THEORY OF MACHINES AND MECHANISMS II

Mechanical IV/I

Chapter 9

Approximate Numerical Methods

When the degrees of freedom of a system become large, the problem of obtaining numerical results becomes difficult. It is necessary to rely on the high-speed electronic computer for the solution.

Although the problem of finding the eigenvalues and eigenvectors of a matrix equation is routinely handled by the electronic computer, there are approximate and the other alternative procedures which are often useful.

Some of the numerical methods used for solving multidegrees of freedom system are

- Rayleigh method,
- Dunkerley method,
- Rayleigh-Ritz method,
- Matrix iteration method, etc.

9.1 Rayleigh's Method

This method developed by Rayleigh is very handy for finding the fundamental frequency of a multi degree of freedom system. It is based upon equating the maximum kinetic energy of the vibrating system to the maximum potential energy.

For multi-degree of freedom system there are many masses and thus many components of kinetic and potential energies, but all the masses will have simple harmonic motions passing through their mean position at the same instant, for any principal mode of vibration.

Good estimate of the fundamental frequency can be made by assuming the suitable deflection curve according to the boundary conditions. The maximum kinetic energy is then equated to the maximum potential energy of the system to get determine the natural frequency *i.e.*,

$$\frac{1}{2}\sum M_i y_i^2 \omega_n^2 = \frac{1}{2}\sum M_i g y_i$$

$$\omega_n^2 = \frac{g\sum M_i y_i}{\sum M_i y_i^2} \qquad \ldots\ldots\ldots\ldots (1)$$

where M_i and y_i are the mass and the deflection at point i.

9.2 Dunkerley's Method

Rayleigh's principle, which gives the upper bound to the fundamental frequency, can now be complemented by Dunkerley's formula, which results in a lower bound to the fundamental frequency.

Dunkerley's equation can be written as

$$\frac{1}{\omega_n^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{\omega_s^2} \qquad \ldots\ldots\ldots\ldots (2)$$

where ω_n is the fundamental natural frequency of the system; ω_1, ω_2, ω_3, …. are the natural frequencies of the system with each mass acting separately at its point of application in the absence of other masses; and ω_s is the natural frequency of the shaft alone due to its distributed mass.

9.3 Rayleigh-Ritz Method

The Ritz method is essentially the Rayleigh method in which the single shape function is replaced by a series of shape functions multiplied by constant coefficients. The coefficients are adjusted by minimizing the frequency with respect to each of the coefficients, which results in n equations in ω2. The solution of these equations then gives the natural frequencies and mode shapes of the system.

9.4 Matrix Iteration Method

With the use of *flexibility matrix* in the differential equations, this method is used when only the lowest eigenvalue and eigenvector of a multidegree of freedom system are desired. The advantage of this method is that the iterative process here results in the principal mode of vibration of the system and the corresponding natural frequency.

The equation of motion in terms of flexibility matrix can be written as

$$[A][M]\{\ddot{x}\} + \{x\} = 0 \quad \ldots\ldots\ldots\ldots (3)$$

Using $\{x\} = \{X\} \sin\omega t$ the above equation results into

$$\{X\} = \omega^2 [A] [M] \{X\} \quad \ldots\ldots\ldots\ldots (4)$$

which may be written as

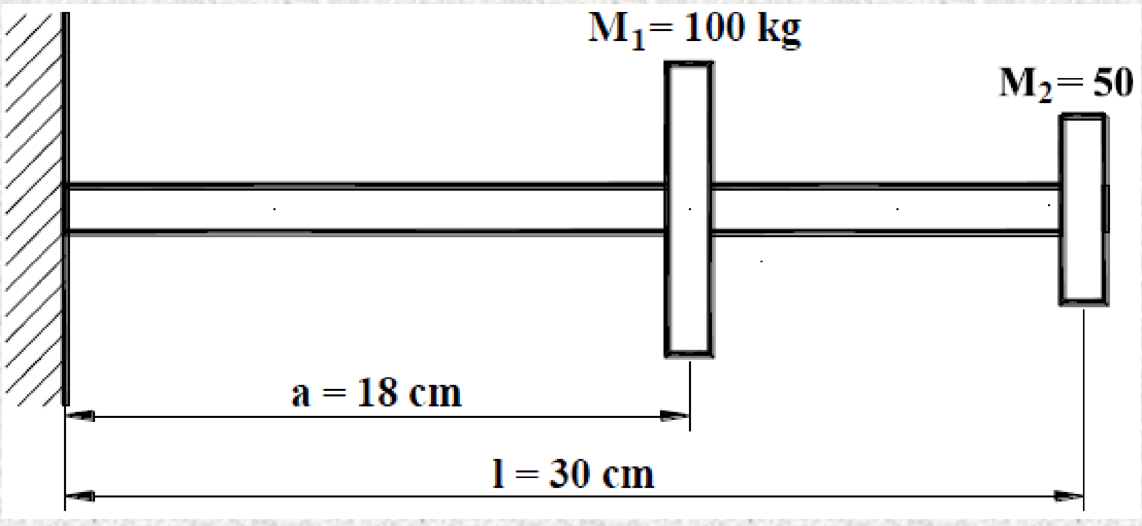
$$\{X\} = \omega^2 [B] \{X\} \quad \ldots\ldots\ldots\ldots (5)$$

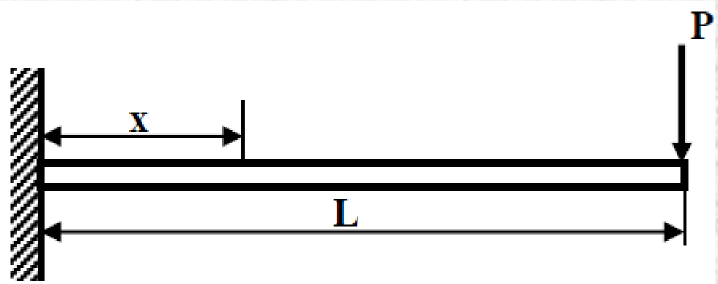
where $[B] = [A] [M]$

Example 9.1

Find the lower natural frequency of vibration for the system shown in **Figure E9.1** by

a) Rayleigh method
b) Dunkerley method. *Take* $E = 1.96 \times 10^{11} N/m^2$, $I = 4 \times 10^{-7} m^4$.

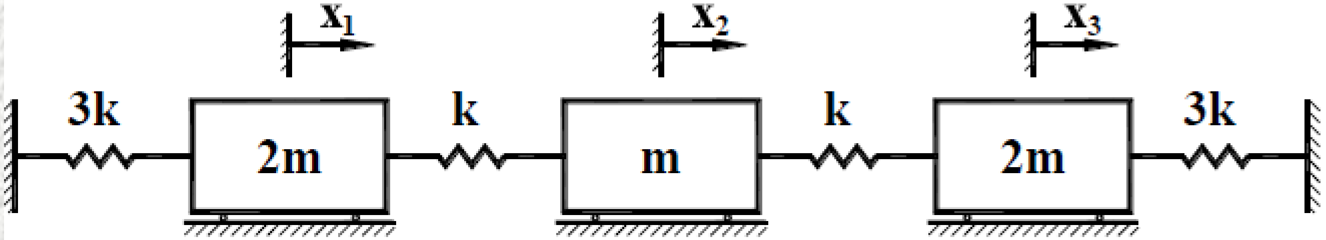




$$y_L = \frac{PL^3}{3EI}$$

$$y_x = \frac{Px^2(3L - x)}{6EI}$$

354.94 rad/s, 348.8 rad/s

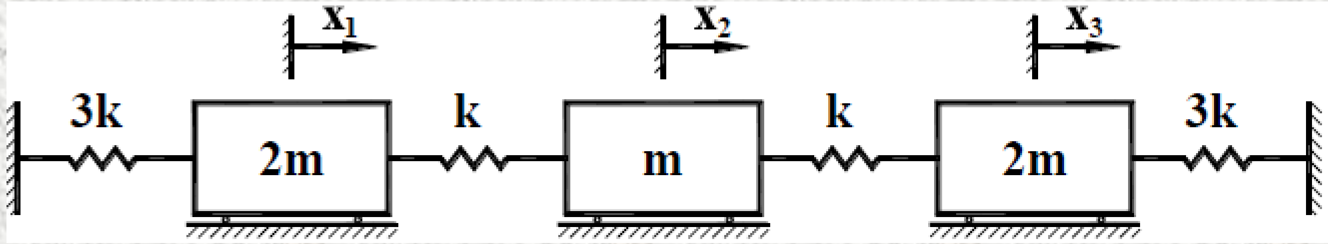


$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 4k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 4k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$[A] = \begin{bmatrix} \frac{7}{24k} & \frac{1}{6k} & \frac{1}{24k} \\ \frac{1}{6k} & \frac{2}{3k} & \frac{1}{6k} \\ \frac{1}{24k} & \frac{1}{6k} & \frac{7}{24k} \end{bmatrix}$$

Example 9.2

For the system shown in **Figure E9.2**, determine the fundamental natural frequency and corresponding mode shape.



$$\begin{bmatrix} \frac{k}{m} \\ \frac{2k}{m} \\ \frac{3k}{m} \end{bmatrix}, \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$