Power Analysis

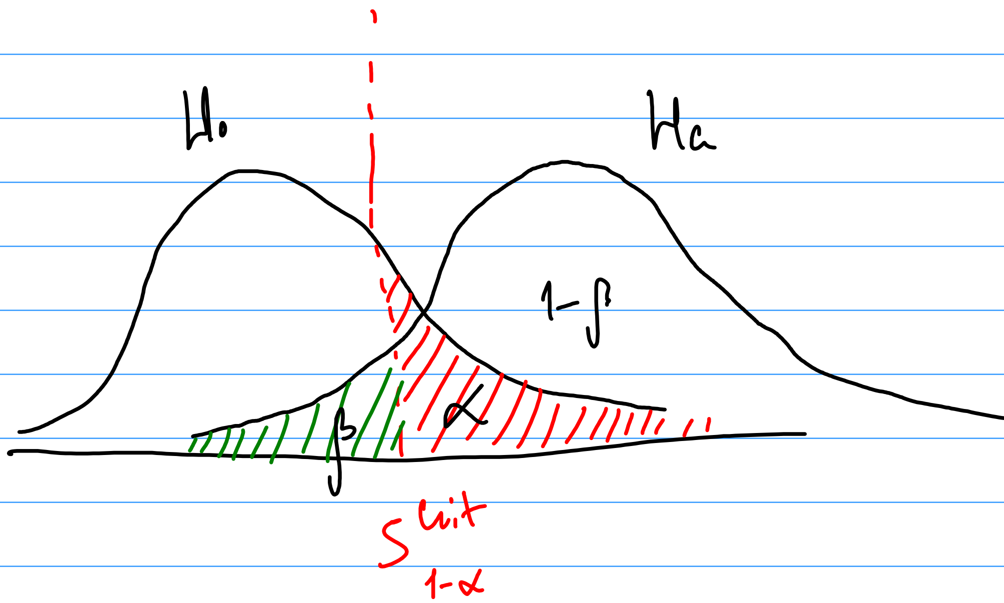
Type I error : $P(S^{obs} \in P^{crit} \mid H_0) = \alpha$

Type II error : $P(S^{obs} \notin P^{crit} \mid H_a) = \beta$

$\Updownarrow$

Power: $1-\beta$

α and β are inversely related:



Hypothesis testing:

- Significance level
- Power
- Effect size (Cohen's d)
- Sample size

Sample size

CLT: $$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

$$P\left(z_{\alpha/2} \le \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \le z_{1-\alpha/2} \right) = 1 - \alpha$$

$$P\left(\bar{X} + \underbrace{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}} \le \mu \le \bar{X} + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right) = 1 - \alpha$$

Margin of error (E)

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le E$$

$$n \geq \frac{z_{\alpha/2}^2 \cdot \sigma^2}{E^2}$$

For proportions: $$\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0,1)$$

$$\hat{p} + \underbrace{z_{\alpha/2} \cdot \sqrt{p(1-p)/n}}_{ME} \le p \le \hat{p} + z_{1+\alpha/2} \cdot \sqrt{p(1-p)/n}$$

$$z_{\alpha/2} \cdot \sqrt{p(1-p)/n} \le E$$

$$n \geq \frac{z_{\alpha/2}^2}{E^2} \cdot p(1-p)$$

Sample → with replacement

$X_1 = 2 \quad w.p. \; p_1$

$X_2 = 2 \quad w.p. \; p_1$

$\cdots$

Sample → w/o replacement

$X_1 = 2 \quad w.p. \; 1/3$

$[2, 4, 8]$

$X_2 = 2 \quad w.p. \; 0$

$$Cov(X_i, X_j) \neq 0$$

$$Cov(X_i, X_j) = -\frac{\sigma^2}{N-1}$$

n - sample size

N - population size

se(.)	with replacement	w/o replacement
$\bar{x}$	$\sqrt{\sigma^2/n}$	$\sqrt{\sigma^2/n\left(1-\frac{n-1}{N-1}\right)} \approx \sqrt{s^2/n\left(1-\frac{n}{N}\right)}$
$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$\sqrt{\frac{p(1-p)}{n}\left(1-\frac{n-1}{N-1}\right)} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}\left(\frac{n}{n-1}\right)\cdot\left(1-\frac{n}{N}\right)}$

Percentage error:

$$E = R \cdot \mu \quad \leftarrow \text{percentage error}$$

$$c = \sigma / \mu$$

$$n = \frac{\sigma^2 \cdot z^2}{E^2} = \frac{\sigma^2/\mu^2 \cdot z^2}{R^2} = \frac{c^2 \cdot z^2}{R^2}$$

for $\hat{p}$:

$$n = \frac{z^2 \cdot (1-\pi)}{R^2 \cdot \pi}$$