Common Factor Analysis (CFA)

CFA vs PCA

1) PCA for maximising variance

   FA for maximising covariance

2) PCA : fully orthogonal components

   FA : orthogonality is not required

3) PCA : each component is a l.c. comb. of observed variables

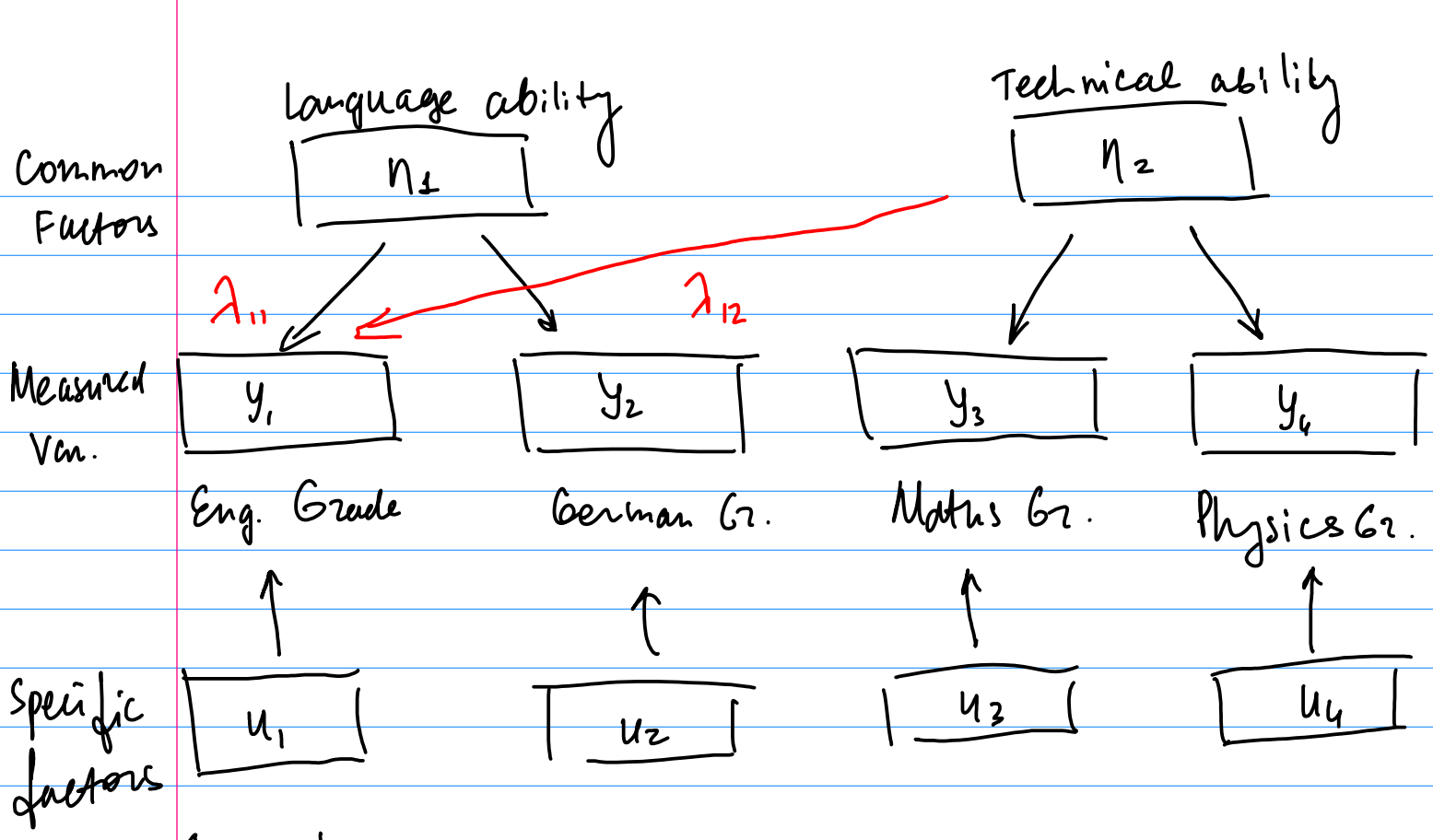
   FA : target variables is lin. comb. of latent factors

4) PCA : components are not interpretable

   FA : factor are labellable and interpretable

5) PCA : dimension reduction / observational

   FA : latent variable method / modelling technique



$$y_1 = \lambda_{11}\eta_1 + \lambda_{12}\eta_2 + u_1$$

$$\vdots$$

$$y_4 = \lambda_{41}\eta_1 + \lambda_{42}\eta_2 + u_2$$

Common | Unique

$$\bar{y} = \Lambda \bar{\eta} + \bar{u}$$

Factor model:

$$X_i = \lambda_{i1} \cdot F_1 + \lambda_{i2} F_2 + \dots + \lambda_{im} F_m + V_i U_i$$

X_i - standardised variable

F - latent common factors

U - unique factors

$$F_i = W_{i1} X_1 + W_{i2} \cdot X_2 + \dots + W_{ik} X_k$$

W_i - weights / factor scores

Factor loading - matrix shows relationship between variables and common factors

Eigenvalues - represents variance explained

Communalities = sum of loadings squared

↳ common variance explained

from 0 to 1 => share

Factor Rotation - orthogonal or oblique

weights of factors are redistributed

Assumptions:

1) No outliers in data

2) Sample size is greater then factor number

3) No perfect multicollinearity
(should be imperfect multicollinearity)

PCA: decomposion of correlation matrix

CFA: — // —

diagonal elements are replaced with $\overline{|1 - \text{var}(d)|}$

$$\hat{\beta} = \frac{\hat{cov}}{\hat{var}}$$

Steps in Factor Analysis

$X_1, \ldots, X_p$ – p measured variables

$F_1, \ldots, F_m$ – m common factors

$$m < p$$

$$X_i = \lambda_{i1} F_1 + \ldots + \lambda_{im} F_m + \varepsilon_i$$

1) Calculate initial factor loadings

- PCA
- Principal Axis Factoring

1 Factor : find λ_{i1} s.t. max $\text{Var}(\hat{x})$

2 Factor : find λ_{i2} s.t. max residual variance

...

2) Factor Rotation

- othogonal (varimax method)
- oblique

re-distribute weights s.t. for each variable only one factor has high loading

3) Calculate Factor scores

- select how many factors should be left

① Fix threshold for variance explained:

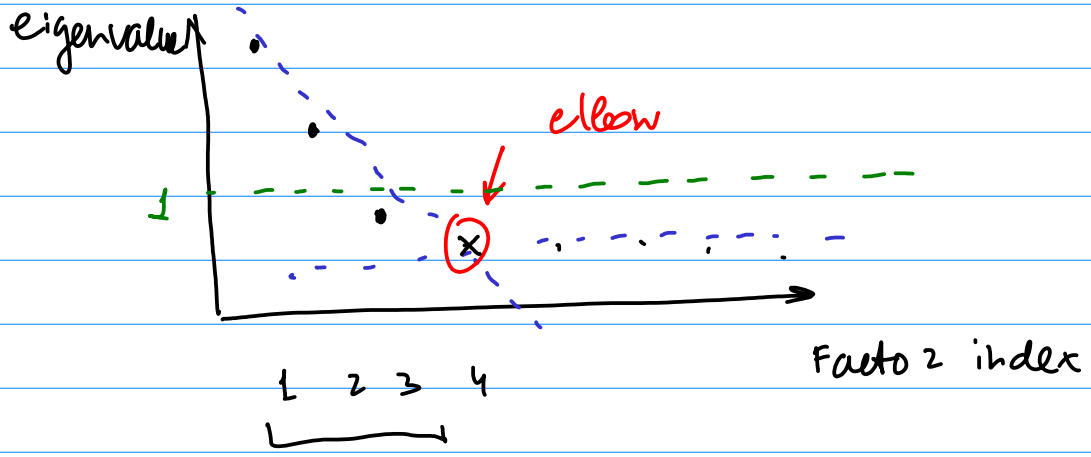
(e.g. 75%)

② Chose only factors with eigenvalues ≥ 1

→ construct c.i. and

chose factors with c.i. ≥ 1

③ Chose by scree plots:



Testing:

① Bartlett test for Sphericity

Ho: correlation matrix is identity matrix

Ho should be rejected

② Kaiser - Meyer - Olkin (KMO) statistic

shows common variance share

KMO > 0,6 => CFA can be done

Variance and communalities