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11741 HW0 Minimum background test

1. $\frac{\nabla y}{\nabla x} = \sin(z) e^{-x} - x\sin(z)e^{-x}$

2. $Xy = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$

$X^{-1} = \begin{bmatrix} \frac{3}{2} & -2 \\ -\frac{1}{2} & 1 \end{bmatrix}$ invertible $|X| \neq 0$

$\text{rank}(X) = 2$

3. $\bar{X} = \frac{3}{5}$

$\text{var}(x) = \frac{6}{25}$ (biased) $\text{var}(X) = \frac{3}{10}$ (unbiased)

$P(\text{this data}) = \frac{1}{32}$

when $P(x=1) = 0.6$ prove: $f(x_1 \cdots x_5 | p) = p^3 \cdot (1-p)^2$

when $\frac{\nabla f}{\nabla p} = 0$ $p = \frac{3}{5}$

$P(x=T \mid y=b) = \frac{0.1}{0.25} = \frac{2}{5} = 0.4$

Modest background test

1. (a) false (b) true (c) false (d) false (e) true

Multivariate Gaussian $\frac{1}{\sqrt{(2\pi)^d \Sigma}} \exp(-\frac{1}{2} - (x-\mu)^T \Sigma (x-\mu))$

Exponential $\lambda e^{-\lambda x}$ when $x \geq 0$; 0 otherwise

Uniform $\frac{1}{b-a}$ when $a \leq x \leq b$; 0 otherwise

(要么成功要么失败) Bernoulli $p^x (1-p)^{1-x}$

n次的Bernoulli分布 Binomial $\binom{n}{x} p^x (1-p)^{n-x}$

(a) mean p variance $p(1-p)$ entropy $-p\log p - (1-p)\log(1-p)$

(b) $4\sigma^2$ σ^2

(a) $E(XY) = \int_{X,Y} xy\, f_{X,Y}(x,y) = \int_X x f_X(x) \int_Y y f_Y(y) = E(X)E(Y)$

(b) No, Yes

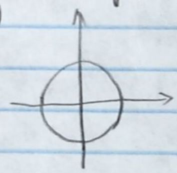
(a) by Law of Large Numbers, the empirical probability will converge to the theoretical probability

(b) by Lindeberg-Lévy CLT $\sqrt{n}(\bar{X} - \mu) \xrightarrow{d} N(0, \sigma^2)$

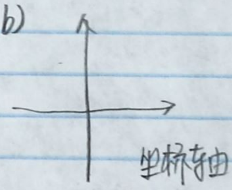
$\mu = \frac{1}{2}$ $\sigma^2 = \frac{1}{4}$

Linear Algebra

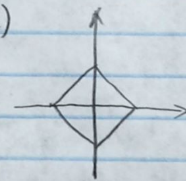
(a)



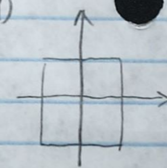
(b)



(c)



(d)



(a) $A \in \mathbb{R}^{n\times n}$ $\quad Ax = \lambda x$

(b)

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix} x = 0 \qquad (2-\lambda)^2 - 1 = 0 \qquad \lambda = 1 \text{ or } 3$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} x = x \qquad x_1 + x_2 = 0 \qquad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} x = x \qquad x_2 = 2x_1 \qquad \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$

(c) We know $A - \lambda_i I = 0$ for every i

Then $A^k = A^{k-1}(\lambda_i I) = \lambda_i (A^{k-1} I) = \lambda_i^k I^k = \lambda_i^k I$

Suppose $Ax_j = \lambda_i x_j$

Then $A^k x_j = A^{k-1}(Ax_j) = A^{k-1}\lambda_i x_j = \lambda_i A^{k-1} x_j$

$= \lambda_i^2 A^{k-1} x_j \cdots = \lambda_i^k x_j$

Each eigenvector, as x_j is still an eigenvector of A^k

(a)

(a) $a \in \mathbb{R}^{n\times 1}$

(b) $2Ax$ (A is symmetric) $\quad (A+A^T)x$ (A is not symmetric)

$2A$ (A is symmetric) $\quad (A+A^T)$

Geometry

(a) Consider two points $\vec{x_1}, \vec{x_2}$ lie on the line

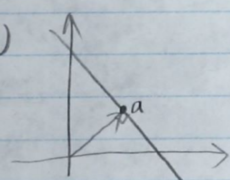
$w^T x_1 + b = 0$ ①

$w^T x_2 + b = 0$ ②

①−② $\quad w^T \underbrace{(x_1 - x_2)}_{\text{the line}} = 0$

w is orthogonal to the line

(b)



suppose there's a point $a^* = \operatorname{argmin} a^T a$

s.t. $w^T a^* + b = 0$

$$L = a^{*T} a^* - \lambda (w^T a^* + b)$$

$$\frac{\nabla L}{\nabla a^*} = 2a^* - \lambda w$$

$$a^* = \frac{\lambda w}{2}$$

$$\frac{\lambda}{2} w^T w + b = 0$$

$$\lambda = \frac{-2b}{w^T w}$$

$$a^* = -\frac{bw}{w^T w}$$

$$\|a^*\|_2 = \sqrt{a^{*T} a^*} = \sqrt{\frac{b^T b (w^T w)}{(w^T w)^2}}$$

$$= \frac{b}{\|w\|}$$