1. Problem

I have a sample X_1, \ldots, X_{80} . I generate one naive bootstrap sample X_1^*, \ldots, X_{80}^* .

What is the probability that the first observation will be present in the bootstrap sample more than 2 times?

2. Problem

We have data of an AB experiment: $\bar{X}_a = 6$, $\bar{X}_b = 5.4$, $n_a = 13$, $n_b = 11$, $\sum (X_i^a - \bar{X}_a)^2 = 860$, $\sum (X_i^b - \bar{X}_b)^2 = 850$.

Calculate the estimate of variance of $\bar{X}_a - \bar{X}_b$ for the Welch test.

3. Problem

I have five results of two runners A and B for the 5 km race:

20:26 (A), 19:53 (A), 20:59 (B), 20:28 (B), 20:56 (B), 19:29 (B), 19:48 (A).

Calculate Mann-Whitney statistic U_A that tests the null-hypothesis of equal distributions of time.

(The statistic U_A should positively depend on the ranks of the runner A).

4. Problem

I have 30 hypothesis with independent statistics. The null hypothesis for all 30 cases is actually true, but I don't know this.

I calculate all p-values. If the 2 lowest p-value are simultaneously lower than 0.01 I wrongly conclude that not all H_0 are true. Otherwise I correctly conclude that all H_0 are true.

What is the probability that I will get the wrong conclusion?

5. Problem

My target variable is binary and I wish minimal detectable effect equal to 0.04, probability of I-error not greater than 0.01, probability of II-error not greater than beta. The control and experimental group are of the same size equal to n.

Which minimal value of n is sufficient in the worst case?

6. Problem

Vasiliy loves to eat shaurma. He has 5 local shaurma dealers. Vasiliy bought 4 shaurmas from each dealer and measured their weight. He would like to test the hypothesis that mean weight is the same for all dealers.

Total sum of squares is 600, between sum of squares is 100 Calculate the F -statistic to test the hypothesis.