Binary Choice Models

$$y_i = G(x_i^T \theta) + u_i$$

$G(.)$: $\Lambda(z) = \dfrac{e^z}{1+e^z} = \dfrac{1}{1+e^{-z}}$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$$

1 Likelihood Ratio test

$$LR = -2(\ln L_R - \ln L_{UR}) \sim \chi^2(q)$$

H_0 : q - linear restrictions
$\beta_1 = \dots = \beta_q = 0$

H_a : $\exists\, \beta_i \neq 0$

2 pseudo - R^2 (R^2 McFaden)

$\ln(L)$

$\ln(L_0)$ - log of max of likelihood function for model with constant only

$$\text{pseudo-}R^2 = 1 - \frac{\ln(L)}{\ln(L_0)} \in [0, 1]$$

$$\text{pseudo-}R^{2\,(*)} = 1 - \frac{1}{1 + \frac{2(\ln L - \ln L_0)}{n}}$$

Example 1

X - years of experience

gender - dummy (gender = 1 for male)

Black - dummy (= 1 for afroamericans)

	Model 1	Model 2	Model 3
X	-	0,49 (.)	0,49 (.)
Gender	-	-	0,15 (.)
Black	-	-	-0,32 (.)
Const	-0,32 (0,2)	-1,02 (.)	-0,9 (.)
ln L	-68	-62	-61
pseudo-R^2	0	0,09	0,1

(a) $\text{pseudo-}R^2 = 1 - \dfrac{\ln(L)}{\ln(L_0)} = 1 - \dfrac{-68}{-68} = 0$

$\text{pseudo-}R^2 = 1 - \dfrac{62}{68} = 0{,}09$

$\text{pseudo-}R^2 = 1 - \dfrac{61}{68} = 0{,}10$

(b) $LR = -2(\ln L_R - \ln L_{UR}) =$

$= -2(-68 - (-61)) = 14$

$LR \sim \chi^2(3)$

$LR_{crit,\,5\%} = 7{,}81$

(c) $LR = -2(-62 + 61) = 2$

$LR_{crit} = \chi^2(2;\, 0{,}95) = 5{,}99$

(d) Marginal effect of X for average worker (Model 2) $(\bar{X} = 1{,}4)$

$$P(y_i = 1) = \frac{1}{1 + e^{-z_i}}$$

$$\frac{d\hat{P}}{dx} = \frac{e^{-(\hat{\beta}_1 + \hat{\beta}_2 x)}}{\left(1 + e^{-(\hat{\beta}_1 + \hat{\beta}_2 x)}\right)^2} \cdot \hat{\beta}_2 =$$

$$\left\{ \begin{array}{l} \hat{\beta}_1 = -1{,}02 \\ \hat{\beta}_2 = 0{,}49 \\ \bar{x} = 1{,}4 \end{array} \right\} = 0{,}12$$

Probit Model

$$P(y_i = 1) = \Phi(z_i) \quad , \; z_i = \beta_1 + \beta_2 x_{2i} + \ldots + \beta_k x_{ki}$$

$$\frac{\partial P(y_i = 1)}{\partial x_j} = \Phi'(z_i)\,\beta_j = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \cdot \beta_j$$

Example 2

Lectures = number of lectures attended

Male = dummy

	Probit
Lectures	0,2 (0,03)
Male	− 0,5 (0,02)
Lectures × Male	− 0,05 (0,02)
Constant	− 1,0 (0,12)

(a) all coef. are statist. significant
(5% sign. level)

(b) Student A: Lectures = 10, male = 1

Student B: Lectures = 10, male = 0

$\hat{z}_A = 0{,}2 \cdot 10 - 0{,}5 \cdot 1 - 0{,}05 \cdot 10 \cdot 1 - 1 = 0$

$\hat{P}_2(y_A = 1) = \Phi(\hat{z}_A) = \Phi(0) = 0{,}5$

$\hat{z}_B = 0{,}2 \cdot 10 - 0{,}5 \cdot 0 - 0{,}05 \cdot 10 \cdot 0 - 1 = 1$

$\hat{P}_2(y_B = 1) = \Phi(1) = 0{,}84$

(c) Marginal effect from lectures for student B

$$\frac{\partial \hat{P}(y_i = 1)}{\partial \text{Lectures}} = \frac{1}{\sqrt{2\pi}} e^{-\hat{z}^2/2} \cdot \hat{\beta}_{Lect.} =$$

$$\frac{1}{\sqrt{2\pi}} e^{-1^2/2} \cdot 0{,}2 = 0{,}24 \cdot 0{,}2 = 0{,}048$$