Rules: online test in lms, no proctoring, 20 questions, 60 minutes, only numerical answers are checked, two digits after decimal point are requested, use anything you want (calculators, python/r code, google, ...), don't cheat.

1. (bootstrap) I have a sample X_1, ..., X_{100}.

   I generate one naive bootstrap sample X_1^*, ..., X_{100}^*.

   What is the probability that the first observation will be present in the bootstrap sample 2 times or more?

2. (bootstrap) Nature generates random variables X_1, ..., X_{100} independently and uniformly on $[0; 10]$.

   I generate one naive bootstrap sample X_1^*, ..., X_{100}^*.

   Find the variance $\text{Var}(X_1^*)$.

3. (welch) We have data for an AB-experiment $\bar{X}_a = 10$, $\bar{X}_b = 12$, $n_a = 20$, $n_b = 30$, $\sum(X_i^a - \bar{X}_a)^2 = 100$, $\sum(X_i^b - \bar{X}_b)^2 = 200$.

   Calculate the standard error of $\bar{X}_a - \bar{X}_b$ for the Welch test.

4. (welch) Assume that X_i are independent and identically normally distributed $\mathcal{N}(\mu, \sigma^2)$, sample size is $n = 10$.

   Find $\text{Var}(\sum(X_i - \bar{X})^2/(n-1))$.

5. (mw test)

6. (mw test)

7. (cuped)

8. (cuped)

9. (matching)

10. (matching)

11. (multiple comparison)

12. (multiple comparison)

13. (sample size)

14. (sample size)

15. (contingency table) I eated 10 M&Ms: 2 green, 1 red, 4 yellow, 1 green, 2 red.

    Only these three colors are possible. I assume that yellow and green colors are equally probable.

    Calculate the maximal log likelihood for my model.

16. (contingency table) Consider the following contingency table

	$B = 1$	$B = 2$
$A = 1$	10	20
$A = 2$	30	40

    Calculate LR statistic that checks the hypothesis that A and B are independent against dependency alternative.

Short rules: 120 minutes, A4 cheat sheet and calculator are allowed, W_t always denotes a Wiener process.

17. (anova 1+2)

18. (anova 1+2)

19. (partial correlation) The variables X and Y are jointly normal with zero means, unit variances and $\text{Corr}(X, Y) = 0.8$.

    Find α such that $X^* = X - \alpha Y$ is not correlated with Y.

20. (partial correlation) The variables X_1, X_2, ... are independent and identically distributed with mean 5 and variance 7.

    Find $\text{pCorr}(X_1, X_2; S)$ where $S = X_2 + X_3$.