Chapter - 4

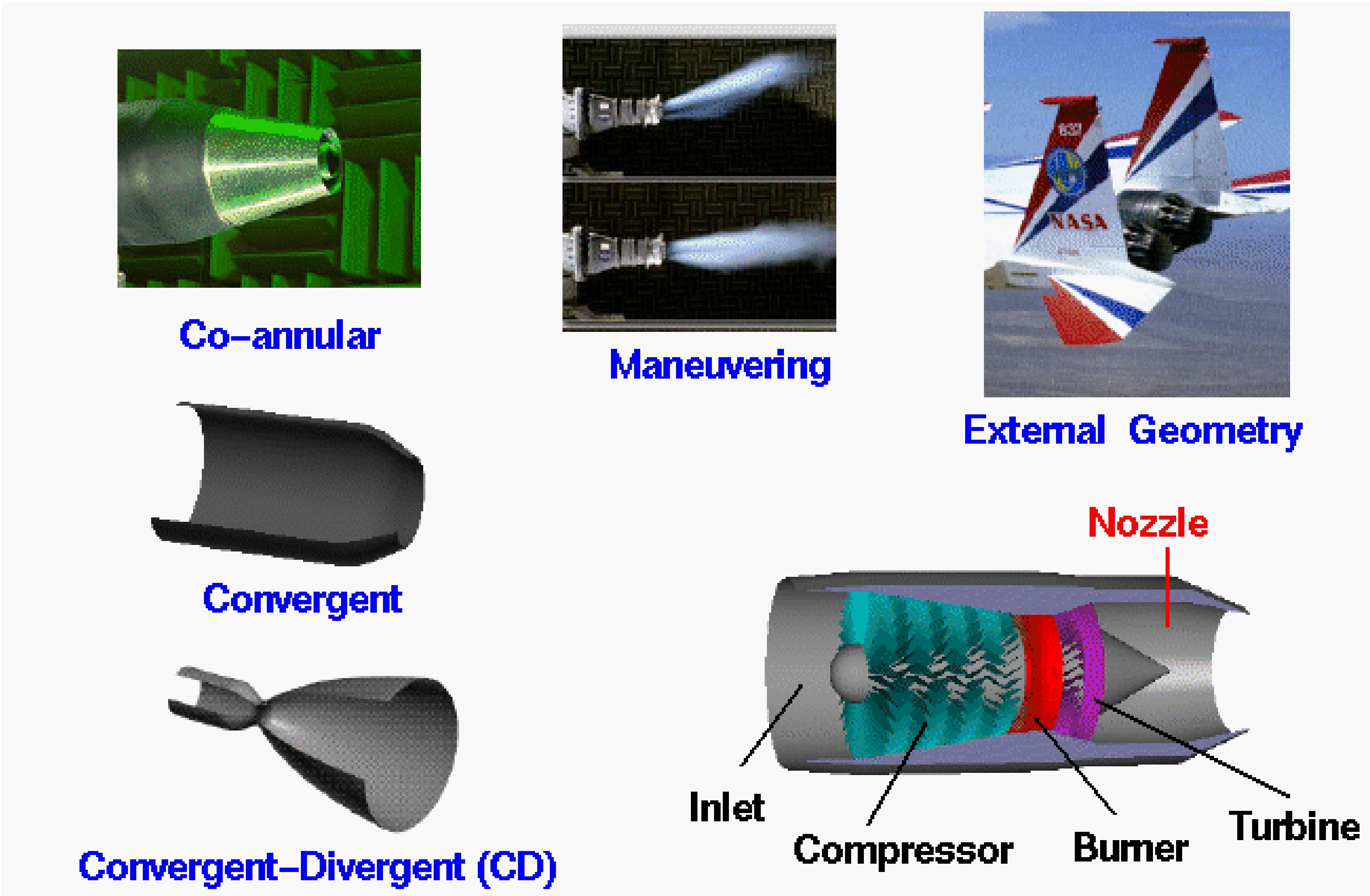
Gas Turbine Nozzle

Rudra Ghimire

Gas Turbine Nozzle

- The nozzle is the exhaust duct of the engine. This is the engine part which actually produces the thrust for the plane. The energy depleted airflow that passed the turbine, in addition to the colder air that bypassed the engine core, produces a force when exiting the nozzle that acts to propel the engine, and therefore the airplane, forward.
- The combination of the hot air and cold air are expelled and produce an exhaust, which causes a forward thrust. The nozzle may be preceded by a **mixer**, which combines the high temperature air coming from the engine core with the lower temperature air that was bypassed in the fan. The mixer helps to make the engine quieter.

Figures of Nozzle



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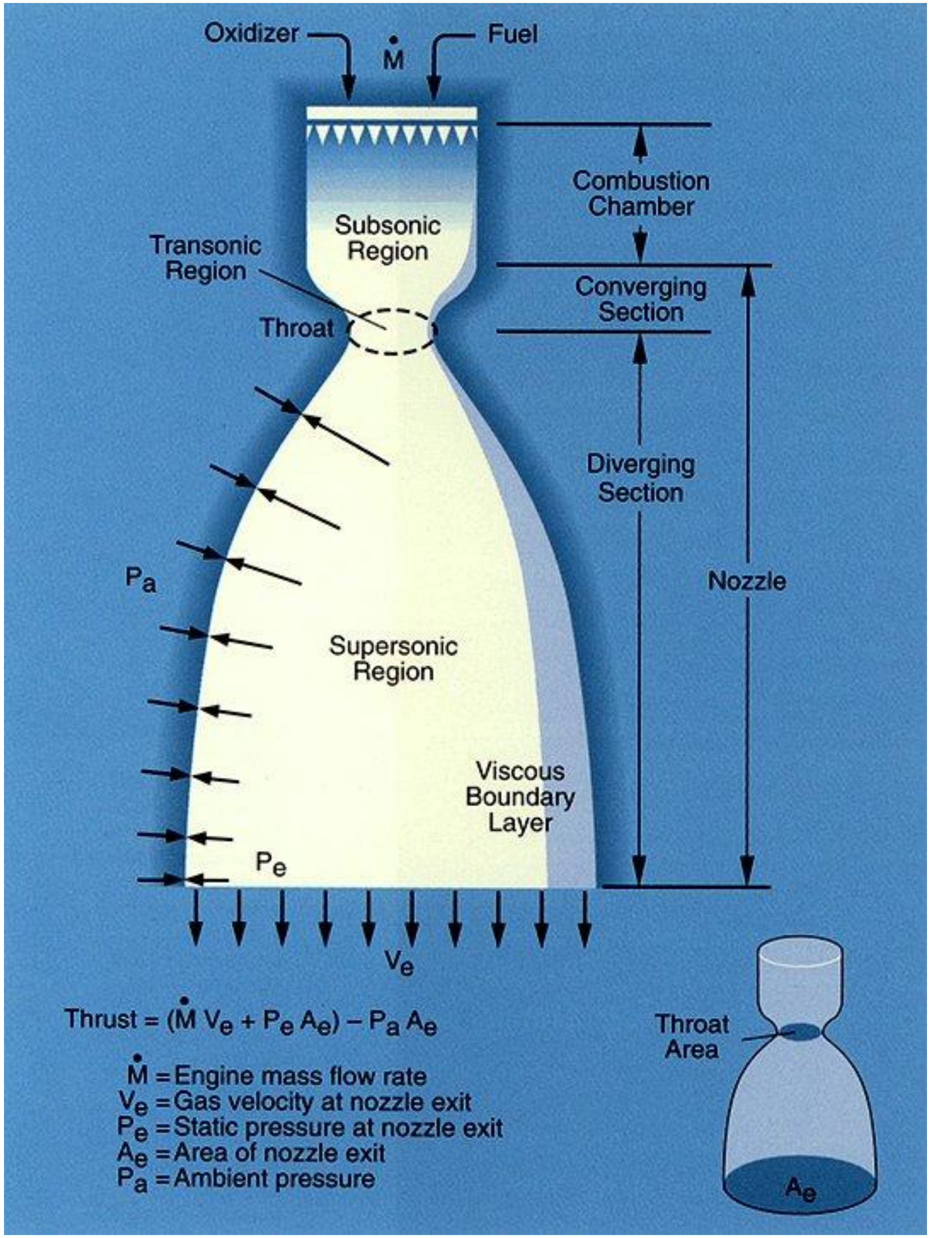
NOZZLE BASICS REVIEW



- Nozzle produces thrust
- Convert thermal energy of hot chamber gases into kinetic energy and direct that energy along nozzle axis
- Exhaust gases from combustion are pushed into throat region of nozzle
- Throat is smaller cross-sectional area than rest of engine → gases are compressed to high pressure
- Nozzle gradually increases in cross-sectional area allowing gases to expand and push against walls creating thrust
- Mathematically, ultimate purpose of nozzle is to expand gases as efficiently as possible so as to maximize exit velocity

$$F = \dot{m}_e V_e + \left(P_e - P_a\right) A_e$$

$$F = \dot{m}_e V_e$$

NOZZLE BASICS REVIEW

- Expansion Area Ratio:
  - Most important parameter in nozzle design is expansion area ratio, ε

$$\varepsilon = \frac{A_{exit}}{A_{throat}} = \frac{A_e}{A^*}$$

  - Fixing other variables (primarily chamber pressure) → only one ratio that optimizes performance for a given altitude (or ambient pressure)
  - However, rocket does not travel at only one altitude
    - Should know trajectory to select expansion ratio that maximizes performance over a range of ambient pressures

- Other factors must also be considered
  - Nozzle weight, length, manufacturability, cooling (heat transfer), and aerodynamic characteristics.

Different type of nozzle

According to the function

- Aerospike Nozzle

A rocket nozzle design that allows combustion to occur around the periphery of a spike (or center plug). The thrust-producing, hot-exhaust flow is then shaped and adjusted by the ambient (atmospheric) pressure. It's also called a plug nozzle or a spike nozzle.

- Liquid Nozzles

Liquid nozzles, such as those from fire hoses, are called converging because the area decreases along the length of the nozzle to increase the speed. Typical liquid nozzles have a simple conical shape and are designed to a specific ratio of inlet to outlet areas.

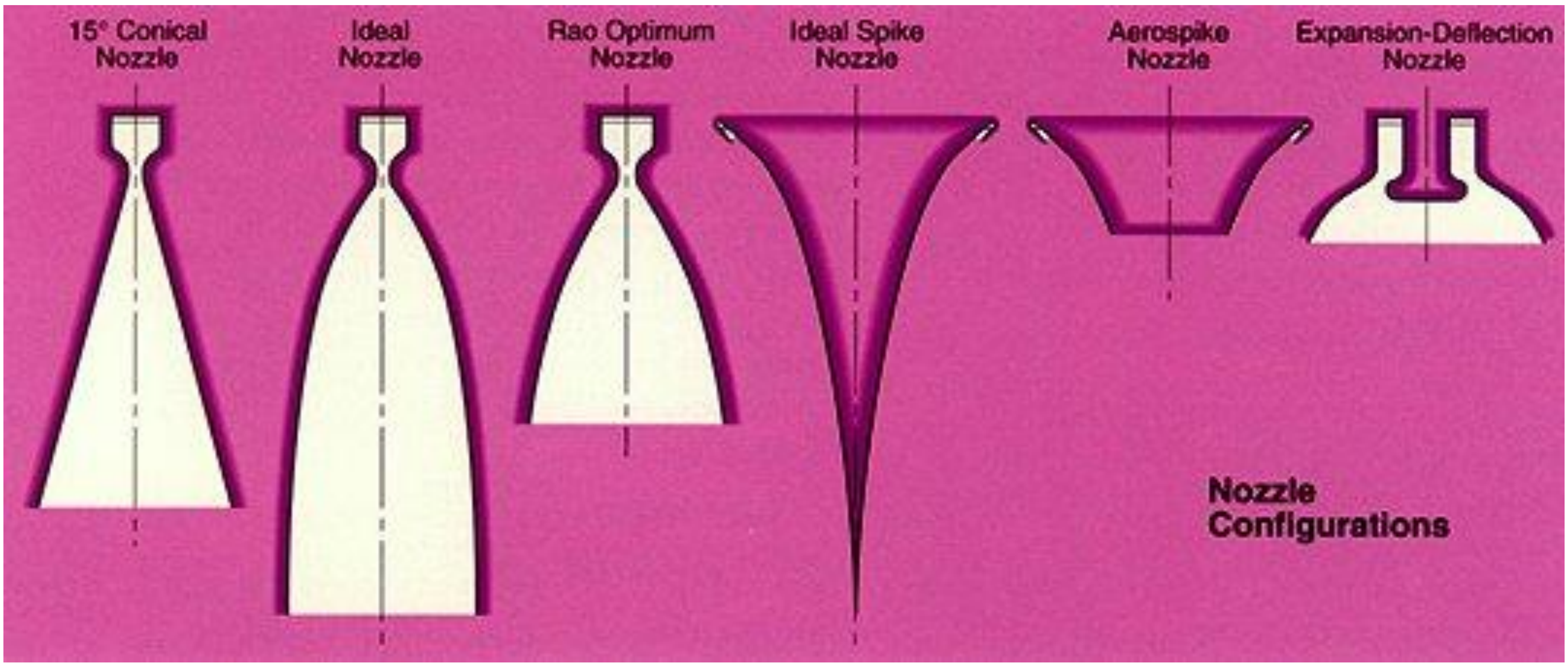
- High-speed Gas Nozzles

In the case of gas nozzles, the gas density can change dramatically from the pressure between the inlet and outlet of the nozzle. At very high gas speeds, this effect is so significant that the basic shape of the nozzle must change to a converging-diverging form. The diverging portion is necessary to accommodate the expansion of the gas as it accelerates to lower pressure.

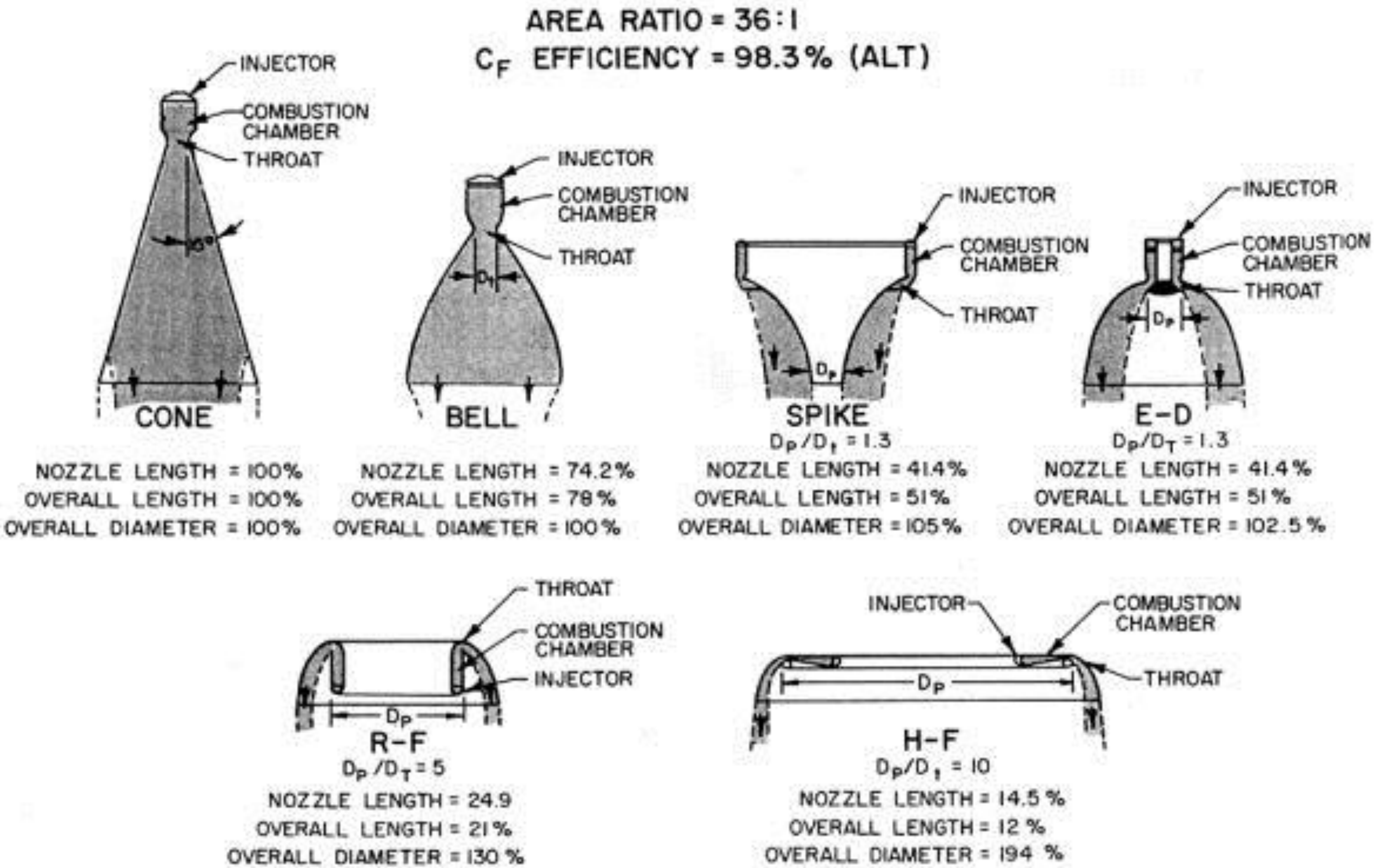
NOZZLE TYPES

primary groups of nozzle types

1. Cone (conical, linear)
2. Bell (contoured, shaped, classic converging-diverging)
3. Annular (spike, aerospike, plug, expansion, expansion-deflection)

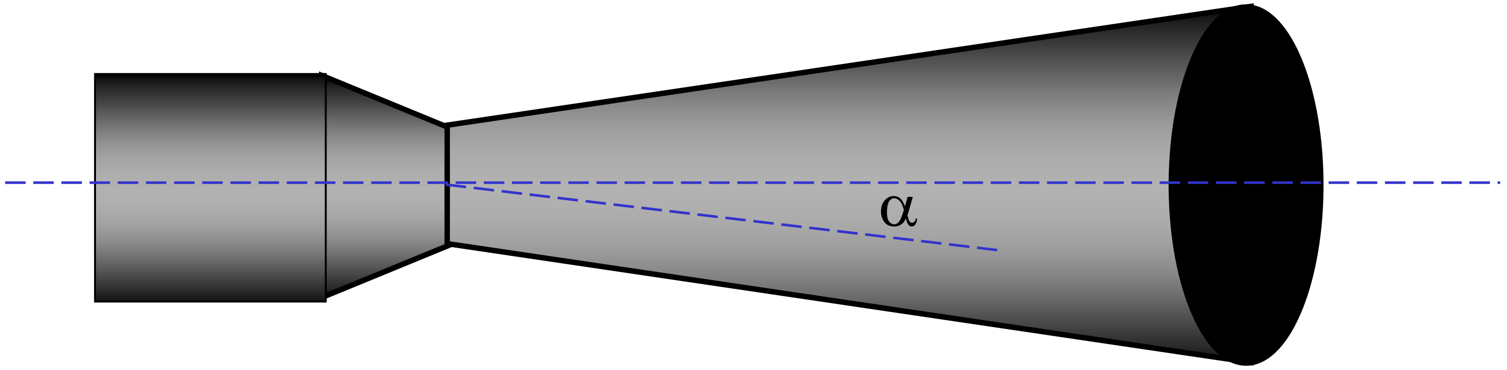


NOZZLE EXAMPLES



1. CONICAL NOZZLES

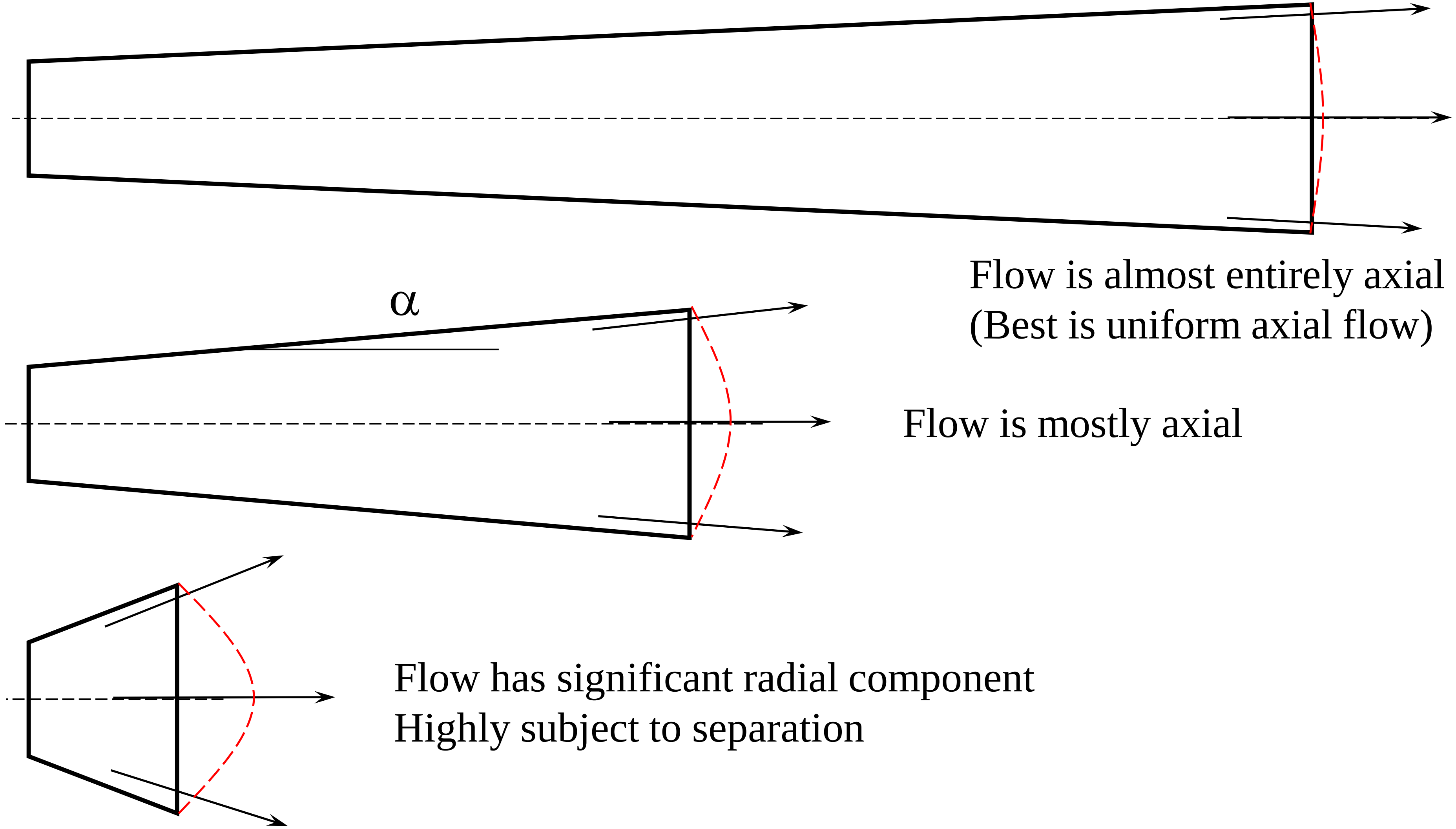
- Used in early rocket applications because of simplicity and ease of construction
- Cone gets its name from the fact that the walls diverge at a constant angle
- A small angle produces greater thrust, because it maximizes the axial component of exit velocity and produces a high specific impulse
- Penalty is longer and heavier nozzle that is more complex to build
- At the other extreme, size and weight are minimized by a large nozzle wall angle
  - Large angles reduce performance at low altitude because high ambient pressure causes overexpansion and flow separation
- Primary Metric of Characterization: Divergence Loss



CONICAL NOZZLES: SOME DETAILS

All 3 nozzles have same Ae/A*

Red dashed lines indicate contours of normal flow



SIZING OF CONICAL NOZZLES

For a conical nozzle with half-angle α, length L, and diameter D^*, the ratio of exit to throat area is:

$$\frac{A_e}{A^*} = \left(\frac{D^* + 2L\tan\alpha}{D^*} \right)^2$$

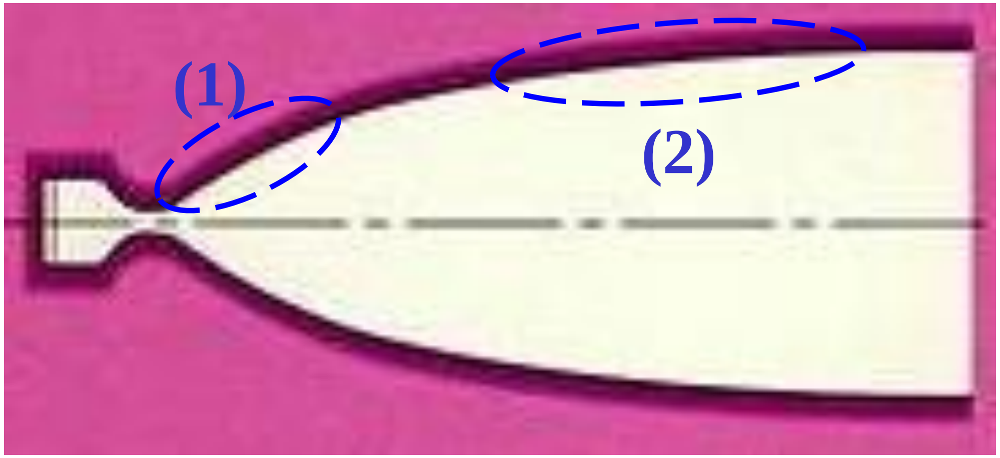
Solving for the Length of the nozzle knowing the area ratio, throat diameter and desired nozzle half angle

$$L = \frac{D^* \left(\sqrt{\frac{A_e}{A^*}} - 1 \right)}{2\tan\alpha}$$

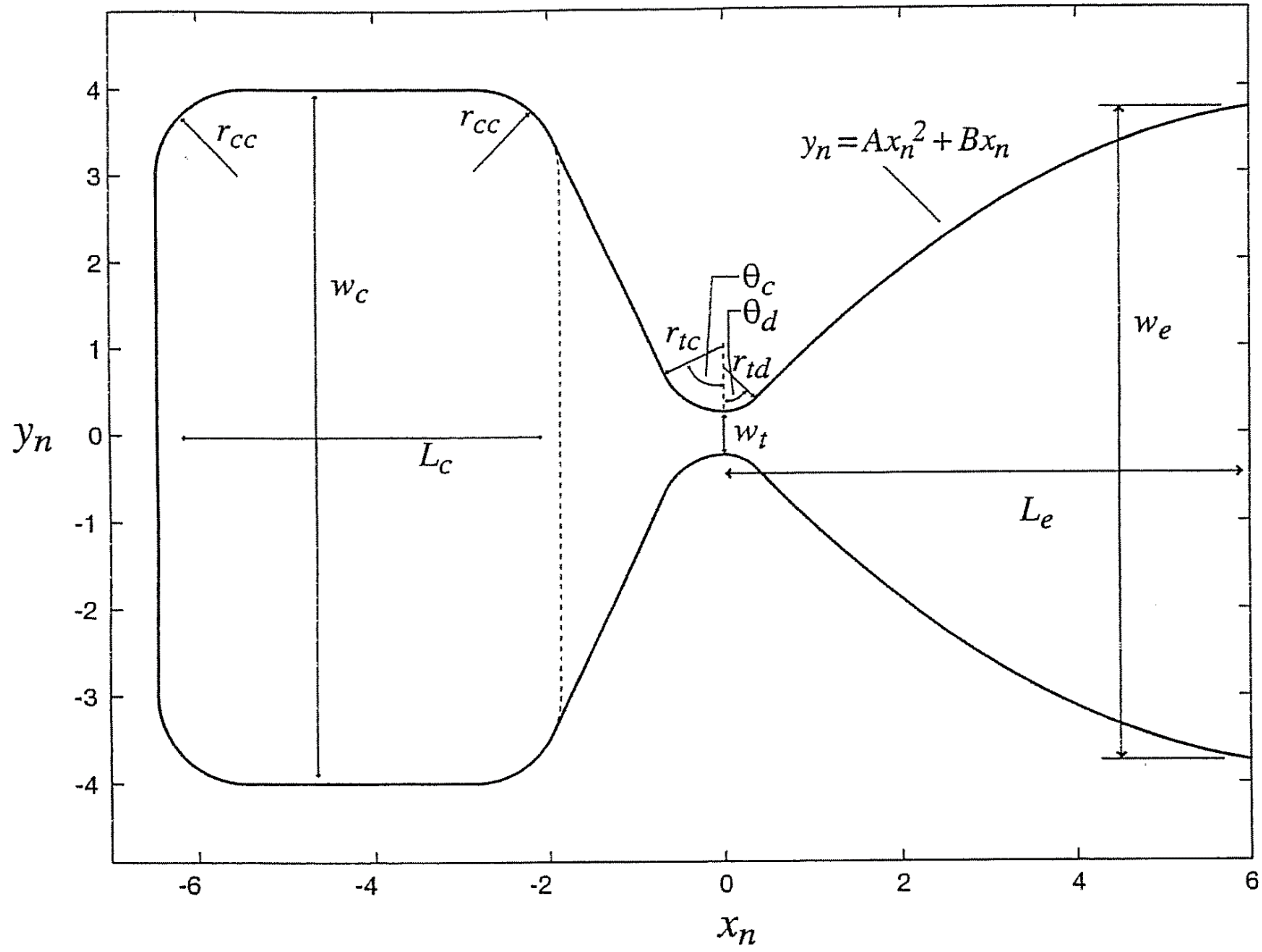
- Throat diameter D^* is fixed by combustion chamber conditions and desired thrust
  - Nozzle length and mass are strongly dependent on α
- **Example:**
  - For area ratio of 100, $L/D^* = 7.8$ for 30º and 16.8 for 15º
  - Reducing α from 30º to 15º would more than double the mass of divergent portion of the nozzle

2. BELL NOZZLES

- Bell is most commonly used nozzle shape
- Offers significant advantages over conical nozzle, both in size and performance
- Bell consists of two sections
  - Near throat, nozzle diverges at relatively large angle, **(1)**
  - Degree of divergence tapers off further downstream
  - Near nozzle exit, divergence angle is very small ~2º-8º, **(2)**
  - Minimize weight / maximize performance ~10-25% shorter than conic
- Issue is to contour nozzle to avoid oblique shocks and maximize performance
- Remember: Shape only optimum at one altitude



SIMPLIFIED NOZZLE DESIGN



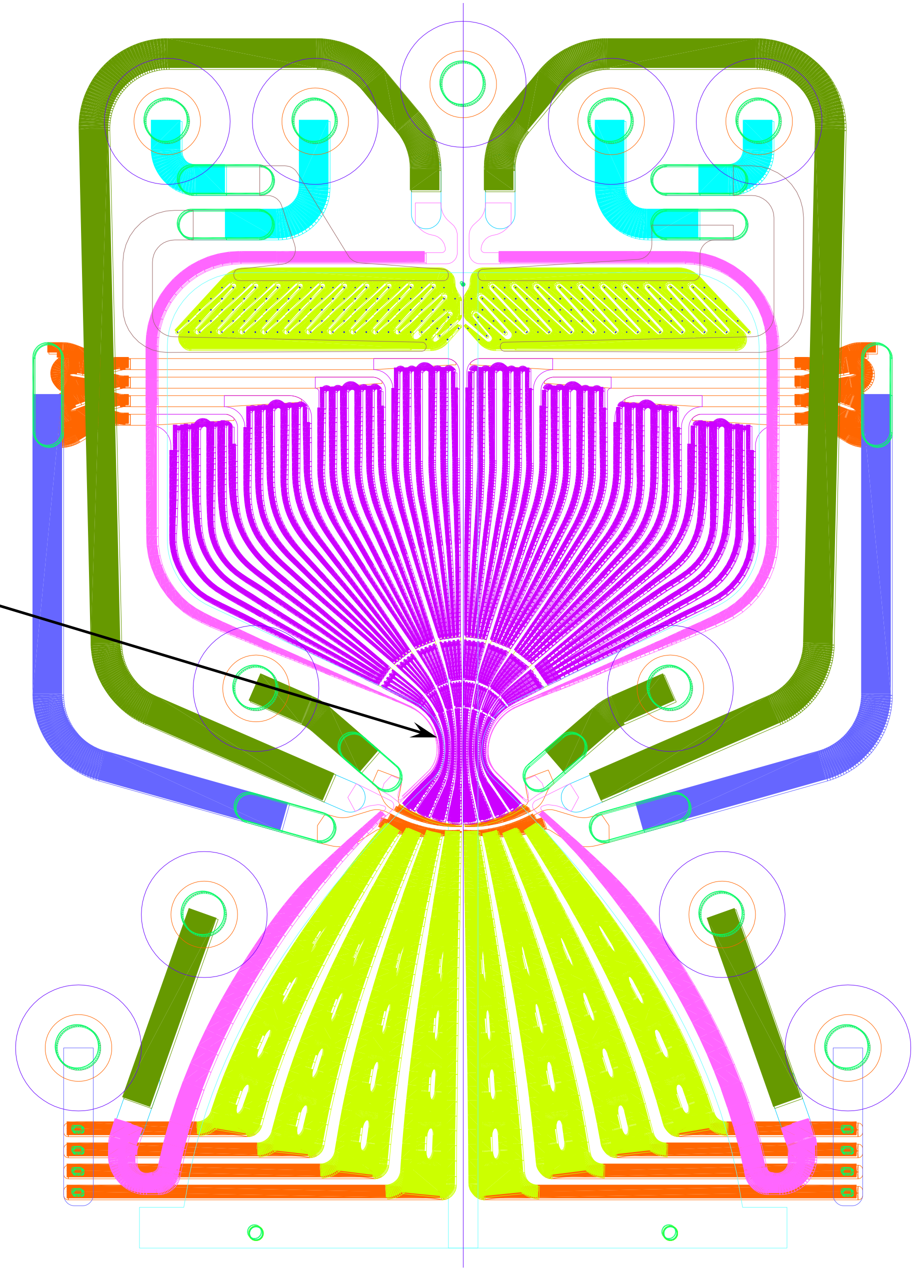
- Simplified parameters used to define chamber and nozzle geometry

COOLING CHANNELS / HEAT EXCHANGER

μRocket has a complex set of cooling channels, flow paths, and coolant injection options

Regions of highest heat load are near throat

Multiple options on how to distribute coolant mass flow, where to use each propellant, and taking into account temperature limit on H_2O_2



3. ANNULAR NOZZLES

- Annular (plug or altitude-compensating) nozzle
  - Least employed due to greater complexity, actually be best in theory
  - Annular: combustion occurs along ring, or annulus, around base of nozzle
  - Plug: refers to centerbody that blocks flow from what would be center portion of traditional nozzle
  - Primary advantage: Altitude-compensating

- Expansion ratio: area of centerbody must be taken into account

$$\varepsilon = \frac{A_{exit} - A_{plug}}{A_{throat}}$$

- Another parameter annular diameter ratio, D_{plug} / D_{throat}
  - Ratio is used as a measure of nozzle geometry for comparison with other plug nozzle shapes

INTERESTING (?) NOTE

Q: Why do U.S. nozzles look more like a polynomial contour and Soviet nozzles look more conical?

A: (Jim Glass, Rocketdyne)

Interestingly, Soviet nozzle designs have a 'different' look to them than typical US designs. US designs are '**truncated Rao optimum**' bells, usually designed by method-of-characteristics methods. Soviet nozzles, to US eyes, look more conical than ours. Ours have that nice 'parabolic' look to them - less conical. One would suppose the Russians are fully capable of running M-O-C and CFD codes and thus their nozzles, if optimum, should look 'just like' ours. Since they don't, I've always wondered if they know something we do not. In my experience, the US is better at combustion engineering (minimal C-star losses) but has fairly substantial losses in the nozzle (aerodynamic losses). The Russians tend to reverse this, throwing away huge gobs of energy due to incomplete combustion and then using a very efficient expansion process to get some of it back. The bottom line is both design approaches appear to yield roughly the same Isp efficiency... One wonders what would happen if one were to mate a US combustor to a Russian nozzle…

MORE COMMENTS ON ANNULAR NOZZLES

- Two major types of annular nozzles have been developed to date
- Distinguished by method in which they expand exhaust: (1) outward or (2) inward

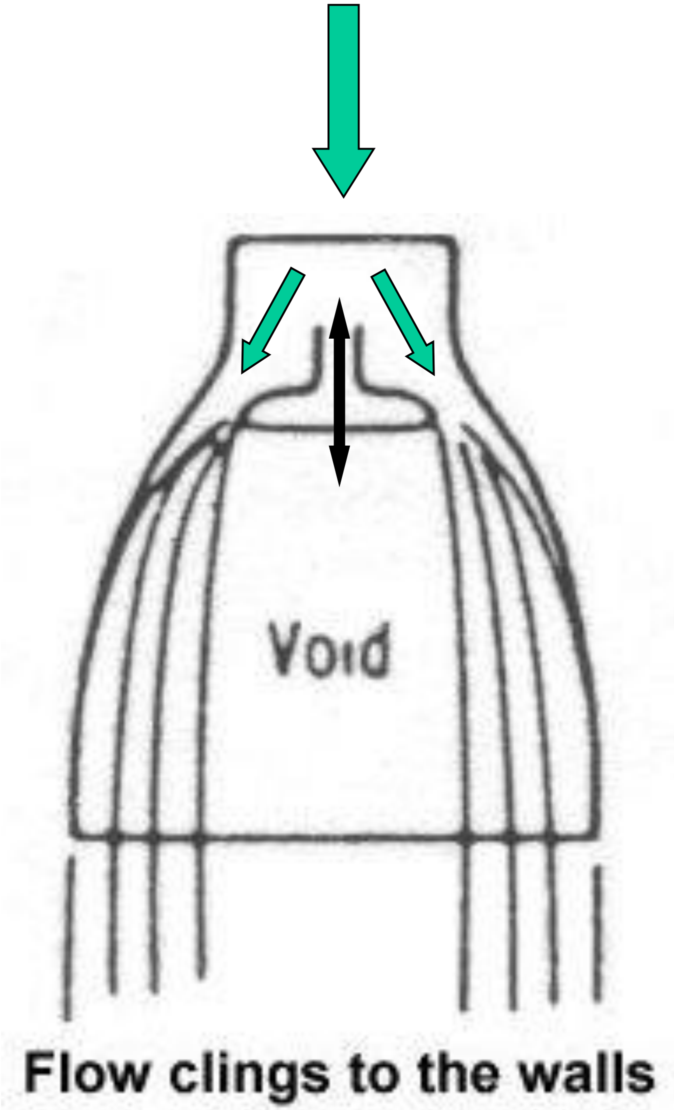
1. **Radial Out-Flow Nozzles**
   - Examples of this type are the expansion-deflection (E-D), reverse-flow (R-F), and horizontal-flow (H-F) nozzles

2. **Radial In-Flow Nozzles**
   - Spike nozzles, linear-aerospike nozzle for X-33

- Annular nozzles receiving significant research attention
  - Several publications call these concepts ‘new’
  - In actuality, these ideas have been around for quite some time
  - As we will see, there are serious challenges with implementation

NOT SO NEW TECHNOLOGY: Messerschmitt Me 262

RADIAL OUT-FLOW NOZZLES



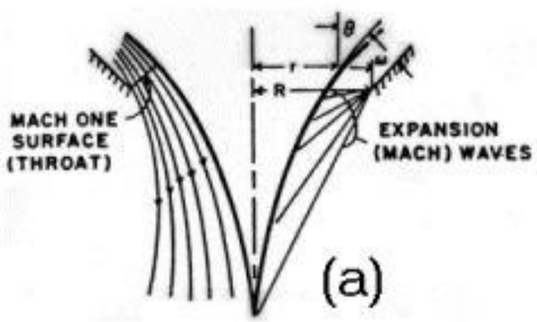
- Picture shows an example of an Expansion-Deflection (E-D) nozzle
- Expansion-deflection nozzle works much like a bell nozzle
- Exhaust gases forced into a converging throat before expanding in a bell-shaped nozzle
- Flow is deflected by a plug, or centerbody, that forces the gases away from center of nozzle and to stay attached to nozzle walls
- Centerbody position may move to optimize performance
- As altitude or back-pressure varies, flow is free to expand into 'void'
  - This expansion into void allows the nozzle to compensate for altitude
  - Pe adjusts to Pb within nozzle

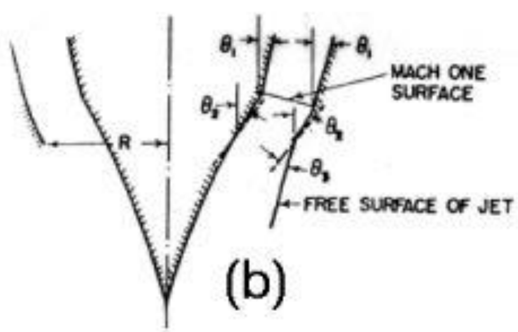
RADIAL OUT-FLOW NOZZLES: DETAILS

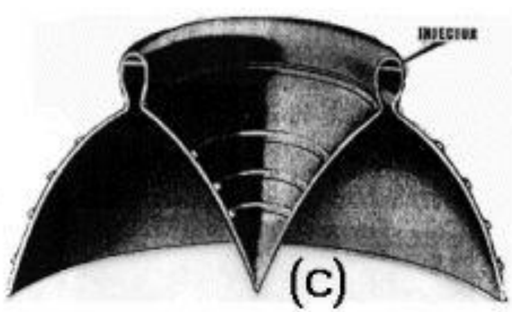
- Name of each of these nozzles indicates how it functions
- Expansion-deflection nozzle works much like a bell nozzle since the exhaust gases are forced into a converging throat region of low area before expanding in a bell-shaped nozzle. However, the flow is deflected by a plug, or centerbody, that forces the gases away from the center of the nozzle. Thus, the E-D is a radial out-flow nozzle.
- The reverse-flow nozzle gets its name because the fuel is injected from underneath, but the exhaust gases are rotated 180° thereby reversing their direction. Similarly, the fuel in the horizontal-flow nozzle is injected sideways, but the exhaust is rotated 90°.
- The E-D, has been one of the most studied forms of annular nozzles. While similar in nature to the bell nozzle, the most notable difference is the addition of a centerbody. As shown below, this "plug" may be located upstream of, downstream of, or in the throat, with each location resulting in better performance for a given set of operating conditions.
- The purpose of the centerbody is to force flow to remain attached to, to stick to nozzle walls
- This behavior is desirable at low altitudes because the atmospheric pressure is high and may be greater than pressure of exhaust gases. When this occurs, the exhaust is forced inward and no longer exerts force on the nozzle walls, so thrust is decreased and the rocket becomes less efficient. The centerbody, however, increases the pressure of the exhaust gases by squeezing the gases into a smaller area thereby virtually eliminating any loss in thrust at low altitude.

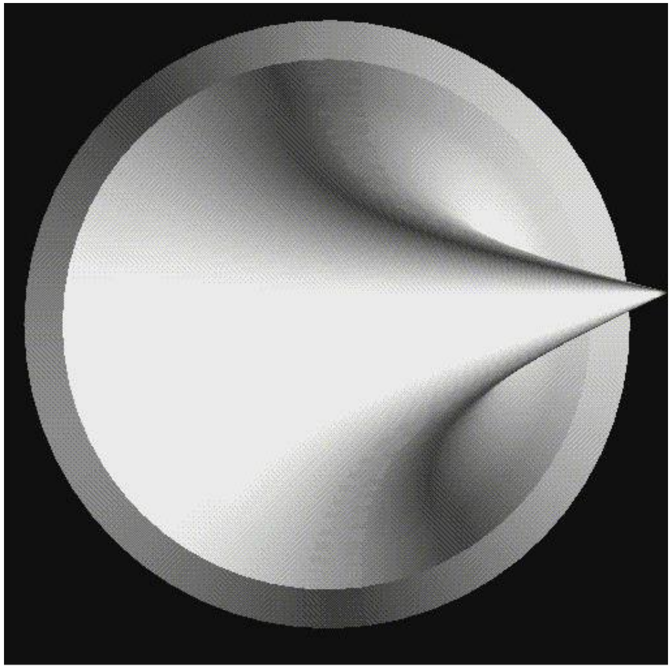
RADIAL IN-FLOW NOZZLES

- Often referred to as spike nozzles
  - Named for prominent spike centerbody
  - May be thought of as a bell turned inside out
  - Nozzle is only one of many possible spike configurations
    - (a) traditional curved spike with completely external supersonic expansion
    - (b) similar shape in which part of the expansion occurs internally
    - (c) design similar to E-D nozzle in which all expansion occurs internally



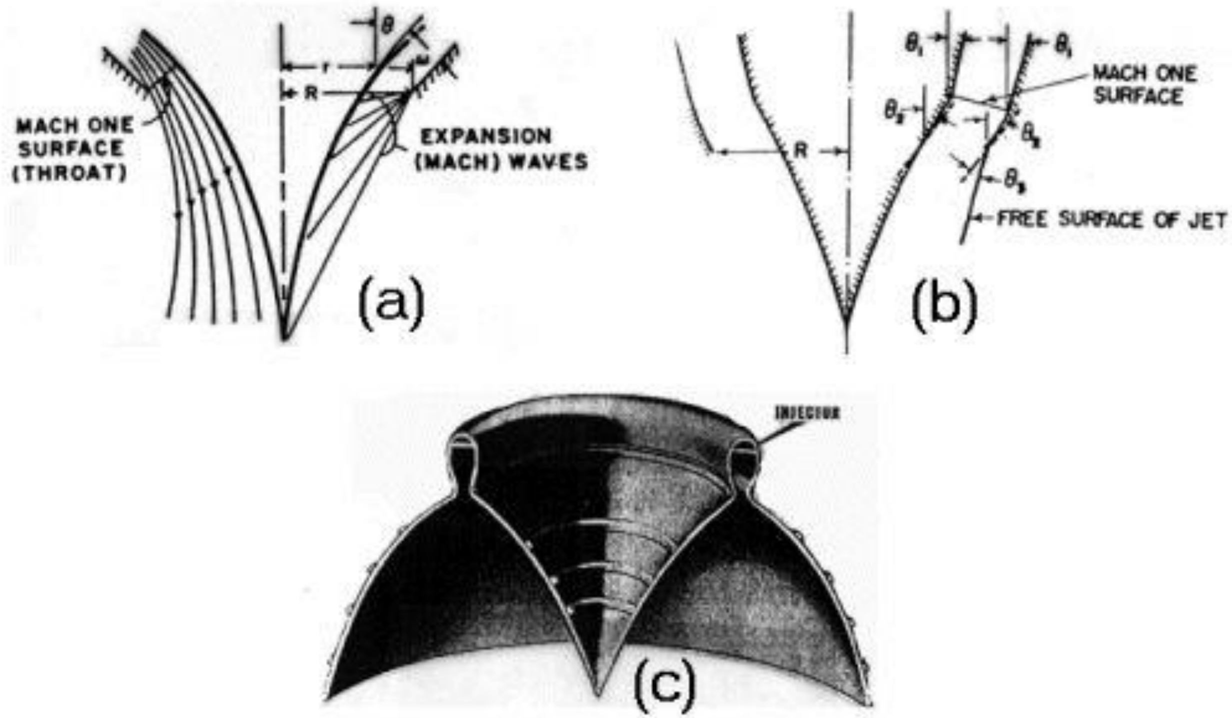






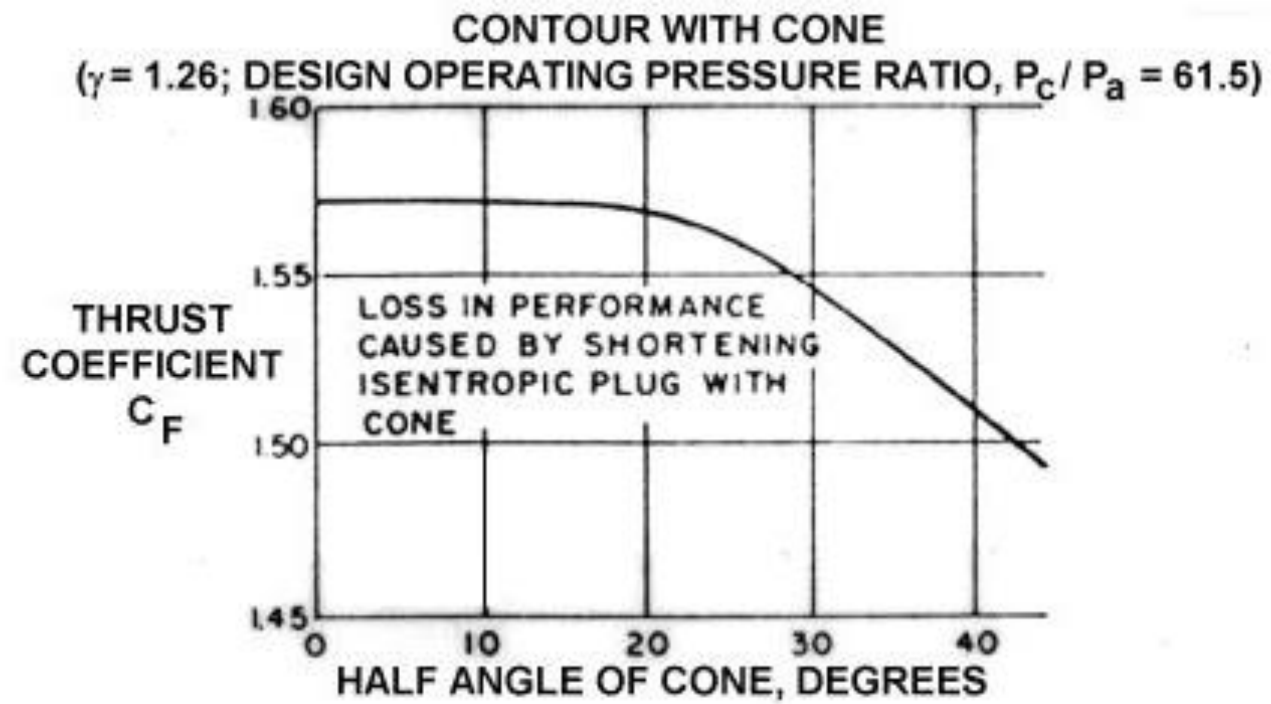
The ideal 100% length aerospike nozzle

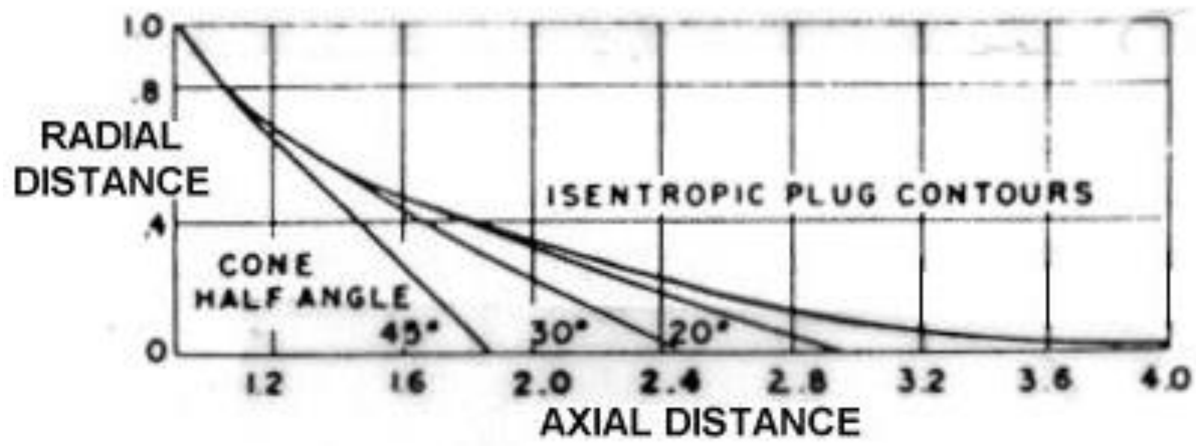
SPIKE NOZZLES



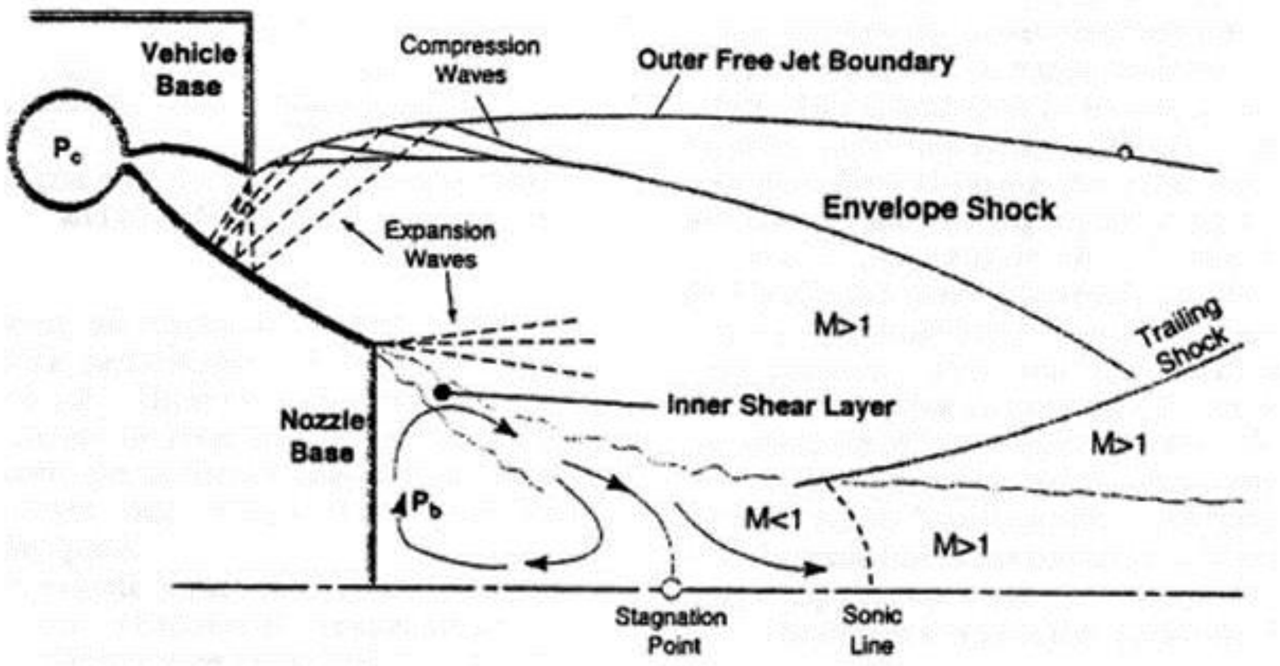
- Each of spike nozzles features a curved, pointed spike
  - Most ideal shape
- Spike shape allows exhaust gases to expand through isentropic process
- Nozzle efficiency is maximized and no energy is lost because of turbulent mixing
- Isentropic spike may be most efficient but tends to be prohibitively long and heavy
- Replace curve shape by shorter and easier to construct cone ~1% performance loss

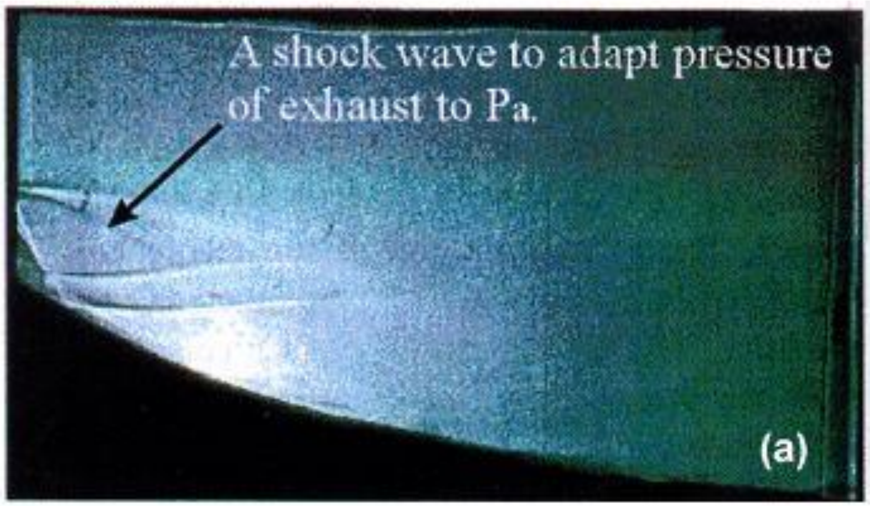
SPIKE NOZZLE PERFORMANCE

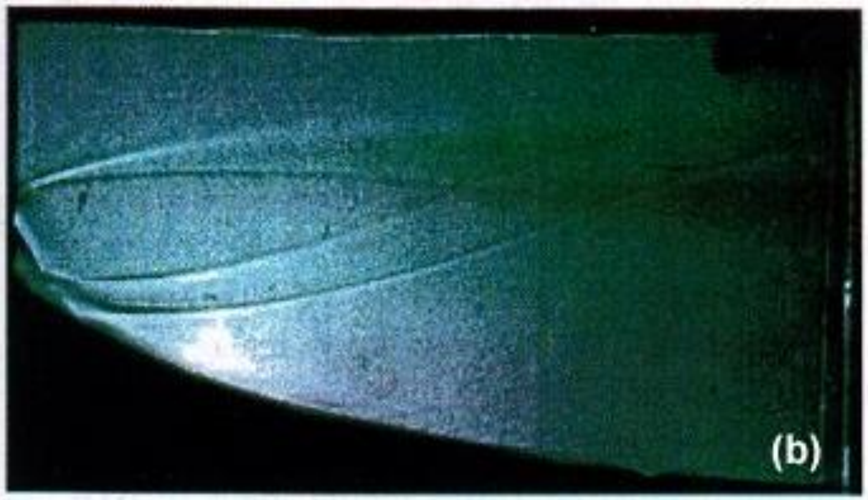




SPIKE: EXTERNAL SUPERSONIC COMPRESSION

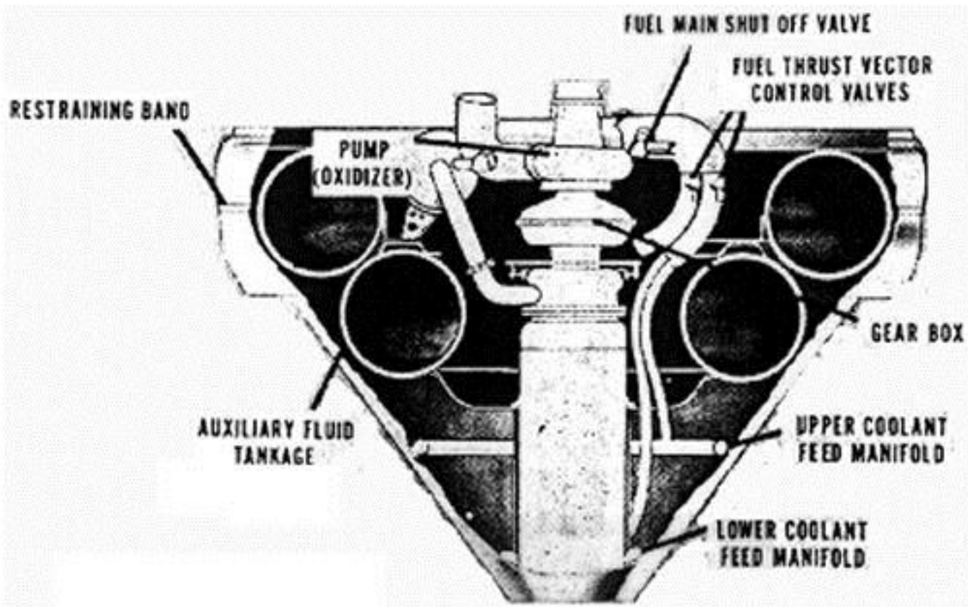


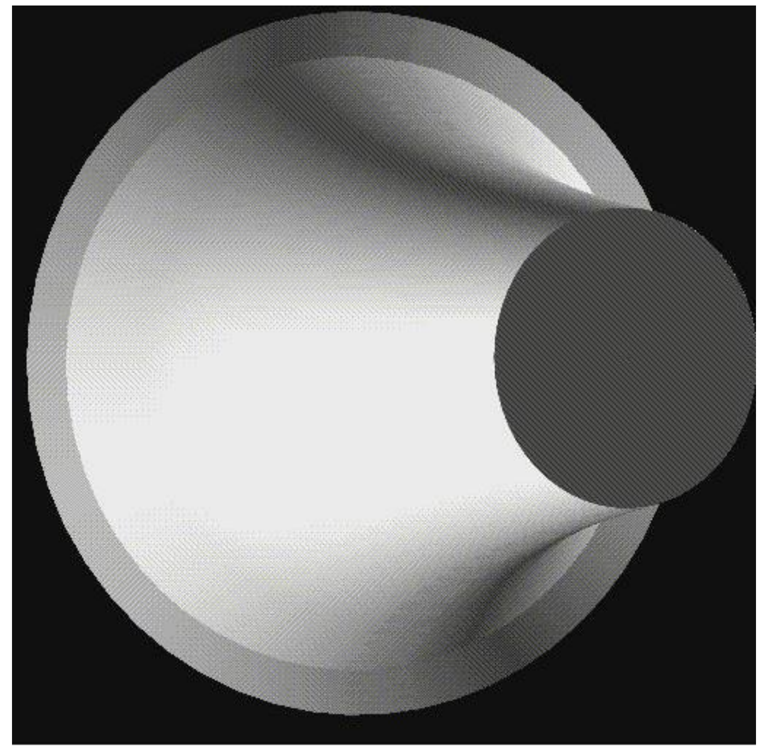




AEROSPIKE NOZZLES

- Further subclass of radial in-flow family of spike nozzles is known as aerospike
- Go even further by removing pointed spike altogether and replace with a flat base
  - This configuration is known as a truncated spike

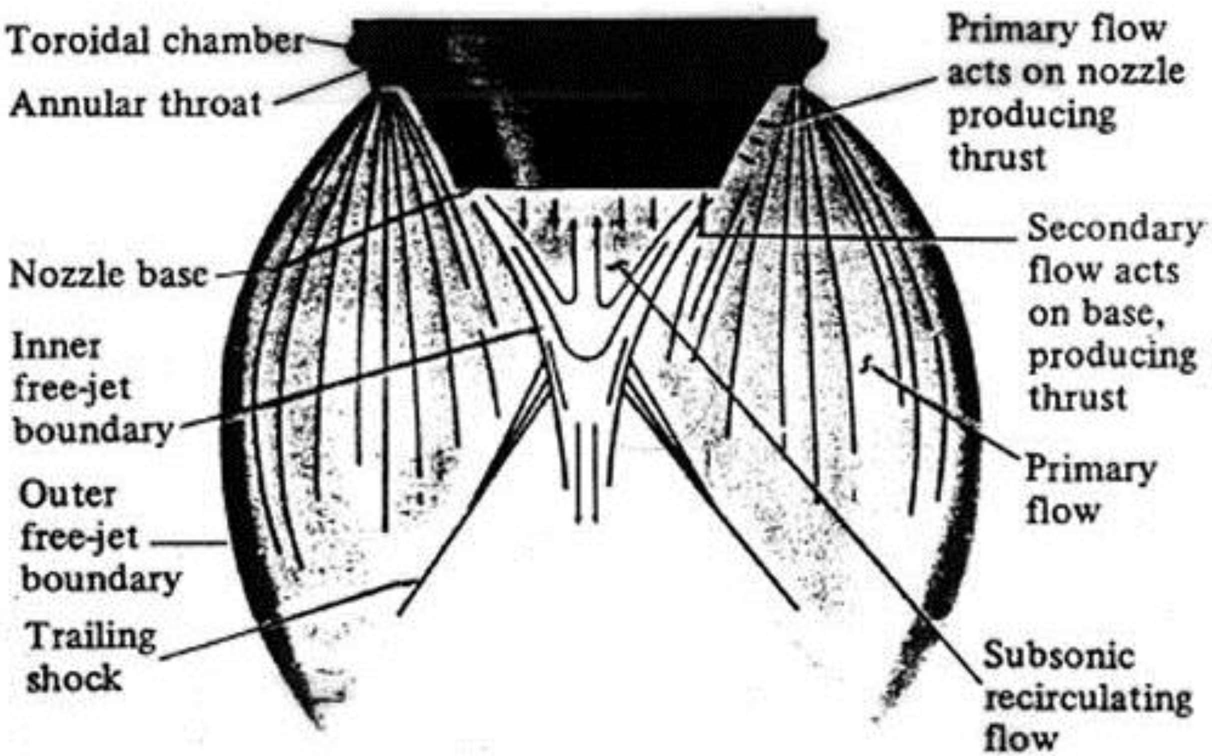




40% length aerospike nozzle

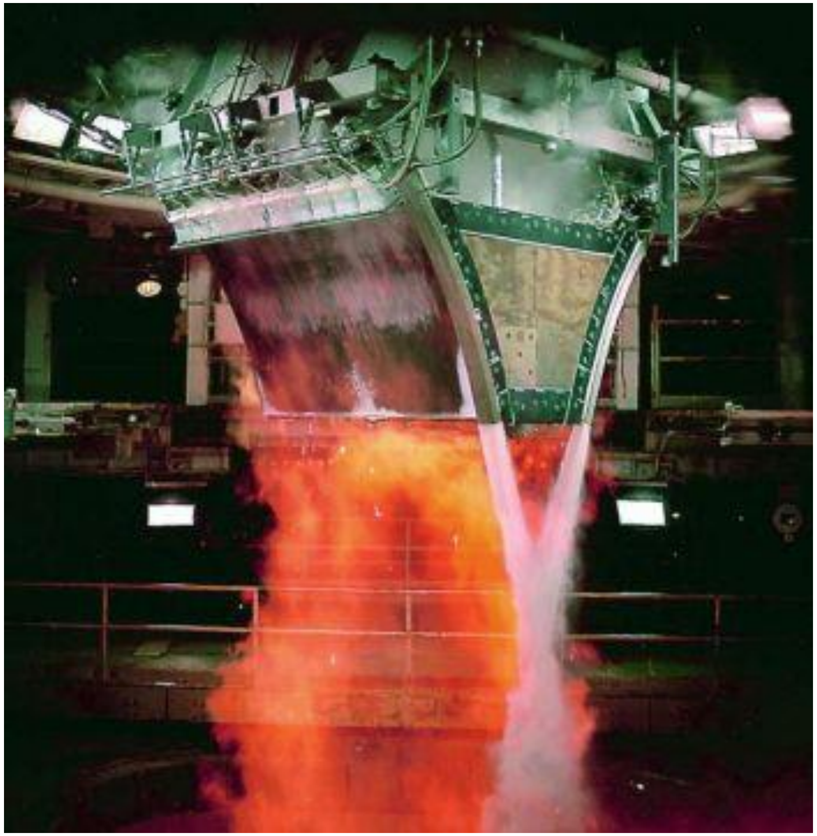
TRUNCATED AEROSPIKE NOZZLES

- Disadvantage of "flat" plug is turbulent wake forms aft of base at high altitudes resulting in high base drag and reduced efficiency
- Alleviated by introducing a "base bleed," or secondary subsonic flow
- Circulation of this secondary flow and its interaction with the engine exhaust creates an "aerodynamic spike" that behaves much like the ideal, isentropic spike
- Secondary flow re-circulates upward pushing on base to produce additional thrust
- It is this artificial aerodynamic spike for which the aerospike nozzle is named

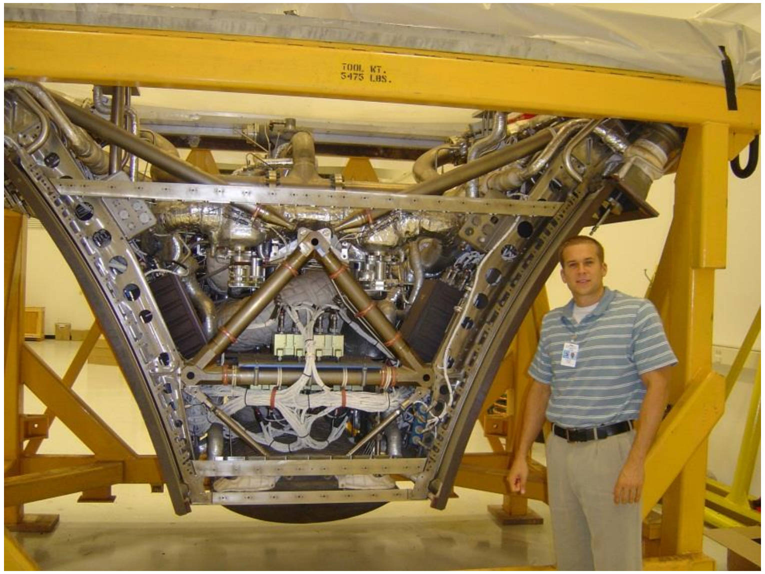


LINEAR AEROSPIKE NOZZLE

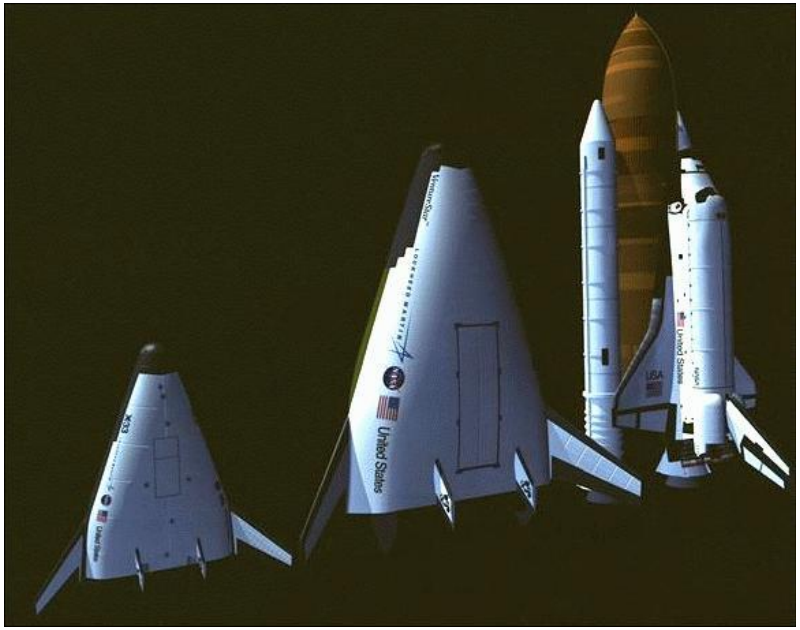
- Still another variation of aerospike nozzle is linear (instead of annular)
- Linear Aerospike pioneered by Rocketdyne (now division of Boeing) in 1970’s
- Places combustion chambers in a line along two sides of nozzle
- Approach results in more versatile design
  - Use of lower-cost modular combustors
  - Modules can be combined in varying configurations depending on application.

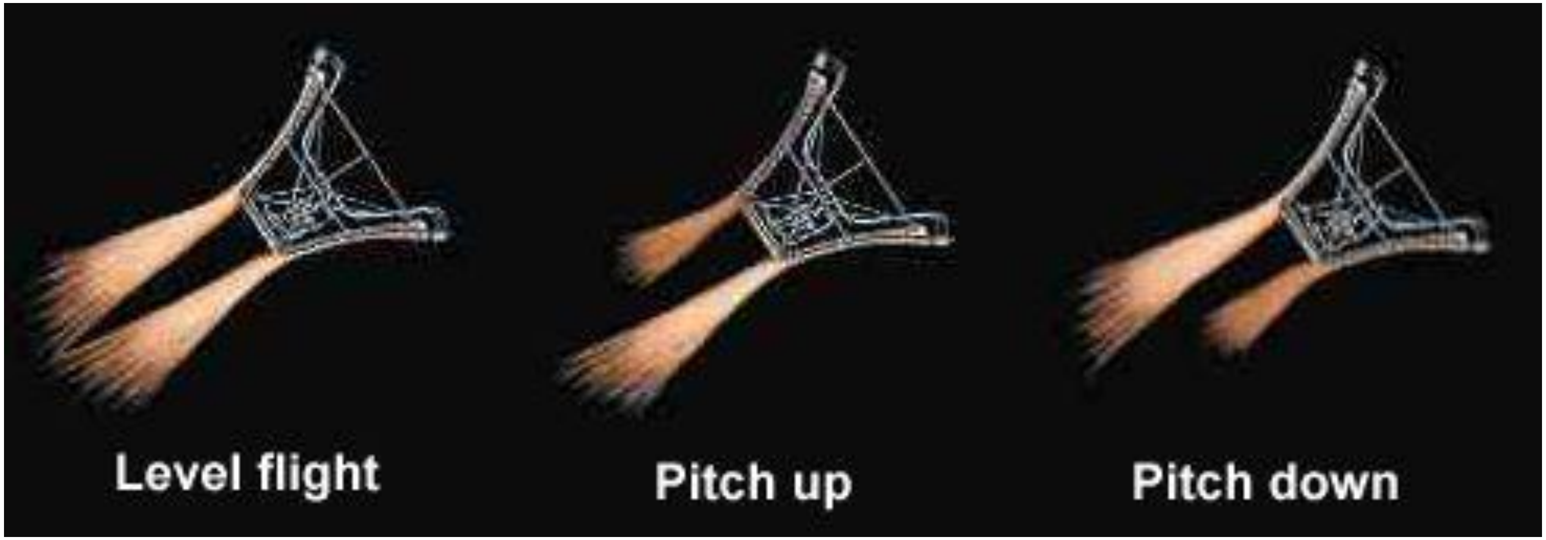


X-33 LINEAR AEROSPIKE NOZZLE



LINEAR AEROSPIKE NOZZLE





SUMMARY: SHAPED (BELL) NOZZLES

ADVANTAGES

- Structural Considerations
  - Essentially only hoop or tangential stresses which are easiest to design for
- Cooling
  - Fabricated with walls of simple tubular construction that enables cooling in a straightforward way
- Matching to combustion chamber
  - Relatively easy to match the combustor, which is most naturally a simple cylinder

DISADVANTAGES

- Over-Expansion Thrust Loss
- Flow instability when over-expanded
  - May lead to uncertainty or unsteadiness of the thrust direction and dangerous high frequency wobble

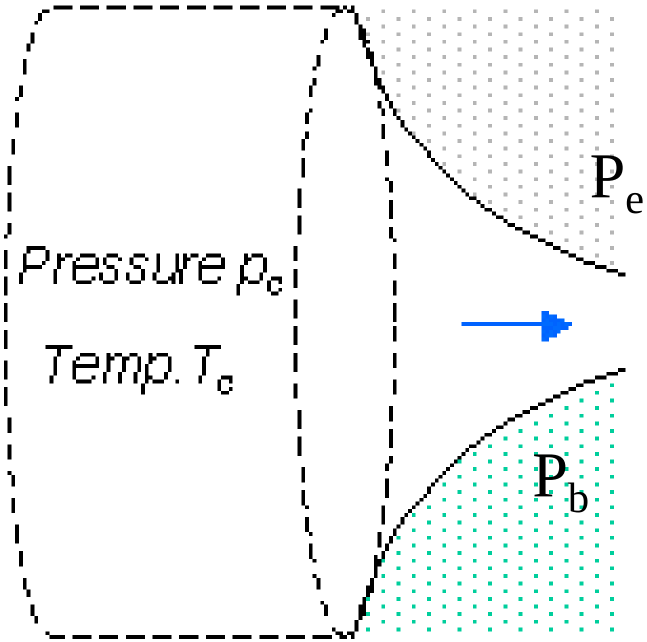
ANNULAR: ADVANTAGES

- Smaller nozzle
  - Truncated spike far smaller than typical bell nozzle for same performance
  - Spike can give greater performance for a given length
- Altitude compensation results in greater performance (no separation at over-expanded)
- Less risk of failure
  - Aerospike engine use simple gas generator cycle with a lower chamber pressure
  - Low pressures → reduced performance, high expansion ratio makes up for deficiency
- Lower vehicle drag
  - Aerospike nozzle fills base portion of vehicle thereby reducing base drag
- Modular combustion chambers
  - Linear aerospike engine is made up of small, easier to develop, less expensive thrusters
- Thrust Vectoring
  - Combustion chambers controlled individually
  - Vehicle maneuvered using differential thrust vectoring
  - Eliminates heavy gimbals and actuators used to vary direction of nozzles

ANNULAR: DISADVANTAGES

- Cooling
  - Central spike experiences far greater heat fluxes than does a bell nozzle
  - Addressed by truncating spike to reduce exposed area and by passing cold cryogenically-cooled fuel through spike
  - Secondary flow also helps cool centerbody.
- Manufacturing
  - Aerospike is more complex and difficult to manufacture than bell nozzle
  - More costly
- Flight experience
  - No aerospike engine has ever flown in a rocket application
  - Little flight design experience has been gained

OPERATION OF CONVERGING NOZZLES



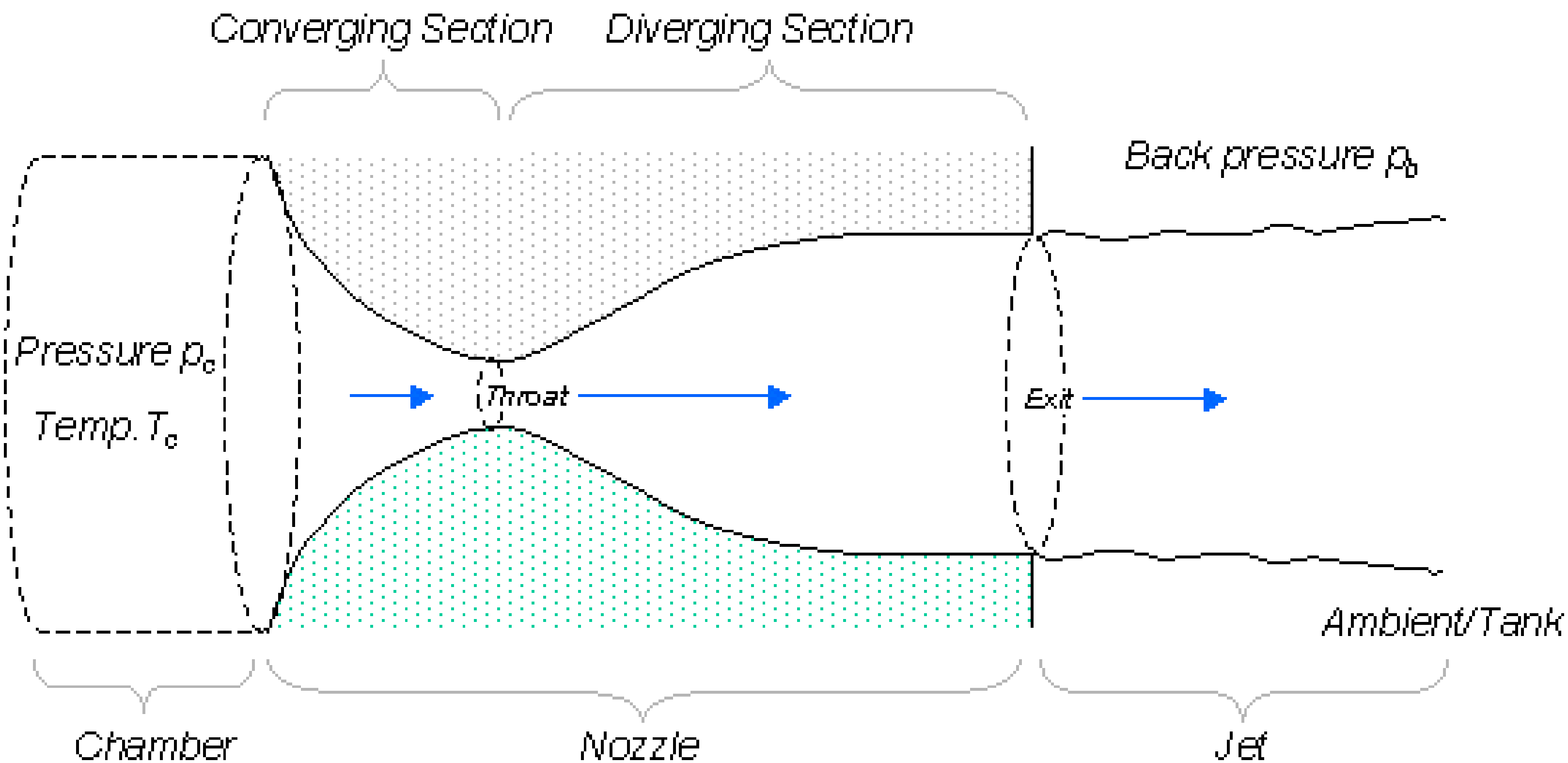
- A converging nozzle will choke when P_b is reduced to a critical value P* (found from isentropic relations)
- P_e = P* is called design pressure ratio
- For P_b > P*
  - Flow will be subsonic
  - $P_e = P_b$
- For any P_b < P*
  - Flow will be sonic at nozzle exit
  - $P_e > P_b$
  - Flow is choked and P_e = P*

Questions?

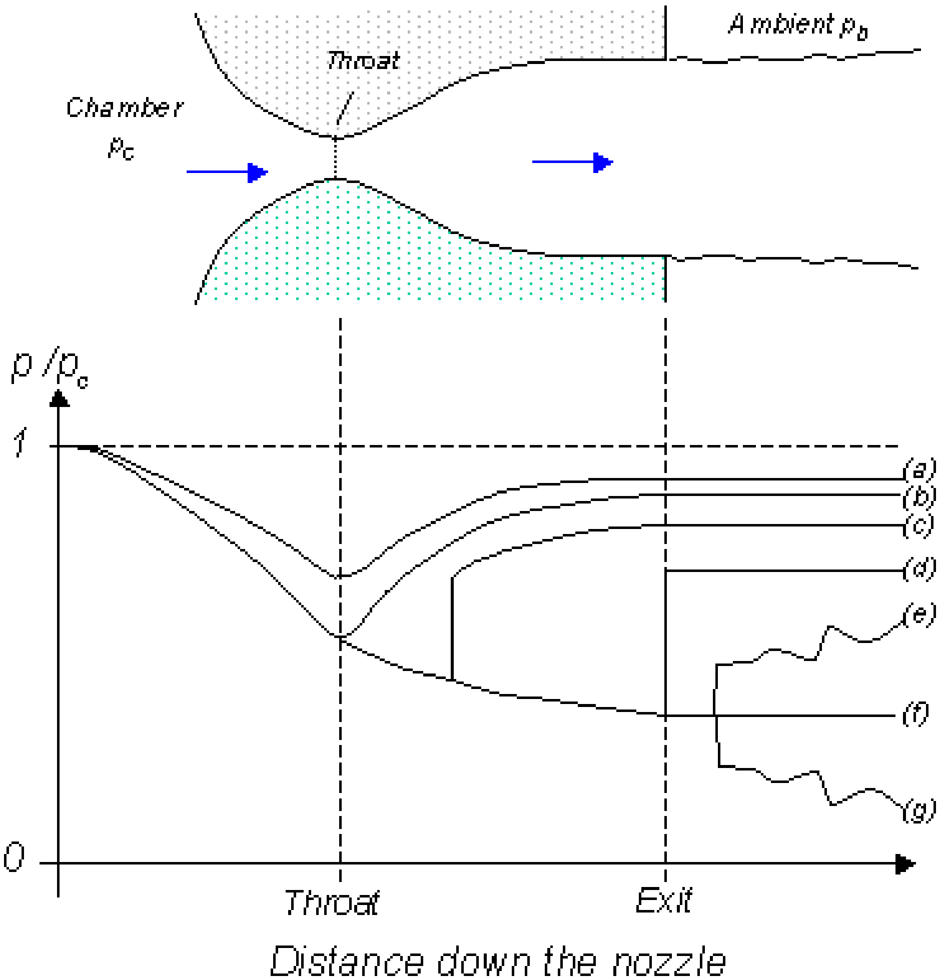
- Can this nozzle ever produce a supersonic flow in the converging portion?
- Can this nozzle ever produce a supersonic region in the exhaust? Is this region expanding or being compressed?
- Can this nozzle ever be over-expanded, such that $P_e < P_b$?

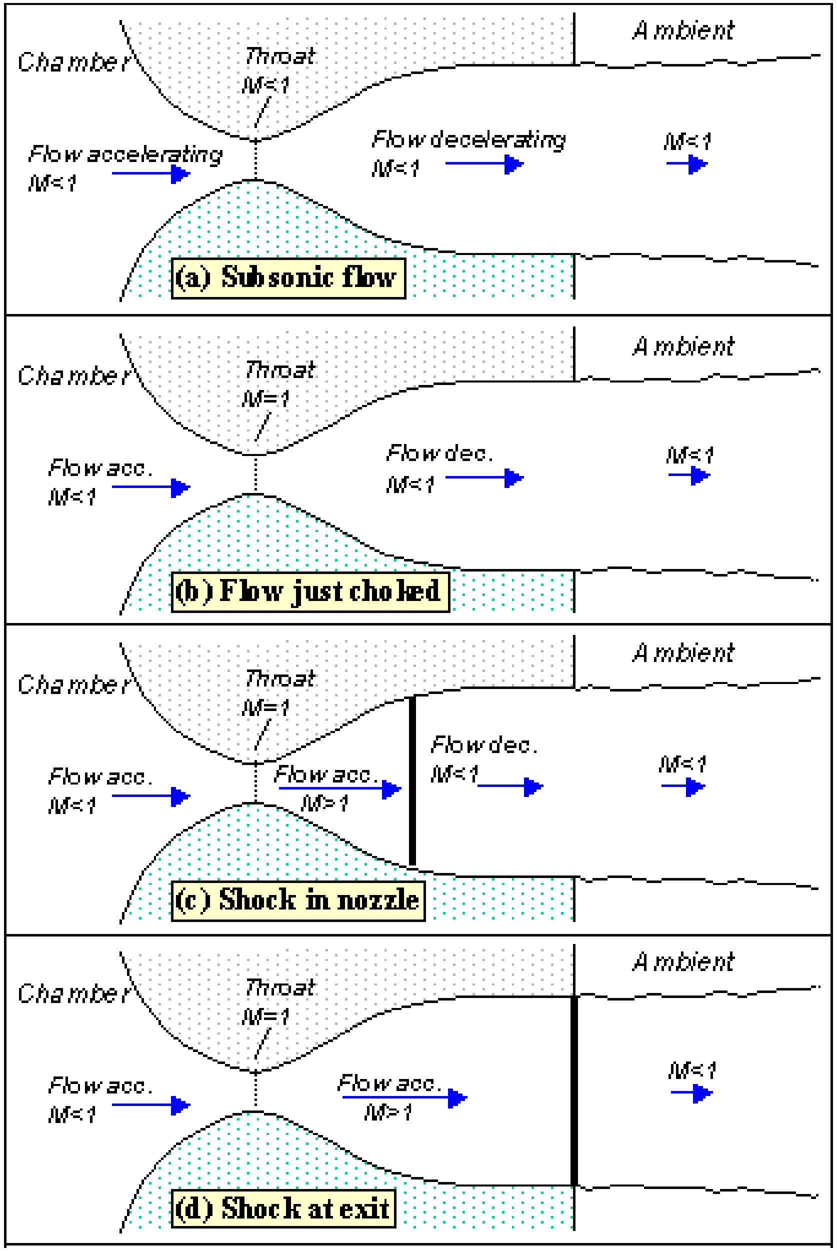
OPERATION OF CD NOZZLES

- Configuration for converging-diverging (CD) nozzle is shown below
- Gas flows through nozzle from region of high pressure (chamber) to low pressure (ambient)
- The chamber is taken as big enough so that any flow velocities are negligible
- Gas flows from chamber into converging portion of nozzle, past the throat, through the diverging portion and then exhausts into the ambient as a jet
- Pressure of ambient is referred to as back pressure

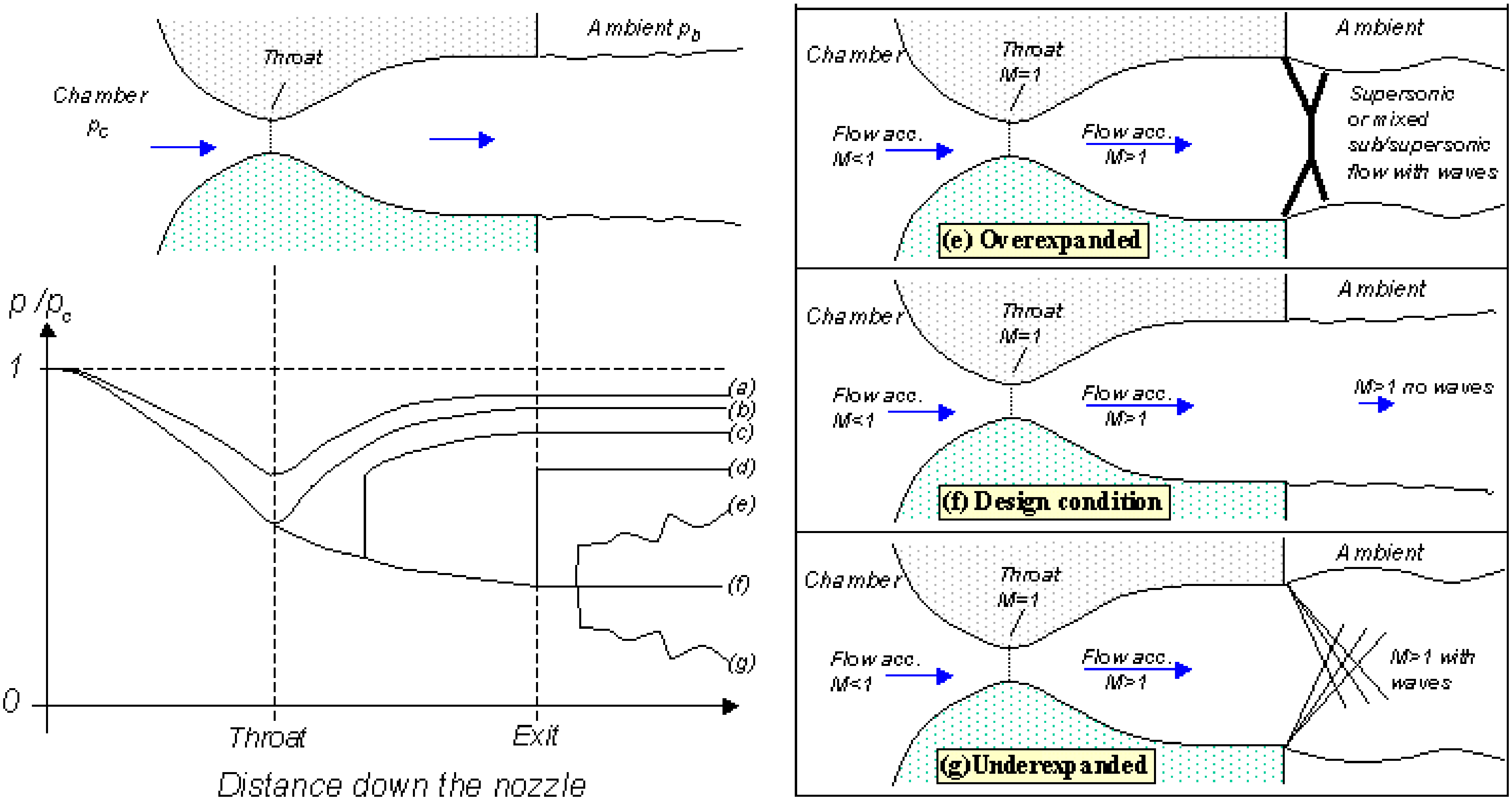


OPERATION OF CD NOZZLES





OPERATION OF CD NOZZLES



All practical rockets operate in regimes (e)-(g)

OPERATION OF CD NOZZLES

- Figure (a) shows the flow through the nozzle when it is completely subsonic (i.e. nozzle isn't choked). The flow accelerates out of the chamber through the converging section, reaching its maximum (subsonic) speed at the throat. The flow then decelerates through the diverging section and exhausts into the ambient as a subsonic jet. Lowering the back pressure in this state increases the flow speed everywhere in the nozzle.

- Further lowering p_b results in figure (b). The flow pattern is exactly the same as in subsonic flow, except that the flow speed at the throat has just reached Mach 1. Flow through the nozzle is now choked since further reductions in the back pressure can't move the point of M=1 away from the throat. However, the flow pattern in the diverging section does change as the back pressure is lowered further.

- As p_b is lowered below that needed to just choke the flow a region of supersonic flow forms just downstream of the throat. Unlike a subsonic flow, the <u>supersonic flow accelerates as the area gets bigger</u>. This region of supersonic acceleration is terminated by a normal shock wave. The shock wave produces a near-instantaneous deceleration of the flow to subsonic speed. This subsonic flow then decelerates through the remainder of the diverging section and exhausts as a subsonic jet. In this regime if the back pressure is lowered or raised the length of supersonic flow in the diverging section before the shock wave increases or decreases, respectively.

OPERATION OF CD NOZZLES

- If p_b is lowered enough the supersonic region may be extended all the way down the nozzle until the shock is sitting at the nozzle exit, figure (d). Because of the very long region of acceleration (the entire nozzle length) the flow speed just before the shock will be very large. However, after the shock the flow in the jet will still be subsonic.

- Lowering the back pressure further causes the shock to bend out into the jet, figure (e), and a complex pattern of shocks and reflections is set up in the jet which will now involve a mixture of subsonic and supersonic flow, or (if the back pressure is low enough) just supersonic flow. Because the shock is no longer perpendicular to the flow near the nozzle walls, it deflects it inward as it leaves the exit producing an initially contracting jet. We refer to this as **over-expanded** flow because in this case the pressure at the nozzle exit is lower than that in the ambient (the back pressure)- i.e. the flow has been expanded by the nozzle too much.

- A further lowering of the back pressure changes and weakens the wave pattern in the jet. Eventually, the back pressure will be lowered enough so that it is now equal to the pressure at the nozzle exit. In this case, the waves in the jet disappear altogether, figure (f), and the jet will be uniformly supersonic. This situation, since it is often desirable, is referred to as the 'design condition‘, Pe=Pa.

OPERATION OF CD NOZZLES

- Finally, if the back pressure is lowered even further we will create a new imbalance between the exit and back pressures (exit pressure greater than back pressure), figure (g). In this situation, called **under-expanded**, expansion waves that produce gradual turning and acceleration in the jet form at the nozzle exit, initially turning the flow at the jet edges outward in a plume and setting up a different type of complex wave pattern.

- Summary Points to Remember:
  - When the flow accelerates (sub or supersonically) the pressure drops
  - The pressure rises instantaneously across a shock
  - The pressure throughout the jet is always the same as the ambient (i.e. the back pressure) unless the jet is supersonic and there are shocks or expansion waves in the jet to produce pressure differences
  - The pressure falls across an expansion wave

EXAMPLE: SPACE SHUTTLE MAIN ENGINE, $\varepsilon \sim 77$

- Static pressure at exit of Space Shuttle Main Engine nozzle is considerably less than ambient pressure at sea level

- Mismatch in pressure gives rise to Mach "disc" in nozzle exhaust

- Extremely strong shock wave that creates a region of subsonic flow and produces a characteristic white luminescent glow

- Flow in picture is over-expanded (lift-off)

DETAILS: OVER-EXPANDED DESCRIPTION

- The rocket's nozzle is designed to be efficient at altitudes above sea level, and, at engine start, the flow is **over-expanded**, that is, the exhaust gas pressure, pe, is higher than the supersonic isentropic exit pressure but lower than the ambient pressure, pa. This causes an oblique shock to form at the exit plane (A) of the nozzle. To reach ambient pressure, the gases undergo compression as they move away from the nozzle exit and pass through the oblique shock wave standing at the exit plane. The flow that has passed through the shock wave will be turned towards the center line (2). At the same time, the oblique shock wave, directed toward the center line of the nozzle, cannot penetrate the center plane since the center plane acts like a streamline. This causes the oblique shock wave to be reflected outward (B) from the center plane. The gas flow goes through this reflected shock and is further compressed but the flow is now turned parallel (3) to the centerline. This causes the pressure of the exhaust gases to increase above the ambient pressure. The reflected shock wave (see diagram below) now hits the free jet boundary called a contact discontinuity (or the boundary where the outer edge of the gas flow meets the free stream air). Pressure is the same across this boundary and so is the direction of the flow. Since the flow is at a higher pressure than ambient pressure, the pressure must reduce. Thus, at the reflected shock wave-contact discontinuity intersection, expansion waves of the Prandtl-Meyer (P-M) type are set up (C) to reduce the pressure to pa. These expansion waves are directed towards the centerline of the nozzle. The gas flow passing through the Prandtl-Meyer expansion waves turn away from the centerline (4). The Prandtl-Meyer expansion waves in turn reflect from the center plane towards the contact discontinuity (D). The gas flow passing through the reflected Prandtl-Meyer waves is now turned back parallel to the centerline but undergoes a further reduction of pressure.

Principle of operation of nozzle

- For every action there is equal and opposite reaction

General Thrust Equation

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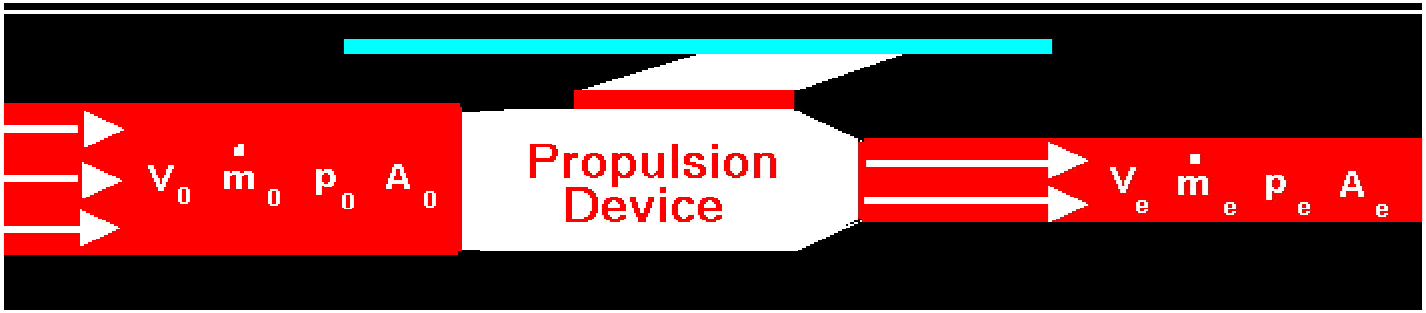


Thrust is a force.

Force = change in momentum with time

$$F = \frac{([mV]_2 - [mV]_1)}{(t_2 - t_1)}$$

$\dot{m}$ = mass flow rate = mass / time

$\dot{m} = r \times V \times A$ where r = density, V = velocity, A = area

If $p_e \neq p_0$: $F = \dot{m}_e V_e - \dot{m}_0 V_0 + (p_e - p_0) A_e$

If $p_e = p_0$: $F = \dot{m}_e V_e - \dot{m}_0 V_0$

- Thrust is the force which moves an aircraft through the air. Thrust is generated by the propulsion of the airplane.

How is thrust generated?

- Thrust is a mechanical **force** which is generated through the **reaction** of accelerating a mass of gas, as explained by Newton's third law of motion. A gas or **working fluid** is accelerated to the rear and the engine and aircraft are accelerated in the opposite direction. To accelerate the gas, we need some kind of propulsive system.
- From Newton's second law of motion, we can define a force F to be the change in momentum of an object with a change in time. Momentum is the object's mass m times the velocity V. So, between two times t1 and t2, the force is given by:

- $F = (m * V_2 - m * V_1) / (t_2 - t_1)$

- If we are dealing with a solid, keeping track of the mass is relatively easy; the molecules of a solid are closely bound to each other and a solid retains its shape. But if we are dealing with a fluid (liquid or gas) and particularly if we are dealing with a moving fluid, keeping track of the mass gets tricky. For a moving fluid, the important parameter is the mass flow rate. Mass flow rate is the amount of mass moving through a given plane over some amount of time. Its dimensions are mass/time (kg/sec, slug/sec, ...) and it is equal to the density r times the velocity V times the area A. Aerodynamicists denote this parameter as m dot (m with a little dot over the top).

$\dot{m} = \rho * V * A$ then we have,

$F = \dot{m} * V_e - \dot{m} * V_0$

This is the design theory behind Propeller aircraft and high bypass turbofan engines. A large amount of air is processed each second, but the velocity is not changed very much. The other way to produce high thrust is to make the exit velocity very much greater than the incoming velocity. This is the design theory behind pure turbojet and turbojets with afterburners, and rockets. A moderate amount of flow is accelerated to a high velocity in these engines. If the exit velocity becomes very high, there are other physical processes which become important and affect the efficiency of the engine. These effects are described in detail on other pages at this site

There is a different simplified version of the general thrust equation that can be used for rocket engines. Since a rocket carries its own oxygen on board, the free stream mass flow rate is zero and the second term of the general equation drops out.

$$F = \dot{m} * V_e + (p_e - p_0) * A_e$$

We have to include the pressure correction term since a rocket nozzle produces a fixed exit pressure which in general is different than free stream pressure. There is a useful rocket performance parameter called the specific impulse Isp, that eliminates the mass flow dependence in the analysis.

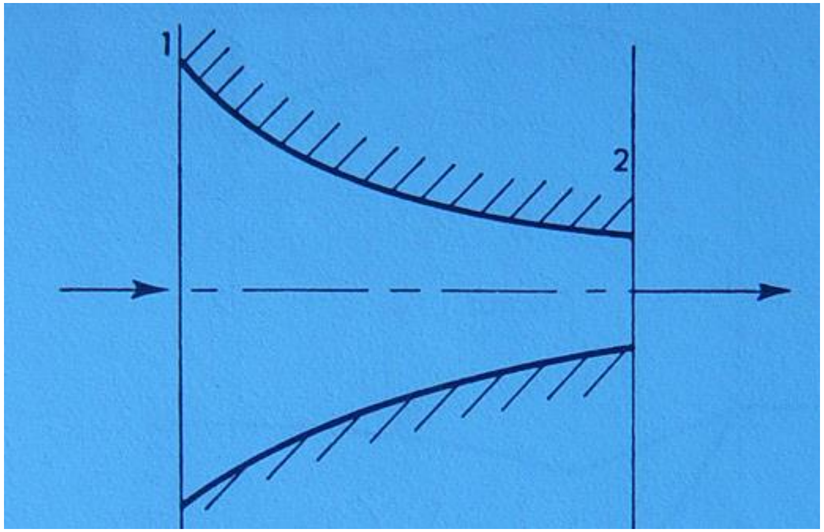
$$Isp = Veq / g$$

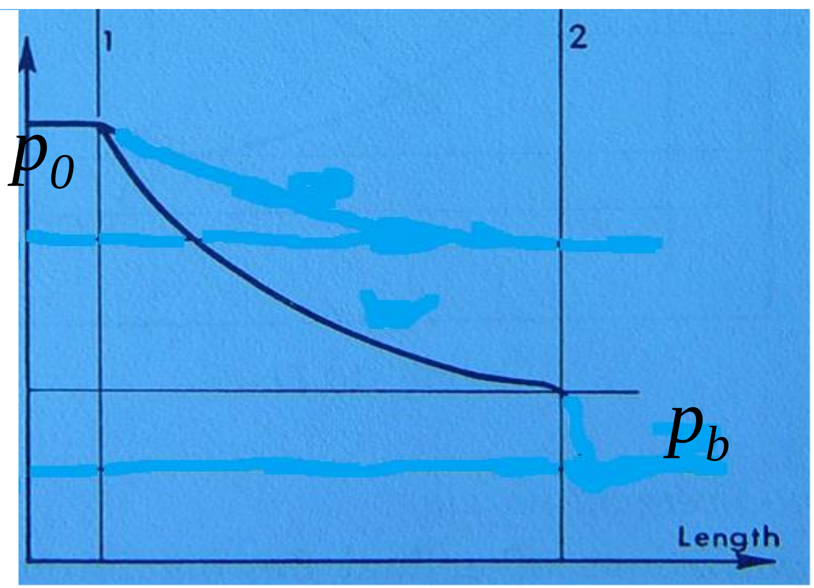
where Veq is the equivalent velocity, which is equal to the nozzle exit velocity plus the pressure-area term, and g is the gravitational acceleration.

- For both rockets and turbojets, the nozzle performs two important roles. The design of the nozzle determines the exit velocity for a given pressure and temperature. And because of flow choking in the throat of the nozzle, the nozzle design also sets the mass flow rate through the propulsion system. Therefore, the nozzle design determines the thrust of the propulsion system as defined on this page. You can investigate nozzle operation with our interactive thrust simulator.

- You can view a short movie of "Orville and Wilbur Wright" discussing the thrust force and how it affected the flight of their aircraft. The movie file can be saved to your computer and viewed as a Podcast on your podcast player.

Converging Nozzle





p_b = *Back Pressure*

Design Variables: $\dot{m}, p_0, T_0$

Outlet Condition:

p_b *or* A_{exit} *or* m_{exit}

Designed Exit Conditions

$$\frac{p_0}{p_{exit}} = \left\{\frac{T_0}{T_{exit}}\right\}^{\frac{\gamma}{\gamma-1}} = \left\{1 + \frac{\{\gamma-1\}}{2} M_{exit}{}^2\right\}^{\frac{\gamma}{\gamma-1}}$$

$$\dot{m} = \sqrt{\frac{\gamma}{R}} \frac{p_0}{\sqrt{T_0}} \frac{A_{exit} M_{exit}}{\left\{1 + \frac{\gamma-1}{2} M_{exit}{}^2\right\}^{\frac{(\gamma+1)}{2(\gamma-1)}}}$$

Under design conditions the pressure at the exit plane of the nozzle is applied back pressure.

$$\frac{p_0}{p_b} = \left\{\frac{T_0}{T_{exit}}\right\}^{\frac{\gamma}{\gamma-1}} = \left\{1 + \frac{\{\gamma-1\}}{2} M_{exit}{}^2\right\}^{\frac{\gamma}{\gamma-1}}$$

Full Capacity Convergent Nozzle

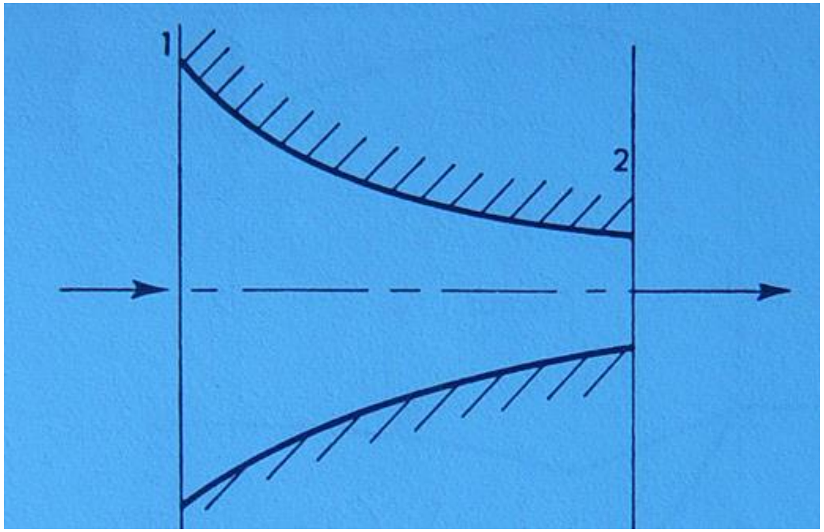
$$\frac{p(x)}{p_{b,critical}} = \left\{ \frac{\gamma + 1}{2 + \{\gamma - 1\} M(x)^2} \right\}^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{A(x)}{A_{exit,critical}} = \frac{1}{M(x)} \left\{ \frac{1 + \frac{\gamma - 1}{2} M(x)^2}{\frac{\gamma + 1}{2}} \right\}^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

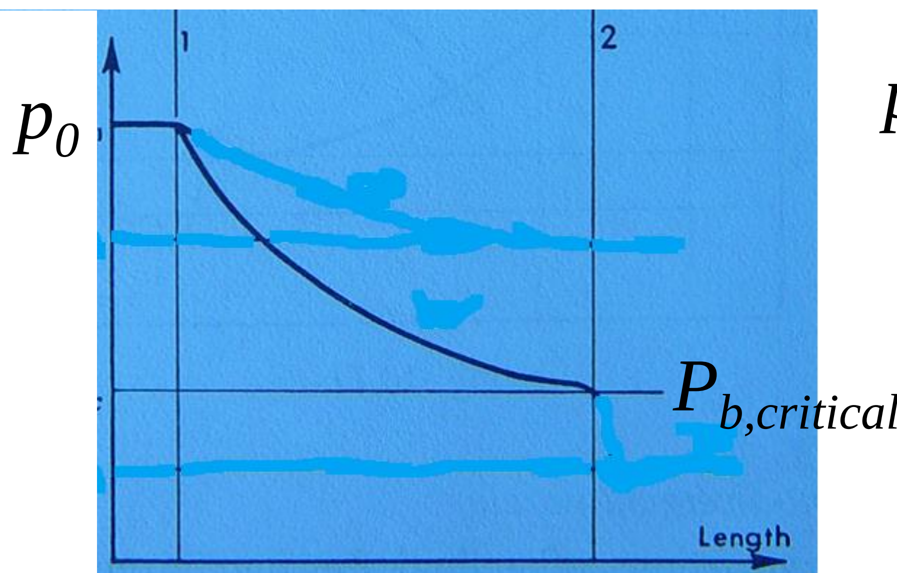
Remarks on Isentropic Nozzle Design

- Length of the nozzle is immaterial for an isentropic nozzle.
- Strength requirements of nozzle material may decide the nozzle length.
- Either Mach number variation or Area variation or Pressure variation is specified as a function or arbitrary length unit.
- Nozzle design attains maximum capacity when the exit Mach number is unity.

Converging Nozzle



$$\frac{p_0}{p_{b,critical}} = \left\{\frac{\gamma+1}{2}\right\}^{\frac{\gamma}{\gamma-1}}$$

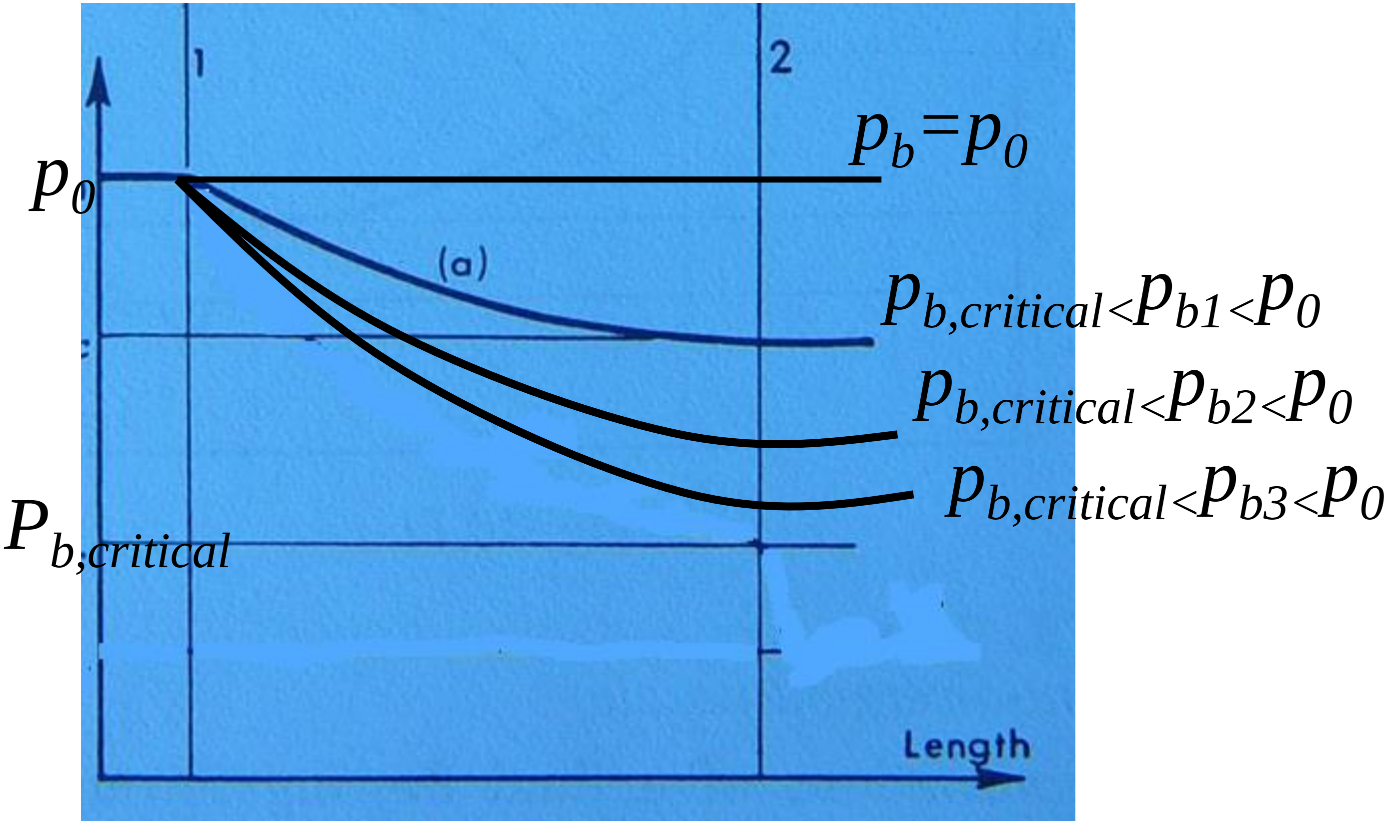


$$p_{b,critical} = p_0\left\{\frac{2}{\gamma+1}\right\}^{\frac{\gamma}{\gamma-1}}$$

Operational Characteristics of Nozzles

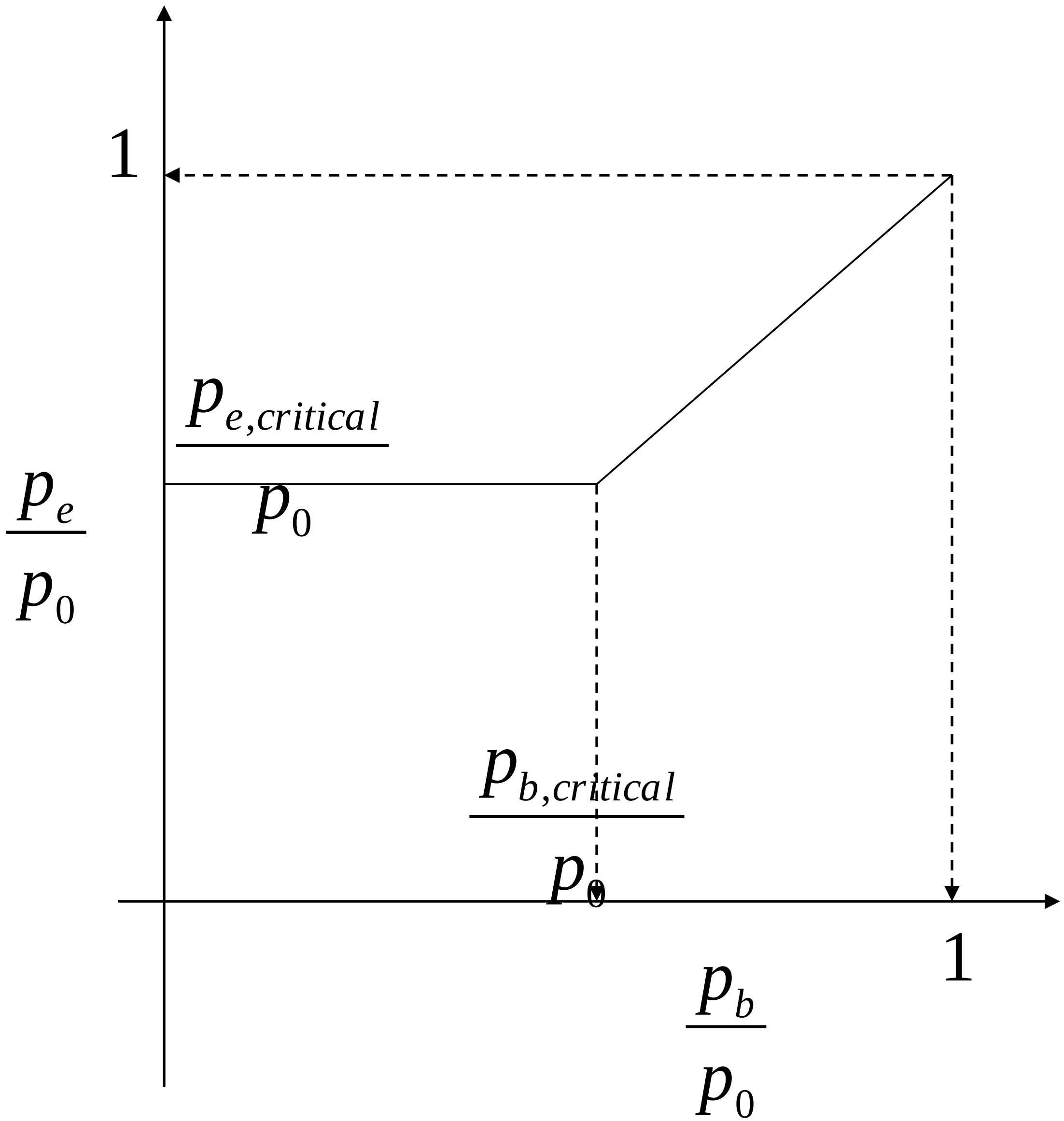
- A variable area passage designed to accelerate the a gas flow is considered for study.
- The concern here is with the effect of changes in the upstream and downstream pressures
- on the nature of the inside flow and
- on the mass flow rate through a nozzle.
- Four different cases considered for analysis are:
- Converging nozzle with constant upstream conditions.
- Converging-diverging nozzle with constant upstream conditions.
- Converging nozzle with constant downstream conditions.
- Converging-diverging nozzle with constant downstream conditions.

Pressure Distribution in Under Expanded Nozzle

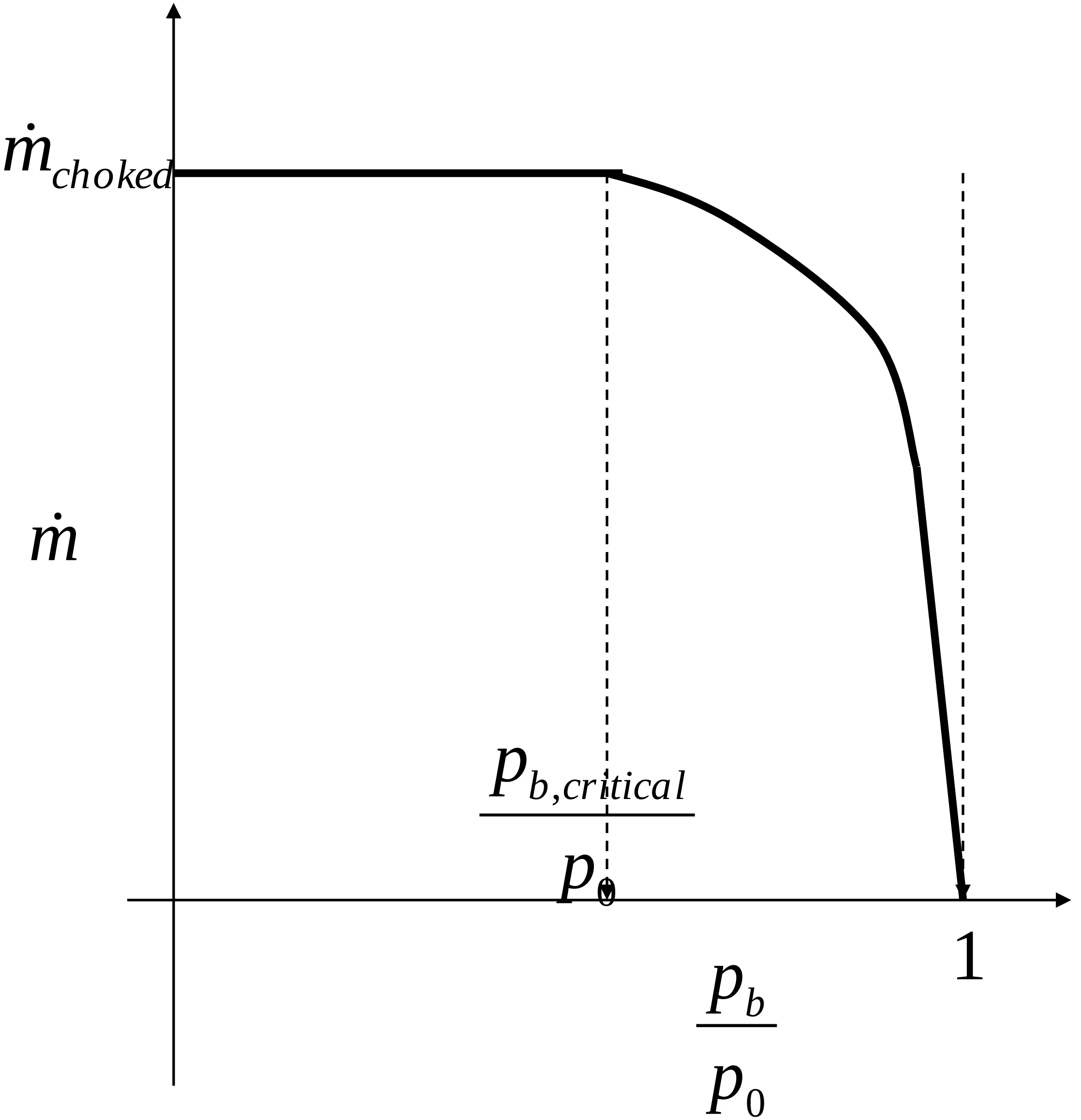


At all the above conditions, the pressure at the exit plane of nozzle, $p_{exit} = p_b$.

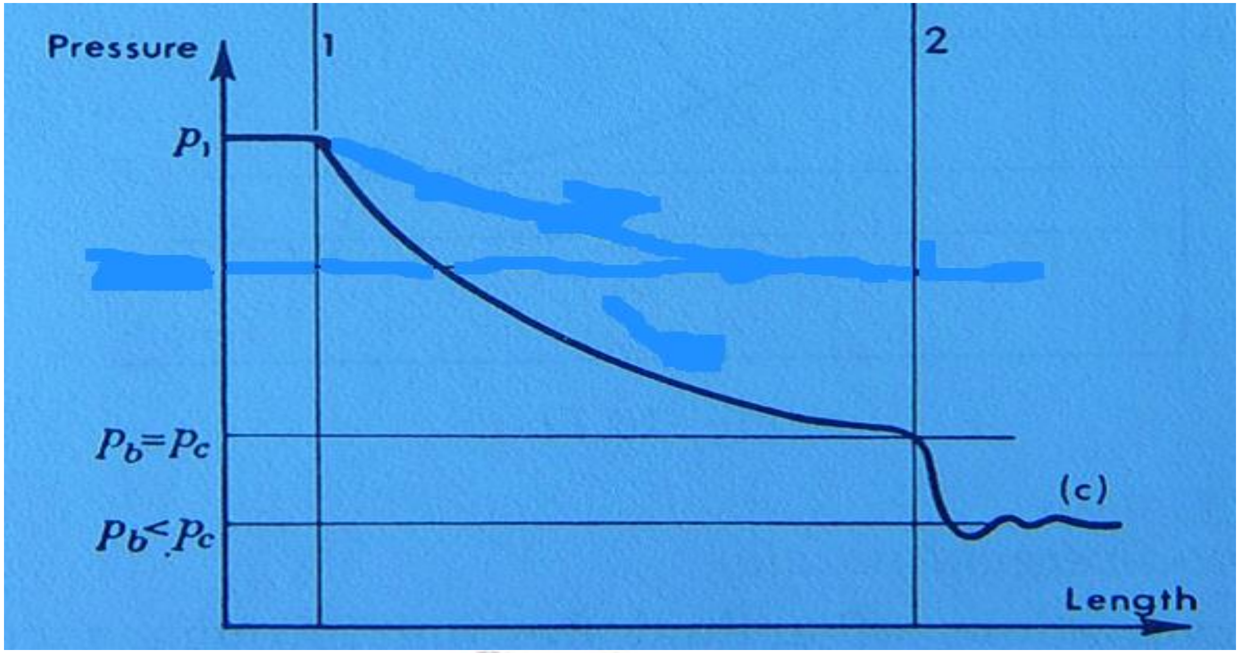
Variation of in Exit Pressure

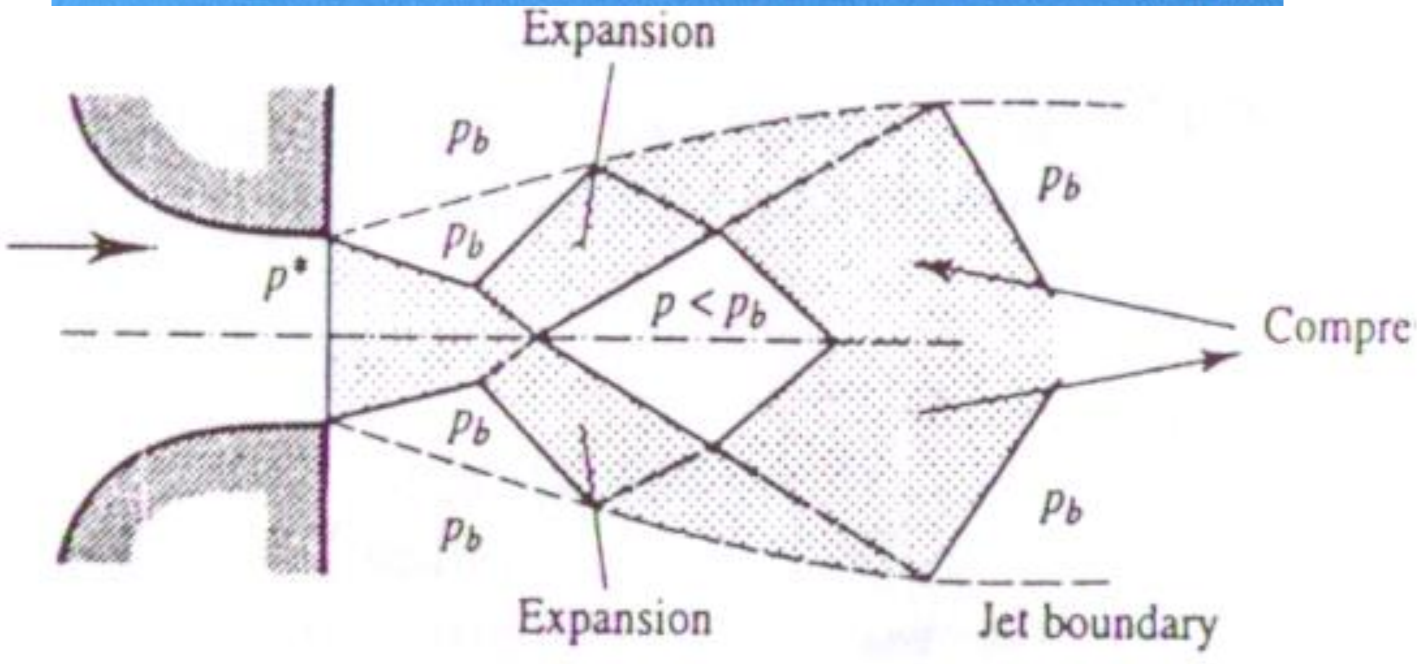


Variation of in Mass Flow Rate

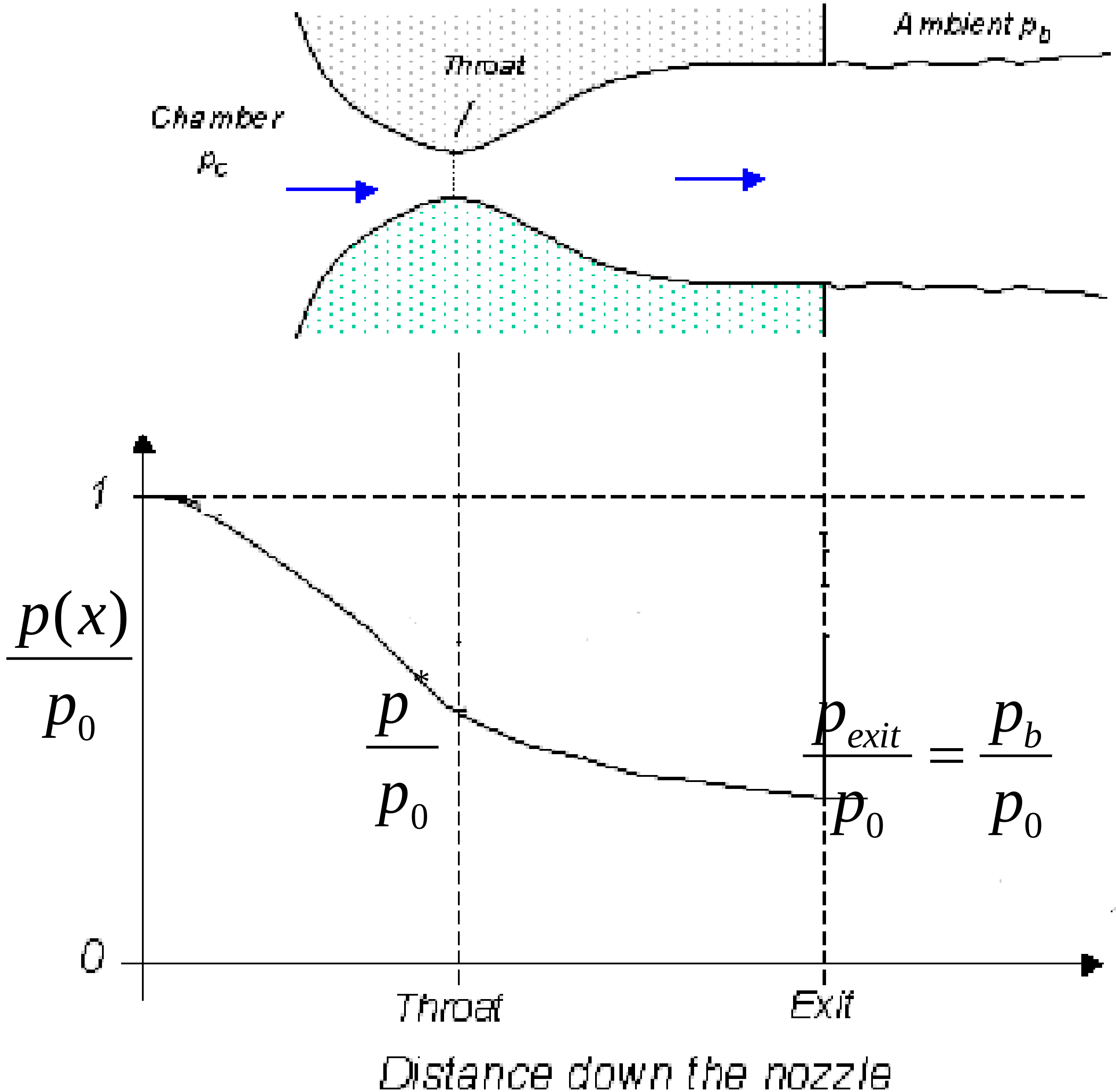


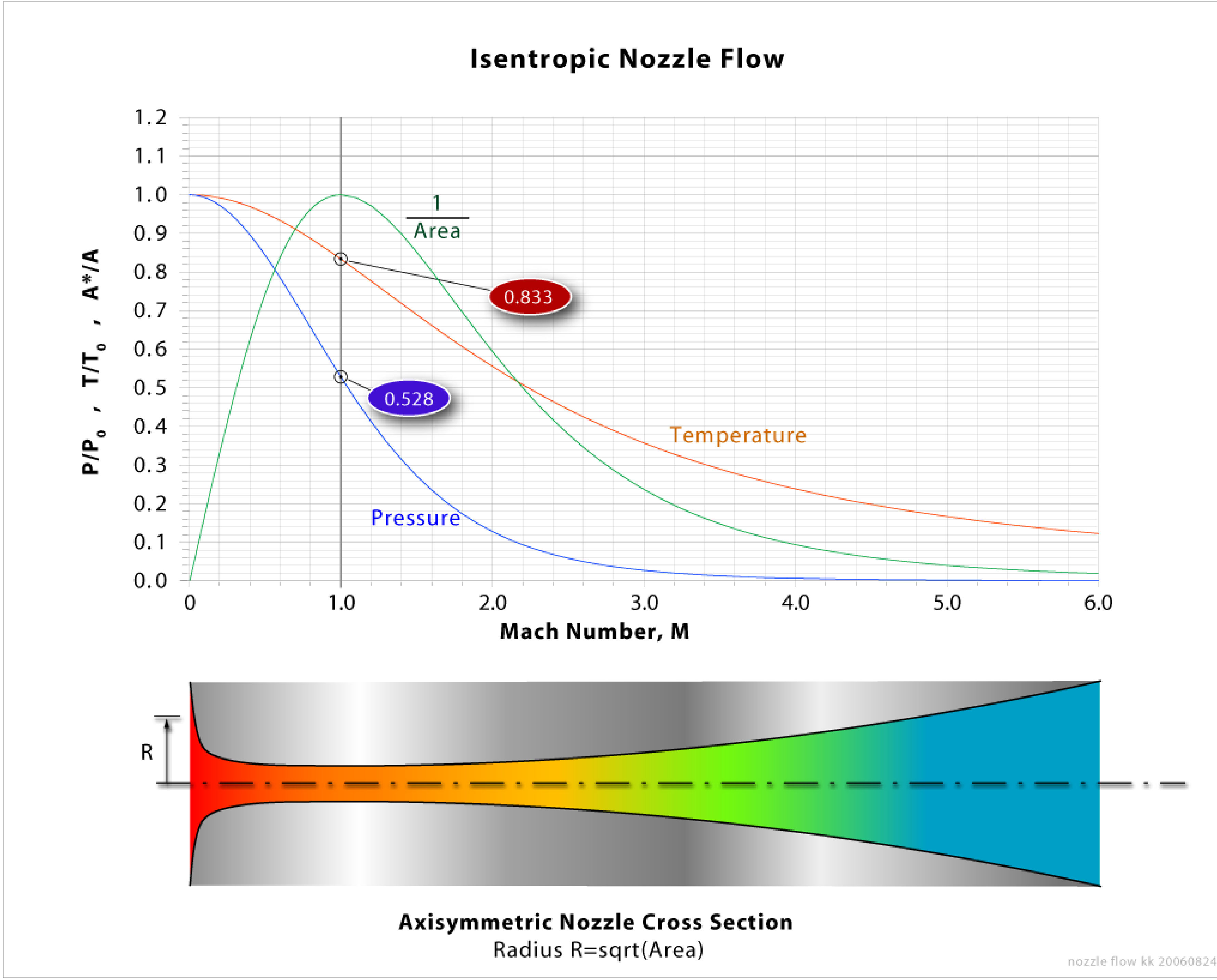
Low Back Pressure Operation





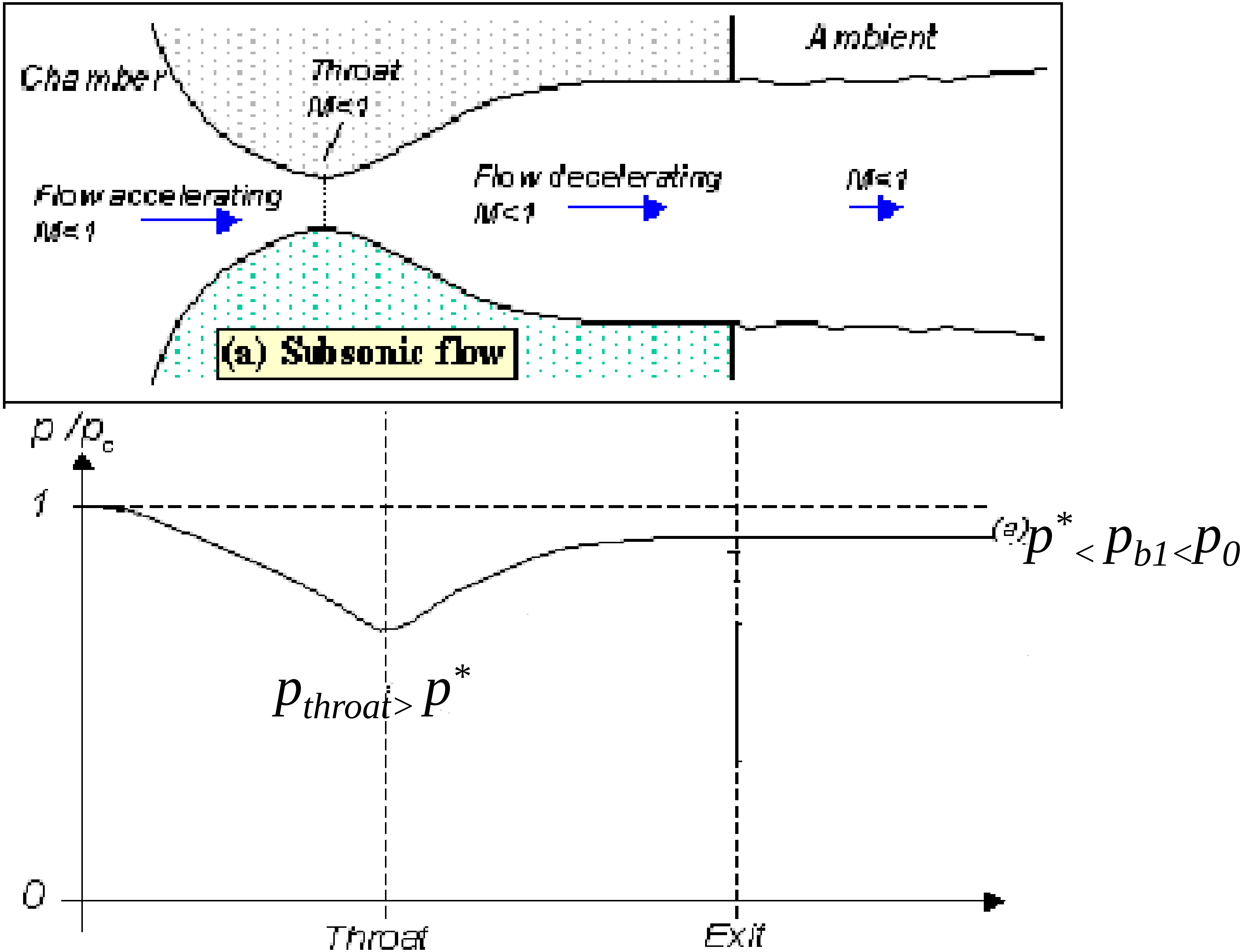
Convergent-Divergent Nozzle Under Design Conditions





Isentropic Nozzle Flow
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
P/P0 , T/T0 , A*/A
1
Area
0.833
0.528
Temperature
Pressure
0
1.0
2.0
3.0
4.0
5.0
6.0
Mach Number, M
R
Axisymmetric Nozzle Cross Section
Radius R=sqrt(Area)
nozzle flow kk 20060824

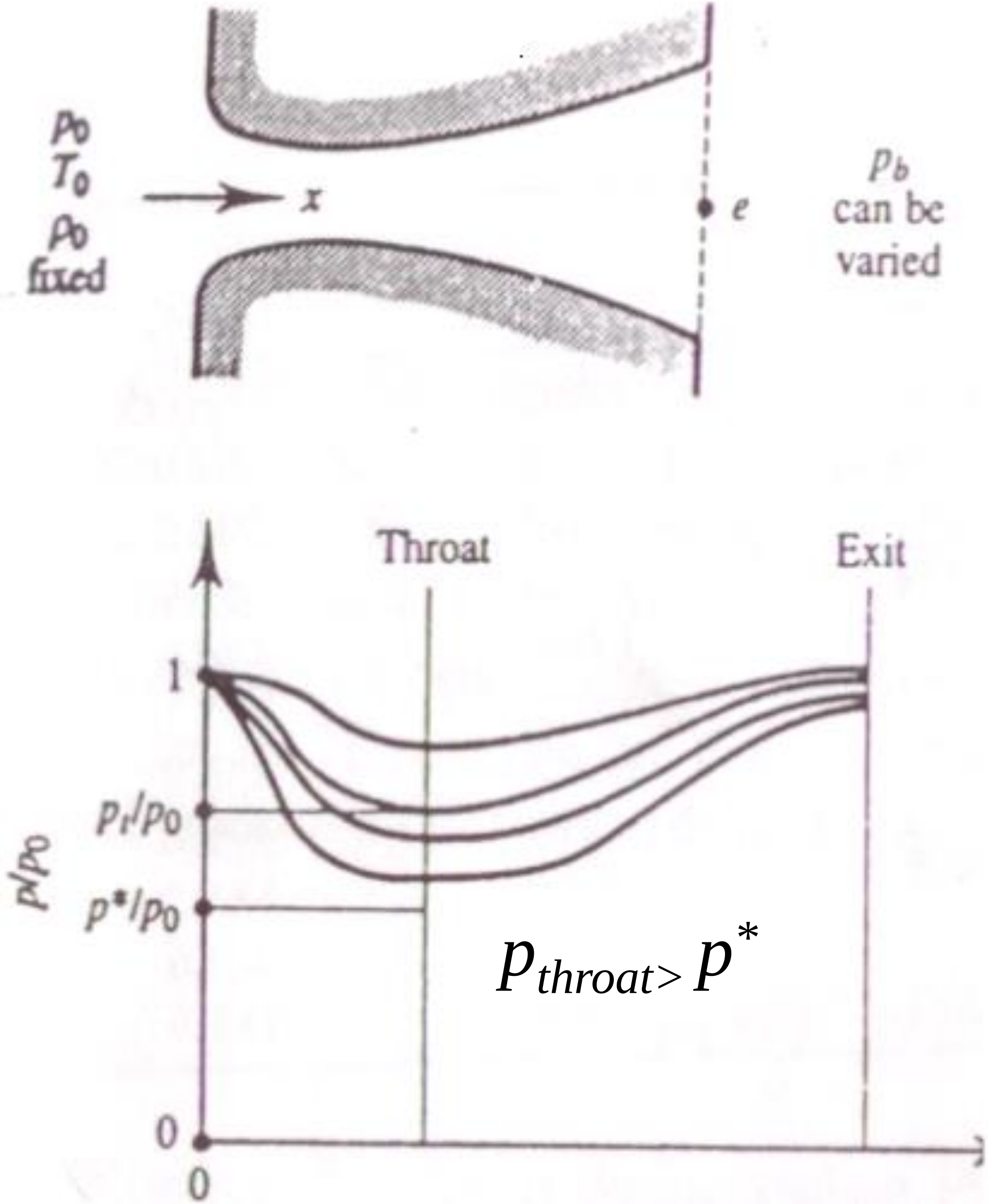
Convergent-Divergent Nozzle with High Back Pressure



Convergent-Divergent Nozzle with High Back Pressure

- When p_b is very nearly the same as p_0 the flow remains subsonic throughout.
- The flow in the nozzle is then similar to that in a venturi.
- The local pressure drops from p_0 to a minimum value at the throat, p_{throat} , which is greater than p^*.
- The local pressure increases from throat to exit plane of the nozzle.
- The pressure at the exit plate of the nozzle is equal to the back pressure.
- This trend will continue for a particular value of back pressure.

Convergent-Divergent Nozzle with High Back Pressure



At all these back pressures the exit plane pressure is equal to the back pressure.

$$C_p T(x) + \frac{u(x)^2}{2} = C_p T_0$$

$$\frac{c^2(x)}{\gamma - 1} + \frac{u(x)^2}{2} = \frac{c_0^2}{\gamma - 1}$$

$$u^2(x) = 2\left\{\frac{c_0^2}{\gamma - 1} - \frac{c^2(x)}{\gamma - 1}\right\} = \frac{2}{\gamma - 1}\left\{c_0^2 - c^2(x)\right\}$$

$$u^2(x) = \frac{2c_0^2}{\gamma - 1}\left\{1 - \frac{c^2(x)}{c_0^2}\right\}$$

$$\frac{c(x)}{c_0} = \left\{\frac{T(x)}{T_0}\right\}^{\frac{1}{2}} = \left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{2[\gamma - 1]}} = \left\{\frac{\rho(x)}{\rho_0}\right\}^{\frac{1}{2[\gamma - 1]}}$$

$$u^2(x) = \frac{2c_0^2}{\gamma - 1}\left\{1 - \left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}$$

$$u^2(x) = \frac{2\gamma RT_0}{\gamma - 1}\left\{1 - \left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}$$

$$u^2(x) = \frac{2\gamma p_0}{(\gamma - 1)\rho_0}\left\{1 - \left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}$$

$$u(x) = \left[\frac{2\gamma p_0}{(\gamma - 1)\rho_0}\left\{1 - \left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}\right]^{1/2}$$

$$\dot{m} = \rho(x)A(x)u(x)$$

$$\dot{m} = \rho_0 \frac{\rho(x)}{\rho_0} A(x)u(x)$$

$$\dot{m} = \rho_0 \frac{\rho(x)}{\rho_0} A(x)\left[\frac{2\gamma p_0}{(\gamma-1)\rho_0}\left\{1-\left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}\right]^{1/2}$$

$$\dot{m} = \rho_0 \left\{\frac{p(x)}{p_0}\right\}^{\frac{1}{\gamma}} A(x)\left[\frac{2\gamma p_0}{(\gamma-1)\rho_0}\left\{1-\left\{\frac{p(x)}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}\right]^{1/2}$$

At exit with high back pressure p_b

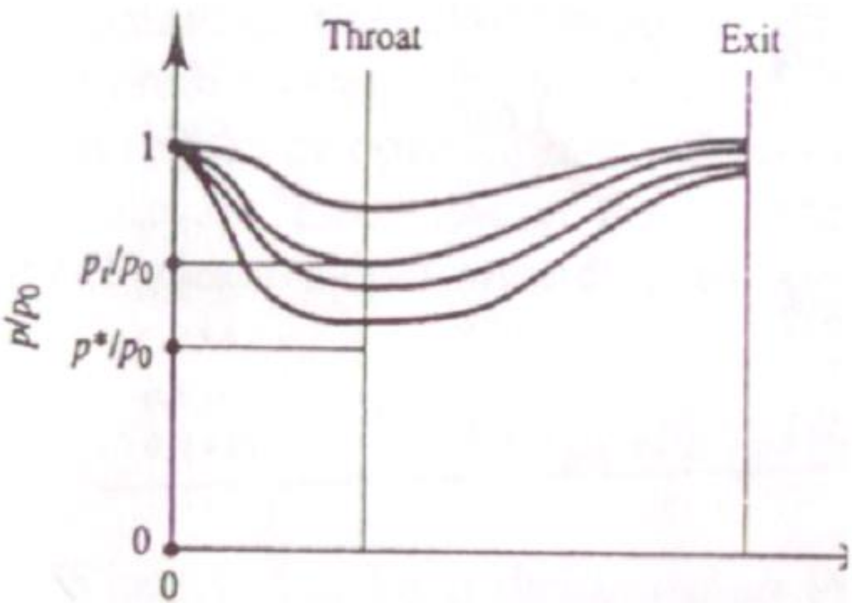
$$\dot{m} = \rho_0 \left\{ \frac{p_b}{p_0} \right\}^{\frac{1}{\gamma}} A_{exit} \left[\frac{2\gamma p_0}{(\gamma - 1)\rho_0} \left\{ 1 - \left\{ \frac{p_{exit}}{p_0} \right\}^{\frac{\gamma}{[\gamma - 1]}} \right\} \right]^{1/2}$$

At throat with high back pressure p_b

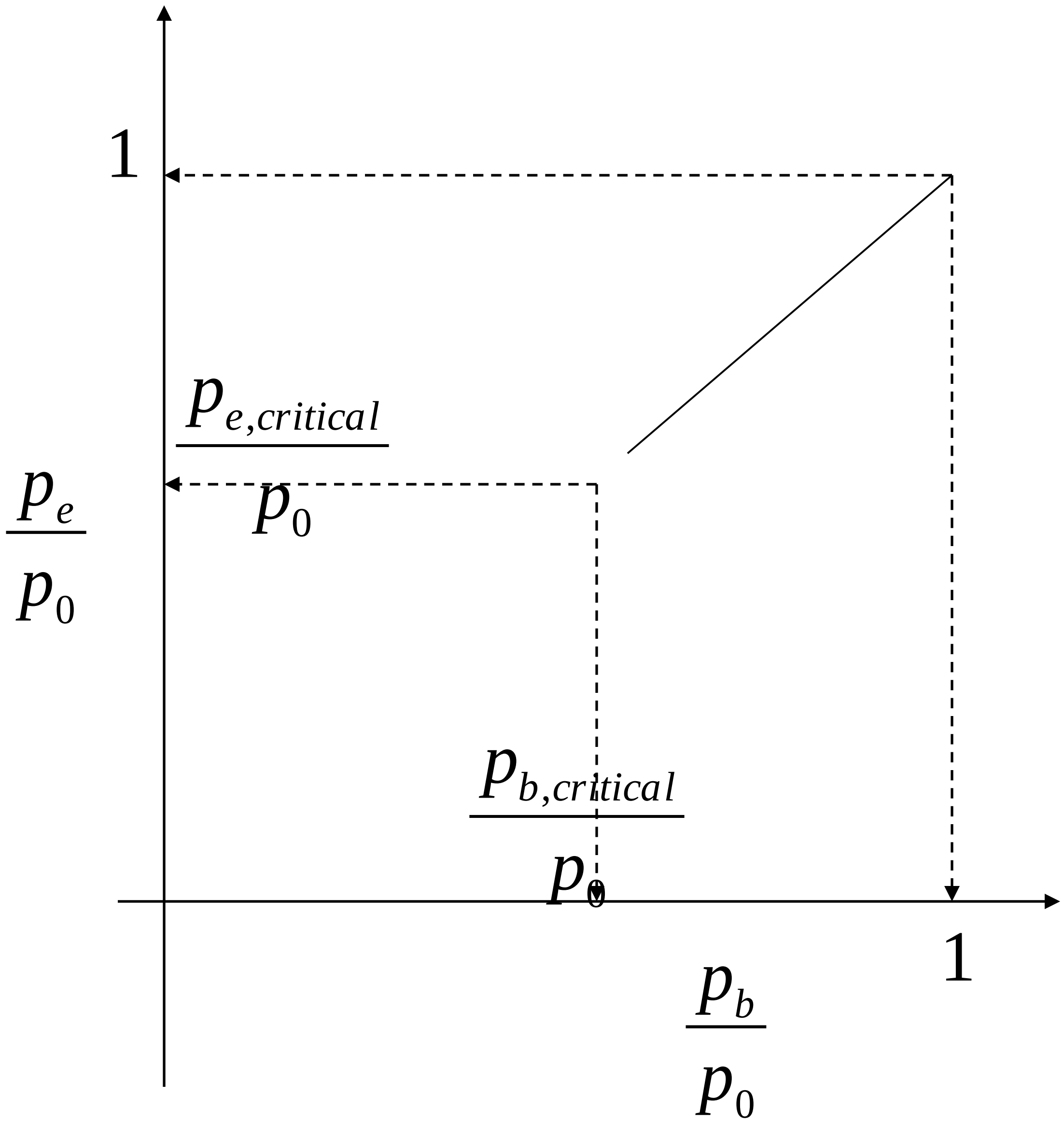
$$\dot{m} = \rho_0 \left\{ \frac{p_t}{p_0} \right\}^{\frac{1}{\gamma}} A^* \left[\frac{2\gamma p_0}{(\gamma - 1)\rho_0} \left\{ 1 - \left\{ \frac{p_t}{p_0} \right\}^{\frac{\gamma}{[\gamma - 1]}} \right\} \right]^{1/2}$$

$$\left\{\frac{p_b}{p_0}\right\}^{\frac{1}{\gamma}} A_{exit}\left\{1-\left\{\frac{p_b}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}^{1/2}=\left\{\frac{p_t}{p_0}\right\}^{\frac{1}{\gamma}} A^{*}\left\{1-\left\{\frac{p_t}{p_0}\right\}^{\frac{\gamma}{[\gamma-1]}}\right\}^{1/2}$$

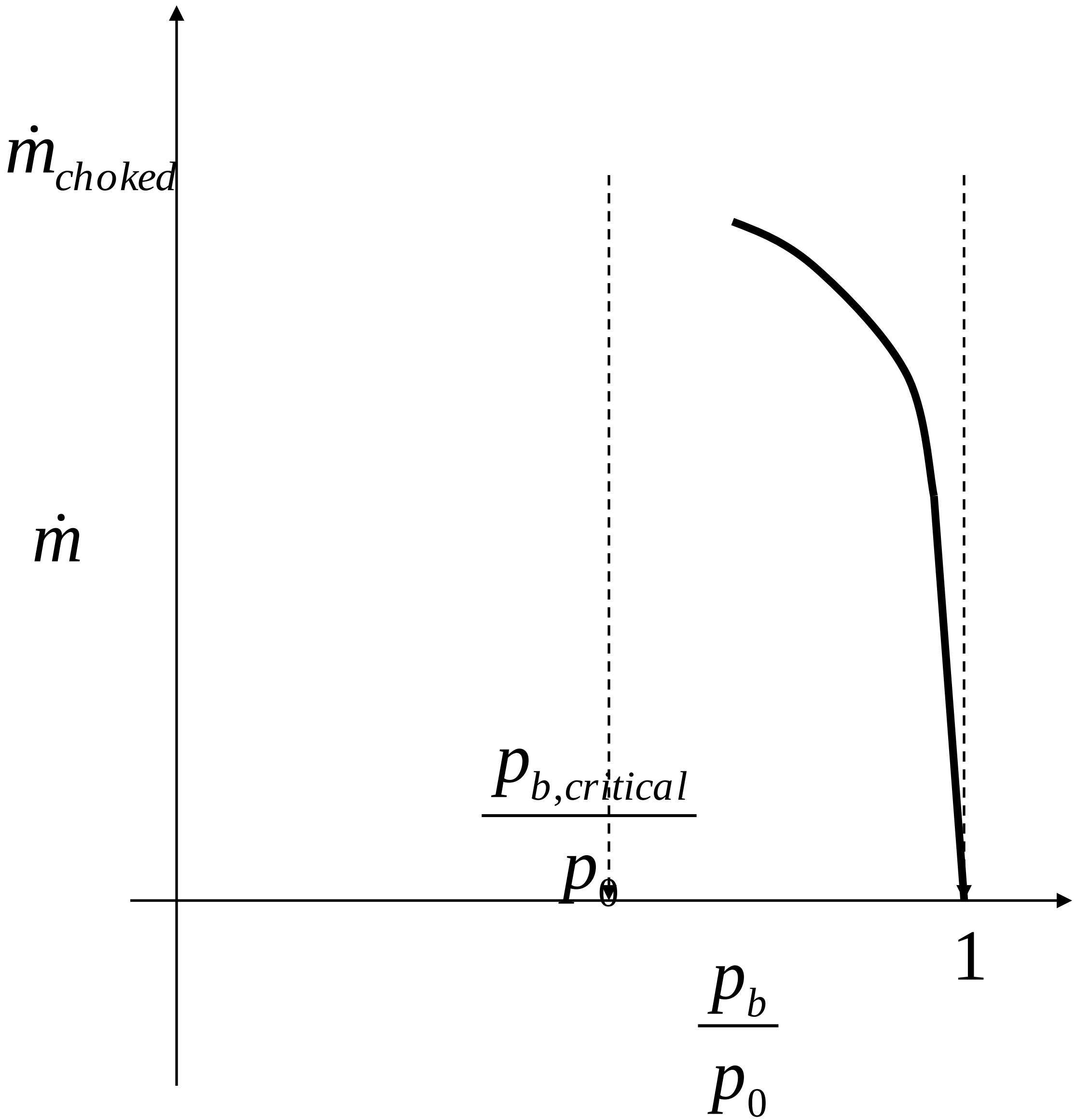
- For a given value of high back pressure corresponding throat pressure can be calculated.
- As exit area is higher than throat area throat pressure is always less than exit plane pressure.
- An decreasing exit pressure pro pressure



Variation of Mass Flow Rate in Exit Pressure



Variation of in Mass Flow Rate



Mach Number

The Mach number is the ratio of the speed of the aircraft, or the speed of the gas, to the speed of sound in the gas. The speed of sound is equal to the speed of transmission of small, isentropic disturbances in the flow. To determine the role of the Mach number on compressibility effects. we begin with the conservation of momentum equation:

$$\rho * V\, dV = -\, dp \qquad \ldots\ldots\ldots\ldots(a)$$

Where,

ρ - fluid density,

V- velocity, and

p -pressure

Mach Number

- $dp/p = \gamma * d\rho/\rho$ or, $dp = \gamma * p / \rho * d\rho$……………….(1)

where γ - specific heat ratio.

We can use the ideal equation of state to simplify the expression on the right:

- $p = \rho * R * T$ or, $dp = \gamma * R * T * d\rho$……………….(2)

where R- specific gas constant and T is the absolute temperature.

$\gamma * R * T = a^2$…………………..(3)

where a- speed of sound. So,

$dp = a^2 * d\rho$ from (2 & 3) ………………..(4)

Substituting this expression for the change of pressure into the conservation of momentum equation gives:

- $\rho * V dV = - a^2 d\rho$ from (4 &1)
- $- (V^2 / a^2) dV / V = d\rho / \rho$

$- M^2 dV / V = d\rho / \rho$

What does this expression tell us about the role of the Mach number in compressible flows?

where **M** is the Mach number.

- For low speed, or subsonic conditions, the Mach number is less than one, **M < 1** and the square of the Mach number is very small. Then the left hand side of the equation is very small, and the change in density is very small. For the low subsonic conditions, compressibility can be ignored.
- As the speed of the object approaches the speed of sound, the flight Mach number is nearly equal to one, **M = 1**, and the flow is said to be transonic If the Mach number is near one, the square of the Mach number is also nearly equal to one. For transonic flows, the change in density is nearly equal to the change in velocity, and compressibility effects can not be ignored.
- As the speed increases beyond the speed of sound, the flight Mach number is greater than one **M > 1** and the flow is said to be supersonic or hypersonic. For supersonic and hypersonic flows, the density changes faster than the velocity changes by a factor equal to the square of the Mach number. Compressibility effects become more important with higher Mach numbers.

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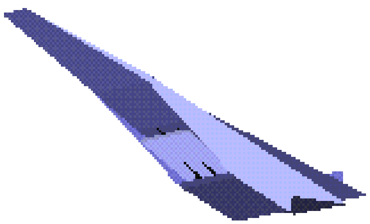
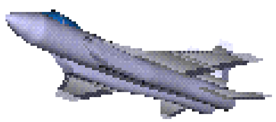
Mach Number

$$\text{ratio} = \frac{\text{Object Speed}}{\text{Speed of Sound}} = \text{Mach Number}$$



Hypersonic

Mach > 5.0



Supersonic

Mach > 1.0

Transonic

Mach = 1.0

Subsonic

Mach < 1.0

Nozzle Performance

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p = pressure

T = temperature

h = specific enthalpy

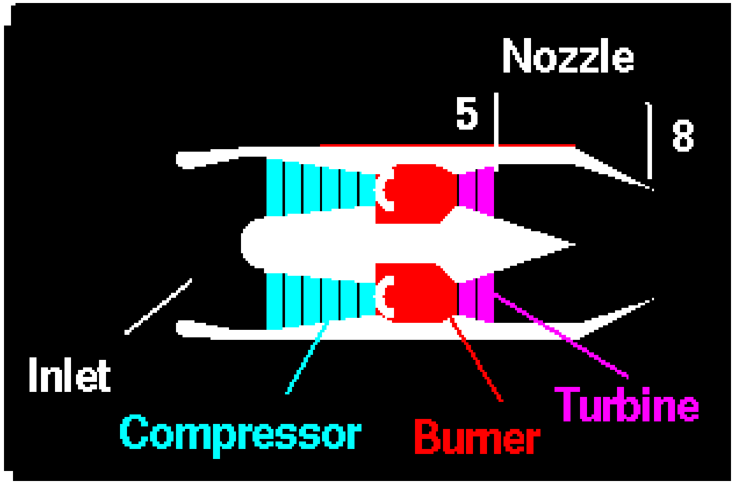
V = velocity

c_p = specific heat

γ = specific heat ratio

η_n = adiabatic nozzle efficiency

NPR = nozzle pressure ratio = pt8/p8



station 5 – turbine exit

station 8 – nozzle throat

Nozzle Total Temperature & Pressure

$$\frac{pt_8}{pt_5} = \left(\frac{Tt_8}{Tt_5}\right)^{\frac{\gamma}{\gamma - 1}} = 1$$

Nozzle Energy Equation

$$h = c_p T \qquad ht_8 = h_8 + \frac{V_8^2}{2\,\eta_n}$$

$$\boxed{V_e = V_8 = \mathrm{sqrt}(2\, c_p\, Tt_8\, \eta_n\, [1 - \{1 / NPR\}^{(\gamma - 1)/\gamma}])}$$

Nozzle Performance

- As shown on slide, the exit velocity depends on the nozzle pressure ratio and the nozzle total temperature. The nozzle pressure ratio depends on the exit static pressure and the nozzle total pressure. We can determine the nozzle total pressure from the free stream conditions and the engine pressure ratio, EPR. EPR depends on the pressure ratio of all the other engine components. We can also determine the nozzle total temperature from the engine temperature ratio , ETR. ETR depends on the temperature ratio of all the other engine components. With this information, we can solve for the thrust developed by a jet engine.

- The nozzle performance equations work just as well for rocket engines except that rocket nozzles always expand the flow to some supersonic exit velocity. You can explore the design and operation of turbojet and rocket nozzles with our interactive nozzle simulator program which runs on your browser.

REFERENCES

- http://www.aerospaceweb.org/design/aerospike/main.shtml
- http://www.engineeringatboeing.com/articles/nozzledesign.jsp
- http://www.engapplets.vt.edu/fluids/CDnozzle/cdinfo.html
- http://batman.mech.ubc.ca/~mech380/notes/node43.html
- http://www.allstar.fiu.edu/aero/