Multiple Hypothesis Testing

H_1 : factor x_1 influences on y?

⋮

H_n : factor x_m influences on y?

$\alpha = 5\%$

$m = 20$ $P(\text{at least } 1 \text{ sig. result}) =$

$1 - P(\text{no sig. result}) =$

$1 - (1 - 0.05)^{20} \approx 0.64$

Corrections to control

(1) FWER or (2) FDR

I $FWER = P(V \geq 1) =$

$= 1 - P(V = 0)$

	not sig	sig	
H_0 true	U TN	V FP	m_0
H_0 false	T FN	S TP	$m - m_0$
	$m - R$	R	m

$FWER \leq \alpha$

- Bonferroni correction

$$\alpha_{Bon} = \alpha / m \quad m - \# \text{ of hypothesis}$$

	p-value	k
H_1	0,03	2
H_2	0,01	1
H_3	0,05	3

- Holm - Bonferroni :

$$\alpha_{HB} = \frac{\alpha}{m + 1 - k} \quad k - \text{rank of } H_i$$

- Sidak

$$\alpha_s = 1 - (1-\alpha)^{1/m}$$

- Sidak – Holm:

$$\alpha_{SH} = 1 - (1-\alpha)^{\alpha/m-k+1}$$

$$\alpha_1 = 1 - (1-\alpha)^{\alpha/m}$$

$$\alpha_m = \alpha$$

if statistics are jointly independent (when controlling FWER) has highest power

II $FDR = E\left(\frac{V}{V+S}\right) = E\left(\frac{V}{R}\right)$

	not sig	sig	
H_0 true	U TN	V FP	m_0
H_0 false	T FN	S TP	$m-m_0$
	$m-R$	R	m

$FDR \leq FWER$

$FDR = FWER$ only when $\forall H_i$, $i = \overline{1,m}$ are true

- Benjamini – Hochberg:

$$\alpha_{Benj} = \frac{k}{m}\alpha$$

k – rang

m – # hyp.

- Hochberg:

$$\alpha_H = \frac{\alpha}{m-k+1}$$