- #paper/read ~ 2021 CE ~ Embedding, Modularity
  - **Evaluating Word Embeddings with Categorical Modularity**
  - https://arxiv.org/abs/2106.00877
  - https://github.com/enscma2/categorical-modularity
  - Mentioned papers:
    - Distributed Representations of Words and Phrases
    - GloVe
    - Enriching Word Vectors with Subword Information
    - Cross-lingual Word Embedding Models Survey
    - Word Translation Without Parallel Data
    - Human Brain Activity for Machine Attention

Summary

- Evaluation of embeddings may be *extrinsic* (downstream task performance is measured) or *intrinsic* (direct testing of how well embeddings capture Semantic or syntactic properties).
- **Categorical modularity metric** employs 500 words drawn from brain-based semantic categories. All words are translated into 29 Languages.

The technique

1. Calculate some distance Function for all embedding pairs.
   - Cosine Similarity is used in the paper.
   - The resulting Matrix M_D is *symmetrical*.
2. For a given $k \in \mathbb{Z}_+$, build an adjacency Matrix M_N for the resulting kNN Graph.
   - This one is *asymmetrical* though!
3. Let m be the total number of edges in the kNN graph.
   - To calculate it from M_N, let's count all the edges in the symmetrical version of the matrix and divide that by two: `m = np.sum(np.fmax(M_N, M_N.T)) // 2`.
4. The fraction of the expected number of edges within the category c:

$$a_c = \frac{1}{2m} \sum_{i,j} M_{N_{i,j}} \mathbb{1}(c_i = c)$$

5. The fraction of edges that connect words of the same semantic category c:

$$e_c = \frac{1}{2m} \sum_{i,j} M_{N_{i,j}} \mathbb{1}(c_i = c) \mathbb{1}(c_j = c)$$

6. The overall modularity Q is calculated as follows:

$$Q = \sum_c (e_c - a_c^2)$$

7. Finally, it should be normalized by setting:

$$Q_{max} = 1 - \sum_c a_c^2$$

$$Q_{norm} = \frac{Q}{Q_{max}}$$

8. A higher value of Q_{norm} indicates that a higher number of words that belong to the same categories are connected in the graph.

Notes

- Categorical modularity seems to reveal how well models map to the human Brain.
  - This is especially true of Regression tasks such as Word Similarity.
  - It may hint at how linguistic Information is encoded in the brain.
