Sem 5.

$$D_i = \begin{cases} 1, & NY \text{ - treatment} \\ 0, & P \text{ - control} \end{cases} \qquad t_i = \begin{cases} 1, & \text{after} \\ 0, & \text{before} \end{cases}$$

y_i - employment

$\uparrow$ min wage $\overset{SR}{\Rightarrow}$ employment $y \downarrow$

1) $\bar{y}_{NY,\,after} - \bar{y}_{NY,\,before}$?

$\hookrightarrow$ No Time effect (GDP$\uparrow$, D$\uparrow$)

2) $\bar{y}_{NY,\,after} - \bar{y}_{P,\,after}$

$\hookrightarrow$ No State effect

$$\underset{NY/P \quad 0/1}{y_{ist}} = \alpha_s + \mu_t + \delta \cdot D_{ist} + \varepsilon_{ist}$$

NY: $\nearrow$ before $\quad \cancel{\alpha_{NY}} + \mu_0$

$\searrow$ after $\quad \cancel{\alpha_{NY}} + \mu_1 + \delta$ $\quad \Big\}\ \Delta_1 = \cancel{\mu_1} + \delta - \cancel{\mu_0}$

P: $\nearrow$ before $\quad \cancel{\alpha_P} + \mu_0$

$\searrow$ after $\quad \cancel{\alpha_P} + \mu_1$ $\quad \Big\}\ \Delta_0 = \cancel{\mu_1} - \cancel{\mu_0}$

$$\Delta_1 - \Delta_0 = \delta$$

$\hat{\delta}$ - "Diff-in-diff"

$$\hat{\delta} = [\bar{y}_{NY,1} - \bar{y}_{NY,0}] - [\bar{y}_{P,1} - \bar{y}_{P,0}]$$

T1

$$y_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot D_i$$

$E(X^4)$ $E(X - E(X))^4$

$Var(X) = E(X - E(X))^2$

$= E(X^2) - E^2(X)$

$$\hat{\beta}_2 = \bar{y}_1 - \bar{y}_0 \quad ?$$

$\hat{Var}(X) = \frac{1}{n-1}\sum(x_i - \bar{x})^2$

$$\hat{\beta}_2 = \frac{\hat{Cov}(y, D)}{\hat{Var}(D)} = \frac{\cancel{\frac{1}{n-1}} \sum (y_i - \bar{y})(D_i - \bar{D})}{\cancel{\frac{1}{n-1}} \sum (D_i - \bar{D})^2} = \left\{ \begin{array}{l} \sum y_i = n \cdot \bar{y} \\ \sum D_i = n \bar{D} \end{array} \right\}$$

$$= \frac{\overline{yD} - \bar{y} \cdot \bar{D}}{\overline{D^2} - (\bar{D})^2} = \frac{\overline{yD} - \bar{y} \cdot p}{p - p^2} =$$

$= p(1-p) = pq = \frac{n_1}{n} \cdot \frac{n_0}{n}$

$n_1 + n_0 = n$

$p + q = 1$

$\bar{D}$ - ?

$$D = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \qquad \bar{D} = \frac{\sum D_i}{n} = \frac{n_1}{n} = p$$

$$D^2 = D \Rightarrow \overline{D^2} = \bar{D} = p$$

$$\overline{yD} = \frac{\sum y_i \cdot D_i}{n} = \frac{\sum_{D_i = 1} y_i}{n}$$

$$= \frac{\dfrac{n \cdot \sum_{D_i=1} y_i}{n \cdot \cancel{n}} - \dfrac{\sum_{D_i=1} y_i + \sum_{D_i=0} y_i}{\cancel{n}} \cdot \dfrac{n_1}{\cancel{n}}}{\dfrac{n_0 \cdot n_1}{n \cdot \cancel{n}}} =$$

$$= \frac{n \cdot \sum_{D_i=1} y_i - n_1 \sum_{D_i=1} y_i - n_1 \cdot \sum_{D_i=0} y_i}{n_0 n_1} =$$

$$= \frac{\underbrace{(n - n_1)}_{n_0} \sum_{D_i=1} y_i - n_1 \cdot \sum_{D_i=0} y_i}{n_0 n_1} =$$

$$= \frac{\sum_{D_i=1} y_i}{n_1} - \frac{\sum_{D_i=0} y_i}{n_0} = \boxed{\bar{y}_1 - \bar{y}_0}$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \cdot \underset{\frac{n_1}{n}}{\overset{\shortparallel}{\bar{D}}} = \frac{\sum_{D_i=1} y_i + \sum_{D_i=0} y_i}{n} - \frac{\sum_{D_i=1} y_i}{\cancel{n_1}} \cdot \frac{\cancel{n_1}}{n} + \frac{\sum_{D_i=0} y_i}{n_0} \cdot \frac{n_1}{n} =$$

$$= \frac{n_0 \sum_{D_i=0} y_i}{n_0\ n} + \frac{\sum_{D_i=0} y_i \cdot n_1}{n_0 \cdot n} = \frac{\sum_{D_i=0} y_i \,(\cancel{n_0 + n_1})^{n}}{n_0 \cdot \cancel{n}} =$$

$$= \frac{\sum_{D_i=0} y_i}{n_0} = \bar{y}_0$$

$$y_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot D_i$$

$$\Delta y_i = \hat{\beta}_2 \cdot \delta X_i$$

$$\hat{\beta}_1 = \bar{y}_0$$

$$\hat{\beta}_2 = \overline{\Delta y_0}$$

$$\hat{\beta}_2 = \bar{y}_1 - \bar{y}_0$$

$$\hat{\delta} = \overline{\Delta y_1} - \overline{\Delta y_0}$$

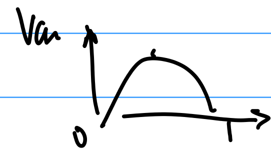
$$\operatorname{Var}(\hat\beta) = \sigma^2 (X'X)^{-1}$$

T2

$$\operatorname{Var}(\hat\beta_2) = \frac{\sigma^2}{\sum (D_i - \bar D)^2} = \frac{\sigma^2}{\sum D_i^2 - 2\sum D_i \bar D + \sum \bar D^2} =$$

$$\sum D_i = n \cdot \bar D$$

$$= \frac{\sigma^2}{n \cdot \bar D - \cancel{2} \cdot n \cdot \bar D \cdot \bar D + \cancel{\bar D^2} \cdot n} =$$



$p(1-p)$

$$= \frac{\sigma^2}{n \bar D (1 - \bar D)} = \frac{\sigma^2}{n\, p\, q}$$

$$\bar D = \frac{n_1}{n} = p$$

$$p = 1/2$$

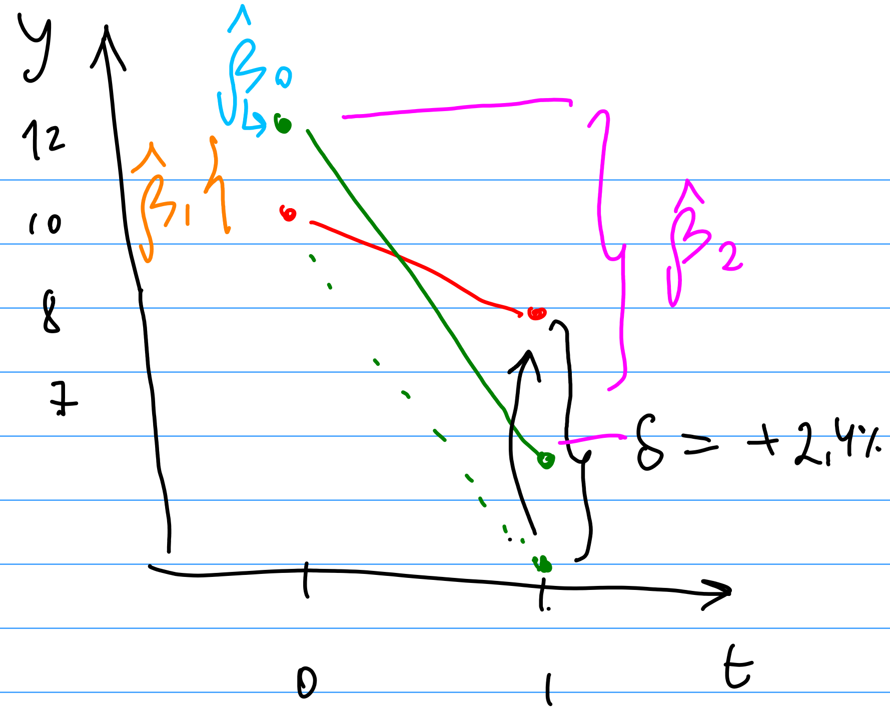
Big S.S.		Small S.S.	
$n_1 = 100$	$n_1 = 100$	$n_1 = 4$	$n_1 = 4$
$n_2 = 900$ →	$n_1 = 100$	$n_2 = 5$	$n_2 = 4$

$$\operatorname{Var}(\hat\beta_2) = \frac{1}{1000 \cdot 0.1 \cdot 0.9} = \frac{1}{90} \qquad \operatorname{Var}(\hat\beta_2) = \frac{1}{9 \cdot \frac{4}{9} \cdot \frac{5}{9}} = \frac{1}{2.2}$$

$$\wedge \qquad\qquad \wedge$$

$$\operatorname{Var}(\hat\beta_2) = \frac{1}{200 \cdot 0.5 \cdot 0.5} = \frac{1}{50} \qquad \operatorname{Var}(\hat\beta_2) = \frac{1}{8 \cdot 0.5 \cdot 0.5} = \frac{1}{2}$$

b)



c) $X_i = \begin{cases} 1, & \text{treatment} \\ 0, & \text{control} \end{cases}$

$Z_t = \begin{cases} 1, & \text{after} \\ 0, & \text{before} \end{cases}$

$$y_{it} = \beta_0 + \beta_1 \cdot X_i + \beta_2 \cdot Z_t + \delta X_i Z_t + \varepsilon_{it}$$

$$\hat{\delta}_{OLS} = \hat{\delta}_{DD}$$

$X_i = 0$

$Z_i = 0$

$$\hat{\delta}_{DD} = [\bar{y}_{T,1} - \bar{y}_{T,0}] - [\bar{y}_{C,1} - \bar{y}_{C,0}]$$

$$\hat{\beta}_0 = \bar{y}_{C,0}$$

$\Delta X_i = 1$ $\quad\quad$ $\hat{\beta}_1 = \bar{y}_{T,0} - \bar{y}_{C,0}$

$\Delta Z_i = 1$ $\quad\quad$ $\hat{\beta}_2 = \bar{y}_{C,1} - \bar{y}_{C,0}$