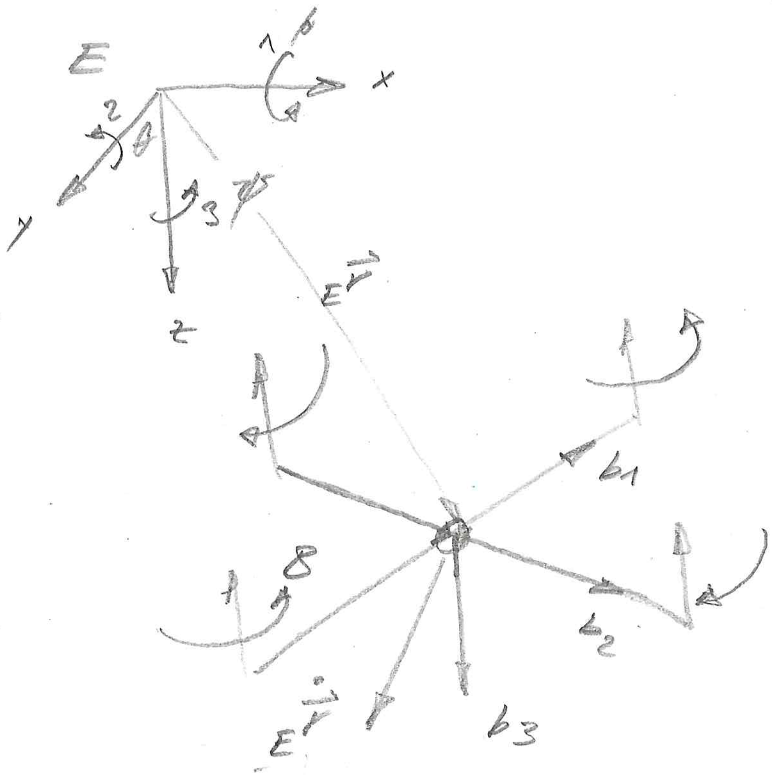
Kardan-Winkel



$$\underbrace{R_z(\psi)\, R_y(\theta)\, R_x(\phi)}_{C_{EB}}$$

earth ← body

$x \mapsto \phi$: Rollen / roll

$y \mapsto \theta$: Nicken / pitch

$z \mapsto \psi$: gieren / yaw

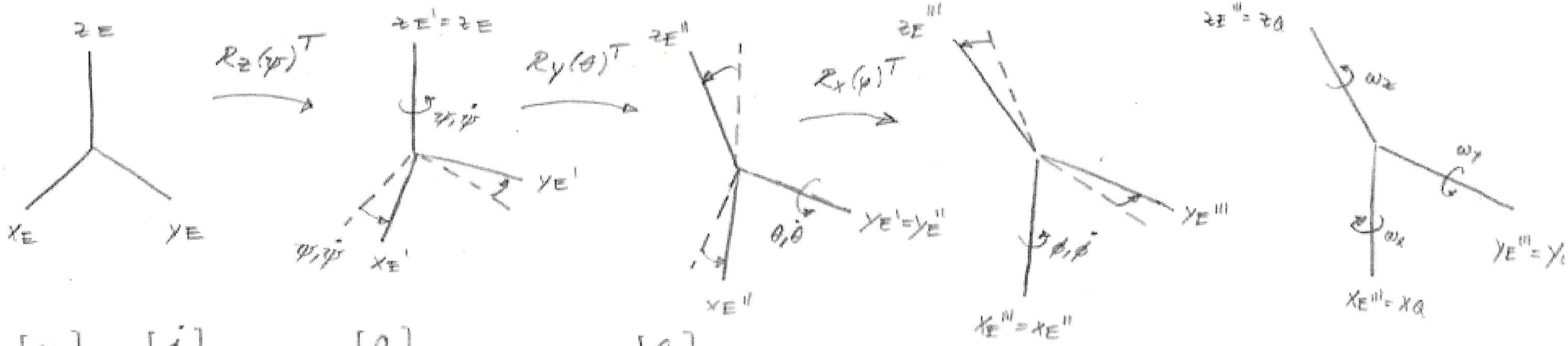
$$C_{BE} = C_{EB}^T = R_x^T(\phi)\, R_y^T(\theta)\, R_z^T(\psi)$$

body ← earth

$${}_B\dot{\vec{r}} = C_{BE}\; {}_E\dot{\vec{r}}$$

Projektion in Quadrokopter-Ebene

$${}_B\dot{\vec{r}}_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot {}_B\dot{\vec{r}}$$



$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}\dot\phi\\ 0\\ 0\end{bmatrix} + R_x(\phi)^T\begin{bmatrix}0\\ \dot\theta\\ 0\end{bmatrix} + R_x(\phi)^T R_y(\theta)^T\begin{bmatrix}0\\ 0\\ \dot\psi\end{bmatrix}$$

Herleitung Transformation Winkelgeschw.

$$\rightarrow \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0 & -s\theta\\ 0 & c\phi & c\theta s\phi\\ 0 & -s\phi & c\theta c\phi\end{bmatrix}}_{\Omega}\begin{bmatrix}\dot\phi\\ \dot\theta\\ \dot\psi\end{bmatrix}$$

$$\begin{bmatrix}\dot\phi\\ \dot\theta\\ \dot\psi\end{bmatrix} = \underbrace{\begin{bmatrix}1 & (s\phi s\theta)/c\theta & (c\phi s\theta)/c\theta\\ 0 & c\phi & -s\phi\\ 0 & s\phi/c\theta & c\phi/c\theta\end{bmatrix}}_{\Omega^{-1}}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}1 & s\phi\, t\theta & c\phi\, t\theta\\ 0 & c\phi & s\phi\\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta}\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

$$g_i = \omega_i + \beta_{gi} + w_{gi} \longleftrightarrow \omega_i = g_i - \beta_{gi} - w_{gi}$$

$$\dot{\beta}_{gi} = -\frac{1}{\tau_{gi}} \beta_{gi} + w_{\beta gi}$$

$$i = x, y$$

$$\dot{\phi} = \omega_x + \sin\phi \cdot \tan\theta \cdot \omega_y + \cos\phi \cdot \tan\theta \cdot \omega_z$$

$$\dot{\theta} = \cos\phi \cdot \omega_y + \sin\phi \cdot \omega_z$$

$$\dot{\phi} = (g_x - \beta_{gx} - w_{gx}) + \sin\phi \cdot \tan\theta \cdot (g_y - \beta_{gy} - w_{gy}) + \cos\phi \cdot \tan\theta \cdot (g_z - \beta_{gz} - w_{gz})$$

$$\dot{\theta} = \cos\phi \cdot (g_x - \beta_{gx} - w_{gx}) + \sin\phi \cdot \tan\theta \cdot (g_y - \beta_{gy} - w_{gy})$$