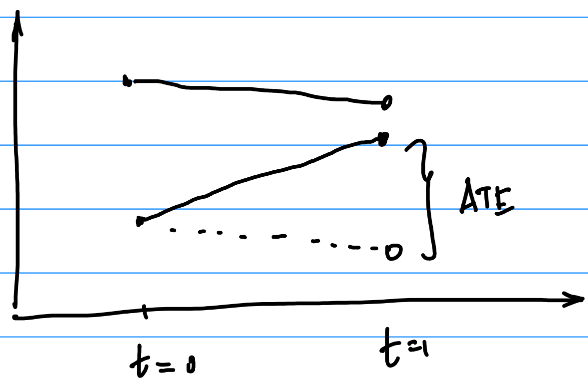
Diff-in-diff

$$y_i = \begin{cases} y_i(1), & D_i = 1 \\ y_i(0), & D_i = 0 \end{cases}$$

$$\underbrace{y_i(1) - y_i(0)}_{\text{individual TE}}$$

$$\underbrace{E(y_i(1) - y_i(0))}_{\text{ATE}}$$



$$E(y_i \mid D_i = 1) - E(y_i \mid D_i = 0) =$$

$$E(y_i(1) \mid D_i = 1) - E(y_i(0) \mid D_i = 0) =$$

$$E(y_i(1) \mid D_i = 1) - E(y_i(0) \mid D_i = 1)$$

$$+ E(y_i(0) \mid D_i = 1) - E(y_i(0) \mid D_i = 0) =$$

$$= \underbrace{E(y_i(1) - y_i(0) \mid D_i = 1)}_{\text{ATET}}$$

$$+ \underbrace{E(y_i(0) \mid D_i = 1) - E(y_i(0) \mid D_i = 0)}_{\text{selection bias}}$$

$$E(y_i \mid D_i = 1) - E(y_i \mid D_i = 0) = \text{ATET} + \text{Selection Bias}$$

$$\widehat{\text{ATET}} = \bar{y}_1 - \bar{y}_0$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \cdot D_i$$

$$\overset{\#n}{\sum y_i} = \overset{\#n_0}{\sum_{D_i=0} y_i} + \overset{\#n_1}{\sum_{D_i=1} y_i}$$

$$\hat{\beta}_1 = \bar{y}_1 - \bar{y}_0$$

Ex. 1 $$\hat{\beta}_1 = \frac{\widehat{Cov}(D,y)}{\widehat{Var}(D)} = \frac{\overline{D\cdot y} - \bar{D}\cdot\bar{y}}{\overline{D^2} - (\bar{D})^2}$$

$$\overline{D\cdot y} - \bar{D}\cdot\bar{y} = \frac{\cancel{0\cdot\sum_{D_i=0} y_i} + 1\cdot\sum_{D_i=1} y_i}{n} - \frac{n_1}{n}\cdot\frac{\sum_{D_i=0} y_i + \sum_{D_i=1} y_i}{n} =$$

$$\frac{(n_0+\cancel{n_1})\cdot\sum_{D_i=1} y_i - n_1\cdot\sum_{D_i=0} y_i + \cancel{n_1\cdot\sum_{D_i=1} y_i}}{n^2} =$$

$$\frac{n_0\cdot\sum_{D_i=1} y_i - n_1\sum_{D_i=0} y_i}{n^2}$$

$$\overline{D^2} - (\bar{D})^2 = \frac{n_1}{n} - \left(\frac{n_1}{n}\right)^2 = \frac{n_1(n_0+n_1) - n_1^2}{n^2} = \frac{n_1 n_0}{n^2}$$

$$\hat{\beta}_1 = \frac{n_0\cdot\sum_{D_i=1} y_i - n_1\sum_{D_i=0} y_i}{n^2} : \frac{n_1 n_0}{n^2} =$$

$$= \frac{\sum_{D_i=1} y_i}{n_1} - \frac{\sum_{D_i=0} y_i}{n_0} = \bar{y}_1 - \bar{y}_0$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{D} = \frac{\sum_{D_i=0} y_i + \sum_{D_i=1} y_i}{n} - \left(\frac{\sum_{D_i=1} y_i}{n_1} - \frac{\sum_{D_i=0} y_i}{n_0} \right) \cdot \frac{n_1}{n} =$$

$$= \frac{1}{n}\left(1 + \frac{n_1}{n_0}\right) \cdot \sum_{D_i=0} y_i = \frac{\sum_{D_i=0} y_i}{n_0} = \bar{y}_0$$

$$\hat{\beta}_0 = \bar{y}_0 \qquad \hat{\beta}_1 = \bar{y}_1 - \bar{y}_0$$

Ex. 2 $\quad Var(\hat{\beta}_1) = \dfrac{\sigma^2}{\sum (D_i - \bar{D})^2} =$ $\qquad \bar{D} = \hat{\alpha} = \dfrac{n_1}{n}$

$$= \frac{\sigma^2}{\sum (D_i - \alpha)^2} = \frac{\sigma^2}{n_1(1-\alpha)^2 + n_0(\alpha^2)} =$$

$$= \frac{\sigma^2}{n\alpha(1-\alpha)^2 + n(1-\alpha)\alpha^2} = \frac{\sigma^2}{n\alpha(1-\alpha)}$$

$$\alpha = 1/2$$

DID:

D.V. for treatment

D.V. for $t=1$

$$y_{it} = \beta_0 + \beta_1 X_i + \beta_2 \cdot z_t + \delta \cdot X_i \cdot z_t + \varepsilon_{it}$$

DID:

$$\Delta y_i = \beta_0 + \delta \cdot X_i + \varepsilon_i$$

Matching

$ATET = E(y_i(1) - y_i(0) \mid D_i = 1)$

$y_{1,k}$ – Treatment group $\qquad y_i(1) \mid D_i = 1$

$y_{0,k}$ – control group $\qquad$ matched: $\hat{y}_i(0) \mid D_i = 1$

① Simple matching

$$\Delta^M = \sum_k w_k (\bar{y}_{1,k} - \bar{y}_{0,k})$$

② NN - matching $\qquad i \leftrightarrow j$

$$A_i = \{ j \mid \min_j \| x_i - x_j \| \}$$

③ Propensity Score Matching

$$P(D_i = 1 \mid X_i^{(1)}, \ldots, X_i^{(k)})$$

↑ logit / probit