Stat 4 MR:

Can you see? hear?

sample size calculation

Sample A $\quad Y_1, Y_2 \ldots Y_{n_A}$
Sample B $\quad X_1, X_2 \ldots X_{n_B}$

many cases.

a) $Y_i \in \{0,1\} \quad X_j \in \{0,1\}$
b) $Y_i \in \mathbb{R} \quad X_j \in \mathbb{R}$

$H_A: \mu_X - \mu_Y \neq 0 \qquad H_A: \mu_X - \mu_Y > 0 \ldots$

subcases.

n_A? n_B?

H_0
H_A

	H_0 is not rej	H_0 is rej
H_0 is true	:)	I type error
H_A is true	II type error	:)

$\alpha = P(H_0 \text{ is rej} \mid H_0 \text{ is true})$
$\beta = P(H_0 \text{ is not rej} \mid H_A \text{ is true})$

Problem ①.

exp. group $X_1, \dots, X_{n_B}$ $\qquad X_i \in \{0,1\}$

contr. group $Y_1 \dots Y_{n_A}$ $\qquad Y_i \in \{0,1\}$

All var-s are indep.

$X_i \sim \text{Bernoulli}(p_B)$

$Y_i \sim \text{Bernoulli}(p_A)$

„1" - good

„0" - bad

My"

$H_0: p_B = p_A$

$H_A: p_B > p_A$

Budget const $\quad n_A + n_B = n$ fixed

Choose „best" n_A and n_B?

$\alpha\downarrow \;\leftrightarrow\; \beta\uparrow$

for desired value of α the value of β is minimized.

Let's recall the ord. test.

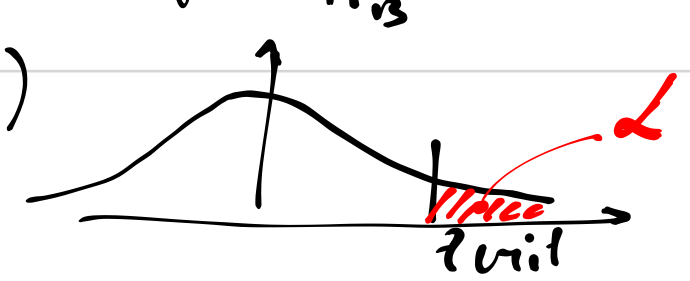
$E(\hat p_B - \hat p_A) = 0$ under H_0

$$\xi = \frac{\hat p_B - \hat p_A - 0}{se(\hat p_B - \hat p_A)} \begin{cases} > \boxed{z_{crit}} & \text{reject } H_0 \\ \le z_{crit} & \text{do not reject } H_0 \end{cases}$$

α

$$se(\hat p_B - \hat p_A) = \sqrt{\hat{Var}(\hat p_B - \hat p_A)} = \sqrt{\hat{Var}(\hat p_B) + \hat{Var}(\hat p_A)} = \sqrt{\frac{\hat p_B(1-\hat p_B)}{n_B} + \frac{\hat p_A(1-\hat p_A)}{n_A}}$$

under H_0: $\xi \overset{asy}{\sim} N(0;1)$



where is β?

$$H_A: p_B - p_A = \Delta \qquad \Delta > 0$$

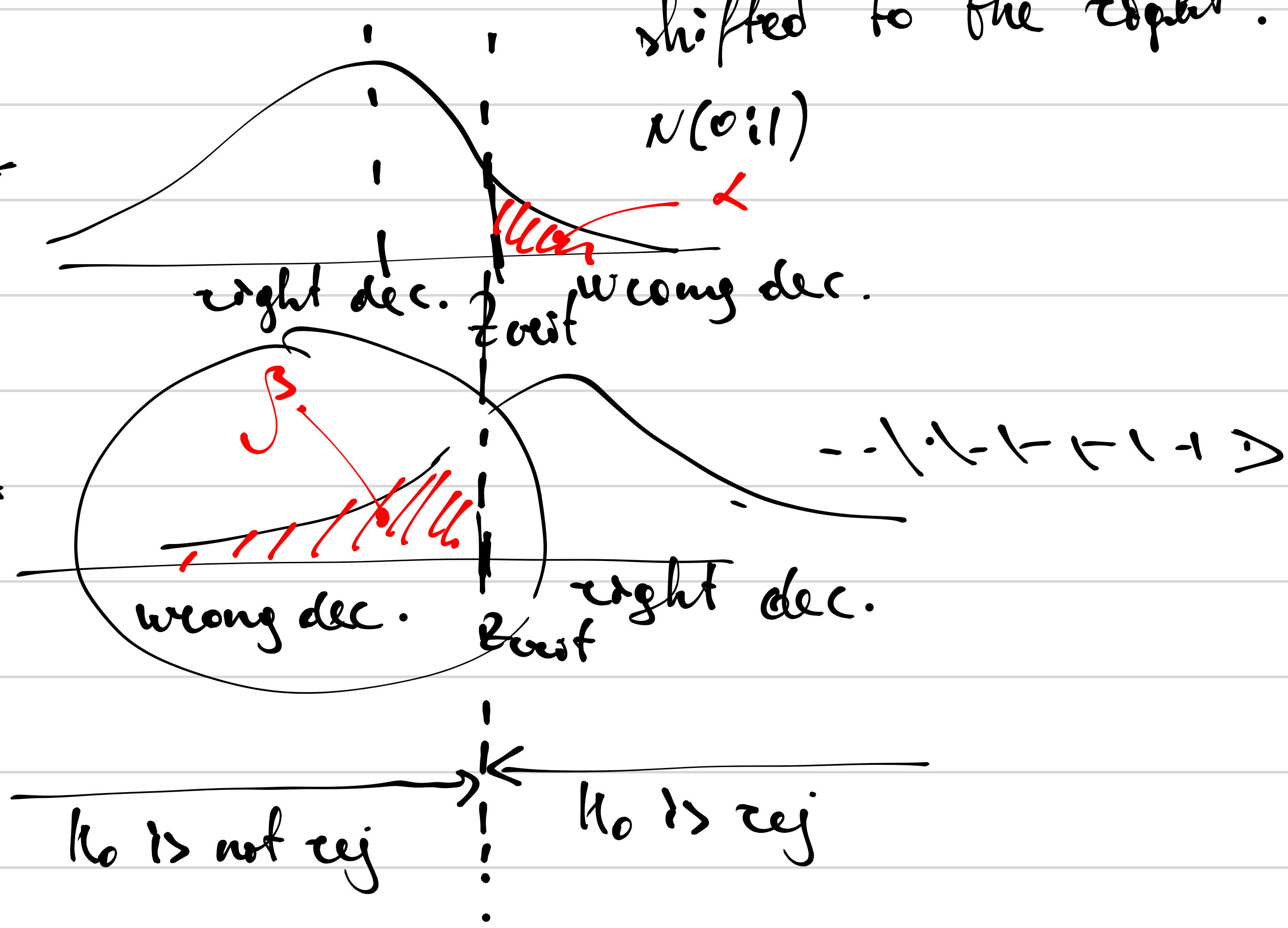
$$\xi = \frac{\hat{p}_B - \hat{p}_A - 0}{se(\hat{p}_B - \hat{p}_A)}$$

(!) subtract 0 but you should subtract Δ.

shifted to the right.

under H_0

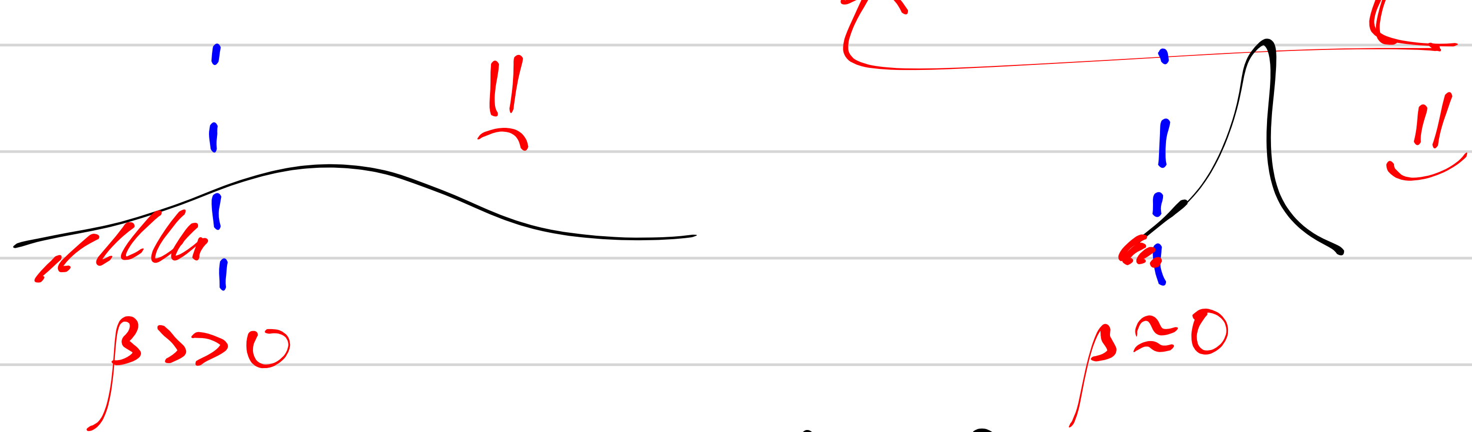
under H_A



$$\hat{Var}(\hat{p}_B - \hat{p}_A) = \frac{\hat{p}_B(1-\hat{p}_B)}{n_B} + \frac{\hat{p}_A(1-\hat{p}_A)}{n_A}$$

!!

!!



$$Var(\hat{p}_B - \hat{p}_A) = \frac{p_B(1-p_B)}{n_B} + \frac{p_A(1-p_A)}{n_A}$$

$$n_B + n_A = n$$

$$\left.\begin{aligned} n_B &= q \cdot n \\ n_A &= (1-q)\cdot n \end{aligned}\right\}$$

$$\min_q \; \frac{p_B(1-p_B)}{q\cdot n} + \frac{p_A(1-p_A)}{(1-q)\cdot n}$$

$$\min_q \frac{1}{n}\left[\frac{p_A(1-p_A)}{1-q} + \frac{p_B(1-p_B)}{q}\right]$$

$f(q)$

$$f'(q) = \frac{p_A(1-p_A)}{(1-q)^2} - \frac{p_B(1-p_B)}{q^2}$$

$$\frac{p_A(1-p_A)}{p_B(1-p_B)} = \frac{(1-q)^2}{q^2}$$

$$\frac{1-q}{q} = \frac{\sqrt{p_A(1-p_A)}}{\sqrt{p_B(1-p_B)}} = \frac{\sigma_A}{\sigma_B}$$

x	0	1
$P(X_i = x)$	$1-p_B$	p_B

$$E(X_i) = 0\cdot(1-p_B) + 1\cdot p_B = p_B$$

$$Var(X_i) = E(X_i^2) - E(X_i)^2 = p_B - p_B^2$$

$$\sigma_B = \sqrt{p_B(1-p_B)}$$

$$\frac{1-q}{q} = \frac{\sigma_A}{\sigma_B}$$

$$(1-q)\,\sigma_B = q\,\sigma_A$$

$$\boxed{q = \frac{\sigma_B}{\sigma_B + \sigma_A}}$$ // theor. answ.?

in practice?

- → guess
- → preliminary study. $\hat{q} = \frac{\hat{\sigma}_B}{\hat{\sigma}_A + \hat{\sigma}_B}$

use $\hat{q}$ in the main stage

Problem 2

! what is the case considered !

$$\begin{cases} \alpha \leq 0.05 \\ \beta \leq 0.20 \\ MDE = 0.1 = p_B - p_A \\ n_A = \frac{1}{2} n \\ n_B = \frac{1}{2} n \end{cases}$$

$H_0: p_B - p_A = 0$

$H_A: p_B - p_A \geq 0.1$

minimal detectable effect

$$\begin{cases} p_B - p_A \\ \mu_B - \mu_A \end{cases}$$

min n?

aim at the worst case.

$\alpha = 0.05$

$\beta = 0.20$

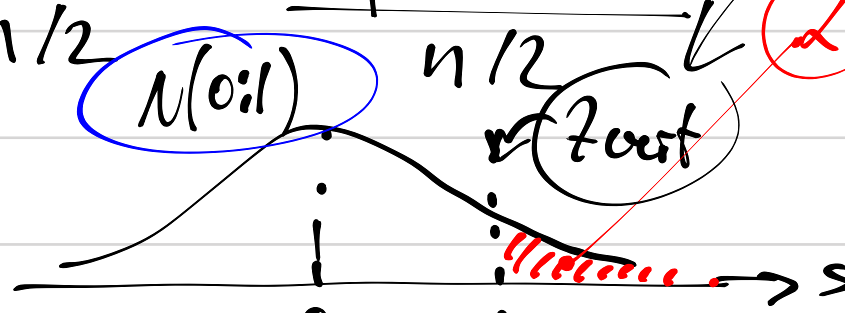
$H_0: p_B - p_A = 0$

$H_A: p_B - p_A = 0.1$

$$S = \frac{\hat{p}_B - \hat{p}_A - 0}{se(\hat{p}_B - \hat{p}_A)}$$

$$se(\hat{p}_B - \hat{p}_A) = \frac{\hat{p}_B(1-\hat{p}_B)}{n/2} + \frac{\hat{p}_A(1-\hat{p}_A)}{n/2}$$

under H_0

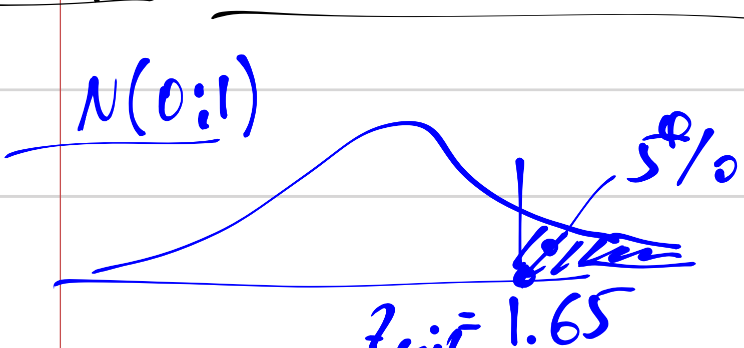


under H_A

n?

Step 1. Find $z_{crit}(\alpha)$. Step 2. $\boxed{P(S' < z_{crit} \mid H_A) = \beta}$

solve this for n.



Step 2

1.65 MDE

$$P\left(S' < z_{crit} \mid H_A: p_B - p_A = 0.1\right) = \beta$$

i subtract wrong value!

$$P\left(\frac{\hat p_B - \hat p_A - 0}{se(\hat p_B - \hat p_A)} < z_{crit} \mid H_A\right) = \beta = 0.2$$

$\not\sim N(0{:}1)$

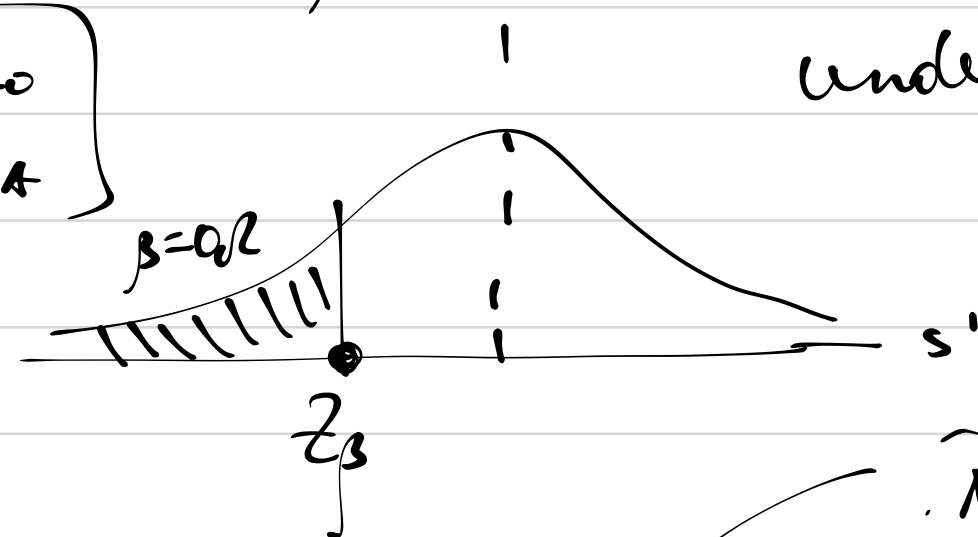
$$P\left(\frac{\hat p_B - \hat p_A - MDE}{se(\hat p_B - \hat p_A)} < z_{crit} - \frac{MDE}{se(\dots)} \mid H_A\right) = \beta$$

$p_B - p_A = MDE = 0.1$

S' $\quad \sim N(0{:}1)$

$S \sim N(0{:}1)$ under H_0
$S' \sim N(0{:}1)$ under H_A

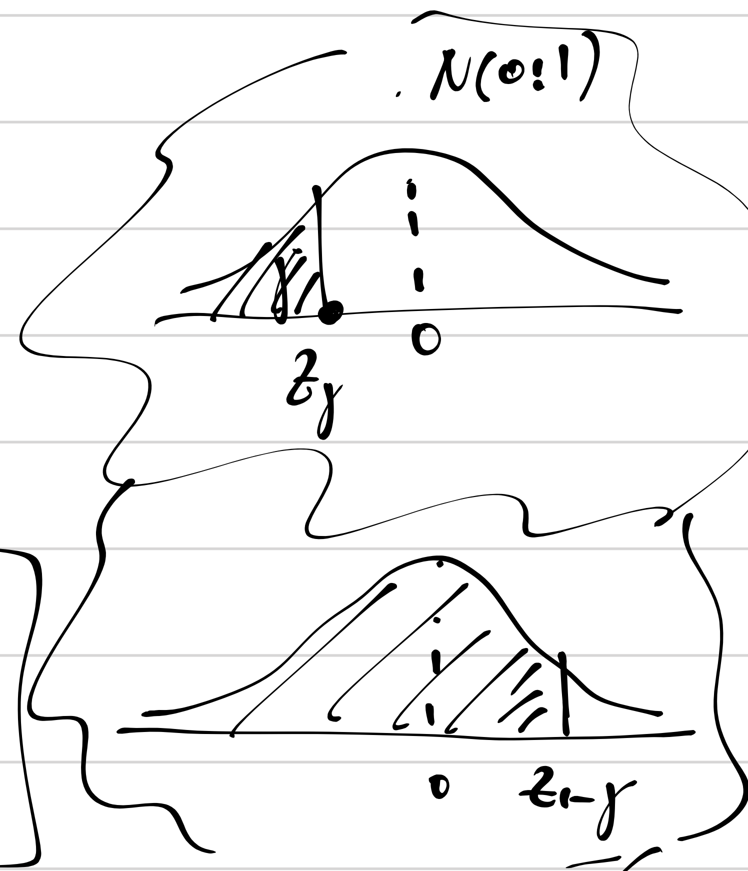
under H_A



$$z_{1-\alpha} - \frac{MDE}{se(\hat p_B - \hat p_A)} = z_\beta$$

$$z_{1-\alpha} - z_\beta = \frac{MDE}{se(\hat p_B - \hat p_A)}$$

$$\boxed{z_{1-\alpha} + z_{1-\beta} = \frac{MDE}{\sqrt{\frac{\hat p_B(1-\hat p_B)}{n/2} + \frac{\hat p_A(1-\hat p_A)}{n/2}}}}$$



$z_\gamma = -z_{1-\gamma}$

Goals: $\alpha, \beta, MDE, n.$

$$z_{1-\alpha} + z_{1-\beta} = \frac{\sqrt{n} \cdot MDE}{\sqrt{2} \cdot \sqrt{\hat p_B(1-\hat p_B) + \hat p_A(1-\hat p_A)}}$$

$$n = 2\left(\hat p_B(1-\hat p_B) + \hat p_A(1-\hat p_A)\right) \cdot (z_{1-\alpha} + z_{1-\beta})^2 / MDE^2$$