Bootstrap

1) Sample size is small

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

"large enough"?

Solutions ↗ collect more data

↘ Bootstrap

$X_1, \ldots, X_n \quad \bar{X}$

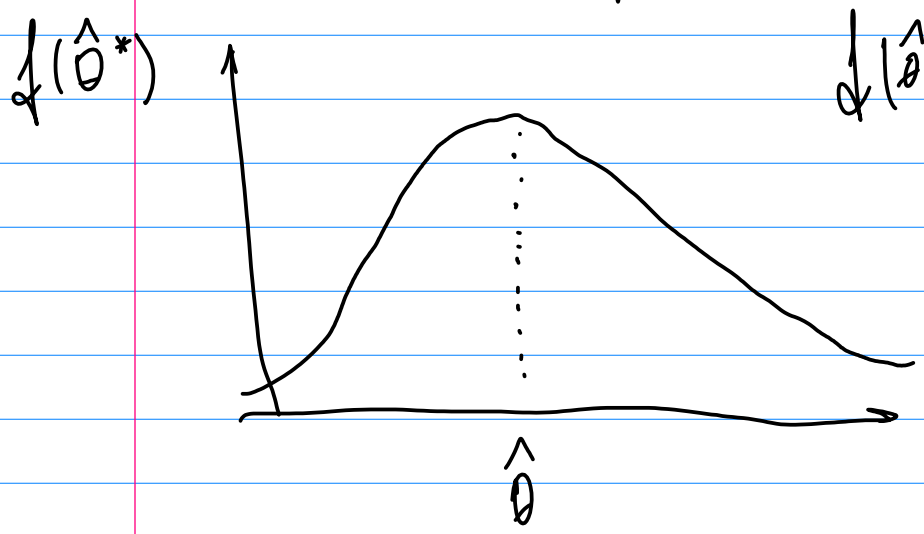
$B = 100, \quad i = \overline{1, B}$

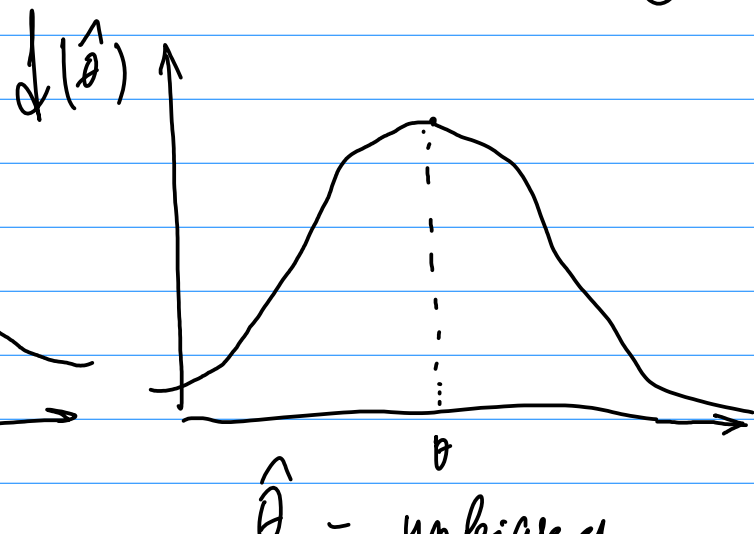
$X_1^*, \ldots, X_n^* \Rightarrow \bar{X}_i^*$

sampling with replacement

$\bar{X}^* \sim$ Bootstrap distr.

2) Hard compute the distribution analitically





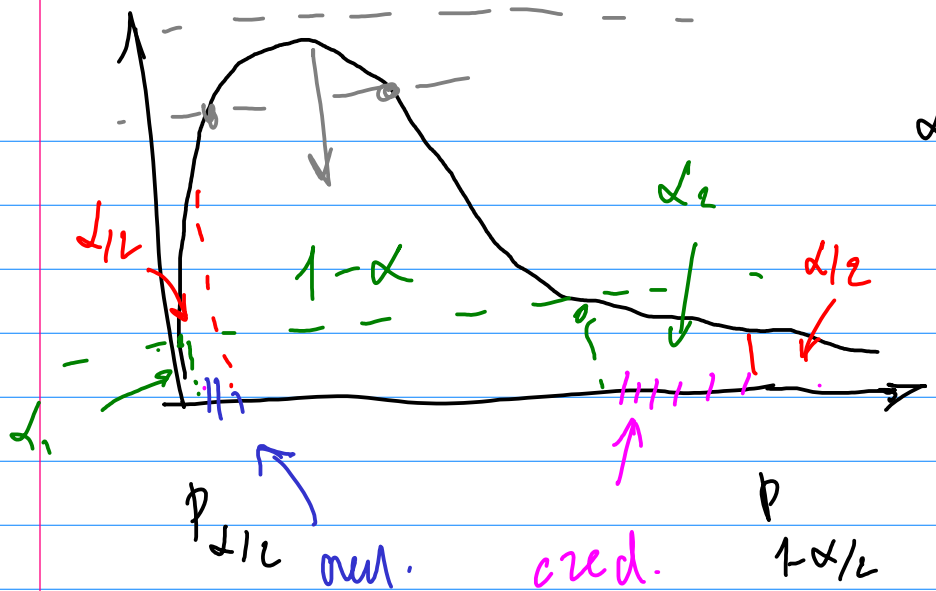
$\hat{\theta}$ - unbiased

$E(\hat{\theta}^*) = \hat{\theta}$

$E(\hat{\theta}) = \theta$

- centered at $\hat{\theta}$ not θ
- variance and skewness

$Var(\hat{\theta}^*) \approx Var(\hat{\theta})$



α-CI :

ETI

(equal tail int)

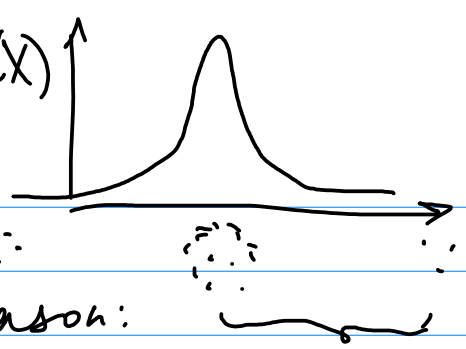
HPI

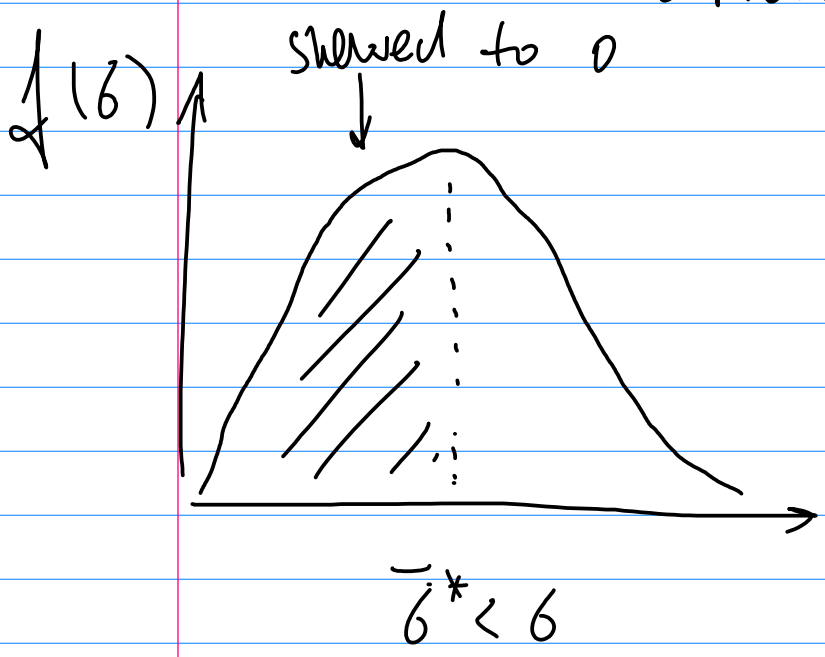
$\alpha_1 + \alpha_2 = \alpha$

own. is higher ($\notin$ ETI)

cred. is smaller ($\in$ ETI)

Inherently biased parameters





Reason:

$X_1^*, \ldots, X_n^*$

With replacement

Balanced Bootstrap

add $(\theta - \bar{\theta}^*)$

to each bootstrapped value

Bootstrap Int.:

1) Percentile CI

$$\left[\hat{\theta}^*_L ; \hat{\theta}^*_u\right]$$

2) Basic CI

(numerical)

$$\left\{\hat{\theta} \pm \left(\hat{\theta} - \hat{\theta}^*\right)\right\}$$

$$\tilde{\theta} \quad d^*$$

3) t - bootstrap

instead d^* calculate t^*

$$t^* = \frac{\hat{\theta} - \hat{\theta}^*}{se^*}$$

$$\left\{\bar{X} \pm z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}\right\}$$

$$\downarrow$$

$$t^*_L \text{ and } t^*_u$$

$$S = \frac{E[k - k_1]}{6}$$