Jackknife

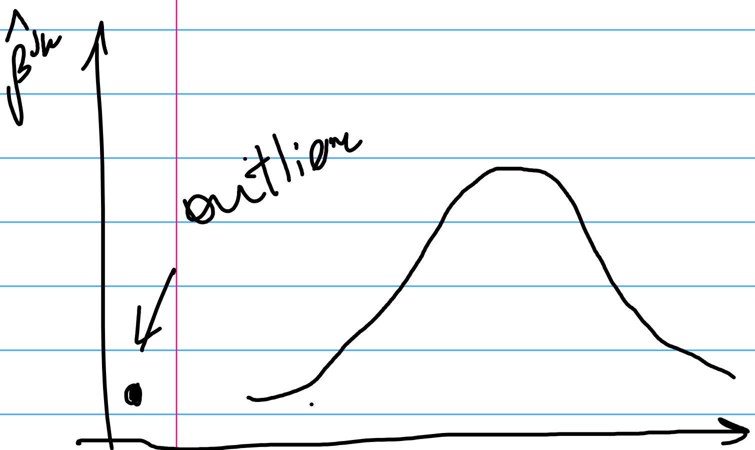
$\{x_1, \dots, x_N\}$

1) Leave-one-out

$X_i = \{x_1, \dots, x_{i-1}, x_{i+1}, x_N\}$

# of subsamples = N

2) $\hat{\theta}_i^{JK}$ $\quad i = \overline{1, N}$



Non-Parametric Bootstrap

1) Subsamples ($B = 1000$)

sampling with replacement

2) $\hat{\theta}_i^{B}$ $\quad i = \overline{1, B}$

Parametric Bootstrap

Assume $X_i \sim Law(\Theta)$

Calculate $\hat{\Theta}$

1) Subsamples

$X_i^b \sim Law(\hat{\Theta})$

2) $\hat{\theta}_i^{B}$ $\quad i = \overline{1, B}$

Non-Parametric

(+) Easy to compute and easily extended

- multiple coefficent
- covariation estimation
- predicting

(−) Only in-sample inference

- when n is small $Var_{sample} < Var_{pop}$

Pair Bootstrap

$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$
$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$$

1) (x_i, y_i) - pairs are fixed

subsamples:

resample indexes $\{1, N\}$ with replacement

2) $\hat{\beta}^B_{OLS}$

Fixed X bootstrap (Bootstrap in residuals)

1) $\hat{y}_i = f(X, \hat{\beta}_{OLS}) \Rightarrow \hat{\varepsilon} = y_i - \hat{y}$

↑ fixed

2) $y_i^B = f(X, \hat{\beta}_{OLS}) + \hat{\varepsilon}_i^B$

Non-parametric

draw samples with replacement from $\hat{\varepsilon}$

Assume: $p(\hat{\varepsilon}_i) = \frac{1}{N}$

Parametric

$\hat{\varepsilon}_i^B \sim Norm(0, \hat{\sigma}_\varepsilon^2)$

Wild Bootstrap (Wu - bootstrap)

1) $\hat{y}_i = f(X, \hat{\beta}_{OLS}) \Rightarrow \hat{\varepsilon} = y_i - \hat{y}_i$

2) $y_i = f(X, \hat{\beta}_{OLS}) + \dfrac{t^* \cdot \hat{\varepsilon}_i}{\sqrt{1 - h_{ii}}}$

leverages

$H = X(X'X)^{-1}X^T$ — projection matrix (hat matrix)

$h_{ii} = x_i^T (X'X)^{-1} x_i$ — leverages

$\mathrm{diag}(H)$

t^*

Parametric: any distribution mean 0
var 1

Non-parametric: t_i^* sampled with replacement a

$$a_i = \frac{\hat{\varepsilon}_i - \bar{\hat{\varepsilon}}_i}{\sqrt{\frac{1}{n}\sum(\hat{\varepsilon}_i - \bar{\hat{\varepsilon}})^2}}$$