THEORY OF MECHANISM AND MACHINE II

Mechanical IV/I

Chapter 2

Turning Moment Diagram and Flywheel

1.1 Turning Moment Diagram

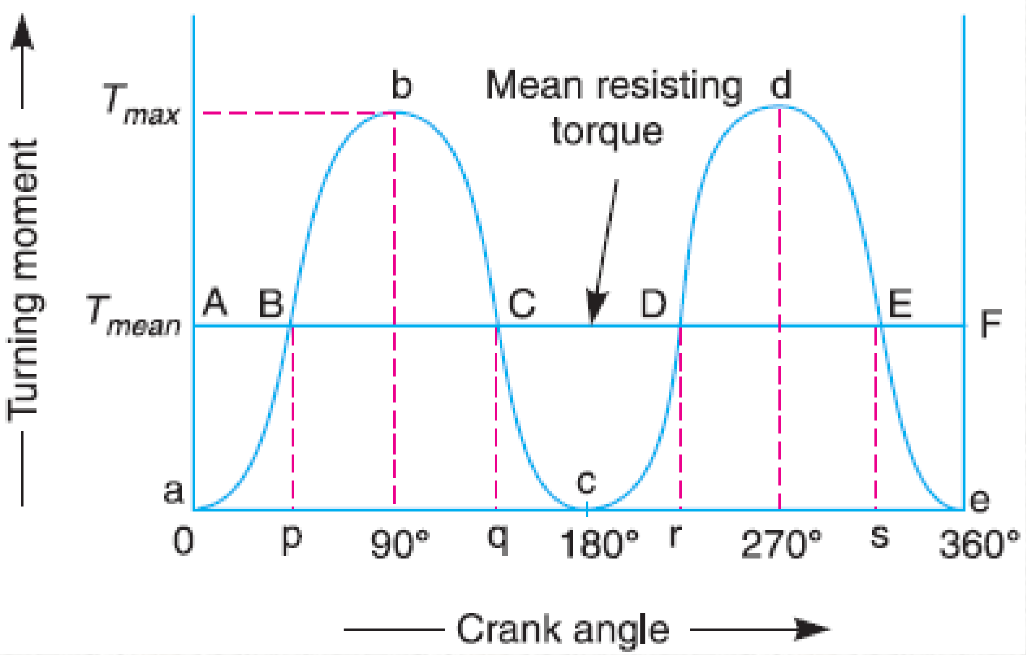
We have discussed in previous chapter that the turning moment on the crankshaft,

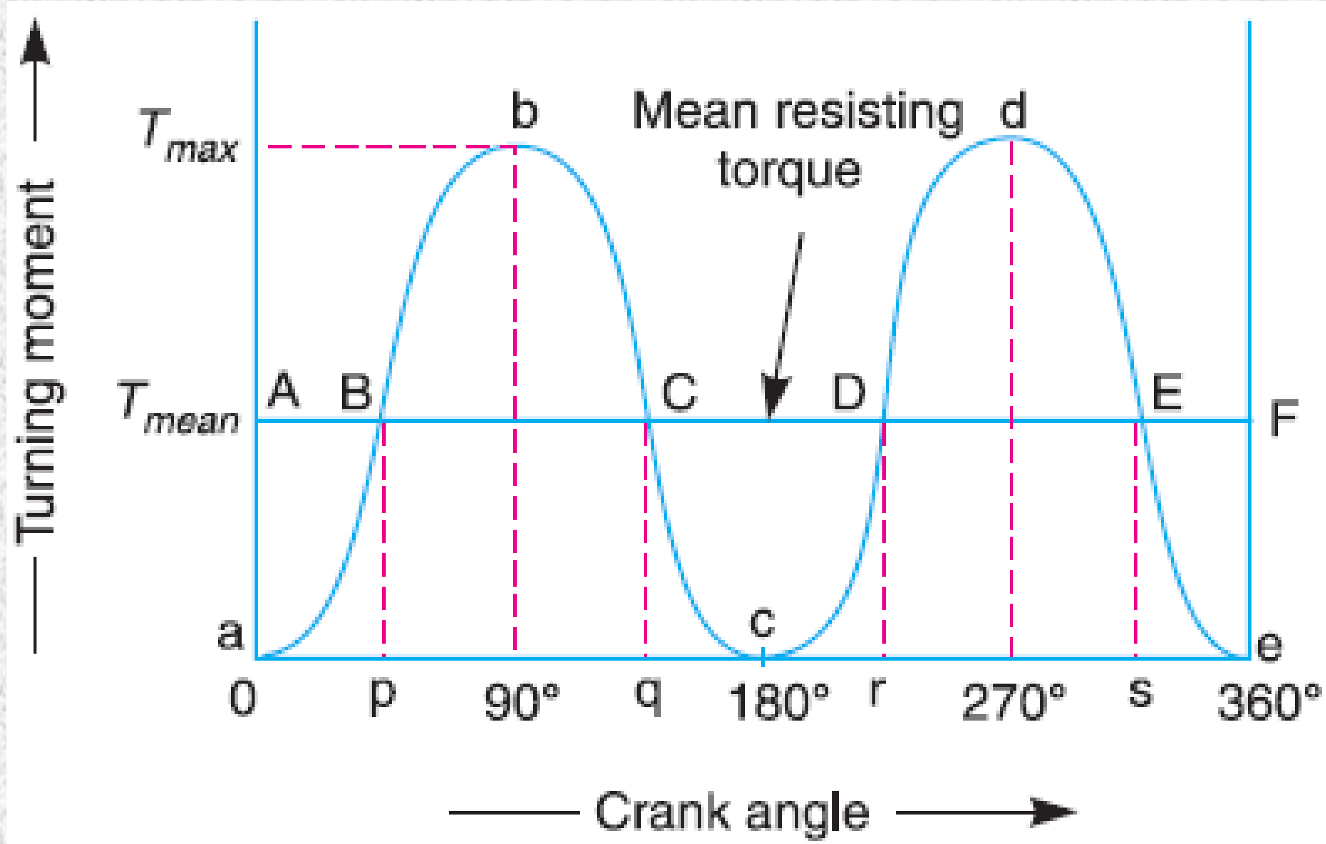
$$T = F_P \times r\left(sin\theta + \frac{sin2\theta}{2\sqrt{n^2 - sin^2\theta}}\right)$$

The turning moment diagram (also known as crank effort diagram) is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on Cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.

Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

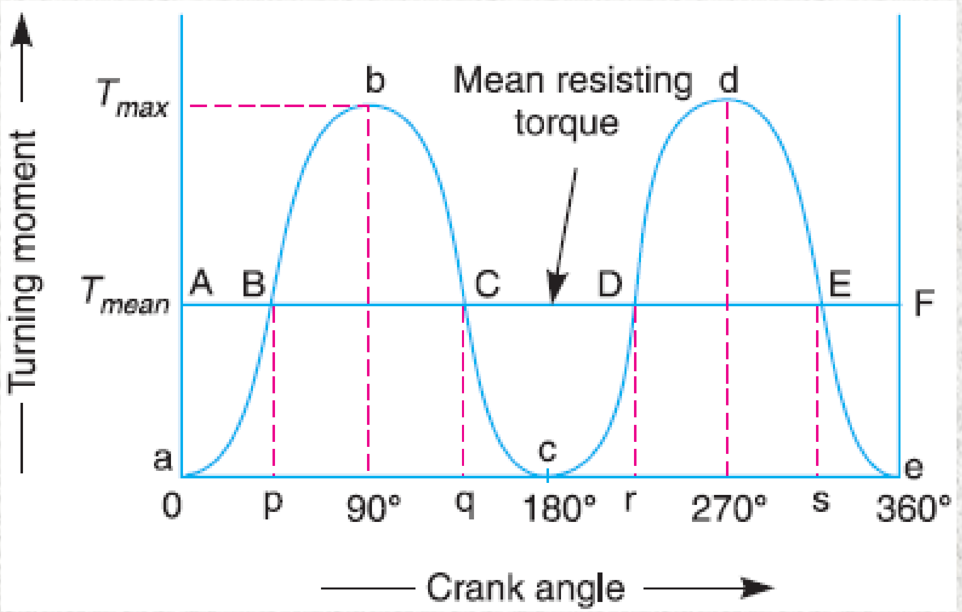
From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.





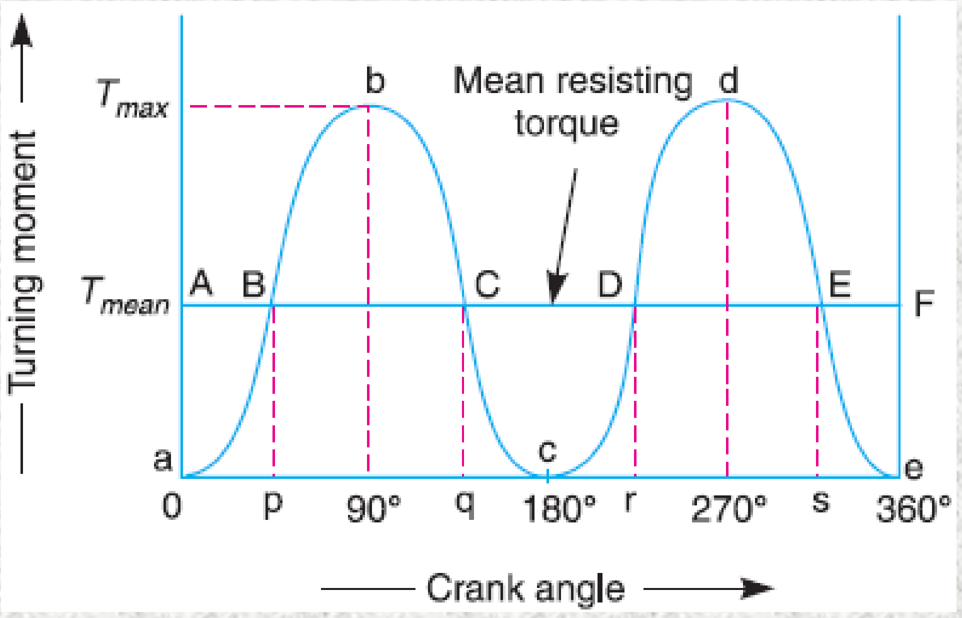
This is shown by the curve *abc* in **Figure** and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution.



In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *aA* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aAFe* is proportional to the work done against the mean resisting torque.

When the turning moment is positive (i.e. when the engine torque is more than the mean resisting torque) as shown between points *B* and *C* (or *D* and *E*) in **Figure**, the crankshaft accelerates and the work is done by the steam.

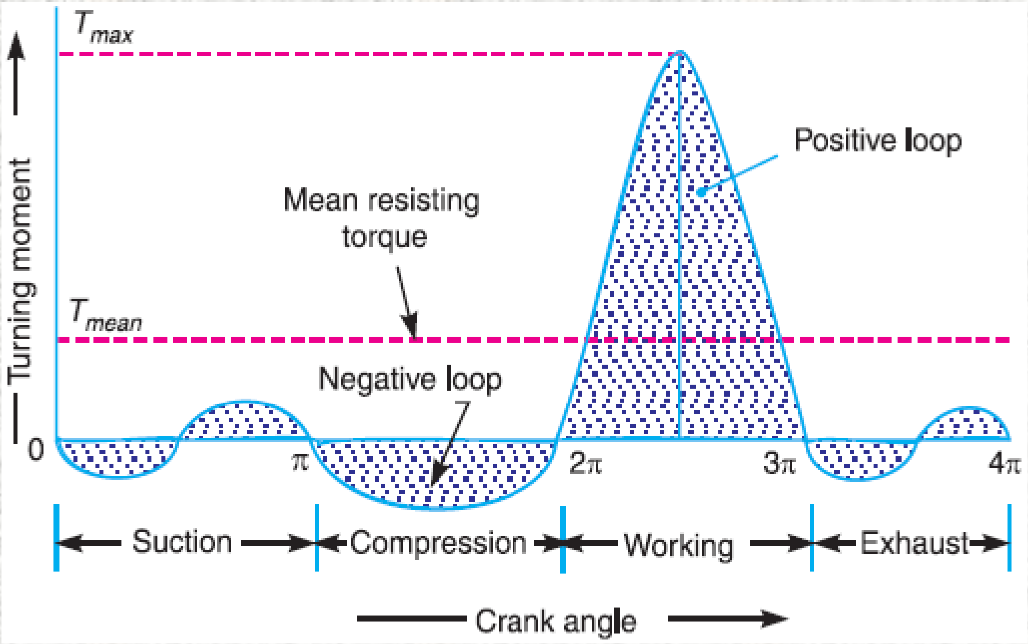


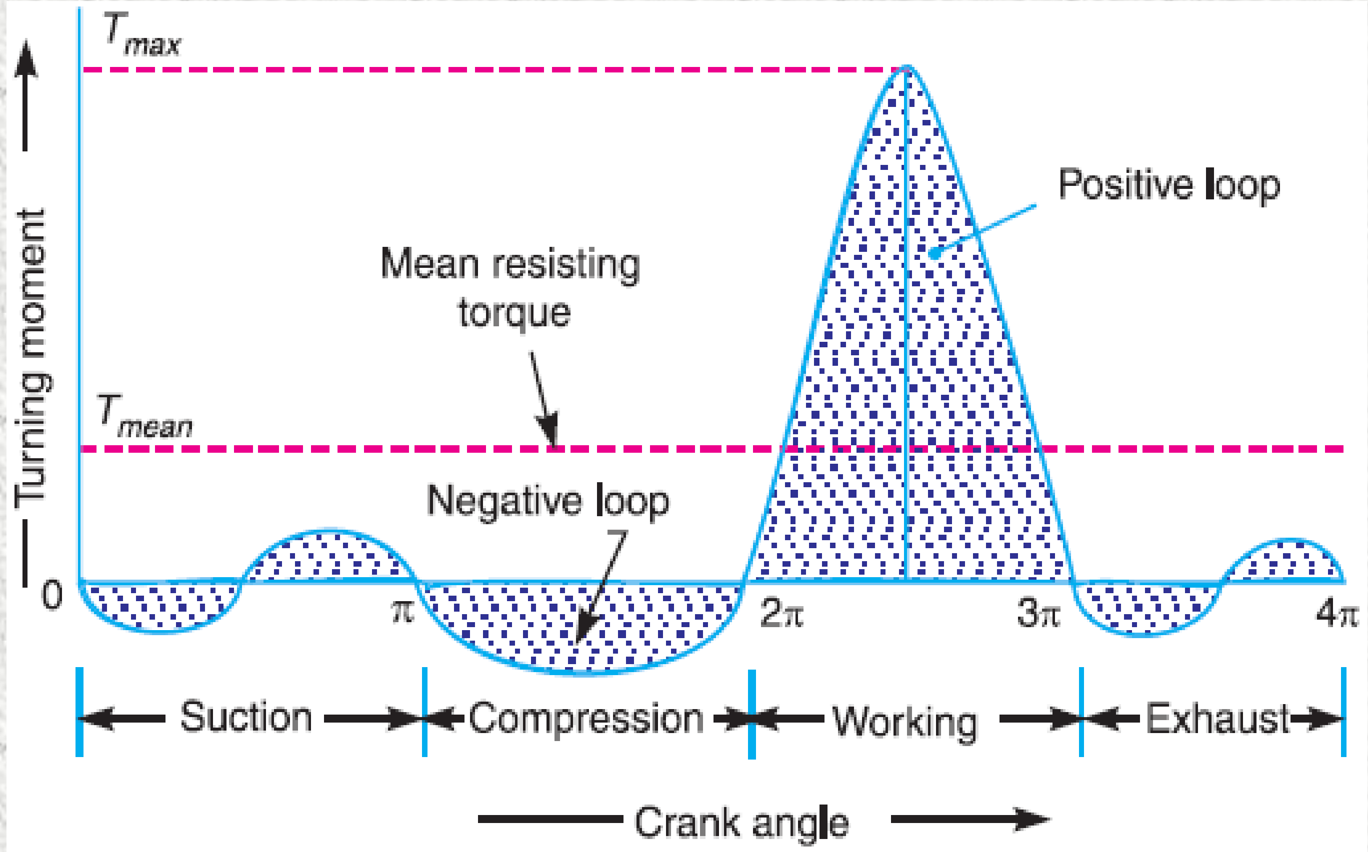
When the turning moment is negative (i.e. when the engine torque is less than the mean resisting torque) as shown between points *C* and *D* in **Figure**, the crankshaft retards and the work is done on the steam.

If $(T - T_{mean})$ is positive, the flywheel accelerates and if $(T - T_{mean})$ is negative, then the flywheel retards.

Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine

A turning moment diagram for a four stroke cycle internal combustion engine is shown in **Figure**. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4 π radians).





Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in **Figure**.

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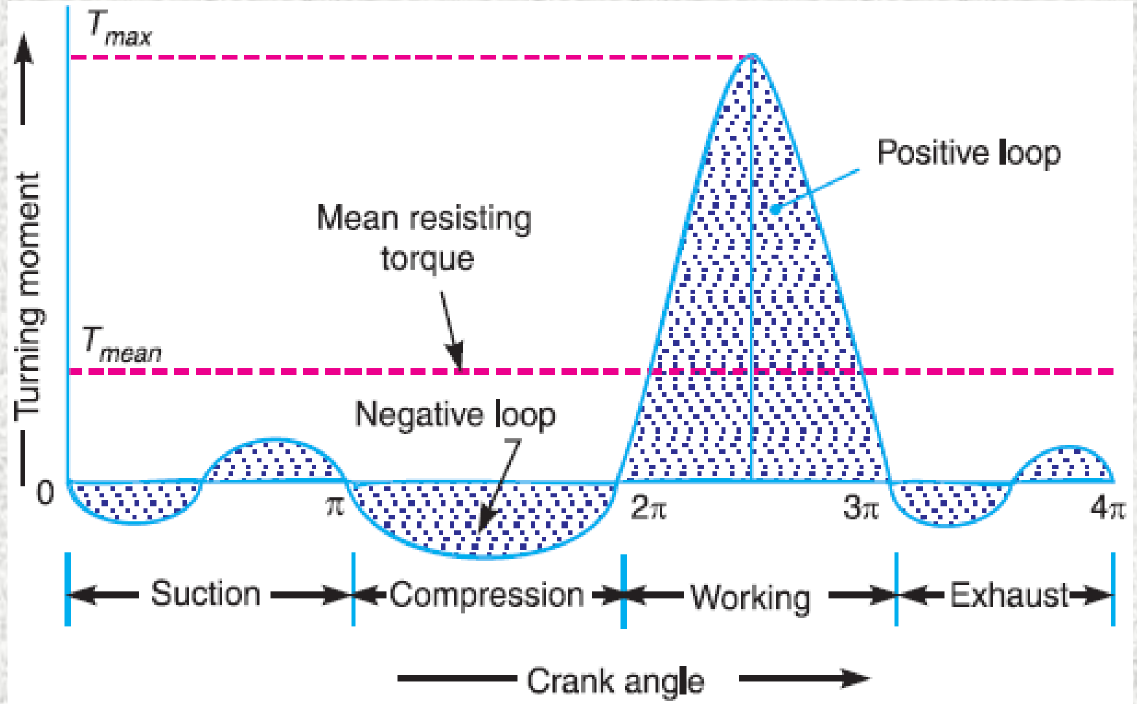


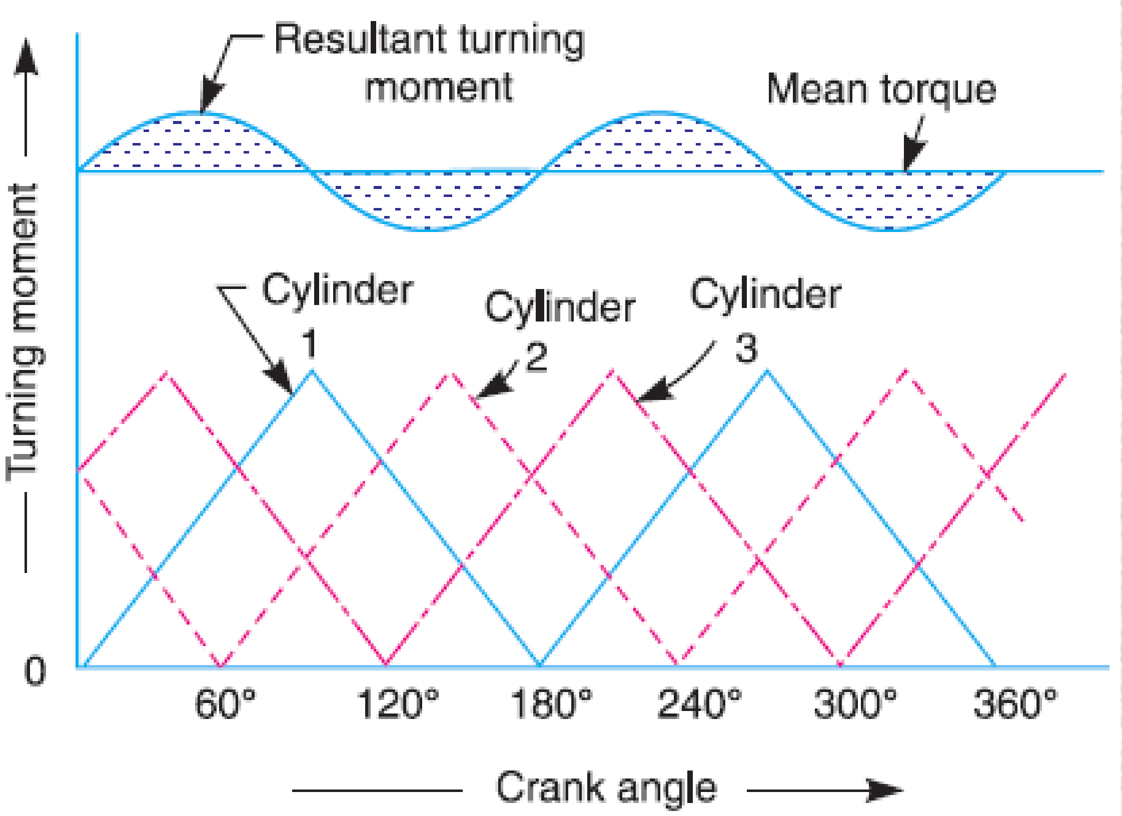
During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained.

During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases.

During exhaust stroke, the work is done on the gases, therefore a negative loop is formed.

Turning Moment Diagram for a Multi-cylinder Engine

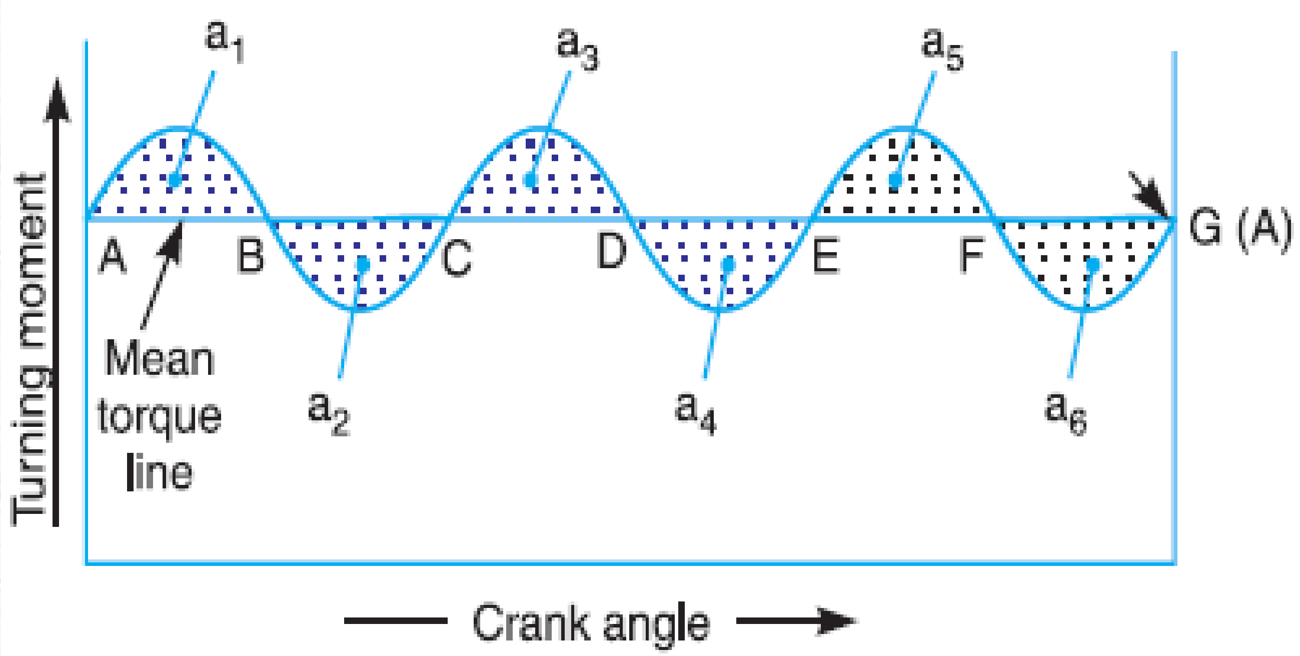
A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in **Figure**. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders.

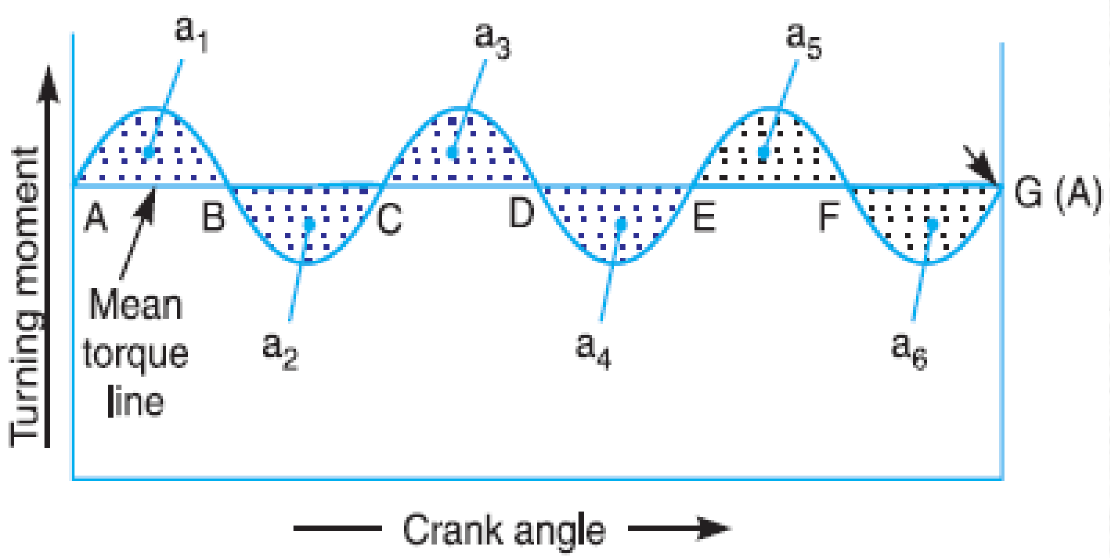


2.2 Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation.

The difference between the maximum and the minimum energies is known as maximum fluctuation of energy.





$$
\begin{aligned}
\text{Energy at } B &= E + a_1 \\
\text{Energy at } C &= E + a_1 - a_2 \\
\text{Energy at } D &= E + a_1 - a_2 + a_3 \\
\text{Energy at } E &= E + a_1 - a_2 + a_3 - a_4 \\
\text{Energy at } F &= E + a_1 - a_2 + a_3 - a_4 + a_5 \\
\text{Energy at } G &= E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 \\
&= \text{Energy at } A \text{ (i.e. cycle repeats after } G\text{)}
\end{aligned}
$$

Let us now suppose that the greatest of these energies is at *B* and least at *E*. Therefore,

$$\text{Maximum energy in the flywheel} = E + a_1$$

$$\text{Minimum energy in the flywheel} = E + a_1 - a_2 + a_3 - a_4$$

Maximum fluctuation of energy,

$$\begin{aligned} \Delta E &= \text{Maximum energy} - \text{Minimum energy} \\ &= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) \\ &= (a_2 - a_3 + a_4) \end{aligned}$$

Coefficient of Fluctuation of Energy

It may be defined as the ratio of the maximum fluctuation of energy to the work done per cycle.

$$C_E = \frac{Maximum\ fluctuation\ of\ energy}{Work\ done\ per\ cycle}$$

Work done per cycle can be determined by using the relation,

$$Work\ done\ per\ cycle = T_{mean}\ \theta$$

where

T_{mean} = mean torque,

θ = Angle turned in one revolution.

= 2π, in case of steam engine and two stroke internal combustion engines

= 4π, in case of four stroke internal combustion engines

2.3 Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

In machines where the operation is intermittent like punching machines, shearing machines, rivetting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

The flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. It does not control the speed variations caused by the varying load.

2.4 Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the **maximum fluctuation of speed**. The ratio of the maximum fluctuation of speed to the mean speed is called the **coefficient of fluctuation of speed**.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

N = Mean speed in r.p.m. $= \dfrac{N_1 + N_2}{2}$

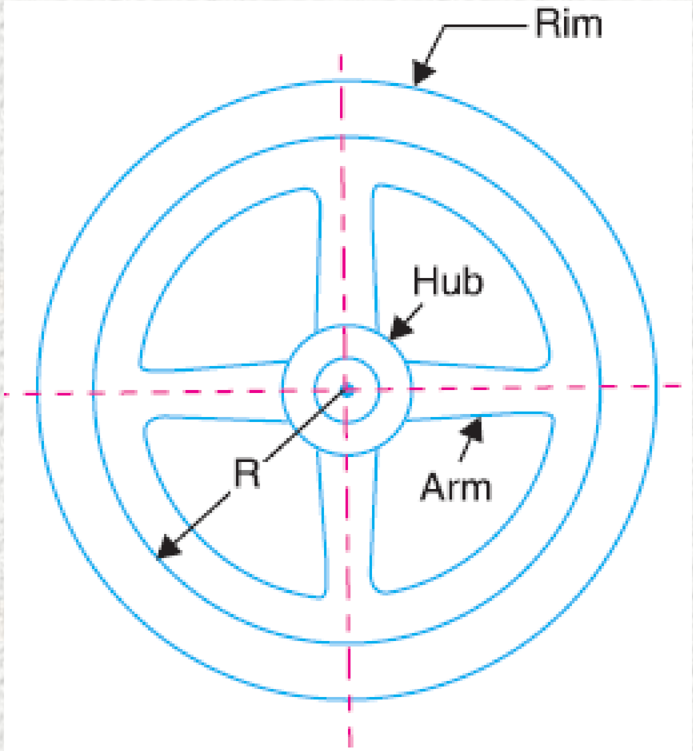
Coefficient of fluctuation of speed, $C_S = \dfrac{N_1 - N_2}{N} = \dfrac{2(N_1 - N_2)}{N_1 + N_2}$

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$ (In terms of angular speeds)

The reciprocal of the coefficient of fluctuation of speed is known as **coefficient of steadiness** and is denoted by ***m***.

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

2.5 Energy Stored in a Flywheel



The mean kinetic energy of the flywheel,

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2} mk^2\omega^2$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = KE_{maximum} - KE_{minimum}$$

$$= \frac{1}{2} I{\omega_1}^2 - \frac{1}{2} I{\omega_2}^2 = \frac{1}{2} I({\omega_1}^2 - {\omega_2}^2)$$

$$= \frac{1}{2} I(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I\omega(\omega_1 - \omega_2)$$

$$= I\omega^2 \frac{(\omega_1 - \omega_2)}{\omega} = I\omega^2 C_S = mk^2\omega^2 C_S = 2EC_S$$

The above expressions can be expressed in terms of N_1 and N_2 as,

$$= I\omega(\omega_1 - \omega_2) = I\frac{2\pi N}{60}\left(\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right)$$

$$= I\frac{\pi^2}{900}N(N_1 - N_2)$$

$$= I\frac{\pi^2}{900}N^2\frac{(N_1 - N_2)}{N}$$

$$= I\frac{\pi^2}{900}N^2 C_S$$

$$= \frac{\pi^2}{900}mk^2N^2C_S$$

Example 2.1

A vertical double acting steam engine develops 75 kW at 250 r.p.m. The maximum fluctuation of energy is 30 per cent of the work done per stroke. The maximum and minimum speeds are not to vary more than 1 percent on either side of the mean speed. Find the mass of the flywheel required, if the radius of gyration is 0.6 m.

547.134 kg

Example 2.2

The torque delivered by a two stroke engine is represented by

$$T = (1000 + 300sin2\theta - 500cos2\theta)\ N.m$$

where θ is the angle turned by the crank from the inner dead center. The engine speed is 250 rpm. The mass of flywheel is 400 kg and radius of gyration is 400 mm. Determine:

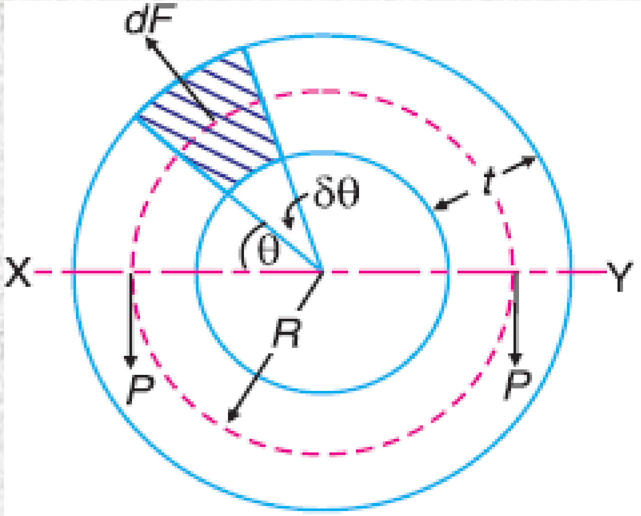
a) **The power developed,**
b) **The total fluctuation of speed**
c) **The angular acceleration of flywheel when the crank has rotated through an angle of 60^0 from the inner dead center.**

26.18 kW

1.329 %

7.9657 rad/s^2

2.6 Dimensions of the Flywheel Rim



Volume of the small element $= AR\delta\theta$

Mass of the small element

$$dm = Density \times Volume = \rho\, AR\delta\theta$$

Centrifugal force on the element, acting radially outwards,

$$dF = dm\omega^2 R = \rho\, A\omega^2 R^2 \delta\theta$$

Vertical component of dF $= dF sin\theta = \rho\, A\omega^2 R^2 \delta\theta sin\theta$

Total vertical upward force tending to burst the rim across the diameter X Y

$$= \rho\, A\omega^2 R^2 \int_0^{\pi} sin\theta\, d\theta = \rho\, A\omega^2 R^2 [-cos\theta]_0^{\pi} = 2\rho\, A\omega^2 R^2$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by 2*P*, such hat

$$2P = 2\sigma A$$

or, $$2\rho A\omega^2 R^2 = 2\sigma A$$

or, $$\rho \omega^2 R^2 = \sigma$$

or, $$\rho v^2 = \sigma$$

$$v = \sqrt{\frac{\sigma}{\rho}}$$

where v is the linear velocity at the mean radius in m/s.

$$v = \omega R = \frac{\pi D N}{60}$$

Mass of the rim, $m = \pi D A \rho$

Cross-sectional area of the rim,

$$A = \frac{m}{\pi D \rho}$$

where, $A = b \times t$

b = Width of the rim, and

t = Thickness of the rim.

Example 2.3

A multi-cylinder engine is to run at a speed of 600 r.p.m. On drawing the turning moment diagram to a scale of 1 mm = 250 N-m and 1 mm = 3°, the areas above and below the mean torque line in mm^2 are : + 160, – 172, + 168, – 191, + 197, – 162

The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel. Determine the suitable dimensions of a rectangular flywheel rim if the breadth is twice its thickness. The density of the cast iron is 7250 kg/m^3 and its hoop stress is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.

32.6599 kgm^2 58.62 mm 117.24 mm