Simulation and estimation of macroeconomic models in Dynare

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1   Introduction

This chapter provides a brief guide to using Dynare. Dynare is a free software (available at http://www.dynare.org/) able to simulate and estimate Dynamic Stochastic General Equilibrium (DSGE) models. Dynare is available for Windows, Mac and Linux. Matlab version 7.0 (R14) or above is required to run Dynare. As a free alternative to Matlab it is also possible to run Dynare on GNU Octave version 3.0.0 or above. No additional toolboxes are necessary but having Matlab's 'optimization toolbox' installed allows for additional options when using some commands. Dynare's current development team consists of Stéphane Adjemian, Houtan Bastani, Michel Juillard, Frédéric Karamé, Junior Maih, Ferhat Mihoubi, George Perendia, Marco Ratto and Sébastien Villemot.

1.1   Installation

Start by downloading (or execute the installer directly from the website) the latest version of Dynare for Matlab (Windows) from http://www.dynare.org/. Unzip the compressed file which is now on your computer. The default destination folder is 'c:\dynare\4.x.y' where x and y represent the version number upgrade. This directory contains several sub-directories: 'doc', 'dynare++', 'examples', 'matlab' and 'mex'. The next step is to add the Dynare 'matlab' subdirectory to the MATLAB path. To do this go to Matlab's "File" menu and select the "Set Path" entry, then click on "Add Folder. . . ", and select the 'matlab' subdirectory of your Dynare installation. Finally click on "Save". One must verify that there is no directory coming from another version of Dynare on the Matlab path other than

that one intends to use.

For Octave installation instructions please consult the Dynare's present manual Adjemian et al. (2011) or the Dynare Wiki.

1.2 Dynare invocation

The first step is to write a file containing the description of the economic model and the computing tasks required (for example, simulation or estimation of the model). To do this go to Matlab's "File" menu and select the "New" entry, then click on "M-File". Once the *model file* is written (details on how to do this are the aim of the remaining sections of this chapter) one must save it with the filename extension '.mod'. Dynare can then be invoked by writing dynare filename.mod in Matlab's "Command Window". Take care to make sure that the folder (you can have it located in any directory on your computer you choose) containing the *model file* is in the Matlab's "Current Directory" window. You can do this by clicking in the "Current Directory" window, or typing the path directly in the "Current Directory: " box on the toolbar of Matlab.

When Matlab has concluded the computing tasks required it will show the figures (such as priors/ posterior distributions of the model's parameters or impulse response functions) corresponding to the command(s) used in the '.mod' file. You will see several results displayed in the "Command Window"; these could include outputs such as, depending on the command(s) used, parameter estimates or the model's theoretical moments. The results seen in the "Command Window" will be stored in a 'filename.log' Text Document. In the "Workspace" window you can find several other Dynare output results. In particular, the MATLAB workspace will contain the following output variables: 'M_' (structure containing various informations about the model), 'options_' (structure contains the values of the various options used by Dynare during the computation) and 'oo_' (structure containing the various results of the computations). The 'M_', 'oo_' and 'options_' structures are saved in a file called 'filename_results.mat'. The figures, '.log' and '_results.mat' files will be automatically saved in the folder containing the '.mod' file used (as can be seen in Matlab's

"Current Directory" window).

2   Simulation of DSGE models

This section explains the basics of how to write a *model file* in Dynare and use it to simulate DSGE models. The first subsection will describe a simple DSGE model, consisting of a standard Real Business Cycle (RBC) model similar to that presented by King and Rebelo (1999), Romer (2011) and Campbell (1994) and the second subsection will show how to implement it in Dynare.[1]

2.1   A basic Real Business Cycle model

2.1.1   The Model

Consider an economy with a continuum of infinitely lived agents and firms on the interval [0,1]. The social planner maximizes the expected value of the representative household utility function:

$$U = E_t \sum_{i=0}^{\infty} \beta^i (\log C_{t+i} + v \log L_{t+i}) \tag{1}$$

subject to the economy's aggregate budget constraint:

$$Y_t = C_t + I_t + G_t \tag{2}$$

and the household's time endowment:

$$L_t + N_t = 1, \tag{3}$$

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[1]The essential difference between the model presented here and that in King and Rebelo (1999) is the introduction of exogenous government spending. The only difference relative to Romer (2011) and Campbell (1994) is that I ignore population growth (since in the following section I make use of per capita data to estimate the model).

where C_t is consumption, L_t is leisure, Y_t is output, I_t is investment, G_t is government expenses and N_t is hours worked. The parameter $v > 0$ measures the utility from leisure and $0 < \beta < 1$ is the household's subjective discount factor.

The aggregate production function is given by:

$$Y_t = A_t K_t^\alpha (\gamma^t N_t)^{1-\alpha} \tag{4}$$

and the stock of capital (K_t) evolves according to:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{5}$$

where K_t is capital and A_t is Total Factor Productivity (TFP) which follows a stationary process. The parameter α is the capital share of output, γ represents a deterministic labour augmenting technological process and δ is the depreciation rate of capital.

The social planner problem can be solved by first replacing (4) and (5) in (2) and then replacing (2) and (3) in (1). We then take the first order conditions with respect to K_{t+1} and N_t:

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} [MPK_{t+1} + (1-\delta)] \right\} \tag{6}$$

$$\frac{MPN_t}{C_t} = \frac{v}{1-N_t}. \tag{7}$$

where MPK_t and MPN_t stand respectively for the marginal productivities of capital and labor:

$$MPK_{t+1} = \alpha A_{t+1} K_{t+1}^{\alpha-1} (\gamma^{t+1} N_{t+1})^{1-\alpha} \tag{8}$$

$$MPN_t = (1-\alpha)\gamma^t A_t K_t^\alpha (\gamma^t N_t)^{-\alpha}. \tag{9}$$

Since the non-stationary technology process γ^t induces a common trend in output, capital, investment, marginal productivity of labor, consumption and government expenditures, it is convenient to express the model in terms of the detrended variables $\widetilde{Y}_t = (Y_t/\gamma^t)$, $\widetilde{K}_t =$

(K_t/γ^t), $\tilde{I}_t = (I_t/\gamma^t)$, $\widetilde{MPN}_t = (MPN_t/\gamma^t)$, $\tilde{C}_t = (C_t/\gamma^t)$ and $\tilde{G}_t = (G_t/\gamma^t)$. The model's equations are then changed in the following way:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t \tag{10}$$

$$\tilde{Y}_t = A_t \tilde{K}_t^\alpha N_t^{1-\alpha} \tag{11}$$

$$\tilde{I}_t = \gamma \tilde{K}_{t+1} - (1-\delta)\tilde{K}_t \tag{12}$$

$$\gamma \frac{1}{\tilde{C}_t} = \beta E_t \left\{ \frac{1}{\tilde{C}_{t+1}} [MPK_{t+1} + (1-\delta)] \right\} \tag{13}$$

$$\frac{\widetilde{MPN}_t}{\tilde{C}_t} = \frac{v}{1 - N_t} \tag{14}$$

$$MPK_{t+1} = \alpha A_{t+1} \tilde{K}_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} \tag{15}$$

$$\widetilde{MPN}_t = (1-\alpha) A_t \tilde{K}_t^\alpha N_t^{-\alpha} . \tag{16}$$

Finally we conclude the description of our model by specifying the stochastic processes for the exogenous variables A_t and $\tilde{G}_t$. TFP and government spending are both assumed to follow a first-order autoregressive process:

$$\ln(A_t) = (1 - \rho_a)\ln(A) + \rho_a \ln(A_{t-1}) + u_t^a \tag{17}$$

$$\ln(\tilde{G}_t) = (1 - \rho_g)\ln(\tilde{G}) + \rho_g \ln(\tilde{G}_{t-1}) + u_t^g. \tag{18}$$

Where u_t^a and u_t^g represent independent shocks with normal distributions of mean zero and respective standard deviations σ_a and σ_g.

2.1.2 Steady State

In a stochastic setting models need to be linearized before being solved. To do this Dynare needs to know the model's steady state. Dynare can solve for the steady state of a model (see the examples in Collard, 2001, and Griffoli, 2007) using numerical methods but it is usually

only successful if the initial values entered are close to the true steady state. Since this can prove difficult even for simple models I normally prefer to enter the steady state solution by hand in Dynare or to write a Matlab program to find the model's steady state. In the case of the Real Business Cycle model described in this section it is simple to find the model's steady state or balanced growth path of the economy (in which output, capital, investment, marginal productivity of labor, consumption and government expenditures all grow at a constant common rate, the exogenous growth rate of the labour augmenting technological process).

Let's start by normalizing $A = 1$. In the steady state all the variables of the detrended model are constant (i.e., $\tilde{Y}_{t+1} = \tilde{Y}_t = \tilde{Y}$):

$$\tilde{Y} = \tilde{C} + \tilde{I} + \tilde{G} \tag{19}$$

$$\tilde{Y} = \tilde{K}^\alpha N^{1-\alpha} \tag{20}$$

$$\tilde{I} = \gamma\tilde{K} - (1-\delta)\tilde{K} \tag{21}$$

$$\gamma\frac{1}{\tilde{C}} = \beta\frac{1}{\tilde{C}}[MPK_t + (1-\delta)] \tag{22}$$

$$\frac{\widetilde{MPN}}{\tilde{C}} = \frac{v}{1-N} \tag{23}$$

$$MPK = \alpha\tilde{K}^{\alpha-1}N^{1-\alpha} \tag{24}$$

$$\widetilde{MPN} = (1-\alpha)\tilde{K}^\alpha N^{-\alpha} \tag{25}$$

Given values for the parameters $\beta, \alpha, \gamma, \delta$, steady state labour N and $sg = \tilde{G}/\tilde{Y}$ it is easy to calculate exact values for the steady state of the remaining variables. One can simplify (22) to obtain:

$$MPK = \frac{\gamma}{\beta} - (1-\delta) \tag{26}$$

After this it is easy to calculate steady state capital by solving (24) for $\widetilde{K}$:

$$\widetilde{K} = (\frac{MPK}{\alpha N^{1-\alpha}})^{\frac{1}{(\alpha-1)}} \tag{27}$$

Then it becomes simple to obtain $\tilde{Y}, \tilde{I}$ and $\widetilde{MPN}$ from (20), (21) and (25). Steady state government expenditures are given by $\tilde{G} = sg\tilde{Y}$. Consumption can now easily be calculated by using (19):

$$\tilde{C} = \tilde{Y} - \tilde{I} - \tilde{G} \tag{28}$$

Finally, using (23) it is very simple to solve for v:

$$v = \frac{\widetilde{MPN}}{\widetilde{C}}(1 - N) \tag{29}$$

2.1.3   Log-linearization of the model

There is no closed form solution to the equations for this model (an exact analytical solution is possible only in the special case of full depreciation of capital and when agents have log utility). An alternate solution method is needed. The strategy is therefore to look for an approximate analytical solution by transforming the model into a system of log-linear difference equations in the unknowns. Log-linearization transforms the domain with a log function, and then approximates with a first order Taylor expansion in the neighborhood of the steady state balanced growth path. Because in economics the relevant functions in many cases are locally more like exponential functions (such as in growth models) than linear functions; log-linearization provides the best of both worlds – closeness of exponential approximations, and tractability of linear approximations.

Log-linearization is employed as follows. Let X_t be a strictly positive variable, X its steady state and $x_t = log(X_t) - log(X)$ the logarithmic deviation. First notice that, for x near zero, $log(1 + x) \simeq x$, thus:

$$x_t = log(X_t) - log(X) = log(X_t/X) = log(1 + \%change) \simeq \%change.$$

Also notice that you can write $X_t = X(X_t/X) = Xe^{log(X_t/X)} = Xe^{x_t}$. Taking a first order Taylor approximation around the steady state $x = 0$ yields:

$$Xe^{x_t} \simeq Xe^0 + Xe^0(x_t - 0) \simeq X(1 + x_t) \ .$$

By the same logic, you can write:

$$X_tY_t \simeq X(1 + x_t)Y(1 + y_t) \simeq XY(1 + x_t + y_t + x_ty_t)$$

where $x_ty_t \simeq 0$, since x_t and y_t are both numbers close to zero. Let's now look at an example. Take the production function:

$$\tilde{Y}_t = A_t\tilde{K}_t^\alpha N_t^{1-\alpha}.$$

Log-linearizing the above expression yields:

$$\tilde{Y}(1 + \tilde{y}_t) = A\tilde{K}^\alpha N^{1-\alpha}(1 + a_t + \alpha\tilde{k}_t + (1 - \alpha)n_t)$$

where lower case letters denote variables in log deviation from the steady state. Simplifying this expression results in:

$$\widetilde{y}_t = a_t + \alpha\tilde{k}_t + (1 - \alpha)n_t.$$

The Dynare guides by Collard (2001) and Griffoli (2007) focus on examples of models entered in the original non-linear form. My preference however is to write the model in log-linear form, expressing variables as percent deviations from the steady state (the initial values for the model's variables are therefore zero). This also seems to be the favorite choice of many researchers (i.e., 46 of the total of 53 models available in the Macro Model Data Base of Wieland et al., 2011, are entered in linear form).

The log-linearized equations of the basic RBC model described in this section are presented below. As in the production function example, I will use lower case letters to denote

variables in log deviation from the steady state. Log-Linearization of equations (10)-(18) yields:

$$\tilde{Y}\widetilde{y}_t = \tilde{C}\widetilde{c}_t + \tilde{G}\widetilde{g}_t + \tilde{I}\widetilde{\imath}_t \tag{30}$$

$$\widetilde{y}_t = a_t + \alpha\tilde{k}_t + (1-\alpha)n_t \tag{31}$$

$$\tilde{I}\widetilde{\imath}_t = \gamma\tilde{K}\tilde{k}_{t+1} - (1-\delta)\tilde{K}\tilde{k}_t \tag{32}$$

$$\widetilde{c}_t = E_t\widetilde{c}_{t+1} - \frac{MPK}{MPK + (1-\delta)}mpk_{t+1} \tag{33}$$

$$\widetilde{mpn}_t - \widetilde{c}_t = \frac{N}{1-N}n_t \tag{34}$$

$$mpk_{t+1} = a_{t+1} + (\alpha-1)\tilde{k}_{t+1} + (1-\alpha)n_{t+1} \tag{35}$$

$$\widetilde{mpn}_t = a_t + \alpha\tilde{k}_t - \alpha n_t \tag{36}$$

$$a_t = \rho_a a_{t-1} + u_t^a \tag{37}$$

$$\tilde{g}_t = \rho_g\tilde{g}_{t-1} + u_t^g \tag{38}$$

2.2 The Dynare code

As mentioned previously the estimation and simulation of an economic model in Dynare involves writing a *model file*.[2] This can be done by using an external or internal editor to Matlab (go to Matlab's "File" menu and select the "New" entry, then click on "M-File"). Once the *model file* is written it must saved with the filename extension '.mod'.

It is useful to think of the structure of the '.mod' file as having four distinct blocks:

1) the preamble which lists variables (endogenous and exogenous) and parameters;

2) the model which outlines the model's equations;

3) the shocks which can be deterministic (temporary or permanent) or stochastic

4) finally computation which instructs Dynare to undertake certain tasks (such as simu-

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[2]The complete '.mod' files (and the data) used to simulate and estimate the standard RBC model described here are available at: https://sites.google.com/site/joaoantoniorodriguesmadeira/home/dynare.

lation, estimation or forecasting).

The first non-comment line (write two forward slashes '//' before entering any comments in Dynare, this can be useful to help others, and even oneself, understand better the code written) in your Dynare code should be: "var" followed by the model's variables names (all variables apart from the exogenous shocks) and culminating with a semicolon. The dynare code will not run unless each entry is followed by a semicolon to suppress the output (note that an entry may occupy more than one line). The second line of code should be "varexo" followed by the names of the exogenous shocks. The next command should be "parameters" followed by the names of the model's parameters (which also include the model's steady state values when the model is entered in Dynare in linear form, as is the case here). Below this command one must specify the values of these parameters. Thus, the preamble for the RBC model outlined in this section is:

```
var  y c i k n mpk mpn a g;
varexo e_A e_G;

parameters sg beta Nss alpha delta lambdaDT gamma rhoA rhoG MPKss Kss Yss Iss
MPNss Gss Css v;

sg=0.2; // same as in Romer (2011); in King and Rebelo (1999) sg=0
beta=0.984;// Same as in King and Rebelo (1999);
//Romer (2011) uses 1.5% real interest rate (r*),
//since MPKss=r*+delta then r*=(gamma/beta)-1 this implies beta=gamma/(1.015)
Nss=0.2;// same as in King and Rebelo (1999); Romer (2011) uses 1/3
alpha=1/3;// same as in Romer (2011); alpha=0.333 in King and Rebelo (1999)
delta=0.025;// same as in Romer (2011) and as in King and Rebelo (1999)
lambdaDT=0.4;// same as in King and Rebelo (1999); Romer (2011) uses 0.5
gamma=1+(lambdaDT/100);
rhoA=0.95;// same as in Romer (2011); King and Rebelo (1999) use 0.979
rhoG=0.95;// same as in Romer (2011)
MPKss=(gamma/beta)-(1-delta);
Kss=(MPKss/(alpha*(Nss^(1-alpha))))^(1/(alpha-1));
Yss=(Kss^alpha)*(Nss^(1-alpha));
Iss=gamma*Kss-(1-delta)*Kss;
MPNss=(1-alpha)*(Kss^(alpha))*(Nss^(-alpha));
Gss=sg*Yss;
Css=Yss-Iss-Gss;
v=(MPNss/Css)*(1-Nss);
```

The values for the structural parameters $\beta, \alpha, \gamma, \delta$, steady state labour N, $sg = \tilde{G}/\tilde{Y}$,

ρ_a and ρ_g are similar to those found in the literature. I have entered comments to compare the values chosen to those used by Romer (2011) and King and Rebelo (1999).

Writing the model block is very straightforward, since Dynare enables the user to do this by entering the model's equations in a manner similar to the way they are written in an academic paper. This bloc starts with the instruction "model;" and concludes with "end;". Because the model's equations have already been log-linearized in the previous subsection it is necessary is to write the term "(linear)" next to the command "model". To write the model block it is important to be aware of a few notational conventions. A variable x with a t subscript is simply written as "x". A variable with a $t+n$ subscript is written as "x(+n)". Similarly, a variable with a $t-n$ subscript is written as "x(-n)". Hence, the model block (consisting of equations 30 to 38) is as follows:

```
model (linear);
Yss*y=Css*c+Gss*g+Iss*i;
y=a+alpha*k(-1)+(1-alpha)*n;
Iss*i=gamma*Kss*k-(1-delta)*Kss*k(-1);
c=c(+1)-(MPKss/(MPKss+(1-delta)))*mpk;
mpn-c=(Nss/(1-Nss))*n;
mpk=a(+1)+(alpha-1)*k+(1-alpha)*n(+1);
mpn=a+alpha*k(-1)-alpha*n;

//exogenous shocks
a=rhoA*a(-1)+e_A;
g=rhoG*g(-1)+e_G;

end;
```

In Dynare, the default convention is that the timing of a variable reflects when this variable is decided. For example, $\tilde{k}_t$ is a predetermined variable in the RBC model in the previous section (because it reflects investment decisions made previously, in the case of the model considered here capital is the result of investment decisions implemented at date $t-1$) and is thus written as "k(-1)".

To specify the shocks block one needs to declare only the nonzero elements of the covariance matrix. Start with the command "shocks;" and conclude with "end;". For the RBC model considered here this block can be written as follows:

```
shocks;
var e_A;
stderr 0.01;
var e_G;
stderr 0.01;
end;
```

The model is then solved and simulated using the "stoch_simul;" command:

```
stoch_simul (irf=40,hp_filter=1600) a g y c i k n mpk mpn;
```

This command will compute the model's impulse response functions and various descriptive statistics (moments, variance decomposition, correlation and autocorrelation coefficients). I made use of some options of the "stoch_simul;" command. The option "irf = integer" allows one to chose the number of periods plotted in the impulse response functions. The default value is 40. The figure Dynare creates for the impulse response functions to a 1% productivity shock is shown below:
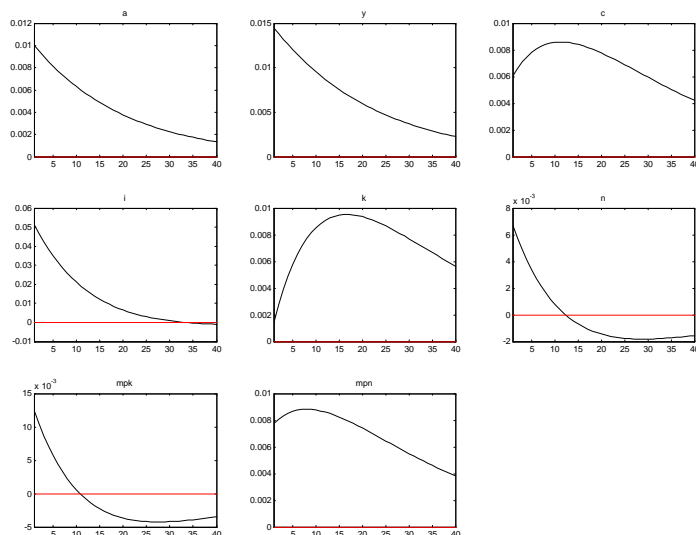
Notice that Dynare plots the variables being decided at date t. For this reason the initial value of the capital stock is displayed as jumping on impact (the variable plotted is $\tilde{k}_{t+1}$ and

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not $\tilde{k}_t$). If a list of variables is specified after "stoch simul()" then results are displayed only for those variables and in the order in which they are listed (for this reason the order of variables in the impulse response functions is the same as that of the list).

The option "hp_filter = integer" produces theoretical moments (variances, covariances, autocorrelations) after HP filtering the data (the default is to apply no filter to the data), just as in Table 2 of King and Rebelo (1999). This is a very useful option because in order to isolate the business cycle component of economic time series many researchers employ the HP filter. So, one must apply the HP filter to the model's theoretical moments if one wants to compare them to those of HP filtered data from aggregate time series (such as the moments reported in Table 1 of King and Rebelo, 1999). The integer is a number corresponding to the smoothing parameter in the HP filter. For quarterly data (which is typically the frequency of interest for economists studying business cycles) Hodrick and Prescott (1981) recommended a value of 1600.

3   Model Estimation

In this section we will look at how to estimate the DSGE model outlined in the previous section using Bayesian techniques as in Smets and Wouters (2003; 2007). To undertake Bayesian estimation it is necessary to have at least as many shocks as there are observable variables. Since the model outlined in the previous section has two exogenous shocks (to productivity and government expenses) the dataset used can contain at most two time series. To estimate the model I used a dataset consisting of seasonally adjusted quarterly US real GDP growth and real investment (so that the time series used are stationary). These same time series were used in Smets and Wouters (2007) but I updated the dataset to include observations for more recent years. Following Gali et al. (2011), I estimated the model for the period 1966Q1 to 2007 Q4 due to concerns that the nonlinearities induced by the zero lower bound on the federal funds rate could distort the estimates for some of the parameters (whereas Smets and Wouters, 2007, estimated their model with data from 1966Q1 to 2004Q4).

The corresponding measurement equations are:

$$dlGDP_t = \bar{\lambda} + [\tilde{y}_t - \tilde{y}_{t-1}], \tag{39}$$

$$dlINV_t = \bar{\lambda} + [\tilde{i}_t - \tilde{i}_{t-1}], \tag{40}$$

where dl stand for log difference and $\bar{\lambda}$ is the common quarterly trend growth rate to real GDP and investment. The parameter $\bar{\lambda}$ is related to the steady state of the model economy as follows: $\gamma = 1 + \frac{\bar{\lambda}}{100}$.

As in the prior section writing the *model file* in Dynare for estimation purposes starts with declaring the model's variables and parameters. This is done exactly as previously described and the preamble bloc looks very similar (the only difference is that the growth rate of real GDP and investment were added to the endogenous variable list in the first line of the code since the parameter $\bar{\lambda}$ was already previously listed):

```
var  y c i k n mpk mpn a g dy dinve;
varexo e_A e_G;

parameters sg beta Nss alpha delta lambdaDT gamma rhoA rhoG MPKss Kss Yss Iss
MPNss Gss Css v;

sg=0.2; // same as in Romer (2011); in King and Rebelo (1999) sg=0
beta=0.984;// Same as in King and Rebelo (1999);
//Romer (2011) uses 1.5% real interest rate (r*),
//since MPKss=r*+delta then r*=(gamma/beta)-1 this implies beta=gamma/(1.015)
Nss=0.2;// same as in King and Rebelo (1999); Romer (2011) uses 1/3
alpha=1/3;// same as in Romer (2011); alpha=0.333 in King and Rebelo (1999)
delta=0.025;// same as in Romer (2011) and as in King and Rebelo (1999)
lambdaDT=0.4;// same as in King and Rebelo (1999); Romer (2011) uses 0.5
gamma=1+(lambdaDT/100);
rhoA=0.95;// same as in Romer (2011); King and Rebelo (1999) use 0.979
rhoG=0.95;// same as in Romer (2011)
MPKss=(gamma/beta)-(1-delta);
Kss=(MPKss/(alpha*(Nss^(1-alpha))))^(1/(alpha-1));
Yss=(Kss^alpha)*(Nss^(1-alpha));
Iss=gamma*Kss-(1-delta)*Kss;
MPNss=(1-alpha)*(Kss^(alpha))*(Nss^(-alpha));
Gss=sg*Yss;
Css=Yss-Iss-Gss;
v=(MPNss/Css)*(1-Nss);
```

Again, the second step is writing the model bloc (to which the measurement equations must be added) of the Dynare code. Therefore the model bloc now includes equations 30 to 40 and looks now as follows:

```
model (linear);
Yss*y=Css*c+g+Iss*i;
y=a+alpha*k(-1)+(1-alpha)*n;
Iss*i=gamma*Kss*k-(1-delta)*Kss*k(-1);
c=c(+1)-(MPKss/(MPKss+(1-delta)))*mpk;
mpn-c=(Nss/(1-Nss))*n;
mpk=a(+1)+(alpha-1)*k+(1-alpha)*n(+1);
mpn=a+alpha*k(-1)-alpha*n;

//exogenous shocks
a=rhoA*a(-1)+e_A;
g=rhoG*g(-1)+e_G;

//measurement equations
dy=lambdaDT+(y-y(-1));
dinve=lambdaDT+(i-i(-1));
end;
```

As in Smets and Wouters (2007) I normalize the exogenous government spending process. I do this by defining a new variable $\tilde{g}_t^* = \tilde{G}\tilde{g}_t$. In this way it is easier to choose a reasonable prior for the standard deviation and will make it easier to compare the estimate for σ_g obtained for the RBC model described in the prior section with that obtained by Smets and Wouters (2007). As Justiniano et al.(2008) point out this is "a practical way to impose correlated priors across coefficients, which is desirable in some cases" and frequently helps "improve the convergence properties of the MCMC algorithm".

I now must declared which variables are observable for the estimation procedure. This is done using the command "varobs" followed by the names of the observable variables used in the estimation:

```
varobs dy dinve;
```

The next step is to declare the parameters to be estimated. I choose to estimate values for $\gamma, \alpha, \rho_a, \rho_g, \sigma_a$ and σ_g (the remaining parameters I maintain fixed in the estimation procedure since it would be difficult to identify well all parameters from the data). This is done by writing "estimated_params;" followed by the parameters to be estimated (one in each line) and concluding with "end;":

```
estimated_params;
lambdaDT,    0.4,0.1,0.8,       NORMAL_PDF,0.4,0.10;
alpha,       0.33,0.01,1.0,     NORMAL_PDF,0.3,0.05;
rhoA,        0.95,.01,.9999,    BETA_PDF,0.5,0.2;
rhoG,        0.95,.01,.9999,    BETA_PDF,0.5,0.2;
stderr e_A, 0.013,0.01,5,       INV_GAMMA_PDF,0.1,2;
stderr e_G, 0.021,0.01,5,       INV_GAMMA_PDF,0.1,2;
end;
```

To estimate the RBC model using maximum likelihood (as in Ireland, 2004) one would use the following syntax (fields listed between "[...]" are optional) for each estimated parameter: "parameter name, initial value [, lower bound, upper bound];". However, estimating the model with maximum likelihood resulted in an unrealistically high level ($\alpha = 0.6125$) for the capital share. This is a common problem when estimating DSGE models and for this reason the use of Bayesian methods has become very popular among macroeconomic researchers. In Bayesian estimation, each line is written accordingly to the following syntax: "parameter name, [initial value [, lower bound, upper bound]], prior shape, prior mean, prior standard error [, prior third parameter, prior fourth parameter];". I choose prior distributions for the model's parameters to be the same as those in Smets and Wouters (2007). The possible prior shapes are: "beta_pdf", "gamma_pdf", "normal_pdf", "uniform_pdf", "inv_gamma_pdf", "inv_gamma1_pdf" and "inv_gamma2_pdf". Note that "inv_gamma_pdf" is equivalent to "inv_gamma1_pdf". For the "beta_pdf", "gamma_pdf", "normal_pdf", "inv_gamma_pdf", "inv_gamma1_pdf" and "inv_gamma2_pdf" shapes one must specify values for the prior mean and prior standard error of the distribution. The prior third parameter is used for generalized beta distribution, generalized gamma and for

the uniform distribution. The prior fourth parameter is used for the generalized beta distribution and for the uniform distribution. As one uses fields more towards the end of the list, all previous fields must be filled (use empty values, if the parameters don't apply). For instance, to specify a uniform distribution between 0 and 1 for α one would enter "alpha, 0.33,0.01,1.0,   uniform pdf, , , 0,1;" since the uniform distribution only takes the third and fourth parameters as arguments.

Finally, the model is estimated using the "estimation();" command:

```
estimation(optim=('MaxIter',200),datafile=rbc_model_data,mode_compute=4,
first_obs=71,nobs=172,presample=4,lik_init=2,mode_check,prefilter=0,
mh_replic=250000,mh_nblocks=2,mh_jscale=1.08,mh_drop=0.2);

stoch_simul (hp_filter=1600,irf=40) a g y c i k n mpk mpn;
```

This command will display results from posterior optimization, graphs with prior, posterior and mode, marginal log data density, graphs of smoothed shocks, smoothed observation errors, smoothed and historical variables. After estimation with Metropolis iterations Dynare will display the mean and confidence interval from posterior simulation and Metropolis-Hastings convergence graphs.

A result that I consider important to highlight is the marginal log data density. The marginal likelihood of the model gives an indication of the overall likelihood of the model given the data and reflects its prediction performance. It therefore forms a natural benchmark for comparing the overall fit of DSGE models. This can be done by computing the Bayes factor. The Bayes factor (BF) of model 1 against model 2 is the difference of their log marginal likelihoods. Kass and Raftery (1995) suggest that values of 2 logBF above 10 can be considered very strong evidence in favor of model 1. Values between 6 and 10 represent strong evidence, between 2 and 6 positive evidence, while values below 2 are 'not worth more than a bare mention'.

I made use of some options of the "estimation();" command which are explained briefly in the paragraphs below:

The option "optim = (fmincon options)" allows one to set options for fmincon (a function available in MATLAB's optimization toolbox).

The option " datafile = filename" specifies the data file (which can be either a '.m' file, a '.mat' file or an '.xls' file) to be used in the estimation process. The data file used 'rbc_model_data.m' contains two vectors of observations: real GDP and real investment growth.[3] This file must be located in the same folder as the '.mod' file.

The option "mode_compute = integer" allows the user to choose the optimizer for the mode computation: 0 the mode isn't computed (mode_file option must be specified); 1 uses fmincon optimization routine (not available under Octave); 2 value no longer used; 3 uses fminunc optimization routine; 4 (default value) uses Chris Sims's csminwel; 5 uses Marco Ratto's newrat; 6 uses a Monte-Carlo based optimization routine (see Dynare wiki for more details); 7 uses fminsearch an optimization routine available under MATLAB (if the optimization toolbox is installed) and Octave (if the optim package from Octave-Forge is installed).

The "first obs = integer" option allows one to choose the number of the first observation to be used (default: 1). Even though the observations in my data file start at 1946Q3 with this option I'm able to chose to use only a subsample of the data (in this case I wish to make use only of data from 1966Q1 onwards to make my results more easy to compare with those of Smets and Wouters, 2007).

The "nobs = integer" option allows the user to specify the number of observations to be used (default: all observations in the datafile). I chose a value of 172 so that Dynare uses data only up to 2007Q4 (while the datafile contains observations up to 2010Q4).

The option "presample = integer" specifies the number of observations to be skipped before evaluating the likelihood (default: 0).

The option "lik_init = integer" specifies the type of initialization of the Kalman filter (default value: 1): 1 is for stationary models while 2 and 3 are for nonstationary models.

---

[3]To obtain time series for real GDP and investment growth per capita I made use of data on the Civilian Population age 16 and over, Real GDP, the GDP price deflator and Fixed Private Investment. This data was obtained from the St. Louis Fed website (http://research.stlouisfed.org/fred2/). Further details are provided in the 'readme.doc' and 'rbc_model_data.xls'.

The option "mode_check" tells Dynare to plot the posterior density for values around the computed mode for each estimated parameter. This is helpful to diagnose problems with the optimizer.

If the "prefilter = integer" is set to one then the estimation procedure demeans the data (default: 0, that is no prefiltering).

The option "mh replic = integer" sets the number of replications for the Metropolis Hasting algorithm. The default is 20,000. I opted to create a sample of 250 000 draws as in Smets and Wouters (2007).

The option "mh_nblocks = integer" specifies the number of parallel chains for the Metropolis-Hastings algorithm (default: 2).

The "mh_jscale = double precision number" option is used to set the scale of the jumping distribution in the Metropolis-Hastings algorithm (the default is 0.2). The value used should be adjusted to yield an acceptance rate of approximately 23%, the optimal rate proposed by Gelman et al. (1996). For the RBC model estimated in this section a value of 1.08 resulted in an acceptance rate of 23.2%.

The "mh_drop = double precision number" specifies the fraction of initially generated parameter vectors to be dropped before using posterior simulations (default: 0.5). I chose a value of 0.2 as in Smets and Wouters (2007).

The posterior means and confidence intervals obtained are presented below:

```
parameters
          prior mean post. mean   conf. interval  prior     pstdev

lambdaDT      0.400      0.4238    0.3921  0.4548  norm      0.1000
alpha         0.300      0.3791    0.3152  0.4409  norm      0.0500
rhoA          0.500      0.9480    0.9257  0.9716  beta      0.2000
rhoG          0.500      0.9500    0.9389  0.9606  beta      0.2000

standard deviation of shocks
          prior mean post. mean   conf. interval  prior     pstdev

e_A           0.100      0.4595    0.3805  0.5339  invg      2.0000
e_G           0.100      0.5411    0.4767  0.6009  invg      2.0000
```

The parameter estimates for the RBC model considered are very much in line with the values commonly used in the literature for calibration purposes (as can be seen by comparing them to those used in the previous section) and very similar to the estimates by Smets and Wouters (2007) for equivalent parameters (the respective posterior means found by these author for $\gamma, \alpha, \rho_a, \rho_g, \sigma_a$ and σ_g were 0.43, 0.19, 0.95, 0.97, 0.45 and 0.53).

If one wishes it is also possible to add the "stoch simul;" command at the end as shown above. I normally chose to do this so that Dynare uses the estimated results to compute the model's impulse response functions, theoretical moments, variance decomposition, matrix of correlations and coefficients of autocorrelation. For maximum likelihood estimation the parameters and the variance matrix of the shocks are set to the mode. For Bayesian estimation the parameters and the variance matrix of the shocks are set to the posterior mode computation in the case without Metropolis iterations or to the posterior mean after estimation with Metropolis iterations.

4 Useful Resources

Dynare can also be used for many other applications which were not explored here such as deterministic shocks (permanent and temporary), Bayesian VAR, forecasting, optimal policy or sensitivity and identification analysis. Besides the Dynare guides by Collard (2001) and Griffoli (2007), the document by Bhandari et al. (2010) available at the Thomas Sargent webpage (https://files.nyu.edu/ts43/public/research/AP_tom16.pdf) is also a valuable resource where users of Dynare can find examples of '.mod' files for several of these applications. Other resources that I find to be of great use are the forums in the Dynare webpage. These forums allow users to ask questions and get feedback from Dynare developers and fellow researchers. It is often best to start by searching prior posts as it is likely that one's query has been raised previously and that the forum already has answers for most issues an user may experience.

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