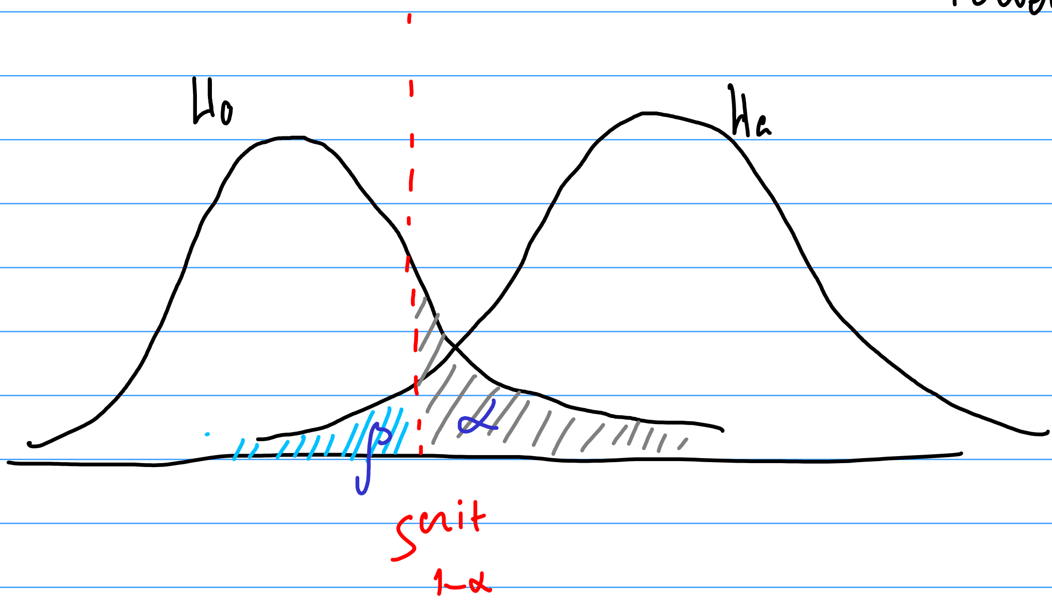
Power Analysis

Type I error $\quad P(S^{obs} \in D^{crit} \mid H_0) = \alpha$

Type II error $\quad P(S^{obs} \notin D^{crit} \mid H_a) = \beta$

$\Updownarrow$

Power $= 1 - \beta$



Hypothesis testing:

- Significance level
- Power
- Effect size (Cohen's d)
- Sample size

Sample size

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$P\left(z_{\alpha/2} \le \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \le z_{1-\alpha/2} \right) = 1-\alpha$$

$$P\left(\bar{X} + \underbrace{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{\text{Margin of error } (E)} \le \mu \le \bar{X} + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right) = 1-\alpha$$

$$z_{\alpha/2} \cdot \sigma/\sqrt{n} \le E$$

$$n \geq \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$$\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0,1)$$

$$\hat{p} + \underbrace{z_{\alpha/2} \cdot \sqrt{p(1-p)/n}}_{E} \leq p \leq \hat{p} + z_{1-\alpha/2} \cdot \sqrt{p(1-p)/n}$$

$$z_{\alpha/2} \cdot \sqrt{p(1-p)/n} \leq E$$

$$n \geq \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p)$$

Samples $\nearrow$ with replacement

$X_1 = 4$, w.p. p_1

$X_2 = 4$, w.p. p_1

Samples $\searrow$ w/o replacement

$[4, 1, 2]$

$X_1 = 4$, w.p. $1/3$

$X_2 = 4$, w.p. 0

$$\mathrm{Cov}(X_i, X_j) \neq 0$$

$$\mathrm{Cov}(X_i, X_j) = -\frac{\sigma^2}{N-1}$$

n - sample size

N - pop. size

se(.)	With replacement	W.O. replacement
$\bar{X}$	$\sqrt{\sigma^2/n}$	$\sqrt{\sigma^2/n\left(1-\frac{n-1}{N-1}\right)} \approx \sqrt{\frac{s^2}{n}\left(1-\frac{n}{N}\right)}$
$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$\sqrt{\frac{p(1-p)}{n}\left(1-\frac{n-1}{N-1}\right)}$

Percentage Error:

$$E = R \cdot \mu$$

R – percentage

$C = \sigma / \mu$ ← coefficient of variation

$\bar{X}$: $$n = \frac{\sigma^2 \cdot z^2}{E^2} = \frac{(\sigma^2/\mu^2)\, z^2}{R^2} = \frac{C^2 \cdot z^2}{R^2}$$

$\hat{p}$: $$n = \frac{z^2 \cdot (1-\pi)}{R^2 \; \pi}$$