1. **Problem**
   Match ETS models with their formulas:

   A:
   $$y_t = \ell_{t-1} + \phi b_{t-1} + \varepsilon_t$$
   $$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha\varepsilon_t$$
   $$b_t = \phi b_{t-1} + \beta\varepsilon_t$$

   B:
   $$y_t = \ell_{t-1}s_{t-m} + \varepsilon_t$$
   $$\ell_t = \ell_{t-1} + \alpha\varepsilon_t/s_{t-m}$$
   $$s_t = s_{t-m} + \gamma\varepsilon_t/\ell_{t-1}$$

   s C:
   $$y_t = \mathcal{E}_{t-1}(1 + \mathcal{E}_t)$$
   $$\ell_t = \ell_{t-1}(1 + \alpha\mathcal{E}_t)$$

   D:
   $$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m})(1 + \varepsilon_t)$$
   $$\ell_t = \ell_{t-1} + b_{t-1} + \alpha(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_l$$
   $$b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_l$$
   $$\ell_t = s_{t-m} + \gamma(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$$

   (a) ETS(ANM)
   (b) ETS(AAdN)
   (c) ETS(MAA)
   (d) ETS(MNN)

   Write down in the solution the sequence of numbers without spaces or delimeters corresponding to the formulas for the ETS models in each case (e.g. 4231).

   Solution
   Check lectures ETS Model (I) and (II).

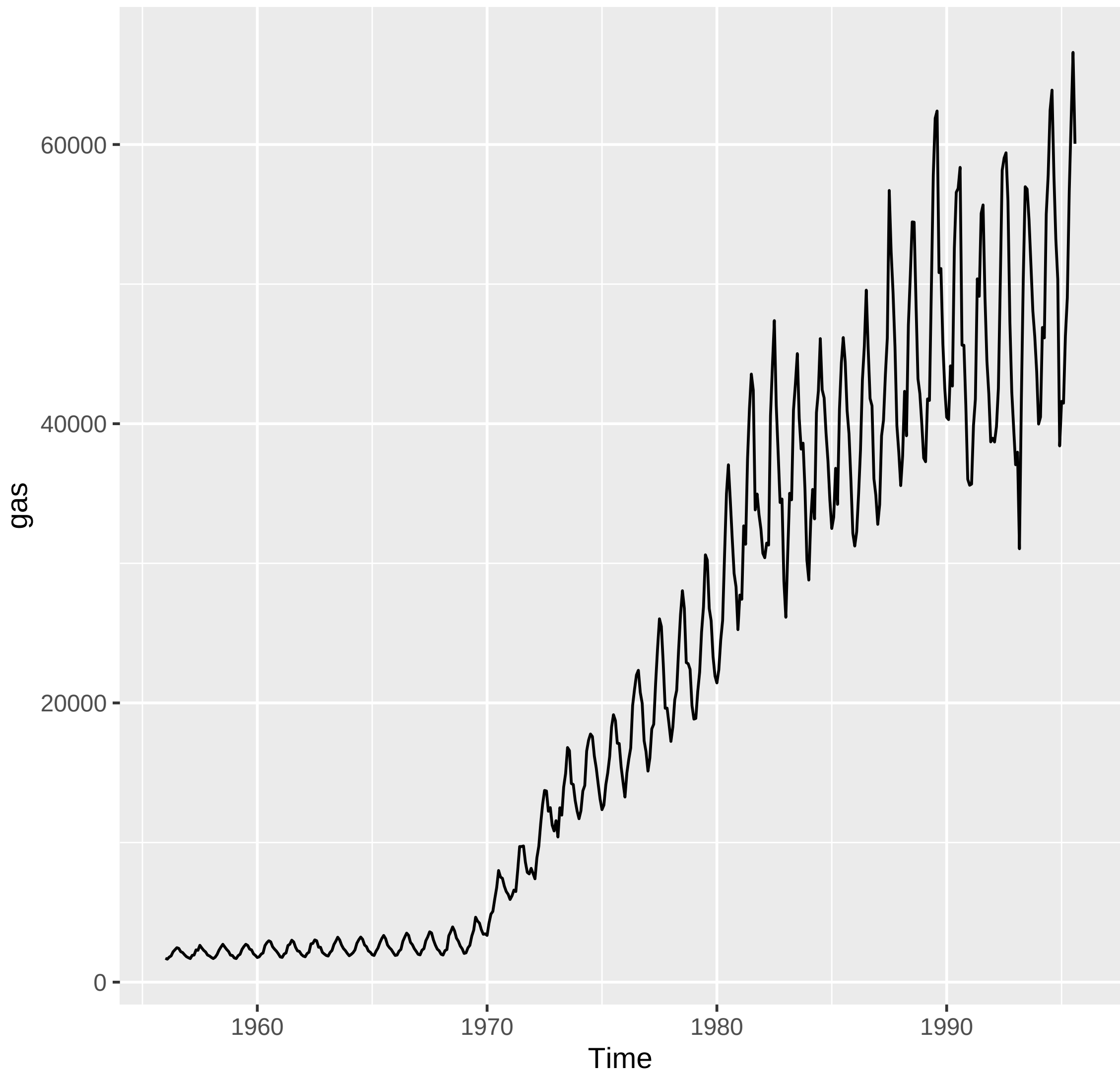
2. **Problem**
   Which of the following statements are true?

   (a) To calculate ACF(2) you can either evaluate a paired regression of y_t on y_{t-2} or calculate the sample correlation between them.
   (b) You can't cluster time series just by using ACF and PACF features.
   (c) PACF(12) for monthly data would show how the change in the value of the series one year ago would influence current value if all the other values between were being constant.
   (d) PACF(3) shows how y_{t-3} influences y_t on average.

   Solution

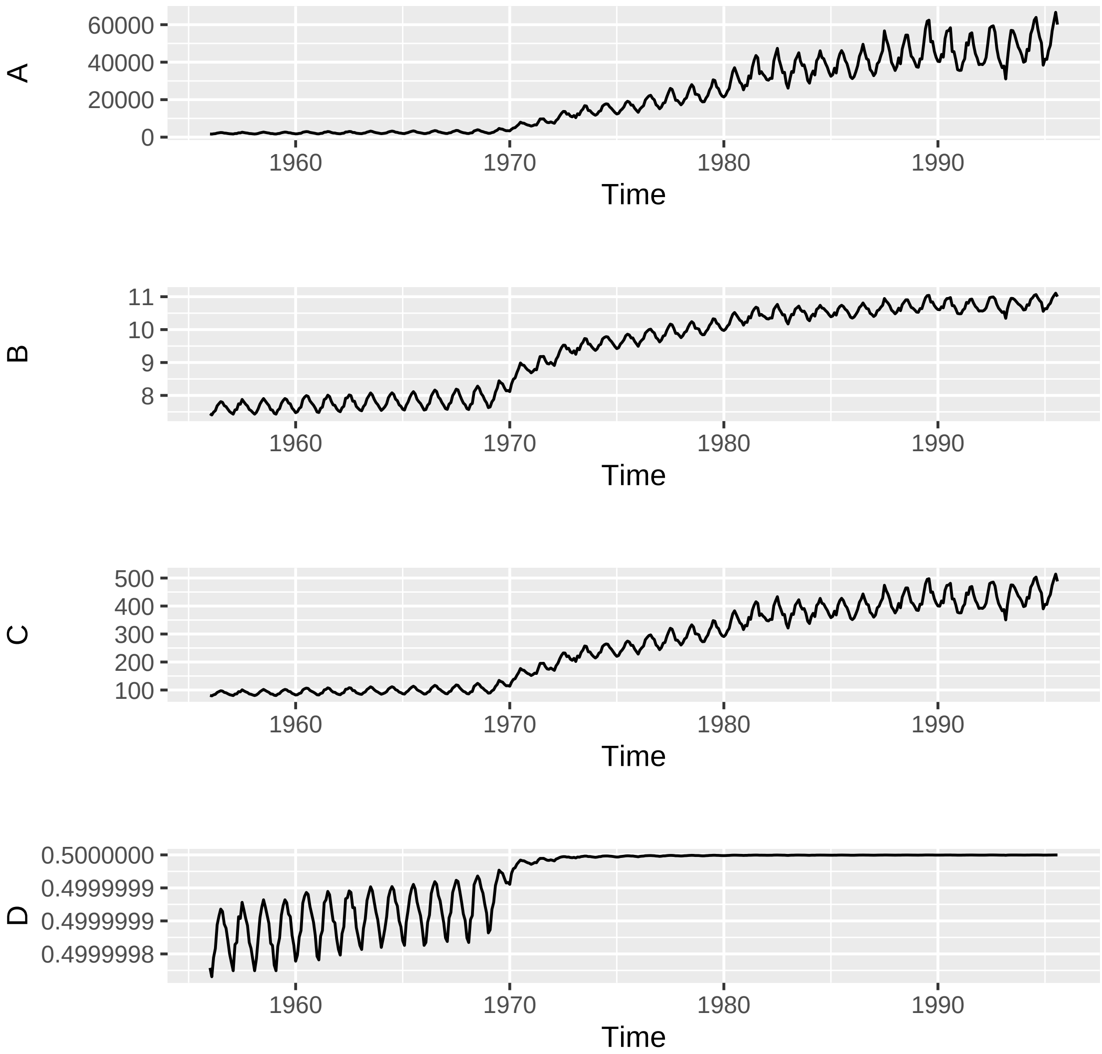
   (a) True.
   (b) False. You can!
   (c) True.
   (d) False. This is ACF(3)

3. **Problem**
   Consider the following time series on Australian monthly gas production:



. 1: plot of chunk unnamed-chunk-1

We have applied Box-Cox tranformation with different parameters:



. 2: plot of chunk unnamed-chunk-2

(a) $\lambda = -2$
(b) $\lambda = 0$
(c) $\lambda = 1$
(d) $\lambda = 0.5$

Write down in the solution the sequence of numbers without spaces or delimeters corresponding to parameters of Box-Cox transformation $\lambda_A, \lambda_B, \lambda_C, \lambda_D$ applied in each case (e.g. 4231).

Solution
Suppose y^{bc} is transformed data. If $\lambda = 0$, the transformation is equivalent to $y^{bc} = \ln(y)$, if $\lambda = 1$ it's $y^{bc} = y$, if $\lambda = 0.5$ it's $y^{bc} = \sqrt{y}$, if $\lambda = -2$ it's $y^{bc} = y^{-2}$.

4. **Problem**
Select the correct statements about STL.

(a) The algorithm contains two loops: outer and inner loop
(b) Increasing the number of iterations of the outer loop can help in case of severe outliers
(c) Within the inner loop the low-frequency component is extracted using LOESS applied to double moving average of the original time series
(d) The weights for the outer loop should be chosen by cross-validation

Solution

(a) True.
(b) True.
(c) False. No it's applied to seasonal subcomponents C_t
(d) False. The weights are recalculated on each iteration of the algorithm depending on R_t.

5. **Problem**
Consider gas dataset. You need to do STL with 2 inner and 1 outer loop for log(gas). Calculate the strength of trend using the practical formula for the first 12 years of observations and for the last 12 years. Give the absolute value of the difference between them as an answer. Provide the answer rounded up to 4 decimal digits.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

Solution
Calculate strength of trend according to the formulas given in the lecture.

6. **Problem**
Select the characteristics of STL.

(a) STL does not have an underlying model
(b) STL can only provide the confidence interval for the predicted values
(c) STL can provide the confidence interval for the seasonal component
(d) Seasonal component should not change over time
(e) Errors should be uniformely distributed

Solution

(a) True.
(b) False. STL is not a model
(c) False. STL is not a model
(d) False. No assumptions for DGP are required
(e) False. No assumptions for DGP are required

7. **Problem**
Consider gas dataset. You need to do STL with 2 inner and 1 outer loop for log(gas). Next check whether the variance of the first half of the seasonal component equals to the second using F-test. Provide the p-value rounded up to 2 decimal digits.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

Solution
You can use var.test(x, y, alternative = "two.sided").

8. **Problem**
   Which of the following statements are true?

   (a) You can improve forecast quality by averaging forecasts of a complex model and a naive one.

   (b) There is no model to approximate dynamics of non-stationary time series.

   (c) You can model stationary process using random walk model.

   (d) A very complex model can perform worse than a naive independent observations model.

   Solution

   (a) True.

   (b) False. The stationary series models are similar to the independent observations model; non-stationary series models are similar to a random walk.

   (c) False. The stationary series models are similar to the independent observations model; non-stationary series models are similar to a random walk.

   (d) True.

9. **Problem**
   Consider gas dataset. First, take logarithm, first difernece and seaasonal difference. Then calculate how many lags in ACF are significant.

   You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

   Solution
   Use acf(x, plot = FALSE) function.

10. **Problem**
    How many parameters should be estimated in ETS(MAdM) with 4 seasons?

    $y_t = (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (1 + \varepsilon_t)$
    $\ell_t = (\ell_{t-1} + \phi b_{t-1})(1 + \alpha\varepsilon_t)$
    $b_t = \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1})\varepsilon_t$
    $s_t = s_{t-m}(1 + \gamma\varepsilon_t)$

    Solution
    4 model parameters + variance of the error term + (6-1) starting observations

11. **Problem**
    Consider an independent observations model with $u_t \sim (0, 0.01)$. Construct two 95% prediction interval for random walk model and independent observations model 100 steps ahead. How many times the width of the random walk PI is bigger than of independent observations PI?

    Provide the answer with 2 decimal digits.

    Solution
    Check lecture "Naive models".

12. **Problem**
    Consider a random walk model. You have a sample $y^T = [-2.34, -1.7, -1.18, -3.26, -4.02]$ and you want to check if forecast 2 steps ahead would be non-negative. Calculate left hand side of your confidence interval, corresponding to a required test.

    Provide the answer with 2 decimal digits.

    Solution
    Check lecture "Naive models".

13. **Problem**
    Consider gas dataset. You need to use seasonal naive model to forecast gas production one year ahead. Calculate the width of prediction interval 2 and 12 steps ahead. Give the absolute value of the difference of these widths as an answer.

    You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

    Solution
    Check lecture "Naive models".

14. **Problem**
    Select the characteristics of STL.

    (a) Forecast 10 steps ahead for ETS(ANN) is the last observation
    (b) Formally there are 30 options for ETS model
    (c) ETS is estimated using MLE
    (d) To estimate ETS(ANN) model you only need to estimate one parameter in the trend component ℓ_t

    Solution

    (a) True.
    (b) True.
    (c) True.
    (d) False. You also need to estimate starting obrevation ℓ_0 and variance of the error term σ^2

15. **Problem**
    Consider ETS(AAdA) with $\alpha = 0.2, \beta = 0.1, \gamma = 0.3, \phi = 0.7, \sigma^2 = 1, l_T = 20, b_T = 2$, $s_T = 0.8, s_{T-1} = 0.7, s_{T-2} = 0.1, s_{T-3} = 0, s_{T-4} = -0.1, s_{T-5} = -0.7, s_{T-6} = -0.8$ and weekly seasonality.

    Construct a 95%-interval forecast 2 steps ahead and calculate it's width. Provide the answer with 2 decimal digits.

    Solution
    Check lectures ETS Model (I) and (II).

16. **Problem**
    Consider the following time series on Australian wine sales from wineind dataset. You need to use naive model to forecast gas production in November, 1994. Provide the answer with 2 decimal digits.

    You can load the gas wineind in R by importing forecast library or if you use other programming languages you can download it here.

    Solution
    Check lecture "Naive models".

17. **Problem**
    Consider ETS(AAdN) model with $\ell_0 = 90, b0 = 10, \phi = 0.9, \alpha = 0.1, \beta = 0.2, \sigma^2 = 10$. We know that $y_1 = 100$.

    Find smoothed value ℓ_1.

    Provide the answer with 2 decimal digits.

    Solution
    Plug everything in a model and derive ℓ_1.

18. **Problem**
    What is correct about LOESS?

    (a) You can only use normal Kernel function
    (b) The bigger the window h the smoother the resulting curve
    (c) You can predict using LOESS
    (d) The higher the degree of the polynomial the smoother the resulting curve

    Solution

    (a) False. You can use any kernel function
    (b) True.
    (c) True.
    (d) False. It works other way around

19. **Problem**
    Consider gas dataset. Estimate ETS model without a trend and automatically selected error and seasonality specification. Calculate point forecast 2 years ahead. Provide the answer with 2 decimal digits.

    You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

    Solution
    You can use ets(x,"ZZZ") function.

20. **Problem**
    Consider ETS(AAN) model with $\ell_0 = 30, b0 = 1, \alpha = 0.2, \beta = 0.3, \sigma^2 = 16$. We know that $y_1 = 32, y_2 = 35$.

    Find smoothed values ℓ_1, ℓ_2 and give their sum as an answer.

    Provide the answer with 2 decimal digits.

    Solution
    Check lecture "Naive models".

21. **Problem**
    Consider a seasonal random walk model for the following sample $y^T = [1, 3, 2, -1, 2, 4, 2, -3]$ of quarterly observations. Construct a 95% prediction interval 19 steps ahead. Provide upper margin of thw interval as the answer.

    Provide the answer with 2 decimal digits.

    Solution
    Check lecture "Naive models".

22. **Problem**
    Consider LakeHuron dataset. You need to calculate how many lags in ACF are significant.

    You can load the LakeHuron data set in R by issuing the following command at the console data("LakeHuron") or if you use other programming languages you can download it here.

    Solution
    Use acf(x, plot = FALSE) function.

23. **Problem**
    Select the characteristics of STL.

    (a) Trend in ETS model can't be negative

(b) If the seasonal oscillation amplitude is decreasing over time it's better to use ETS without seasonal component

(c) In long run in model with damped trend the trend disappears

(d) If the seasonal oscillation amplitude is increasing over time you can use Box-Cox transformation

Solution

(a) False. It can!

(b) False. It's better to use model with multiplicative seasonality

(c) True.

(d) True.

24. **Problem**
Consider gas dataset. You need to use seasonal naive model to forecast gas production in December, 2099. Provide the answer with 2 decimal digits.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

Solution
Check lecture "Naive models".

25. **Problem**
How many parameters should be estimated in ETS(MAdA) for weekly data?

$$\begin{aligned}
&y_t = (\ell_{t-1} + \phi b_{t-1} + s_{t-m})(1 + \varepsilon_t) \\
&\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t \\
&b_t = \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t \\
&s_t = s_{t-m} + \gamma(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t
\end{aligned}$$

Solution
Check lecture "STL Decomposition"

26. **Problem**
Consider ETS(MAM) with $\alpha = 0.1, \beta = 0.3, \gamma = 0.2, \sigma^2 = 0.5, l_T = 10, b_T = 1, s_T = 0.1, s_{T-1} = 0.5, s_{T-2} = -0.1, s_{T-3} = -0.5$ and quarterly seasonality

$$\begin{aligned}
&y_t = (\ell_{t-1} + b_{t-1})s_{t-m}(1 + \varepsilon_t) \\
&\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t) \\
&b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t \\
&s_t = s_{t-m}(1 + \gamma\varepsilon_t)
\end{aligned}$$

Construct a point forecast 2 steps ahead.

Solution
Check lectures ETS Model (I) and (II).

27. **Problem**
Consider an independent observations model with $u_t \sim (0, 4)$. Construct a 95% prediction interval for independent observations model 100 steps ahead. Give the width of the interval as an answer.

Provide the answer with 2 decimal digits.

Solution
Check lecture "Naive models".

28. **Problem**
Consider gas dataset. Estimate ETS model without a trend and automatically selected error and seasonality specification. Provide initial state for trend component as an answer. Provide the answer with 2 decimal digits.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

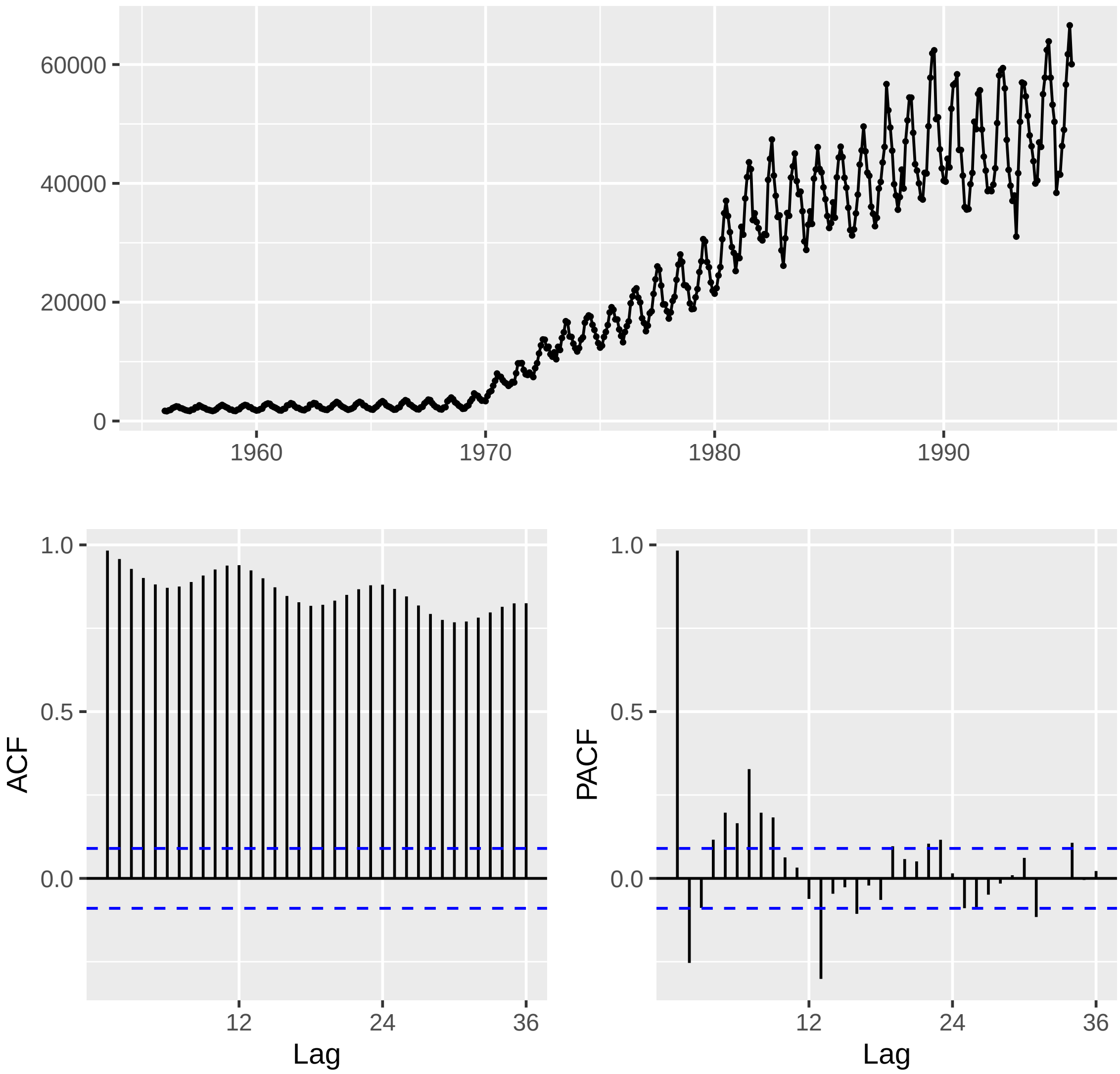
Solution
You can use ets(x,"ZZZ") function.

29. **Problem**
Consider a seasonal random walk model for the following sample $y^T = [1, 3, 2, -1, 2, 4, 2, -3]$ of quarterly observations. Construct a 95% prediction interval 19 steps ahead. Provide upper margin of thw interval as the answer.

Provide the answer with 2 decimal digits.

Solution
Check lecture "Naive models".

30. **Problem**
Consider the following time series on Australian monthly gas production:



. 3: plot of chunk unnamed-chunk-1

Which ETS model is more appropriate to use?

(a) ETS(AAdA)
(b) ETS(MAdM)
(c) ETS(MAM)
(d) ETS(AAA)

Solution

(a) False. Does not account for changing seasonality
(b) True.
(c) True.

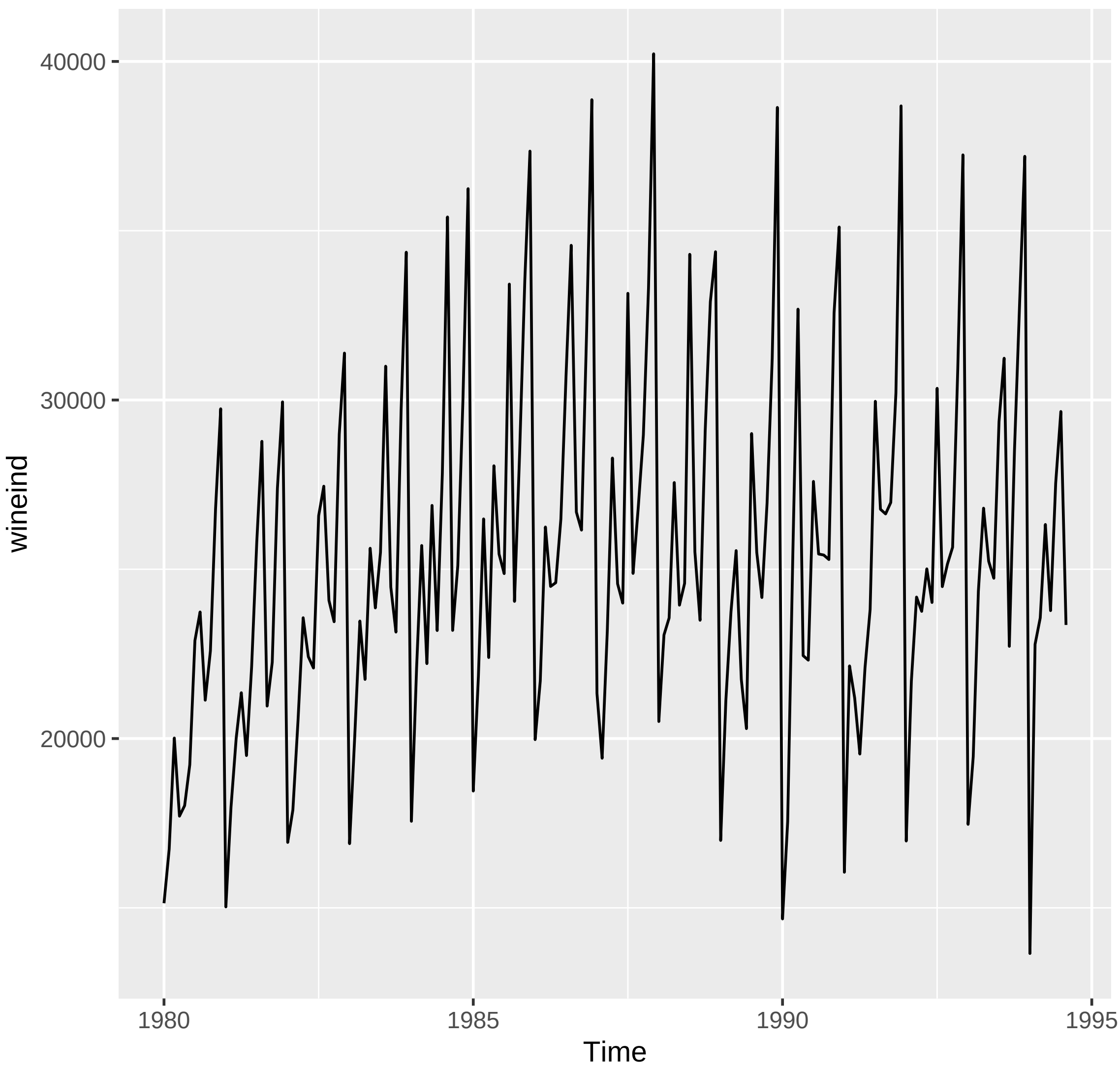
(d) False. Does not account for changing seasonality

31. **Problem**
    Consider an independent observations model with $u_t \sim (0, 0.01)$. Construct two 95% prediction interval for random walk model and independent observations model 100 steps ahead. How many times the width of the random walk PI is bigger than of independent observations PI?

    Provide the answer with 2 decimal digits.

    Solution
    Check lecture "Naive models".

32. **Problem**
    Consider the following time series on Australian wine sales:



. 4: plot of chunk unnamed-chunk-1

Which parameter for Box-Cox transformation can be used for this time series to deal with growing amplitude of seasonality?

(a) $\lambda = 1$ You can load the gas wineind in R by importing forecast library or if you use other programming languages you can download it here.

(b) $\lambda = 0$

(c) $\lambda = -2$

(d) $\lambda = 0.5$

Solution

(a) False. Oscillations amplitude grows over time

(b) True.

(c) False. Oscillations are unstable

(d) True.

33. **Problem**

(a) Naive random walk can't predict better than ETS(AAdA)

(b) To estimate STL components generalized method of moments is used

(c) Seasonal component is observed within one period, while cyclical component can have periodicity varying over time and span over several periods

(d) By l'Hopital's rule Taking a limit for of a Box-Cox transformation at $\lambda = 0$ will be equal to a logarithm

Solution

(a) False. Depends on the series

(b) False. It's an iterative procedure

(c) True.

(d) True.

34. **Problem**
Match ETS models with their formulas:

A:

$$y_t = (\ell_{t-1} + b_{t-1})(1 + \varepsilon_t)$$
$$\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t)$$
$$b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t$$

B:

$$y_t = (\ell_{t-1} + \phi b_{t-1})(1 + \varepsilon_t)$$
$$\ell_t = (\ell_{t-1} + \phi b_{t-1})(1 + \alpha\varepsilon_t)$$
$$b_t = \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1})\varepsilon_l$$

C:

$$y_t = (\ell_{t-1} + b_{t-1})s_{t-m} + \varepsilon_t$$
$$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t/s_{t-m}$$
$$b_t = b_{t-1} + \beta\varepsilon_t/s_{t-m}$$
$$s_t = s_{t-m} + \gamma\varepsilon_t/(\ell_{t-1} + b_{t-1})$$

D:

$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$
$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
$b_t = b_{t-1} + \beta\varepsilon_t$
$s_t = s_{t-m} + \gamma\varepsilon_t$

(a) ETS(MAdN)

(b) ETS(AAA)

(c) ETS(AAM)

(d) ETS(MAN)

Write down in the solution the sequence of numbers without spaces or delimeters corresponding to the formulas for the ETS models in each case (e.g. 4231).

Solution
Check lectures ETS Model (I) and (II).

35. **Problem**
Consider a random walk model. You have a sample $y^T = [0.47, 3.42, 5, 2.85]$ and you want to check if forecast 2 steps ahead would be non-positive at 5% significance level. Calculate right hand side of your confidence interval, corresponding to a required test.

Provide the answer with 2 decimal digits.

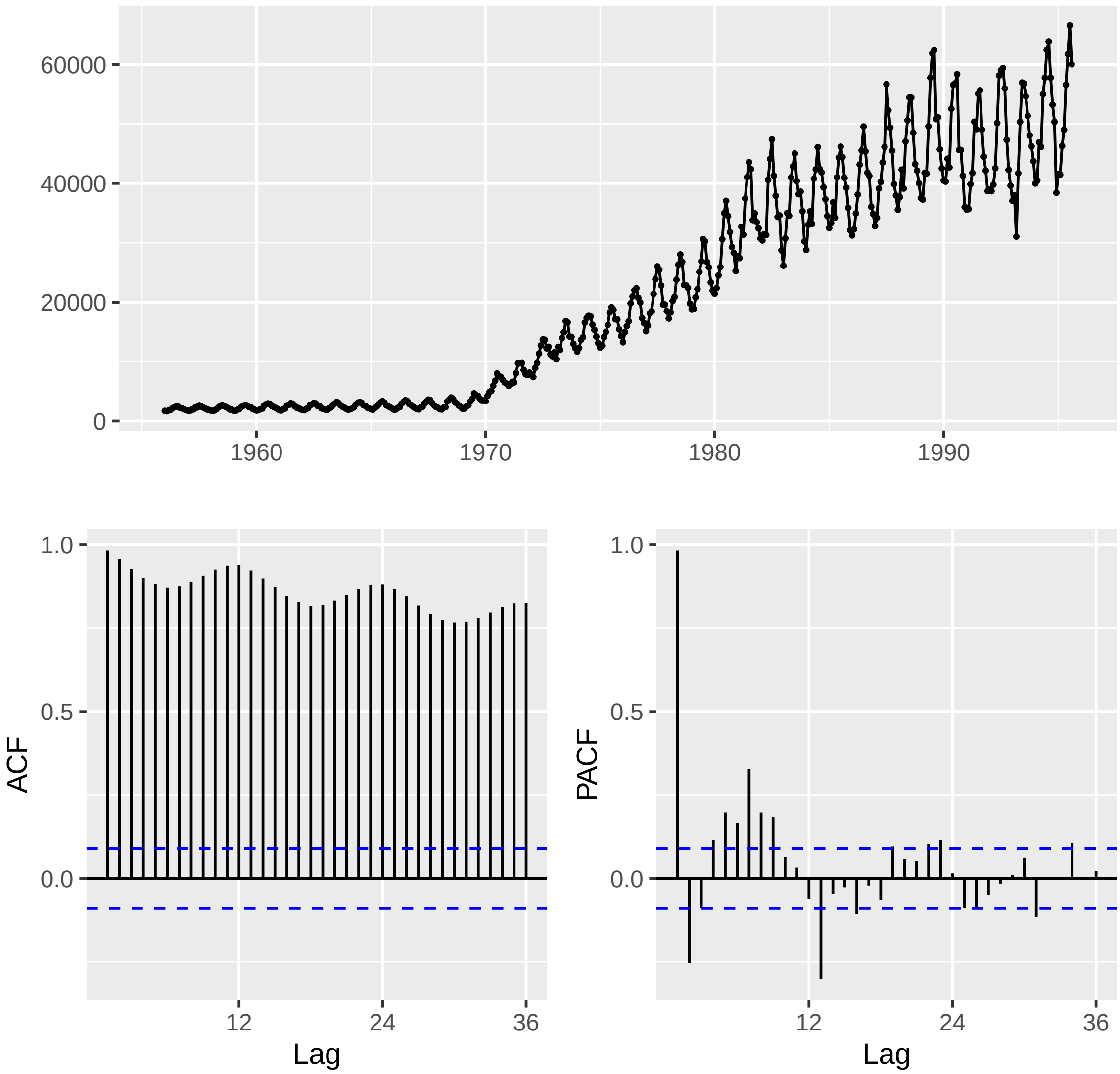
Solution
Check lecture "Naive models".

36. **Problem**
Consider LakeHuron dataset. You need to calculate sample ACF(6). Provide the answer with 4 decimal digits.

You can load the LakeHuron data set in R by issuing the following command at the console data("LakeHuron") or if you use other programming languages you can download it here.

Solution
Use acf(x, plot = FALSE) function.

37. **Problem**
Consider the following time series on Australian monthly gas production:



. 5: plot of chunk unnamed-chunk-1

Which model is more appropriate to use?

(a) Use $ETS(AAA)$

(b) Use models with built in Box-Cox transformation

(c) Use $\ln y_t = \ln trend_t + \ln seas_t + \ln remainder_t$

(d) Use $y_t = trend_t \cdot seas_t \cdot remainder_t$

Solution

(a) False. Does not account for changing seasonality

(b) True.

(c) True.

(d) True.

38. **Problem**

Consider gas dataset. You need to do STL with 2 inner and 1 outer loop for log(gas). Calculate the strength of seasonality using the ideal and practical formula. Choose the one that is bigger as an answer. Provide the answer rounded up to 4 decimal digits.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

Solution

Calculate strength of seasonality according to the formulas given in the lecture.