Lecture 9:
Chi-square test

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Plan

Plan

1) Contingency tables

2) Chi-square test

3) Partial correlation

Plan

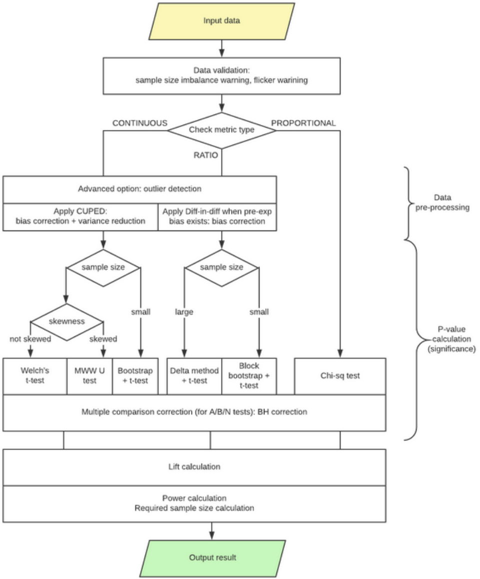

Figure: Uber's statistics engine is used for A/B/N experiments and dictated by fixed horizon

Cross-tabulation

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Cross-tabulation results in tables which reflect the **joint distribution** of two or more variables with a limited number of categories or distinct values, for example:

Internet usage	Gender		Row total
	Male	Female	
Light	5	10	15
Heavy	10	5	15
Column total	15	15	

Cross-tabulation

Since two variables have been cross-classified, percentages could be computed either *column-wise*, based on column totals, or *row-wise*, based on row totals. The general rule is to compute the percentages in the direction of the **independent variable**, across the dependent variable.

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(i) Column-wise percentages: internet usage is dependent variable (meaningful, gender is consistent)

	Gender	
Internet usage	Male	Female
Light (1)	33.3%	66.7%
Heavy (2)	66.7%	33.3%
Column totals	100%	100%

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Heavy (2)	66.7%	33.3%
Column totals	100%	100%

(ii) Row-wise percentages: gender is dependent variable (not meaningful)

	Gender total		
Internet usage	Male	Female	Row total
Light (1)	33.3%	66.7%	100%
Heavy (2)	66.7%	33.3%	100%

Perks and limitations of cross-tabulations

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- **Clarity** - i.e. the clarity of interpretations provides a stronger link between the research results and managerial action.

- **Simplicity** - i.e. cross-tabulation analysis is simple to conduct and appealing to the less sophisticated researcher.

Cross-tabulation is *seldom used* in computations involving more than three variables, since the interpretation becomes quite complex.

Also, since the number of cells increases multiplicatively, maintaining an adequate number of participants in each cell becomes problemat·ic. Consequently, the statistics computed could be *unreliable*.

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4. It can indicate no change in the initial association.

Refined initial relationship

	Marital status	
Purchase of luxury branded clothing	Married	Unmarried
High	31%	52%
Low	69%	48%
Column	100%	100%
Number of participants	700	300

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52% of unmarried participants fell in the high-purchase category, as opposed to 31% of the married participants. Before concluding that unmarried participants purchase more luxury branded clothing than those who are married, a third variable, the buyer's gender, was introduced into the analysis.

	Gender			
Purchase of luxury branded clothing	Male marital status	Female marital status		
	Married	Unmarried	Married	Unmarried
High	35%	40%	25%	60%
Low	65%	60%	75%	40%
Column	100%	100%	100%	100%
Number of participants	400	120	300	180

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In the case of females, 60% of the unmarried participants fall in the high-purchase category compared with 25% of those who are married. On the other hand, the percentages are much closer for males.

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Hence, the introduction of gender (third variable) has **refined the relationship** between marital status and purchase of luxury branded clothing (original variables).

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Hence, the introduction of gender (third variable) has **refined the relationship** between marital status and purchase of luxury branded clothing (original variables).

Unmarried participants are more likely to fall into the high-purchase category than married ones, and this effect is much more pronounced for females than for males.

Initial relationship was spurious

	Education	
Own expensive car	Degree	No degree
Yes	32%	21%
No	68%	79%
Column	100%	100%
Number of participants	250	750

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Conclusion: education influenced ownership of expensive cars.

However, income may also be an important factor for determining car ownership.

	Income			
Own expensive car	Low-income education		High-income education	
	Degree	No degree	Degree	No degree
Yes	20%	20%	40%	40%
No	80%	80%	60%	60%
Column totals	100%	100%	100%	100%
Number of participants	100	700	150	50

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Own expensive car	Education: Degree	Education: No degree
Yes	32%	21%
No	68%	79%
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No	80%	80%	60%	60%
Column totals	100%	100%	100%	100%
Number of participants	100	700	150	50

The percentages of those with and without university degrees who own expensive cars are the same for each income group.

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Column totals	100%	100%	100%	100%
Number of participants	100	700	150	50

The percentages of those with and without university degrees who own expensive cars are the same for each income group.

When the data for the high-income and low-income groups are examined separately, the association between education and ownership of expensive cars disappears, indicating that the **initial relationship** observed between these two variables was **spurious**.

Reveal suppressed association

	Age	
Desire to travel abroad	Under **45**	**45** or older
Yes	50%	50%
No	50%	50%
Column totals	100%	100%
Number of participants	500	500

Cross-tabulation indicate no association.

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Number of participants	500	500

Cross-tabulation indicate no association.

Let's introduced gender as the third variable.

	Gender			
	Male age		Female age	
Desire to travel abroad	Under **45**	**45** or older	Under **45**	**45** or older
Yes	60%	40%	35%	65%
No	40%	60%	65%	35%
Column totals	100%	100%	100%	100%
Number of participants	300	300	200	200

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Yes	60%	40%	35%	65%
No	40%	60%	65%	35%
Column totals	100%	100%	100%	100%
Number of participants	300	300	200	200

Among men, 60% of those under 45 indicated a desire to travel abroad compared with 40% of those 45 or older. The pattern was reversed for women.

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Number of participants	300	300	200	200

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Since the association between desire to travel abroad and age runs in the *opposite direction* for males and females, the relationship between these two variables is masked when the data are aggregated across gender.

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Since the association between desire to travel abroad and age runs in the *opposite direction* for males and females, the relationship between these two variables is masked when the data are aggregated across gender.

But when the effect of gender is controlled, the **suppressed association** between preference and age is **revealed** for the separate categories of males and females.

No change in initial relationship

	Family size	
Eat frequently in fast-food restaurants	Small	Large
Yes	65%	65%
No	35%	35%
Column totals	100%	100%
Number of participants	500	500

No change in initial relationship

	Family size	
Eat frequently in fast-food restaurants	Small	Large
Yes	65%	65%
No	35%	35%
Column totals	100%	100%
Number of participants	500	500

No association is observed.

No change in initial relationship

Eat frequently in fast-food restaurants	Family size: Small	Family size: Large
Yes	65%	65%
No	35%	35%
Column totals	100%	100%
Number of participants	500	500

No association is observed.

The participants were further classified into high- or low-income groups based on a median split.

Eat frequently in fast-food restaurants	Income: Low-income family size		Income: High-income family size	
	Small	Large	Small	Large
Yes	65%	65%	65%	65%
No	35%	35%	35%	35%
Column total	100%	100%	100%	100%
Number of participants	250	250	250	250

No change in initial relationship

Eat frequently in fast-food restaurants	Family size: Small	Family size: Large
Yes	65%	65%
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Column total	100%	100%	100%	100%
Number of participants	250	250	250	250

Again, no association was observed.

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	Family size	
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Eat frequently in fast-food restaurants	Income			
	Low-income family size		High-income family size	
	Small	Large	Small	Large
Yes	65%	65%	65%	65%
No	35%	35%	35%	35%
Column total	100%	100%	100%	100%
Number of participants	250	250	250	250

Again, no association was observed.

In some cases, the introduction of the third variable **does not change the initial relationship** observed, regardless of whether the original variables were associated.

This suggests that the third variable does not influence the relationship between the first two.

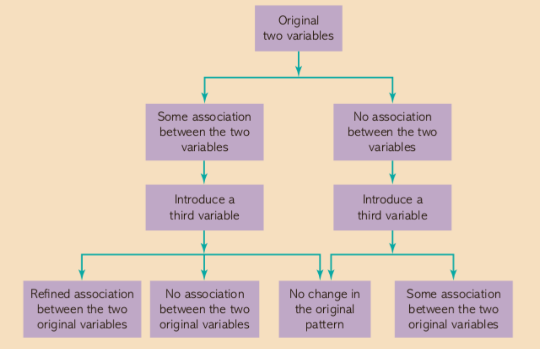

Figure: The introduction of a third variable in cross-tabulation

Spurious association – i.e. the introduction of a third variable in the cross-tabulation reveals that there is no association between the two variables despite the observed initial association.

Suppressed association – i.e. after introducing a third variable, the cross-tabulation reveals association between the two variables although no association was initially observed.

Chi-square test

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H_0 : there is no association between the variables.

H_a : there is association between the variables.

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The chi-square statistic (χ^2) is used to test the statistical significance of the observed association in a cross-tabulation.

H_0 : there is no association between the variables.

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Idea: compare the cell frequencies that would be expected if no association were present between the variables, given the existing row and column totals.

f_e – expected cell frequencies

f_o – actual observed frequencies.

The greater the discrepancies between the expected and observed frequencies, the larger the value of the statistic.

Chi-square test

The expected frequency for each cell can be calculated by using a simple formula:

$$f_e = \frac{n_r n_e}{n}$$

where

$$\begin{aligned} n_r = & \text{ total number in the row} \\ n_c = & \text{ total number in the column} \\ n = & \text{ total sample size.} \end{aligned}$$

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Then the value of χ^2 is calculated as follows:

$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e} \sim \chi^2_{(r-1)\times(c-1)}$$

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The chi-square distribution is a skewed distribution whose shape depends solely on the number of degrees of freedom.

As the number of degrees of freedom increases, the chi-square distribution becomes more symmetrical.

Example

Internet usage	Gender		Row total
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The expected frequencies for the cells, going from left to right and from top to bottom, are:

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The value of χ^2 is calculated as:

$$\begin{aligned}
\chi^2 &= (5-7.5)^2/7.5 + (10-7.5)^2/7.5 + (10-7.5)^2/7.5 + (5-7.5)^2/7.5 \\
&= 0.833 + 0.833 + 0.833 + 0.833 \\
&= 3.333
\end{aligned}$$

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Number of degree of freedom: $df = (2-1) \times (2-1) = 1$

Critical value at 5%: $\chi^2_{1,0.95} = 3.841$

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Critical value at 5%: $\chi^2_{1,0.95} = 3.841$

The null hypothesis of no association cannot be rejected, indicating that the association is not statistically significant at the 0.05 level.

Note that this lack of significance is mainly due to the small sample size (30).

If, instead, the sample size were 300 and each data entry were multiplied by 10, test statistics would be multiplied by 10 $\chi^2_{obs} = 33.33$, which is significant at the 0.05 level.

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The following is Yates's corrected version of Pearson's chi-squared statistics:

$$\chi^2_{\text{Yates}} = \sum_{i=1}^{N} \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$

where: O_i = an observed frequency E_i = an expected (theoretical) frequency, asserted by the null hypothesis N = number of distinct events

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In some cases, Yates's correction may adjust too far, and so its current use is limited.

Example

Internet usage	Gender		Row total
	Male	Female	
Light	5	10	15
Heavy	10	5	15
Column total	15	15	

The value of Yates χ^2 is calculated as:

$$\chi^2 = (\mid 5 - 7.5 \mid -0.5)^2/7.5 + (\mid 10 - 7.5 \mid -0.5)^2/7.5 + (\mid 10 - 7.5 \mid -0.5)^2/7.5 + (\mid 5 -$$
$$= 2.133$$

Critical value at 5%: $\chi^2_{1,0.95} = 3.841$

Result: H_0 is not rejected

Test assumptions

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$$P(X \leq x) = P(X < x + 1)$$

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for any $x \in \{0, 1, 2, \ldots n\}$.

If np and $np(1-p)$ are large (sometimes taken as both ≥ 5), then the probability above is fairly well approximated by

$$P(Y \leq x + 1/2)$$

where Y is a normally distributed random variable with the same expected value and the same variance as X, i.e., $\mathrm{E}(Y) = np$ and $\mathrm{var}(Y) = np(1-p)$.

This addition of $1/2$ to x is a continuity correction.

Goodness-of-fit

The chi-square statistic can also be used in goodness-of-fit tests to determine whether certain models fit the observed data.

These tests are conducted by calculating the significance of sample deviations from assumed theoretical (expected) distributions and can be performed on cross-tabulations as well as on frequencies (one-way tabulations).

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(i) Equal proportion hypothesis:

$$H_0 : p_1 = p_2 = p_3 = p_4 = p_5$$

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We calculate the test statistic using the formula below:

$$\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{n-1}$$

Example

Consider a categorical variable which is the flavors of candy.

We collect a random sample of ten bags. Each bag has 100 pieces of candy and five flavors. Our hypothesis is that the proportions of the five flavors in each bag are the same.

Each bag has 100 pieces of candy. Each bag has five flavors of candy. We expect to have equal numbers for each flavor: $100 / 5 = 20$ pieces of candy in each flavor from each bag.

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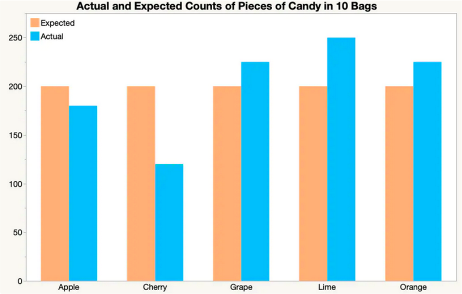
For 10 bags in our sample, we expect 10 x 20 = 200 pieces of candy in each flavor.



Are the number of pieces “close enough” for us to conclude that across many bags there are the same number of pieces for each flavor?

Example

Let's start by listing what we expect if each bag has the same number of pieces for each flavor. Above, we calculated this as 200 for 10 bags of candy.

Flavor	Number of Pieces of Candy (10 bags)	Expected Number of Pieces of Candy	Observed-Expected	Squared Difference	Squared Difference / Expected Number
Apple	180	200	$180 - 200 = -20$	400	$400/200 = 2$
Lime	250	200	$250 - 200 = 50$	2500	$2500/200 = 12.5$
Cherry	120	200	$120 - 200 = -80$	6400	$6400/200 = 32$
Orange	225	200	$225 - 200 = 25$	625	$625/200 = 3.125$
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Finally, we add the numbers in the final column to calculate our test statistic:

2+12.5+32+3.125+3.125=52.75

Critical value: $\chi^2_{5-1=4,0.95} = is 9.488$

Since $52.75 > 9.488$, we reject the null hypothesis that the proportions of flavors of candy are equal.

1) **Phi coefficient** (ϕ)

– works for the special case of a table with two rows and two columns (a 2×2 table)

For a sample of size n, this statistic is calculated as:

$$\phi = \sqrt{\frac{\chi^2}{n}}$$

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Example: in our case, because the association was not significant at the 0.05 level, we would not normally compute the phi value. However, for the purpose of illustration, the value of phi is:

$$\phi = \sqrt{3.333/30} = 0.333$$

Thus, the association is not very strong.

Strength of association

2) **Contingency coefficient** (C)

– a more general case involving a table of any size

This index is also related to chi-square, as follows:

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The contingency coefficient varies between 0 and 1.

The value of 0 occurs in the case of no association (i.e. the variables are statistically independent), but the maximum value of 1 is never achieved.

The maximum value of the contingency coefficient *depends on the size of the table* (number of rows and number of columns), hence, it should be used only to compare tables of the *same size*.

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Example: The value of the contingency coefficient:

$$C = \sqrt{3.333/(3.333 + 30)} = 0.31$$

This value of C indicates that the association is not very strong.

3) **Cramer's** V

– a modified version of the phi correlation coefficient used in tables larger than 2×2.

For a table with r rows and c columns:

$$V = \sqrt{\frac{\phi^2}{\min(r-1),(c-1)}}$$

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When phi is calculated for a table larger than 2×2, it has no upper limit. Cramer's V is obtained by adjusting phi for either the number of rows or the number of columns in the table, based on which of the two is smaller. Hence, V will range from 0 to 1.

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Example: The value of Cramer's V:

$$V = \sqrt{(3.333/30)/1 = 0.333}$$

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As can be seen, in this case $V = \phi$, which is always the case for a 2×2 table.

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Strength of association

4) **Lambda coefficient**

The lambda coefficient assumes that the variables are measured on a nominal scale.

Asymmetric lambda – measures the percentage improvement in predicting the value of the dependent variable, given the value of the independent variable.

Lambda also varies between 0 and 1. A value of 1 indicates that the prediction can be made without error.

Asymmetric lambda can be computed for each of the variables (treating it as the dependent variable). In general, the two asymmetric lambdas are likely to be different, since the marginal distributions are not usually the same.

Symmetric lambda – does not make an assumption about which variable is dependent. It measures the overall improvement when prediction is done in both directions.

Example: The value of asymmetric lambda, with usage as the dependent variable, is 0.333. This indicates that knowledge of gender increases our predictive ability by the proportion of 0.333 , i.e. a 33% improvement.

The symmetric lambda is also 0.333 .

Strength of association

5) **Non-parametric coefficients of association**

Note that in the calculation of the chi-square statistic, the variables are treated as being meaured only on a nominal scale.

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All these statistics use information about the ordering of categories of variables by considering every possible pair of cases in the table.

Each pair is examined to determine whether its relative ordering on the first variable is the same as its relative ordering on the second variable (concordant), the ordering is reversed (discordant), or the pair is tied.

Strength of association

The manner in which the ties are treated is the basic difference between these statistics:

tau b – makes an adjustment for ties and is the most appropriate when the table of variables is square.

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tau *b* value varies between +1 and -1 . Thus the direction (positive or negative) as well as the strength (how close the value is to 1) of the relationship can be determined.

Gamma also varies between +1 and -1 and generally has a higher numerical value than tan *b* or tau *c*.

Example: for our case as gender is a nominal variable, it is not appropriate to calculate ordinal statistics.

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5. Translate the results of hypothesis testing, strength of association and pattern of association into managerial implications and recommendations.

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