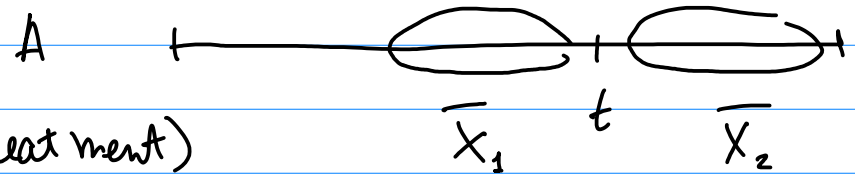
A|B Testing

1) Time Effect

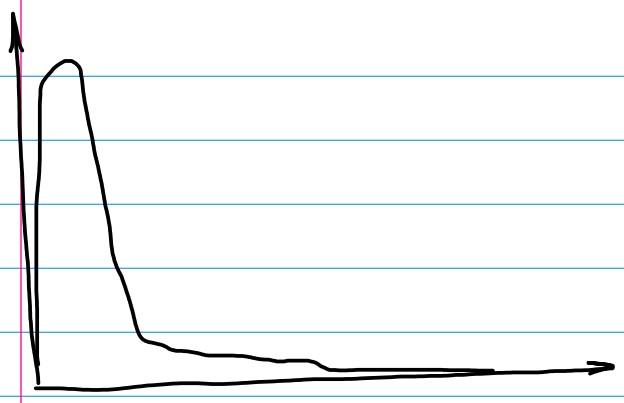


(treatment) $\bar{X}_1$ $\bar{X}_2$ $\bar{X}_1 \overset{?}{\neq} \bar{X}_2$

(control) $\bar{Y}_1$ $\bar{Y}_2$

$\bar{X}_1 = \bar{Y}_1$ $\bar{X}_2 \overset{?}{=} \bar{Y}_2$

2) Skewed



⊖ Remove outliers

⊕ Transformation

- Logarithmic

(Normality test ?)

- Box-Cox Transformation

3) Too big data

- "Bucketing"

$$\bar{x}_j = \sum_{i=1}^{N_j} x_{ji}$$

N_j # obs in j bucket

$$\frac{s^2}{N} \approx \frac{s_b^2}{B}$$

$j = \overline{1, B}$

Types of Metrics

1) User - level $U = \frac{\sum X_i}{N}$ $\frac{3 + \dots + 5}{1 + \dots + 1}$

2) Ratio $R = \frac{\sum X_i}{\sum Y_i}$ $\frac{2 + \dots + 2}{10 + \dots + 3}$

X, Y - r.v. $\Rightarrow$ X/Y - r.v.

1. Bootstrapping
2. Linearization $E(R) \approx E(Z)$; $Var(R) \approx Var(Z)$

   Taylor expansion of X/Y at $(E(X), E(Y))$

$$Z = \frac{E[X]}{E[Y]} + \frac{1}{E[Y]}\left(X - \frac{E[X]}{E[Y]} \cdot Y\right)$$

3. Delta - method

Decision about H_0	Hypothesis H_0 True	Hypothesis H_0 False
Not reject	TN (confidence) $1-\alpha$	FN (Type II error) β
Reject	FP (Type I error) α	TP (power) $1-\beta$

t-test

- normality
- $\sigma_x = \sigma_y = \sigma_p$

$n_x = n_y = n$

$$t = \frac{\bar{x} - \bar{y}}{s_p \cdot \sqrt{\frac{2}{n}}} \sim t_{2n-2} \qquad s_p = \sqrt{\frac{s_x^2 + s_y^2}{2}}$$

$n_x \neq n_y$

$$t = \frac{\bar{x} - \bar{y}}{s_p \cdot \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \sim t_{n_1 + n_2 - 2}$$

Welch test

- normality
- $\sigma_x \neq \sigma_y$

$n_x \neq n_y$

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} \sim t_{\wedge \alpha}$$

Mann - Whitney's Test

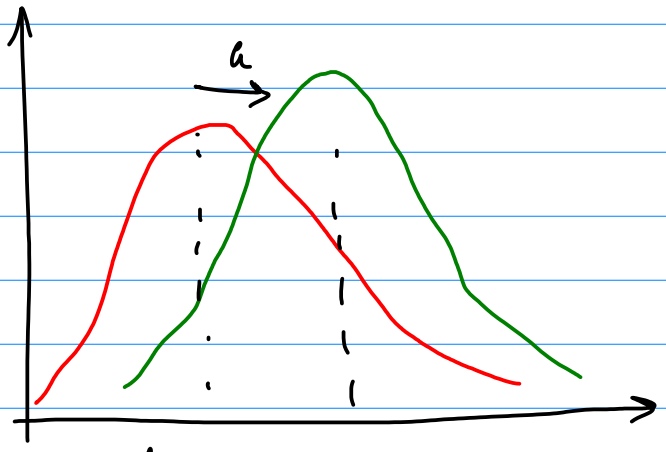
H_0: is there a difference in rank sums

$\Updownarrow$

MW checks for a shift

$H_0: f_x(x) = f_y(y)$

$H_1: f_x(x) = f_y(y+a)$



Exact:

1) $U_x = R_x - \frac{n_x(n_x+1)}{2}$

R_x - sum of all ranks

$U_y = R_y - \frac{n_y(n_y+1)}{2}$

2) $\min(U_x, U_x)$

3) compare to U_{crit}

Asymptotic:

$$U = \sum_j \sum_i S(x_i, y_j)$$

$$S(x,y) = \begin{cases} 1, & x > y \\ 0, & x \le y \end{cases}$$

$$z = \frac{U - m_U}{\sigma_U} \qquad m_U = \frac{n_x n_y}{2}$$

$$\sigma_U = \sqrt{\frac{n_x n_y (n_x + n_y + 1)}{12}}$$

Levene's Test (one-way ANOVA)

$H_0: \sigma_1 = \sigma_2 = \dots = \sigma_k$

$H_a: \exists\, i, j \quad \sigma_i = \sigma_j$

$$W = \frac{\text{between group var}}{\text{within group var}} = \frac{(N-k)\cdot \sum N_i (\bar{Z}_{i\cdot} - \bar{Z}_{\cdot\cdot})^2}{(k-1)\cdot \sum\sum (Z_{ij} - \bar{Z}_{i\cdot})^2} \sim F(k-1; N-k)$$