① t-test, Welch test, MW

② t1 error rate, power

③ CUPED, stratification

④ diff-in-diff & matching

⑤ Bootstrapping

(1)

$1/60$ draw X_1

$59/60$ draw something else

$$C_{60}^{2}\left(\frac{1}{60}\right)^{2}\cdot\left(\frac{59}{60}\right)^{58} = P(N=2)$$

$$N \sim Bin(60, 1/60)$$

(2)

$$60\cdot\left(\frac{59}{60}\right)^{60}$$

(3)

$$t = \frac{\bar{X}_a - \bar{X}_b}{se(\bar{X}_a - \bar{X}_b)} \sim t_{n-1}$$

$$t^{W} = \frac{\bar{X}_a - \bar{X}_b}{se(\bar{X}_a - \bar{X}_b)} \sim t_{\hat{\nu}}$$

$$se(\bar{X}_a - \bar{X}_b) = \sqrt{\frac{s_A}{n_A} + \frac{s_B}{n_B}}$$

$$= \sqrt{\frac{840/17}{18} + \frac{820/11}{12}}$$

(4) $\mathrm{Var}\left(\sum (x_i - \bar{X})^2\right) =$

$2 \cdot 3^2 \cdot (20-1)$

(5) 1 2 3 4 5

A B

$C_1^{3=4-1}$

$C_2^{5=2+3}$

(6) $E(U) = \frac{n_1 \cdot n_2}{2}$

(7)

$$\text{plim } \hat{\text{cov}}(x,z) = \boxed{\begin{array}{l} z_i = \begin{cases} 0 \\ 1 \end{cases} \\ z_i^2 = z_i \end{array}}$$

$$= \text{cov}(x_i, z_i) =$$

$$= \text{cov}(\beta_i (1 + 0{,}08 \cdot z_i),\ z_i) =$$

$$= 0{,}08 \cdot \text{cov}(\beta_i z_i ;\ z_i) =$$

$$= 0{,}08 \left(E(\beta_i \cdot z_i^2) - E(\beta_i z_i) \cdot E(z_i) \right)$$

$$= 0{,}08 \cdot E(\beta_i \cdot z_i) \cdot (1 - E(z_i))$$

$$= 0{,}08 \cdot 800 \cdot 0{,}4 \cdot 0{,}6$$

(8)

$$y_i = \alpha + \beta z_i + \gamma \cdot x_i + \varepsilon_i$$

1) $\hat{y}_i^{cup} = y_i - \hat{\theta} x_i$

2) t-test for y_i^{cup}

$$\hat{\text{cov}}(z,z) = \hat{\text{cov}}(y - \hat{\alpha}_1 - \hat{\alpha}_2 \cdot x,\ z) =$$

$$= \hat{\text{cov}}(y,z) - \hat{\alpha}_2 \hat{\text{cov}}(x,z) =$$

$$= \hat{cov}(y,z) - \frac{\hat{cov}(y,x)}{\hat{Var}(x)} \cdot \hat{cov}(x,z) =$$

$$= 0,75 - \frac{2,64}{1,17} \cdot 0,26 = \ldots$$

(9) 6

(10)

$$\hat{p}_{yellow} = 2 \cdot \hat{p}_{green}$$

$$\hat{p}_{green} = \frac{3}{5 + 5 \cdot 2}$$

$$\hat{p}_{red} = 1 - \hat{p}_{green} - \hat{p}_{yellow}$$