Department of Mechanical Engineering, Pulchowk campus, Institute of Engineering, Tribhuvan University

ENGINEERING ECONOMICS

Project Evaluation Techniques

Annual Equivalent Value Analysis

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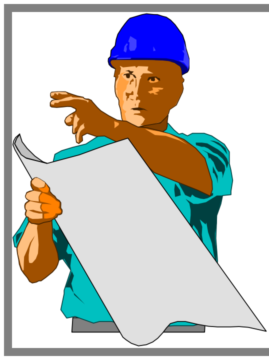
2018

Lecture 8



Annual Equivalent Value Analysis

The annual equivalent value (AE) criterion is a basis for **measuring investment value by determining equal payments** on an **annual basis**.

First, we have to **find the NPV** of the project and **then convert** it to **equal annual payments**.

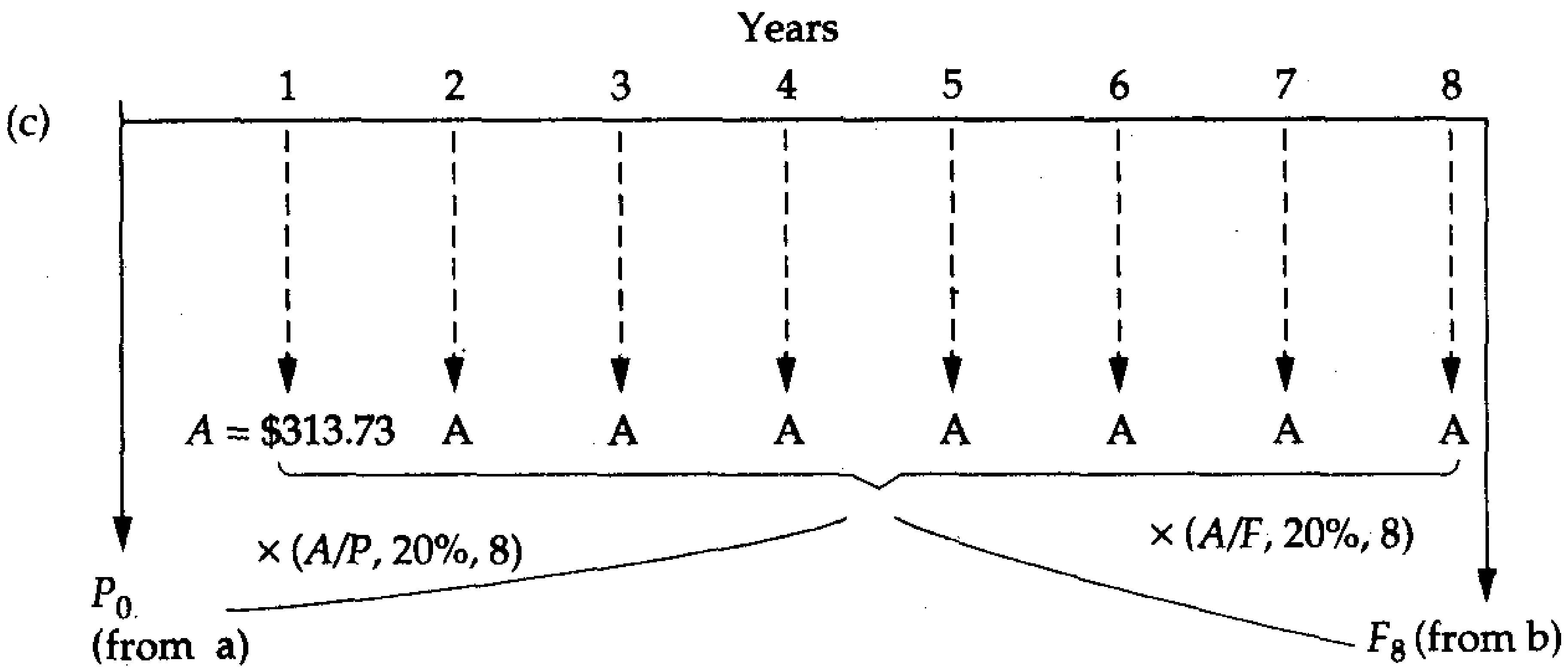
$$AE(i) = PV(i)(A/P, i, n)$$

If AE>0, accept the project

If AE =0, remain indifferent

If AE <0, reject the project

Annual Equivalent Value



Benefits of AE analysis

1. **Consistency of report format:** Financial and engineering managers may prefer to work on yearly costs rather than overall costs.

2. **Need for unit costs:** In many situations, project must be broken down into unit costs for comparison and ease.

3. **Unequal project lives:** Comparing projects with unequal service lives is complicated in calculations, but using AE analysis, this problem can be easily solved.

Example: Make or Buy Option

Ampex Corporation currently **produces** both **videocassette cases** and **metal particle magnetic tape** for commercial use.

An increased demand for metal particle tapes is projected, and Ampex is deciding between

- **increasing the internal production of empty cassette cases** and magnetic tape or
- **purchasing empty cassette cases** from an outside vendor.

If it purchases the cases from a vendor, the **company must also buy specialized equipment to load the magnetic tapes**, since its current loading machine is not compatible with the cassette cases produced by the vendor under consideration.

The projected **production rate of cassettes is 79,815 units per week for 48 weeks of operation per year**. The planning horizon is **seven years**. **MARR is 14%**, calculate the **unit cost under each option.**

Example: Make or Buy Option

- Make option (annual costs):

Labor	$1,445,633
Materials	$2,048,511
Incremental overhead	$1,088,110
Total annual cost	$4,582,254

- Buy option:

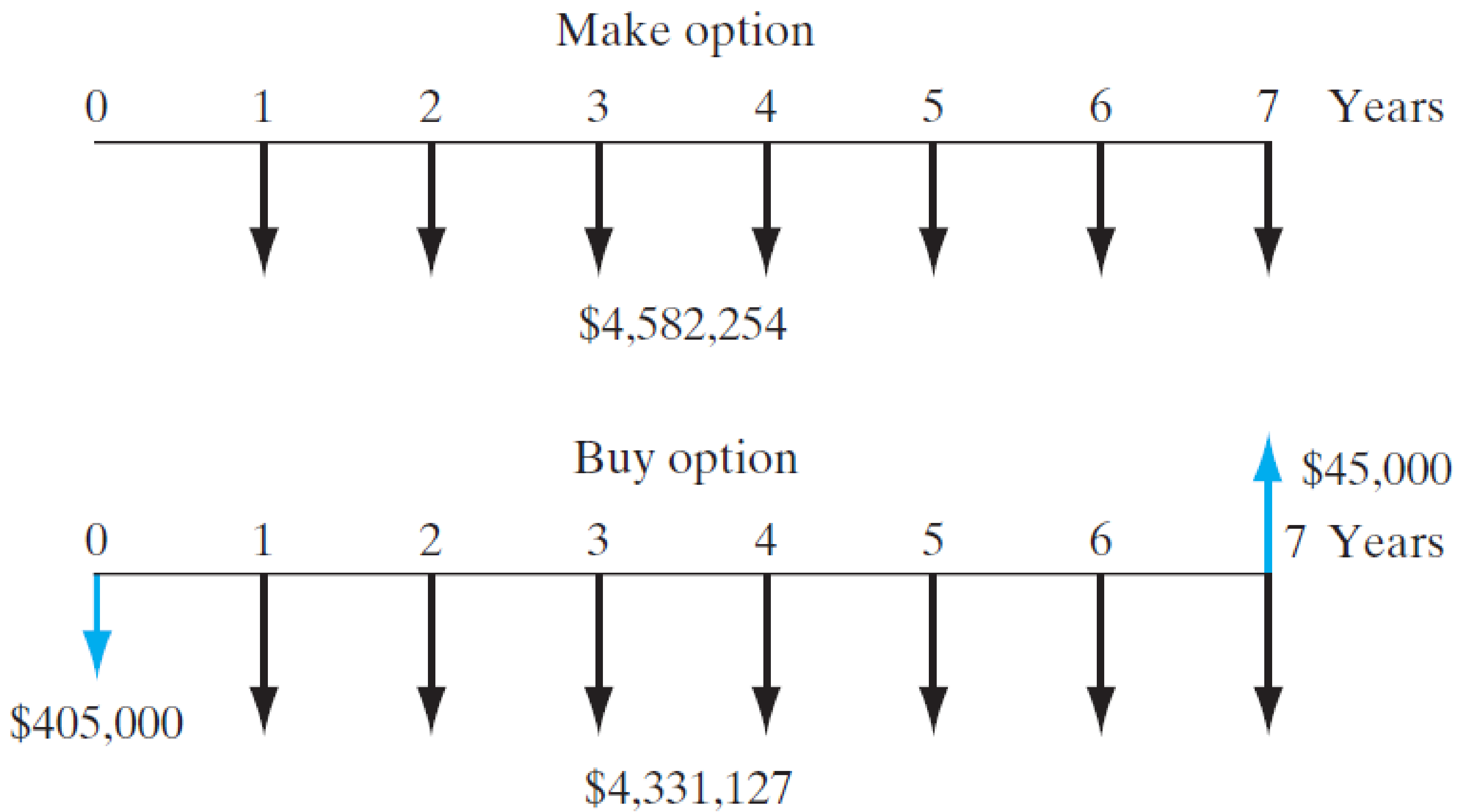
Capital costs:

Acquisition of a new loading machine	$405,000
Salvage value at end of 7 years	$ 45,000

Annual operating costs:

Labor	$ 251,956
Purchasing empty cassette ($0.85/unit)	$3,256,452
Incremental overhead	$ 822,719
Total annual operating costs	$4,331,127

Example: Make or Buy Option



Example: Make or Buy Option

- **Make option.** Since the "make option" is already given on an annual basis, the equivalent annual cost will be

$$AEC(14\%)_{Make} = \$4{,}582{,}254.$$

- **Buy option.** The two cost components are capital cost and operating cost.

Capital cost:

$$\begin{aligned} CR(14\%) &= (\$405{,}000 - \$45{,}000)(A/P, 14\%, 7) \\ &\quad + (0.14)(\$45{,}000) \\ &= \$90{,}249 \end{aligned}$$

$$AEC(14\%)_1 = CR(14\%) = \$90{,}249.$$

Operating cost:

$$AEC(14\%)_2 = \$4{,}331{,}127.$$

Total annual equivalent cost:

$$AEC(14\%)_{Buy} = AEC(14\%)_1 + AEC(14\%)_2 = \$4{,}421{,}376.$$

Example: Make or Buy Option

$$79{,}815 \text{ units/week} \times 48 \text{ weeks} = 3{,}831{,}120 \text{ units per year.}$$

- **Make option:**

$$\text{Unit cost} = \$4{,}582{,}254/3{,}831{,}120 = \$1.20/\text{unit.}$$

- **Buy option:**

$$\text{Unit cost} = \$4{,}421{,}376/3{,}831{,}120 = \$1.15/\text{unit.}$$

Example: Unit cost/saving

Tiger Machine Tool Company is considering the proposed acquisition of a new **metal-cutting machine**. The required initial investment of $75,000 and the projected cash benefits' over the project's 3-year life are as follows. Suppose that the machine will be operated for 2000 hours per year. Compute the equivalent savings per machine hour at i = 15%.

Given: NPW = $3553, N = 3 years, i = 15% per year, 2000 machine hours per year

Find: Equivalent savings per machine hour

Solution,

NPW(15%) = -75,000

+ 24,400 (P/F, 15%, 1)

+ 27,340 (P/F, 15%, 2)

+ 55,760 (P/F, 15%, 3)

= 3,553

End of Year	Cash Flow
0	−$75,000
1	24,400
2	27,340
3	55,760

$$AE(15\%) = \$3553(A/P, 15\%, 3) = \$1556.$$

With an annual usage of 2000 hours, the equivalent savings per machine hour would be

$$\text{Savings per machine hour} = \$1556/2000 = \$0.78/\text{hr}.$$

If Annual Operating Hours **Fluctuate**,

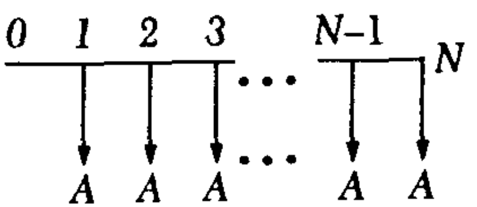
then we have to find out the **effective Annual Operating Hours**

Suppose: 1st Year= 1,500, 2nd Year= 2,500 and 3rd Year = 2,000

Effective Annual Operating Hour = **[** (1,500)(P/F, 15%, 1) + (2,500)(P/F, 15%, 2) + (2,000)(P/F, 15%, 3) **]** (A/P, 15%, 3)

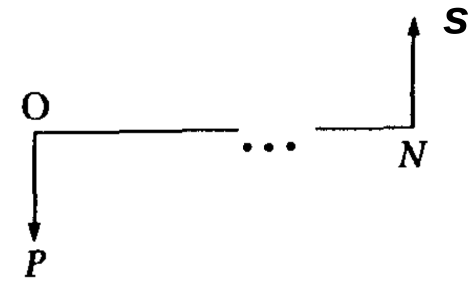
=1,976.16

Operating costs and capital costs



Operating costs are incurred by the operations of the plant or factory.

Capital costs are incurred during the start of the project like initial investment or initial borrowing.



Capital costs are **incurred only one time** in the project life, where as **operating costs incur annually**.

The **annual equivalent** of the capital cost is **capital recovery cost** 'CR'.

Initial Investment

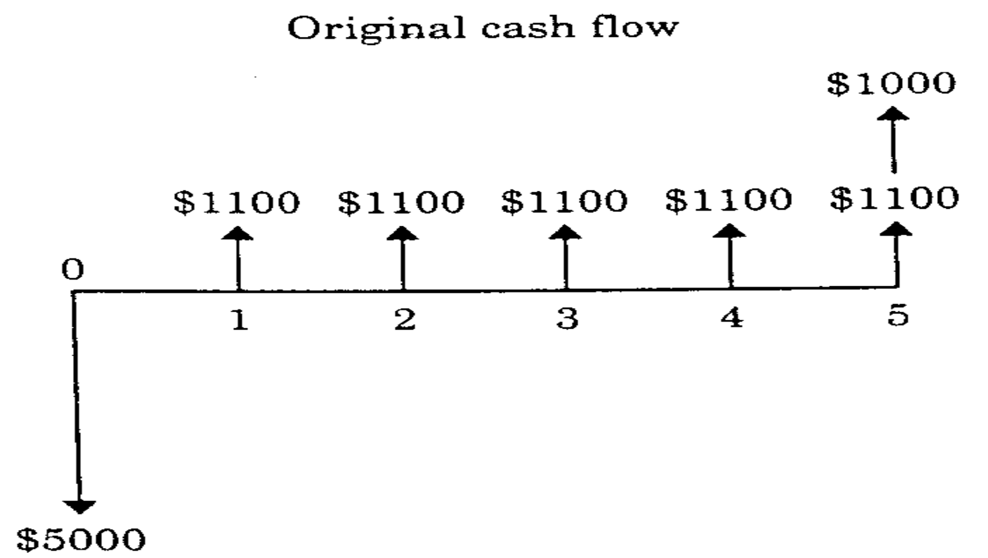
$$CR(i) = P(A/P,i,n) - S(A/F,i,n)$$

Salvage Value

$$CR(i) = (P-S)(A/P,i,n) - i \times S$$

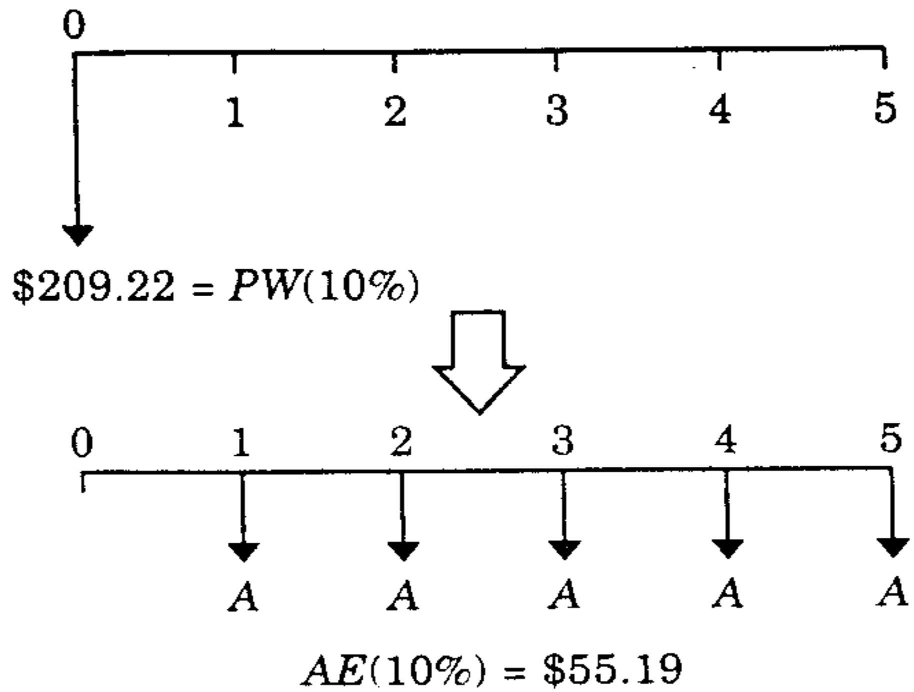
Example 1

Consider a machine that costs $5000 and has a 5-year useful life. At the end of the 5 years, it can be sold for $1000 after tax adjustment. If the firm could earn an after-tax revenue of $1100 per year with this machine, should it be purchased at an interest rate of 10%? **Find: AE, and determine whether to purchase**



Given: I = $5000,
S = $1000, A = $1100,
N = 5 years, i = 10% per year

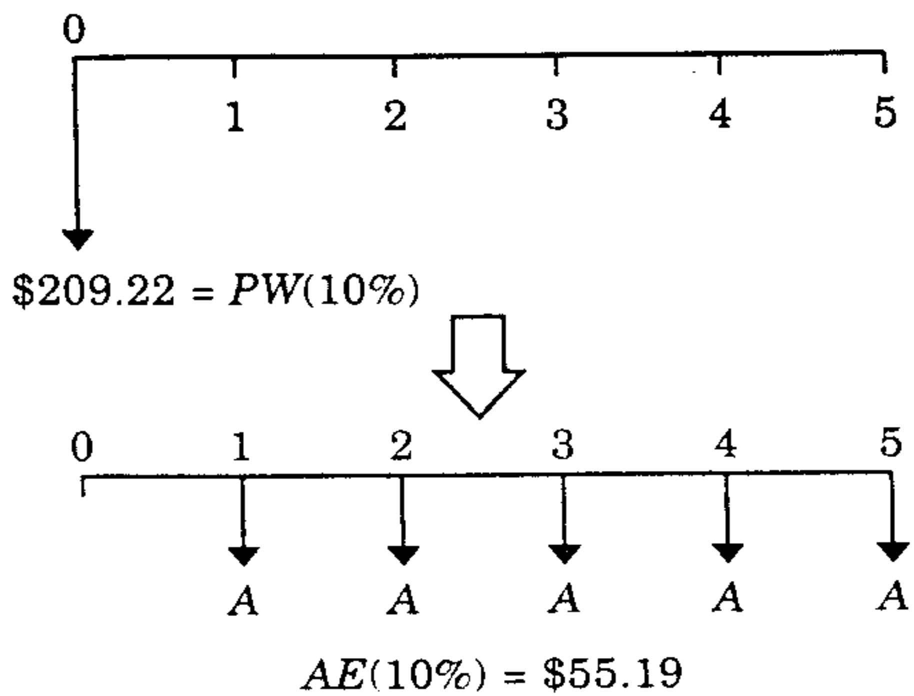
Method 1:



10.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000	0.0000	0.0000	1
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264	2
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291	3
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812	4.3781	4
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101	6.8618	5
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296	2.2236	9.6842	6
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054	2.6216	12.7631	7
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874	3.0045	16.0287	8
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736	3.3724	19.4215	9
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627	3.7255	22.8913	10
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540	4.0641	26.3963	11
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468	4.3884	29.9012	12
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988	33.3772	13
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955	36.8005	14
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789	40.1520	15

Method 1:

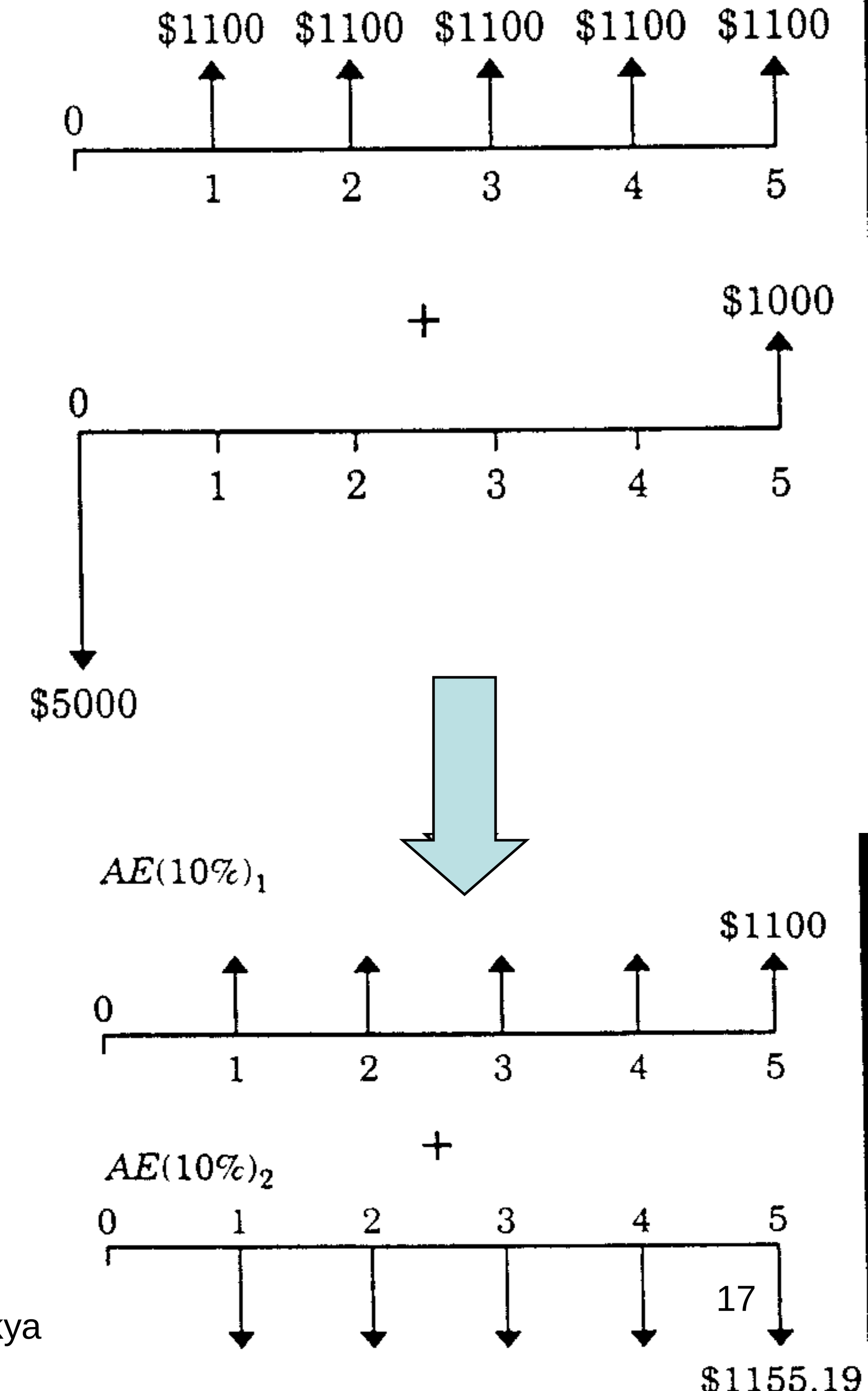


$$\begin{aligned}
PW(10\%) &= -\$5000 + \$1100(P/A, 10\%, 5) \\
&\quad + \$1000(P/F, 10\%, 5) \\
&= -\$5000 + \$1100(3.7908) + \$1000(0.6209) \\
&= -\$209.22 \\
AE(10\%) &= -\$209.22(A/P, 10\%, 5) = -\$55.19.
\end{aligned}$$

Method 2:

$$CR(i) = (P - S)(A/P, i, N) + iS$$

$$AE(i)_1 = -CR(i)$$



10.0%

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2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264	2
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291	3
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812	4.3781	4
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101	6.8618	5
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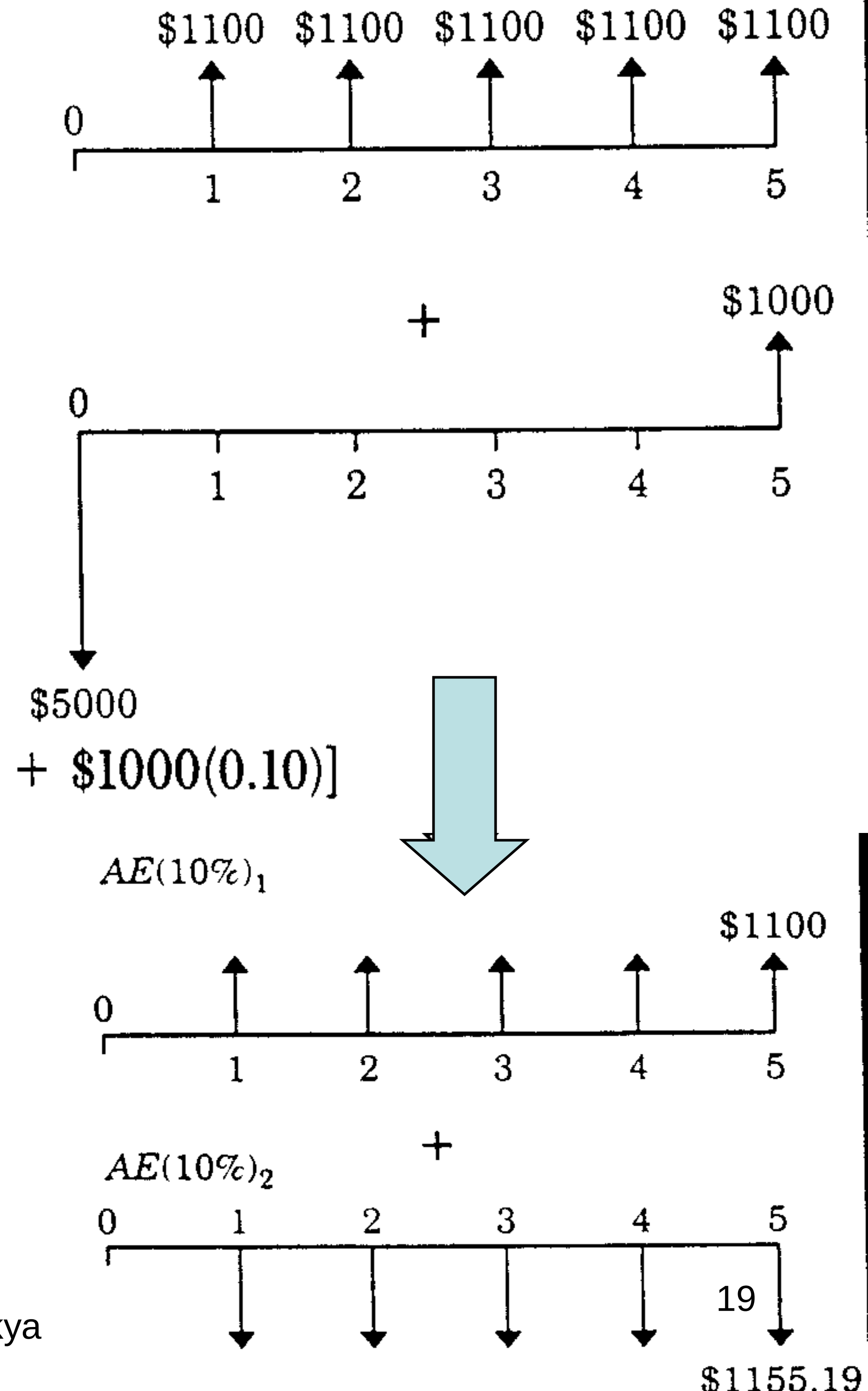
Method 2:



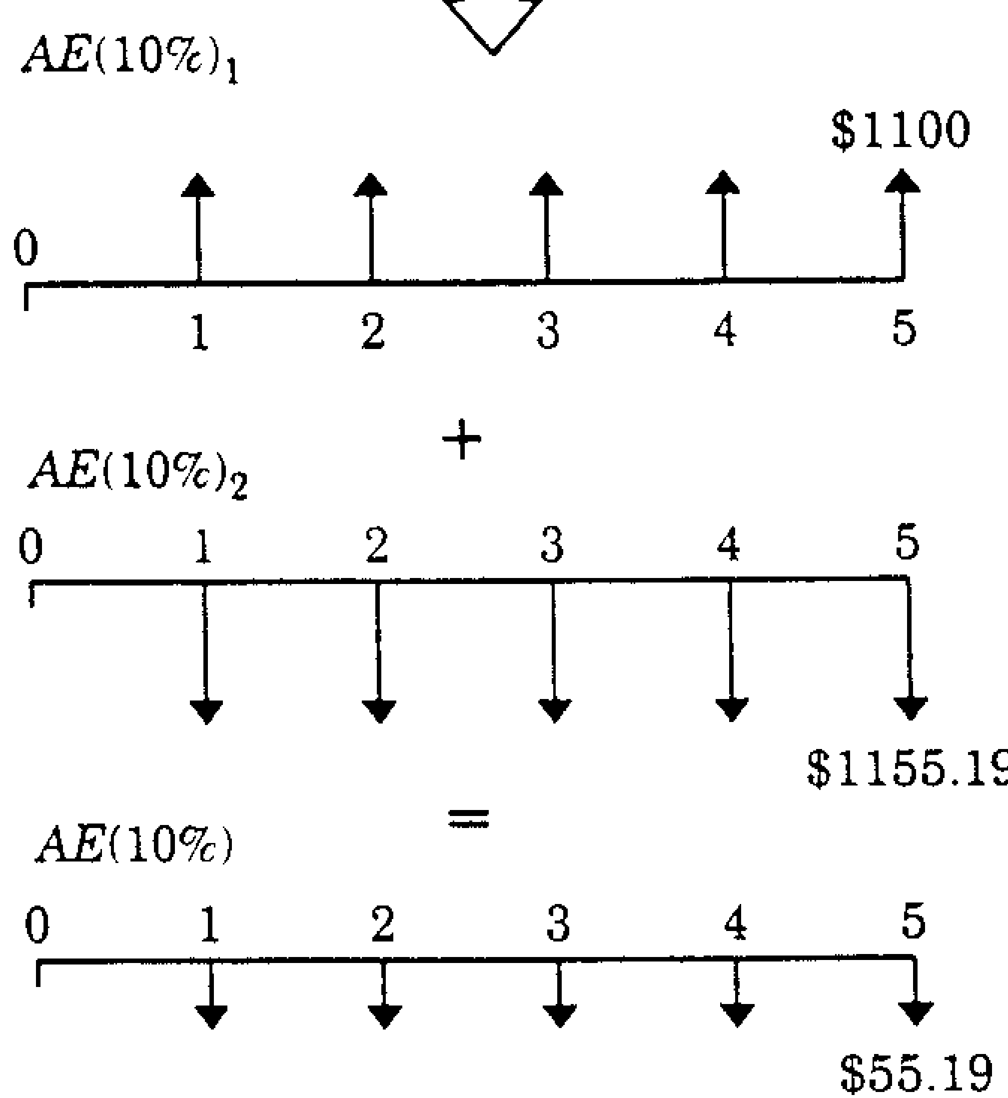
$$CR(i) = (P - S)(A/P,i,N) + iS$$

$$AE(i)_1 = -CR(i)$$

$$= -[(\$5000 - \$1000)\,(A/P, 10\%, 5) + \$1000(0.10)]$$

$$= -\$1155.19$$

$$AE(i)_2 = \$1100$$



$$
\begin{aligned}
AE(10\%) &= AE(i)_1 + AE(i)_2 \\
&= -\$1155.19 + \$1100 \\
&= -\$55.19.
\end{aligned}
$$

Example: Break-even point-per unit of Equipment use

Sam purchased van worth $11,000 for office work use. If I = 6%, what should be the reimbursement rate per kilometer that should be claimed to his office so that Sam can break even?

	First Year	Second Year	Third Year
Expected kilometres driven	14,500	13,000	11,500
Depreciation	2,879	1,776	1,545
Scheduled maintenance	100	153	220
Insurance	635	635	635
Registration and taxes	78	57	50
Total ownership cost	$ 3,692	$ 2,621	$ 2,450
Nonscheduled repairs	35	85	200
Replacement tires	35	30	27
Accessories	15	13	12
Gasoline and taxes	688	650	522
Oil	80	100	100
Parking and tolls	135	125	110
Total operating costs	$ 988	$ 1,003	$ 971
Total of all costs	$ 4,680	$ 3,624	$ 3,421

Suppose Company pays Sam \$X per kilometer for his personal car

Year	Total Kilometres Driven	Reimbursement (\$)
1	14,500	$(X)(14{,}500) = 14{,}500X$
2	13,000	$(X)(13{,}000) = 13{,}000X$
3	11,500	$(X)(11{,}500) = 11{,}500X$

the annual equivalent reimbursement would be **AE(6%)**

$$[14{,}500X(P/F,6\%,1) + 13{,}000X(P/F,6\%,2) + 11{,}500X(P/F,6\%,3)]\,(A/P,6\%,3) = 13{,}058X.$$

The annual equivalent costs of owning and operating would be

$$[\$4680(P/F,6\%,1) + \$3624(P/F,6\%,2) + \$3421(P/F,6\%,3)](A/P,6\%,3) = \$3933.$$

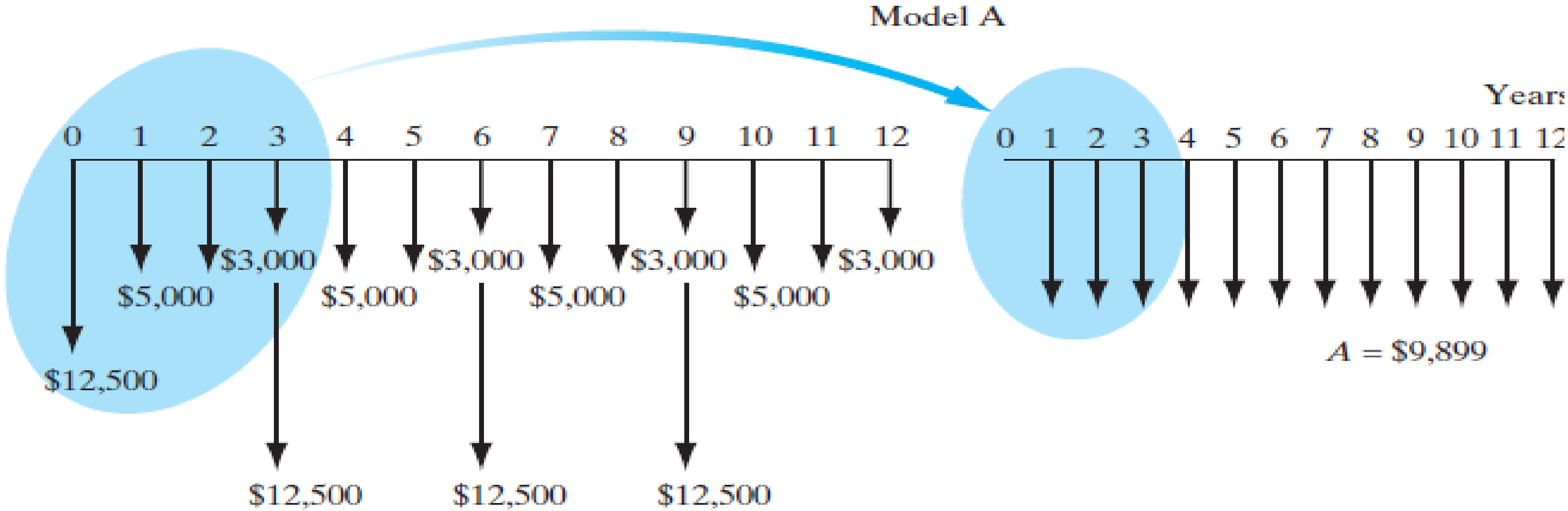
Then, the minimum reimbursement rate should be

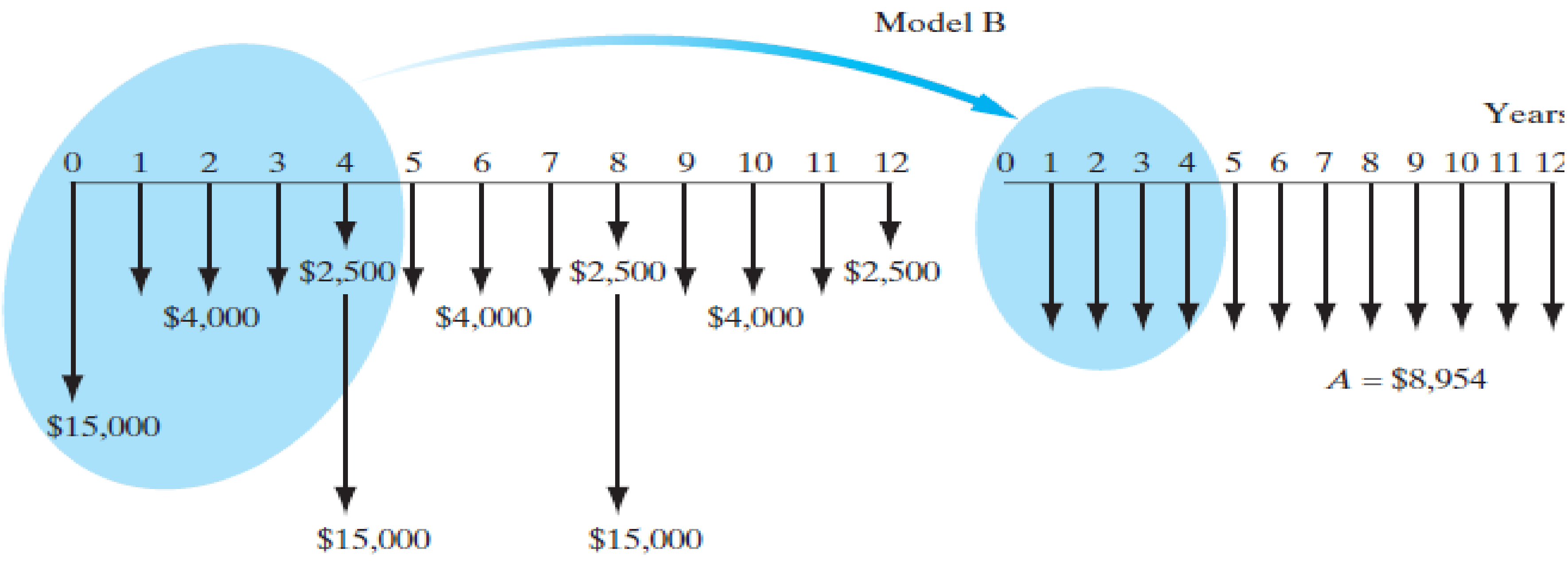
$$13{,}058X = \$3933$$

$$X = 30.12 \text{ cents per kilometre.}$$

Example: Comparison of projects

i = 15% per year **Find: AE, and which alternative is preferred**

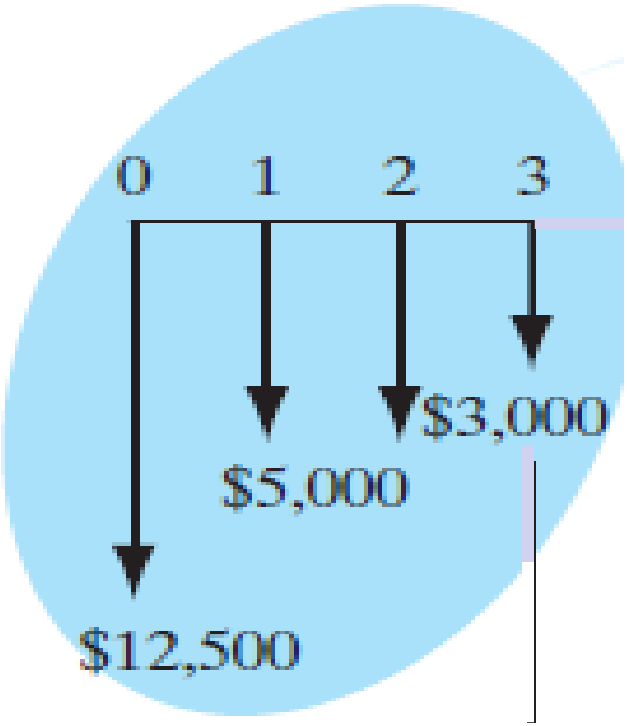




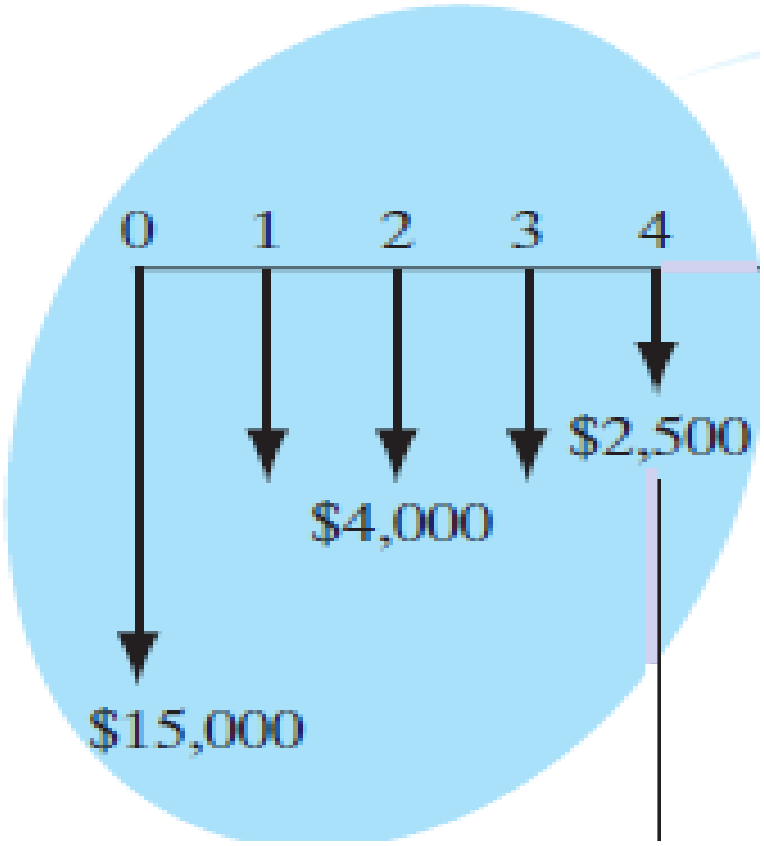
Example: Comparison of projects

i = 15% per year **Find: AE, and which alternative is preferred**

Model A



Model B



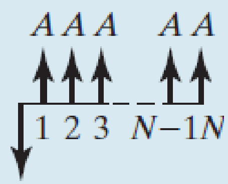
15.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	**Compound Amount Factor *(F/P,i,N)***	**Present Worth Factor *(P/F,i,N)***	**Compound Amount Factor *(F/A,i,N)***	**Sinking Fund Factor *(A/F,i,N)***	**Present Worth Factor *(P/A,i,N)***	**Capital Recovery Factor *(A/P,i,N)***	**Gradient Uniform Series *(A/G,i,N)***	**Gradient Present Worth *(P/G,i,N)***	***N***
1	1.1500	0.8696	1.0000	1.0000	0.8696	1.1500	0.0000	0.0000	1
2	1.3225	0.7561	2.1500	0.4651	1.6257	0.6151	0.4651	0.7561	2
3	1.5209	0.6575	3.4725	0.2880	2.2832	0.4380	0.9071	2.0712	3
4	1.7490	0.5718	4.9934	0.2003	2.8550	0.3503	1.3263	3.7864	4
5	2.0114	0.4972	6.7424	0.1483	3.3522	0.2983	1.7228	5.7751	5
6	2.3131	0.4323	8.7537	0.1142	3.7845	0.2642	2.0972	7.9368	6
7	2.6600	0.3759	11.0668	0.0904	4.1604	0.2404	2.4498	10.1924	7
8	3.0590	0.3269	13.7268	0.0729	4.4873	0.2229	2.7813	12.4807	8
9	3.5179	0.2843	16.7858	0.0596	4.7716	0.2096	3.0922	14.7548	9
10	4.0456	0.2472	20.3037	0.0493	5.0188	0.1993	3.3832	16.9795	10

Present worth $(P/A, i, N)$

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$=\mathrm{PV}(i, N, A,, 0)$



Capital recovery $(A/P, i, N)$

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$=\mathrm{PMT}(i, N,, P)$

- Model A:
  For 3-year life:

$$
\begin{aligned}
PW(15\%) &= -\$22{,}601 \\
AE(15\%) &= -\$22{,}601(A/P, 15\%, 3) \\
&= -\$9899.
\end{aligned}
$$

For the 12-year period (computed for the complete analysis period):

$$
\begin{aligned}
PW(15\%) &= -\$53{,}657 \\
AE(15\%) &= -\$53{,}657(A/P, 15\%, 12) \\
&= -\$9899.
\end{aligned}
$$

- Model B:
  For 4-year life:

$$
\begin{aligned}
PW(15\%) &= -\$25{,}562 \\
AE(15\%) &= -\$25{,}562(A/P,\ 15\%,\ 4) \\
&= -\$8{,}954.
\end{aligned}
$$

For the 12-year period (computed for the complete analysis period):

$$
\begin{aligned}
PW(15\%) &= -\$48{,}534 \\
AE(15\%) &= -\$48{,}534(A/P,\ 15\%,\ 12) \\
&= -\$8{,}954.
\end{aligned}
$$

Solve

Consider the following sets of investment projects:

	Project's Cash Flow ($)			
n	A	B	C	D
0	−$2,500	−$4,500	−$8,000	−$12,000
1	400	3,000	−2,000	2,000
2	500	2,000	6,000	4,000
3	600	1,000	2,000	8,000
4	700	500	4,000	8,000
5	800	500	2,000	4,000

Compute the equivalent annual worth of each project at $i = 10\%$, and determine the acceptability of each project.

10.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	**Compound Amount Factor (F/P,i,N)**	**Present Worth Factor (P/F,i,N)**	**Compound Amount Factor (F/A,i,N)**	**Sinking Fund Factor (A/F,i,N)**	**Present Worth Factor (P/A,i,N)**	**Capital Recovery Factor (A/P,i,N)**	**Gradient Uniform Series (A/G,i,N)**	**Gradient Present Worth (P/G,i,N)**	**N**
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2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264	2
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291	3
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812	4.3781	4
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13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988	33.3772	13
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955	36.8005	14
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789	40.1520	15

Practice

- 6.3. 6.5, 6.6, 6.11, 6.16, 6.20, 6.24, 6.27, 6.28, 6.39