SV

$\dot{\omega}$ $\quad J\cdot\dot{\omega} + \omega\,[J\cdot\omega]_x = \sum_i {}_B M_i$

$\dot{R}$ $\quad \dot{R} = R\cdot[\omega]_x$

$R = C_{IB}$

$R^T = C_{BI}$

$$[a]_x = \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix}$$

${}_I\dot{v}$ $\quad m\,{}_I\dot{v} = \sum_i {}_I F_i$

${}_B\dot{v}$ $\quad m\cdot{}_B\dot{v} + \omega\cdot[m\cdot{}_B v]_x = \sum_i {}_B F_i$

${}_I p$ $\quad {}_I\dot{p} = {}_I v = C_{IB}\,{}_B v$

${}_B x$: x w.r.t (B) : body

${}_I x$: x w.r.t (I) : inertial

(E) : earth

J : inertia

ω : angular rate

M : Torque

R : Rotation matrix

m : mass

F : Force

v : velocity

p : position

$$\frac{1}{m}\sum_i {}_B F_i = \underbrace{C_{BI}\cdot g}_{\text{gravity w.r.t body}} - \underbrace{\begin{pmatrix} k_x & & 0 \\ & k_y & \\ 0 & & \underbrace{k_z}_{\approx 0} \end{pmatrix}\cdot {}_B v}_{\text{drag (normalized with m)}} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \frac{T}{m} \end{pmatrix}}_{\text{thrust}}$$

(no wind)

Acc measurement (IMU), no noise, no bias, no wind

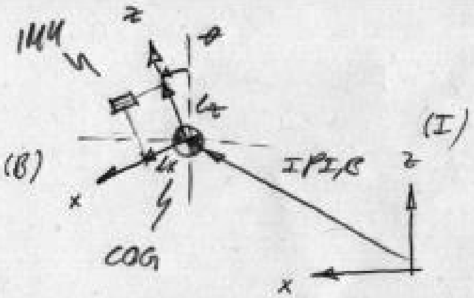
$${}_B y_a = \frac{1}{m}\sum_i {}_B F_i - C_{BI}\cdot g = \Big|_{k_z=0} \begin{pmatrix} -k_x\cdot{}_B v_x \\ -k_y\cdot{}_B v_y \\ \frac{T}{m} \end{pmatrix} \quad //$$

↳ acceleration measured from the quad w.r.t body frame

gravity gets canceled, while hovering we measure $\frac{T}{m} \approx 9.81\ m/s^2$

2D (x,z)

$${}_B y_a = \begin{pmatrix} -k_x\cdot{}_B v_x \\ \frac{T}{m} \end{pmatrix} \quad //$$



$$_I P_{I,IMU} = {}_I P_{I,B} + {}_I P_{B,IMU}$$

$$= \begin{pmatrix} x \\ z \end{pmatrix} + \underbrace{C_{EB} \cdot \begin{pmatrix} l_x \\ l_z \end{pmatrix}}_{\text{offset of IMU rel. to COG w.r.t. body}} \qquad C_{EB} = R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} l_x \cos\theta + l_z \sin\theta \\ -l_x \sin\theta + l_z \cos\theta \end{pmatrix}$$

$$_I \dot{P}_{I,IMU} = \begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} + \begin{pmatrix} -l_x \sin\theta\,\dot{\theta} + l_z \cos\theta\,\dot{\theta} \\ -l_x \cos\theta\,\dot{\theta} - l_z \sin\theta\,\dot{\theta} \end{pmatrix}$$

$$_I \ddot{P}_{I,IMU} = \begin{pmatrix} \ddot{x} \\ \ddot{z} \end{pmatrix} + \begin{pmatrix} -l_x \cos\theta\,\dot{\theta}^2 - l_z \sin\theta\,\dot{\theta}^2 - l_x \sin\theta\,\ddot{\theta} + l_z \cos\theta\,\ddot{\theta} \\ l_x \sin\theta\,\dot{\theta}^2 - l_z \cos\theta\,\dot{\theta}^2 - l_x \cos\theta\,\ddot{\theta} - l_z \sin\theta\,\ddot{\theta} \end{pmatrix}$$

$$= \begin{pmatrix} \ddot{x} \\ \ddot{z} \end{pmatrix} + \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} -l_x \\ -l_z \end{pmatrix} \cdot \dot{\theta}^2 + \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} l_z \\ -l_x \end{pmatrix} \cdot \ddot{\theta}$$

$$= \begin{pmatrix} \ddot{x} \\ \ddot{z} \end{pmatrix} + C_{EB} \Bigg(\underbrace{\begin{pmatrix} -l_x \\ -l_z \end{pmatrix} \dot{\theta}^2 + \begin{pmatrix} l_z \\ -l_x \end{pmatrix} \ddot{\theta}}_{\text{this is acceleration due to the offset w.r.t. body}} \Bigg) \quad //$$

→ Acc measurement (IMU), no noise, no bias, no wind
+ offset to COG

$$_B y_a = \begin{pmatrix} -k_x \cdot {}_B v_x - l_x \dot{\theta}^2 + l_z \ddot{\theta} \\ \frac{T}{m} - l_z \dot{\theta}^2 - l_x \ddot{\theta} \end{pmatrix} \quad //$$