Kalman filter

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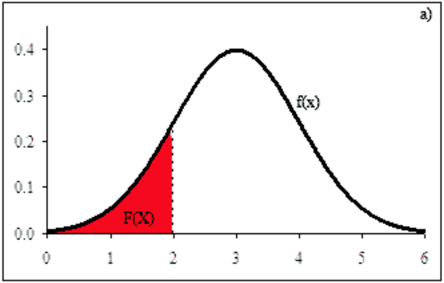
Plan

Basic probability

- Random value
- Probability density function
- Cumulative distribution function



Central limit theorem

Suppose $\{X_1, X_2, \ldots, X_n\}$ is a sequence of i.i.d. random variables with $E[X_i] = \mu$ and $Var[X_i] = \sigma^2 + \inf$. Then as n approaches infinity, the random variables $\sqrt{(S_n - \mu)}$ converge in distribution to a normal $N(0, \sigma^2)$ $\sqrt{n}\left(\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) - \mu\right) \xrightarrow{d} N\left(0, \sigma^2\right)$.

Task

Random value x_k is given (e.g. velocity, temperature).
$x_k = x_{k-1} + v_k * dt + \xi_k$ Sensor: $z_k = x_k + \eta_k$ v_k – value known from a physical model
ξ_k and η_k – random values (independent of time)
$E\xi_k = E\eta_k = 0$
$Var[\xi_k] = \sigma_\xi^2$
$Var[\eta_k] = \sigma_\eta^2$

Kalman algorithm

- Inductive
- x_k^{opt} is the optimal approximation
- $x_{k+1} = x_k + u_k + \xi_k$
- $x_{k+1}^{opt} = K * z_{k+1} + (1 - K) * (x_k^{opt} + u_k)$, where k is different for different iterations
- trying to minimize $e_{k+1} = x_{k+1} - x_{k+1}^{opt}$, by minimizing Ee_k^2
- $e_{k+1} = (1 - K)(e_k + \xi_k) - K * \eta_{k+1}$

Expectation of squared error

Theorem: $Ee_{k+1}^2 = (1\text{ - K})^2(Ee_k^2 + \sigma^2) + K^2 * \sigma_n^2$

Theorem: $\min\limits_K(E(e_{k+1}^2)) = \dfrac{Ee_k^2 + \sigma_\xi^2}{Ee_k^2 + \sigma_\xi^2 + \sigma_\eta^2}$

Practical questions

- What if we don't know physical model?
- What is the time of convergence?
- What is about multidimensional case?

Practical applications

- Self-driving cars
- Tracking
- Price forecasting