min. sample size

$$n \geq \frac{\sigma^2 Z^2_{\alpha/2}}{m^2}$$

margin of error

$$n \geq \frac{\hat{p}(1-\hat{p})z^2}{m^2}$$

1) Stratification

↳ by region

$$\hat{Y}_{strat} = \sum w_k \cdot \overline{Y}_k$$

$$E(\hat{Y}_{strat}) = \sum p_k \, E(\overline{Y}_k) = \sum p_k \mu_k = \mu$$

within → $$\mathrm{var}(\hat{Y}_{stat}) = \sum p_k^2 \, \mathrm{var}(\overline{Y}_k) =$$

$$= \sum p_k^2 \cdot \frac{\sigma_k^2}{n_k} = \sum \frac{n_k^{\cancel{2}}}{n^2} \cdot \frac{\sigma_k^2}{\cancel{n_k}} =$$

$$= \frac{1}{n} \sum p_k \cdot \sigma_k^2$$

$$\mathrm{var}(\overline{Y}) = \frac{\sigma^2}{n} \qquad E(\overline{Y}) = \mu$$

$$\mathrm{Var}_{2S}(\bar{Y}) = E(\mathrm{Var}(Y|Z)) + \mathrm{Var}(E(Y|Z)) = E\left(\sum \sigma_k^2 I(Z=k)\right) + \mathrm{Var}\left(\sum \mu_k I(Z=k)\right) =$$

$$\sum \sigma_k^2 \underbrace{E(I(Z=k))}_{= p_k} + \sum \mu_k^2 E(I^2(Z=k)) - \mu^2$$

$\mathrm{Var} = E(X^2) - E(X^2)$

$$= \sum \sigma_k^2 p_k + \underbrace{\sum \mu_k^2 p_k - \mu^2}_{\Downarrow}$$

$$= \underbrace{\sum \sigma_k^2 p_k}_{\text{Var within}} + \underbrace{\sum p_k (\mu_k - \mu)^2}_{\text{Var between}}$$

II Cuped

$$\begin{array}{ccc} Z_i^{0/1} & & X_i \\ & \searrow \quad \swarrow & \\ & Y_i & \end{array}$$

$$Y_{cuped} = Y - \hat{\theta} X$$

$$\hat{\theta} = \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)}$$

$$\mathrm{Var}(Y_{cuped}) = \mathrm{Var}(Y) + \theta^2 \cdot \mathrm{Var}(X) - 2\theta \cdot \mathrm{Cov}(X,Y)$$

$$\mathrm{Var}(Y_{cuped}) = \mathrm{Var}(Y)(1 - \rho^2)$$

(I) $\underline{1\ шаг}$

$$Y_{cuped} = r_i = Y_i - \hat{\beta} X_i$$

$\underline{2\ шаг}$

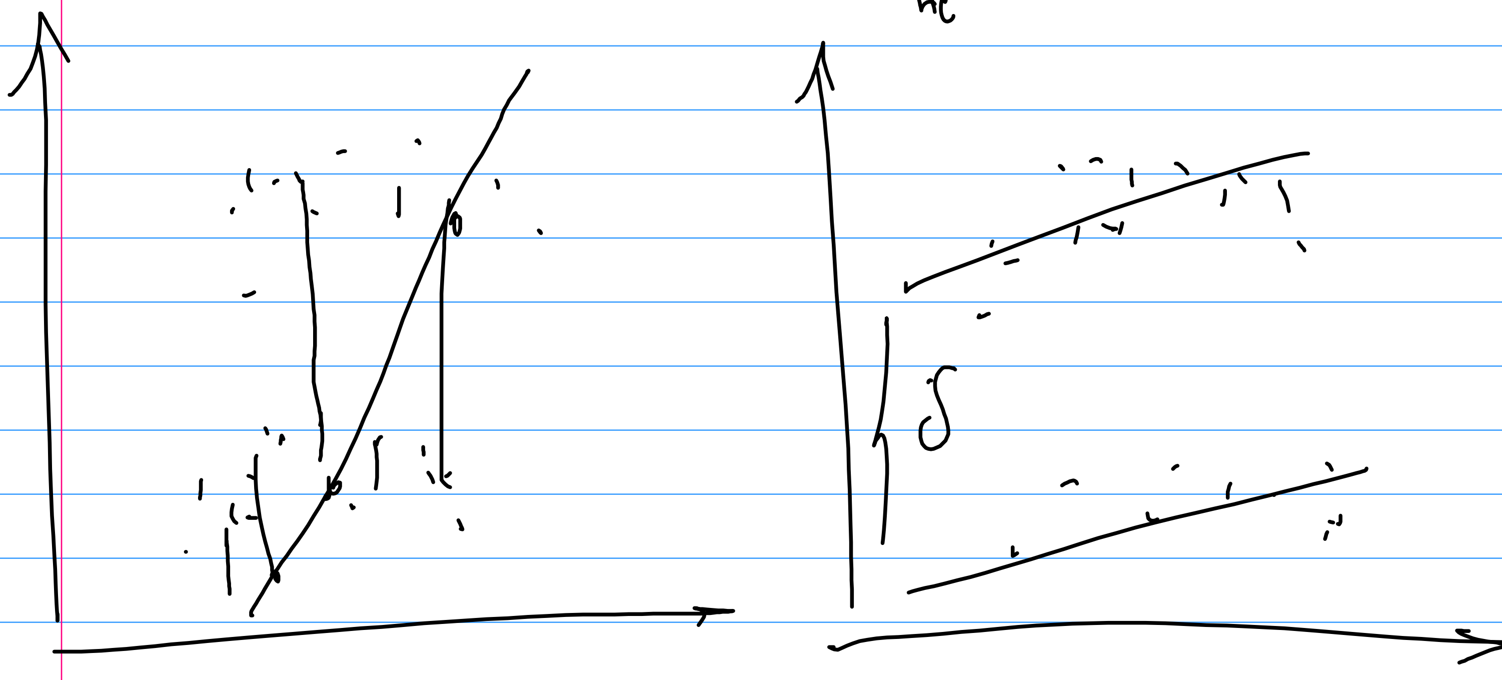
$$CI_{HC}(\gamma_2)$$

$$r_i = \gamma_1 + \gamma_2 \cdot Z_i + v_i$$

(II) $$Y_i = \alpha + \beta \cdot X_i + \delta Z_i + v_i$$

$$CI_{HC}(\delta)$$

(I)



Control is discrete

$\Rightarrow$ Cuped = Stratification

$$X \begin{matrix} \nearrow 0 \\ \searrow 1 \end{matrix} \qquad w = E(X)$$

$$\hat{Y}_{strat} = w\bar{Y}_1 + (1-w)\bar{Y}_0$$

$$\hat{Y}_{cuped} = \bar{Y} - \hat{\theta}\bar{X} + \hat{\theta} w \; =$$

$$\hat{\theta} = \frac{cov(X,Y)}{var(X)} = \bar{Y}_1 - \bar{Y}_0$$

$$= \; \bar{Y} - (\bar{Y}_1 - \bar{Y}_0)\bar{X} + (\bar{Y}_1 - \bar{Y}_0) w =$$

$$= (1-\bar{X})\bar{Y}_0 + \bar{Y}_0\bar{X} + (\bar{Y}_1 - \bar{Y}_0) w =$$

$$= w\bar{Y}_1 + (1-w)\bar{Y}_0 = \hat{Y}_{strat}$$