1. **Problem**
Suppose we have splitted time series into training and test sets and estimated three different models. Which of the following statements is correct?

   (a) Growing window cross-validation is better than the sliding window cross-validation if the process is stationary
   (b) For sliding window cross-validation a selected window size should be as small as possible
   (c) Approximate cross-validation by one step forward based on MAPE can be done using Akaike Information Criterion
   (d) Sliding window cross-validation can be used on a small dataset

   Solution

   (a) True.
   (b) False. A short data sample increases the chance that your parameter estimates are imprecis
   (c) False. This is true for RMSE based CV
   (d) False. Either the number of observations for a training set or the number of CV-iterations will be small

2. **Problem**
Consider a process $y_t = -1 + u_t + 0.8u_{t-1} - 0.4u_{t-2}$, where u_t is a white noise with variance 4.

   Construct an 95% prediction interval 2 steps ahead, given that $\hat{u}_T = 0.5, \hat{u}_{T-1} = -0.3$. Provide the variance of the point forecast rounded to 3 decimal digits as the answer.

   Solution
The variance of the point forecast will be equal to $Var(y_{t+2} \mid \mathcal{F}_t) = Var(u_{t+2} + 0.8u_{t+1} \mid \mathcal{F}_t)$

3. **Problem**
Suppose we have splitted time series into training and test sets and estimated three different models. Which of the following statements ARE correct?

   (a) This approach can be preferred to a cross-validation if the dataset is small
   (b) We cannot choose the best model using this method since forecasts have different horizons
   (c) Since forecasts have different horizons we cannot differentiate models which are better for short-term forecasts and long-term forecasts

   Solution

   (a) True.
   (b) False. We certainly can!
   (c) True.

4. **Problem**
Which of the following is regressors should be chosen for modelling yearly seasonality in daily data?

   (a) Either $\cos\left(\frac{2\pi}{365} \cdot t\right)$ or $\sin\left(\frac{2\pi}{365} \cdot t\right)$
   (b) 365 dummies for each day of the year
   (c) Both $\cos\left(\frac{2\pi}{365} \cdot t\right)$ and $\sin\left(\frac{2\pi}{365} \cdot t\right)$

Solution

(a) Check lecture "More predictors".

(b) NA

(c) NA

5. **Problem**

Match forecast quality metrics' formulas

A: $\frac{\sum_{k=1}^{H} \left| \frac{y_{t+k} - \hat{y}_{t+k}}{y_{t+k}} \cdot 100 \right|}{H}$

B: $\frac{\sum_{k=1}^{H} |y_{t+k} - \hat{y}_{t+k}|}{H}$

C: $\frac{\sum_{k=1}^{H} \left| \frac{y_{t+k} - \hat{y}_{t+k}}{MAE^{naive}} \right|}{H}$

D: $\frac{\sum_{k=1}^{H} \left| \frac{y_{t+k} - \hat{y}_{t+k}}{0.5 y_t + 0.5 \hat{y}_t} \cdot 100 \right|}{H}$

and their names:

(a) Mean Absolute Error

(b) Mean Absolute Scaled Error

(c) Mean Absolute Percentage Error

(d) Symmetric Mean Absolute Percentage Error

Write down in the solution the sequence of numbers without spaces or delimeters corresponding to the formulas in each case (e.g. 4231).

Solution

Check lecture Model Quality.

6. **Problem**

Which of the following statements about Diebold-Mariano-test ARE correct?

(a) If model A has smaller RMSE than a naive model then model A should always be preferred

(b) Diebold-Mariano-test can be used to compare short and long-term forecasts

(c) Diebold-Mariano-test can be used to compare 3 or more forecasts to each other

(d) Diebold-Mariano-test is based on the idea of testing that the square loss by model A equals to the square loss by model B

Solution

(a) False. The simpler the better, if by Diebold-Mariano-test the forecasts aren't different, probably a naive model can be preferred

(b) True.

(c) False. DM-test is not suitable for pairwise comparison of multiple forecasts

(d) True.

7. **Problem**
   Consider a process $y_t = 7 + 0.2y_{t-1} + u_t$, where u_t is a white noise with variance 10.

   Construct a point forecast 5 steps ahead given that $y_T = 1$.
   Provide the answer with 3 decimal digits.

   Solution
   Check lecture about ARMA model.

8. **Problem**
   Suppose you want to predict future values of y_t. Which of the following statements is correct?

   (a) When working with weekly data adding $L^7 y_t$ as a regressor can improve forecast quality
   (b) Averaging forecasts from random forest algorithm and ETS model will improve forecast quality
   (c) You cannot make predictions for future values of y_t using linear regression model
   (d) An example of growing window function is mean of 2 previous values

   Solution

   (a) True.
   (b) False. It depends on data
   (c) False. You can!
   (d) False. It's a sliding window function

9. **Problem**
   Which of the following statements ARE correct?

   (a) MAPE can be used to compare models performances between datasets
   (b) IF MASE is greater than 1 then a naive model is better
   (c) For SMAPE if the actual value or forecast value is 0, the value of error will approach 100%
   (d) RMSE is robust to outliers

   Solution

   (a) True.
   (b) False. IF MASE is greater than 1 then a naive model is worse
   (c) True.
   (d) False. RMSE is sensitive to outliers

10. **Problem**
    Suppose you want to predict future values of y_t. Which of the following statements is correct?

    (a) In a regression with a constant term you can add 7 dummies for each day of the week to model weekly seasonality
    (b) If you suppose that the linear trend slows down over time you can apply Box-Cox transformation and use a linear trend as a regressor for the transformed data
    (c) You can model yearly seasonality for daily data using fourier series
    (d) In gradient boosting, if you suppose that the linear trend slows down over time you should use square root of trend instead of linear trend

Solution

(a) False. It's a trap!
(b) False. Box-Cox transformation will be useful if the variance of the time series monotonically changes over time
(c) True.
(d) False. Decision trees are invariant under monotonic transformations of time

11. **Problem**
Match ACF and PACF with their corresponding ARMA processes:

Picture A

Picture B

Picture C

Picture D

(a) ARMA(2,0)
(b) ARMA(1,0)
(c) ARMA(1,2)
(d) ARMA(0,2)

Write down in the solution the sequence of numbers without spaces or delimeters corresponding to pairs of ACF and PACF in each case (e.g. 4231).

Solution
A gradual geometrically declining ACF and a PACF that is significant for only a few lags indicate an AR process. MA process shows a gradually geometrically declining PACF and the ACF has a few significant lags. An ARMA process is indicated by geometrically filling ACF and PACF.

12. **Problem**
Select all of the assumptions for ARMAX model

$$y_t = c + \gamma a_t + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + u_t + \alpha_1 u_{t-1} + \ldots + \alpha_q u_{t-q}$$

where a_t is a predictors and (u_t) is white noise

(a) $E(a_t^4) < \infty$
(b) Series (y_t) is non-stationary
(c) Predictor (a_t) is stationary
(d) $E(u_t \mid a_{t-1}, b_{t-1}, y_{t-1}, a_{t-2}, b_{t-2}, y_{t-2}, \ldots) = 0$
(e) Predictor (a_t) is non-stationary, but $\Delta(a_t)$ is stationary
(f) Series (y_t) is stationary

Solution

(a) True.
(b) False. (y_t) and (a_t) should be stationary
(c) True.
(d) True.

(e) False. (y_t) and (a_t) should be stationary

(f) True.

13. **Problem**
Consider LakeHuron dataset. You need to estimate a linear regression model with two regressors: sliding window for mean applied to 2 previous values and growing window for mean. Provide the R squared for the estimated model as an answer with 3 decimal digits.

You can load the LakeHuron data set in R by issuing the following command at the console data("LakeHuron") or if you use other programming languages you can download it here.

Solution
You can use cummean() and lag() functions to create regressors.

14. **Problem**
Consider gas dataset. You need to estimate an ARIMA and ETS model with automatically chosen parameters. Compare out-of-sample forecasts 6 steps ahead using MAPE. Provide MAPE for the best of the compared models with 3 decimal digits as an answer.

You can load the LakeHuron data set in R by issuing the following command at the console data("LakeHuron") or if you use other programming languages you can download it here.

Solution
You can use dm.test(residuals(f1),residuals(f2),h=1) function.

15. **Problem**
Consider a process $y_t = 2 + u_t + 0.2u_{t-1} - 0.2u_{t-1}$, where u_t is a white noise with variance 5.

Check that the process is invertible. If the process is invertible give the sum of the characteristic polynomial roots as an answer, else give 0 as an answer. Provide the answer with 3 decimal digits.

Solution
The process is invertible.

16. **Problem**
Suppose you want to predict future values of y_t. Which of the following statements ARE correct?

(a) In a regression without a constant term you can add 4 dummies for quarter of the year to model yearly seasonality

(b) When working with monthly data adding $L^7 y_t$ as a predictor can significantly improve forecast quality

(c) Random forest with linear trend used as a predictor will perform exactly the same if logarithm of trend would be used instead

(d) To model yearly seasonality in daily data the only predictors required are $\cos\left(\frac{4\pi}{365} \cdot t\right)$ and $\sin\left(\frac{4\pi}{365} \cdot t\right)$

Solution
Check lecture about Forecasting using predictors.

17. **Problem**
Match forecast quality metrics' formulas

A: $\dfrac{\sum_{k=1}^{H} \left| \frac{y_{t+k} - \hat{y}_{t+k}}{y_{t+k}} \cdot 100 \right|}{H}$

B: $\dfrac{\sum_{k=1}^{H} |y_{t+k} - \hat{y}_{t+k}|}{H}$

C: $\dfrac{\sum_{k=1}^{H} \left| \frac{y_{t+k} - \hat{y}_{t+k}}{MAE^{naive}} \right|}{H}$

D: $\dfrac{\sum_{k=1}^{H} \left| \frac{y_{t+k} - \hat{y}_{t+k}}{0.5 y_t + 0.5 \hat{y}_t} \cdot 100 \right|}{H}$

and their names:

(a) Mean Absolute Error

(b) Mean Absolute Scaled Error

(c) Mean Absolute Percentage Error

(d) Symmetric Mean Absolute Percentage Error

Write down in the solution the sequence of numbers without spaces or delimeters corresponding to the formulas in each case (e.g. 4231).

Solution
Check lecture Model Quality.

18. **Problem**
Suppose you forecast the price of a flat calcucated in euros. Which of the following statements is correct?

(a) RMSE is calculated in the euros squared

(b) sMAPE is calculated in percents

(c) MASE is calculated in percents

(d) MAPE is calculated in the euros

Solution

(a) NA

(b) NA

(c) NA

(d) Check lecture Model Quality.

19. **Problem**
Consider a process $y_t = 1 + 0.3y_{t-1} + u_t$, where u_t is a white noise with variance 5.

Find it's autocovariance function γ_k. Give γ_5 as an answer rounded to 3 decimal digits.

Solution
To find autocovariance function consider MA(∞) representation of an AR(1) process.

20. **Problem**
Consider gas dataset. You need to estimate a linear regression model for the logarithm of gas series staring from 1970. Use square root of the trend as a regressor. Provide the forecast 10 steps ahead for the log(gas) rounded up to 3 decimal digits as an answer.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

Solution
You can use lm() and predict() functions.

21. **Problem**
Which of the following statements ARE correct?

(a) ARMAX can work with non-stationary predictors
(b) In regression with ARMA errors you can use no more than two predictors
(c) In ARDL model lags of the predictors are used instead of noise lags u_t
(d) ARDL model can be used if you want to find a long-term relationships between time series

Solution

(a) False. In ARMAX regressors should be stationary
(b) False. You can use as many predictors as you like (unless you have enough data)
(c) True.
(d) True.

22. **Problem**
Suppose you want to predict future values of monthly observations for China's GDP y_t using the following set of regressors: seasonal lag, lag of a second difference, mean as a growing window and median as a sliding window of 3 values. What should the prediction sample 1 step ahead be?

(a) $(y_{T-11}, (y_T - y_{T-1}) - (y_{T-1} - y_{T-2}), \text{Mean}\{y_{T-2}, y_{T-1}, y_T\}, \text{Med}\{y_1, \cdots, y_T\})$
(b) $(y_{T-12}, (y_T - y_{T-1}) - (y_{T-1} - y_{T-2}), \text{Mean}\{y_{T-2}, y_{T-1}, y_T\}, \text{Med}\{y_1, \cdots, y_T\})$
(c) $(y_{T-12}, y_T - y_{T-1}, \text{Mean}\{y_1, \cdots, y_T\}, \text{Med}\{y_{T-2}, y_{T-1}, y_T\})$
(d) $(y_{T-11}, (y_T - y_{T-1}) - (y_{T-1} - y_{T-2}), \text{Mean}\{y_1, \cdots, y_T\}, \text{Med}\{y_{T-2}, y_{T-1}, y_T\})$

Solution

(a) NA
(b) Check lecture "Forecasting without a model".
(c) NA
(d) NA

23. **Problem**
Consider the process $x_t = 0.5x_{t1} + 2u_{t1} + u_t$, where u_t is a white noise process.

(a) non-invertible and stationary
(b) invertible and stationary
(c) invertible and non-stationary
(d) non-invertible and non-stationary

Solution
Non-invertible since root of MA polynomial is not outside unit circle. Non-stationary, since root of AR polynomial is outside unit circle.

24. **Problem**
Consider ARDL model

$$y_t = c + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + x_t + \alpha_1 x_{t-1} + \ldots + \alpha_q x_{t-q} + u_t.$$

Which of the following statements about ARDL model ARE correct?

(a) ARDL model can be used if you want to find a long-term relationships between time series

(b) ARDL model can model non-stationary y_t

(c) $(u_t) \sim ARMA(p, q)$ w.r.t. white noise

(d) ARDL model allows only one predictor for y_t

Solution

(a) Check lecture about prediction with AR type models

(b) NA

(c) NA

(d) NA

25. **Problem**

The variables X_1, X_2, ... are independent with mean 5 and variance 7.

Let $S = X_2 + X_3 + \ldots + X_7$.

Find partial correlation $\mathrm{pCorr}(X_1, X_1 + X_2; S)$.

Solution

Find the correlation between X_1 and $X_1 + X_2 - \beta(X_2 + X_3 + \ldots + X_7)$. Note, that $\hat{\beta}_{OLS} \to \beta$, as $n \to \infty$

26. **Problem**

Which of the following statements ARE correct?

(a) MAPE is scale-independent metric

(b) Diebold-Mariano-test can be used for pairwise comparison of 3 or more forecasts

(c) For growing window cross-validation there is a trade-off between the need for larger window size for precision and smaller window size to ensure the same time series dynamics persists

(d) IF MASE is greater than one indicate then in-sample one-step forecasts from the naive method perform better than the forecast from the considered model

Solution

(a) Check lecture Model Quality.

(b) NA

(c) NA

(d) NA

27. **Problem**

Consider the following ARMA equation $y_t = 2 - 0.4y_{t-1} + 0.04y_{t-2} + u_t + 0.5u_{t-1} + 0.06u_{t-2}$, where u_t is a white noise with variance 1. Which of the following is true?

(a) The equation is irreducible

(b) The process is invertible

(c) The ARMA equation has infinite number of solutions

(d) The corresponding ARMA process is stationary

Solution
There is one common factor, hence in fact it is an ARMA(1,1). If you check the roots for this reduced process, it will be clear the process is stationary and invertible.

28. **Problem**
Consider LakeHuron dataset. You need to estimate an ARIMA and ETS model with automatically chosen parameters. Compare in-sample one-step forecasts using Diebold-Mariano test applied to model residuals. Provide the DM statistics if the forecasts are different with 3 decimal digits, else give 0 as an answer.

You can load the LakeHuron data set in R by issuing the following command at the console data("LakeHuron") or if you use other programming languages you can download it here.

Solution
You can use dm.test(residuals(f1),residuals(f2),h=1) function.

29. **Problem**
Which of the following statements ARE true?

(a) If the irreducible ARMA equations has lag polynomial roots for the AR part which equal to 1, there are no stationary solutions

(b) $y_t = 3 + 3t + u_t$ where u_t is a white noise is trend-stationary

(c) If the ARMA equation has common roots, than it's irreducible

(d) If the irreducible ARMA equations has lag polynomial roots for the AR part which are greater than 1, there are no stationary solutions

Solution
Check lection about ARMA and ARIMA processes.