Policy Evaluation

Groups $\nearrow$ Treatment $\quad D_i = 1$

$\searrow$ Control $\quad D_i = 0$

$Y_i(1) - Y_i(0)$ — ind. treatment effect

$$ATE = E(Y_i(1) - Y_i(0))$$

$$Y_i = \begin{cases} Y_i(1), & D_i = 1 \\ Y_i(0), & D_i = 0 \end{cases}$$

$$Y_i = Y_i(0) + D_i \cdot (Y_i(1) - Y_i(0))$$

$$E(Y_i \mid D_i = 1) - E(Y_i \mid D_i = 0) =$$

$$= E(Y_i(1) \mid D_i = 1) - E(Y_i(0) \mid D_i = 0) =$$

$$= E(Y_i(1) \mid D_i = 1) - E(Y_i(0) \mid D_i = 1)$$
$$+ E(Y_i(0) \mid D_i = 1) - E(Y_i(0) \mid D_i = 0)$$

$$= E(Y_i(1) - Y_i(0) \mid D_i = 1) +$$
$$+ \left[E(Y_i(0) \mid D_i = 1) - E(Y_i(0) \mid D_i = 0)\right]$$

$$ATET = E(Y_i(1) - Y_i(0) \mid D_i = 1)$$

Selection bias $= \left[E(Y_i(0) \mid D_i = 1) - E(Y_i(0) \mid D_i = 0)\right]$

Random assignment: $E(Y_i(0) \mid D_i = 1) =$

$$= E(Y_i(0) \mid D_i = 0) = E(Y_i(0))$$

$\hookrightarrow$ no selection bias $\Rightarrow$

$$ATET = E(Y_i \mid D_i = 1) - E(Y_i \mid D_i = 0)$$

$$\widehat{ATET} = \bar{Y}_1 - \bar{Y}_0$$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot D_i$$

$$\hat{\beta}_2 = \bar{Y}_1 - \bar{Y}_0$$

Diff - in - diff (DID)

$$Y_i = \alpha_s + \mu_t + \delta \cdot D_i + \varepsilon_i$$

α_s - state effect

α_0 - contr.

α_1 - treat.

μ_t - time effect

μ_0 - before

μ_1 - after

$$Y_i = \alpha_s + \mu_t + \delta \cdot D_i + \varepsilon_i$$

$$E(Y_i \mid s = 1, t = 0) = \mu_0 + \alpha_1$$

$$E(Y_i \mid s = 1, t = 1) = \mu_1 + \alpha_1 + \delta$$

$$\Delta_{TRET} = \mu_1 - \mu_0 + \delta$$

$$E(Y_i \mid s = 0, t = 0) = \mu_0 + \alpha_0$$

$$E(Y_i \mid s = 0, t = 1) = \mu_1 + \alpha_0$$

$$\Delta_{CONTR} = \mu_1 - \mu_0$$

$$\delta = \Delta_{TRET} - \Delta_{CONTR}$$

$$\hat{\delta} = \left[\bar{Y}_{T,A} - \bar{Y}_{T,B}\right] - \left[\bar{Y}_{C,A} - \bar{Y}_{T,B}\right]$$

$$\Delta Y_i = \beta_0 + \delta \cdot D_i + u_i$$

$$\delta Y_i = Y_{i1} - Y_{i0}$$

Matching

Treatment | Control

"green" bonds | "brown" bonds

1) Simple Matching

$$\Delta^M = \sum_k w_k (\bar{Y}_{1,k} - \bar{Y}_{0,k})$$

w_k - share of objects in kth group

Alternative:

$$\Delta^M = \frac{1}{N_T} \sum \left(y_i - \frac{1}{N_{c,i}} \sum_{j \in (D=0)} y_j \right)$$

N_T - # in Treatment group

$N_{c,i}$ - # obs in c_{th} part of control group for ith obs.

II) Nearest - neighbour matching

$$A_i = \{ j \mid \min_j \| x_i - x_j \| \}$$

(III) Propensity Score Matching

$$PS = P(D_i = 1 \mid x_i^{(1)}, \dots, x_i^{(k)})$$

1) Calculate PS using Logit model

2) $PS_{T,i} \approx PS_{C,i}$