1. **Problem**
   Consider a process $y_t = 5 + 0.3y_{t-1}$, where u_t is a white noise with variance 2.

   Construct a point forecast 4 steps ahead given that $y_T = 2$.
   Provide the answer with 3 decimal digits.

   Solution
   The point forecast will be equal to $E(y_{t+4} \mid \mathcal{F}_t) = (1 + 0.3 + 0.3^2 + 0.3^3) * 5 + 0.3^4 * 2$.

2. **Problem**
   Let's continue conducting a Box-Jenkins procedure for log(gas) time series.

   You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

   Conduct a Dickey-Fuller unit root test without trend and constant to check whether after applying both first and seasonal differences the series become stationary. If the process is stationary give DF statistic as an answer, else give 0 as an answer.

   Provide the answer with 2 decimal digits.

   Solution
   You can use ur.df(x, type = 'none') function.

3. **Problem**
   Consider a process $y_t = 5 + 0.3y_{t-1} + u_t$, where u_t is a white noise with variance 2.

   Construct a point forecast 4 steps ahead given that $y_T = 2$.
   Provide the answer with 3 decimal digits.

   Solution
   The point forecast will be equal to $E(y_{t+4} \mid \mathcal{F}_t) = (1 + 0.3 + 0.3^2 + 0.3^3) * 5 + 0.3^4 * 2$.

4. **Problem**
   Consider a process $y_t = 4 + 0.8y_{t-1} + u_t$, where u_t is a white noise with variance 3.

   Find it's autocorrelation function ρ_k. Give ρ_3 as an answer.

   Solution
   To find autocorrelation function consider MA(∞) representation of an AR(1) process.

5. **Problem**
   Which of the following is correct?

   (a) Initial conditions can determine stationarity of the process
   (b) If the ARMA equation has common roots, than it's irreducible
   (c) Stationarity of the process is determined by AR part and invertibility by MA part
   (d) If the irreducible ARMA equations has lag polynomial roots which are smaller than one by its absolute value, there are no stationary solutions

   Solution
   Check ARMA lectures.

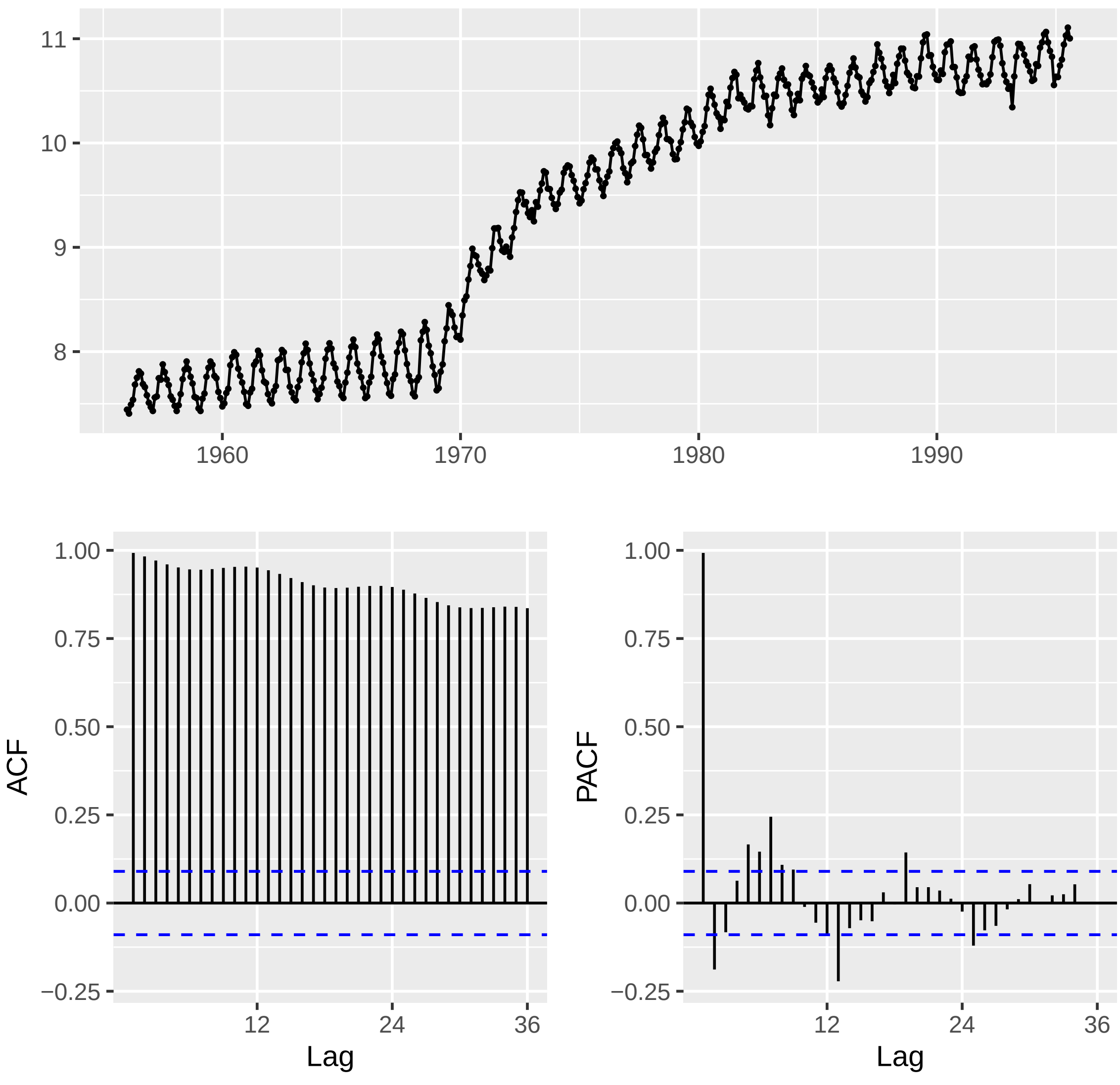
6. **Problem**
   Consider the following ARMA equation $y_t = 0.4y_{t-1} + 0.45y_{t-2} + u_t + u_{t-1} + 0.25u_{t-2}$, where u_t is a white noise with variance 1. Which of the following is true?

   (a) The process is non-invertible
   (b) The ARMA equation has a unique solution
   (c) The equation is irreducible
   (d) The corresponding ARMA process satisfying the is stationary

Solution
There is one common factor, hence in fact it is an ARMA(1,1). If you check the roots for this reduced process, it will be clear the process is stationary and invertible.

7. **Problem**
Let's continue conducting a Box-Jenkins procedure for log(gas) time series.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.



. 1: plot of chunk unnamed-chunk-1

Is the process stationary?

(a) Yes, we can proceed with log(gas) time series
(b) No, seasonal differencing should be applied

(c) No, first differencing should be applied

(d) No, both first and seasonal differencing should be applied

Solution
Only if we apply both first and seasonal differences the mean will be stable over time. From analysing the ACF plot it is clear that there is a unit root and a seasonal unit root, but you can also check this using unit root tests.

8. **Problem**
Consider the following process $y_t = 3t + y_{t-1} + u_t$, where u_t is a whte noise. Which of the weakly stationarity conditions are violated.

   (a) Autocovariance function depends only on the difference between time indexes

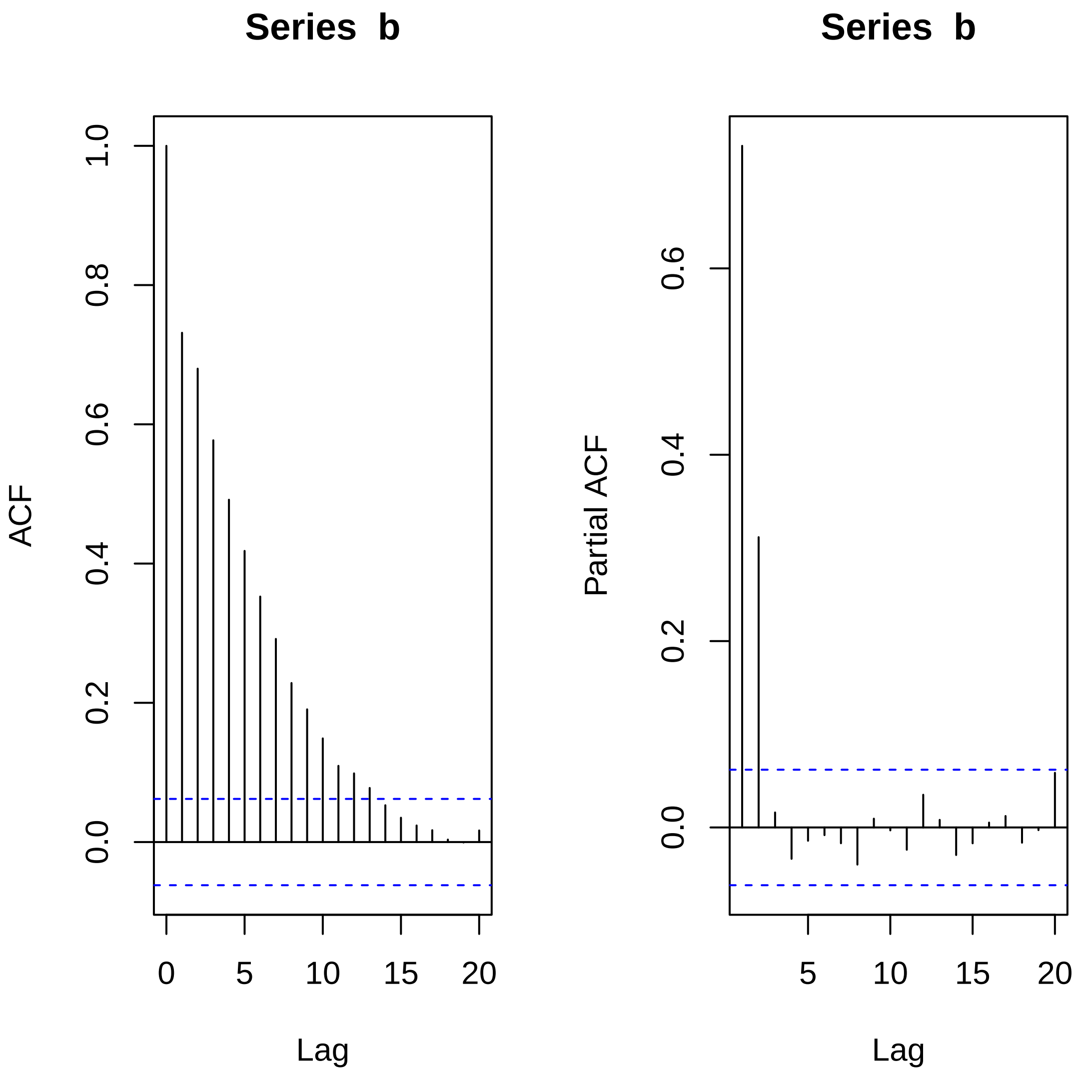
   (b) Constant mean

   (c) None of the conditions are violated, the process is stationary

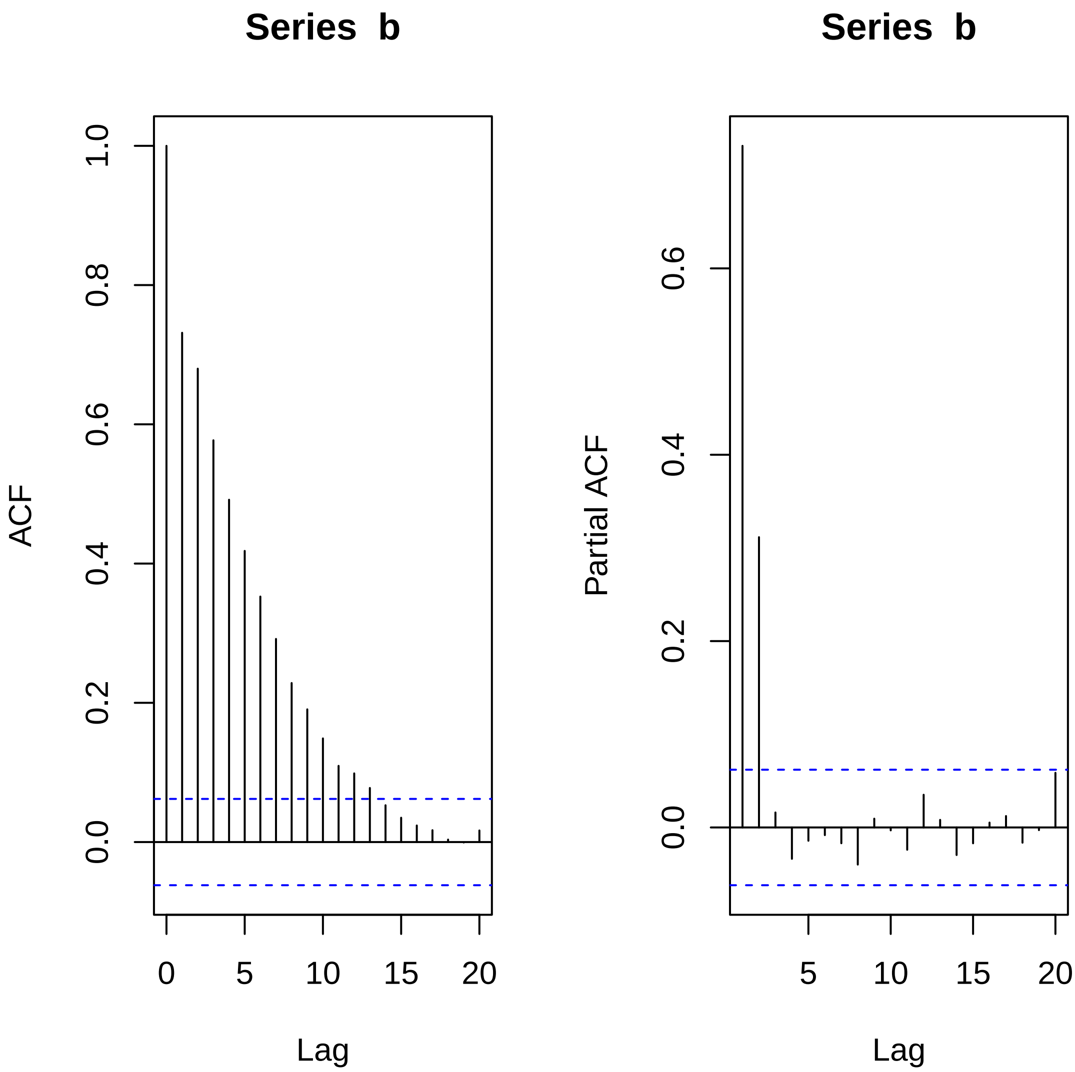
   (d) Constant variance

   Solution
   Check mean and covariance functions.

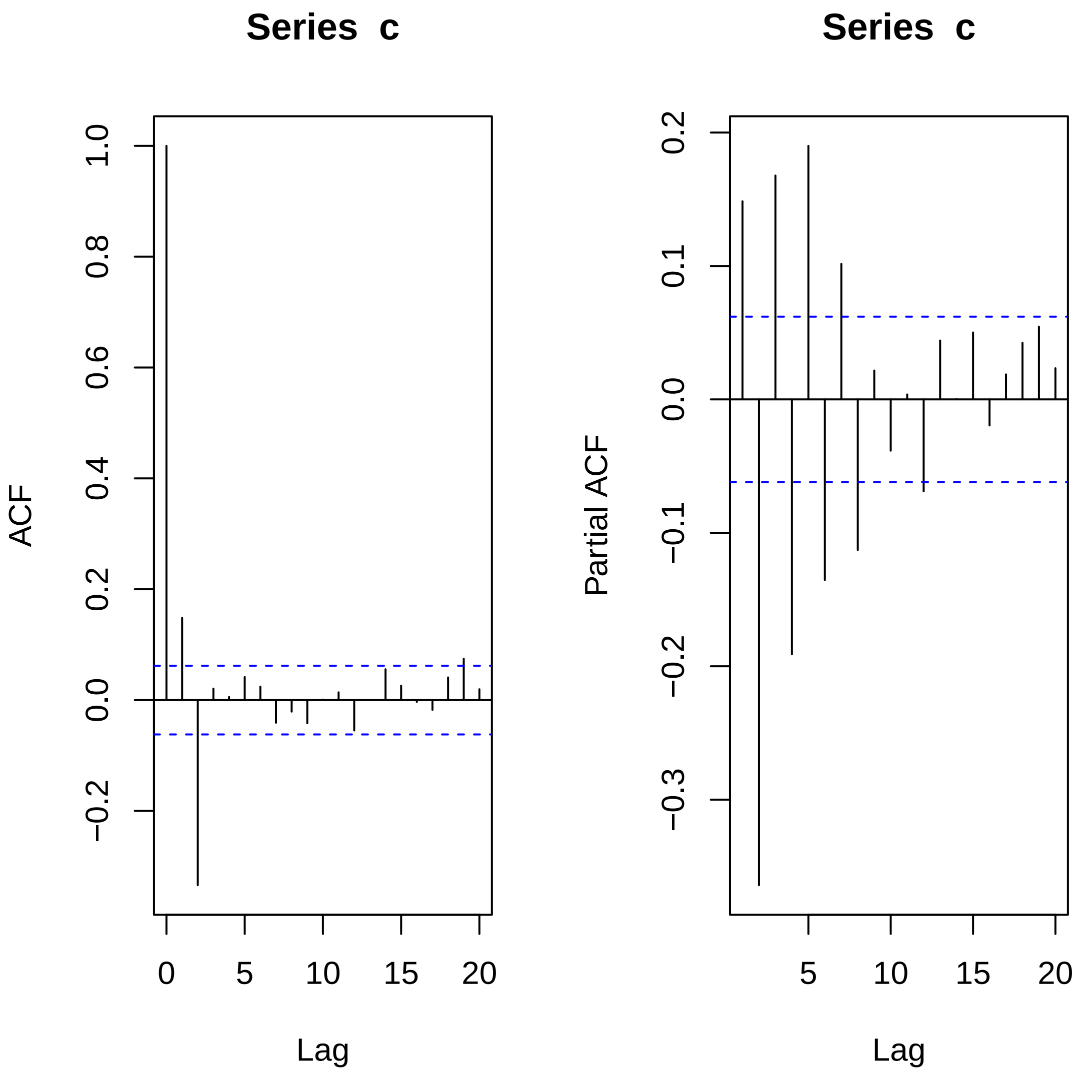
9. **Problem**
Match ACF and PACF with their corresponding ARMA processes:



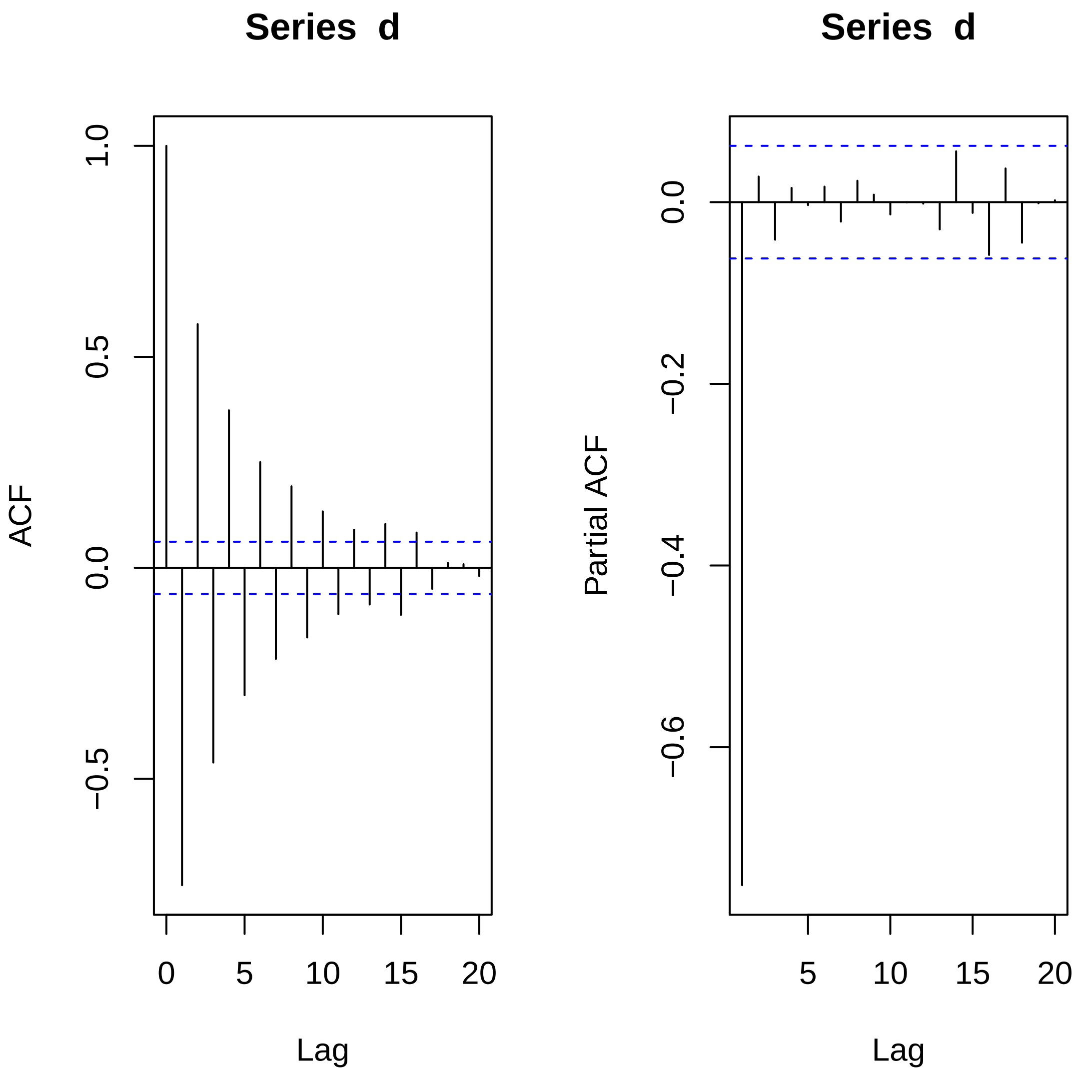
. 2: plot of chunk unnamed-chunk-1



. 3: plot of chunk unnamed-chunk-1



. 4: plot of chunk unnamed-chunk-1



. 5: plot of chunk unnamed-chunk-1

(a) ARMA(2,0)
(b) ARMA(1,0)
(c) ARMA(0,2)
(d) ARMA(2,2)

Write down in the solution the sequence of numbers without spaces or delimeters corresponding to pairs of ACF and PACF in each case (e.g. 4231).

Solution
A gradual geometrically declining ACF and a PACF that is significant for only a few lags indicate an AR process. MA process shows a gradually geometrically declining PACF and the ACF has a few significant lags. An ARMA process is indicated by geometrically filling ACF and PACF.

10. **Problem**
Consider a process $y_t = 4 + 0.8y_{t-1} + u_t$, where u_t is a white noise with variance 3.

Find it's autocovariance function γ_k. Give γ_0 as an answer. Provide the answer with 3 decimal digits.

Solution
You are need to find the variance of y_t. Note, that the process is stationary, so its variance does not change over time.

11. **Problem**
Consider a process $y_t = 4 + 0.8y_{t-1} - 0.2y_{t-2} + 0.6y_{t-12} + u_t - 0.5u_{t-1} + 0.2u_{t-2} - 0.1u_{t-12}$.

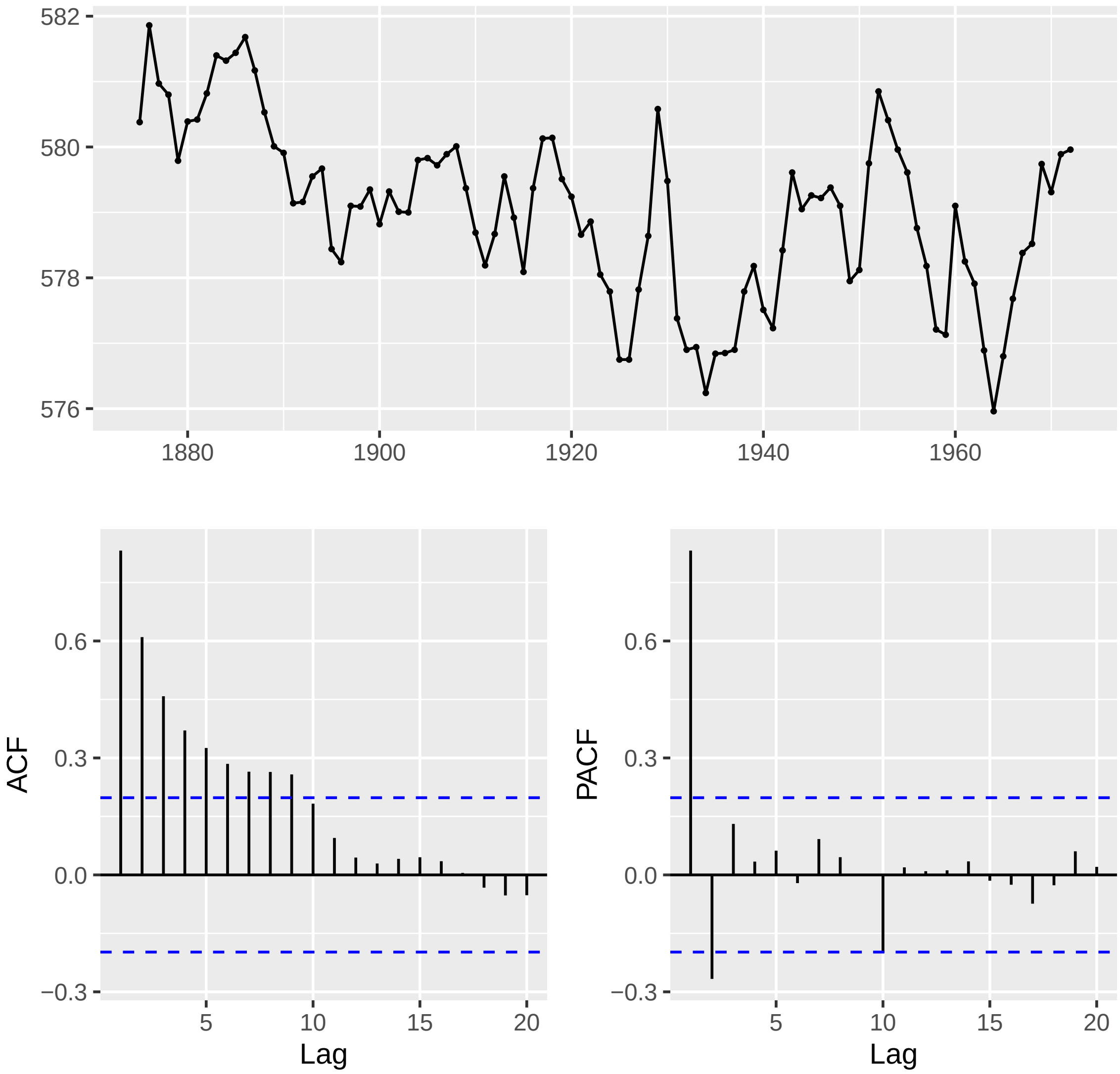
Which SARIMA(p,d,q)[P,D,Q] model describes y_t.

(a) SARIMA(1,0,1)[2,0,2]
(b) SARIMA(12,0,12)[0,0,0]
(c) SARIMA(2,0,2)[1,0,1]
(d) SARIMA(2,1,2)[1,1,1]

Solution
Check lecture about SARIMA.

12. **Problem**
Consider LakeHuron time series.

You can load the LakeHuron data set in R by issuing the following command at the console data("LakeHuron") or if you use other programming languages you can download it here.



. 6: plot of chunk unnamed-chunk-1

By analysing ACF and PACF functions answer, what can the underlying ARMA(p,q) process be?

(a) ARMA(0,1)
(b) ARMA(1,0)
(c) ARMA(1,1)
(d) ARMA(2,0)
(e) ARMA(0,0)
(f) ARMA(0,2)

Solution
A gradual geometrically declining ACF and with PACF that is significant for only 2 lags

indicate an AR process. Since the number of observations is not large enough the significance of PACF(2) is questionable.

13. **Problem**
    Let's continue conducting a Box-Jenkins procedure for the differenced log(gas) time series.

    You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

    At last we should estimate the chosen SARMA(0,1)[0,1] for the transformed and differenced gas series and analyse the model.

    Which of the following are true?

    (a) Both MA and SMA coefficients are significant
    (b) Residuals are uncorrelated (autocorrelation of 1st order)
    (c) Residuals are seasonally uncorrelated
    (d) The residual ACF and PACF are not significant
    (e) Distribution of residuals is normal

    Solution

    (a) Check model summary
    (b) To check if residual are correlated try Box.test(x) and Box.test(x, lag=12)
    (c) To check if residual are correlated try Box.test(x) and Box.test(x, lag=12)
    (d) ACF(11) and PACF(11) are sigificant
    (e) To check normality you can use shapiro.test(x) and jarque.bera.test(x) funcions Also, you can use checkresiduals(model) function to analyse the residuals

14. **Problem**
    Consider a process $y_t = 6 + u_t + 0.7u_{t-1} - 0.1u_{t-1}$, where u_t is a white noise with variance 2.

    Check that the process is invertible. If the process is invertible give the sum of the lag polynomial roots as an answer, else give 0 as an answer. Provide the answer with 3 decimal digits.

    Solution
    The process

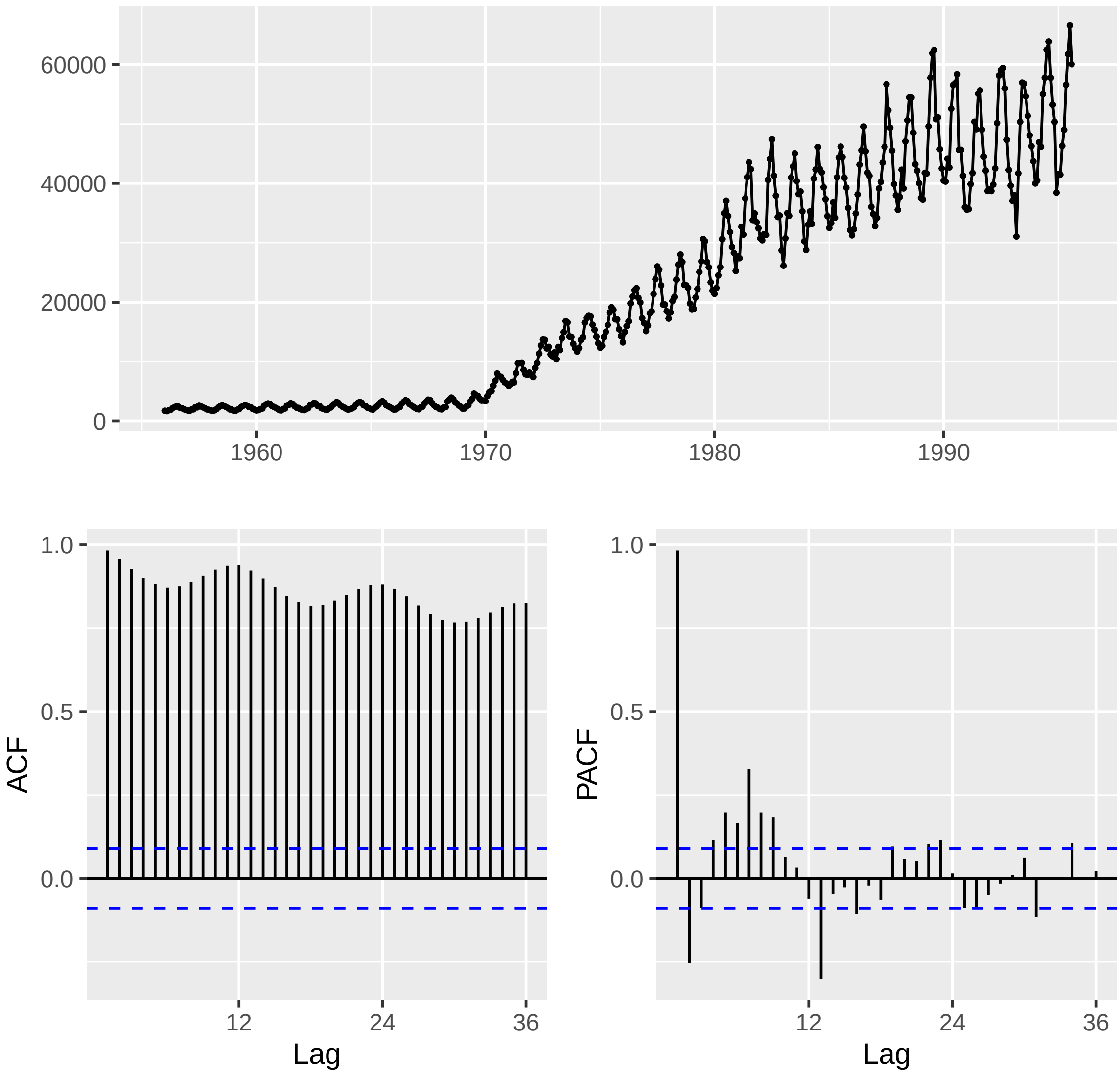
15. **Problem**
    Consider gas dataset. Estimate MA(12) model for the first differene of gas series. Find point forecast 12 steps aheaad. Provide the answer with 2 decimal digits.

    You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

    Solution
    You can use Arima(x, order = c(0, 0, q)) function.

16. **Problem**
    Let's conduct a Box-Jenkins procedure for gas time series to determine the specification of SARIMA model.

    You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.



. 7: plot of chunk unnamed-chunk-1

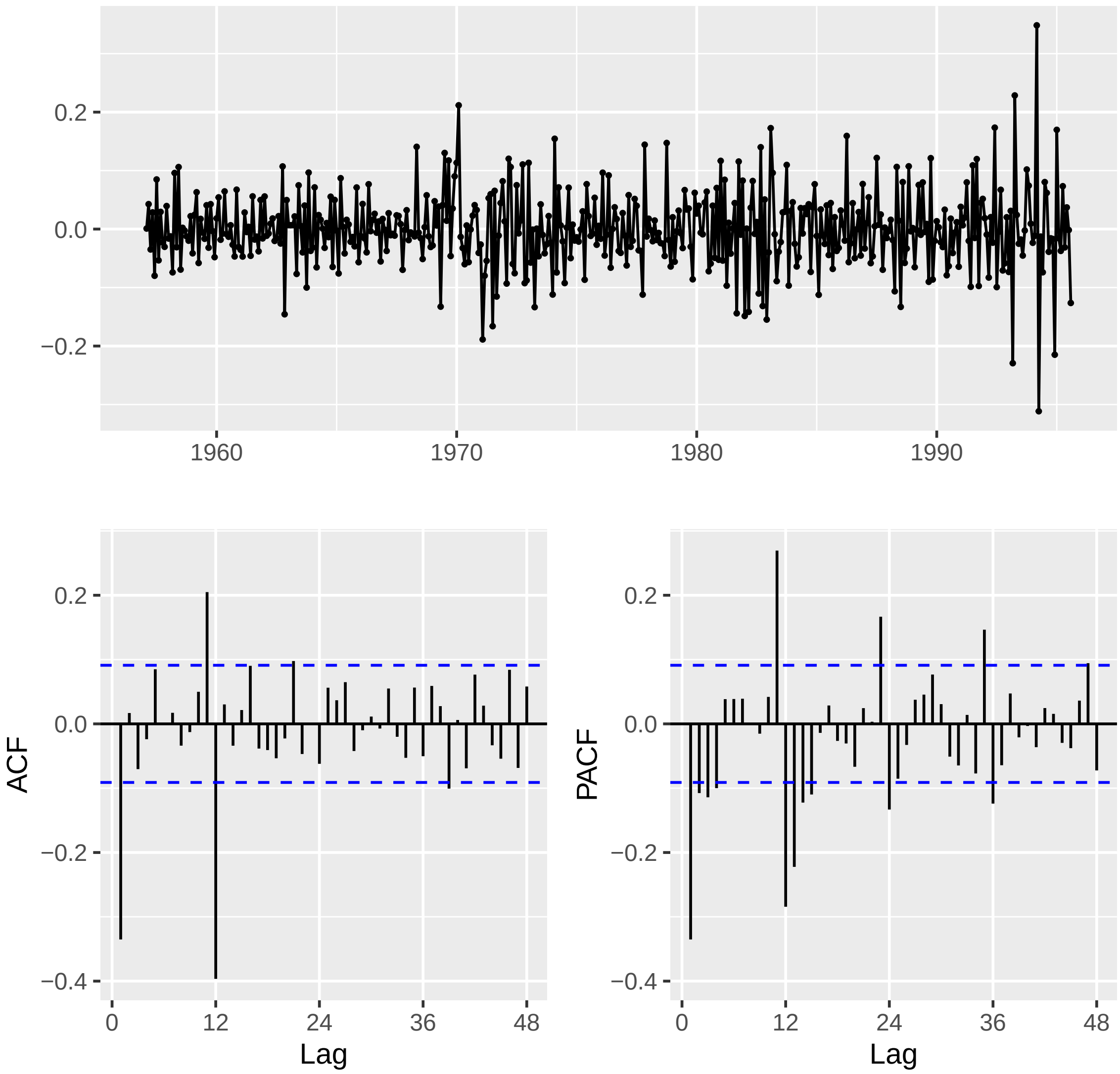
Is the variance stable?

(a) Yes, we can proceed with gas time series
(b) No, the logarithmic transformation should be applied

Solution
The variance grows over time, a transformation is required.

17. **Problem**
Let's continue conducting a Box-Jenkins procedure for the differenced log(gas) time series.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.



. 8: plot of chunk unnamed-chunk-1

Now let's determine the parameters for SARIMA model.

By analysing ACF and PACF functions answer, what can the underlying SARMA(p,q) process for the transformed and differenced gas series be?

(a) SARMA(1,0)[1,0]
(b) SARMA(0,1)[0,0]
(c) SARMA(0,1)[0,1]
(d) SARMA(0,0)[1,0]
(e) SARMA(0,0)[0,1]
(f) SARMA(1,0)[0,0]

Solution
MA process shows a gradually declining PACF and the ACF has a 1st significant lag corresponding to MA part and 12th significant lag corresponding to seasonal MA part.

18. **Problem**
At last, if you aren't satisfied with prediction quality you can modify the model. What can be the reasonable suggestions for model modification?

    (a) In ACF(11) and PACF(11) for residuals were significant, better use ARMA(0,12) instead of SARMA.

    (b) Probably logarithm wasn't the optimal transformation, better use Box-Cox transformation with parameter other than 0.

    (c) There is still seasonal correlation in the residuals, so you can consider other parameter for SARMA part.

    (d) The process could have been trend-stationary, you can remove trend instead of applying first differencing

    Solution

    (a) Increasing number of estimated parameters from 3 to 13 is not a good idea

    (b) It can work

    (c) It can work

    (d) It can work, but note, that trend here is non-linear

19. **Problem**
The variables X_1, X_2, ... are independent with mean 5 and variance 7.

    Let $S = X_2 + X_3 + \ldots + X_5$.

    Find partial correlation $\text{pCorr}(X_1, X_1 + X_2; S)$.

    Solution
    Find the correlation between X_1 and $X_1+X_2-\beta(X_2+X_3+\ldots+X_5)$. Note, that $\hat{\beta}_{OLS} \to \beta$, as $n \to \infty$

20. **Problem**
Consider a process $y_t = 4 + 0.8y_{t-1} + u_t$, where u_t is a white noise with variance 3.

    Find it's autocovariance function γ_k. Give γ_4 as an answer.

    Solution
    To find autocovariance function consider MA(∞) representation of an AR(1) process.

21. **Problem**
Consider LakeHuron dataset. Estimate ARMA(2,0) model for the LakeHuron series. Forecast 100 steps ahead and compare the width of prediction interval 100 steps ahead vs 1 step ahead. Give the absolute value of the difference of the widths as an answer.

    Provide the answer with 2 decimal digits.

    You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

    Solution
    You can use Arima(x, order = c(p, 0, 0)) function.

22. **Problem**
Consider a process $y_t = 6 + 0.3y_{t-1} + u_t + 0.7u_{t-1} - 0.1u_{t-1}$, where u_t is a white noise with variance 3.

Construct a point forecast 2 steps ahead, given that $y_T = 2, \hat{u}_T = 1, \hat{u}_{T-1} = -1$.
Provide the answer with 3 decimal digits.

Solution
The point forecast will be equal to $E(y_{t+2} \mid \mathcal{F}_t) = 6 + 0.3E(y_{t+1} \mid \mathcal{F}_t) - 0.1\hat{u}_T$

23. **Problem**
Consider the process $x_t = x_{t1} + \frac{1}{4}Z_{t1} + Z_t$, where Z_t is a white noise process.

(a) invertible and non-stationary
(b) invertible and stationary
(c) non-invertible and non-stationary
(d) non-invertible and stationary

Solution
Invertible since roots of MA polynomial is outside unit circle. Non-stationary, since root of AR polynomial is not outside unit circle.

24. **Problem**
Final step of the Box Jenkins procedure would be forecasting using the chosen model and analysing the prediction quality.

You can load the gas dataset in R by importing forecast library or if you use other programming languages you can download it here.

Calculate forecast 2 years ahead for gas time series using the model selected on the previous steps.

Provide the answer with 2 decimal digits.

Solution
Don't forget you have applied transformation and differencing to the initial time series.