Rules: online test in lms, no proctoring, 20 questions, 60 minutes, only numerical answers are checked, two digits after decimal point are requested, use anything you want (calculators, python/r code, google, ...), don't cheat.

1. (bootstrap) I have a sample X_1, ..., X_{100}.

   I generate one naive bootstrap sample X_1^*, ..., X_{100}^*.

   What is the probability that the first observation will be present in the bootstrap sample 2 times or more?

2. (bootstrap) Nature generates random variables X_1, ..., X_{100} independently and uniformly on $[0; 10]$.

   I generate one naive bootstrap sample X_1^*, ..., X_{100}^*.

   Find the variance $\mathrm{Var}(X_1^*)$.

3. (welch) We have data for an AB-experiment $\bar{X}_a = 10$, $\bar{X}_b = 12$, $n_a = 20$, $n_b = 30$, $\sum(X_i^a - \bar{X}_a)^2 = 100$, $\sum(X_i^b - \bar{X}_b)^2 = 200$.

   Calculate the standard error of $\bar{X}_a - \bar{X}_b$ for the Welch test.

4. (welch) Assume that X_i are independent and identically normally distributed $\mathcal{N}(\mu, \sigma^2)$, sample size is $n = 10$.

   Find $\mathrm{Var}(\sum(X_i - \bar{X})^2/(n-1))$.

5. (mw test) I have five results of two runners A and B for the 5 km race: 25:12 (A), 26:34 (B), 27:43 (A), 28:12 (A), 29:05 (B).

   Calculate Mann-Whitney statistic U_A that tests the null-hypothesis of equal distributions of time.

   (The statistic U_A should positively depend on the ranks of the runner A).

6. (mw test) I have five results of two runners A and B for the 5 km race: three results for A and two results for B. Assume that the running time for both runners are continuously distributed and their distribution are equal.

   What is the probability that the running times of the runner A will get the ranks 1 and 5?

7. (cuped) Consider three variables: target variable y_i, predictior x_i and indicator of treatment $z_i \in \{0, 1\}$. The treatment z_i was assigned independently of x_i, total $n = 200$.

   The matrix of all cross products (sums of the form $\sum a_i b_i$) C is provided. The order of variables is y, 1, x and z. For example, $\sum 1 \times y_i = 10$:

$$C = \begin{pmatrix} 500 & 10 & 2 & 8 \\ 10 & 200 & 100 & 100 \\ 2 & 100 & 100 & 40 \\ 8 & 100 & 40 & 100 \end{pmatrix}$$

   Consider CUPED with first regression given by $\hat{y}_i = \hat{\alpha}_1 + \hat{\alpha}_2 x_i$ with residuals $r_i = y_i - \hat{y}_i$.

   What is the cross-product $\sum r_i z_i$?

Short rules: 120 minutes, A4 cheat sheet and calculator are allowed, W_t always denotes a Wiener process.

8. (cuped) Consider three variables: target variable y_i, predictior x_i and indicator of treatment $z_i \in \{0, 1\}$. The treatment z_i was designed to be independent of x_i, but in fact $x_i = f_i \cdot (1 + 0.01 z_i)$.

   We suppose that z_i are Bernoulli 0.5, $f_i \sim \mathcal{N}(1; 1)$ and they are independent.

   Find the probability limit
   $$\text{plim} \frac{\sum (x_i - \bar{x})(z_i - \bar{x})}{n - 1}.$$

9. (matching)

10. (matching)

11. (multiple comparison)

12. (multiple comparison)

13. (sample size)

14. (sample size)

15. (contingency table) I eated 10 M&Ms: 2 green, 1 red, 4 yellow, 1 green, 2 red.

    Only these three colors are possible. I assume that yellow and green colors are equally probable.

    Calculate the maximal log likelihood for my model.

16. (contingency table) Consider the following contingency table

	$B = 1$	$B = 2$
$A = 1$	10	20
$A = 2$	30	40

    Calculate LR statistic that checks the hypothesis that A and B are independent against dependency alternative.

17. (anova 1+2) Vasiliy loves to eat shaurma. He has three local shaurma dealers. Vasiliy bought 7 shaurmas from each dealer. and measured their weight. He would like to test the hypothesis that mean weight is the same for all dealers.

    Total sum of squares is 1000, between sum of squares is 500.

    Calculate the F-statistic to test the hypothesis.

18. (anova 1+2) Vasiliy loves to eat shaurma. He has three local shaurma dealers with two types of shaurma. Vasiliy bought 7 shaurmas of each type from each dealer and measured their weight. He would like to test the hypothesis that weight depenends on the shaurma type and not on the dealer. He assumes no interaction.

    Total sum of squares is 1000, sum of squares explained by dealers is 300, sum of squares explained by type is 500.

    Calculate the F-statistic to test the hypothesis.

19. (partial correlation) The variables X and Y are jointly normal with zero means, unit variances and $\text{Corr}(X, Y) = 0.8$.

    Find α such that $X^* = X - \alpha Y$ is not correlated with Y.

Short rules: 120 minutes, A4 cheat sheet and calculator are allowed, W_t always denotes a Wiener process.

20. (partial correlation) The variables X_1, X_2, ... are independent and identically distributed with mean 5 and variance 7.

    Find $\mathrm{pCorr}(X_1, X_2; S)$ where $S = X_2 + X_3$.