

Consider the following table:

Desire to travel abroad	Age	
	Under 45	45 or older
Yes	50%	50%
No	50%	50%
Column totals	100%	100%
Number of participants	500	500

(a) Is there significant association between variables?

(b) Measure the strength of association.

E.g.

y

N

< 45    ≥ 45

100	x	500
x	x	500
500	1000	

$H_0$ : no association

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{(2-1) \times (2-1)}$$

$O_i$	< 45	≥ 45	
y	250	250	= 500
N	250	250	

$(n_r - 1) \times (h_c - 1)$

1	3	.
2	2	.
.	.	.
Σ	Σ	

$E_i$	< 45	≥ 45
y	250	250
N	250	250

$$E_{11} = \frac{500 \cdot 500}{1000} = \frac{n_r \cdot n_c}{N}$$

$$\alpha = 0,05$$

$$\chi^2_{obs} = 0 < \chi^2_{crit, 1, 0,95}$$

$$\approx 8$$

$\Rightarrow$  not rejected  $H_0$

$$\chi^2_n = \sum_{i=1}^n \xi^2$$

$$\xi \sim N(0,1)$$

$$\max \chi^2 :$$

$$500$$

$$0$$

$$0$$

$$500$$

$$\chi^2 = \frac{250^2}{250} \cdot 4 = 1000]$$

# Effect size

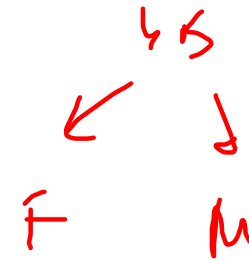
Cont:  $\hat{\rho}_{\text{pearson}}$ ,  $\hat{\rho}_{\text{spearman}}$

Cat:  $\hat{\phi} = \sqrt{\frac{\chi^2}{N}} \in [0, 1]$

$$\phi = 0,1 \quad ; \quad \phi = 0,9$$

Let's introduced gender as the third variable.

Desire to travel abroad	Gender			
	Male age		Female age	
	Under 45	45 or older	Under 45	45 or older
Yes	60%	40%	35%	65%
No	40%	60%	65%	35%
Column totals	100%	100%	100%	100%
Number of participants	300	300	200	200



(c) Is there significant association between variables within the subgroups?

(d) Measure the strength of association.

(e) Calculate Pearson residuals for Female under 45.

↗ binary / prop.

→ cont.

↘ ratio

$$\frac{0/1 + \dots + 0/1}{1 + \dots + 1}$$

$$\frac{X_1 + \dots + X_n}{1 + \dots + 1}$$

X

$$\frac{X_1 + \dots + X_n}{Y_1 + \dots + Y_n}$$

$\frac{X}{Y}$

$$2 \times 2 \times 2 = 8$$

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(e) Calculate Pearson residuals for Female under 45.

Goal: Any relation

H

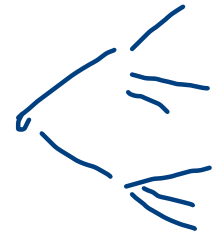
$$E_{ij} = (n_{c.}) \cdot \frac{n_{.i}}{N}$$

$$= n_{.j} \cdot \frac{n_{.i}}{N}$$

$$\chi^2_{obs.} = \frac{30^2}{100} \cdot 4 = 36 \sim \chi^2_{1, 0.95} \quad \begin{array}{l} \text{prop in column} \\ \text{v.r.t. to rows} \end{array}$$

$$\begin{array}{c|cc} & <45 & \geq 45 \\ O_i y & 70 & 130 \\ N & 130 & 70 \end{array}$$

$$\begin{array}{c|cc} & <45 & \geq 45 \\ E_i y & 100 & 100 \\ N & 100 & 100 \end{array}$$



$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \sum \chi_{ij}^2$$

	1	2	3
k	1	1	1
l	1	1	1

$$\chi_{ij} = \frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}}} = \frac{30}{10} = 3$$

	$-3 < -1,96$	
$\chi_{ij}$	$-3$	3
	3	$-3$

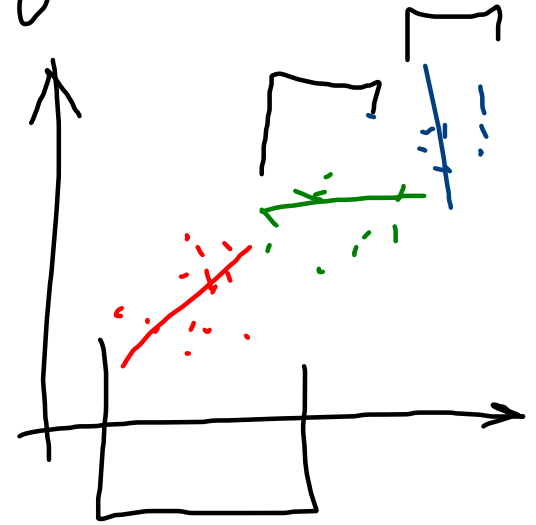
$$\begin{matrix} i=2 \\ j=1 \end{matrix}$$

$$\chi_{ij} \sim N(0,1)$$

① Unobserved heterogeneity



$\pi \uparrow$



$\pi \uparrow \uparrow \uparrow$

② Seasonality

$\nearrow 1$   
 $\searrow 0$

$\uparrow \uparrow \uparrow \uparrow \uparrow \downarrow \downarrow$   
 $\sim$

Holidays / One-time events

↳ no counterfactual

↳ matching

↳  $X_1, \dots, X_k$

exogenous

$\perp$  with  $D_t$



② Bucketization

$$E(R_i) \approx E(R_B)$$

$$\textcircled{R^1} = \frac{X_1^1 + \dots + X_n^1}{y_1^1 + \dots + y_n^1}$$

$$\overset{||}{E}(R_i)$$

$$R^B = \frac{X_1^1 + \dots + X_n^1 + X_1^2 + \dots + X_n^2 + \dots}{y_1^1 + \dots + y_n^1 + y_1^2 + \dots + y_n^2 + \dots}$$

$1, \dots, K \in [B] \quad \text{i.i.d.}$