

CUPED (Controlled
experiments using pre-experiment data)
&

Diff-in-diff

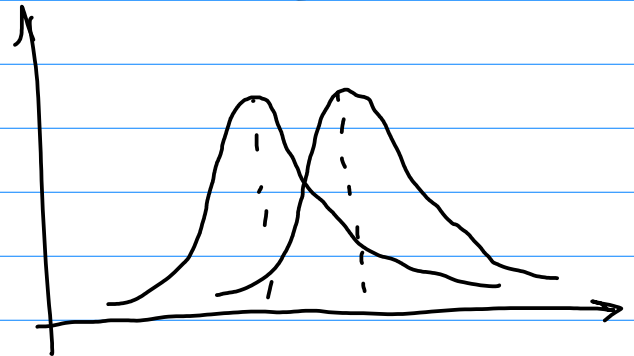


(✓)



(x)

↓ CUPED



Naive approach: $\Delta = \bar{y}_T - \bar{y}_C$

Example: Δu after policy implementation ΔW .

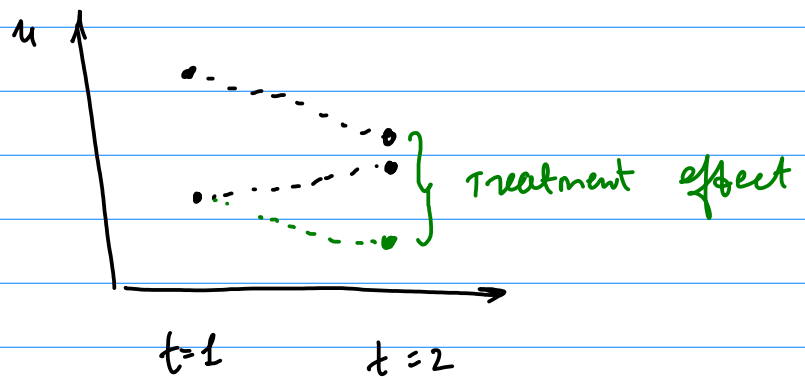
① $\Delta = \bar{u}_2 - \bar{u}_1$

other variates
changing over time
aren't controlled
for

② $\Delta = \bar{u}_T - \bar{u}_C$

states can be
different

"
Diff-in-diff



$$E(y | T, t=1) = \mu_1 + \alpha_T$$

$$E(y | T, t=2) = \mu_2 + \alpha_T + \delta$$

↑
unknown TE

$$\Delta^T = \mu_2 - \mu_1 + \delta$$

$$E(y | T, t=1) = \mu_1 + \alpha_T$$

$$E(y | C, t=2) = \mu_2 + \alpha_C$$

$$\Delta^C = \mu_2 - \mu_1$$

$$\Delta^T - \Delta^C = \delta$$

"Diff-in-Diff"

$$\Delta = \beta_2 + \delta \cdot D_T + \epsilon_i$$

CUPEB (Related to Frisch-Waugh)

y - explained data

X - covariate (pre-experiment)

$$y | D, X \quad \hat{\theta}$$

$$1) \hat{y} | X \Rightarrow \hat{y}^c \quad \updownarrow$$

$$2) \hat{y}^c | D \quad \hat{\theta}$$

$$1) y^{cupeb} = y - \theta \cdot X$$

$$E[y] = E[\bar{y}] = E[\bar{y} - \theta X] + \theta E[X] =$$

$$= E[\bar{y} - \theta \bar{X}] + \theta E[X]$$

$$\hat{y} = \bar{y} - \theta \bar{X} + \theta E[X]$$

$$\theta = \frac{\text{Cov}(X, y)}{\text{Var}(X)}$$

$$\hat{\theta} = \frac{\hat{\text{Cov}}(X, y)}{\hat{\text{Var}}(X)}$$

$$2) \bar{y}^{cupeb} - \bar{y}^c$$

$$\text{Var}(y^{cupeb}) = (1 - \rho^2) \text{Var}(y) \leq \text{Var}(y)$$

$$y = \beta + \delta \cdot D_T + \theta \cdot X + \varepsilon_i$$

variance
exp.

var.
ind.

var

unexplained

$$\text{Generalize: } y = \beta + \delta_1 D_{T1} + \delta_2 D_{T2} \dots +$$

$$+ \theta_1 \cdot X_1 + \dots + \varepsilon_i$$

Stratification:

$$\text{Var}(\bar{Y}) = \sum \frac{W_k}{n} \sigma_k^2 + \sum \frac{W_k}{n} (\mu_k - \mu)^2 \geq \sum \frac{W_k}{n} \sigma_k^2 = \text{Var}(\hat{Y}_{\text{strat}})$$

within between

(=) Cuped for X - categorical

$$X = \begin{cases} 1, & p_1 = w \\ 0, & p_2 = 1-w \end{cases}$$

$$\hat{Y}_{\text{STRAT}} = w \bar{y}_1 + (1-w) \bar{y}_0$$

$$\hat{Y}_{\text{cuped}} = \bar{y} - \hat{\theta} \bar{X} + \hat{\theta} w$$

$$\hat{\theta} = \frac{\widehat{\text{Cov}}(X, Y)}{\widehat{\text{Var}}(X)} = \bar{y}_1 - \bar{y}_0 \quad (\text{since } X \text{ - dummy})$$

$$\bar{y} = \bar{X} \cdot \bar{y}_1 + (1-\bar{X}) \bar{y}_0$$

$$\hat{Y}_{\text{cuped}} = \bar{y} - (\bar{y}_1 - \bar{y}_0) \bar{X} + (\bar{y}_1 - \bar{y}_0) w =$$

$$(=) (1-\bar{X}) \bar{y}_0 + \bar{y}_1 \bar{X} + (\bar{y}_1 - \bar{y}_0) w =$$

$$= w \bar{y}_1 + (1-w) \bar{y}_0 = \hat{Y}_{\text{STRAT}}$$