4. (a) The dietary characteristics of 200 respondents were recorded in a survey. Respondents were asked to rate the extent to which their diet agreed with the following descriptors:

- fast food
- filling
- hearty
- low meat
- vegetarian.

Figure 3 (spread over the next two pages) presents selected SPSS output from a factor analysis with principal components extraction, using the varimax rotation procedure. Interpret the output. In your analysis, be sure to address at least the following:

- Explain how you determine the number of factors and interpret the extracted factors.
- Explain qualitatively and quantitatively how the fit of the factor analysis model should be examined.
- Briefly discuss for what modelling purpose(s) extracted factors could be used.

(20 marks)

CFA 1) PCA

2) Rotation for interpretability

3) Interpretation

1

$$X = U \cdot \Lambda \cdot V^T$$

weights

$$X = \begin{bmatrix} u_1^1 & u_1^2 & \cdots \\ \vdots & \vdots & \vdots \\ u_n^1 & u_n^2 & \cdots \end{bmatrix} \begin{bmatrix} 2{,}8 & & \\ & 1{,}6 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \cdots \\ \cdots \end{bmatrix}$$

$$X = \left[\begin{array}{c|c} \text{comp} & \text{noise} \end{array}\right] \left[\begin{array}{c|c} C. & \\ \hline & N \end{array}\right] \left[\begin{array}{c} \text{comp.} \\ \hline \text{Noise} \end{array}\right]$$

$$V_i = \underbrace{\overset{?}{\lambda_{v1}} F_1 + \overset{?}{\lambda_{v2}} F_2}_{} + U_v$$

Specific Factor

$$L_i = \dots$$

$$\hat{V} \mid \overset{PCA}{u_1}, \overset{PCA}{u_2}$$

$$FF_i = .$$

$$\hat{V} \mid F_1, F_2$$

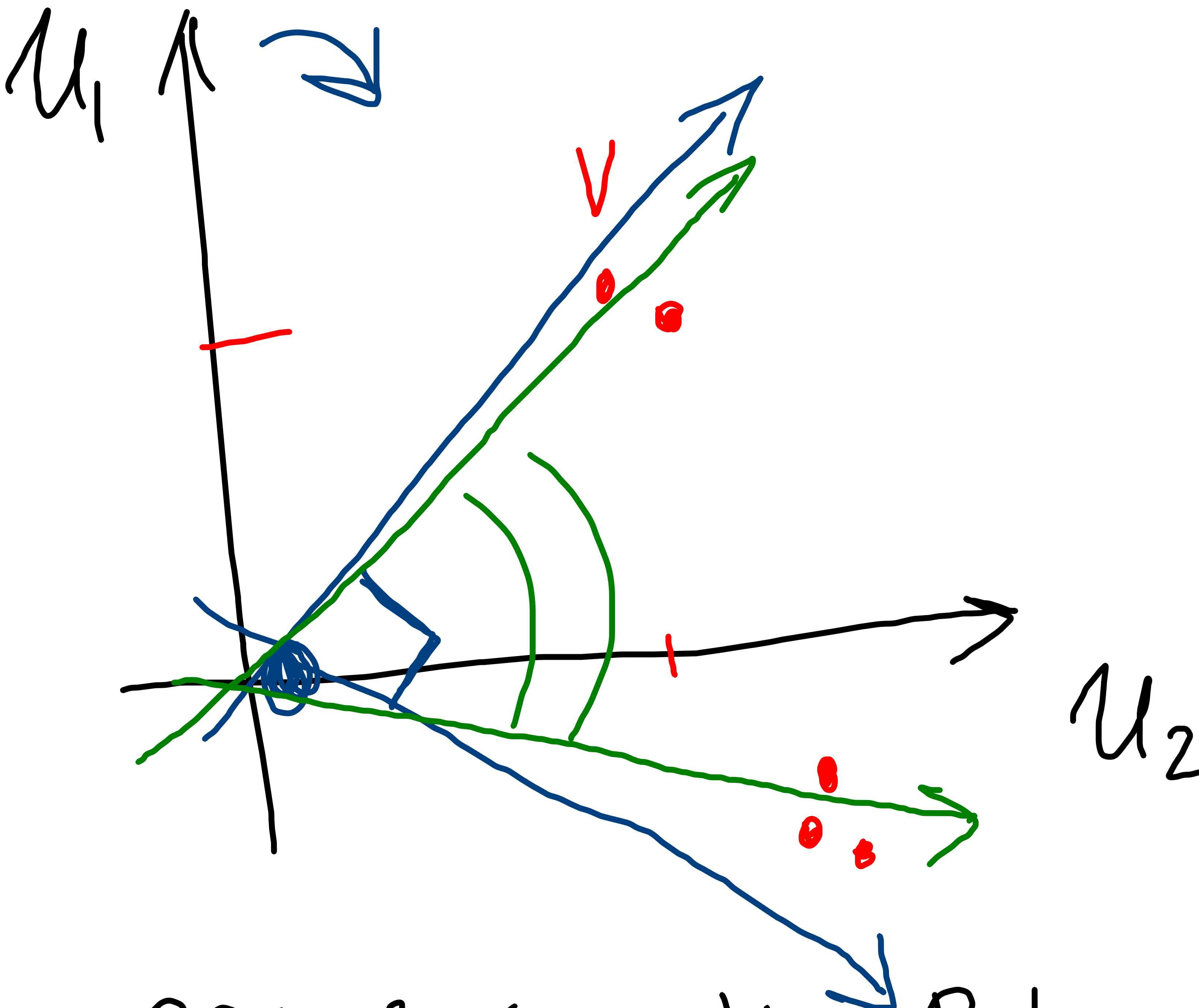
$$F_i =$$

$$X_{n \times 5} = F \Lambda + U$$

$$H_i =$$

Λ – loading factors

$$\begin{cases} \max \text{Var} \\ \text{s.t. } u^T u = 1 \end{cases}$$



	PC1 u_1	PC2 u_2	$\cancel{u_3}$
V	0,5	0,5	0.1
LM	0,5	0,5	-0,1
F	-0,2	0,7	-0.
FF	-0,2	0,7	+0.
H ⇔	-0,2	0,7	0,9

Exp Var PCA ? Exp. Var Rot.

$\sum \max \sum \mathrm{Var}_i^2 \quad \lambda_i^2$

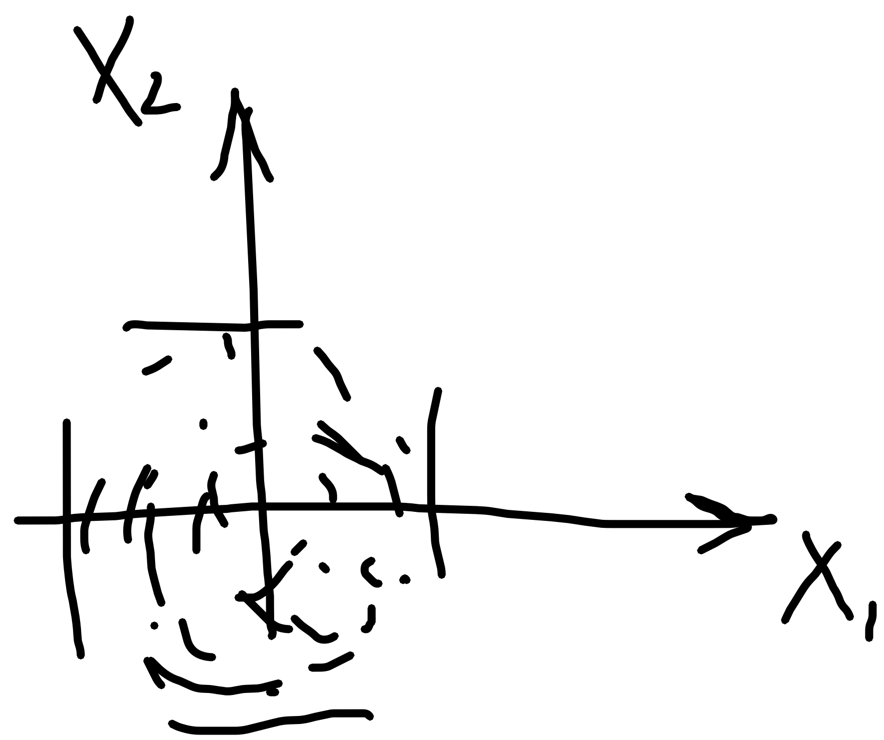
Orthogonal rotation : Varimax

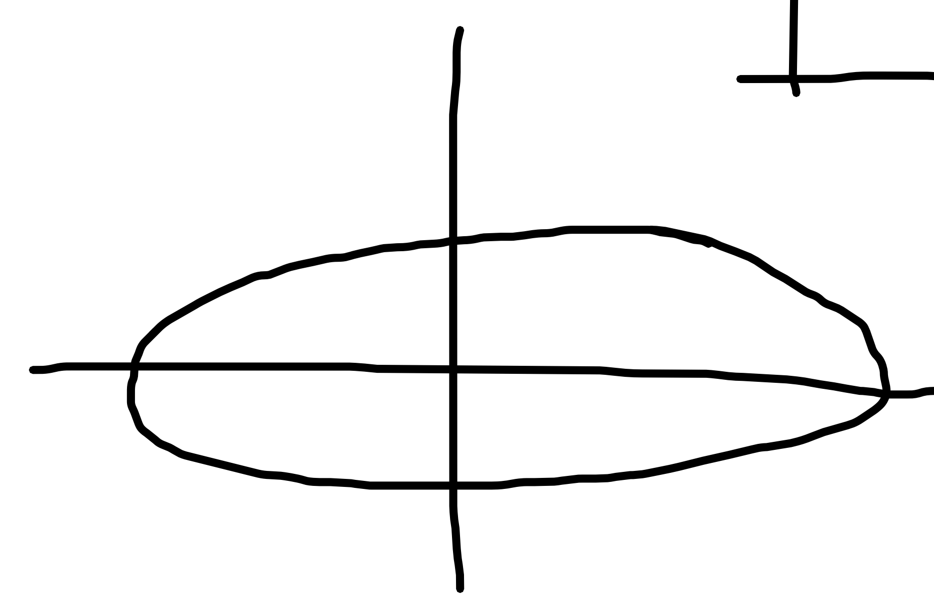
Oblique rotation

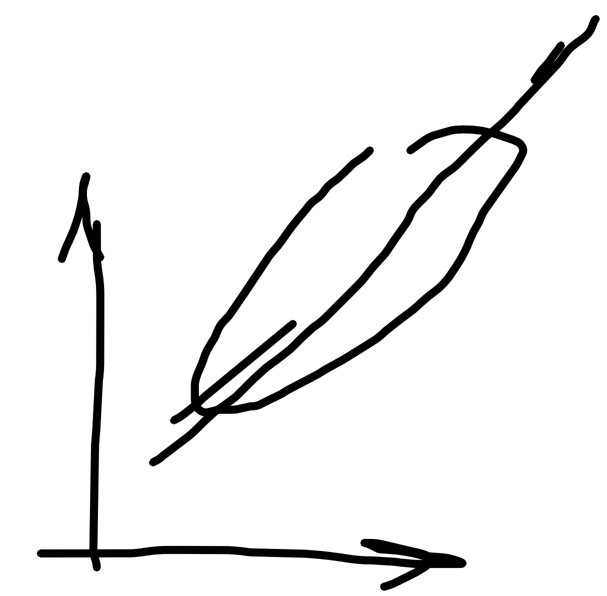
(2) Bartlett: $H_0: \sigma_1^2 = \dots = \sigma_k^2$ $\qquad \chi^2$

Bartlett for sphericity: $\qquad \chi^2$

$H_0: \quad \tilde{X}^T \tilde{X} = \mathbb{1}_{n \times n} \; ; \; \tilde{X} = \dfrac{X - \bar{X}}{se(X)}$







③ $$KMO = \frac{\sum r_{ij}}{\sum\limits_{i \neq j} r_{ij} + \underbrace{\sum\limits_{i \neq j} \rho_{ij}}}$$

$KMO > 0{,}6$

r_{ij} – Pearson cor.

ρ_{ij} – partial correlation

$$\begin{tikzcd}
X_{t-2} \arrow[r] \arrow[rr, bend left] & X_{t-1} \arrow[r] & X_t
\end{tikzcd}$$

1 | $X, y \perp Z$

$\rho_{x,y|z} = r_{xy}$

2 | $x, y \uparrow\uparrow z$

$\rho_{x,y|z} \leq r_{xy}$

X, Y, Z

$X \mid Z \Rightarrow X - \hat{X}$

$y \mid Z \Rightarrow y - \hat{y}$

(4) Communality

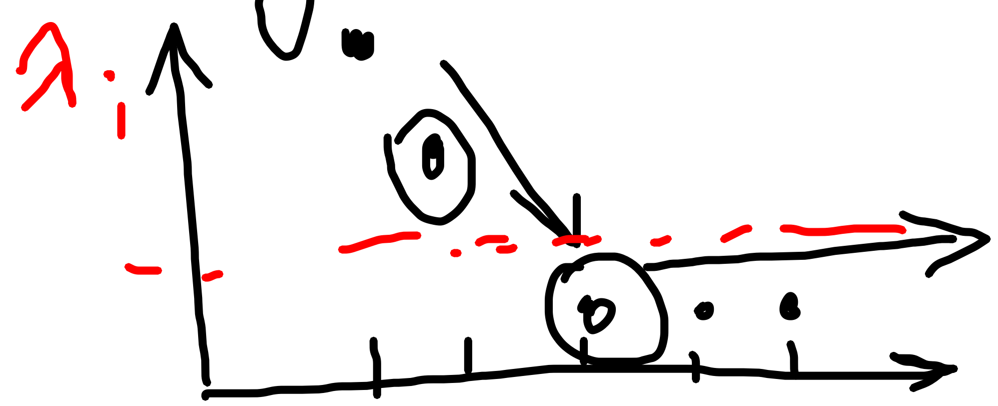
$$C_v = \lambda_{v1}^2 + \lambda_{v2}^2$$

1) % Tot. Var ≈ 75%

2) Eig. values ≥ 1

Scree plot

2 or 3

3) 

Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	2.828	56.553	56.553
2	1.631	32.614	89.167
3	.256	5.129	94.296
4	.166	3.316	97.611
5	.119	2.389	100.000

Extraction Method: Principal Component Analysis.