The Calculus of Statistics

Blaise Whitesell Jeremiah Gelb

May 24, 2015

1 Probability Distribution Functions

1.1 Probability Density Functions

A probability density function (**PDF**) describes the relative likelihood $f(x)$ of possible outcomes for a continuous random variable.

- The probability of any x occurring is either positive or zero, so a PDF can never be negative.

$$f(x) \geq 0 \text{ for all } x$$

- The area under the curve must equal 1

$$\int_a^b f(x)\,\mathrm{d}x = 1$$

  where a and b are the lower and upper bounds, often $-\infty$ and ∞.

1.2 Cumulative Distribution Functions

A cumulative distribution function (**CDF**) gives the probability $F(x)$ that the outcome of a continuous random variable will be less than or equal to x.

- A CDF is monotonically increasing on the interval (a,b).

$$F(a) = 0\ ; \quad F(b) = 1$$

1.3 Calculus (placeholder)

2 The Uniform Density Function

2.1 Definition

A uniform density function describes a continuous random variable where every outcome on an interval (a,b) is equally likely.

$$f(x) = \kappa$$

2.2 Implications

- The function looks like a rectangle with length $(b - a)$ and height κ
- Area $= 1 = (b - a) * \kappa$
- Therefore, we have an explicit definition for κ:

$$\kappa = \frac{1}{b - a}$$

2.3 Equation of a uniform density function

The formal equation is piecewise. It depends only on the bounds a and b.

$$f(x) = \begin{cases} 0 & \text{if } x < a, \\ \frac{1}{b-a} & \text{if } a \leq x \leq b, \\ 0 & \text{if } x > b. \end{cases}$$