CUPED.

A/B - tests. — from stat viewpoint: just ordinary tests!

A - old method
B - new method

$H_0: p_A = p_B$
$H_A: p_B > p_A$

Deng „Improving the sensitivity of .... experiments by using pre-experimental data".
CUPED.

Q. I need (90%) CI for θ.

Forecastors: Claude → good mood 95% CI (1/2)
→ Bad mood 85% CI (1/2)
Dmitriy → always make 90% CI

a) Which one do you prefer?

$$P(\underbrace{[\hat{\theta}_C^L ; \hat{\theta}_C^R]}_{CI_C} \ni \theta) = P(\text{Good mood}) \cdot P(CI_C \ni \theta \mid \text{Good}) +$$

$$+ P(\text{Bad mood}) \cdot P(CI_C \ni \theta \mid \text{Bad}) =$$

$$= \frac{1}{2} \cdot 0{,}95 + \frac{1}{2} \cdot 0.85 = 0{,}9.$$

$$P(CI_D \ni \theta) = 0{,}9$$

For stability → prefer CI_D

b) which interval is shorter?
→ sometimes CI_D is wider
sometimes CI_C is wider.

$\mu_A = E(y_i \mid w_i = A)$

$\mu_B = E(y_i \mid w_i = B)$

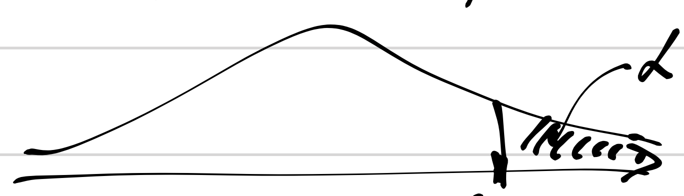
y_i – target variable

$w_i \in \{A, B\}$

control (old meth) experim (new meth)

$H_0: \mu_A = \mu_B$

$H_1: \mu_B > \mu_A$



1.69

Stand. test

„Claude case"

$$z = \frac{\bar{y}_B - \bar{y}_A - 0}{se(\bar{y}_B - \bar{y}_A)}$$

[Under H_0
$z \overset{asy}{\sim} N(0;1)$]

$\boxed{CI \text{ for } (\mu_B - \mu_A)}$

exact match

if $z > z_{crit}$ then rej H_0 (use new meth)

if $z \leq z_{crit}$ then do not rej H_0 (use old method)

simple regression.

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot I(w_i = B)$$

$$t = \frac{\hat{\beta}_2 - 0}{se(\hat{\beta}_2)}$$

$\hat{\beta}_2 = \bar{y}_B - \bar{y}_A$

$se(\hat{\beta}_2) = se(\bar{y}_B - \bar{y}_A)$

if $t > z_{crit}$ then rej H_0

if $t \leq z_{crit}$ do not rej H_0.

$$\frac{\hat{\beta}_2 - 0}{se_{HC}(\hat{\beta}_2)}$$

x_i – pre experimental information.

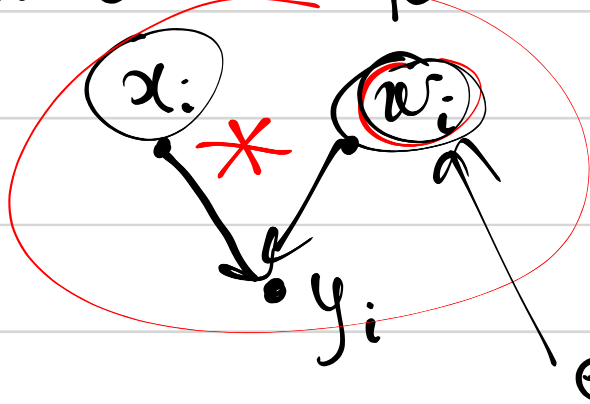
(!) $\boxed{x_i \text{ should be indep-t of } w_i}$

we are org-ing the experiment

we should honestly randomize w_i

How can improve our tests?

We have



$\in \{A, B\}$

no link between x_i and w_i

$I(w_i = B)$

„Priority case"

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_2 \cdot I(w_i = B) + \underbrace{\hat{\beta}_3 \cdot x_{i1} + \hat{\beta}_4 \cdot x_{i2}}_{\text{other predictors!}}$$

may be I'm wrong

$H_0: \ \beta_2 = 0$

$H_A: \ \beta_2 > 0.$

$$t = \frac{\hat{\beta}_2 - 0}{se(\hat{\beta}_2)}$$

CI for β_2

Message 1: coverage probab. for CI for $(\mu_B - \mu_A)$ and CI for (β_2) is the same. provided all assumpl.

Message 2: regression method has one problem!

heteroskedasticity

$$Var(u_i \mid predictors_i) \neq const.$$

Remedy is simple!!

use robust se

$$\frac{\hat{\beta}_2 - 0}{se_{HC}(\hat{\beta}_2)}$$

Message 3: we may be wrong in the description how y_i depends on x_i

CUPED

Step 1 [2 variants of CUPED]

Step 1a

$$\hat{y}_i = \hat{\beta}_1 + x_i^T \hat{\beta}_x$$

semiresidual

$$r_i = y_i - x_i^T \cdot \hat{\beta}_x$$

Step 1b

$$\boxed{\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot w_i + x_i^T \cdot \hat{\beta}_x}$$

$$r_i = y_i - x_i^T \cdot \hat{\beta}_x$$

Step 1.c.

intuit „the part of y_i non-explained by pre-experimental characteristics"
clean part of y_i

Step 2. OLS: $\quad w_i \in \{A, B\}$

$$\boxed{\hat{r}_i = \hat{\beta}_1 + \hat{\beta}_2 \, \mathbb{I}(w_i = B)}$$

→ CI for β_2

$$\boxed{\begin{array}{l} H_0: \beta_2 = 0 \\ H_1: \beta_2 > 0 \end{array}}$$

$$t = \frac{\hat{\beta}_2 - 0}{se(\hat{\beta}_2)}$$

if $t > t_{crit} \Rightarrow$ reject H_0

* Main advantage (over stand test)
  It stabilizes coverage prob. ~~fy~~.

* Pract evidence: in most cases it will shorten the CI. [not in all]

* Adv over mult. regression on w_i, x_i
  generalize step 1.

variance reduction. $\boxed{RSS < TSS}$