Bootstrapping

1) Hard to analitically derive
2) When the sample size is small

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

n should be $\underbrace{\text{"large enough"}}_{?}$

$X_1, \ldots, X_n \sim \text{Norm}$

Solution → Get more data (?)
→ bootstrapping

$X_1, \ldots, X_n$

→ Generate bootstrap sample of size n (with replacement)

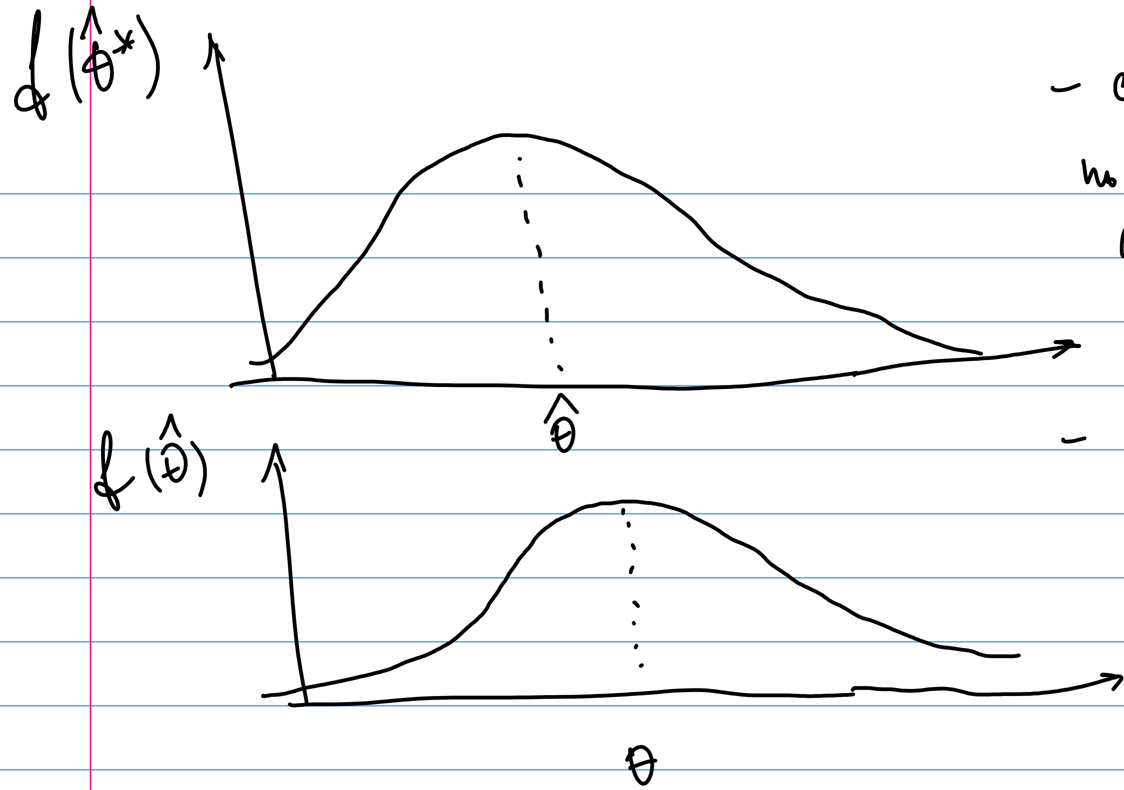
Variance of $\bar{X}$
= Variance of $\bar{X}^*_i$

→ Calc. $\bar{X}^*_i$ $\quad i = 1, B$

B - # bootstrap samples

$B = 100$

→ $\bar{X}^* \sim$ Bootstrap dist.



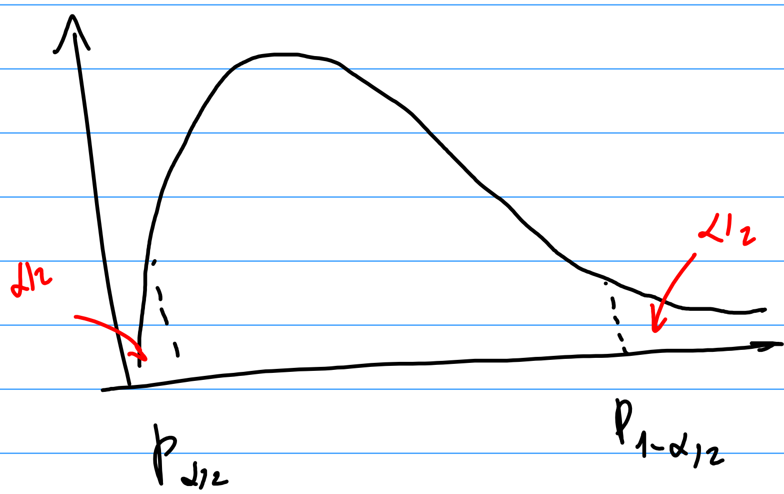
- center ✗

  no centered at $\hat{\theta}$

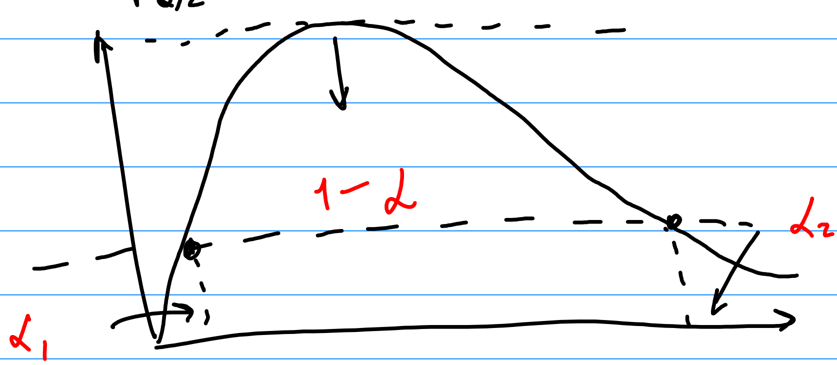
  not θ

- spread ✓

  and skewness



CI:

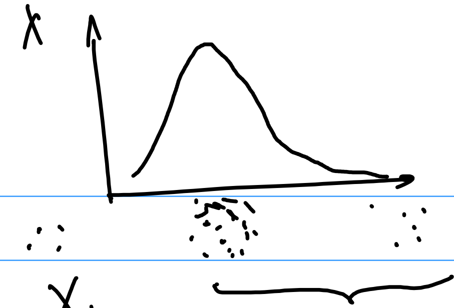


HDI:

$\alpha_1 + \alpha_2 = \alpha$

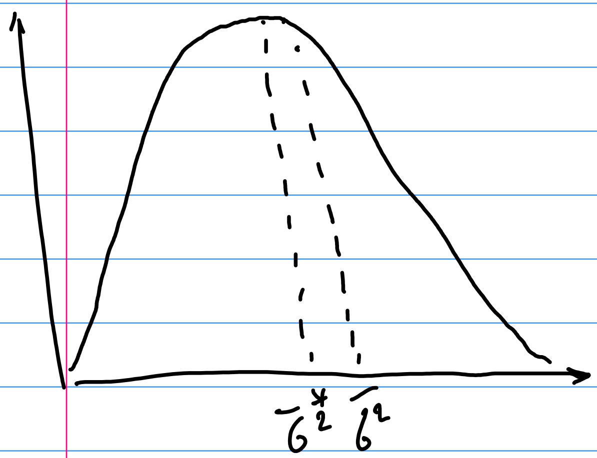
- in case of skewed dist => narrower CI

Inherently Biased Parameters



$X_1, \ldots, X_n$

- new samples with replacement



Solution: Add $(\bar{\sigma}^2 - \bar{\sigma}^{*2})$ to each bootstrapped value

Balanced Bootstrap

1. Percentile Bootstrap

$$[\hat{\theta}^*_l ; \hat{\theta}^*_u]$$

2. Basic Bootstrap

(Numerical)

$$[\hat{\theta} + (\underbrace{\hat{\theta} - \hat{\theta}^*_u}_{d^*}) ; \hat{\theta} + (\hat{\theta} - \hat{\theta}^*_l)]$$

3. t - bootstrap

instead of d^*

calculate $t^* = \dfrac{\hat{\theta}^* - \hat{\theta}}{se^*}$

$$\left(\bar{X} \pm z_{\alpha/2} \cdot \frac{se(\bar{X})}{\sqrt{n}}\right)$$

$$\left(\bar{X} \pm t^*_{\alpha/2} \cdot \frac{se(\bar{X})}{\sqrt{n}}\right)$$

$$S = \frac{E[R - R_f]}{\sigma}$$