ satisfies the appropriate regularity conditions, we can write (1) and (2) as:

$$d(P_t - f(t)) = \kappa(f(t) - P_t)\, dt + \sigma\, dZ \tag{3}$$

which shows that when P_t deviates from the deterministic term $f(t)$ it is pulled back to it at a rate proportional to its deviation. In this model, the only source of uncertainty comes from the stochastic behavior of X_t as described by (2), so we can call X_t the state variable.

The process followed by P_t can be expressed as the solution to the following stochastic differential equation (provided again that the function $F = f(t)$ satisfies appropriate regularity conditions):

$$dP_t = \kappa(a(t) - P_t)\,dt + \sigma\,dZ \tag{4}$$

where $a(t)$ is the deterministic function of t defined by:

$$a(t) \equiv \frac{1}{\kappa}\frac{df}{dt}(t) + f(t). \tag{5}$$

This can be seen as a particular version of the extended-Vasicek model by Hull and White (1990).[24]

The assumed simple one-factor model is analytically very tractable. An explicit solution for (2) can be obtained, which together with (1) gives:

$$P_t = f(t) + X_0 e^{-\kappa t} + \sigma \int_0^t e^{\kappa(s-t)}\,dZ(s). \tag{6}$$

Hence we have that the conditional distribution of P_t is normal with conditional mean and variance given by (using $X_0 = P_0 - f(0)$):

$$\begin{aligned} E_0(P_t) &\equiv E(P_t|X_0) = f(t) + (P_0 - f(0))e^{-\kappa t} \\ Var_0(P_t) &\equiv Var(P_t|X_0) = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}), \qquad \kappa > 0. \end{aligned} \tag{7}$$

These results are helpful in developing some intuition for the price process. P_t tends to a mean value of $f(t)$ in the long run, given its value at a previous moment P_0. The higher the value of κ (assuming $\kappa > 0$), the faster the convergence. The variance in turn decreases with the time horizon and has a finite limit as this horizon tends to infinity.

For the purposes of derivative securities valuation, and following Cox and Ross (1976) and Harrison and Kreps (1979), we need the risk-neutral or risk-adjusted process for the state variable X_t, instead of the real one given by (2). Taking into account the non-tradable nature of X_t, standard arbitrage arguments with two derivative assets allow us to obtain the risk-neutral process for X_t. This is given by the equation:

$$dX_t = \kappa(\alpha^* - X_t)dt + \sigma\,dZ^* \tag{8}$$

with

$$\alpha^* \equiv -\lambda\sigma/\kappa \tag{9}$$

where dZ^* stands for an increment to Z_t^*, a standard Brownian motion under the risk-neutral probability measure, and λ denotes the market price per unit risk linked to the state variable X_t. We assume that λ is constant. In general, it could be a function of the state variable X_t and t.

Following the same steps as before we can derive some important results for the risk-neutral process. The explicit solution for the stochastic differential equation (8) allows us to obtain:

$$P_t = f(t) + X_0 e^{-\kappa t} + \alpha^*(1 - e^{-\kappa t}) + \sigma \int_0^t e^{\kappa(s-t)} dZ^*(s) \tag{10}$$

with α^* as defined in (9). From this we have that P_t is conditionally normal under the risk-neutral measure, with the following conditional mean:

$$E_0^*(P_t) = f(t) + X_0 e^{-\kappa t} + \alpha^*(1 - e^{-\kappa t}). \tag{11}$$

Now, the value of any derivative security must be the expected value, under the risk-neutral measure, of its payoffs discounted to the valuation date at the risk-free rate, which we assume to be constant. The value at time zero of a forward contract on the spot price maturing at time T must be:

$$v_0(X_T, T) = e^{-rT} E_0^*[P_T - F_0(P_0, T)] \tag{12}$$

where $F_0(P_0, T)$ stands for the forward price set at time zero for a contract maturing at time T, and r is the riskless continuously compounded interest rate. Since the value of a forward contract must be zero when it is first entered into, we finally have the following closed form solution for the forward (futures) price, using (11) and (1) for $t = 0$:[25]

$$F_0(P_0, T) = E_0^*(P_T) = f(T) + (P_0 - f(0))e^{-\kappa T} + \alpha^*(1 - e^{-\kappa T}) \tag{13}$$

with $\alpha^* = -\lambda\sigma/\kappa$.

3.2. One Factor Model Based on the Log Spot Price

The second group of models comes from working directly with the natural logarithm of the spot price instead of the spot price itself. To be precise, we assume that the log-price process $\ln P_t$ can be written as:

$$\ln P_t = f(t) + Y_t \tag{14}$$

for $t \in [0, \infty)$, where $F = f(t)$ is as before a known deterministic function of time, and Y_t is a stochastic process whose dynamics are given by:

$$dY_t = -\kappa Y_t\, dt + \sigma\, dZ \tag{15}$$

with $\kappa > 0$ and $Y(0) = y_0$.[26] In this type of model, the log-price detached from the deterministic component $f(t)$ follows a zero-mean reverting process. This implies the following process for the price, under suitable conditions for $f(t)$:

$$dP_t = \kappa(b(t) - \ln P_t)P_t\,dt + \sigma P_t\,dZ \tag{16}$$

with

$$b(t) \equiv \frac{1}{\kappa}\left(\frac{\sigma^2}{2} + \frac{df}{dt}(t)\right) + f(t).$$

From the process for Y_t (15) we have that in this class of models it is assumed that the log of the price, $\ln P_t$, has a conditional normal distribution with conditional mean and variance:

$$\begin{aligned} E_0(\ln P_t) &= f(t) + (\ln P_0 - f(0))e^{-\kappa t} \\ Var_0(\ln P_t) &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}), \quad \kappa > 0. \end{aligned} \tag{17}$$

Therefore, the log-price have the same properties as the price in the previous subsection. Hence, the spot price P_t has a conditional lognormal distribution, and from the properties of the lognormal distribution, we have:

$$\begin{aligned} E_0(P_t) &= \exp\big(E_0(\ln P_t) + \tfrac{1}{2}Var_0(\ln P_t)\big) \\ &= \exp\left(f(t) + (\ln P_0 - f(0))e^{-\kappa t} + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa t})\right) \end{aligned} \tag{18}$$

and

$$\begin{aligned} Var_0(P_t) &= \exp(2E_0(\ln P_t) + Var_0(\ln P_t))[\exp(Var_0(\ln P_t)) - 1] \\ &= E_0(P_t)^2\left[\exp\left(\frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})\right) - 1\right]. \end{aligned}$$

From which we have that the conditional mean of the price converges to $f(\infty) + \alpha^2/4\kappa$ in the long run, and the conditional variance also converges to a function of the value of the deterministic component at infinity.

We also have that, under the risk-neutral measure:

$$dY_t = \kappa(\alpha^* - Y_t)dt + \sigma\,dZ^* \tag{19}$$

with

$$\alpha^* \equiv -\lambda\sigma/\kappa$$

where we assume λ to be constant.

From the explicit solution to (19) together with (14) we can write the following explicit solution for $\ln P_t$ under the risk neutral measure:

$$\ln P_t = f(t) + Y_0 e^{-\kappa t} + \alpha^*(1 - e^{-\kappa t}) + \sigma \int_0^t e^{\kappa(s-t)} dZ^*(s) \tag{20}$$

from which we have that $\ln P_t$ has a conditional normal distribution under the risk neutral measure, with conditional mean and variance given by:

$$\begin{aligned} E_0^*(\ln P_t) &= f(t) + Y_0 e^{-\kappa t} + \alpha^*(1 - e^{-\kappa t}) \\ Var_0^*(\ln P_t) &= \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa t}\right), \quad \kappa > 0. \end{aligned} \tag{21}$$

Hence, the price P_t has a lognormal distribution with mean given by:

$$E_0^*(P_t) = \exp\left(E_0^*(\ln P_t) + \tfrac{1}{2} Var_0^*(\ln P_t)\right). \tag{22}$$

And applying the same argument as before, the forward / futures price must be:

$$\begin{aligned} F_0(P_0, T) &= E_0^*(P_T) \\ &= \exp\left[f(T) + (\ln P_0 - f(0))e^{-\kappa T} + \alpha^*\left(1 - e^{-\kappa T}\right) + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa T}\right)\right] \end{aligned} \tag{23}$$

with $\alpha^* = -\lambda\sigma/\kappa$.

Note that in the models derived in this section, the deterministic component of the behavior of the spot price (log-price) appears directly in the price of the forward and futures contracts (see (13) and (23)), that term being an important determinant of the shape of the forward / futures curve. Also, in both types of single-factor models all forward / futures prices are perfectly correlated.

3.3. Two Factor Model Based on the Spot Price

As mentioned above, one of the key limitations of all single-factor models is that they imply that changes in spot prices and in forward and futures prices for all maturities are perfectly correlated. This means that these prices always move in the same direction, a fact that clearly contradicts the data. This limitation can be avoided and the fit of the models to the data can be improved if changes in spot prices are allowed to depend on more than one factor. Two and three factor models have been shown to perform significantly better than one-factor models for commodities like copper and oil (see for example Schwartz (1997)).

In this subsection we extend the one-factor models developed in the previous subsections by adding a second stochastic factor in the spirit of Schwartz and Smith (2000). They model the stochastic behavior of oil prices as having a short-term mean reverting component and a long-term equilibrium price level. An important implication of adding a second factor in the model is that changes in prices of futures contracts with different maturities are not perfectly correlated, as is the case for all one-factor models. We now briefly describe the two factor models.

The model based on the price described by equations (1) and (2) now becomes:

$$P_t = f(t) + X_t + \varepsilon_t \tag{24}$$

$$dX_t = -\kappa X_t \, dt + \sigma_X \, dZ_X \tag{25}$$

$$d\varepsilon_t = \mu_\varepsilon \, dt + \sigma_\varepsilon \, dZ_\varepsilon \tag{26}$$

$$dZ_X \, dZ_\varepsilon = \rho \, dt. \tag{27}$$

In this model the second state variable, ε_t, follows an arithmetic Brownian motion, and the two Wiener processes dZ_X and dZ_ε are correlated through equation (27).

The risk-adjusted processes for these state variables are:

$$dX_t = \kappa(\alpha^* - X_t)dt + \sigma_X \, dZ_X^* \tag{28}$$

$$d\varepsilon_t = \mu_\varepsilon^* \, dt + \sigma_\varepsilon \, dZ_\varepsilon^* \tag{29}$$

where

$$\alpha^* \equiv -\lambda_X \sigma_X / \kappa$$

$$\mu_\varepsilon^* \equiv \mu_\varepsilon - \lambda_\varepsilon \sigma_\varepsilon$$

$$dZ_X^* dZ_\varepsilon^* = \rho \, dt$$

and λ_X and λ_ε are the market prices of risk for each state variable which are assumed to be constant.

Following the same steps as before, it can be proven that the futures prices are given by

$$F_0(P_0, T) = E_0^*(P_T) = f(T) + e^{-\kappa T} X_0 + \varepsilon_0 + \left(1 - e^{-\kappa T}\right)\alpha^* + \mu_\varepsilon^* T \tag{30}$$

with: $\alpha^* = -\lambda_X \sigma_X / \kappa$ and $\mu_\varepsilon^* = \mu_\varepsilon - \lambda_\varepsilon \sigma_\varepsilon$.

3.4. *Two Factor Model Based on the Log Spot Price*

Proceeding similarly for the log spot price we define:

$$\ln P_t = f(t) + X_t + \varepsilon_t$$

where the stochastic factors follow the same processes as in the previous subsection.[27]

The futures price is given by the expression (see Schwartz and Smith(2000)):

$$\begin{aligned} F_0(P_0, T) &= E_0^*(P_T) \\ &= \exp\Bigg(f(T) + e^{-\kappa T} X_0 + \varepsilon_0 + \left(1 - e^{-\kappa T}\right)\alpha^* + \mu_\varepsilon^* T + \left(1 - e^{-2\kappa T}\right)\frac{\sigma_X^2}{4\kappa} \\ &\qquad + \frac{1}{2}\sigma_\varepsilon^2 T + \left(1 - e^{-\kappa T}\right)\frac{\rho\sigma_X\sigma_\varepsilon}{\kappa}\Bigg) \end{aligned} \tag{31}$$

with $\alpha^* = -\lambda_X \sigma_X / \kappa$ and $\mu_\varepsilon^* = \mu_\varepsilon - \lambda_\varepsilon \sigma_\varepsilon$.

Note that in this case the futures price depends on the volatilities and the correlation between the two factors.

3.5. The Deterministic Component of the Models

In order to implement the general models described above, we need to specify the deterministic time function $F = f(t)$. Here there are several choices available. Recall that this function tries to capture any relevant predictable component of the electricity prices behavior arising from genuine regularities along time. This implies that the final selection should be based on the nature and basic characteristics of the time-series properties of the price variable under scrutiny, as well as modeling considerations such as parsimony.

The simplest deterministic function is a constant function for all time *t*. In this case, we would have a constant-mean reverting process for the price (or the log-price). In our context, this possibility implies to accept that, although prices tend to be higher during the coldest seasons and lower during the warmest ones, departures from the "usual" behavior are so frequent and significant that neither the duration of the seasons nor the differences in prices between them can be meaningfully predicted. As such, the seasonal behavior would be better described by a process fluctuating randomly around a constant long-run mean. This case also discards other possible regularities of prices.

We may also consider to include a deterministic general trend. The simplest one would be a linear time trend. This, applied to the log-price, which implies an exponential trend for the price itself, gives the "trending Ornstein–Uhlenbeck process" by Lo and Wang (1995).

We may also be interested in incorporating some kind of periodic behavior, such as seasonality, into the function. For instance, as suggested by Pilipovic (1998), a sinusoidal function like the cosine function could be used to reflect the general seasonal pattern of the price time series.

The use of a constant piece-wise or step function in order to approximate the periodic components and to incorporate them in the implementation stage of the models may also be worth being considered.[28] This can be accomplished by means of using dummy variables in the implementation process. Dummy variables are intuitive and easy to interpret. Also, compared with fixing any *a priori* well behaved functional form to capture periodic components into the model, the use of dummy variables can potentially provide some necessary flexibility. On the other hand, as a result of such flexibility, the use of dummy variables in order to capture genuine regularities in the behavior of the underlying variable suffers from the potentially serious drawback that they are especially sensitive to anomalies in the sample, such as outliers. Additionally, the use of seasonal dummy variables with high frequency data can always be seen as an approximation since the number of steps and the placing of every step point are both arbitrarily fixed, and some precision is lost in order to keep the model parsimonious enough to be of practical use.[29]

According to the results in Section 2, we decided to include two terms into the deterministic function $F = f(t)$, in addition to a constant. The additional terms try to capture, respectively, the variation in the level of prices between working and non-working days, and the seasonal evolution of prices throughout the year.[30] We finally propose two versions of the deterministic function. In the first one, the function takes the following form:

$$f_1(t) = \alpha + \beta D_t + \sum_{i=2}^{12} \beta_i M_{it} \tag{32}$$

where

$$D_t = \begin{cases} 1 & \text{if date } t \text{ is holiday or weekend} \\ 0 & \text{otherwise} \end{cases}$$
$$M_{it} = \begin{cases} 1 & \text{if date } t \text{ belongs to the } i\text{-th calendar month} \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for } i = 2, \ldots, 12$$

and α, β, and β_i for $i = 2, \ldots, 12$ are all constant parameters.

In this case, the beta parameters try to capture the changes on the level of the variable for holidays and weekends, and for the different months of the year, respectively, with respect to the general long-run level (assumed constant) for the working days during January.

The second version of the deterministic function takes the following form:

$$f_2(t) = \alpha + \beta D_t + \gamma \cos\left((t + \tau)\frac{2\pi}{365}\right) \tag{33}$$

where

$$D_t = \begin{cases} 1 & \text{if date } t \text{ is holiday or weekend} \\ 0 & \text{otherwise} \end{cases}$$

cos stands for the cosine function measured in radians, and α, β, γ and τ are all constant parameters. Here, the coefficient β tries to capture the changes in the level of the variable for weekends and holidays. The cosine function is expected to reflect the seasonal pattern in the evolution of the relevant variable throughout the year, hence it has annual periodicity.

4. Estimation of the Stochastic Process for the One Factor Models

In this section we estimate the stochastic process for the one factor models from the spot price data. In order to estimate the models using discrete observations, we need to express them fully in discrete form. We choose the empirical estimation to be based on the following simple discretization of equation (2):

$$X_t = (1-\kappa)X_{t-1} + \xi_t \tag{34}$$

for $t = 0, 1, 2, \ldots, N$, and where the innovations ξ_t are i.i.d. normal random variables with mean zero and variance σ^2. The same discretization is used for the process Y_t defined in equation (15).[31]

To summarize, we have four one factor models to estimate that differ in the time series chosen to be directly modeled (price and log-price, respectively), and in the way the deterministic component is approximated. Namely,

Model 1

$$\begin{aligned} P_t &= \alpha + \beta D_t + \sum_{i=2}^{12} \beta_i M_{it} + X_t \\ X_t &= \phi X_{t-1} + u_t \end{aligned} \tag{35}$$

Model 2

$$\begin{aligned} P_t &= \alpha + \beta D_t + \gamma \cos\left((t+\tau)\frac{2\pi}{365}\right) + X_t \\ X_t &= \phi X_{t-1} + u_t \end{aligned} \tag{36}$$

Model 3

$$\begin{aligned} \ln P_t &= \alpha + \beta D_t + \sum_{i=2}^{12} \beta_i M_{it} + Y_t \\ Y_t &= \phi Y_{t-1} + u_t \end{aligned} \tag{37}$$

Model 4

$$\begin{aligned} \ln P_t &= \alpha + \beta D_t + \gamma \cos\left((t+\tau)\frac{2\pi}{365}\right) + Y_t \\ Y_t &= \phi Y_{t-1} + u_t \end{aligned} \tag{38}$$

with the dummy variables as defined in (24) and (25), and $\phi \equiv 1 - \kappa$.

For each one of these models, we estimate all the parameters simultaneously by nonlinear least squares methods. To be precise, let's write any of the four previous models in the following general form:

$$
\begin{aligned}
y_t &= f(\Theta, x_t) + \xi_t \\
\xi_t &= \phi\xi_{t-1} + u_t.
\end{aligned}
\tag{39}
$$

The first equation expresses the dependent variable y_t, i.e. the price or the log-price variable, as a function of a vector of parameters, Θ, and the vector of explanatory variables, x_t. The second equation is the first-order autoregressive structure of the disturbance term ξ_t in the first equation. By substituting for ξ in the second equation, and rearranging terms, we get:

$$
y_t = \phi y_{t-1} + f(\Theta, x_t) - \phi f(\Theta, x_{t-1}) + u_t \tag{40}
$$

whose parameters ϕ and Θ are estimated simultaneously using a numerical nonlinear least squares procedure. Finally, we take $\hat{\kappa} = 1 - \hat{\phi}$ as the estimate of the parameter κ, and the standard error of the regression as the estimate of σ. The estimation results for the whole spot sample period described in Section 2 are reported in Table 2.

In the four models the independent coefficient α is significatively different from zero. The sign of every dummy variable is negative in all models, as expected from the results of Section 2. Nevertheless, their level of significance differs. The coefficient β corresponding to the dummy variable D_t is significatively different from zero in the four models, but not all the coefficients of the monthly dummy variables are significative.

The estimates of the coefficients β, ϕ, and σ are virtually indistinguishable between Models 1 and 2, and between Models 3 and 4. The null hypothesis of $\phi = 1$ is rejected by the usual t-test for every model. This means that the estimate for the reversion coefficient κ, though very small, turns out to be significative in all cases.[32]

Figure 4 plots the residuals corresponding to the equation (40), also called the one-period-ahead prediction errors, for the four models (see also the bottom part of Table 2). The results are virtually indistinguishable between Models 1 and 2, as well as between Models 3 and 4. The models for the price have mean absolute percentage error of 5%, while in the case of the models for the log-price the mean absolute percentage error is 1%, for the whole sample period.

5. Futures and Forward Valuation Using One Factor Models

In this section we analyze the empirical performance of the valuation one factor models discussed previously by comparing a sample of futures and forward market prices from the Nord Pool with the theoretical values provided by the pricing models.

The derivatives sample was obtained from the Nord Pool's FTP server files. The sample consists of the observation of the complete term structure of futures and forward prices every four weeks, from December 1998 to November 1999. The observation dates were selected to be the first trading day of the second week of each block delivery period. This left us with thirteen observation dates. The sample includes the closing prices for all traded

Table 2. Estimation Results for Alternative Models for the System Price (1993–1999)

	Models Based on the Price				Models Based on the Log-price			
	Model 1		Model 2		Model 3		Model 4	
Parameter	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic
α	153.051	8.146	145.732	8.670	4.938	38.711	4.867	46.192
β	−9.514	−28.085	−9.542	−28.277	−0.090	−28.339	−0.090	−28.523
γ			29.735	2.336			0.306	2.986
τ			6.691	0.269			0.836	0.043
β_2	−2.527	−0.754			−0.027	−0.878		
β_3	−4.511	−0.998			−0.041	−0.977		
β_4	−3.484	−0.664			−0.041	−0.849		
β_5	−13.248	−2.317			−0.185	3.480		
β_6	−12.656	−2.114			−0.097	−1.744		
β_7	−7.038	−1.157			−0.062	1.093		
β_8	−8.109	−1.347			−0.101	−1.807		
β_9	−10.061	−1.740			−0.094	−1.749		
β_{10}	−9.597	−1.795			−0.067	−1.352		
β_{11}	−7.304	−1.566			−0.052	−1.190		
β_{12}	−6.019	−1.674			−0.057	−1.703		

Note: The estimation is based on daily system price observations from January 1, 1993 to December 31, 1999 (2,556 observations). The parameters are estimated by transforming the following general specification of the models (refer to the main body of the text for the specific details):

$$y_t = f(\Theta, x_t) + \varepsilon_t$$
$$\varepsilon_t = \phi\varepsilon_{t-1} + u_t$$

into the following alternative specification:

$$y_t = \phi y_{t-1} + f(\Theta, x_t) - \phi f(\Theta, x_{t-1}) + u_t$$

and then estimating the coefficients ϕ and Θ simultaneously by nonlinear least squares methods (first derivative iterative optimization algorithms have been used). The following estimator for κ is used: $\hat{\kappa} = 1 - \hat{\phi}$. The log likelihood function is evaluated (assuming normally distributed errors) at the estimated values of the coefficients. The errors refer to the residuals $\hat{u}_t$; corresponding to the innovations u_t in the above equations (the one-period-ahead forecast errors). M.A.E. and M.A.P.E. stand for the mean absolute error and the mean absolute percentage error, respectively.

Table 2. Estimation Results for Alternative Models for the System Price (1993–1999) *(Continued)*

	Models Based on the Price				Models Based on the Log-price			
	Model 1		Model 2		Model 3		Model 4	
Parameter	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic
ϕ	0.990	355.4	0.989	340.4	0.986	299.0	0.984	277.5
κ	0.010		0.011		0.014		0.016	
S.E. of Regression	9.001		9.222		0.086		0.086	
Adjusted R^2	0.981		0.981		0.974		0.973	
Log likelihood	−9294.2		−9299.0		2652.9		2640.2	
Errors:								
M.A.E.	5.847		5.855		0.053		0.053	
M.A.P.E.	4.980		5.000		1.176		1.179	

Note: The estimation is based on daily system price observations from January 1, 1993 to December 31, 1999 (2,556 observations). The parameters are estimated by transforming the following general specification of the models (refer to the main body of the text for the specific details):

$$y_t = f(\Theta, x_t) + \varepsilon_t$$
$$\varepsilon_t = \phi\varepsilon_{t-1} + u_t$$

into the following alternative specification:

$$y_t = \phi y_{t-1} + f(\Theta, x_t) - \phi f(\Theta, x_{t-1}) + u_t$$

and then estimating the coefficients ϕ and Θ simultaneously by nonlinear least squares methods (first derivative iterative optimization algorithms have been used). The following estimator for κ is used: $\hat{\kappa} = 1 - \hat{\phi}$. The log likelihood function is evaluated (assuming normally distributed errors) at the estimated values of the coefficients. The errors refer to the residuals $\hat{u}_t$; corresponding to the innovations u_t in the above equations (the one-period-ahead forecast errors). M.A.E. and M.A.P.E. stand for the mean absolute error and the mean absolute percentage error, respectively.

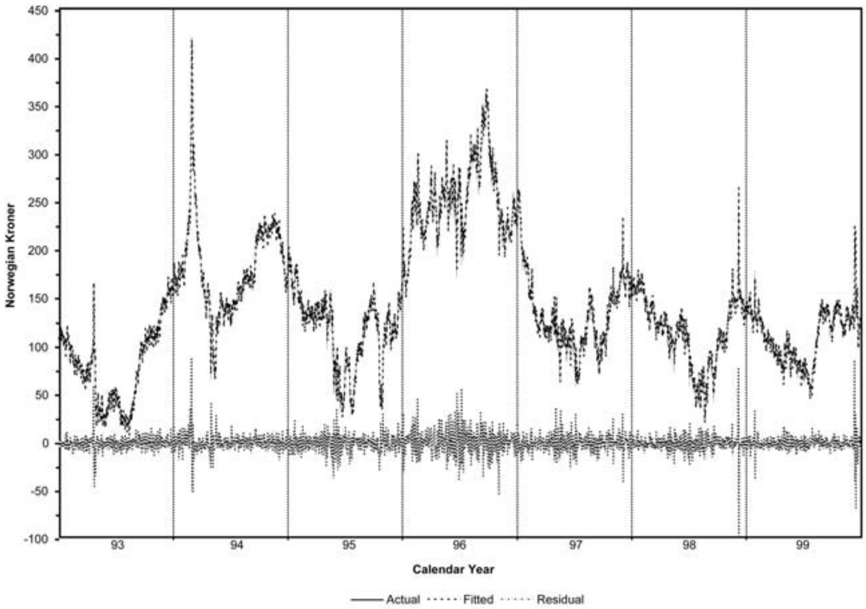

Figure 4(a). System Price vs. One Factor Model 1 Prices (1993–1999). The figure plots the actual daily system price (Actual) against the model estimates (Fitted), as well as the associated errors (Residual).

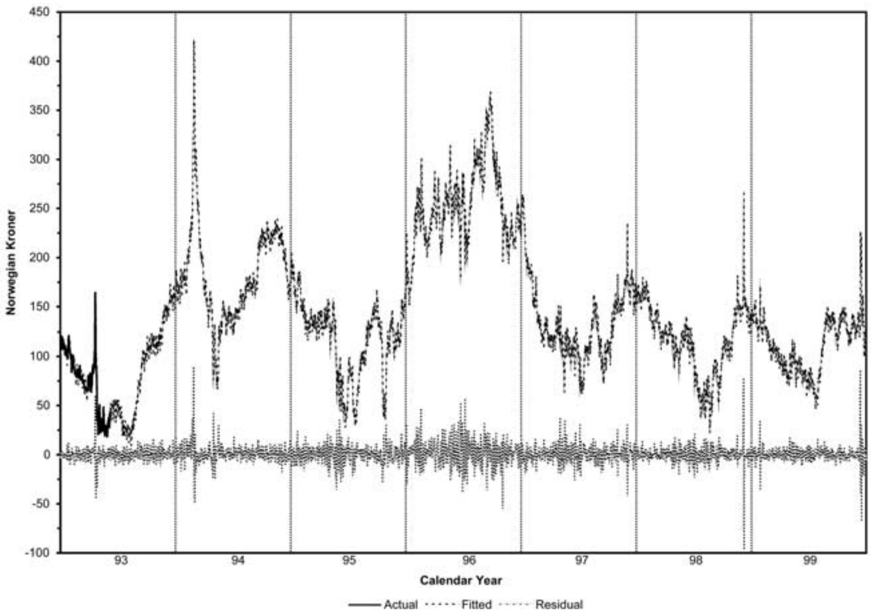

Figure 4(b). System Price vs. One Factor Model 2 Prices (1993–1999). The figure plots the actual daily system price (Actual) against the model estimates (Fitted), as well as the associated errors (Residual).

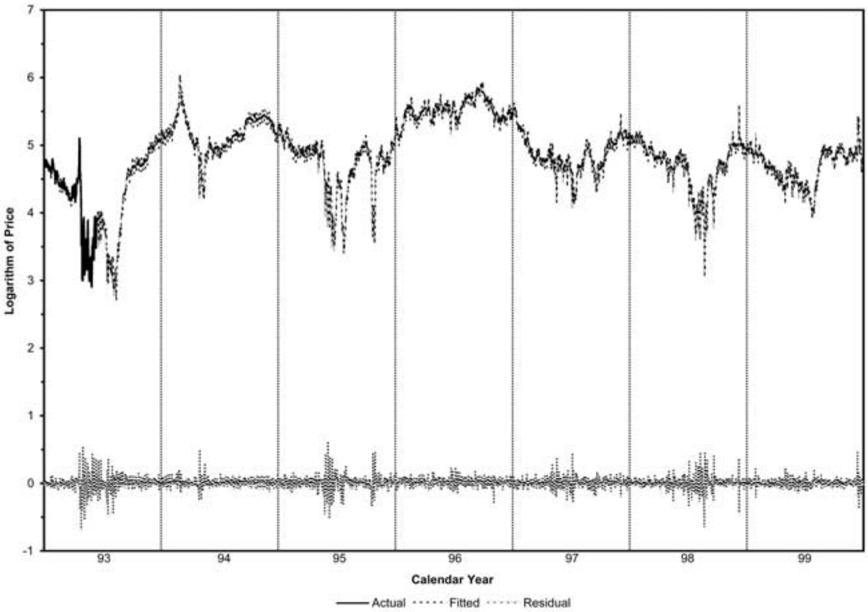

Figure 4(c). System Price vs. One Factor Model 3 Prices (1993–1999). The figure plots the actual daily system price (Actual) against the model estimates (Fitted), as well as the associated errors (Residual).

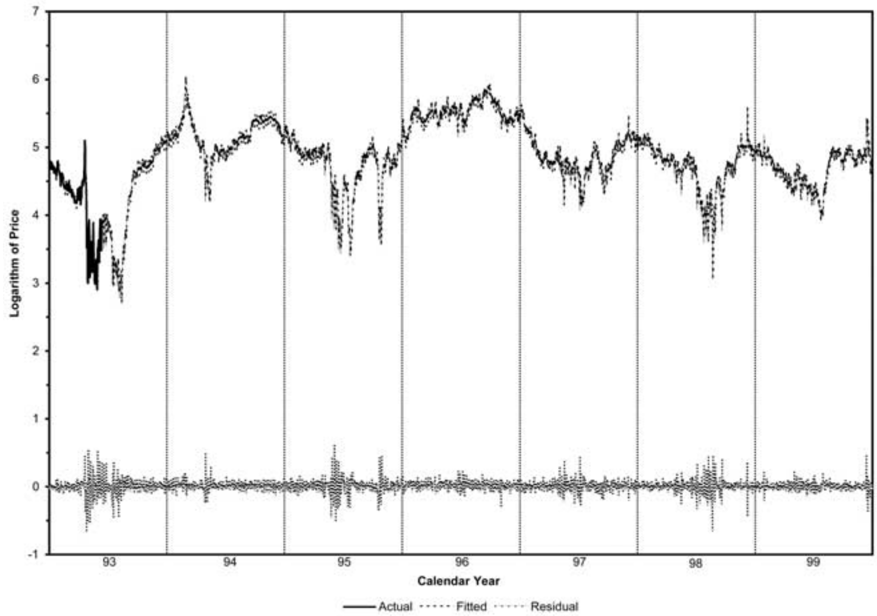

Figure 4(d). System Price vs. One Factor Model 4 Prices (1993–1999). The figure plots the actual daily system price (Actual) against the model estimates (Fitted), as well as the associated errors (Residual).

futures contracts (week, block, and season) excluding daily contracts (which started trading in September 1999), and all forward contracts (season and year). Note that we only consider the closing prices for those listed contracts actually traded on any observation date. Prices are given in NOK (per each contracted megawatt and each hour during the delivery period). Other relevant information extracted from the data files is the first day and the last day of the delivery period for each contract.

Table 3 summarizes the basic available characteristics of the derivatives sample. The number of different contracts with different delivery periods traded each day ranged from 8 to 29, with a trading volume per day and delivery period that ranges from 1 to 795 MW. They span a variety of expiration and delivery periods (the former ranging from 6 days to almost 3 years). Note that the period of three years until expiration correspond to forward contracts. The maximum period for the futures contracts in the sample is 525 days.

The Nord Pool settles futures and forward contracts on a daily basis during their delivery period. This means that a liquidation in cash is realized every day based on the difference between the reference price in Elspot for that day and the appropriate futures or forward price. We calculate the price of a futures or forward contract as the arithmetic average of the daily contract prices obtained from the formulas derived in Section 3. To be precise, let's denote $F_0(P_0;\, T_1,\, T_2)$ the futures/forward price, where T_1 and T_2 represent, respectively, the number of days till the beginning and the end of the delivery period. Then,

$$F_0(P_0;\, T_1,\, T_2) = \frac{1}{T_2 - T_1} \sum_{T=T_1}^{T_2} F_0(P_0,\, T) \tag{41}$$

with $F_0(P_0,\, T)$ given by the formula (13) or (23), as appropriate.[33]

The required parameters in the formulas were estimated from the historical system price series as shown in Section 4. The last system price included in the estimation is the system price with delivery date on the valuation date. That price was fixed the previous day. This provides us with an out of sample test of the pricing models since the required parameters were estimated for each day using only the historical series of system prices up to the valuation date.

Table 4 summarizes the valuation results for the four models based on equations (35)–(38), assuming that the market price of risk equals zero. The Root Mean Squared Errors (RMSE) and the Mean Errors (ME) are provided as measures of the performance of the models, separately for each day and each group of contracts, in both NOK and percentage terms. Finally, we average the results across the days.

The results vary a lot across days in the sample. In general, every model does a better job in explaining the longest and shortest delivery period contracts. The year forward contracts have a mean RMSE in percentage that ranges from 6.73%, for Model 1, to 9.71%, for Model 4, and mean ME in percentage ranging from 5.09% to 8.68%, respectively, for the same models. The lowest mean RMSE for the week futures contracts is 9.24% (Model 2) and the highest is 15.78% (Model 3). Overall, Models 3 and 4 for the log-price do a worse job in explaining the futures and forward prices, as compared to the

Table 3. Overview of the Futures and Forward Contracts Sample

Type of Contract	Number of Delivery Periods Traded per Day			Daily Trading Volume per Delivery Period (in MW)			Period until Expiration (in days)			Delivery Period (in days)		
	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
Futures												
Week	4.9	3.0	6.0	143.7	2.0	795.0	23.7	6.0	42.0	7.0	7.0	7.0
Block	5.4	2.0	11.0	30.4	4.0	271.0	151.2	48.0	350.0	28.0	28.0	28.0
Season	0.2	0.0	2.0	5.0	5.0	5.0	413.0	357.0	525.0	121.3	84.0	168.0
All	10.5	6.0	19.0	82.8	2.0	795.0	97.4	6.0	525.0	20.3	7.0	168.0
Forward												
Season	3.5	1.0	8.0	30.0	2.0	180.0	332.9	11.0	994.0	120.6	92.0	153.0
Year	1.6	0.0	3.0	26.5	1.0	122.0	442.7	47.0	974.0	365.5	365.0	366.0
All	5.1	2.0	10.0	28.9	1.0	180.0	367.8	11.0	994.0	198.5	92.0	366.0
All contracts	15.6	8.0	29.0	65.3	1.0	795.0	185.3	6.0	994.0	78.2	7.0	366.0

Note: This table summarizes the basic characteristics of the derivative contracts sample. The whole sample consists of every futures and forward contract traded on the first trading day of the second week of each block delivery period throughout December 1998–November 1999 (13 days), excluding contracts with delivery period of one day. The Delivery Period refers to the period of time underlying each contract and used in the cash-settlement procedure. The Expiration date is considered to be the first day of the delivery period.

Table 4 (a). Valuation Results: Model 1

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
	Panel A: In Monetary Units (NOK)											
12/14/98	5.87	10.94		14.89		9.58	−1.78	5.34		12.52		2.76
1/11/99	7.16	26.78	5.05	15.42	7.99	17.54	7.09	23.74	5.05	8.84	7.05	12.44
2/8/99	9.43	22.45		21.21	9.68	19.14	8.78	19.07		14.70	9.68	14.63
3/8/99	6.94	35.66		26.35	16.60	26.26	5.82	33.44		24.92	16.60	21.77
4/6/99	8.11	31.32		23.72	14.34	22.71	7.88	28.28		22.26	13.68	18.83
5/3/99	15.69	19.71	16.61	14.09	6.81	16.43	14.60	12.52	10.84	3.05	1.87	9.45
5/31/99	26.16	27.04		4.56	9.24	22.09	25.80	25.80		4.56	8.17	19.52
6/28/99	27.15	14.08		13.31	8.42	17.67	26.61	8.67		5.60	5.07	11.69
7/26/99	4.58	15.82		11.94	5.86	11.01	−4.39	−14.24		−10.55	5.86	−8.06
8/23/99	4.52	5.36		2.77		4.51	1.85	1.64		2.73		1.99
9/20/99	7.49	2.74		4.98	10.10	6.13	7.42	1.20		4.97	10.10	5.04
10/18/99	9.66	19.70		11.47	4.66	14.02	−9.55	8.01		8.26	−2.22	0.89
11/15/99	18.99	17.86		11.75	5.27	16.80	−18.91	6.27		4.03	−2.83	−2.88
Mean	11.67	19.19	10.83	13.57	9.00	15.68	5.48	12.29	7.95	8.14	6.64	8.31

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(a). Valuation Results: Model 1 *(Continued)*

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
Panel B: In Percentage												
12/14/98	3.76	7.76		11.85		7.05	−0.95	4.05		9.73		2.36
1/11/99	5.02	24.11	3.37	13.03	5.75	15.35	4.97	20.59	3.37	7.51	5.04	10.31
2/8/99	7.36	20.34		20.00	6.99	17.48	6.82	16.73		13.33	6.99	12.70
3/8/99	6.95	39.66		27.07	12.87	28.40	5.77	36.12		24.50	12.87	22.37
4/6/99	8.48	37.53		25.40	11.17	26.12	8.18	32.05		22.68	10.55	19.96
5/3/99	17.15	21.36	14.06	11.08	4.90	16.73	15.90	13.16	9.33	3.35	1.54	9.89
5/31/99	30.96	30.67		3.49	7.03	25.44	30.53	28.41		3.49	6.16	21.64
6/28/99	41.21	16.10		11.38	6.58	23.58	40.35	10.36		5.22	4.02	15.36
7/26/99	5.97	11.17		9.47	4.54	8.61	−5.75	−10.43		−8.26	4.54	−7.04
8/23/99	3.68	4.14		1.99		3.54	1.53	1.45		1.97		1.61
9/20/99	5.76	1.93		3.57	7.56	4.63	5.70	0.88		3.57	7.56	3.81
10/18/99	6.31	16.76		9.03	3.11	11.42	−6.25	8.07		6.64	−1.44	1.82
11/15/99	12.50	15.40		9.24	3.51	13.11	−12.46	6.46		3.74	−1.83	−0.67
Mean	11.93	18.99	8.72	12.05	6.73	15.50	7.26	12.92	6.35	7.50	5.09	8.78

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(b). Valuation Results: Model 2

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
	Panel A: In Monetary Units (NOK)											
12/14/98	6.16	3.70		1.23		4.97	0.93	−2.72		−1.08		−0.43
1/11/99	6.81	19.88	11.87	14.31	7.76	14.35	6.70	17.62	11.87	10.81	6.81	11.48
2/8/99	5.81	28.62		18.51	9.45	20.70	5.29	26.87		16.28	9.45	17.22
3/8/99	2.02	26.65		29.49	17.09	22.10	−1.63	22.34		27.38	17.09	15.64
4/6/99	0.96	30.68		30.62	15.64	23.33	−0.13	27.20		29.08	14.96	17.20
5/3/99	4.14	23.36	17.33	14.75	7.58	17.02	3.67	20.54	15.74	10.91	3.32	12.61
5/31/99	14.70	21.49		33.33	11.61	18.77	14.56	21.28		33.33	10.56	17.25
6/28/99	23.63	32.94		18.96	11.05	22.96	22.53	32.75		16.03	8.11	20.01
7/26/99	5.06	17.53		16.46	11.83	13.02	3.58	16.91		15.90	11.83	10.69
8/23/99	16.91	34.49		25.99		26.88	12.98	34.46		24.18		23.84
9/20/99	22.28	23.61		21.74	11.77	22.12	21.58	22.10		20.53	11.77	20.92
10/18/99	4.81	6.01		3.69	3.87	5.01	2.83	2.49		−2.64	−1.86	1.10
11/15/99	8.92	6.87		6.48	4.94	7.38	−8.25	−3.68		−6.16	−4.10	−5.43
Mean	9.40	21.22	14.60	18.12	10.24	16.82	6.51	18.32	13.80	14.97	7.99	12.47

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(b). Valuation Results: Model 2 *(Continued)*

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
Panel B: In Percentage												
12/14/98	4.15	2.34		0.83		3.32	0.80	−1.76		−0.75		−0.18
1/11/99	4.77	15.36	7.92	10.23	5.59	10.71	4.70	13.95	7.92	7.73	4.87	8.55
2/8/99	4.54	22.44		14.62	6.83	16.24	4.12	21.10		12.75	6.83	13.46
3/8/99	1.93	28.08		24.91	13.25	21.41	−1.55	23.66		24.08	13.25	15.14
4/6/99	1.01	29.82		26.74	12.18	21.80	−0.11	27.05		26.36	11.53	16.20
5/3/99	4.54	20.09	12.17	10.68	5.53	14.08	4.00	17.43	11.41	7.92	2.56	10.40
5/31/99	17.40	23.69		25.52	8.82	18.68	17.23	23.01		25.52	7.93	17.37
6/28/99	35.83	31.02		14.62	8.66	25.12	34.17	30.60		12.18	6.28	20.91
7/26/99	6.19	15.09		12.88	9.17	11.29	4.32	14.02		12.29	9.17	9.27
8/23/99	13.76	25.73		19.25		20.30	10.48	25.61		17.76		17.98
9/20/99	17.03	16.47		15.84	8.81	16.20	16.53	15.34		14.87	8.81	15.32
10/18/99	3.12	4.37		2.88	2.58	3.53	1.82	1.67		−2.10	−1.21	0.67
11/15/99	5.92	5.19		4.55	3.29	5.21	−5.46	−2.42		−4.43	−2.76	−3.63
Mean	9.24	18.44	10.05	14.12	7.70	14.45	7.00	16.10	9.67	11.86	6.12	10.88

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(c). Valuation Results: Model 3

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
Panel A: In Monetary Units (NOK)												
12/14/98	5.76	15.68		17.00		11.73	15.18	8.89		1.76		11.02
1/11/99	12.52	31.10	10.15	16.00	10.05	20.48	12.37	28.62	10.15	11.61	9.31	16.51
2/8/99	14.14	27.04		22.99	12.18	22.43	13.28	24.32		18.17	12.18	18.89
3/8/99	11.32	41.00		33.13	19.77	31.13	10.30	38.54		32.20	19.77	26.92
4/6/99	11.24	40.51		31.49	17.34	29.48	11.07	38.20		30.59	16.67	25.37
5/3/99	26.37	27.53	17.64	14.59	8.56	22.62	23.50	21.54	13.98	7.59	4.95	16.34
5/31/99	39.66	37.37		15.85	12.64	32.75	39.46	36.30		15.85	11.62	29.95
6/28/99	28.98	19.57		14.83	11.06	20.05	28.28	17.08		10.28	8.01	16.09
7/26/99	2.72	7.60		4.98	11.34	6.04	−2.31	−4.35		−0.61	11.34	−1.82
8/23/99	12.01	14.19		11.76		12.82	9.32	12.50		11.64		11.10
9/20/99	15.23	9.23		12.17	13.08	12.78	15.01	8.77		12.15	13.08	12.24
10/18/99	5.06	19.82		11.02	4.50	13.24	−4.76	11.73		9.67	−0.25	4.31
11/15/99	15.29	18.47		11.41	5.47	15.88	−15.09	10.74		8.07	−0.02	1.17
Mean	15.41	23.78	13.89	16.71	11.45	19.34	11.97	19.45	12.06	13.00	9.70	14.47

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(c). Valuation Results: Model 3 *(Continued)*

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
Panel B: In Percentage												
12/14/98	3.89	11.18		13.05		8.56	3.23	9.25		12.37		6.53
1/11/99	8.78	27.54	6.78	13.44	7.22	17.52	8.67	24.52	6.78	9.35	6.63	13.27
2/8/99	11.04	24.17		21.45	8.80	20.12	10.31	20.98		15.87	8.80	16.00
3/8/99	11.26	45.42		33.36	15.33	33.31	10.15	41.56		31.20	15.33	27.44
4/6/99	11.63	47.53		32.86	13.48	33.23	11.45	42.52		30.55	12.84	26.55
5/3/99	28.87	29.27	14.90	11.68	6.29	23.45	25.64	21.03	11.57	6.48	3.71	16.03
5/31/99	46.94	41.95		12.14	9.59	37.51	46.69	39.68		12.14	8.72	32.86
6/28/99	44.02	21.71		12.60	8.67	26.07	42.91	17.97		8.69	6.22	19.26
7/26/99	3.92	5.41		3.85	8.79	4.97	−3.20	−2.61		−0.65	8.79	−1.77
8/23/99	9.77	11.06		8.61		10.01	7.53	9.58		8.49		8.54
9/20/99	11.68	6.48		8.77	9.80	9.56	11.52	6.12		8.74	9.80	9.07
10/18/99	3.33	17.29		8.69	3.08	11.27	−3.12	10.60		7.52	−0.08	4.09
11/15/99	10.08	16.46		9.23	3.78	13.07	−9.95	9.67		6.53	0.14	2.14
Mean	15.78	23.50	10.84	14.59	8.62	19.13	12.45	19.30	9.18	12.10	7.35	13.85

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(d). Valuation Results: Model 4

	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
Observation Date	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
Panel A: In Monetary Units (NOK)												
12/14/98	15.44	10.24		9.16		13.18	15.18	8.89		1.76		11.02
1/11/99	16.85	26.55	25.01	22.80	10.49	22.03	16.78	20.82	25.01	15.16	9.77	17.18
2/8/99	11.31	38.19		25.69	12.53	28.20	10.70	34.42		20.79	12.53	22.91
3/8/99	4.20	25.30		39.06	20.74	24.69	3.53	22.49		34.77	20.74	19.08
4/6/99	3.83	37.02		39.80	18.77	28.80	3.58	32.15		36.20	18.12	21.83
5/3/99	6.11	33.15	24.90	23.21	9.41	24.63	5.84	28.98	17.97	16.88	6.20	18.16
5/31/99	15.42	19.97		47.79	14.52	22.25	15.34	19.69		47.79	13.57	19.52
6/28/99	24.60	46.04		28.68	13.90	30.64	23.63	44.36		22.62	11.32	25.68
7/26/99	4.59	34.31		31.11	16.66	24.24	3.30	33.17		30.83	16.66	18.86
8/23/99	22.12	55.54		42.17		42.25	17.12	55.35		40.11		37.20
9/20/99	33.12	45.62		38.14	15.87	37.94	31.55	43.26		36.74	15.87	35.35
10/18/99	19.24	20.96		12.98	4.38	17.56	16.69	9.69		−3.13	1.50	8.58
11/15/99	10.78	9.72		8.18	4.28	9.51	6.36	0.22		−1.26	0.07	1.99
Mean	14.43	30.97	24.96	28.37	12.87	25.07	13.05	27.19	21.49	23.02	11.49	19.80

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

Table 4(d). Valuation Results: Model 4 *(Continued)*

Observation Date	Root Mean Squared Error						Mean Error					
	Futures			Forward			Futures			Forward		
	Week	Block	Season	Season	Year	All	Week	Block	Season	Season	Year	All
	Panel B: In Percentage											
12/14/98	9.99	6.63		6.50		8.61	9.75	5.79		0.67		7.02
1/11/99	11.80	19.22	16.69	15.74	7.53	15.54	11.75	15.68	16.69	10.17	6.95	12.18
2/8/99	8.83	28.47		18.80	9.05	20.93	8.31	26.20		15.32	9.05	17.26
3/8/99	4.21	26.49		32.25	16.08	22.80	3.50	23.69		30.03	16.08	18.14
4/6/99	4.04	33.88		33.61	14.58	25.55	3.73	31.04		32.07	13.95	20.17
5/3/99	6.67	25.68	16.94	16.43	6.93	18.69	6.36	23.09	12.29	11.56	4.59	14.19
5/31/99	18.25	22.14		36.59	11.00	20.80	18.15	21.35		36.59	10.16	19.14
6/28/99	37.31	39.84		21.69	10.85	29.87	35.83	39.64		16.64	8.66	25.27
7/26/99	5.59	25.87		24.07	12.92	18.63	4.00	25.53		23.67	12.92	15.14
8/23/99	17.99	40.88		31.16		31.48	13.83	40.85		29.40		27.86
9/20/99	25.27	31.65		27.72	11.89	27.41	24.14	29.94		26.57	11.89	25.68
10/18/99	12.46	13.54		9.54	3.06	11.51	10.84	5.36		−3.05	1.11	5.09
11/15/99	7.07	6.92		6.25	2.97	6.59	4.13	−0.46		−1.49	0.17	0.95
Mean	13.04	24.71	16.82	21.57	9.71	19.88	11.87	22.13	14.49	17.55	8.68	16.01

Note: This table summarizes the valuation errors for each observation date (see Table 3) assuming a zero market price of risk. Two measures of the valuation errors are provided: the Root Mean Squared Error, and the Mean Error. Both measures have been calculated, alternatively, on the valuation errors in monetary units (NOK), in Panel A, and on the percentage valuation errors, in Panel B. The error in monetary units is defined as the theoretical model price minus the actual market price (closing price). The percentage error is defined as the error in monetary units times 100 divided by the market closing price.

results for Models 1 and 2 for the system price. The global results for Models 1 and 2 are quite similar. Although Model 1 performs better for block, season and year contracts, it does a worse job in explaining the week contracts. Model 2 shows the lowest mean RMSE in percentage for all contracts (14.45%), under the restrictive assumption of a market price of risk equal to zero.

Finally, we computed an implicit (constant) market price per unit risk independently for each day by minimizing the RMSE in NOK for all contracts. We also calculated an implicit market price of risk for the whole sample in the same way. The implicit market price of risk together with its associated RMSE (in both monetary units and percentage) for each model is reported in Table 5. The implicit lambda is positive for most of the days and models. For the whole sample, it takes value of 0.011 for Model 1, 0.018 for Model 2, 0.019 for Model 3, and 0.033 for Model 4. Models 2 and 4 (models with the sinusoidal seasonal function) experience the larger percentage reduction in the mean RMSE in percentage (23% and 37%, respectively, against 7% and 17% for Models 1 and 3). In addition, the percentage reduction in the RMSE is larger for the models for the log-price as compared to the models for the system price. The resulting mean RMSE in percentage are lower for models based on the price than models for the log-price, and lower for models with the sinusoidal seasonal function. Overall, with a constant market price of risk equal to the implicit market price of risk calculated for the whole sample, Model 2 has the lowest RMSE (11.1%), followed by Model 4 (12.5%), and finally Models 1 and 3 (14.4% and 15.9%, respectively).[34]

In summary, with respect to the type of one-factor models studied in this paper, these results show that those models based on the price do a better job in explaining actual futures and forward prices than those models based on the log-price. Moreover, the simple sinusoidal function is better in incorporating the seasonal pattern observed in spot prices into the futures and forward prices. Additionally, a market price of risk different from zero is required in this market.

In order to gain some insight into the causes for the discrepancies between actual derivatives prices and the theoretical prices, Figure 5 allows for a graphical comparison of the theoretical prices provided by Model 2 with the actual futures and forward prices. This is done for two specific dates: January 11, 1999 (Figure 5(a)), and May 3, 1999 (Figure 5(b)). They belong to a cold and a warm season, respectively, and they were selected based on the large number of different traded contracts. Each horizontal line indicates a different futures or forward price. The length of the line represents the delivery period of the contract. As in Tables 4 and 5, only those contracts actually traded on the selected dates have been considered. In order to make a clear figure, the daily Model 2 theoretical prices have been substituted by a seven days centered moving average. The remaining discontinuities in the lines are due to the concentration of holidays for some periods during the year. The market price of risk is assumed to be, alternatively, zero or the implicit market price of risk for the given day.

It can be seen that the seasonal pattern implicit in the actual derivative prices does not exactly matches the one extracted from historical spot prices (incorporated into the theoretical prices by the model), though it is reasonably close for week and block futures contracts. Another difference is that the actual term structure is increasing while the

Table 5. Implicit Market Price of Risk for the One Factor Models

	Models Based on the Price						Models Based on the Log-price					
	Model 1			Model 2			Model 3			Model 4		
Observation Date	Implicit λ	RMSE (in NOK)	RMSE (in %)	Implicit λ	RMSE (in NOK)	RMSE (in %)	Implicit λ	RMSE (in NOK)	RMSE (in %)	Implicit λ	RMSE (in NOK)	RMSE (in %)
12/14/98	0.012	8.0	5.3	−0.003	4.8	3.2	0.019	5.4	3.6	0.018	9.7	7.0
1/11/99	0.014	13.3	10.9	0.016	8.5	6.2	0.020	13.4	10.8	0.026	12.3	9.2
2/8/99	0.017	13.4	11.6	0.024	9.7	7.5	0.023	13.6	11.6	0.036	11.4	8.9
3/8/99	0.036	11.4	11.7	0.032	9.0	8.5	0.049	11.8	11.9	0.045	7.4	6.6
4/6/99	0.028	13.7	15.3	0.034	10.5	9.3	0.042	15.7	17.1	0.049	10.0	8.3
5/3/99	0.008	15.1	14.7	0.017	11.5	9.2	0.017	18.7	18.8	0.029	13.5	10.2
5/31/99	0.022	17.6	19.5	0.028	10.5	9.7	0.042	23.9	25.9	0.038	10.7	9.5
6/28/99	0.009	16.3	21.3	0.024	15.8	17.8	0.016	16.8	22.1	0.039	17.7	18.3
7/26/99	−0.014	7.2	5.7	0.021	5.6	5.1	−0.002	5.9	4.9	0.042	6.1	5.3
8/23/99	0.004	4.1	3.2	0.051	11.3	8.4	0.019	7.1	5.5	0.073	10.8	7.8
9/20/99	0.007	4.8	3.6	0.035	13.6	10.1	0.017	7.8	5.8	0.062	16.0	11.7
10/18/99	0.007	13.1	10.0	0.000	5.0	3.5	0.010	11.4	9.0	0.014	15.5	11.0
11/15/99	0.005	16.5	12.4	−0.006	6.4	4.8	0.009	14.8	11.4	0.003	9.3	6.7
Mean	0.012	11.9	11.2	0.021	9.4	8.0	0.022	12.8	12.2	0.036	11.6	9.3
Std. Deviation	0.012	4.5	5.7	0.017	3.3	3.8	0.015	5.4	7.0	0.019	3.4	3.3
Whole Sample	0.011	14.8	14.4	0.018	12.9	11.1	0.019	15.8	15.9	0.033	15.4	12.5

Note: This table reports the implicit market price per unit risk for the one factor models. It has been calculated independently for each observation date (the arithmetic average and standard deviation values are also provided), and for the whole sample (the sample is the same as in Table 4). The market price of risk has been obtained by minimizing the Root Mean Squared Error (RMSE) in monetary units for all the contracts. The implicit market price per unit risk is reported together with its associated RMSE for all contracts in monetary units (NOK) and in percentage (%). The RMSE is defined as the square root of the arithmetic average of the squared pricing errors. The pricing error in monetary units is defined as the theoretical model price minus the actual market (closing) price. The pricing error in percentage is the pricing error in monetary units times 100 divided by the actual market price.

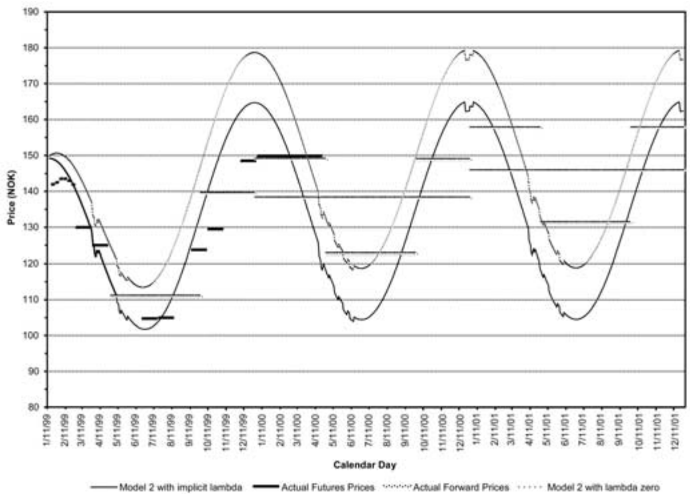

Figure 5(a). Model 2 Theoretical Values vs. Actual Futures and Forward Prices (1/11/1999). The figure plots futures and forward closing prices as well as the daily theoretical prices provided by the (one factor) Model 2, for day January 11, 1999. Only prices of contracts actually traded on those dates have been considered. For the actual prices, the length of the line indicating the price level represents the duration of the delivery period in days. A centered moving average of seven daily theoretical prices has been used to represent the theoretical term structure of futures/forward prices.

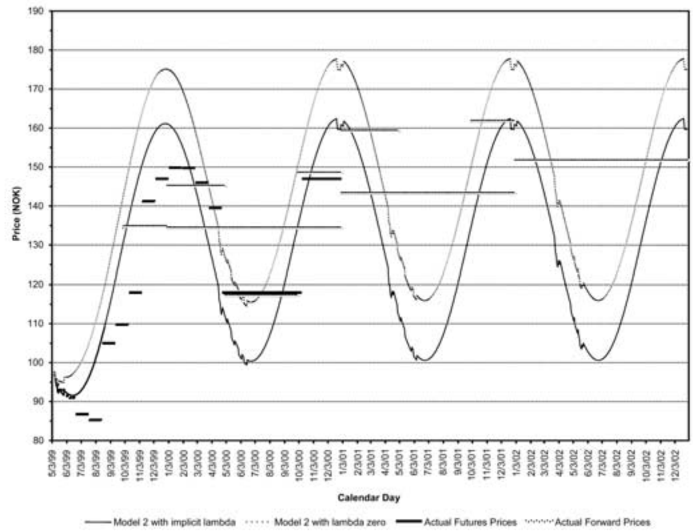

Figure 5(b). Model 2 Theoretical Values vs. Actual Futures and Forward Prices (5/03/1999). The figure plots futures and forward closing prices as well as the daily theoretical prices provided by the (one factor) Model 2, for day May 3, 1999. Only prices of contracts actually traded on those dates have been considered. For the actual prices, the length of the line indicating the price level represents the duration of the delivery period in days. A centered moving average of seven daily theoretical prices has been used to represent the theoretical term structure of futures/forward prices.

theoretical one fluctuates around a constant long-run level. Even though there is no trend in the spot sample data, forward and future market prices seem to imply expectations of a future trend. Alternatively, the market price of risk could be time dependant. In the next section we explore two two-factor models that incorporate a stochastic trend by adding a second state variable.

6. Futures and Forward Valuation Using Two Factor Models

To estimate the stochastic process for the one factor model we only needed a time series of spot prices, which was available from 1993 to 1999. On the other hand, to implement the two factor models we need time series data of both spot and futures/forward prices with several maturities in order to estimate the two unobservable state variables. Schwartz (1997) shows how to estimate unobservable state variables from spot and futures prices by using a Kalman filter procedure. However, the procedure requires liquid futures contracts with homogeneous maturities. Unfortunately, Nord Pool is not liquid enough to provide with a long time series of diverse futures and forward prices with homogenous maturities.

To overcome those difficulties, we implemented the two factor models by first estimating the deterministic components of the models using spot price data prior to the valuation period. By using a long enough time series we are able to obtain reliable estimates for the seasonal components (from January 1993 throughout December 1998). Then, we estimated the remaining parameters together with the state variables implicitly from the spot, futures and forward prices during the valuation period, from December 1998 to November 1999 (see Section 5 for the specific dates).

In order to calculate the implicit parameters, we used an iterative procedure that consists of the following steps. First, for a given set of parameters, for every observation date in the sample we individually find the state variable that minimizes the sum of square errors between model and market prices of spot, futures and forward prices for that day. Second, we estimate the volatilities and correlation parameters from the time series of the estimated state variables. Third, given those implied state variables and parameters, we implicitly estimate the remaining parameters by minimizing the sum of square errors for the whole spot and derivative contracts sample. With this new set of parameters we repeat the process until convergence is achieved.[35]

Given the characteristics of our sample, and in order to avoid the estimation problems pointed out by Schwartz and Smith (2000, Section 6.1), we concentrate on the risk neutral parameters (see Table 6, and Section 3). In addition, based on the results of the previous section we concentrate on the cosine representation (33) of the deterministic seasonal component.

To be able to assess the improvements of the two factor models relative to the corresponding one-factor models, we also estimated the latter using exactly the same iterative procedure described above. Table 6 reports the results of these estimations. The implicit procedure we used does not give us the standard errors of the parameter estimates. The root mean square errors, however, clearly indicate that the two factor models are superior to the one-factor models. For the models using spot prices the

Table 6. Valuation Results by Implicit Methods

Parameter	Price Models		Log-price Models	
α	151.08		4.89	
β	−10.24		−0.10	
γ	30.27		0.31	
τ	3.96		−2.24	
	One Factor	Two Factor	One Factor	Two Factor
κ	0.0014	0.0077	0.0012	0.016
α^*	9.57	−53.74	0.13	−0.17
μ_ε^*		−0.029		-0.06×10^{-3}
σ_X	2.36	5.77	0.018	0.056
σ_ε		3.10		0.019
ρ		−0.81		−0.63
RMSE (NOK)	8.11	6.99	10.85	8.94

Note: This table reports the valuation results for the one and two factor models using a sinusoidal function for the seasonal component. The parameters corresponding to the deterministic seasonal term were obtained by minimizing the sum of the squared errors from the spot price previous to the valuation sample period (the spot prices run from January 1, 1993 throughout December 14, 1998). The remaining parameters where obtained by implicit methods using the whole futures and forward sample reported in Tables 4 and 5, by minimizing the sum of the squared errors (see the main body of the text for details). The Root Mean Squared Errors in NOK for all futures and forwards contracts are also provided in the last row.

RMSE improves from 8.11 to 6.99 (a 14% improvement), and for the models using the log spot prices these figures are 10.85 and 8.94 (an 18% improvement).[36] Once again models based on the spot price perform better than models based on the log spot price.

Comparing the RMSE for the one factor models in Table 6 with those obtained for the same models in Table 5, we note that these are substantially lower when using the iterative procedure. The RMSE improves by 37% for the model based on the price and by 30% for the model based on the log price. This is to be expected since in the iterative procedure more parameters are implicitly estimated from the data, whereas in the previous estimation procedure only the market price of risk is estimated from the futures and forward prices.

This preliminary analysis suggests that there is a substantial improvement to be made in modeling electricity prices using two factor models.[37] The implementation of these more complex models will become feasible when additional futures and forward price data becomes available.

7. Concluding Remarks

In this paper we have studied one and two factor models for the valuation of power derivatives. All of them include a deterministic component that accounts for genuine regularities in the behavior of electricity prices. The inclusion of this term has been motivated by a thorough analysis of the behavior of the underlying electricity price at the Nord Pool, and an inspection of the term structure of futures prices in this market.

The analysis clearly reveals that at least some of those regularities, especially the seasonal pattern, play a central role in explaining the shape of the term structure of futures prices at the Nord Pool. The valuation results are promising. There are, however, inherent difficulties in the estimation of a genuine regular pattern, such as the seasonal component, in the time behavior of a given variable. An accurate estimation the annual seasonal pattern requires a large time series, i.e. a large number of years. To minimize this problem, we have conducted our empirical analysis on spot price data from one of the oldest wholesale electricity markets in the world.

We have estimated the deterministic component of the models from data of the underlying variable, i.e. the relevant electricity spot price. The estimates can then be used to value any electricity derivative contract. Another procedure not explored here consists of extracting that component from the actual term structure of futures or forward prices. This then, can be used to value other derivatives such as option contracts written on them.

Given the limitations of the data available, we conducted a preliminary test of models that consider two stochastic factors. The results of the empirical analysis are encouraging. Other possible specifications of the second stochastic factor are possible. Our analysis of the system price in the Nord Pool has revealed, for instance, that the volatility is consistently different between cold and warm seasons. A mean reverting diffusion process for the volatility could be considered.

Considering the non-storability of electricity, an important extension of the model would be to include jumps in spot prices. Since the supply of electricity is very inelastic in the short run, increases in demand beyond the production and transmission capacity of the system can produce sharp increases in spot prices. Recent events in California show that this problem can become of dramatic proportions.

As the necessary data becomes available, this type of analysis should be extended to other electricity markets. As has been noticed in the introduction, there are important reasons to expect several substantial differences in the behavior of electricity prices corresponding to different geographic locations, hours of the day, or time periods during the year. These differences may require adjustments to the specific structure of the deterministic component of the models.

An important topic for further research is the issue of liquidity in the electricity markets. Of special interest would be the relation between volume of trade and volatility in these markets. Finally, several issues regarding the electricity market microstructure deserves further study.

Notes

1. Additionally, regulatory issues such as market rules and market structure may also have an impact on the behavior of prices in competitive electricity markets and, consequently, on their differences across countries.

2. Other papers that have studied the behavior of the Nord Pool spot prices with different objectives include Wolak (1997) and Johnsen, Verma and Wolfram (1999).
3. The grid operators are responsible for ensuring a well-functioning physical system, including the system balance, that is, the equilibrium required in every instant between the energy produced/imported in any area and the sum of consumption/export and network losses in the same area.
4. Eastern Denmark joined the market on October 1, 2000 under the same conditions applied to western Denmark.
5. According to the Press Release 1/2000 of Nord Pool ASA, in 1999 a total of 75 TWh (terawatt-hour, 1 TWh = 10^6 MWh) were traded in the spot market (this implies a volume growth of 34% over 1998), and implies that more than 20% of the total consumption of electric power in the Nordic countries was traded via Nord Pool. Also, a total of 216 TWh were traded in the market for financial power contracts in 1999 (representing a volume increase of 143% compared with 1998). Finally, a total of 684 TWh of electric power traded in the OTC market was cleared by Nord Pool in 1999 (83% higher than in 1998). At the end of 1999, Nord Pool had got 264 market participants.
6. As a matter of fact, market participants communicate their generation and demand bids for each hour independently for each bidding area. Sweden, Finland, and western Denmark are always independent bidding areas. Additionally, if necessary, the Norwegian system operator divide the country into two or more different bidding areas. These areas used to be fixed and communicated to market participants on a weekly basis until the first week of 2000. They are fixed on a seasonally basis thereafter.
7. Nord Pool is involved in *Elbas*, a physical market for short-term trade launched in March 1999 by the Finish electricity exchange El-Ex, Electricity Exchange Ltd., that allows traders in Sweden and Finland to adjust imbalances after the day-ahead spot market is closed (see El-Ex (1999)).
8. Season futures contracts are not longer traded since the end of 1999. At the beginning of year 2000, Nord Pool announced that the Season 3-2000 contract, which had been split at the end of 1999, would be the last season futures contract listed for trading, and that new block contracts would be listed in accordance with past practice on former block-splitting dates in weeks 1, 17, and 41 (Nord Pool participant information no. 1-00 and no. 17-00).
9. Recorded dates in Elspot data correspond to delivery dates.

   We discarded the initial period of available data, i.e. the last eight months of 1992, because those data corresponded to the period when trading activities were conducted by the so-called "Coordinated Norwegian power works," a membership association based on mutual responsibility. In January 1993 its activities were transferred to the newly created power market, first known as Statnett Marked and later as Nord Pool. Statnett Marked started acting as the counterpart for buyers and sellers in the clearing of market trades, which included both spot market trades and trades in a new "futures market" for delivery on future dates.

   Additionally, the transition from standard time (GMT+1) to summer time (daylight savings time, GMT+2) and vice versa imply either the reduction of one hour or the addition of an extra one, respectively, on transition dates. We standardized to 24 the number of hours per day in the following way. We set the price of any missing value hour equal to the mid point between the two adjacent hours' prices, and any extra hour was eliminated by dividing the price corresponding to the double value hour by two. We work with the resulting standardized series in the sequel.
10. Before we move on to the analysis of the system price time series, here follows some basic figures of interest on the generation side of the Nordic electricity system. Power generating sources vary among the Nordic countries. According to 1998 data published by the national grid operators and Nord Pool (see TSO and Nord Pool (1999)), almost all the Norwegian electricity production is hydroelectric (99%); Sweden relies in equal portions on hydropower (48%) and nuclear power (46%) with a residual production obtained from other thermal (gas and oil) units (6%); Finland has a mixture of hydro (22%), nuclear (31%), and thermal plants using a variety of alternative fuels (47%); Denmark mainly produces thermal (natural gas/coal) power (93%) with some wind production units (7%).
11. A large number of lags, $p = 21$, is necessary in the relevant regression model, in order to account for the serial correlation present in the changes of the relevant variables $\Delta y_t \equiv y_t - y_{t-1}$. Phillips–Perron tests for a unit root (see, e.g. Hamilton, 1994: 17.6) were also conducted and they provided virtually the same evidence as the ADF tests.

12. This is obtained by multiplying the standard deviation by the square root of 365.
13. This fact was previously detected by Johnsen, Verma and Wolfram (1999: p. 34). A cold season is defined here as any period running from October through April, and a warm season from May through September (the latter is the delivery period for summer forward contracts listed in Eltermin). Care has been taken to avoid the calculation of changes in the price and log-price series using a couple of observations one season apart.
14. One reason to expect a somehow lower volatility during cold seasons is that, as during these seasons hydroelectric production depends on the amount of water available in the reservoirs, prices may be less subject to supply side shocks, as argued by Johnsen, Verma and Wolfram (1999: p. 17).
15. These large daily variations occurred at the end of February 1994, and in the middle of December 1998.
16. Kaminski (1997) further elaborates on this idea.
17. See Johnsen, Verma and Wolfram (1999: pp. 48–52) for an account of some important episodes of high system prices and their possible causes.
18. For instance, an autocorrelation at lag seven of the daily change of the price (respectively, the log-price) of 31.5% (41%) implies that the R^2 of a regression of the first-differences of the prices (log-prices) on a constant and its seventh lag is 0.099 (0.168), i.e. the square of the autocorrelation coefficient. Therefore, 10% (17%) of the variation in the daily price (log-price) increments is predictable using the daily increment seven days apart.
19. These facts were also previously pointed out by Johnsen, Verma and Wolfram (1999: p. 17).
20. We took as holidays the official public holidays in Norway excluding Saturdays and Sundays made public by the Norwegian Ministry of Foreign Affairs through ODIN. Note though that the majority of these dates were common holidays in the whole Nordic area. Besides, Norway represents more than 60 per cent of the spot market, based on traded volume (see the Nord Pool Annual Report 1998).
21. Seasonal climatic variations also influence hydroelectric generation through their impact on the level of water stored in the reservoirs. In the Nordic countries a large part of the inflows comes from snow melting during the warm season, so it is predictable to some extent. (This fact somehow facilitates hydro plants to solve their intertemporal profit maximization problem; see Jonhsen, Verma and Wolfram (1999: p. 20) and the references therein.) Publicly available weekly mean data from Nord Pool for the period 1981–1992 show that the minimum level of reservoirs electric production capacity is reached around the seventh week (29.7%) and the maximum around the forty-second week (90%).
22. Figure 3 is based on the closing prices for every listed futures contracts on a given date regardless of whether they were traded on that date or not. This does not represent a problem for our present purposes of motivating the discussion to follow in the next sections. Later, in the empirical examination of the valuation models, only traded contracts will be considered. We also postpone the detailed description of the futures contract sample until that moment.
23. Jaillet, Ronn and Tompaidis (1998) and Manoliu and Tompaidis (1999) use similar models in explaining natural gas futures prices.
24. See also Hull and White (1993). For convenience, we work directly with the variable X_t, i.e. the price detached from the deterministic component, and the function $f(t)$, instead of the price process (4) in terms of a given function $a(t)$.
25. Under the assumption of a constant interest rate, forward and futures prices are equal.
26. Note that although we use the same symbols for the parameters in (2) and (15) to simplify the notation, the parameters values are not the same.
27. To simplify, we use the same notation as in the previous subsection. Of course, the values of the parameters need not be equal.
28. This is the approach followed by Jaillet, Ronn and Tompaidis (1998) and Manoliu and Tompaidis (1999).
29. Of course, we can add two or more of the above mentioned possibilities when appropriate. Pilipovic (1998), for instance, has claimed that some electricity prices may require two sinusoidal functions in order to capture the two peaks, one in winter and the other one in summer, arising respectively from the use of heating and air conditioning. The relative importance of both peaks depend greatly on the geographical location.
30. We also added a linear time trend to the models, but eventually empirical evidence based on the behavior of the spot price led us to discard it.
31. For daily observations, this is a very reasonable approximation to the available exact discretization.

32. The inference results should be carefully interpreted since, although the model accounts for the serial correlation of first order through the inclusion of first-order autoregresive disturbances, there is still significative autocorrelation present in the residuals at lags multiple of seven. This fact is related to the regular pattern within the week, which the models have not accounted for. The inclusion of dummy variables for each day of the week, that was originally discarded based on their significativity, may be reconsidered based on this result.
33. This is an approximation. The correct formula, which requires an estimate for the riskless interest rate, is given by: $F_0(P_0;\ T_1,\ T_2) = (1/\sum_{T=T_1}^{T_2} e^{-rT}) \sum_{T=T_1}^{T_2} e^{-rT} F_0(P_0,\ T)$ with $F_0(P_0,\ T)$ given by the formulas (13) and (23). In order to asses the validity of the approximation, we simulated the theoretical values given by the correct formula for several sets of realistic values of the models parameters and a range of interest rates, and compared them with the theoretical values provided by the approximated formula. The results turned out to be very close.

    Additionally, we work with standardized days of 24 hours each. The Nord Pool's liquidation procedure, however, accounts for the exact number of hours during the delivery period (see Nord Pool (1999c) for the details).
34. It should be pointed out that these RMSE are two to three times larger than the ones reported by Schwartz (1997) in a one-factor model for oil and copper futures. But, it should also be noted that the electricity derivatives are considerable more complex than those of commodities such as copper or oil.
35. We start the procedure with an initial guess for the parameters to be estimated based on the results from previous sections.
36. Once again it should be pointed out that these RMSE are more than three times larger than those reported by Schwartz (1997) for a two factor model for copper and oil futures.

    Additionally, Manoliu and Tompaidis (1999) implement one factor and two factor models with a stepwise seasonal term, and they get a very good fit to the natural gas futures curve (mean absolute errors range from 0.02 to 0.4 dollars depending on the model and the time to expiration of the futures contracts). Nevertheless, it should be noticed that they estimate all the parameters, including the seasonal factors, exclusively from future price data, by using a Kalman filter procedure.
37. To statistically examine the need for a second factor a likelihood ratio tests would be required.

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