

$\{X_1, \dots, X_N\}$

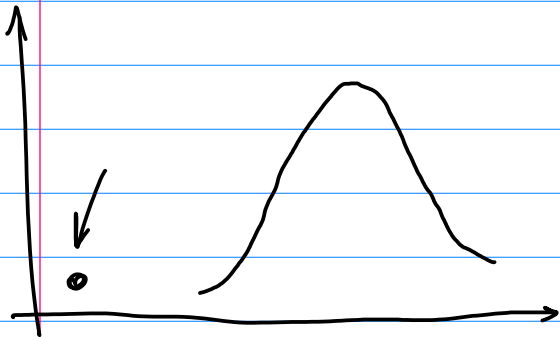
## Jackknife

1) Leaving-one-out samples

$$X_i = \{X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_N\}$$

# subsamples = N

$$2) \hat{\theta}_i^{jk} \quad i = \overline{1, N}$$



## Non-parametric Bootstrap

1) Subsamples (#  $B = 1000$ )

sampling with replacement

$$2) \hat{\theta}_i^B, \quad i = \overline{1, B}$$

## Parametric Bootstrap

1) Assume  $X_i$  is  
draw  $\text{Law}(\hat{\theta})$   
 $\hat{\theta}$

2) Subsamples of size  $n$   
are drawn from  
 $\text{Law}(\hat{\theta})$

## Non-parametric Bootstrap

(+)

Easy to implement

- multiple parameters
- covariance estimation
- predictions

(-)

1 $n$ -sample inference

- small sample:  $\text{Var}_{\text{sample}} < \text{Var}_{\text{pop}}$

Pair Boots trap:

$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

$(x_i, y_i)$

$$\varepsilon_i \sim \text{Norm}(0, \sigma^2)$$

1) Resamples

indexes  $1, \dots, N$  resamples

with replacement

$$2) \hat{\beta}_{OLS, i}^B$$

$$i = \overline{1, B}$$

$$B = 1000$$

Fixed  $X$  - bootstrap

\* Assump<sup>1, 0, 2</sup>

(Bootstrap: in residuals)

$$p(\hat{\varepsilon}_i) = 1/N$$

$$\hat{y}_i = f(x, \hat{\beta}_{OLS}) \Rightarrow \hat{\varepsilon}_i = y_i - \hat{y}_i$$

$\uparrow$  fixed

Sub-samples:

$$y_i^B = f(x, \hat{\beta}_{OLS}) + \varepsilon_i^B$$

sampling with  
replace from  $\hat{\varepsilon}$   
(non-param.)

parametric  
bootstrap  
version

$$\hat{\beta}_{OLS, i}^B$$

$$\hat{\varepsilon}_i^B \sim N(0, \hat{\sigma}_{\varepsilon}^2)$$

# Wild Bootstrap (Wu - Bootstrap)

1)  $\hat{y}_i = f(X, \hat{\beta}_{OLS}) \Rightarrow \hat{\varepsilon}_i$  drawn from with mean 0 var 1

2)  $y_i^b = f(X, \hat{\beta}_{OLS}) + \frac{t_i^* \hat{\varepsilon}_i}{\sqrt{1 - h_{ii}}}$   
↖ leverage

$H = X(X'X)^{-1}X'$  - projection (hat matrix)

$h_{ii} = x_i^T (X'X)^{-1} x_i$  diag(H) or leverage

non-parametric  $t_i^*$  sampling

with replacement from a

$$a_i = \frac{\hat{\varepsilon}_i - \bar{\hat{\varepsilon}}}{\sqrt{1/n \sum (\hat{\varepsilon}_i - \bar{\hat{\varepsilon}})^2}}$$