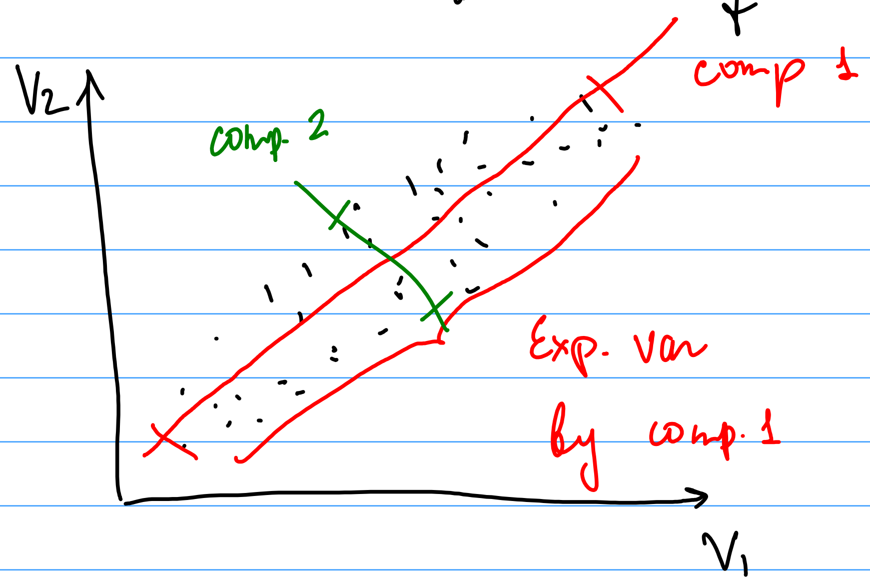
PCA:

Goal - Max Variance of new components



$$Z = Xu$$

Goal : $\max\limits_{u} \ \mathrm{Var}(Z)$

s.t. : $\|u\| = 1$

PCA Algorithms

(I) Sequential

$\{x_1, \dots, x_m\}$ - centered data

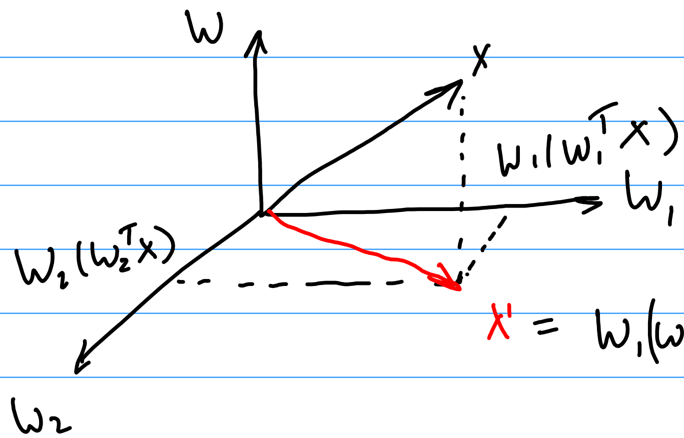
Compute 1^{st} component :

$$w_1 = \underset{\|w\|=1}{\arg\max} \ \frac{1}{m} \sum_{i=1}^{m} (w^T x_i)^2$$

Compute kth component:

$$w_k = \underset{\|w\|=1}{\arg\max} \frac{1}{m} \sum_{i=1}^{m} \Big(w^T \big[\underbrace{x_i - \sum_{j=1}^{k-1} w_j w_j^T x_i}_{} \big]\Big)^2$$

X' - PCA reconstruction



$$X' = w_1(w_1^T x) + w_2(w_2^T x)$$

II Sample covariance matrix

$\{x_1, \dots, x_m\}$

$$\Sigma = \frac{1}{m} \sum (x_i - \bar{x})(x_i - \bar{x})^T \qquad \bar{x} = \frac{1}{m} \sum x_i$$

eigenvectors of Σ = PCA basis vectors

eigenvalues of Σ: higher eigenvalue

=> more important eigenvector

$$\max \quad u^T X X^T u$$

$$\text{s.t.} \quad u^T u = 1$$

$$\mathcal{L} = u^T X X^T u - \lambda u^T u$$

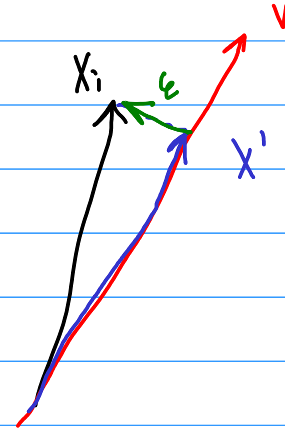
$$\frac{\partial \mathcal{L}}{\partial u} = X X^T u - \lambda u = (X X^T - \lambda \mathbb{1}) u = 0$$

$$u \neq 0 \Rightarrow (X X^T - \lambda \mathbb{1}) u = 0$$

$\Rightarrow$ u – eigenvector of XX^T

λ – eigenvalues

PCA:



1) Max Variance

$$\frac{1}{n} \sum (v^T x_i)^2 = v^T X X^T v$$

2) Min Reconstruction Error

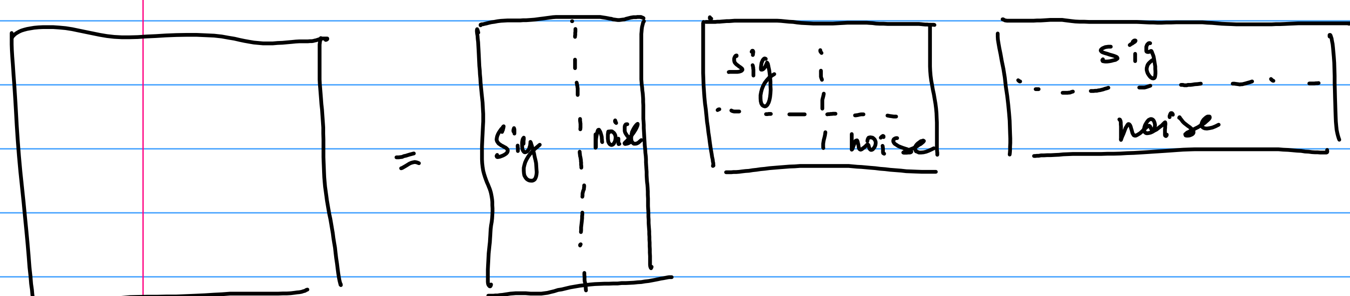
$$\frac{1}{n} \sum \| x_i - (v^T x_i) v \|^2$$

III SVD

$$X = [x_1, \dots, x_m] \in \mathbb{R}^{N \times m}$$

↑ centered data

$$X = U S V^T$$



sample	columns: principal vectors $\{u^{(1)}, \dots, u^{(k)}\}$	diagonal: shows importance of comp.	columns: coefficents for reconstructing X

VIF

$$y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + \varepsilon_i$$

1) $x_j \mid x_{-j} \Rightarrow R_j^2$

2) $$\mathrm{VIF}(\hat{\beta}_j) = \frac{1}{1-R_j^2}$$

VIF > 5 => multicollineanity