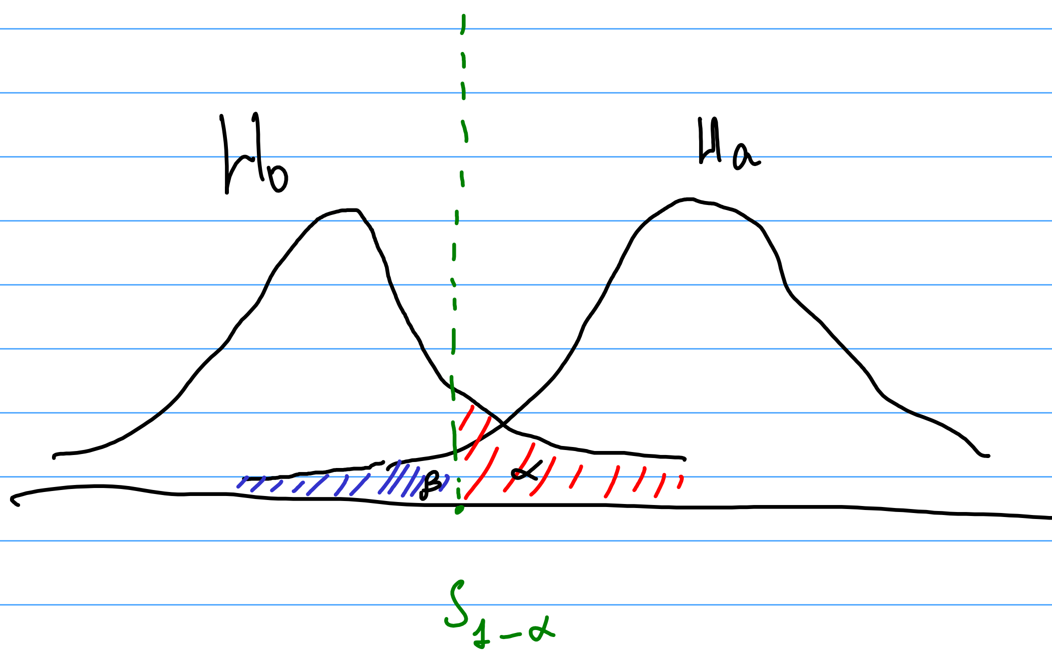
Power Analysis

Type I error $p(S^{obs} \subset D^{crit} \mid H_0) = \alpha$

Type II error $p(S^{obs} \subset D^{crit} \mid H_a) = \beta$

Power $= 1 - \beta$

α and β are inversely related



Testing:

- effect size
- sample size
- significance
- power

Sample size

$\bar{X}$: $$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

$$P\left(Z_{\alpha/2} \leq Z \leq Z_{1-\alpha/2} \right) = 1 - \alpha$$

$$P\left(Z_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \leq Z_{1-\alpha/2} \right) = 1 - \alpha$$

$$P\Big(\bar{X} + \underbrace{Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{\text{Margin of error } (E)} \leq \mu \leq \bar{X} + Z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \Big) = 1 - \alpha$$

$$Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq E$$

$$\Rightarrow n \geq \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

for $\hat{p}$:

$$\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0,1)$$

$$\Downarrow$$

$$\hat{p} + \underbrace{z_{\alpha/2} \cdot \sqrt{p(1-p)/n}}_{ME(E)} \leq p + z_{1-\alpha/2} \cdot \sqrt{p(1-p)/n}$$

$$z_{\alpha/2} \sqrt{p(1-p)/n} \leq E$$

$$n \geq \left(\frac{z_{\alpha/2}}{E}\right)^2 \cdot p(1-p)$$

Samples $\nearrow$ With replacement

$\searrow$ w/o replacement

(fixed sample)

[1, 2, 3]

$\downarrow$

$X_1 = 1 \quad p = 1/3$

$X_2 = 1 \quad p = 0$

$Cov(X_i, X_j) \neq 0$

$$Cov(X_i, X_j) = -\frac{\sigma^2}{N-1}$$

n = sample size

N = pop. size

se($\bar{X}$)	With replacement	w/o replacement
$\bar{X}$	$\sqrt{\sigma^2/n}$	$\sqrt{\sigma^2/n\left(1-\frac{n-1}{N-1}\right)}$
$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$\sqrt{\frac{p\cdot(1-p)}{n}\cdot\left(1-\frac{n-1}{N-1}\right)}$

Percentages:

percentage deviation

$$E = R\mu$$

$$n = \frac{\sigma^2 z^2}{E^2} = \frac{(\sigma^2/\mu^2)\cdot z^2}{R^2} = \frac{c^2 \cdot z^2}{R^2}$$

$c = \sigma/\mu$ - coefficient of variation