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Modeling electricity spot prices: combining mean reversion, spikes, and stochastic volatility

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With the liberalization of electricity trading, the electricity market has grown rapidly over the last decade. However, while spot and future markets are currently rather liquid, option trading is still limited. One of the potential reasons for this is that the electricity spot price process remains a puzzle to researchers and practitioners. In this paper, we propose an approach to model electricity spot prices that combines mean reversion, spikes, negative prices, and stochastic volatility. Thereby, we use different mean reversion rates for ‘normal’ and ‘extreme’ (spike) periods. Furthermore, all model parameters can easily be estimated using historical data. Consequently, we argue that this model does not only extend the academic literature on electricity spot price modeling, but is also suitable for practical purposes, such as an underlying price model for option pricing.

Keywords: electricity; Lévy processes; mean reversion; spikes; stochastic volatility

JEL Classification: G17

1. Introduction

Starting with the deregulation of electricity markets in the 1990s and the liberalization of electricity trading about one decade ago, electricity markets have become exceedingly important. However, electricity has several peculiarities that distinguish it from other types of commodities. The most notable difference is its non-storability – or at least the very high costs associated with its storage. This leads to several problems for price modeling and for the pricing of related derivatives. Because electricity markets are quite young, it is not surprising that research in this vital field is still limited.[1] Several recent studies have addressed the question of how electricity future prices are formed in the market (e.g. Wilkens and Wimschulte 2007; Redl et al. 2009; Botterud, Kristiansen, and Ilic 2010 or Furio and Meneu 2010). Unlike futures markets, which are rather liquid, option trading is still underdeveloped in electricity markets. One of the potential reasons for this is that the price behavior of electricity still puzzles both researchers and practitioners. The price process in this market differs substantially from other commodity markets because of its very high volatility and, even more important, the more common occurrence of spikes (at least partly as a consequence of the non-storeability). Consequently, traditional models, which mostly build on the assumption of Gaussian distributions, are not suitable in this case. However, without a proper understanding of the price process and validated models, the pricing of options is impossible. This is the main

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motivation for our paper: an increased understanding of the price process can foster the liquidity of option trading and the efficiency of the whole electricity market.

In the commodity pricing literature, the most common approach is to model the logarithmic price through a mean reverting process (Schwartz 1997; Lucia and Schwartz 2002). As in the Black–Scholes–Merton model, the mean reverting process is based on the exponential treatment of the stochastic spot price (Black and Scholes 1973; Merton 1973). If these models are applied for electricity, they can capture the mean reversion of electricity prices, but fail to account for the huge and non-negligible spikes in this market. In order to capture the spike behavior of electricity spot prices, it is necessary to extend the model by a jump component. Merton (1976) first introduced this class of jump–diffusion models to model equity dynamics. Cartea and Figueroa (2005) apply this model to the English and Welsh electricity market and find that it offers a proper adjustment to the peculiarities of electricity markets. However, Geman and Roncoroni (2006) address one of the drawbacks of this model, namely that it only uses one mean reversion rate for both the diffusion process and the jump process. In this context, we argue that a single mean reversion rate for these two aspects is only of limited use because the price of electricity shows spikes instead of classical jumps. These spikes tend to revert very quickly, leading to a high rate of mean reversion after the appearance of a spike. In 'normal' times without any spikes, the mean reversion rate is much lower. Consequently, the use of a single mean reversion factor results in too slow of a removal of 'extreme' price movements (spikes) and too fast of a return to the seasonal trend in periods without 'extreme' events. This problem can be solved by separating the mean reversion factors for the 'extreme' and the 'normal' processes. A suitable theoretical approach for this purpose is described by Benth et al. (2003).[2] The model proposed by Benth et al. (2003) was calibrated by Kluge (2006) for the Scandinavian, British, and German electricity market. Thereby, he estimates the parameters for the diffusion process from historical data and assumes a constant volatility over time. However, this approach has several drawbacks. First, the parameters for the spike process are not estimated from the time series, but based on expert opinions. Second, this approach neglects that the volatility in electricity markets is stochastic over time. Deng (2001) compares the jump–diffusion model of Merton (1976) with constant and stochastic volatility and derives prices for different energy derivatives using the Fourier transform, showing that stochastic volatility is important. Escribano, Pena, and Villaplana (2011) provide extensive empirical tests on a wide range of markets and conclude that it is necessary to include jumps and stochastic volatility. In this context, recent academic work by Chan and Gray (2006) and Bowden and Payne (2008) suggests that the EGARCH[3] is the best volatility model for electricity prices. Furthermore, most prior models neglect the empirical observation of negative electricity prices (over short periods of time).

We contribute to the existing literature along several dimensions. *First*, we propose a new approach to model the electricity spot price that overcomes several drawbacks of the existing models. In particular, we are – to the best of our knowledge – the first to present a self-contained model that simultaneously includes (i) separate speeds of mean reversion for the spike and the 'normal' processes, (ii) stochastic volatility, (iii) negative prices, and (iv) estimation based only on historical data. Consequently, we argue that this model is suitable as a price process model, for example, in the context of option pricing, as (i) all distinct characteristics of the electricity spot price are included and (ii) the parameters can easily be estimated with the help of historical price series. *Second*, the model is calibrated for four different electricity markets, that is, France, Germany, Scandinavia, and Great Britain. This is especially important for the development of a generalizable price model, as the price process between these markets are considerably different.[4] *Third*, unlike most prior papers in the field of electricity price modeling, we present an empirical setting to test the validity of the model against several alternatives. We find that the model proposed in this paper is superior to prior models across all the electricity markets considered.

2. Electricity markets

This section focuses on the description of electricity markets and the characteristics of electricity prices. It should be noted that this paper focuses on European electricity markets. Consequently, the results might not be perfectly suitable for other markets such as the USA which differs in several dimensions.

2.1 *Electricity markets*

In the 1990s, the European electricity sector underwent a liberalization which started with the privatization of power utilities in the UK in 1990. In 1997, the European Community launched directive 96/92 EC to improve competition in these markets. The aim was to break up monopolistic structures in electricity markets, separate generation and supply from transmission and distribution to enable third-party access to electricity market and enable customers to choose their electricity supplier. These liberalization activities led to an increase in the trading of electricity. Power exchanges were founded in various countries to organize this increase in trading. In Europe, Norway was the first country to establish a power exchange, the Nord Pool, which expanded into other Nordic countries over time. In 2000 and 2001, power exchanges followed in Germany, France, and Great Britain. Spot and forward electricity contracts are traded at these power exchanges. Spot market contracts are traded with delivery up to 1 day, whereas forward market contracts capture delivery periods ranging from weeks to years into the future. Even though intraday trading in electricity spot markets has increased in some importance in recent years, the electricity spot price usually refers to day-ahead contracts.

2.2 *Data*

This section focuses on the description of the electricity price data used in this paper. Our analysis is based on data from four European electricity markets. We obtain data for the German and French market from EPEXSpot, for the Nordic countries from Nord Pool Spot, and for Great Britain from ELEXON Exchange.[5] These four electricity markets represent the largest market areas based on electricity production and have different price behaviors based on their production technologies. We analyze a time period from 1 January 2004 to 31 December 2009 for which full year data are available for all four markets. Furthermore, it should be noted that we use a daily time resolution for our analysis. Hence, the daily prices represent the arithmetic mean over the particular trading day.

Figure 1 shows the spot price dynamics for all four price processes. In the following sections, we will focus on the procedures and results for the German price process. For the other markets, the procedures are the same and the results can be found in the appendix.

2.3 *Price behavior*

The Black–Scholes–Merton model (Black and Scholes 1973; Merton 1973) assumes the prices are independently identically log-normally distributed, which implies that the log returns of the prices follow a Gaussian distribution. We evaluate the validity of their assumptions in this section.

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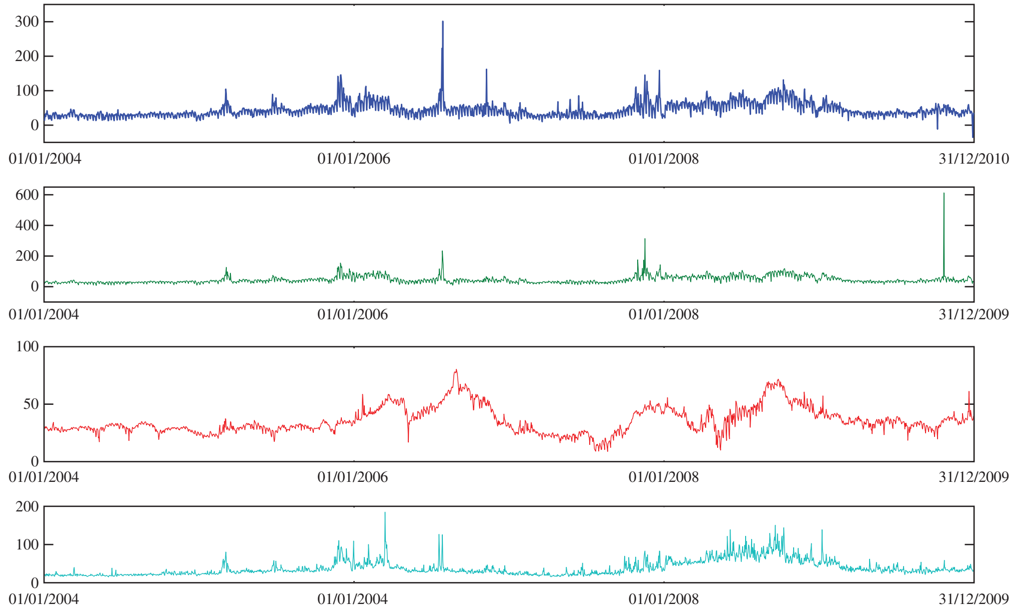

Figure 1. Prices series for the German, French, Scandinavian, and British electricity spot prices from 1 January 2004 to 31 December 2009 in EUR/MWh for the German, French, and Scandinavian market and GBP/MWh for the British market.

2.3.1 Negative prices

If the prices are indeed log-normally distributed, negative prices should not occur. However, an additional distinctive feature of electricity is the existence of negative prices, at least in markets where negative prices are allowed by the power exchange as in EPEXSpot for Germany and France. For example, the German market minimum value over the time series was −35,57 EUR on 26 December 2009. The early morning trading hours are especially vulnerable to negative prices. This is the time of day with the lowest demand and, thus, the lowest prices. The negative prices occur due to low demand and a (simultaneous) supply shock, for example, because of unexpected high electricity transfers from wind turbines. In this case and due to the fact that electricity cannot be stored on large scale, power plants have to be shut down to balance the offered and demanded quantity of electricity. But for power generators, it can be more expensive to shut down a (thermal) power plant than to pay the consumer for electricity usage. The rationale for this is related to the technical specifications of power plants. Many types do not have the flexibility to shut down and ramp up again in a short period of time or they require certain downtimes in case they were shut down. Negative prices lead to several problems with the application of 'classical' price process models, which were usually developed for equity markets, where negative prices cannot occur. To solve this problem, we exclude the negative prices from the time series for the ongoing analysis and add a component that can capture the negative prices at the end.

2.3.2 Seasonality and trend analysis

We analyze the independence assumption for electricity prices using an autocorrelation test. If the data are in fact independently distributed, the autocorrelation coefficient should be close to zero. Figure 2 indicates a strong level of autocorrelation in the data. This observed autocorrelation is a

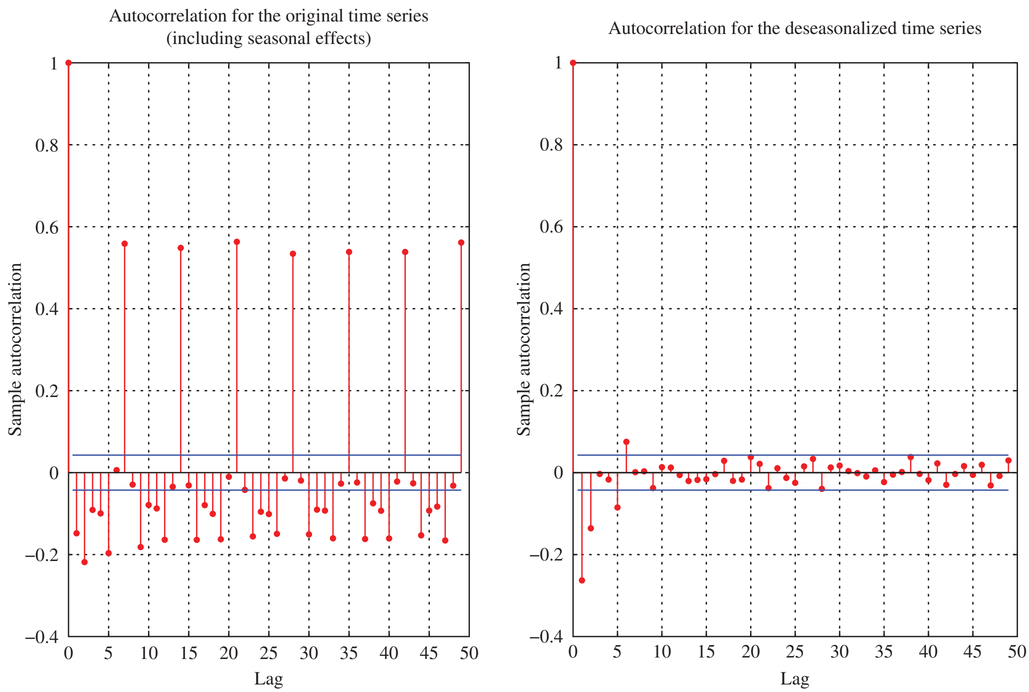

Figure 2. Autocorrelation plots of the daily log returns and the deseasonalized log returns (German baseload process).

result of an underlying seasonality in electricity markets, as discussed in Pindyck and Rubinfeld (1998). In order to estimate the parameters of the price process model properly, we remove this seasonality from the return time series with a filter for weekly and yearly seasonality. The autocorrelation plot in Figure 2 of the filtered time series shows that the seasonality is eliminated by the filtering process.[6] A significant negative autocorrelation remains only at lag 1. However, this is not surprising, since a negative autocorrelation at lag 1 is typical for mean reverting processes such as commodity prices.

2.3.3 Normality test

The last assumption of the Black–Scholes–Merton model is that returns are normally distributed. Although the analysis of financial markets' data reveals a higher probability for an 'extreme' event than predicted by the Gaussian distribution, the assumption is still embedded in most stochastic models.[7]

However, for electricity spot prices, the deviation from normality is more extreme than, for example, for equity and most other types of commodities. Figure 3 shows normality tests for the German baseload[8] price process from 1 January 2004 to 31 December 2009.[9] If the empirical returns are normally distributed, we would expect to observe a straight line in this figure. However, this is not the case. Instead, we find a clear indication of fat tails and, hence, more 'extreme' events than predicted by the Gaussian distribution. As an example, the probability for a daily return of ±50% is virtually zero according to the normal distribution. However, we observe a non-negligible amount of such events in the time series.

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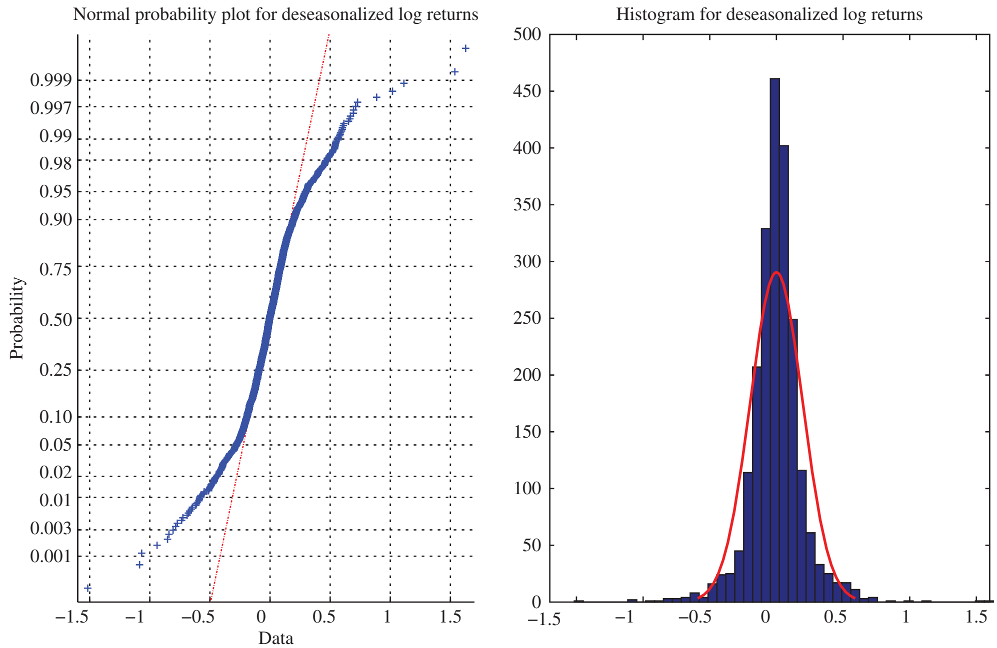

Figure 3. Normal probability plot and histogram of the unseparated price process (German baseload process).

2.4 *Spikes*

Consequently, we argue that electricity spot prices do not follow a Gaussian distribution because too many 'extreme' events occur in the time series. In the next step, we extract these 'extreme' events, the spikes, from the original time series using a numerical algorithm that recursively filters returns with absolute values that are greater than a multiple of the return series standard deviation at the specific iteration. The spikes are removed from the return series, which leads to a smaller standard deviation of the remaining returns. With the smaller standard deviation the range around the mean becomes smaller and new spikes can be detected. This procedure is repeated until it converges, that is, no new spikes can be detected.

The importance of the spikes in the electricity return series is illustrated by a simple comparison of Figures 3 and 4.[10] After the extraction of the spikes from the original series, the assumption of a Gaussian distribution is more accurate. In fact, the fat tails nearly disappear and the deviation of the empirical returns from the returns predicted by a Gaussian distribution becomes negligible.[11]

Our descriptive analysis revealed that electricity prices fluctuate around a long-term deterministic seasonal component and are characterized by price spikes and mean reversion. Furthermore, our analysis suggests that assuming a Gaussian process is inaccurate for electricity spot prices.

Since the seminal work of Samuelson (1965), Black and Scholes (1973), and Merton (1973), most stochastic models have tried to offer a solution to the problem that empirically observed prices do not follow Gaussian distributions. This literature has discussed jump–diffusion processes, stochastic volatility models, and, more recently, the use of Lévy processes.[12] While deviations from the normality assumptions are even common in equity and other commodity markets, their magnitude is far higher for electricity. Consequently, these models cannot be used to model the price process of electricity. Our model, which is described in the next section,

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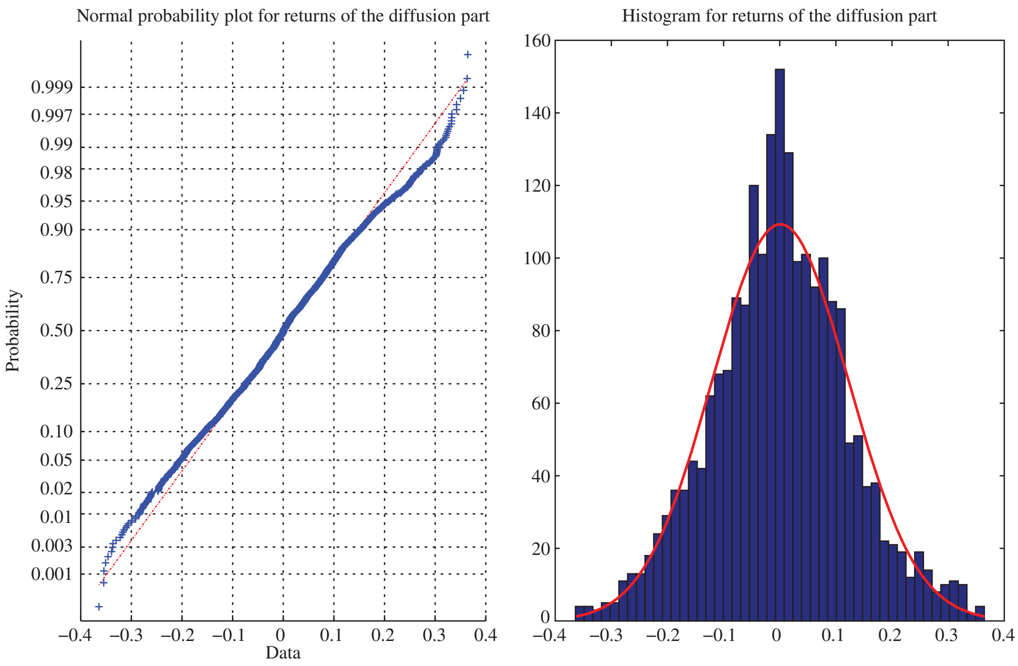

Figure 4. Normal probability plot and histogram of the diffusion part (German baseload process).

addresses this non-normality problem by considering stochastic volatility and separate mean reversion parameters for price spikes and 'normal' price processes.

3. Theoretical framework

The data analysis in the previous section showed three distinct characteristics of electricity markets, which should be accounted for in the model. First, negative prices occur over time. This is problematic for the application of geometric models. To cope with this issue, we exclude the negative prices from the price series and treat them separately. Even though negative prices seldom occur in baseload time series, an electricity price model should account for this characteristic. When the model is used for hourly or off-peak price data, where the occurrence of negative prices is much higher, this becomes even more important.

The second characteristic is a seasonality component, which reflects a (varying) long-term equilibrium level. Schwartz (1997) proposes a model that includes mean reversion to a long-term trend, and Lucia and Schwartz (2002) extend this model by including mean reversion and a deterministic seasonality. The third characteristic of electricity spot prices, namely the observation that randomly fluctuation prices revert slowly back to equilibrium level in times without 'extreme' events while price spikes revert very quickly, is addressed by Benth et al. (2003). Their model includes separate rates of mean reversion for the diffusion process and the spike process. However, none of these models accounts for stochastic volatility. In this paper, we propose a similar model that is able to handle all the characteristics of electricity spot prices. For this, we extend the model of Benth et al. (2003) to (i) stochastic volatility, (ii) negative prices, and (iii) a self-contained estimation of all relevant parameters.

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We require that the standard assumptions for stochastic models hold. Formally, $(\Omega, P, \mathcal{F}, \{\mathcal{F}_t\}_{t\epsilon[0,T]})$ is a complete filtered probability space, with $T < \infty$ a fixed time horizon. If $S(t)$ denotes the spot price of electricity at time t, then we set

$$S(t) = \exp(\Lambda(t) + X(t) + Y(t)), \tag{1}$$

where $\Lambda(t)$ denotes the deterministic seasonality function value at time t, and $X(t)$ and $Y(t)$ the values of two Lévy processes at time t. Process X addresses the 'normal' part of the price behavior, whereas process Y addresses the 'extreme' part with price spikes. The processes X and Y are assumed to be independent.

The deterministic function $\Lambda(t)$ is assumed to consist of a trend component, a yearly seasonal component to capture the effect of temperature, daylight time, etc. over the year, and a weekly seasonal component reflecting the strong differences in demand during work days and on weekends. We assume a linear trend in prices and, therefore, use a logarithmic trend in the geometric model. The seasonal function is given by

$$\Lambda(t) = \log(\beta_1 + \beta_2 \cdot t) + \sum_{j=1}^{12} 1_{\{\mathrm{month}(j)\}}(t) m(j) + \sum_{k=1}^{7} 1_{\{\mathrm{day}(k)\}}(t) d(k), \tag{2}$$

where β_1 and β_2 represent the linear trend in prices. To account for seasonality we include the dummies $m(j)$ for months $j = 1, \dots, 12$ and $d(k)$ for the weekdays $k = 1, \dots, 7$. The dummies for monthly and weekly seasonality have to meet the basic characteristics of seasonal function. In particular, there should be no effect on the price when moving a whole period forward or backward. For weekly seasonality, the resulting constraint is $\sum_{k=1}^{7} d(k) = 0$ and for monthly seasonality, $\sum_{j=1}^{12} m(j) \cdot h_i(j) = 0$, where $h_i(j)$ represents the number of data points in month j of year i. Both constraints require that an integration over the particular period has to be zero.

To account for mean reversion in processes X and Y, we use Ornstein–Uhlenbeck processes, given by the stochastic differential equations

$$\mathrm{d}X(t) = -\alpha_X X(t)\,\mathrm{d}t + \sigma(t)\,\mathrm{d}B(t), \tag{3}$$

$$\mathrm{d}Y(t) = -\alpha_Y Y(t)\,\mathrm{d}t + \mathrm{d}I(t), \tag{4}$$

where α_X and α_Y denote the mean reversion parameters, $\sigma(t)$ the volatility of the diffusion process, $\mathrm{d}B(t)$ stochastic increments of the diffusion process, and $\mathrm{d}I(t)$ increments of a stochastic jump process. The increments of the 'normal' part are modeled with Brownian motion and the spike part with a jump process.[13] Furthermore, we assume that the mean reversion parameters of Equations (3) and (4) are constant over time. To achieve the spikes, that is, high price jumps followed by a fast reversion to the previous level, in process Y, a jump process in combination with a high mean reversion rate is needed. We assume that the volatility $\sigma(t)$ can be either constant or stochastic. In this paper, we apply an EGARCH-type (Nelson 1991) volatility model. However, the implementation of other types of volatility models is straightforward.

The jump process $I(t)$ in Equation (4) is modeled with two separate compound Poisson processes, one for positive and one for negative jumps.

$$I(t) = I^+(t) - I^-(t),$$

with

$$I^{\pm} = \sum_{i=1}^{N^{\pm}(t)} J_i^{\pm}$$

with

$$N^{\pm}(0) = 0,$$
$$E(N^{\pm}(t)) = \lambda^{\pm} \times t,$$
$$\mathrm{Var}(N^{\pm}(t)) = \lambda^{\pm} \times t,$$
$$\ln J_i^{\pm} \sim N(\mu^{\pm}, \sigma^{\pm}).$$

The processes N^+ and N^- are Poisson processes, which represent the number of positive or negative jumps until time t. The jump intensities for the Poisson processes are $\lambda^{\pm}$. The jump height for jump i is described by $J_i^{\pm}$, which is assumed to be log-normally distributed. To achieve negative jumps with log-normally distributed jump heights, we have to subtract I^- when computing the process $I(t)$.

To adjust for negative prices, we expand model (1) with an arithmetic component $D(t)$:

$$S(t) = \exp(\Lambda(t) + X(t) + Y(t)) + D(t), \tag{5}$$

where $D(t)$ follows a stochastic differential equation as Y in Equation (4), except that it only allows negative jumps. Due to the fact that there is a low number of negative prices, the mean reversion speed α_D will be about 1, resulting in the disappearance of negative prices within 1 day.

4. Calibration

In this section, we demonstrate the calibration of all necessary model parameters. The parameters are estimated for the four electricity spot markets to show the usability of the model for various markets with different price behaviors. Several subsequent steps are necessary to calibrate our model. *First*, we remove the negative prices; *second*, we use a seasonal filter to remove seasonal patterns and gain a stationary time series; *third*, we separate processes X and Y as shown in Figure 5; and finally, we estimate the parameters of the stochastic processes. The steps used to calibrate the model can be found in Table A1.

4.1 *Negative prices*

Negative prices seldomly occur in the baseload prices due to the fact that the price is the arithmetic mean of 24 single contracts over the course of day. Therefore, most of the negative prices in hourly contracts cancel out in the baseload calculation. However, there are some negative prices in the German baseload electricity price series that should be considered for price modeling. Therefore, we remove the negative prices and replace them with the 7-day moving average value at the specific points of time. The values removed from the time series are assigned to process D. After

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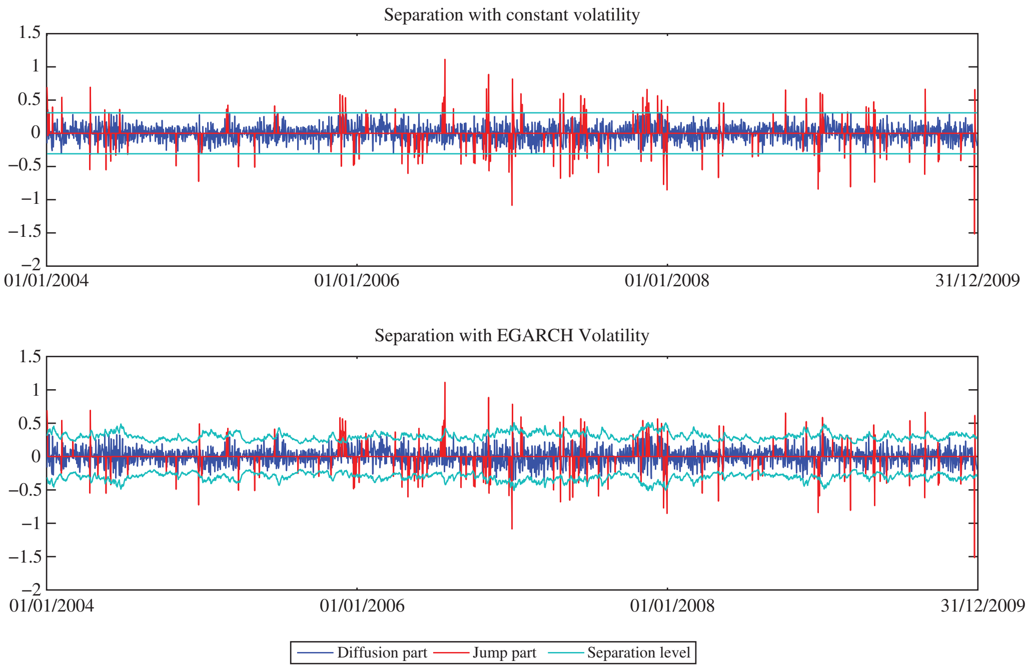

Figure 5. Jump detection for the constant and EGARCH volatility models (German baseload process).

removing the negative prices, we use a logarithmic transformation for the remaining time series with only positive values.

4.2 *Seasonality function*

We start the analysis of deterministic seasonality by removing outliers from the time series. Therefore, all prices that are below the first quartile minus three times the inter-quartile range or above the third quartile plus three times the inter-quartile range are declared as outliers and removed from the time series. As for the negative prices, the removed values are replaced by the 7-day moving average. To estimate the seasonal parameters, we perform a least-square fit of the seasonal function:

$$\begin{aligned}\hat{\Lambda}(t) = &\sum_{i=2004}^{2009} 1_{\{\text{year}(i)\}}(t)\,\text{yr}(i)\\ &+ \sum_{j=1}^{12} 1_{\{\text{month}(j)\}}(t)\,m(j)\\ &+ \sum_{k=1}^{7} 1_{\{\text{day}(k)\}}(t)\,d(k),\end{aligned} \tag{6}$$

where $\text{yr}(i)$ denotes the dummy values for the years $i = 2004, \ldots, 2009$, $m(j)$ for months $j = 1, \ldots, 12$ and $d(k)$ for the days of week $k = 1, \ldots, 7$. To achieve stationarity, the trend has to

Table 1. Seasonality and drift parameters.

	Germany	France	Scandinavia	Great Britain
β_1	32.41	30.62	30.31	20.77
β_2	2.79	3.50	1.41	4.57
January	−0.01	0.06	−0.04	−0.05
February	0.03	0.05	−0.05	−0.01
March	−0.05	0.00	−0.07	−0.02
April	−0.09	−0.11	−0.05	−0.10
May	−0.18	−0.23	−0.08	−0.10
June	−0.05	−0.10	−0.04	−0.03
July	0.02	−0.01	−0.02	0.01
August	−0.08	−0.18	0.09	−0.05
September	0.08	0.05	0.00	0.00
October	0.16	0.15	0.08	0.07
November	0.14	0.18	0.12	0.13
December	0.03	0.17	0.05	0.10
Monday	0.07	0.05	0.02	0.04
Tuesday	0.14	0.13	0.02	0.05
Wednesday	0.14	0.12	0.02	0.04
Thursday	0.12	0.11	0.02	0.05
Friday	0.06	0.07	0.00	0.02
Saturday	−0.15	−0.13	−0.04	−0.07
Sunday	−0.39	−0.36	−0.06	−0.12

be replaced by yearly dummies. From a linear regression of the yearly dummies, we receive the parameters β_1 and β_2 for the seasonal model (2).

Table 1 shows the seasonal function and trend parameters for the four markets.

4.3 *Stochastic component*

After removing the seasonality from the sample time series, we analyze the stochastic part. Since there is no direct way to separate the independent stochastic processes X and Y, due to the fact that dB and dI are not observable, we first have to detect and remove the mean reversion. The procedure is orientated on the proceeding used by Benth, Benth, and Koekebakker (2008) for a process with one mean reversion rate and enhanced for the model with two mean reversion rates with a self-developed method.

4.3.1 Mean reversion rates

We start by determining the mean reversion rates α_X and α_Y. Therefore, we construct Z with

$$Z(t) = X(t) + Y(t) \tag{7}$$

and, therefore,

$$\mathrm{d}Z(t) = \mathrm{d}X(t) + \mathrm{d}Y(t). \tag{8}$$

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Using Equation (3) as well as Equation (4) and the discrete version of Equation (8), it follows that

$$\begin{aligned}
\Delta Z(t) &= \Delta X(t) + \Delta Y(t) \\
&= -\alpha_X X(t)\Delta t + \sigma \Delta B(t) - \alpha_Y Y(t)\Delta t + \Delta I(t) \\
&= -(\alpha_X X(t) + \alpha_Y Y(t))\Delta t + \sigma \Delta B(t) + \Delta I(t) \\
&= -\left(\alpha_X \frac{X(t)}{X(t)+Y(t)} + \alpha_Y \frac{Y(t)}{X(t)+Y(t)}\right)(X(t)+Y(t))\Delta t + \sigma \Delta B(t) + \Delta I(t) \\
&= -\alpha_Z Z(t)\Delta t + (\sigma \Delta B(t) + \Delta I(t)).
\end{aligned}$$

Since the time series consists of daily data, a discretization of the interval length $\Delta t = 1$ is reasonable. Using linear regression of log prices against the log returns as in Equation (9), we find a value for α_Z:

$$\Delta Z(t) = \gamma Z(t)\Delta t + \varepsilon(t), \tag{9}$$

where γ is the regression coefficient and ε describes the daily changes not caused by the mean reversion effect. Using $\alpha_Z = (-1) \cdot \gamma$, it is possible to estimate the mean reversion rate.

4.3.2 Separation of stochastic processes

By removing the mean reversion effect from process Z, we get the process $\varepsilon(t) = \sigma \Delta B(t) + \Delta I(t)$ with increments that have to be modeled stochastically. For this process, we can separate jumps by defining a range of 2.57 times the standard deviation of ε around the mean to classify increments as 'normal' and all movements outside this range as 'extreme', that is, as jump. We remove these jumps and use the standard deviation of the remaining increments to define the new, then smaller range. We repeat this procedure until it converges, that is, no new jumps are detected in the remaining increments. We use 2.57 as the parameter for this recursive procedure as, if the remaining part follows a Gaussian distribution, it accounts for 99% of the increments from this distribution. Consequently, we classify 1% of the most extreme movements from the Gaussian distribution and all movements outside this range as jumps. The jumps, listed in a vector ζ, and the mean reversion rate α_Z are used to separate processes X and Y and to determine their mean reversion rates α_X and α_Y.

For the calculation of X and Y, the stochastic part first needs to be divided between processes $\sigma\,\mathrm{d}B$ and $\mathrm{d}I$. As the final values of $\sigma\,\mathrm{d}B$ and $\mathrm{d}I$ change because of the mean reversion, we initially name the processes $\sigma\,\mathrm{d}B_0$ and $\mathrm{d}I_0$. Generally, we set $\sigma\,\mathrm{d}B_0 = \mathrm{d}Z$ and $\mathrm{d}I_0 = 0$ as the first step. However, at the jump points, that is, the entries of vector ζ, the value of $\sigma\,\mathrm{d}B_0$ is set to 0 and the values of $\mathrm{d}I_0$ at these points are equal to the values of $\mathrm{d}Z$. Afterward, processes X and Y are calculated recursively. We assume $X(0) = Z(0)$ and $Y(0) = 0$ as starting values. The stochastic influences of $\sigma\,\mathrm{d}B$ and $\mathrm{d}I$ have to be adjusted for the mean reversion effect to offset the movement caused by the mean reversion by means of the stochastic components, to achieve the same movement intensity as in the process $\mathrm{d}Z$. Generally, the mean reversion effect is associated with the 'normal' part $\sigma\,\mathrm{d}B$ and is deducted from $\sigma\,\mathrm{d}B_0$. The only exceptions are movements, where an association with $\sigma\,\mathrm{d}B$ would lead to a new jump. In this case, it is associated with jump process $\mathrm{d}I$. The adaption formulas for $\sigma\,\mathrm{d}B$ and $\mathrm{d}I$ are

$$\sigma\,\mathrm{d}B(t) = \sigma\,\mathrm{d}B_0(t) - (-\alpha_X X(t) - \alpha_Y Y(t)) \tag{10}$$

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and, in the case where it would lead to a new jump,

$$\mathrm{d}I(t) = \mathrm{d}I_0(t) - (-\alpha_X X(t) - \alpha_Y Y(t)). \tag{11}$$

Here, $\sigma\,\mathrm{d}B_0(t)$ and $\mathrm{d}I_0(t)$ on the right-hand side of Equations (10) and (11) represent in each case the first assignment, and $\sigma\,\mathrm{d}B(t)$ and $\mathrm{d}I(t)$ on the left-hand side of Equations (10) and (11) represent the final values. With these values, the iterative update formulas for X (Equation (12)) and Y (Equation (13) are

$$\begin{aligned} X(t+1) &= X(t) + \mathrm{d}X(t) \\ &= (1-\alpha_X)X(t) + \sigma\,\mathrm{d}B(t) \end{aligned} \tag{12}$$

$$\begin{aligned} Y(t+1) &= Y(t) + \mathrm{d}Y(t) \\ &= (1-\alpha_Y)Y(t) + \mathrm{d}I(t). \end{aligned} \tag{13}$$

When the recursive filtering procedure is finished, processes X and Y can have different jumps than we originally assumed for the calculation. Therefore, we again determine the jumps in $\sigma\,\mathrm{d}B + \mathrm{d}I$ with a second algorithm. This algorithm classifies all price movements that are larger than 2.57 times the standard deviation of process $\sigma\,\mathrm{d}B$ as jumps, but in contrast to the first algorithm, it does not perform any recursions. Instead, the jumps are calculated solely on the specified volatility. This enables us to take stochastic volatility into account. Contrary to constant volatility, when using stochastic volatility, the range to determine jumps from 'normal' movements is variable due to the varying volatility. Therefore, movements that will be classified as jumps in a constant volatility environment can vary from those classified under stochastic volatility. The reason for this is that in times of high (low) volatility, the stochastic volatility models allow larger (smaller) movements of process $\sigma\,\mathrm{d}B$ without classifying them as jumps. Not using a recursive approach might result in a very low number of price moments not to be identified as jumps; we use this approach because, otherwise, the run-time of the algorithm becomes extraordinarily long and the accuracy does not increase significantly. The required stochastic volatility processes are determined by the maximum-likelihood estimation from process $\mathrm{d}B$.

With the obtained processes X and Y, the mean reversion factors α_X and α_Y can be determined in an analogy to Equation (9). These parameters will then replace the initial values for the mean reversion rates and the calculation of processes X and Y starts again until it converges, that is, the mean reversion rates stay constant and using them as input would end up with the same rates again. In the case of alternating mean reversion factors, those factors are averaged and the calculation of the processes X and Y is done again. The results of this procedure provide us with the mean reversion factors α_X and α_Y, processes X and Y, the jump vector with the entries of the jumps, and the stochastic parts $\sigma\,\mathrm{d}B$ and $\mathrm{d}I$ of processes X and Y. The mean reversion rates for the four markets are shown in Table 2.

4.3.3 Distribution

We analyze the distribution of the stochastic part for $\sigma\,\mathrm{d}B$ and $\mathrm{d}I$. By analyzing the individual stochastic processes, we determine the stochastic processes B and I for each price process.

When analyzing the distribution of $\sigma\,\mathrm{d}B$, we assume that for the EGARCH volatility model a normalized process B, with zero mean and standard deviation of 1, exists. This allows us to use a constant parameter σ, while the volatility of the normalized process B is stochastic. This method is applied for practical purposes to use the same simulation procedures for different volatility models. The parameters for stochastic volatility are estimated from the normalized process $\mathrm{d}B$

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Table 2. Parameters of the stochastic processes.

	Germany	France	Scandinavia	Great Britain
	Mean reversion parameter			
α_X	0.19	0.12	0.01	0.15
α_Y	0.57	0.59	0.27	0.45
	Distribution parameter (diffusion process)			
μ	0.00	0.00	0.00	0.00
σ	0.12	0.12	0.03	0.12
	EGARCH parameter (diffusion process)			
κ	0.00	0.00	0.00	−0.01
ARCH	0.19	0.16	0.08	0.24
GARCH	0.95	0.96	0.98	0.78
Leverage	−0.02	0.00	−0.04	0.20
	Jump distribution parameter (diffusion process)			
μ^+	−0.97	−0.89	−2.04	−0.98
σ^+	0.50	0.59	0.55	0.36
λ^+	0.03	0.03	0.07	0.04
μ^-	−0.92	−1.01	−2.02	−0.92
σ^-	0.49	0.53	0.55	0.53
λ^-	0.04	0.04	0.07	0.01
No. of jumps	159	160	313	110
No. of down jumps	73	74	153	80
No. of up jumps	86	86	160	30

by the maximum-likelihood estimation. The increments of dB are compared with a standard deviation by performing a Kolmogorow–Smirnow test on the distributions. The test statistics are not significant at a significance level of 5% and, therefore, the hypothesis for a Gaussian distribution cannot be rejected. Table 2 shows the distribution parameters and the EGARCH parameters for the price processes.

To analyze the distributions of process I, we have to separate the positive jumps from the negative jumps. Then, we estimate the jump intensities λ^+ and λ^- for the Poisson processes N^+ and N^- by dividing the respective number of jumps by the total number of observations. As previously mentioned, we use log-normally distributed jump heights for modeling the jumps, which is quite reasonable due to the separation procedure and the resulting jump heights. The estimation of the parameters is performed with a maximum-likelihood estimation. The jump distributions' parameters for the compound Poisson processes I^+ and I^- for the four markets are shown in Table 2. It can be seen that the distributions of the markets differ from each other, especially when observing the Nord Pool data where we observe much higher jump intensities, but much lower jump heights compared with the other markets. This results from a much lower volatility of the 'normal' part, as shown in Table 2.

5. Testing the model

In this section, we test the model that was presented in Section 3 and calibrated in Section 4. To test the model, especially its ability to represent the distribution of returns, we compare it with other electricity price models. The selection of models ranges from a simple Ornstein–Uhlenbeck model over a jump–diffusion model, proposed by Cartea and Figueroa (2005), to our model, first with constant volatility and then with EGARCH volatility. All four models are versions of model (1), the Ornstein–Uhlenbeck model, given by

$$S(t) = \exp(\Lambda(t) + X(t)), \tag{14}$$

where X follows the same stochastic differential equation as Equation (3). As the second model for our comparison, we choose the jump–diffusion model (Cartea and Figueroa 2005):

$$S(t) = \exp(\Lambda(t) + Z(t)), \tag{15}$$

where process Z is given by the stochastic differential equation

$$\mathrm{d}Z(t) = -\alpha_Z Z(t) + \sigma \mathrm{d}B(t) + \mathrm{d}I(t). \tag{16}$$

This model is equivalent to our model described in Equation (1) when using the same mean reversion rates for X and Y, that is, $\alpha_Z := \alpha_X = \alpha_Y$. The third and fourth model in our comparison is our model (1), first with constant volatility $\sigma(t) = \sigma$ and then with stochastic volatility, that is, EGARCH volatility. For all models, we use the same seasonal function and, therefore, focus solely on their stochastic part.

We test the performance by analyzing the dispersion of the distribution. We argue that the dispersion is the more important measure for option pricing than exact price forecasts.[14] Therefore, we do not use the root-mean-squared error. This measure is often used in model testing, but leads to strong distortion when spikes do not occur on exactly the same day as predicted. For the same reason, we do not test our model with other metrics that are commonly used to test the accuracy of *price* forecasting models. Instead, we apply the Ansari and Bradley (1960) test, which compares whether the population distribution functions of two series of location-adjusted random variables $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n, n \in \mathbb{N}$, are identical against the alternative that they differ by the dispersion, that is, the variance. The test statistics are based on the absolute differences of the rank $i, i = 1, \ldots, n$, to the average rank $n + 1/2$.[15]

We use the time series from 1 January 2004 to 31 December 2009 as test data. To perform out-of-sample tests within this period, we exclude 1 month from the time series,[16] estimate the parameters with the time series analysis, as in Chapter 4, and start a Monte Carlo simulation for a period of 1 month. We generate 100, 000 simulation paths and compare the distributions of the simulated paths to the distribution of the realized path. Then, we shift our observation window by 1 month and exclude the next month from the original time series. This allows us to generate 12 test results for each year in the data sample, which leads to a total of 72 out-of-sample tests for the whole sample. Obviously, this test procedure is somewhat unrealistic as it uses data from after the test period for calibration. However, the problem is that electricity price time series are rather short. Hence, using only data from before the test period comes at the cost of less tests. As the test is not designed to measure how well the model can forecast the price, but the overall behavior in terms of the price distribution, we decided to use all 72 test opportunities.

Table 3 shows the average p-value and rejection probability for the null hypothesis of samples. We observe increasing p-values and diminishing rejection probabilities for models with higher complexity. The simplest model (14) is less likely to generate sample paths with the same dispersion as the realized path. The model (15) of Cartea and Figueroa, using one rate of mean reversion, performs better than the simple Ornstein–Uhlenbeck model in all analyzed markets. Our proposed model (1) with constant volatility performs better than the two previously mentioned models in all markets and best in the French market. This model with stochastic (EGARCH) volatility performs equal to the model with constant volatility in the Nordic market and better in the German and British markets.

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Table 3. Test results for the four models.

Model	Ornstein–Uhlenbeck	Cartea Figueroa	Constant volatility	EGARCH volatility
Number of simulations		100,000		
Number of periods		72		
		German market		
Rejections	42.1%	33.0%	32.0%	31.8%
Average *p*-value	25.0%	29.9%	30.6%	30.7%
		French market		
Rejections	42.3%	35.4%	32.8%	33.0%
Average *p*-value	26.4%	30.4%	32.1%	32.0%
		Nordic market		
Rejections	34.0%	29.8%	24.2%	24.2%
Average *p*-value	38.5%	42.9%	44.8%	44.7%
		British market		
Rejections	38.3%	33.0%	31.0%	29.9%
Average *p*-value	29.5%	33.2%	34.1%	34.6%
		Total average		
Rejections	39.3%	32.8%	30.0%	29.7%
p-value	29.8%	34.0%	35.3%	35.4%

6. Conclusion

Electricity price modeling is complex and still in its infancy. For example, one of the reasons for the illiquidity of electricity option markets is that suitable price models are still lacking. For practical purposes, two requirements have to be fulfilled: the model must capture all the important characteristics of electricity price behavior, and the time and effort for parameter estimation must be small. In this paper, we presented a self-contained model that simultaneously includes (i) separate speeds of mean reversion for the spike and the 'normal' process, (ii) stochastic volatility, (iii) negative prices, and (iv) estimation based only on historical data. For the calibration and the testing of the validity of the model, we used electricity spot price data from four European markets: France, Germany, Scandinavia, and Great Britain. The empirical tests showed that our model fulfills the practical requirements. In particular, it outperformed simpler models like the models proposed by Cartea and Figueroa and Ornstein–Uhlenbeck across all considered electricity markets. However, the differences between constant and EGARCH volatility are small. Furthermore, the estimation of all necessary parameters is automated and self-explanatory. Consequently, we argue that this model is suitable as an underlying price process model, for example, for option pricing, as it captures electricity price behavior very well and can be easily estimated with historical data. However, future research is necessary to further increase our understanding of electricity spot price behavior. For example, models with an own stochastic process for the volatility might be promising. Furthermore, longer time series will allow for a more accurate testing of different electricity price models in the future.

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We thank the International Graduate School of Science and Engineering (IGSSE) for financial support and the 'Sonderforschungsbereich' (SFB) 649 for data access. This paper greatly benefited from the comments during the review process as well as in-depth discussions with the members of the Energy 2030 research cluster at IGSSE and participants of the Forecasting Financial Markets 2011 conference in Marseilles. The usual caveat applies.

List of symbols

$\mathcal{F}$	Filtration
$\Lambda(t)$	Seasonality function value at time t
$\hat{\Lambda}(t)$	Empirical seasonality function value at time t
$\Phi(\cdot)$	Normal distribution function
Ω	Probability space
α_x	Mean reversion rate of the 'normal' diffusion process
α_y	Mean reversion rate of the 'extreme' spike process
α_z	Mean reversion rate of the unseparated process
β_1	Intercept parameter for linear trend
β_2	Trend parameter for linear trend
γ	Regression coefficient
$\varepsilon(t)$	Daily changes of Z not caused by mean reversion
$\lambda^{\pm}$	Jump intensity
$\sigma(t)$	Volatility function of the 'normal' diffusion process
ζ	Spike vector
$B(t)$	Brownian motion at time t
$D(t)$	Stochastic process covering the negative prices at time t
$I(t)$	Compound Poisson process at time t
J_i	Jump height of the ith Poisson process jump
$N^{\pm}$	Number of jumps at time t
P	Probability measure
$S(t)$	Spot price at time t
T	Time horizon
$X(t)$	'Normal' diffusion process at time t
$Y(t)$	'Extreme' spike process at time t
$Z(t)$	Unseparated stochastic process at time t
$\mathrm{d}B(t)$	Independent increments of the Brownian motion
$\mathrm{d}I(t)$	Independent increments of the compound Poisson process
$d(k)$	Weekly seasonality dummy for day k
$m(j)$	Monthly seasonality dummy for month j
$\mathrm{yr}(i)$	Average value in year i

Notes

1. An overview of the literature about electricity price modeling is provided by Higgs and Worthington (2010).
2. For example, Hambly, Howison, and Kluge (2009) use this approach for a practical application, that is, to value swing options.
3. Exponential general autoregressive conditional heteroskedastic.
4. For example, spikes are less common in the Scandinavian environment where electricity production is mainly based on hydroelectric plants.
5. ELEXON Exchange publishes the APX power prices for Great Britain.
6. See Figures A1–A3 for the French, Nordic, and British markets.
7. The fact that return distributions are often not normal can be found among others in Mandelbrot (1963), Fama (1965), Hull and White (1998), and Fezzi and Bunn (2010).
8. Baseload prices are the arithmetic mean over the 24 hourly prices of 1 day.
9. See Figures A4–A6 for the French, Nordic, and British markets.
10. Results for the French, Nordic, and British markets can be seen in Figures A7–A9.
11. To be more precise, the deviation is still present, but less extreme. The assumption of a Gaussian distribution, despite the fact that small deviations exist, is common, for example, for equity prices.
12. See, for example, Merton (2001), Knight and Satchell (2001), or Shrevel (2004).
13. Consequently, we will denote these 'extreme' events in process Y as spikes and in process I as jumps in the remainder of this paper, due to the fact that a spike is the result of a jump combined with the high speed of mean reversion.
14. Most option traders hedge their price risk by delta hedging.

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15. For a more detailed description, see Duller (2008).
16. Since we are focused on the stochastic part, we removed the seasonality from the time series over the whole period as a first step.

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Appendix

Table A1. Steps to calibrate the electricity price model.

Step	Description	Related variables
1	Remove negative prices and outliers	$\tilde{S}(t) = S(t) - D(t)$
2	Log transform $\tilde{S}$ and remove trend and seasonality	$Z(t) = \log(\tilde{S}(t)) - \Lambda(t) = X(t) + Y(t)$
3	Estimate α_Z as starting value for further analysis	$\Delta Z(t) = -\alpha_Z \Delta t + \epsilon(t)$
4	Set starting values for α_X and α_Y	$\alpha_X = \alpha_Y = \alpha_Z$
5	Identify jumps in epsilon	ζ
6	Separate X and Y	$X_{\text{new}}, Y_{\text{new}}, \alpha_{X_{\text{new}}}, \alpha_{Y_{\text{new}}}, \epsilon_{\text{new}}, \sigma_{\text{d}B}, \text{d}I$
7	Identify jumps in the ϵ_{new}	ζ_{new}
8	Test whether jumps stay same, otherwise use ζ_{new} as starting vector for jumps and go back to 7	$\zeta = \zeta_{\text{new}}$?
9	Estimate α_X and α_Y	$\Delta X(t) = -\alpha_X \Delta t + \sigma \Delta B(t)$, $\Delta Y(t) = -\alpha_Y \Delta t + \Delta I(t)$
10	Test whether mean reversion rates stay same, otherwise use $\alpha_{X_{\text{new}}}$ and $\alpha_{Y_{\text{new}}}$ as starting values for the mean reversion rates and go back to 6	$\alpha_X = \alpha_{X_{\text{new}}}, \alpha_Y = \alpha_{Y_{\text{new}}}$?
11	Estimate the distribution parameters for $\sigma \Delta B$ and $\text{d}I$	$\sigma \Delta B, \Delta I$

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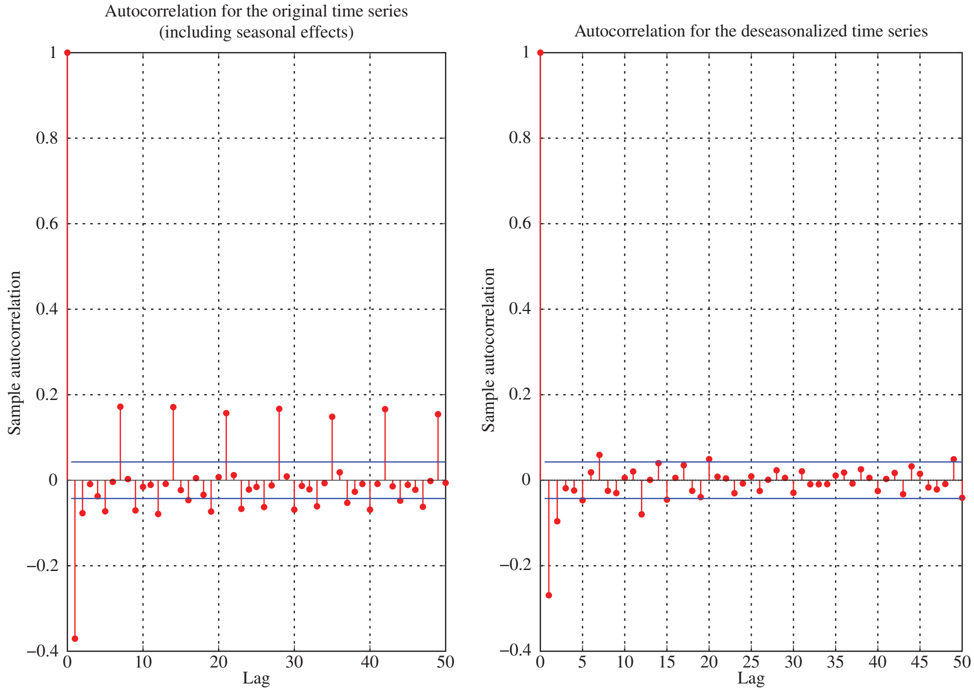

Figure A1. Autocorrelation plots of the daily log returns and the deseasonalized log returns (French baseload process).

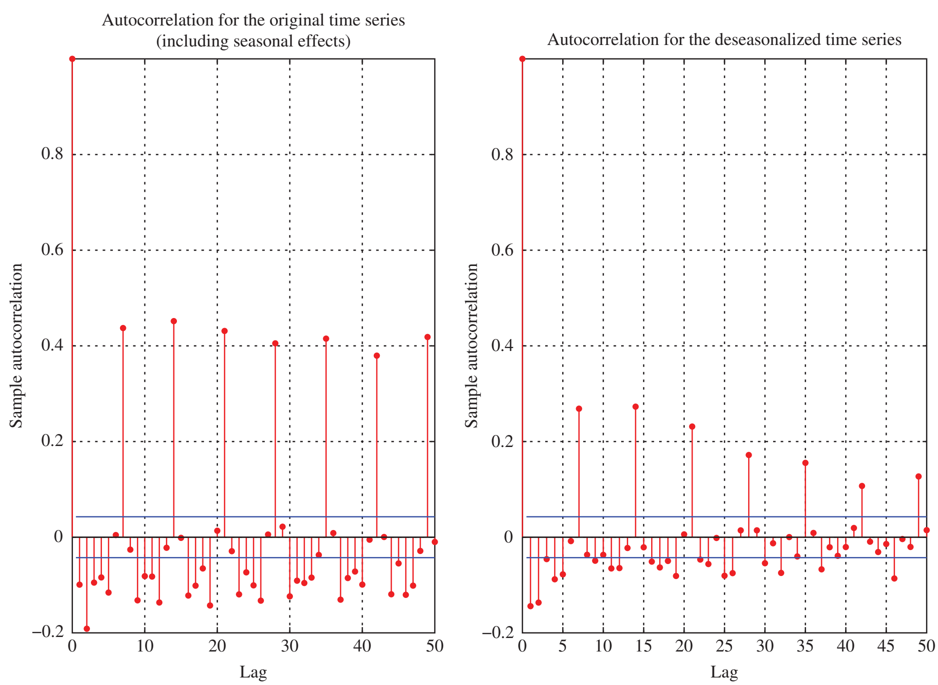

Figure A2. Autocorrelation plots of the daily log returns and the deseasonalized log returns (Nordic baseload process).

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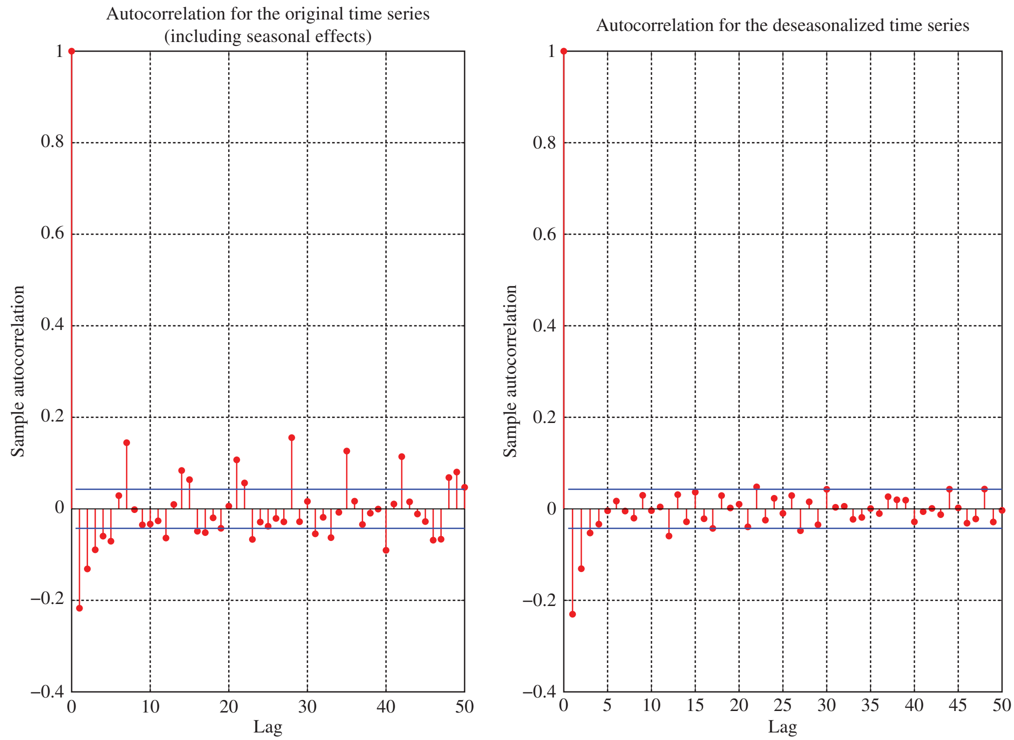

Figure A3. Autocorrelation plots of the daily log returns and the deseasonalized log returns (British baseload process).

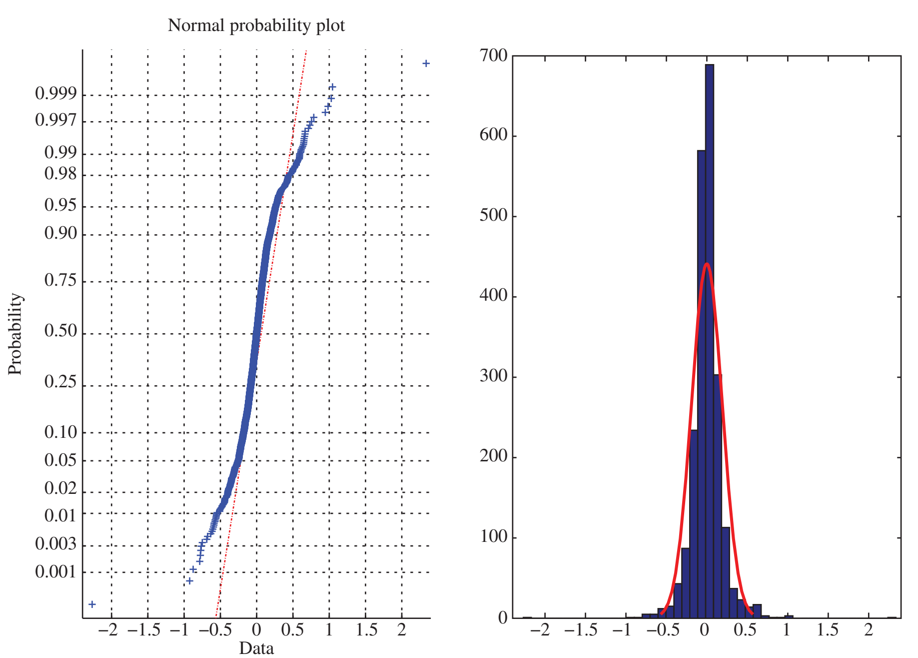

Figure A4. Normal probability plot and histogram of the unseparated price process (French baseload process).

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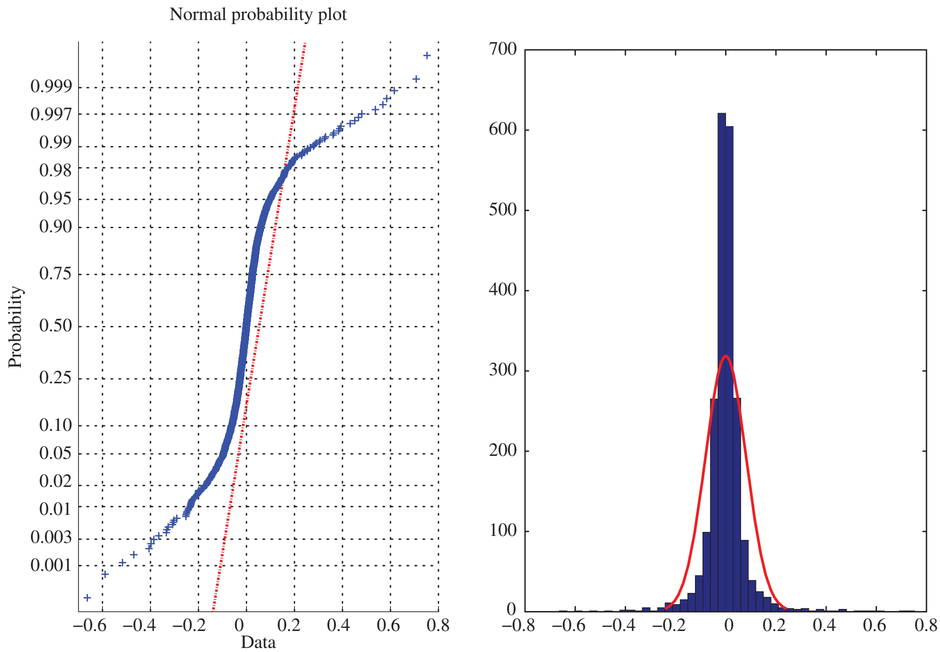

Figure A5. Normal probability plot and histogram of the unseparated price process (Nordic baseload process).

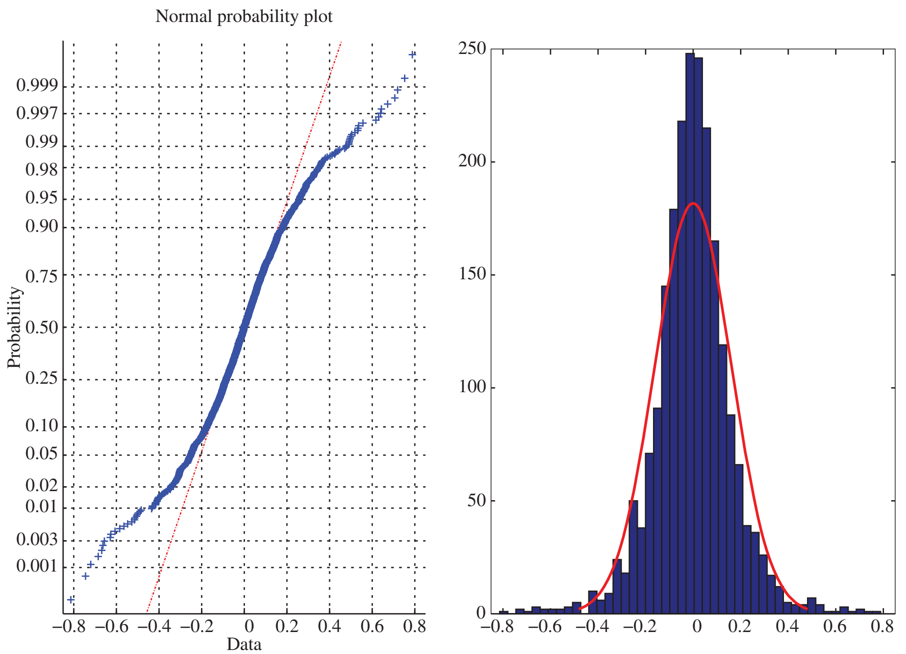

Figure A6. Normal probability plot and histogram of the unseparated price process (British baseload process).

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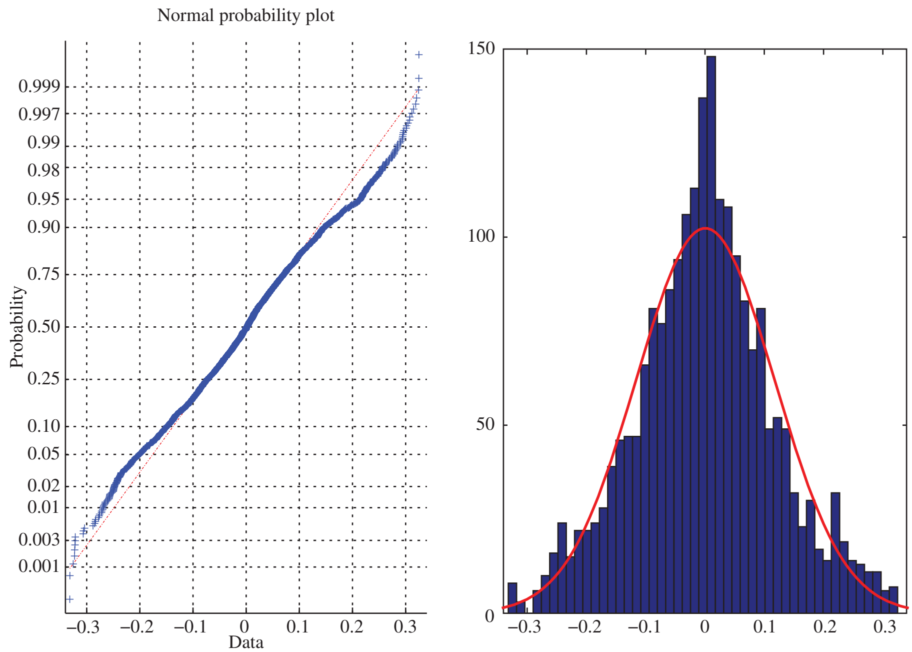

Figure A7. Normal probability plot and histogram of the diffusion part (French baseload process).

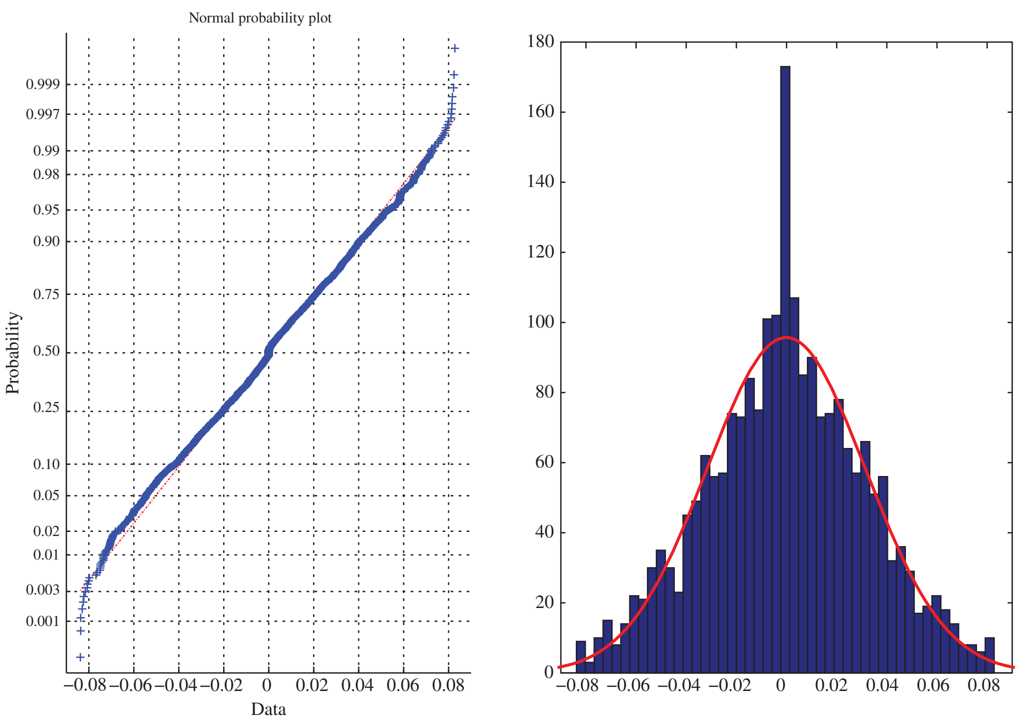

Figure A8. Normal probability plot and histogram of the diffusion part (Nordic baseload process).

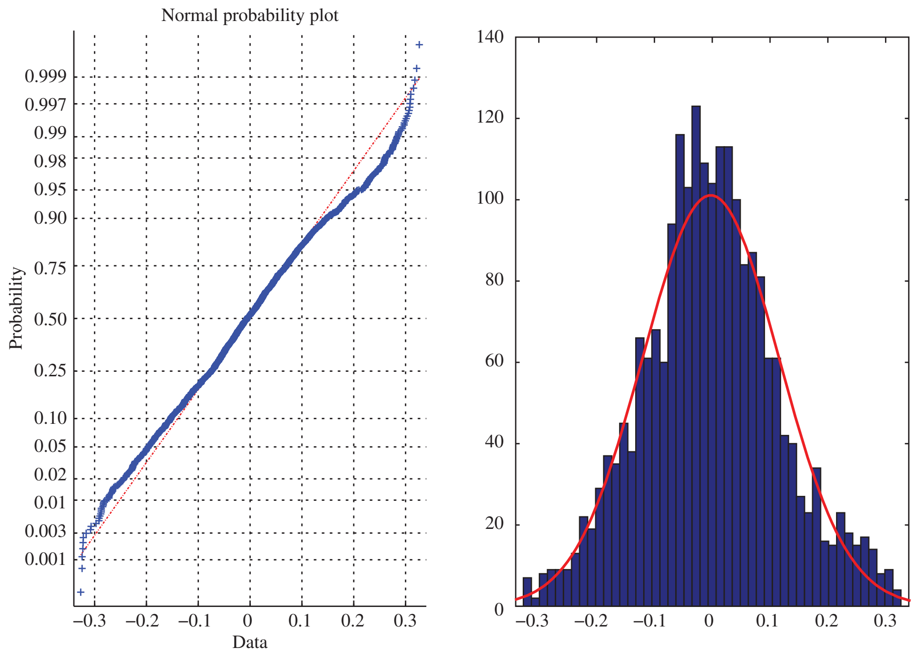

Figure A9. Normal probability plot and histogram of the diffusion part (British baseload process).

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