Chapter 2

Aircraft Performance

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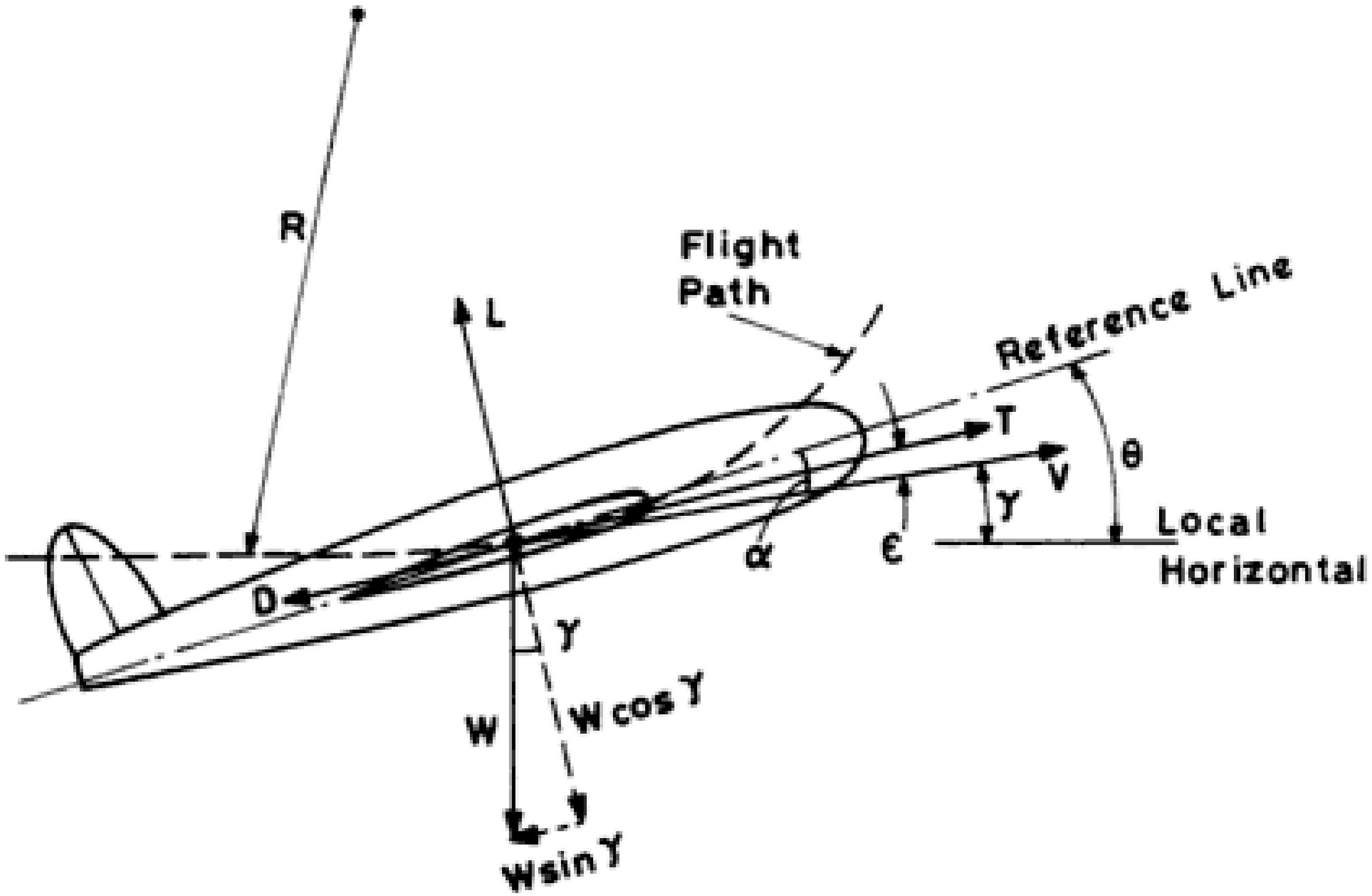
Contents

Introduction

- Performance characteristics depend on the weight of the airplane, aerodynamic characteristics of the airframe, and the thrust or the power developed by the powerplant.
- For a given airplane configuration, aerodynamic characteristics depend on AOA/sideslip, Mach number, and Reynolds number.
- The thrust/power characteristics of the powerplant depend on altitude, flight velocity, and engine operating conditions.
- Therefore, in general, it is not possible to analytically estimate airplane performance considering the arbitrary variations of aerodynamic and propulsive characteristics.
- We will discuss simplifying assumptions so that performance calculations become amenable to the methods of ordinary calculus.

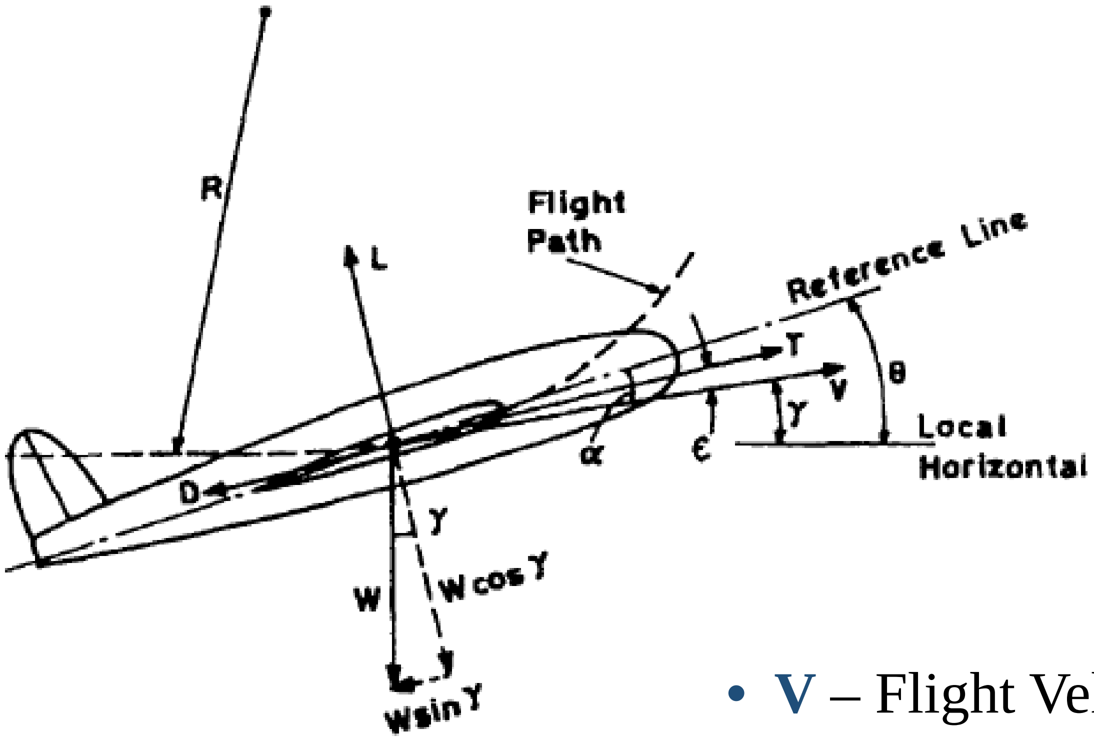
1. Equations of Motion

Forces acting on an airplane in a vertical plane



- **V** – Flight Velocity
- **γ** – Flight Path Angle
- **θ** – Inclination Angle
- **α** – Angle of Attack = **θ - γ**

1. Equations of Motion



Along velocity: $$T\cos\varepsilon - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

Along Lift: $$T\sin\varepsilon + L - W\cos\gamma = \frac{W}{g}\frac{V^2}{R}$$

$$\frac{dV}{dt} : Angular \quad Accleration$$

$$\frac{V^2}{R} : Centrifugal \quad Accleration$$

- **V** – Flight Velocity
- **ε** – Thrust Inclination
- **γ** – Flight Path Angle
- **θ** – Inclination Angle
- **α** – Angle of Attack = **θ - γ**

1. Equations of Motion

Force balance equations

- Generally , thrust inclination, **ε**, is very small. For an airplane with a straight flight path and constant velocity, both the angular and centrifugal accelerations vanish.

$$T - D - W sin\gamma = 0$$

$$L - W cos\gamma = 0$$

Static Performance:

Examples: Steady level flight

Steady Climb

range and endurance in constant-velocity cruise

Kinematic Relations:

$$\dot{x} = V cos\gamma$$

$$\dot{h} = V sin\gamma$$

x and h are the horizontal and vertical distances measured with respect to the origin of a suitable coordinate system fixed on the ground.

1. Equations of Motion

Non-Dimensional Parameters

- Defined non-dimensional parameters:

Lift-to-Drag Ratio

$$E = \frac{C_L}{C_D}$$

Non-dimensional Thrust

$$z = \frac{T E_m}{W}$$

Load Factor

$$n = \frac{L}{W}$$

Non-Dimensional Flight Velocity

$$u = \frac{V}{V_R}$$

1. Equations of Motion

- The aerodynamic force acting on the airplane in the coefficient form.

$$C_L = a\alpha$$

$$C_D = C_{DO} + kC_L^2$$

The variation of the drag coefficient is often called the parabolic drag polar of the airplane.

$$C_{Di} = kC_L^2$$

Induced drag Parameter

$$k = \frac{1}{\pi e A}$$

- At low speeds, C_{D0} induces skin friction and pressure drag of all wetted components of the airplane.
- At high speeds, another form of zero-lift drag arises, known as wave drag.
- For high subsonic and supersonic airplanes, the term C_{D0} will also include the wave drag.
- The induced drag also includes those parts of the skin friction and pressure drag that vary with angle of attack.

- For finte wings, e is the wing planform efficiency factor.
- For elliptical wing planform, e = 1.

1. Equations of Motion

Reference Velocity

➢ The reference velocity (V_R)is the flight velocity in level flight when the drag is minimum, i.e., when

$$\frac{C_D}{C_L} = \frac{C_{DO} + kC_L^2}{C_L}$$

(Drag Polar)

$$\frac{\mathrm{d}}{\mathrm{d}C_L}\left(\frac{C_D}{C_L}\right) = \frac{-C_{DO} + kC_L^2}{C_L^2}$$

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L}\right) = 0$$

$$C_L = \sqrt{\frac{C_{DO}}{k}} \qquad \left(\frac{C_D}{C_L}\right)_{\min} = 2\sqrt{kC_{DO}}$$

$$E_m = \left(\frac{C_L}{C_D}\right)_{\max}$$

$$E = E_m = \frac{1}{2\sqrt{kC_{D0}}}$$

$$C_L = C_L^* = \sqrt{\frac{C_{D0}}{k}}$$

$$V_R = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}}$$

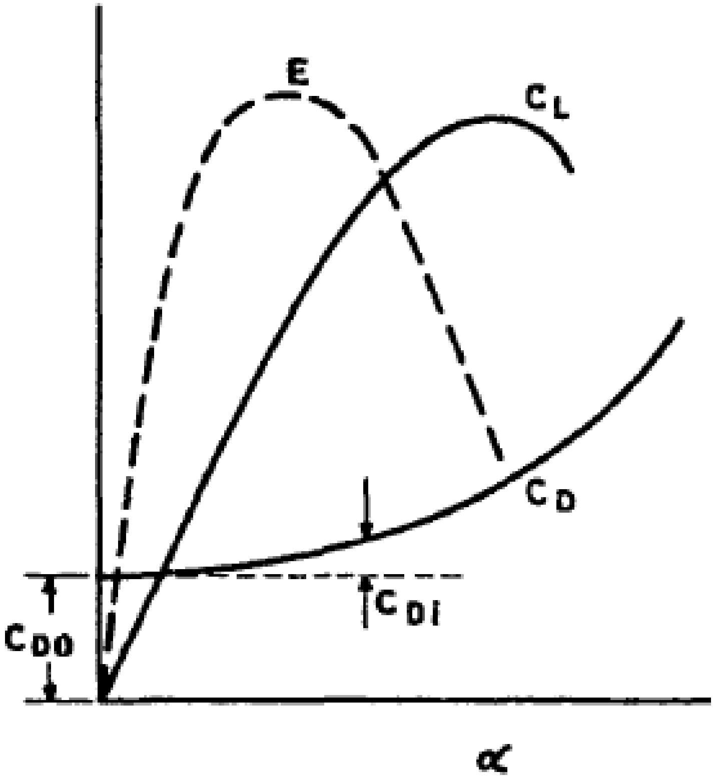
1. Equations of Motion

Non-dimensional level flight equations

For $\boldsymbol{L = nW}$, we have:

$$C_L = \frac{2nW}{\rho V^2 S}$$

$$D = \frac{W}{2E_m}\left(u^2 + \frac{n^2}{u^2}\right)$$



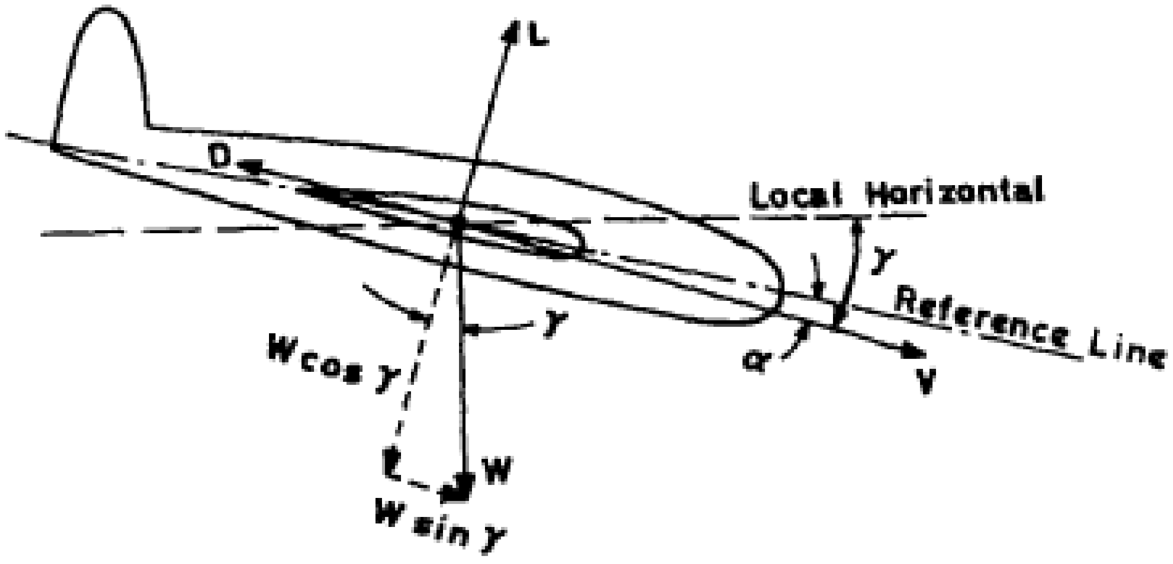
- Thus, the equation for static performance becomes,

$$T - D - W sin\gamma = 2zu^2 - u^4 - n^2 - 2E_m u^2 sin\gamma = 0$$

$$L - W cos\gamma = n - cos\gamma = 0$$

2. Gliding Flight

Mechanism



- Forces acting on a glider are as shown in the picture below.
- The glide angle $\gamma < 0$.

- A glider is an unpowered light airplane. The power to overcome the aerodynamic drag comes at the expense of its potential energy or its height above the ground.
- Because a glider attain the height on its own, it must be towed by another powered airplane to the desired height and then launched to fly on its own.
- Example is the gliding flight of the space shuttle during its atmospheric reentry.
- Gliders were the first test platform for flight mechanism, as studied by Otto Lilienthal, George Cayley and the Wright Brothers.
- In nature, avian scavengers like eagles and vultures use evolutionarily developed gliding capabilities to remain for a long period of time in the sky.

2. Gliding Flight

- For Gliding flight $\gamma < 0$ with $T = 0$

$$D + W \sin\gamma = 0 \qquad \dot{x} = V\cos\gamma$$

$$L - W\cos\gamma = 0 \qquad \dot{h} = V\sin\gamma$$

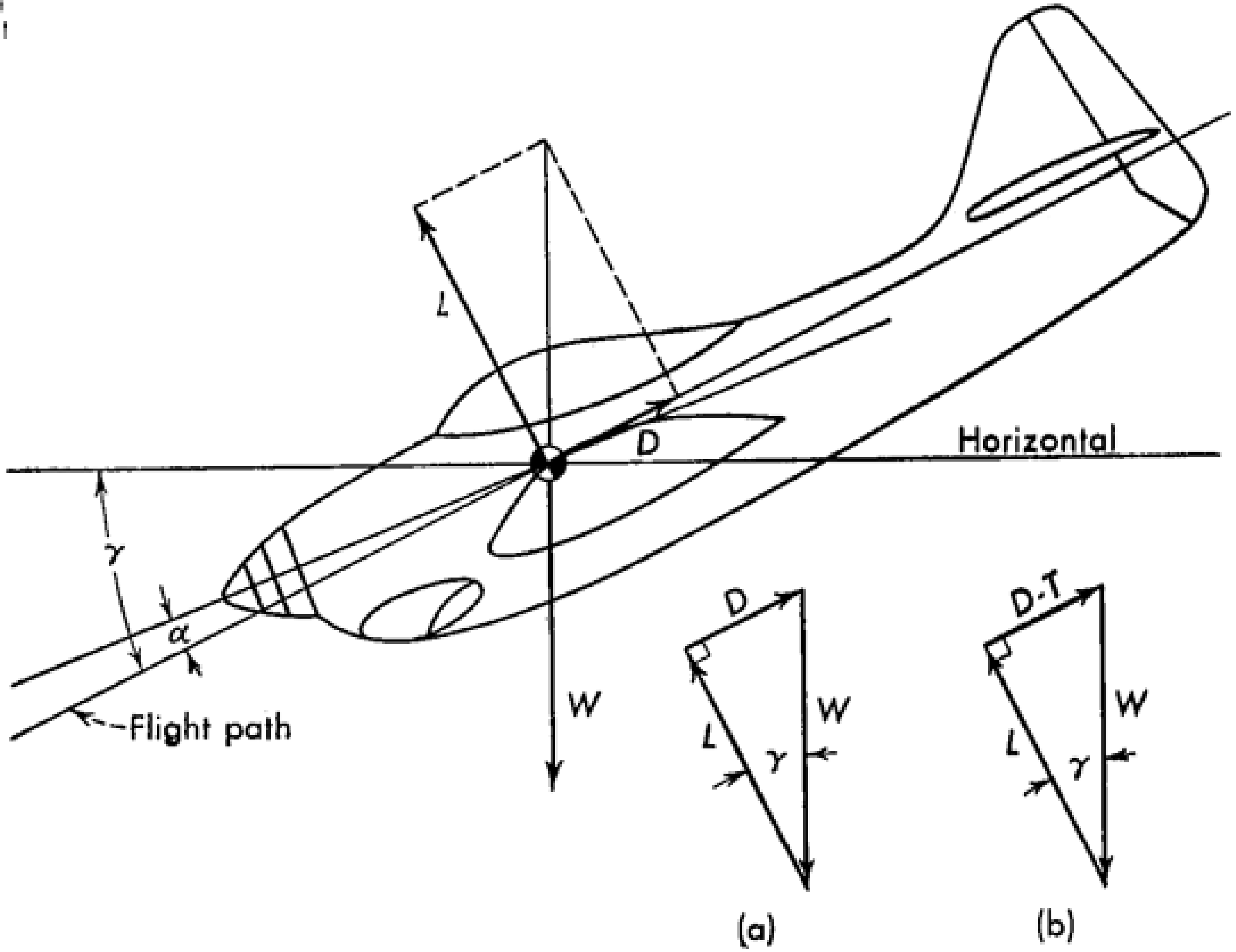
Generally, the glide angle is small so,

$$D + W\gamma = 0$$

$$L = W$$

$$\dot{x} = V$$

$$\dot{h} = V\gamma$$



2. Gliding Flight

Mechanism

Lift and Drag

$$L = \frac{1}{2}\rho V^2 S C_L$$

$$D = \frac{1}{2}\rho V^2 S C_D$$

$$= \frac{1}{2}\rho V^2 S(C_{DO} + kC_L^2)$$

Velocity

$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

Glide Angle

$$\gamma = -\frac{D}{W}$$

$$= -\frac{D}{L}$$

$$= -\frac{C_D}{C_L}$$

$$= -\frac{1}{E}$$

$$\gamma_{\min} = -\frac{D_{\min}}{W}$$

$$= -\frac{1}{E_m}$$

Thus, the flattest glide occurs when the lift-to-drag ratio is maximum.

$$E = E_m,$$

$$C_L = C_L^* = \sqrt{C_{DO}/k}.$$

Flight Velocity for Flattest glide

$$V = \sqrt{\frac{2W}{\rho S C_L^*}}$$

$$= \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{DO}}}$$

2. Gliding Flight

Maximum Range

$$\dot{x} = V = \frac{dx}{dt} = \frac{dx}{dh}\frac{dh}{dt} = V\gamma$$

$$\frac{dx}{dh} = \frac{V}{V\gamma} = \frac{1}{\gamma} = -E$$

$$R = -\int_{h_i}^{h_f} E dh$$

- The range of a glider is directly proportional to its lift-to-drag ratio.
- Range is achieved at the expense of altitude- with a **Glide Ratio** (**E**).
- The **range is maximum for the flattest glide**.
- A head wind reduces the range, whereas a tailwind increases it.
- In sailplane terminology, glide ratio is the ratio between the ground distance traveled and the height lost.
- A high performance sail plane with a glide ratio of 40 can cover 4km with respect to the ground for every 100m height lost.

h_i - h_f is the height lost during the gliding flight.

$$R = -E\Delta h \qquad R_{max} = \frac{\Delta h}{2\sqrt{kC_{D0}}} \qquad V = V_R \qquad C_L = C_L^*$$

2. Gliding Flight

Rate of Sink

$$\dot{h}_s = V\gamma = \frac{DV}{W} = \sqrt{\frac{2W}{\rho S}}\left(\frac{C_D}{C_L^{3/2}}\right)$$

- The term DV represents the power required P_R to sustain the gliding flight. Thus, the **rate of sink** is minimum when the power required per-unit-weight is minimum.
- This corresponds to the case when the following term is minimum:

$$\left(\frac{C_D}{C_L^{3/2}}\right)$$

Maximum Endurance

$$\frac{d\dot{h}_s}{dC_L} = 0 \qquad C_{Di} = 3C_{D0}$$

$$C_{L,max} = \sqrt{\frac{3C_{D0}}{k}} = \sqrt{3}C_L^*$$

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{max} = \frac{1}{4}\sqrt{\frac{27}{k^3 C_{D0}}}$$

Velocity for glide with minimum sink rate is:

$$V_m = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{3C_{D0}}}$$

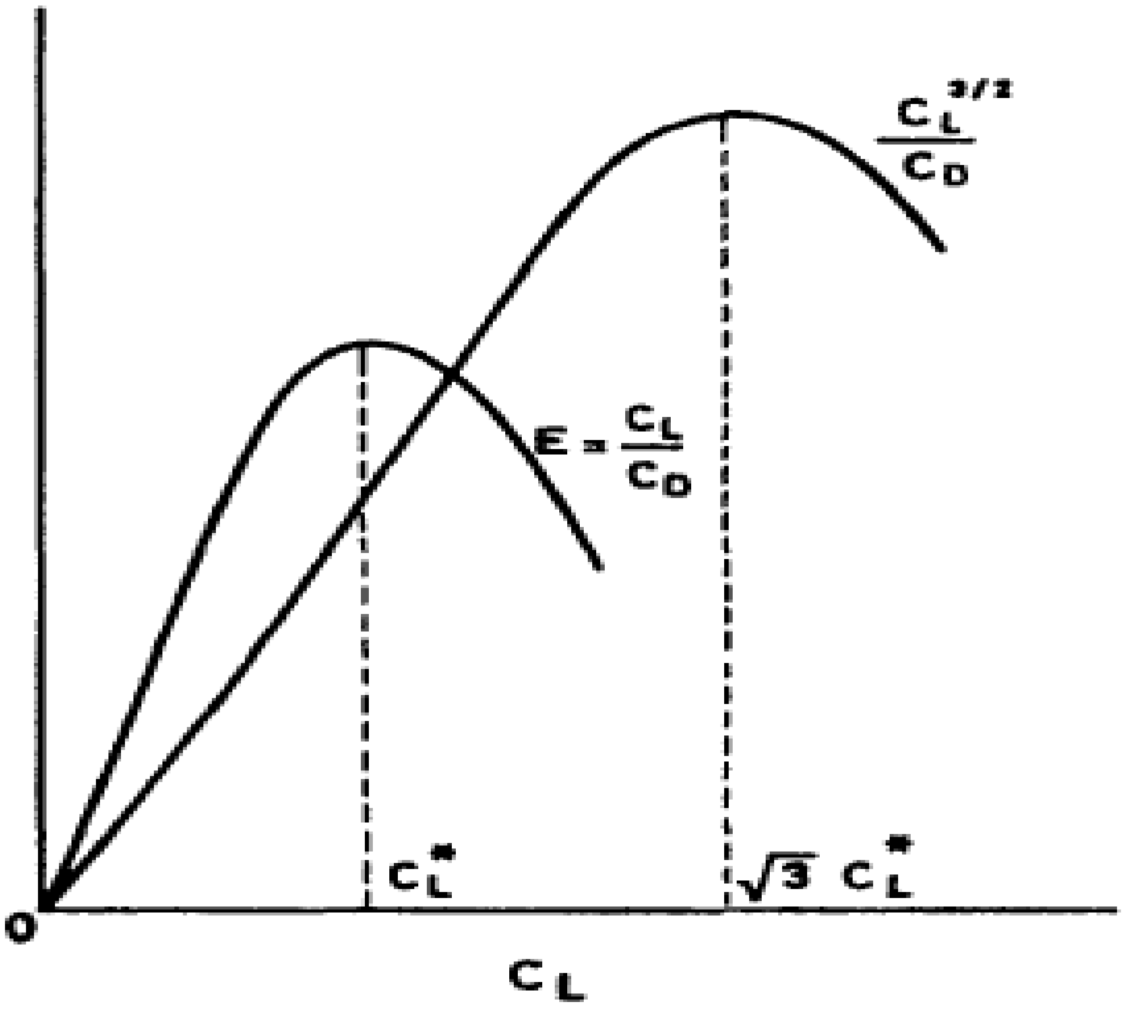
$$V = V_m \simeq 0.76V_R$$

2. Gliding Flight

Rate of Sink

- From the above relations, it is shown that the velocity for the glide with minimum sink rate is about **0.76 times the reference velocity**, or **0.76 times the velocity for flattest glide**.
- Whereas the flattest glide occurs when the drag is minimum, the **glide with minimum sink rate occurs when the power required is a minimum**.
- The **minimum sink rate** is given by:

$$\dot{h}_{s,min} = \sqrt[4]{\frac{2Wk^3C_{D0}}{27\rho S}}$$



2. Gliding Flight

Endurance

- The **endurance is the total time the glider remains in the air**.

$$dt = \frac{dh}{V\gamma}$$

$$t = -\int_{h_i}^{h_f} \sqrt{\frac{\rho S}{2W}} \left(\frac{C_L^{3/2}}{C_D}\right) dh$$

$$t = \sqrt{\frac{\rho S}{2W}} \left(\frac{C_L^{3/2}}{C_D}\right) (h_i - h_f)$$

- For maximum endurance, the glider has to fly at that angle of attack or lift coefficient when the parameter **($C_L^{3/2}/C_D$)** is maximum, which occurs when **$C_L = \sqrt{3} * C_L^*$** and $V = 0.76 V_R$.
- This is also the condition for minimum sink rate. Thus, the endurance is maximum when the sink rate is minimum.

$$t_{max} = \sqrt{\frac{\rho S}{2W}} \sqrt[4]{\frac{27}{k^3 C_{D0}}} \left(\frac{h_i - h_f}{4}\right)$$

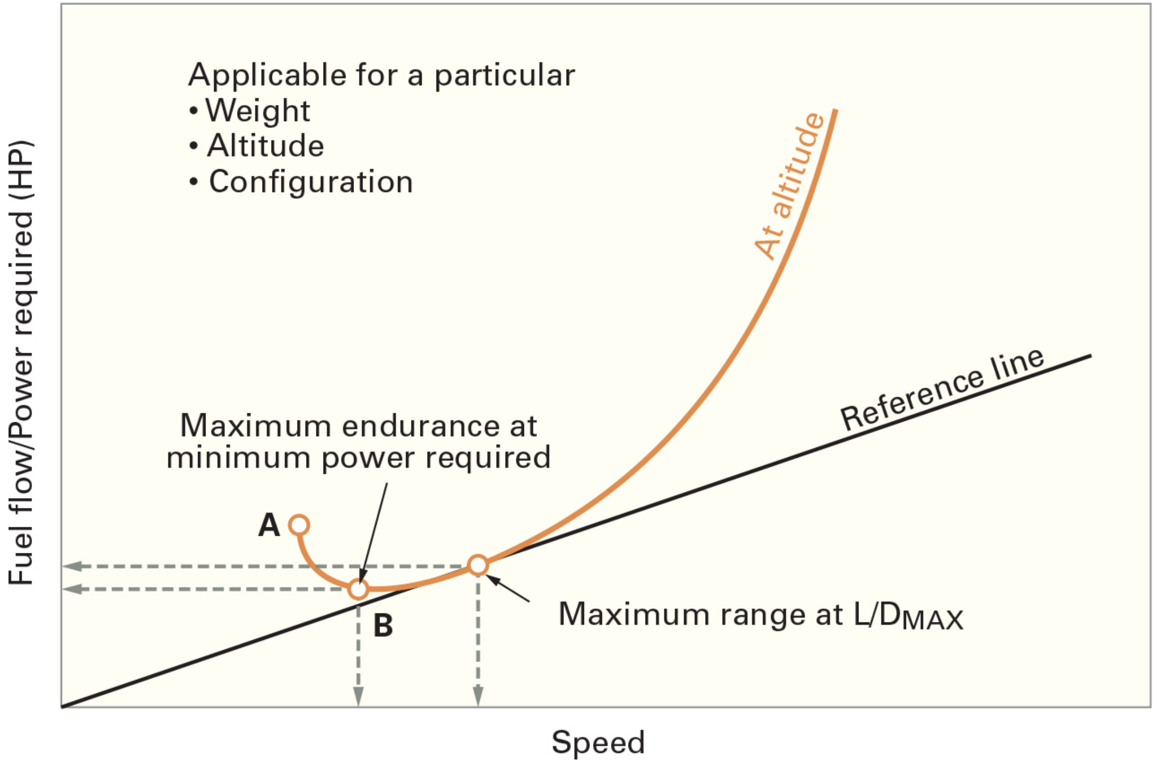
- If the difference between initial and final altitude is significant, then the variation in density may also have to be considered.

$$\rho = \rho_0 e^{-0.000114h}$$

2. Gliding Flight

Range and Endurance

- The range and endurance are important measures of a glider's performance.
- Whereas the maximum range was independent of the weight, the maximum endurance depends on the weight. This calls for the designer to make the glider as light as possible.
- Furthermore, both the maximum range and endurance improve if the aerodynamics parameters **k** and C_{D0} are kept to their lowest possible values.
- Because of this, gliders tend to have an elliptical wing with a high aspect ratio and an efficient low-drag, laminar –flow airfoil section.



Example

A glider having $W = 2000$ N, $S = 8.0$ m^2, $A = 16.0$, $e = 0.95$, and $C_{DO} = 0.015$ is launched from a height of 300 m. Determine the maximum range, corresponding glide angle, forward velocity, and lift coefficient at sea level.

At sea level, we have $\rho_o = 1.225$ kg/m^3. Furthermore,

$$k = \frac{1}{\pi A e} = \frac{1}{\pi * 16 * 0.95} = 0.02$$

$$E_m = \frac{1}{2\sqrt{KC_{DO}}} = \frac{1}{2\sqrt{0.02 * 0.015}} = 28.86$$

$$R_{\max} = (h_f - h_i)E_m = 28.86 * 300 \text{ m} = 8.658 \text{ km}$$

The maximum range occurs when the glide angle is minimum

$$\gamma_{\min} = \frac{1}{E_m} = \frac{1}{28.86} = 0.0346 \text{ rad} = 1.985 \text{ deg}$$

The velocity and lift coefficient for the flattest glide

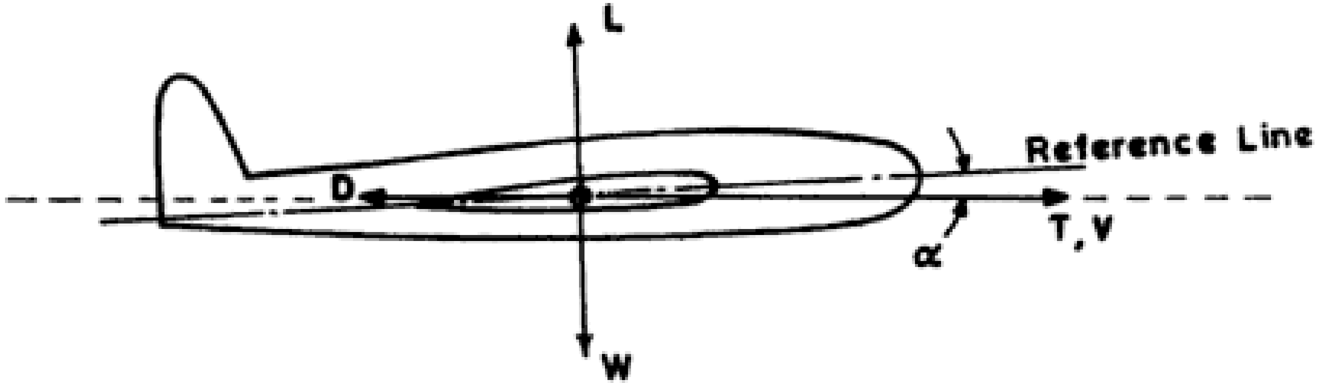
$$V = V_R = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{DO}}} = \sqrt{\frac{2 * 2000.0}{1.225 * 8.0}}\sqrt[4]{\frac{0.02}{0.015}} = 23.328 \text{ m/s}$$

$$C_L = C_L^* = \sqrt{\frac{C_{DO}}{k}} = \sqrt{\frac{0.015}{0.02}} = 0.866$$

3. Level Flight

Mechanism

➢ The forces acting on an airplane in a level flight are shown in figure below:



➢ In level flight, the condition for force balance can be stated in simple terms as the lift is equal to weight and thrust available is equal to thrust required.

$$L - W = 0 \qquad \dot{x} = V$$
$$T - D = 0 \qquad \dot{h} = 0 \qquad n = 1$$

Power Required

➢ With lift = weight, the drag equation can be written as,

$$D = \frac{1}{2}\rho V^2 S C_{D0} + \left(\frac{2kW^2}{\rho V^2 S}\right)$$

➢ Zero-lift drag varies as the square of velocity, whereas the induced drag varies inversely with the square of the velocity.

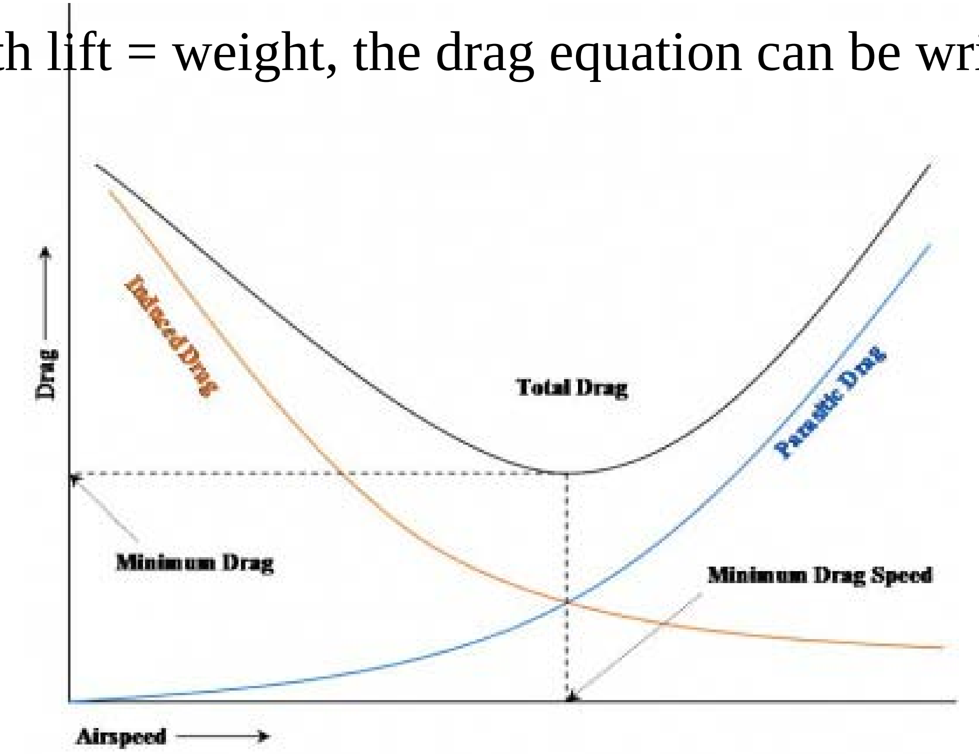
➢ At low speeds, the induced drag is dominant because the lift coefficient needed to sustain level flight is quite high. As the velocity increases, the zero-lift drag rises very rapidly, but the induced drag becomes insignificant.

3. Level Flight

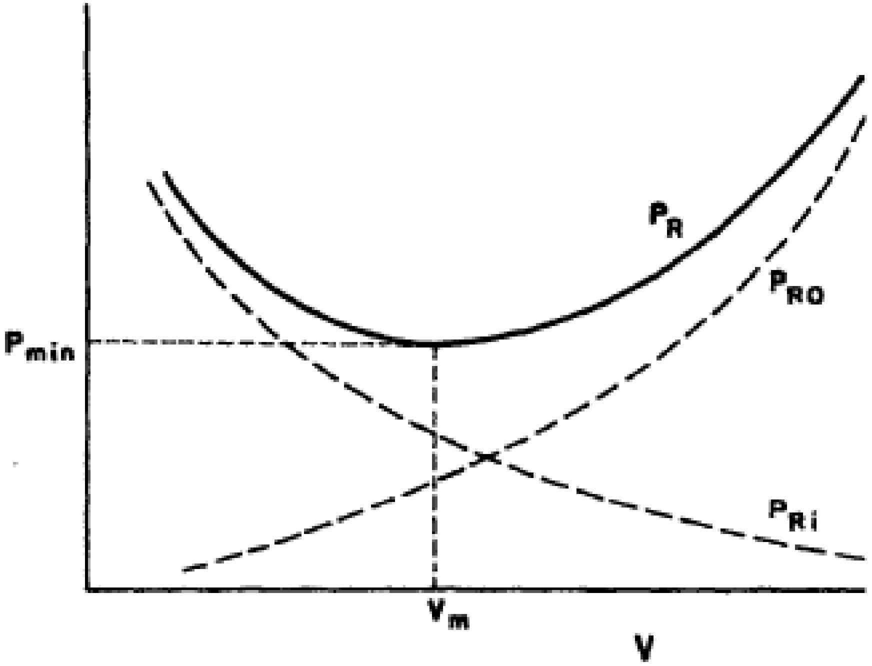
Power Required

- At low subsonic speeds, the zero-lift drag coefficient can be assumed constant with respect to velocity, while at t higher speeds, it varies with Mach number and includes the wave drag component.

- With lift = weight, the drag equation can be written as,



Variation of drag component with velocity



Power-required curves for propeller aircraft

3. Level Flight

Power Required

- As seen in the plots, the total drag assumes a minimum value at a certain velocity.

$$\frac{dD}{dV} = \rho V S C_{D0} - \left(\frac{4kW^2}{\rho V^3 S}\right) = 0$$

$$V = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}}$$

$$D_0 = D_i = W\sqrt{kC_{D0}}$$

$$D_{min} = 2D_0 = 2D_i$$

$$D_{min} = 2W\sqrt{kC_{D0}}$$

- The reference velocity happens to be the velocity for minimum drag.
- When total drag in level flight is minimum, $D_0 = D_i$.
- Power required in level flight

$$P_R = DV$$
$$= \underbrace{\frac{1}{2}\rho V^3 S C_{D0}}_{P_{R_0}} + \underbrace{\left(\frac{2kW^2}{\rho V S}\right)}_{P_{R_i}}$$

Observation: At low speed the power required to overcome induced drag is dominating, whereas, at high speeds, it is the power required to overcome zero-lift drag that assumes significance.

Power Required

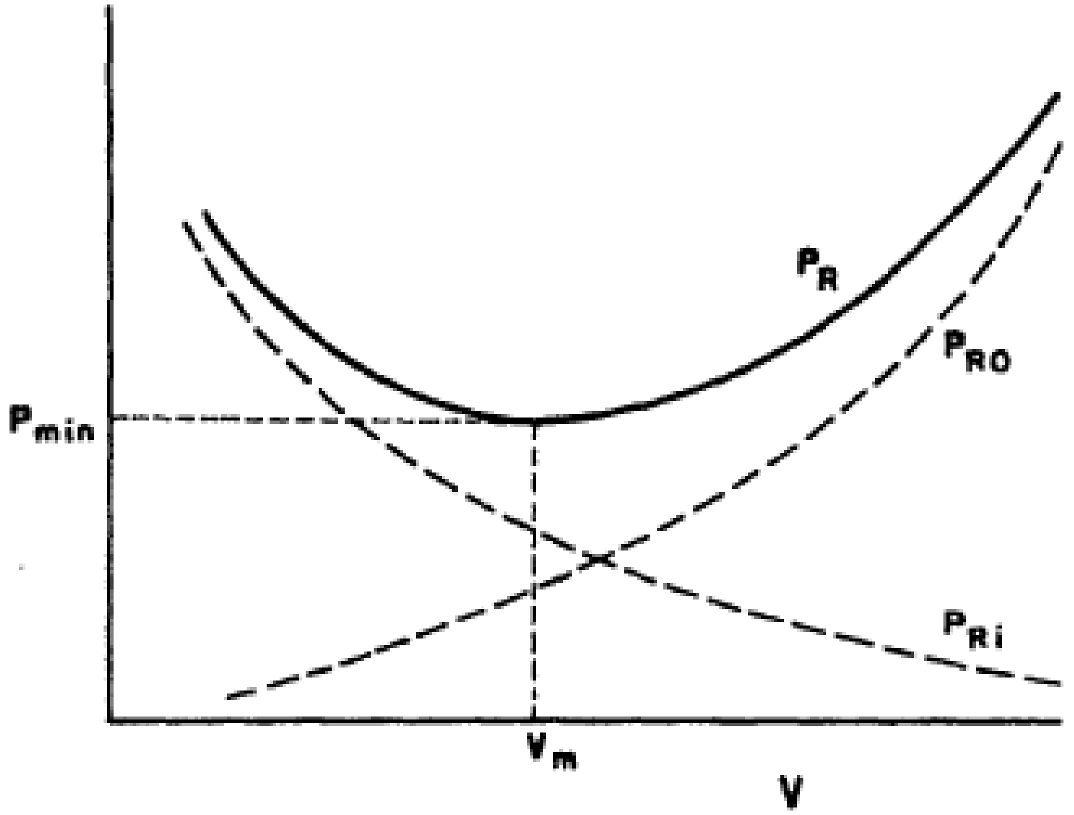
The total power required for level flight assumes a minimum value at some velocity, V_{mp}

$$\frac{dP_R}{dV} = \frac{d}{dV}\left[\frac{1}{2}\rho V^3 S C_{D0} + \left(\frac{2kW^2}{\rho V S}\right)\right] = 0$$

$$= \frac{3}{2}\rho V^2 S C_{D0} - \left(\frac{2kW^2}{\rho V^2 S}\right) = 0$$

$$V_{mp}^4 = \frac{4kW^2}{3\rho^2 S^2 C_{D0}}$$

$$V_{mp} = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{3C_{D0}}} = \frac{1}{\sqrt[4]{3}} V_R$$



3. Level Flight

Power Required: Range Equation

- The lift coefficient in level flight when the power required in level flight is minimum is as follows:

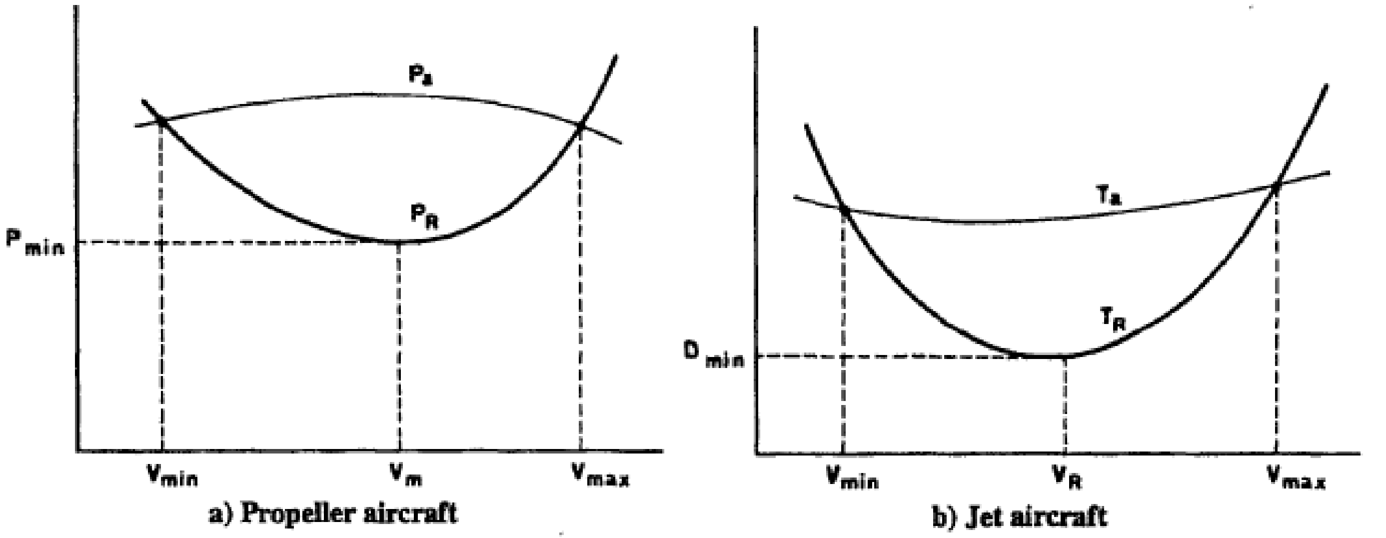
$$C_{L,mp} = \sqrt{3}C_L^*$$

$$E_{mp} = 0.866E_m$$

$$P_{R,min} = \frac{8}{3}\frac{kW^2}{\rho S V_{mp}} = \frac{WV_{mp}}{0.866E_m}$$

- Thus, both thrust- and power-required curves assume their minimum values at some velocities.

- There are two intersection points between power-required and power available curves for propeller planes, and between thrust required and thrust available curves for jet airplane.



Level flight solutions

3. Level Flight

Power Required

- Spending the same amount of fuel and developing the same magnitude of thrust or power, an airplane can fly at either of these two velocities.
- At the low or minimum velocity V_{min}, the thrust power available is essentially used to overcome induced drag, whereas, at high or maximum speed V_{max}, it is mostly used to balance the zero-lift drag.

Analytical Solutions for Propeller Aircraft

- The force (or power) balance equation for a propeller aircraft in level flight has not closed-form analytical solution.
- Thus, to determine the two speed limits in level flights, i.e. V_{min} and V_{max}, it is a common practice to obtain graphical or numerical solution. This is the case even under the assumption that the power developed by the reciprocating engine and propulsive efficiency are independent of flight velocity.

3. Level Flight

Analytical Solutions for Propeller Aircraft

- Define an altitude-independent parameter- **equivalent airspeed** as:

$$V_e = V\sqrt{\sigma}$$

$$\sigma = \frac{\rho}{\rho_0}$$

- The equivalent airspeed can be used to rewrite the drag equation such that it holds for all altitudes.

- The drag equation written in a form that can represent a single curve as a function of equivalent airspeed and altitude can be written as:

$$D = \frac{1}{2}\rho V_e^2 S C_{D0} + \left(\frac{2kW^2}{\rho V_e^2 S}\right)$$

- The thrust available curve intersects the drag curve at two points designated as $\mathbf{V_{e,min}}$ and $\mathbf{V_{e,max}}$.
- At these two points, the level flight eqns are identically satisfied, thus each is a level flight solution.

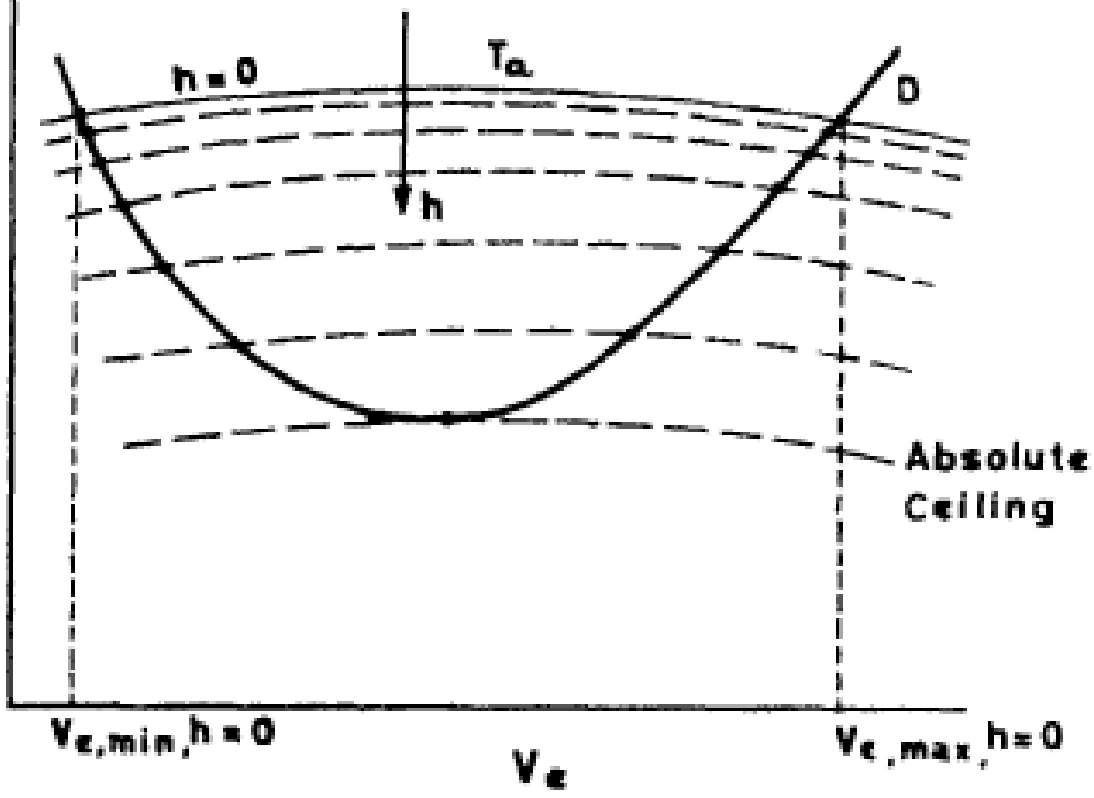
Analytical Solutions for Propeller Aircraft

➢ The true airspeed corresponding to the high-speed solution increases initially with altitude and then begins to decrease, whereas the true air speed corresponding to the low-speed solution increases monotonically with altitude.

➢ At a certain altitude, the two solutions merge, and we have only one level flight solution. This altitude is called the **absolute ceiling** of the airplane.

➢ This is also the point where the thrust-available is tangential to the thrust-required curve.

Several thrust available curves for different altitudes are plotted

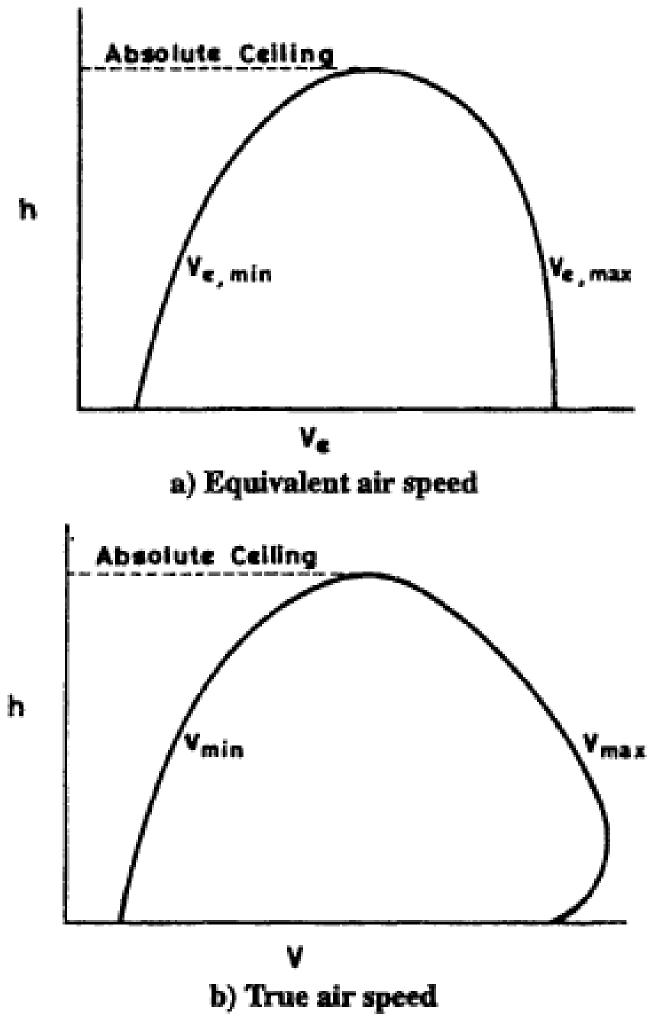


Analytical Solutions for **Propeller Aircraft**

- At absolute ceiling, the thrust-available has dropped so much the level flight is only possible only at one speed.

At absolute ceiling:

- Drag is minimum.
- Rate of climb is zero.
- Beyond it, steady level flight is not possible because the thrust available is not sufficient to balance the aerodynamics drag.
- **Question:** why is true airspeed's high-speed solution initially increasing?



Level flight envelope

3. Level Flight

Analytical Solutions for Jet Aircraft

➢ Assuming that at a given altitude, the thrust developed by a jet engine is independent of flight velocity, the analytical solution for level flight can be obtained as discussed here:

$$T = D = \frac{1}{2}\rho V^2 S C_{D0} + \left(\frac{2kW^2}{\rho V^2 S}\right)$$

$$AV^4 - TV^2 + B = 0$$

$$A = \frac{1}{2}\rho S C_{D0}$$

$$B = \left(\frac{2kW^2}{\rho S}\right)$$

$$V_{max,min} = \sqrt{\frac{T \pm \sqrt{T^2 - 4AB}}{2A}}$$

$$u^4 - 2zu^2 + 1 = 0, \quad n = 1$$

$$u^2 = z \pm \sqrt{z^2 - 1}$$

$$u_{min/max} = \sqrt{z \pm \sqrt{z^2 - 1}}$$

$$V_{min/max} = u_{max/min} V_R$$

3. Level Flight

Analytical Solutions for Jet Aircraft

- Given the variation of thrust available with altitude, using the non-dimensional approach, the level flight envelope for a jet aircraft whose thrust is independent of flight velocity can be constructed.
- At the absolute ceiling, $\mathbf{u_{min} = u_{max}}$, which gives $\boldsymbol{z = 1}$.
- Physically, when $\boldsymbol{z = 1}$, $\mathbf{T = D_{min}}$.
- Thus, **at absolute ceiling, level flight is possible only at one speed where drag is minimum**.
- Whether the flight is possible at V_{min} depends on the magnitude of the level flight stalling velocity:

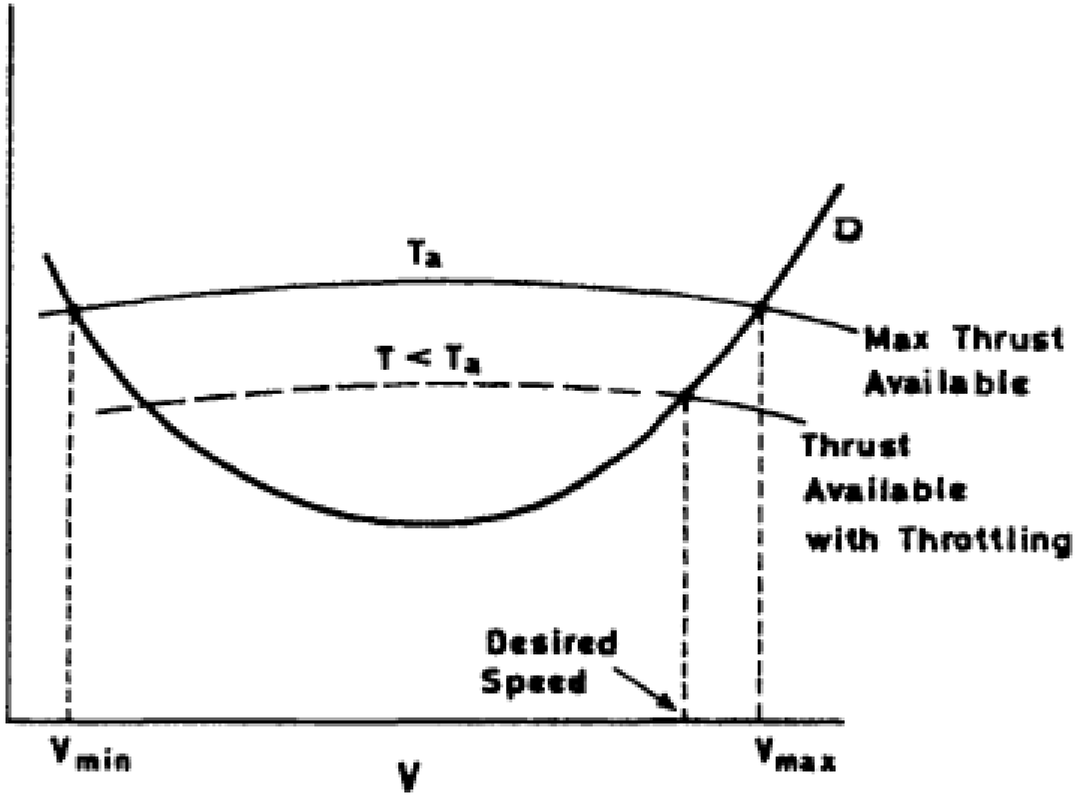
$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,max}}}$$

- If $\mathbf{V_{min} < V_{stall}}$, then the airplane cannot fly at $\mathbf{V_{min}}$.

3. Level Flight

Analytical Solutions for Jet Aircraft

- In cases where V_{min} < V_{stall} , the speed V_{min} itself has no significance and V_{stall} effectively becomes the low-speed solution.
- For flight at any speed V such that $V_{min} < V < V_{max}$, the pilot has to throttle such that the drag at this speed equals the thrust available.



Concept of level flight at any desired speed

4. Climbing Flight

Mechanism

- For all altitudes below the absolute ceiling, two solutions for steady level flight exist.
- For flight at any speed V such that $\mathbf{V_{min} < V < V_{max}}$, the question remains as to what happens when using full thrust or power…
- Since now there is excess thrust, steady flight is not possible. Either one the following two cases may happen:
  - The airplane will accelerate to $\mathbf{V_{max}}$ if the altitude is forced to remain constant.
  - The airplane will climb.
- There can be two methods for climb- a steady or constant velocity climb, and the more general method of accelerated climb.
- For reasonably short climb durations/distance, the weight of the aircraft can be assumed to be constant.

4. Climbing Flight

Steady Climb

- The equations of motion for an aircraft in steady climb are as follows:

$$L - W cos\gamma = 0$$

$$T - D - W sin\gamma = 0$$

$$\dot{x} = V cos\gamma \qquad \dot{h} = V sin\gamma$$

$$sin\gamma = \frac{T - D}{W} = \frac{excess \quad thrust}{weight}$$

- The rate of climb, R/C, is given by:

$$R/C = \dot{h} = V sin\gamma$$

- The climb angle γ is directly proportional to the excess thrust per unit weight. Therefore, the steepest climb γ_{max} occurs when the excess thrust per unit weight is maximum.

$$R/C = \frac{V(T - D)}{W} = \frac{P_a - P_R}{W} = P_s$$

- P_s is the excess power per unit weight. It us usually called the **specific excess power.**

4. Climbing Flight

Analytical solution for climbing flight of propeller aircraft

➢ For propeller aircraft whose power developed by the engine **P(KW)** and propulsive efficiency η_P are independent of flight velocity,

$$R/C = \frac{1}{W}\left(\underbrace{k'\eta_p P(KW)}_{\textbf{Available}} - \underbrace{\frac{1}{2}\rho S C_{D0} V^3 - \frac{2kW^2}{\rho S V}}_{\textbf{Required}}\right)$$

$$\frac{d(R/C)}{dV} = 0 \Longrightarrow V_{R/C,max} = \frac{1}{\sqrt[4]{3}} V_R$$

➢ This is also the speed for minimum power required in level flight. Thus, **the speed at which the rate of climb is maximum is the same as that when the power required in level flight is minimum.**

4. Climbing Flight

Analytical solution for climbing flight of propeller aircraft

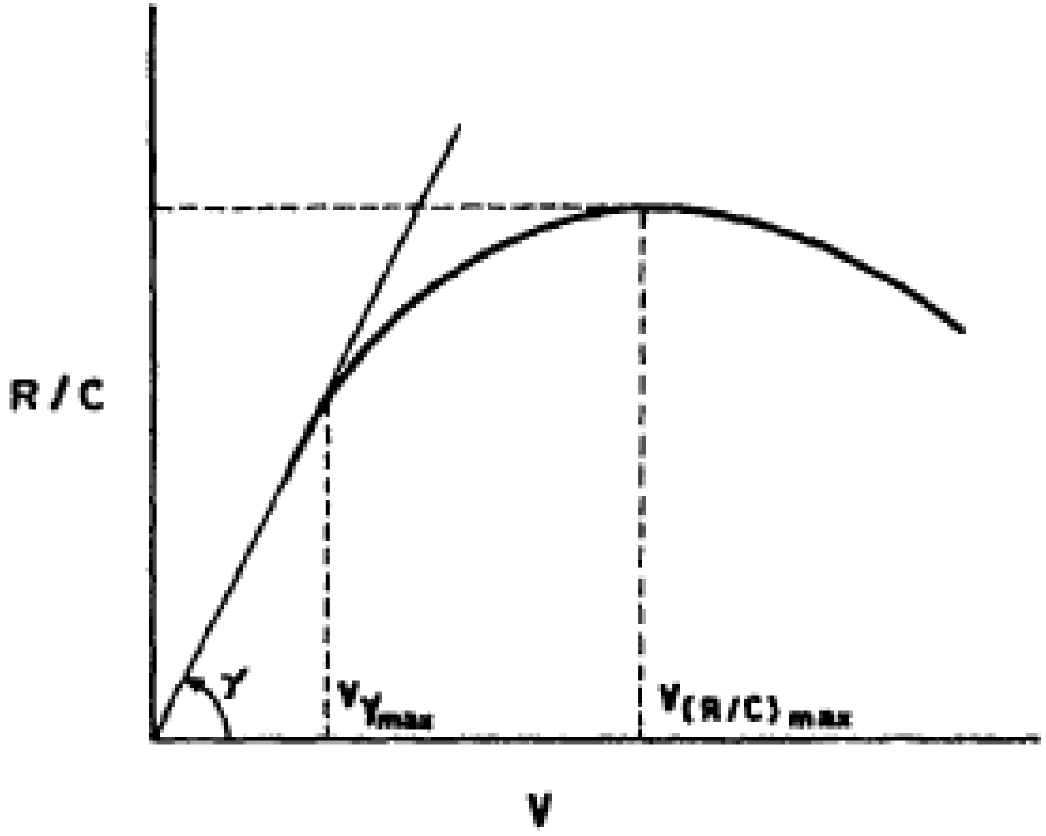
- Observation: The speed at which the rate of climb is maximum is the same as the speed at which power required in level flight is minimum.
- The two speeds are the same because, for the propeller aircraft, we have assumed that the power available is independent of forward speed.
- Thus, essentially, the speed at which the excess power is maximum becomes the same as that when the power required is minimum, i.e. $\mathbf{V_{R/C,\,max} = V_{mp}}$.
- The climb angle is maximum (steepest climb) at that velocity when the excess thrust per unit weight is maximum.
- The angle for steepest climb can be obtained by maximizing the climb angle with respect to velocity.
- However, no analytical solution can be obtained through this method, thus a graphical method is sought.

4. Climbing Flight

Analytical solution for climbing flight of propeller aircraft

➤ The velocity for maximum climb angle is given by the tangent drawn from the origin to the hodograph representing R/C vs. V, as shown here:

$$\gamma = \frac{\dot{h}}{V} = \frac{R/C}{V}$$



Hodograph of climbing flight

4. Climbing Flight

Analytical solution for climbing flight of jet aircraft

- For jet aircraft

$$T - D - W sin\gamma = 0$$
$$L - W cos\gamma = 0$$

Assume climb angle is small, L = W and n = 1

$$\frac{zW}{E_m} - \frac{W}{2E_m}\left[u^2 + \frac{1}{u^2}\right] - W sin\gamma = 0$$

$$sin\gamma = \frac{1}{2E_m}\left[2z - \left(u^2 + \frac{1}{u^2}\right)\right]$$

For γ to be maximum,

$$\frac{dsin\gamma}{du} = 0 \Longrightarrow u = 1$$

For u=1,

$$\gamma_{max} = sin^{-1}\left(\frac{z-1}{E_m}\right)$$

Thus, climb angle is maximum when γ is maximum, which is also the condition for minimum drag in level flight.

4. Climbing Flight

Analytical solution for climbing flight of jet aircraft

- Non-dimensional rate of climb is (**u** ***sinγ***). Velocity for which the rate of climb is maximum is achieved as follows.

$$\frac{d(u sin\gamma)}{du} = 0$$

$$u = u_{max/min} = \sqrt{\frac{z \pm \sqrt{z^2+3}}{3}}$$

$$(u sin\gamma)_{max} = \frac{1}{2E_m}\left[2zu_m - \left(u_m^3 + \frac{1}{u_m}\right)\right]$$

Maximum rate of climb in nondimensional form

- Here, $\mathbf{u_m}$ is the non-dimensional flight velocity for which the rate of climb is maximum. The **maximum rate of climb** is:

$$(R/C)_{max} = (V sin\gamma)_{max} = (u sin\gamma)_{max} V_R$$

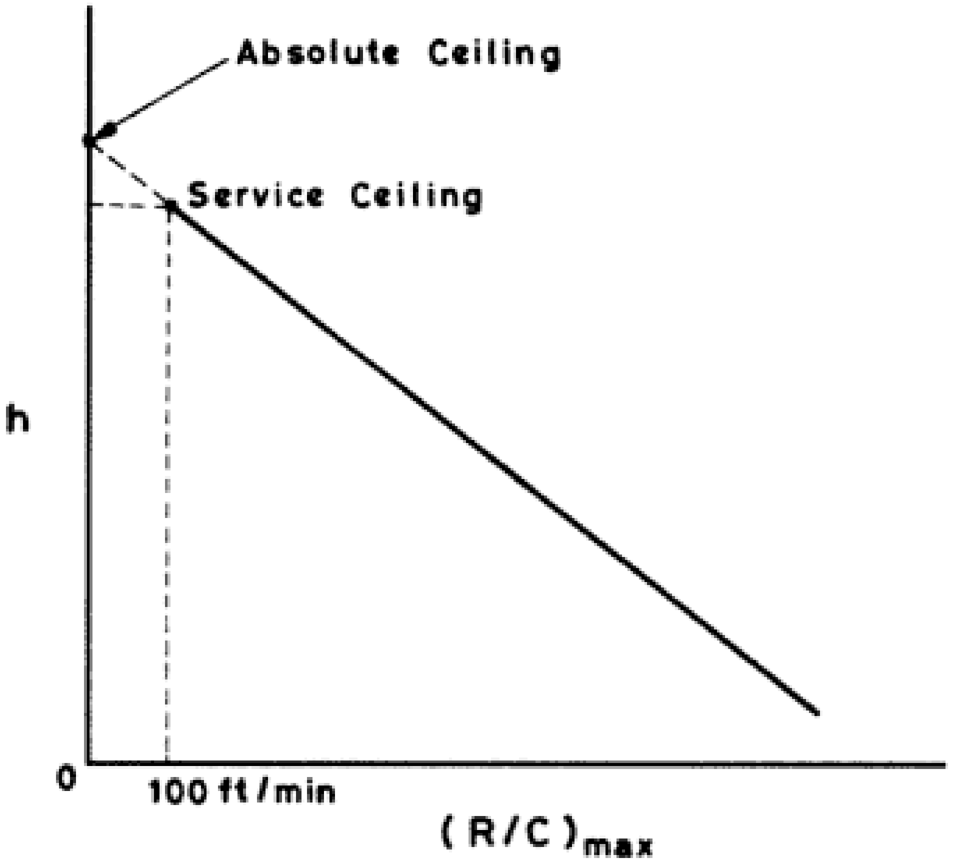
4. Climbing Flight

Absolute and Service Ceilings

- Absolute ceiling was found to be a point where the two level flight solutions merge into one. Based on the rate of climb, an alternative definition of the absolute ceiling can be obtained.

  - The altitude where the $(R/C)_{max}$ drops to 100 ft/min (30.5 m/min) is called **service ceiling**.
  - When $\mathbf{(R/C)_{max}}$ = 0, **absolute ceiling** is reached.
  - For jet aircraft whose thrust is independent of flight velocity, $\mathbf{z}$ = 1 when $\mathbf{(R/C)_{max}}$ = 0.

- For propeller airplanes, the service ceiling is in the range of 4-6 km, whereas for commercial jet airplanes it is 11-17 km.



Service and absolute ceiling.

4. Climbing Flight

Energy Climb Method

- Energy climb is the case of climb flight that involves non-zero accelerations along and normal to the flight path directions.

$$T - D - W sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$L - W cos\gamma = \frac{W}{g}V\frac{d\gamma}{dt}$$

$$\frac{d\gamma}{dt} = \frac{V}{R}$$

- Energy height is defined as,

$$h_e = h + \frac{V^2}{2g}$$

- Such that,

$$\frac{dh_e}{dt} = V\left(\frac{T-D}{W}\right) = P_s$$

- Thus, the specific excess-power is more realistically equal to energy rate of climb rather than the kinematic rate of climb ***dh/dt*** as assumed for steady-state climb.

5. Cruise Flight and Range

- Cruise flight begins at the end of the climb phase when the airplane has reached the desired altitude, and cruise flight ends when the descent phase begins.
- For a given amount of fuel load, the horizontal distance covered with respect to the ground during cruise flight is called **cruise range** , not including any distance covered during the climb or descent.
- Fro physical considerations, the range will be maximum when the airplane cruises at that velocity when the ratio of velocity to fuel consumed per unit time is maximum.
- **Endurance** is the total time that an airplane can remain in air for a given amount of fuel load and is usually expressed in hours. It is of interest to all aircraft during **loiter**, which is defined as that phase of flight where the primary aim is to remain airborne and not worry about the ground distance covered.

5. Cruise Flight and Range

Range of a jet aircraft

➤ **<u>Range of a jet aircraft at constant altitude</u>**: the velocity must vary continuously during the cruise to compensate for the variation in weight.

$$c = -\frac{\dot{W}}{T} \qquad V = \sqrt{\frac{2W}{\rho S C_L}}$$

$$R = \frac{2}{c}\left(\frac{\sqrt{C_L}}{C_D}\right)\sqrt{\frac{2}{\rho S}}\left[\sqrt{W_0} - \sqrt{W_1}\right]$$

Thus, for a given amount of fuel load, the range is maximum when,

$$\left(\frac{\sqrt{C_L}}{C_D}\right) = \left(\frac{\sqrt{C_L}}{C_D}\right)_{max}$$

The **conditions for maximum range** are given as follows:

$$C_L = \sqrt{\frac{C_{D0}}{3k}} = \frac{1}{\sqrt{3}}C_L^*$$

$$V = \sqrt{3}V_R$$

- In general, the thrust available drops with altitude so that there is an altitude where the overall range is maximum. This altitude is called the **cruise altitude**.

5. Cruise Flight and Range

Range of a jet aircraft

- **Range of a jet aircraft at constant velocity**: the range of an airplane cruising at a constant velocity is given by the **Breguet range formula**.

$$R = E\left(\frac{V}{c}\right) ln\left(\frac{W_0}{W_1}\right)$$

- The **Breguet range** is maximum when the aircraft flies at that velocity for which **E = E$_m$** . Thus the maximum range is given by:

$$R_{max} = E_m\left(\frac{V_R}{c}\right) ln\left(\frac{W_0}{W_1}\right)$$

- During this constant velocity cruise the airplane will steadily gain the altitude because of continuous decrease in weight. This method is called **cruise climb**. Height attained is substantial for long cruises.

$$V_R = \sqrt{\frac{2W_0}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}}$$

V_R is based on initial weight W_0

5. Cruise Flight and Range

Range of a propeller aircraft

- For propeller airplanes, the specific fuel consumption (**c**) is the amount of fuel consumed per unit power developed by the engine per unit time.

$$\dot{W} = cP$$

- The power required is equal to the product of drag ***D*** and velocity ***V***.
- The power available is the product of the power developed by the engine and the propulsive efficiency.

For steady level flight condition,

$$\eta_p P = DV \implies P = \frac{DV}{\eta_p}$$

- The range of a propeller aircraft in level flight is given by:

$$R = E\left(\frac{\eta_p}{c}\right) ln\left(\frac{W_0}{W_1}\right)$$

$$R_{max} = E_m\left(\frac{\eta_p}{c}\right) ln\left(\frac{W_0}{W_1}\right)$$

The range of propeller aircraft assumes a maximum value when flying at $E = E_m$

5. Cruise Flight and Range

Range of a propeller aircraft

- Thus, the range of the propeller aircraft assumes a maximum value when flying at $\boldsymbol{E = E_m}$ ($C_L = C_L^*$).
- To determine the velocity for maximum, consider the following 2 options:
  - 1) constant-velocity cruise and
  - 2) constant-altitude cruise.
- In constant-velocity cruise, the cruise velocity is taken equal to the reference velocity based on the initial weight.

$$V = V_R = \sqrt{\frac{2W_0}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}}$$

- In constant-altitude cruise, the flight velocity varies continuously and is equal to the instantaneous value of $\mathbf{V_R}$, based on inst. weight.

$$V = V_R = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}}$$

5. Cruise Flight and Range

Effect of wind on range

- **Tailwind has a beneficial effect and headwind has an adverse effect on the range.**
- The velocity with respect to ground in presence of the wind is given by, (the – and + signs apply for headwind and tailwind, respt.)

$$\frac{dx}{dt} = V \mp V_w$$

- For a given airspeed ***V***, the headwind reduces the Earth-related velocity, whereas tailwind increases it.

$$dx = -(V \mp V_m)\frac{dW}{cT}$$

$$= -(V \mp V_m)E\left(\frac{1}{c}\right)\frac{dW}{W}$$

$$R_w = -\int_{W_0}^{W_1}(V \mp V_w)E\left(\frac{1}{c}\right)\frac{dW}{W}$$

$$= R \mp E\left(\frac{1}{c}\right)V_w ln\frac{W_0}{W_1}$$

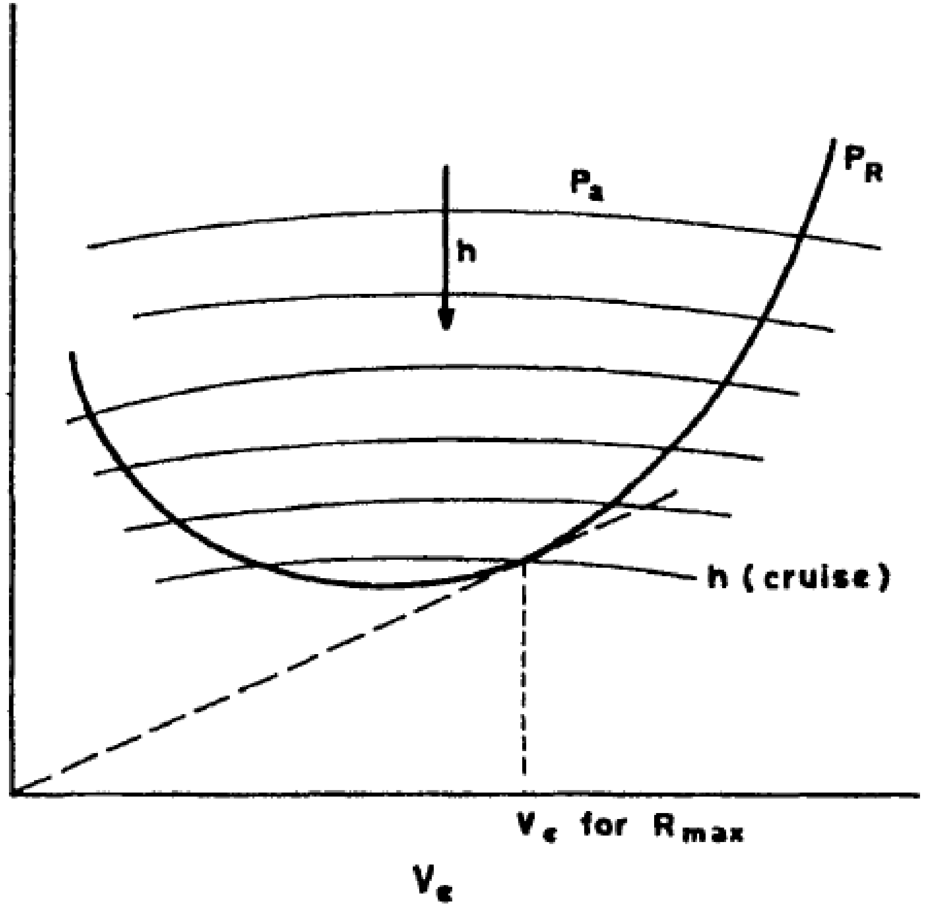
5. Cruise Flight and Range

Estimation of cruise altitude

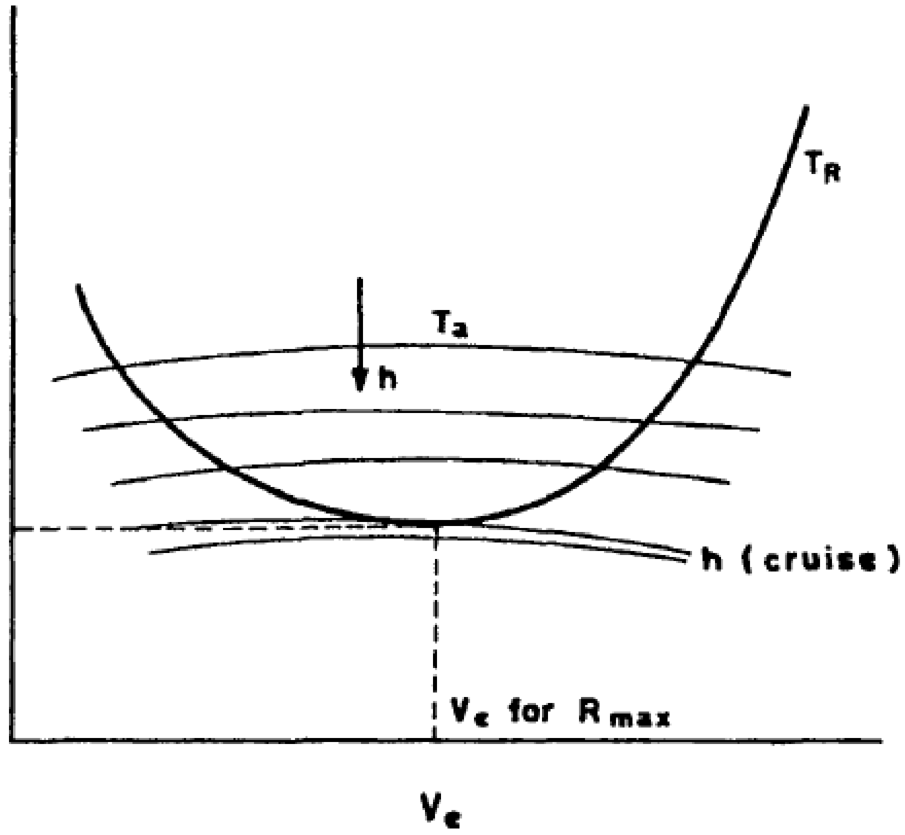
- The cruise, or the most economic altitude, is that altitude where the overall range is maximum. For propeller airplanes, it is in the range of 4-7 km and, for jet airplanes, it is 11-17 km.
- An approximate estimation of the cruise altitude can be made if we ignore the variation of specific fuel consumption with altitude.
- For propeller aircraft, the cruise altitude corresponds to the altitude of that $\mathbf{P_a}$ curve, which intersects the $\mathbf{P_R}$ curve at a point where $\mathbf{V_e} = \mathbf{V_{e,Rmax}}$. The speed $\mathbf{V_{e,Rmax}}$ is obtained by drawing a tangent from the origin to the $\mathbf{P_R}$ curve.
- For jet aircraft, the cruise altitude corresponds to the altitude of that $\mathbf{T_a}$ curve which is tangential the $\mathbf{T_R}$ curve. However, this altitude is not the absolute ceiling since the $\mathbf{T_R}$ is based on the initial weight $\mathbf{W_0}$ and not the instantaneous weight.

5. Cruise Flight and Range

Estimation of cruise altitude



Cruise altitude for propeller aircraft



Cruise altitude for jet aircraft

Estimation of cruise altitude

- The cruise altitude can also be estimated analytically if the thrust is assumed to vary with altitude as $T = T_0 \sigma^{\beta}$ (**β**<1) and ignoring the variation of specific fuel consumption with altitude.

$$T = D \Longrightarrow T_0 \sigma^{\beta} = \frac{W_0 D}{L}$$

$$\sigma = \left(\frac{W_0}{E T_0} \right)^{\frac{1}{\beta}}$$

- The most commercial cruise altitude h in kilometers can be obtained approximately obtained using,

$$h = 44.3(1 - \sigma^{0.235})$$

6. Endurance

Estimation of cruise endurance

- The endurance of an airplane is the total time an airplane remains in the air and is usually expressed in hours.
- For a **jet aircraft**:

$$t_{max} = E_m \left(\frac{1}{c}\right) ln \left(\frac{W_0}{W_1}\right)$$

$$E = E_m \qquad C_L = C_L^*$$

$$V = V_R \qquad D = D_{min}$$

- For a **propeller aircraft**:

$$t = E \left(\frac{2\eta_p}{c}\right) \left(\frac{C_L^{\frac{3}{2}}}{W_1}\right) \sqrt{\frac{\rho S}{2}} \left(\sqrt{\frac{1}{W_1}} - \sqrt{\frac{1}{W_0}}\right)$$

$$C_L = \sqrt{3} C_L^*$$

$$V = V_{mp} = 0.76 V_R$$

7. Turning Flight

Mechanism

- The turning flight can be broadly classified in two categories:
  - **Steady turning flight in a horizontal plane**
  - **General turning flight**
- In a steady or constant velocity turning flight in a horizontal plane, the airplane is at a constant altitude, whereas the general turning flight may involve a gain or loss of altitude.
- The routine flights of commercial transport and general aviation airplanes usually belong to the first category.
- The examples of the second category are the turning flight of a glider and that of a fighter aircraft performing limiting turns exploiting the full aerodynamic and structural capabilities.

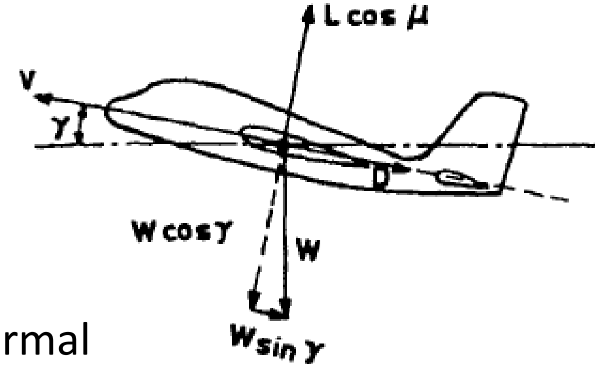
7. Turning Flight

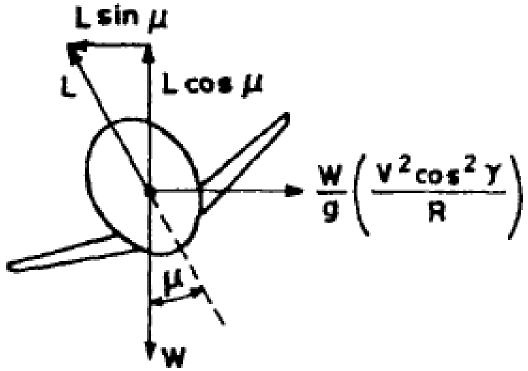
Mechanism

Governing equation for steady turning flight

$$T cos\beta - D - W sin\gamma = 0$$ Along flight path

$$L cos\mu - W cos\gamma = 0$$ Along the principal normal





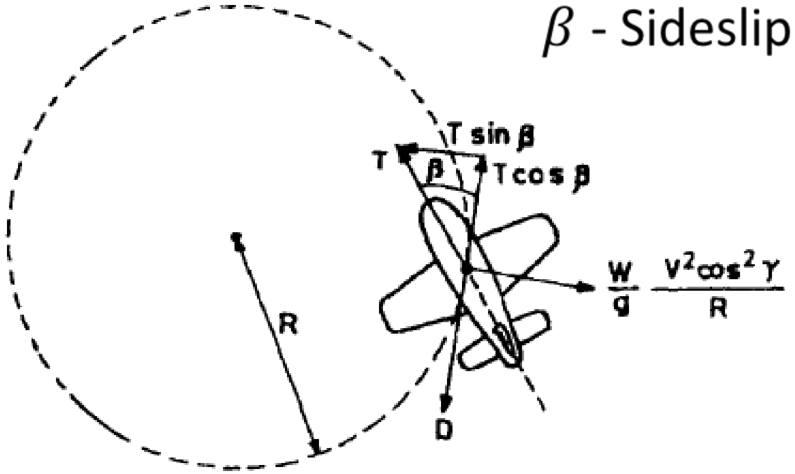
$$\dot{x} = V cos\gamma \qquad \dot{h} = V sin\gamma$$

μ -Bank Angle

β - Sideslip

Along the normal:

$$T sin\beta + L sin\mu - \left(\frac{W V^2 cos^2\gamma}{gR}\right) = 0$$



Aircraft in turning flight in a horizontal plane

7. Turning Flight

Turning flight in a horizontal plane

- For a steady flight in a horizontal plane, **γ = 0**. Ignoring the variation of weight because of fuel consumption during turn the following relations are obtained:

$$T cos\beta - D = 0 \qquad L cos\mu - W = 0$$

$$T sin\beta + L sin\mu - \left(\frac{WV^2}{gR}\right) = 0$$

- The load factor 'n' depends on the bank angle.
- The **higher the bank angle, the higher the load factor that will be developed**.

$$n = \frac{L}{W} = \frac{1}{cos\mu} \qquad tan\mu = \sqrt{n^2 - 1}$$

$$V = \sqrt{\frac{2n(W/S)}{\rho C_L}} = \sqrt{\frac{2(W/S)}{\rho C_L cos\mu}}$$

$$tan\mu = \frac{V^2}{Rg} - \frac{T sin\beta}{W}$$

$$R = \frac{V^2}{g(tan\mu + \frac{T Sin\beta}{W})}$$

7. Turning Flight

Turning flight in a horizontal plane

$$\omega = \frac{V}{R} = \frac{g(tan\mu + \frac{TSin\beta}{W})}{V} \qquad \omega = g\left(tan\mu + \frac{TSin\beta}{W}\right)\sqrt{\frac{\rho C_L}{2n(W/S)}}$$

- The time to complete one complete turn of 2π radians in given by:

$$t_{2\pi} = \frac{2\pi}{\omega} \qquad = \frac{2\pi}{g(tan\mu + \frac{TSin\beta}{W})}\sqrt{\frac{2n(W/S)}{\rho C_L}}$$

- The important metrics of turning performance are the turn rate or the angular velocity in turn **ω** and the radius of turn ***R***.

7. Turning Flight

Turning flight in a horizontal plane

- The aircraft with smaller values of wing loading W/S will have higher turn rates and a lower radius of turn, everything else being equal.
- The expressions for the load factor ***n***, rate of turn **ω**, and the radius of turn ***R*** do not explicitly contain the drag term. It is implied that for these relations to be true, the thrust available balances the drag.
- The sideslip helps the turning flight because of the availability of a thrust component **Tsin*β***, which provides part of the centripetal force.
- However, this benefit may be slightly offset because of the additional drag experienced by the aircraft in sideslip.

7. Turning Flight

Coordinated turning flight in horizontal plane

- In a coordinated turning flight, the sideslip is zero and the aircraft is correctly banked so that the lift vector is tilted away from the plane of symmetry to provide the centripetal force.
- With $\boldsymbol{\beta} = \mathbf{0}$,

$$T - D = 0$$
$$L\cos\mu - W = 0$$
$$L\sin\mu - \frac{WV^2}{gR} = 0$$

$$n = \frac{1}{\cos\mu} \qquad \tan\mu = \frac{V^2}{Rg} \qquad R = \frac{V^2}{g\tan\mu}$$

$$t_{2\pi} = \frac{2\pi}{R} = \frac{2\pi V}{g\tan\mu}$$

- The **turning performance improves with increase in load factor**, all the other things being equal. Increase in the load factor result in better performance in turning flight in terms of ***R*** and ***ω***.

7. Turning Flight

Coordinated turning flight in horizontal plane

- The load factor is a measure of the stress to which both the aircraft and the pilot are subjected. A load factor of 2 means the aircraft structure and the pilot are stressed twice as much in steady level flight with a load factor of unity.
- For transport airplanes, the load factor is limited to about 2.5, whereas for fighter aircraft it can be as high as 9. However, in most of the cases, this limit of a fighter aircraft becomes more of a limitation of the pilot rather than to the machine he is flying.
- As the load factor increases, the pilot experiences what is called “blackout” or “greyout” caused by the blood draining away from his/her brain; then he/she loses color perception. With further increase, he/she will experience loss of peripheral vision and eventually a complete loss of sight; unconscious by the time of 6-7***g***.

7. Turning Flight

Coordinated turning flight in horizontal plane

- The two best conditions- highest possible rate of turn and least possible turning radius, generally do not occur at the same time. The capability to turn as fast as possible is often preferred compared to the capability to make the sharpest turn.
- Some of the (modern) high-performance combat aircrafts such as the F-15 or the F-16 are capable of producing turn rates as high as 20 deg/s.

7. Turning Flight

Maximum Sustained Turn Rate (MSTR):

- The maximum turn rate generated while holding a constant altitude.
- ***Propeller aircraft:*** Numerical or graphical forms of solutions exist. (***ft***- fastest turn solution)

$$\omega_{max} = \frac{g\sqrt{n^2 - 1}}{V_{ft}} \qquad R = \frac{V_{ft}^2}{g \tan\mu} \qquad C_L = \frac{2nWV^2}{\rho S V_{ft}^2}$$

- ***Jet aircraft:*** Analytical solution exists.

$$V = V_R \qquad \omega_{max} = \frac{g\sqrt{2z - 2}}{V_R}$$

7. Turning Flight

Sharpest Sustained Turn:

- The aircraft attempts to make the sharpest turn or the turn with minimum radius of curvature while holding a constant altitude.
- ***Propeller Aircraft:*** No analytical sol^n. (***st***- sharpest turn solution)

$$R_{min} = \frac{V_{st}^2}{g\sqrt{n^2-1}} \qquad C_L = \frac{2nWV^2}{\rho S V_{st}^2}$$

- ***Jet Aircraft:*** Analytical sol^n.

$$V = V_R \qquad R_{min} = \frac{V_R^2}{g\sqrt{z^2-1}} \qquad C_L = \sqrt{\frac{(2z^2-1)C_{D0}}{k}}$$

- For both cases, i.e. ***'ft'*** & ***'st'***, we assume the following conditions hold,

$$n \le n_{lim} \qquad C_L \le C_{L,max}$$

7. Turning Flight

Turn with maximum load factor:

- Because the bank angle and load factor are uniquely related, the turn for maximum load factor is the same as the turn for the maximum bank angle.
- ***Propeller Aircraft:*** The maximum load factor is given by,

$$n_{max} = 0.6874 \left(\frac{[k' n_p P'(kW)]^2 \rho E_m}{k(W/S)} \right)^{\frac{1}{3}}$$

- ***Jet Aircraft:*** The analytical sol^{n} is given as below,

$$n_{max} = z \qquad V = \sqrt{z} V_R \qquad T \leq \frac{n_{lim} W}{E_m} \qquad C_L = \sqrt{\frac{C_{D0}}{k}}$$

7. Turning Flight

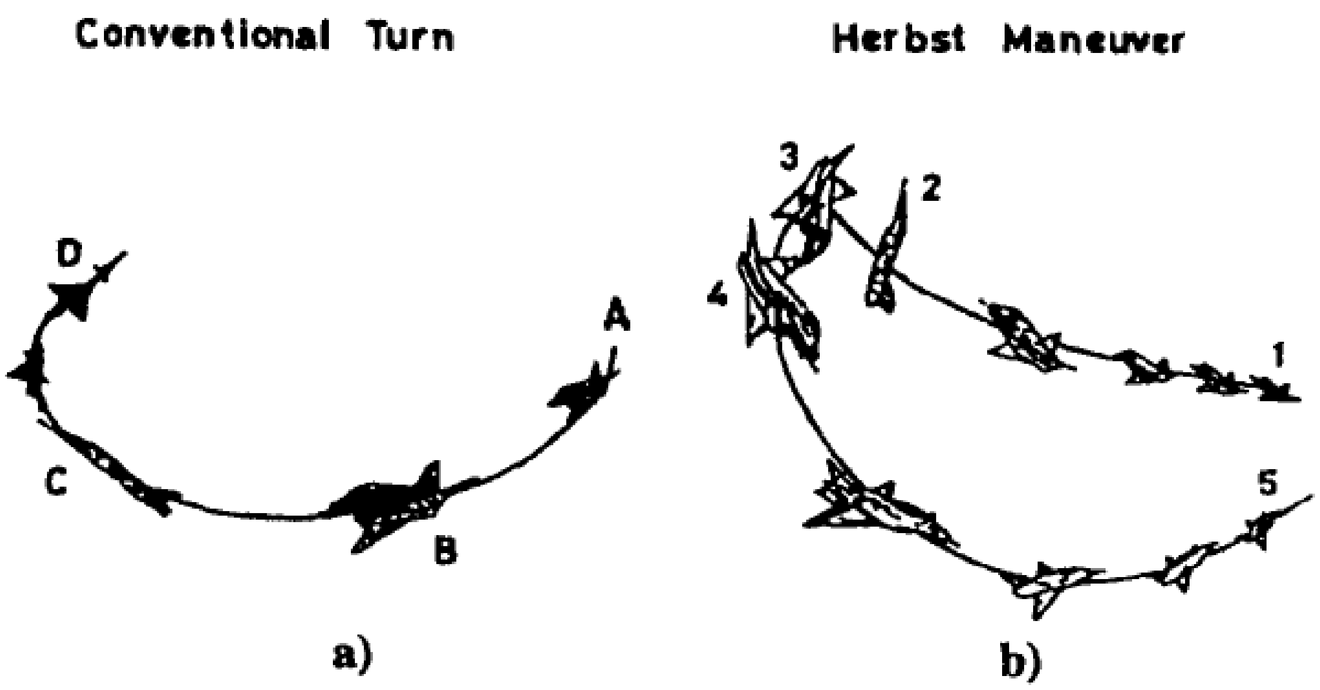
General Turning Flight

- The aircraft can generate higher rates of turn if it is permitted to lose altitude so that it can make use of its height or the potential energy in addition to the aerodynamic and propulsive forces.
- The turn rate so generated can be much higher than MSTR and is called the **maximum instantaneous turn rate** or the **maximum attainable turn rate (MATR)**.
- A high value of MATR is a good measure of the air superiority of combat aircraft because it is of crucial importance in close encounters where a pilot getting first point-and-shoot capability has a better chance to win.
- Because the aircraft will rapidly lose altitude during this maneuver, it is necessary for a pilot to have enough height margin before he initiates this maneuver.

7. Turning Flight

General Turning Flight

- ***Herbst Maneuver:*** During the Herbst maneuver, the pilot decelerates the aircraft by pitching the nose up till the angle of attack goes well beyond the stall and aircraft is nearly perpendicular to the flight path using the entire airframe as the speed brake. The pilot rolls the aircraft about the velocity vector until the nose points in the right direction, levels the nose and accelerates back to the desired airspeed.



Conventional turn and Herbst maneuver

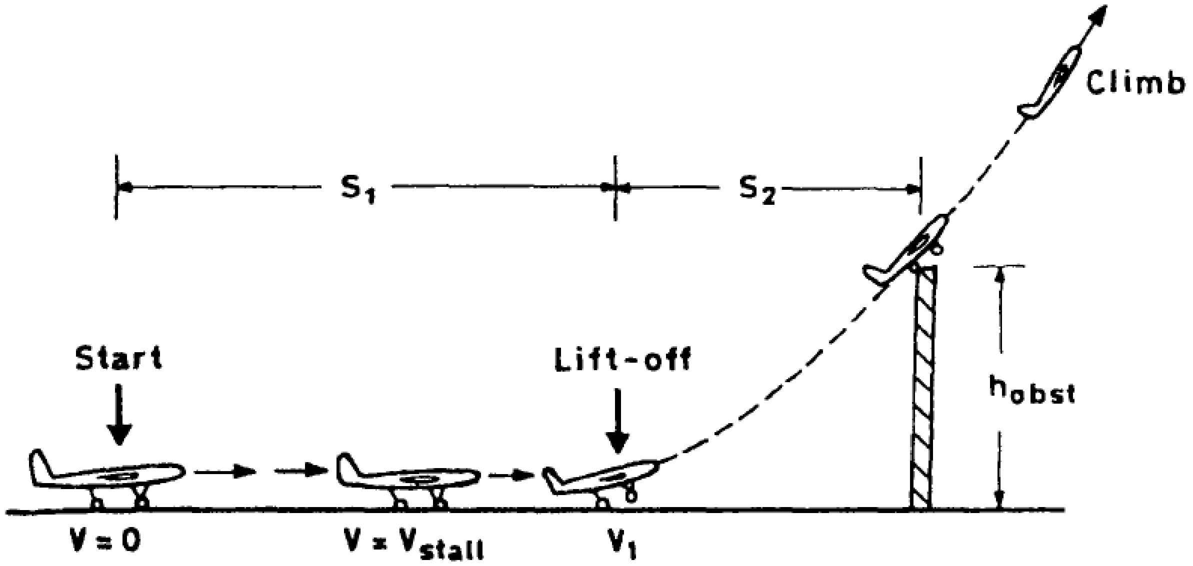
8. Takeoff and Landing

Takeoff

- A typical takeoff phase consists of the following steps:
  - **throttle-up** to maximum power/thrust in order for the aircraft to reach the takeoff speed,
  - on reaching the takeoff speed the aircraft is **rotated noseup** so that the angle of attack increases to generate sufficient lift for liftoff, and
  - the aircraft starts to **climb to clear the specified obstacle height**.
- The takeoff velocity $\mathbf{V_1}$ is usually equal to $\mathbf{1.2*V_{stall}}$. S_1 is the **ground run distance** and S_2 is called the **airborne distance**.
- The Federal Aviation Regulations (FAR), US specifies an **obstacle height** of 35 ft (11.5m), whereas the Royal Aircraft Establishment (RAE), UK specifies it at 50 ft (~15m)

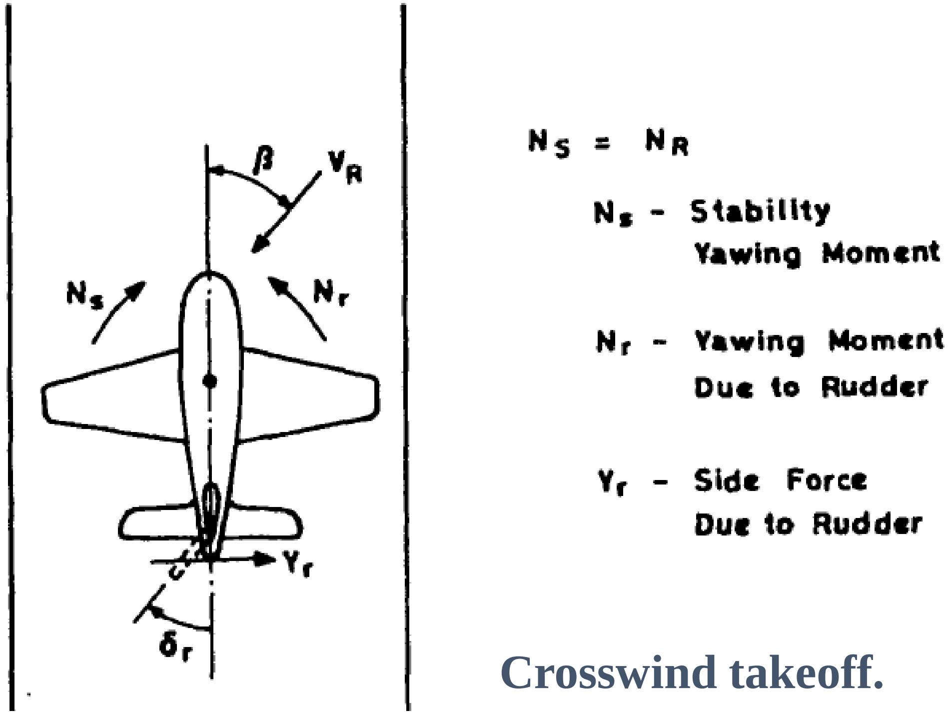
8. Takeoff and Landing

Takeoff



Schematic diagram of an airplane in takeoff

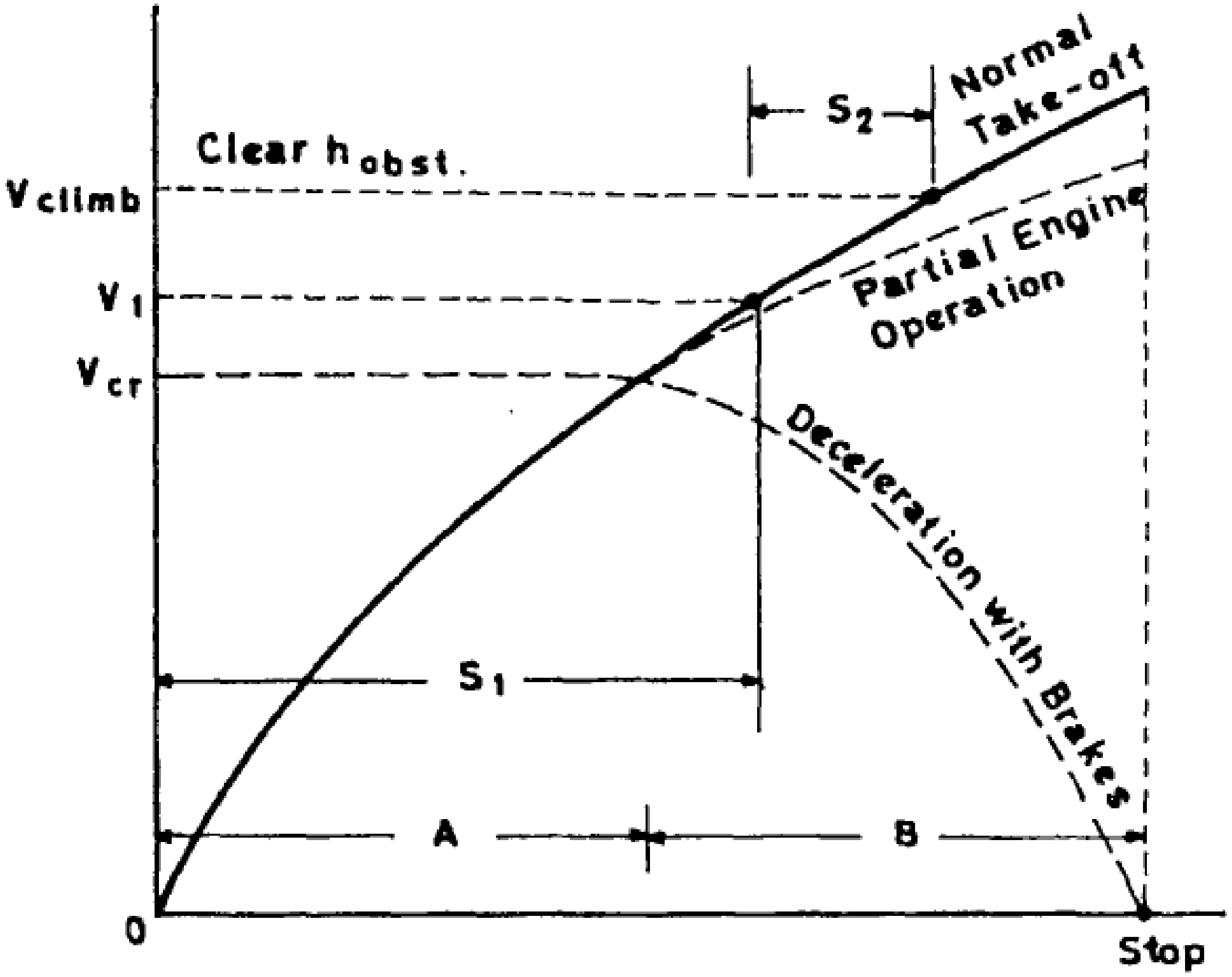
- **Crosswind takeoff**: with sideslip, the drag will be higher and as a result the ground run will be higher. The pilot has to operate the rudder to prevent the airplane from aligning itself along the resultant velocity vector.



Crosswind takeoff.

Takeoff

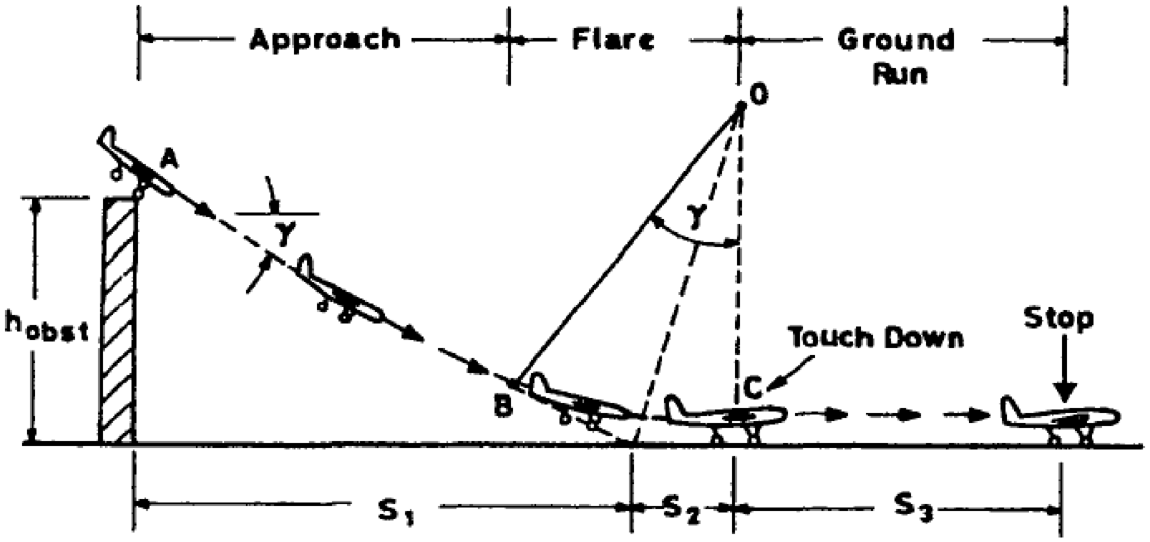
- **Aborted takeoff**: To account for unforeseen situation, like an engine failure, a critical speed is defined for each aircraft to assist the pilot in making a decision whether to abort or continue with the takeoff



Normal takeoff distance and balanced field length.

Landing

- A typical landing phase consists of the following sequences:
  - a **steady shallow glide** approach to the runway holding a constant glide angle, held as small as possible so that the rate of sink is kept down to a minimum. This way it is possible to reduce the energy that has to be dissipated at impact with the ground.
  - **flare**, during which the aircraft is rotated noseup so as to momentarily achieve a level flight condition and to further minimize the rate of sink.
  - **touchdown**, prior to which the pilot may momentarily stall the airplane.
  - **ground run**, during with full reverse thrust (if available). Spoilers or drag parachutes, and brakes are applied to produce maximum retardation and bring the aircraft to a complete halt.



Schematic diagram of airplane landing.

9. Hazards During Landing and Takeoff

Windshear and Microburst

- ***Windshear*** is the change in wind direction and/or speed in a very short distance with respect to the ground. Whereas a change in horizontal wind velocity affects the airspeed, the vertical component of windshear affects the flight path angle.
- They are normally caused by thunderstorm activity, weather fronts, and jet streams. It is most hazardous if encountered during landing and takeoff.
- ***Microburst*** is formed when a column of air at high altitudes quickly cools because of evaporation of ice, snow, or rain and, becoming denser than surrounding air, falls rapidly to the ground. On hitting the ground, this mass of air it spreads radially outward in all directions.
- The life of a microburst is typically 5-15 min.

9. Hazards During Landing and Takeoff

Windshear and Microburst