THEORY OF MACHINES AND MECHANISMS II

Mechanical IV/I

Chapter 3

Gyroscopic Couple

3.1 Gyroscopic Effect

If we try to move the spin axis of the disc, it resists the motion. The resistance to change the direction of the spin axis is called Gyroscopic Effect.

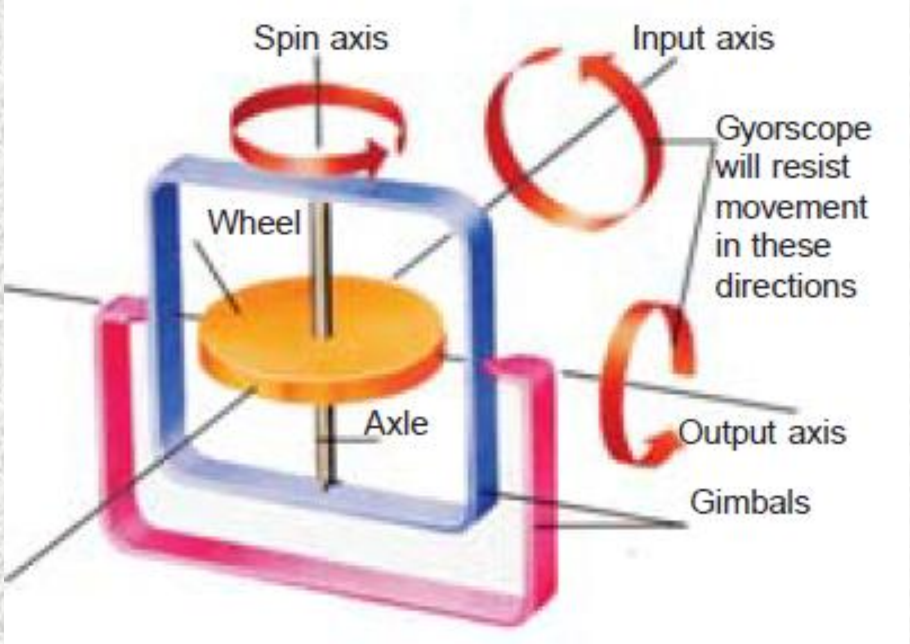


Figure 3.1

Gyroscopic inertia prevents a spinning top from falling sideways.

3.2 Precessional Angular Motion

Consider a disc, as shown in **Figure 3.2 (a)**, revolving or spinning about the axis **OX** (known as axis of spin) in anticlockwise when seen from the front, with an angular velocity **ω** in a plane at right angles to the paper.

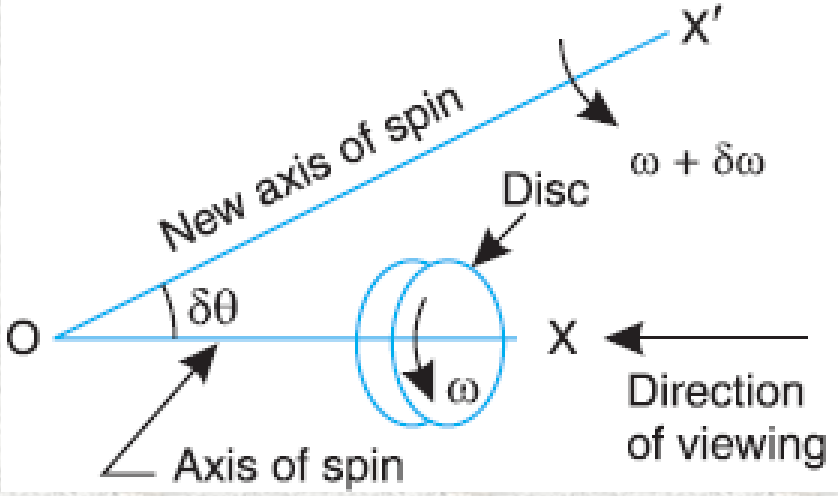


Figure 3.2 (a)

After a short interval of time **δt**, let the disc be spinning about the new axis of spin **OX ′** (at an angle **δθ**) with an angular velocity **(ω + δω)**.

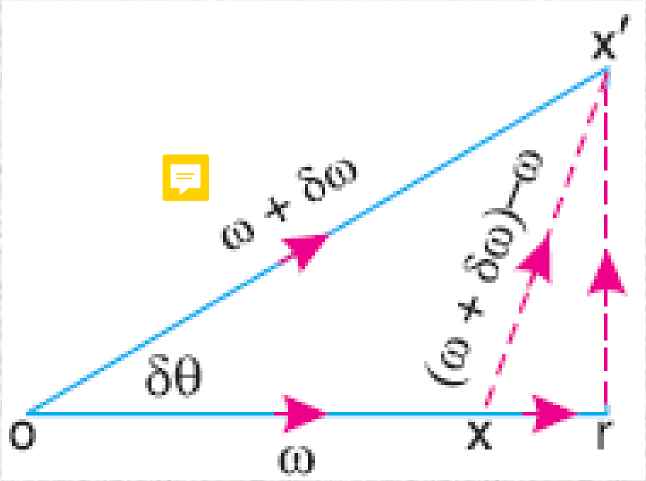


Figure 3.2 (b)

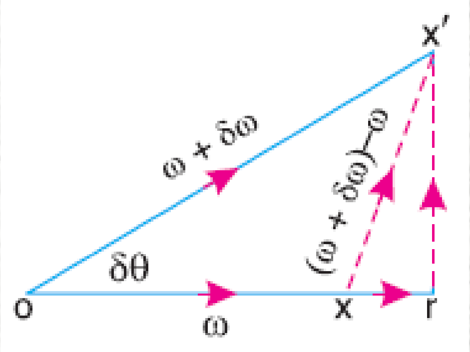
Initial angular velocity of the disc (**ω**) is represented by vector ox; and the final angular velocity of the disc (**ω + δω**) is represented by vector ox′ as shown in **Figure 3.2 (b)**.

The vector **xx′** represents the change of angular velocity in time **δt** i.e. the angular acceleration of the disc. This may be resolved into two components, one parallel to ox and the other perpendicular to **ox**.

Component of angular acceleration in the direction of ox,

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos \delta\theta - ox}{\delta t}$$

$$= \frac{(\omega + \delta\omega)\ \cos\delta\theta - \omega}{\delta t} = \frac{\omega \cos\delta\theta + \delta\omega \cos\delta\theta - \omega}{\delta t}$$



Since **δθ** is very small, therefore substituting **cos δθ =** 1, we have

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

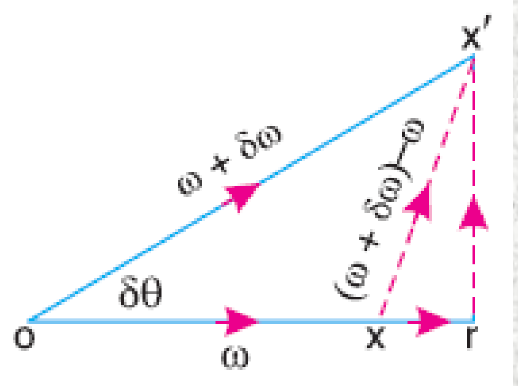
In the limit,

$$\alpha_t = \underset{\delta t \to 0}{\mathrm{Lt}} \left(\frac{\delta\omega}{\delta t} \right) = \frac{d\omega}{dt}$$

Component of angular acceleration in the direction perpendicular to **ox**,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox' \sin\delta\theta}{\delta t} = \frac{(\omega + \delta\omega)\sin\delta\theta}{\delta t}$$

$$= \frac{\omega \sin\delta\theta + \delta\omega.\sin\delta\theta}{\delta t}$$



Since **δθ** in very small, therefore substituting **sin δθ = δθ**, we have

$$\alpha_c = \frac{\omega.\delta\theta + \delta\omega.\delta\theta}{\delta t} = \frac{\omega.\delta\theta}{\delta t}$$

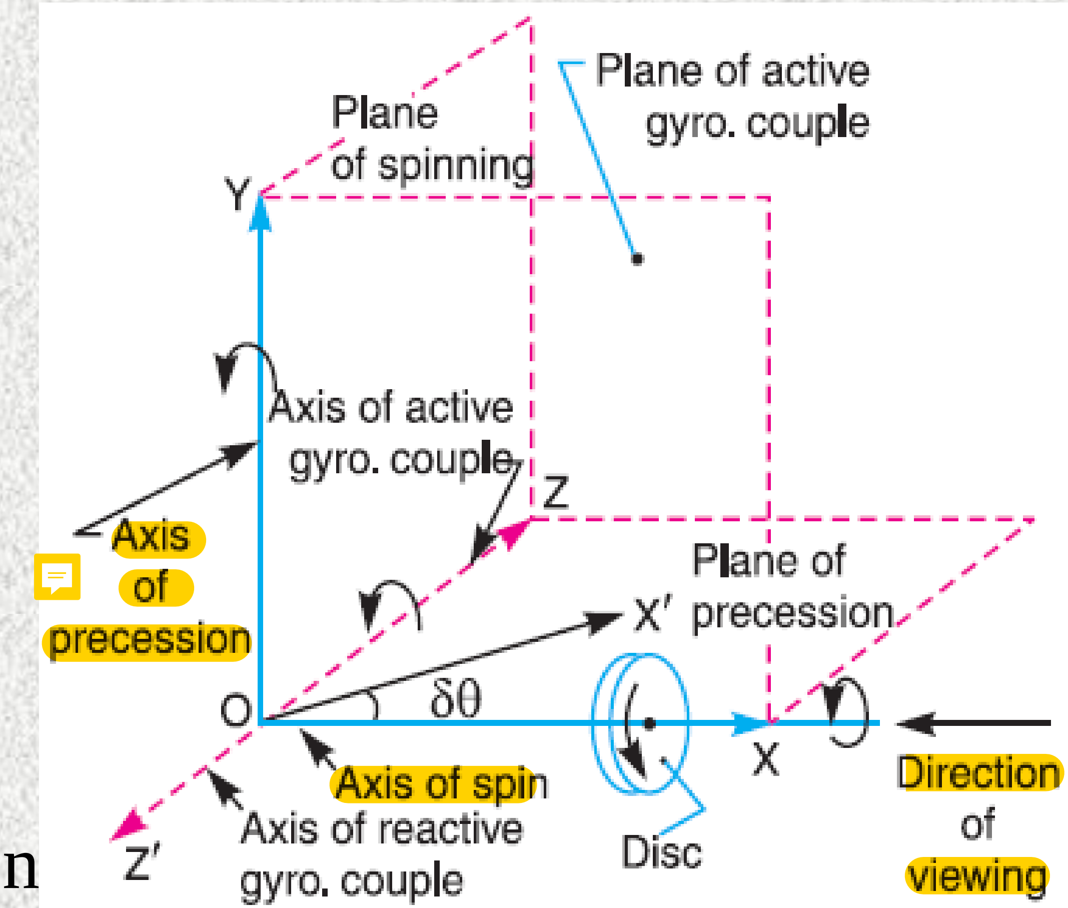
In the limit,

$$\alpha_c = \underset{\delta t \to 0}{\mathrm{Lt}} \frac{\omega.\delta\theta}{\delta t} = \omega \times \frac{d\theta}{dt} = \omega.\omega_P$$

where **dθ/dt** is the angular velocity of the axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. This angular velocity of the axis of spin (i.e. **dθ/dt**) is known as **angular velocity of precession** and is denoted by $\boldsymbol{\omega_P}$. The axis, about which the axis of spin is to turn, is known as **axis of precession**. The angular motion of the axis of spin about the axis of precession is known as **precessional angular motion**.

The angular acceleration $\boldsymbol{\alpha_c}$ is known as **gyroscopic acceleration**.

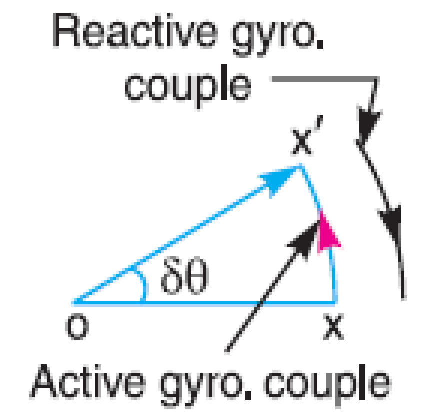
3.3 Gyroscopic Couple



OX: Axis of Spinning
YOZ: Plane of Spinning
OY: Axis of Precession
XOZ: Plane of Precession
OZ: Axis of Active Gyroscopic Couple
XOY: Plane Active Gyroscopic Couple

Angular momentum of the spinning disc

$$= I\omega$$



Change in angular momentum

$$= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox}.\delta\theta$$
$$= I.\ \omega.\delta\theta$$

The rate of change of angular momentum

$$= I.\omega \times \frac{\delta\theta}{dt}$$

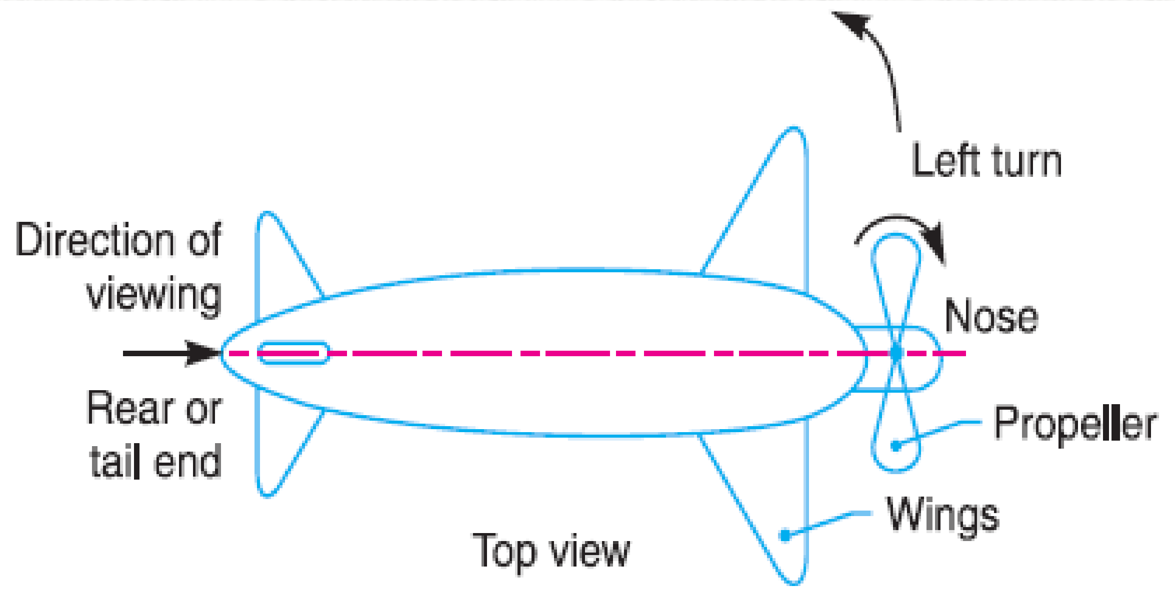
Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession,

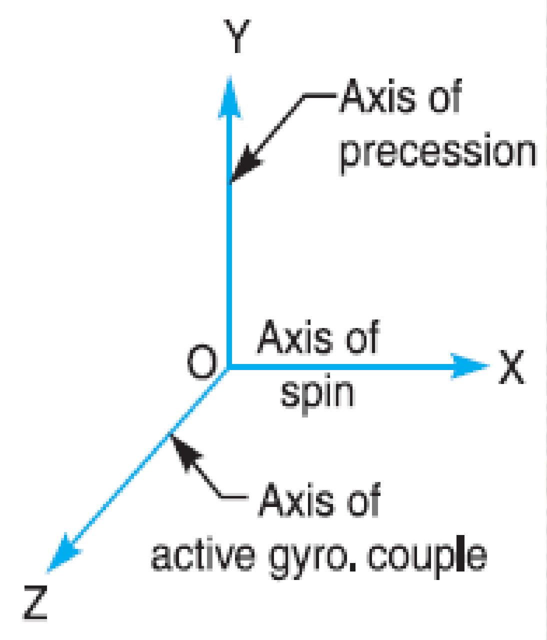
$$C = \underset{\delta t \to 0}{\mathrm{Lt}}\ I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P$$

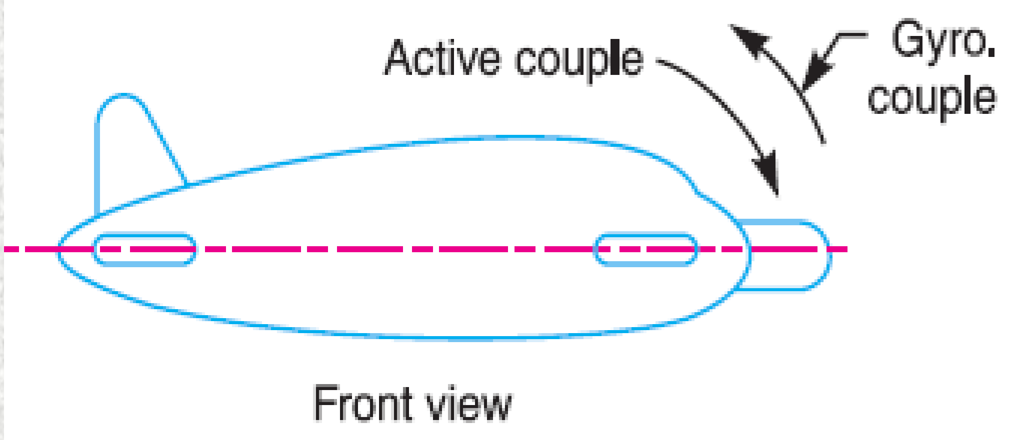
The couple $\mathbf{I.\omega.\omega_p}$, in the direction of the vector $\mathbf{xx'}$ (representing the change in angular momentum) is the **active gyroscopic couple**, which has to be applied over the disc when the axis of spin is made to rotate with angular velocity $\mathbf{\omega_P}$ about the axis of precession.

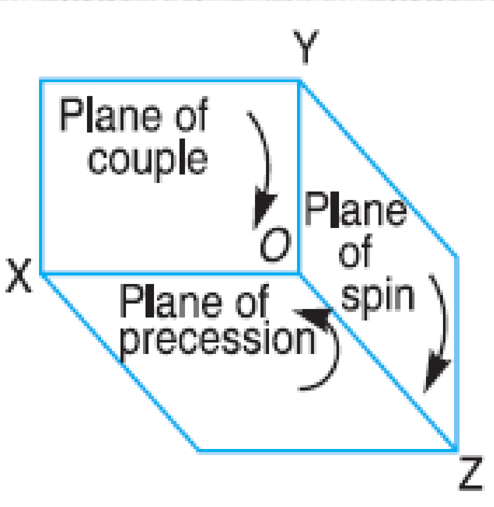
When the axis of spin itself moves with angular velocity $\mathbf{\omega_P}$, the disc is subjected to reactive couple whose magnitude is same (i.e. $\mathbf{I.\ \omega.\omega_P}$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected then the axis of spin rotates about the axis of precession is known as **reactive gyroscopic couple**.

3.4 Effect of the Gyroscopic Couple on an Aeroplane

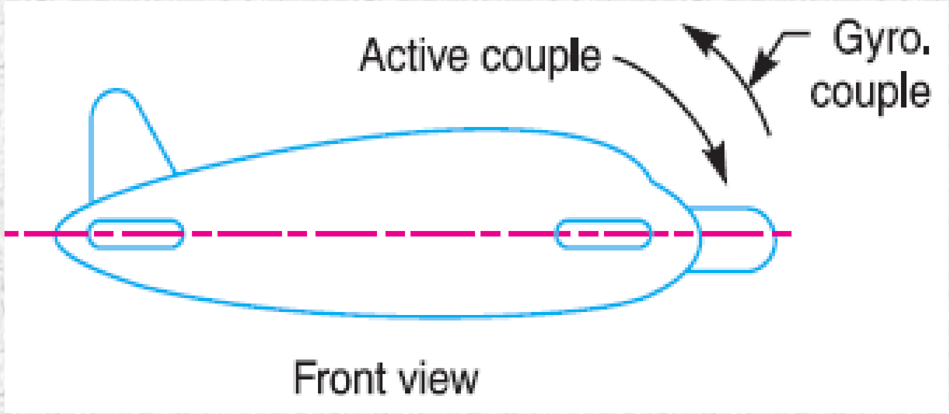


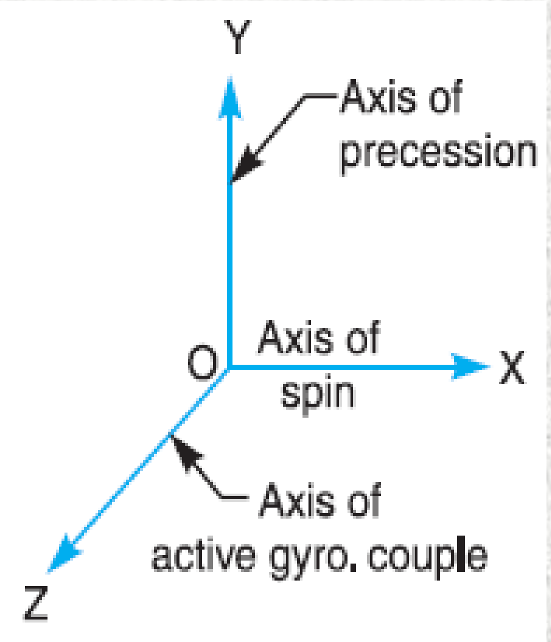






For left hand turning, the active gyroscopic couple on the aeroplane in the axis **OZ** will be clockwise as shown in **Figure.** The reactive gyroscopic couple (equal in magnitude of active gyroscopic couple) will act in the opposite direction (i.e. in the anticlockwise direction) and the effect of this couple is, therefore, to raise the nose and dip the tail of the aeroplane.





3.5 Stability of a Four Wheel Drive Moving in a Curved Path

Effect of the gyroscopic couple

Velocity of precession $\omega_P = \dfrac{V}{R}$

Gyroscopic couple due to 4 wheels,

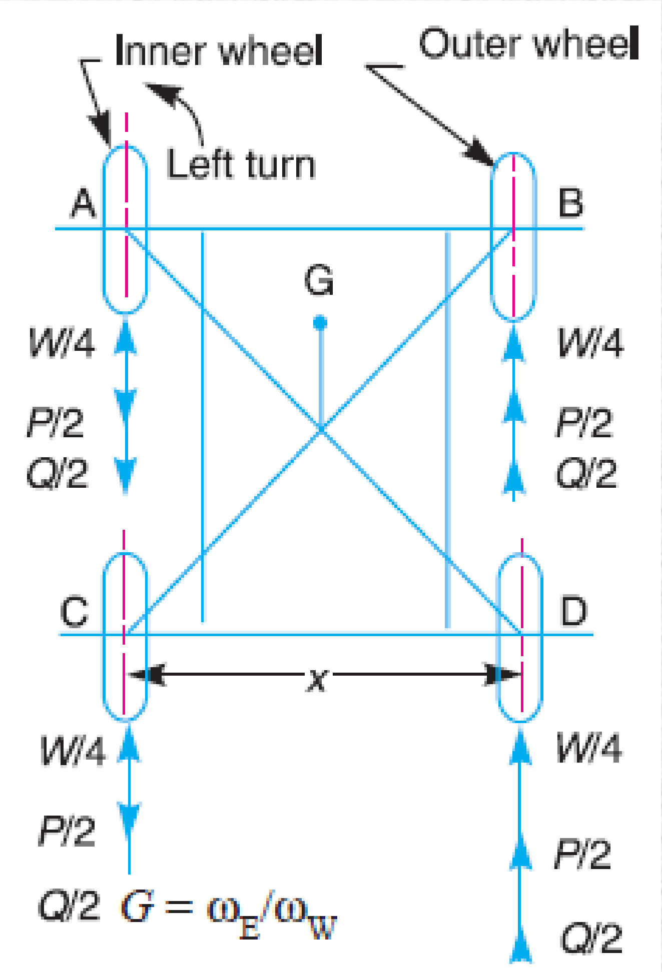
$$C_W = 4\, I_W.\omega_W.\omega_P$$

Gyroscopic couple due to the rotating parts of the engine,

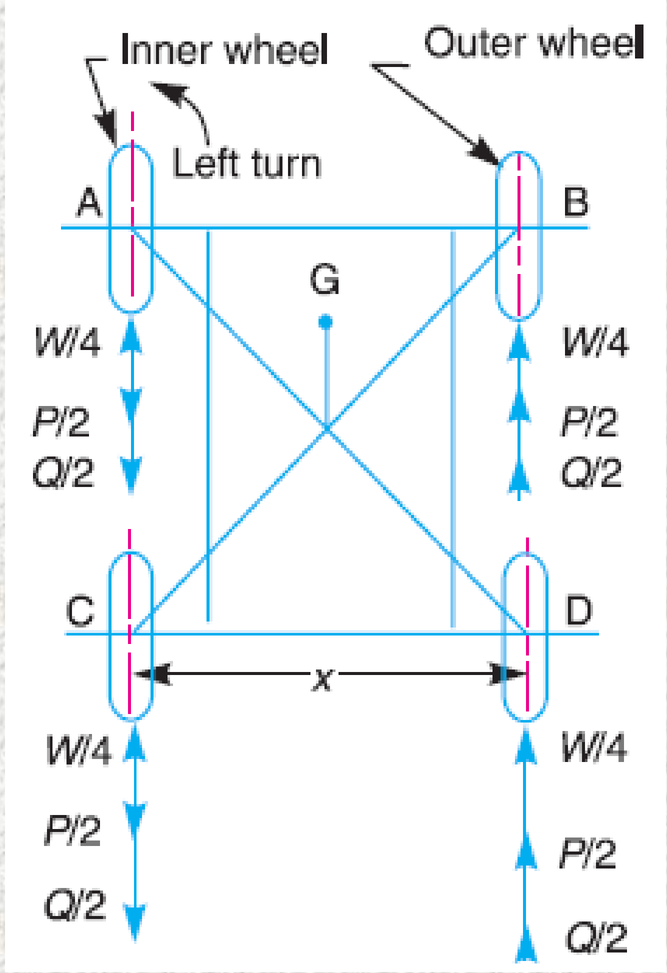
$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P$$

Net gyroscopic couple,

$$C = C_W \pm C_E = 4\, I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P = \omega_W.\omega_P\,(4\, I_W \pm G.I_E)$$



Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be **P** newtons. Then



$$P \times x = C \quad \text{or} \quad P = C/x$$

Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

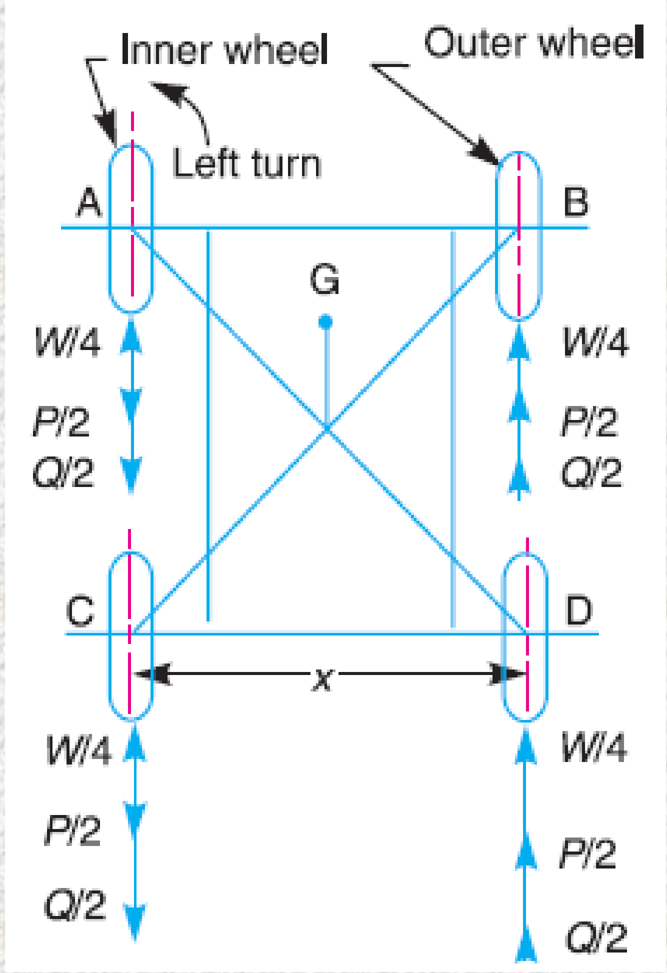
Effect of the centrifugal couple

The effect of this centrifugal force is also to overturn the vehicle.

$$F_C = \frac{m \times v^2}{R}$$

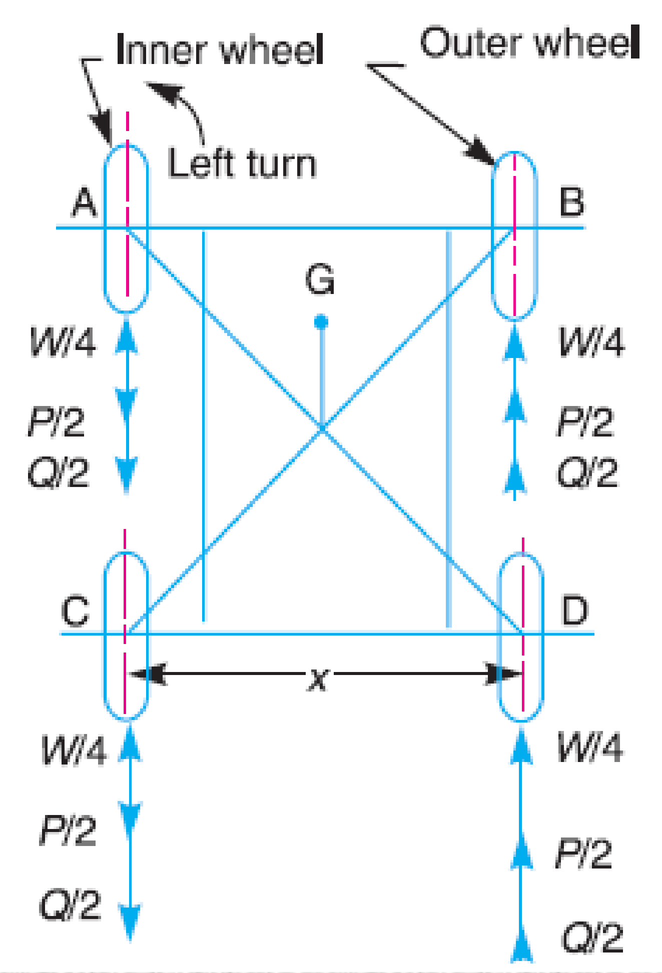
The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$



This overturning couple is balanced by vertical reactions, which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be **Q**. Then

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$



Vertical reaction at each of the outer or inner wheels,

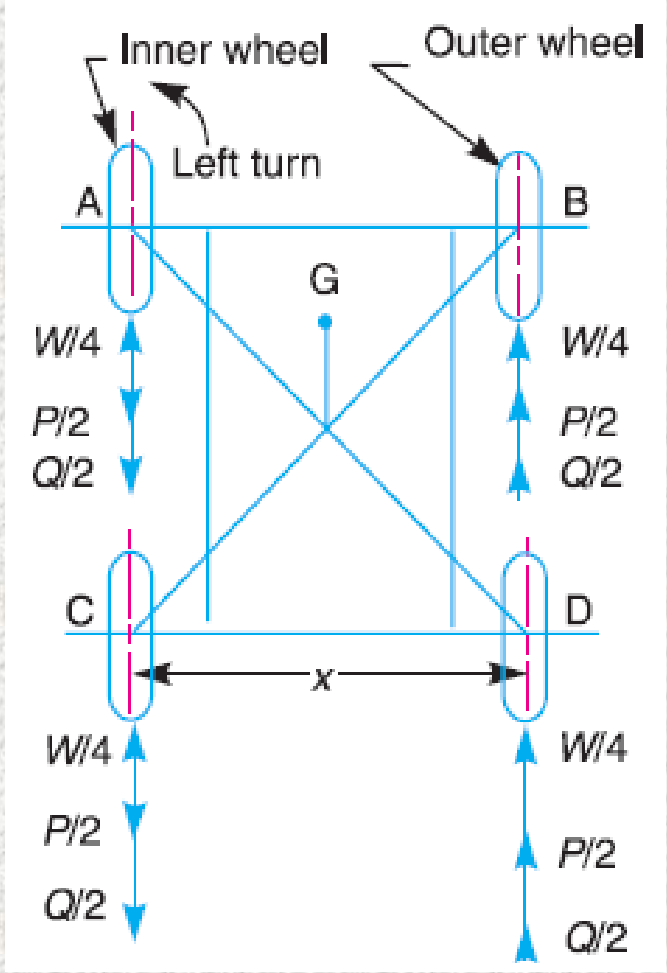
$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

Total vertical reaction at each of the outer wheel,

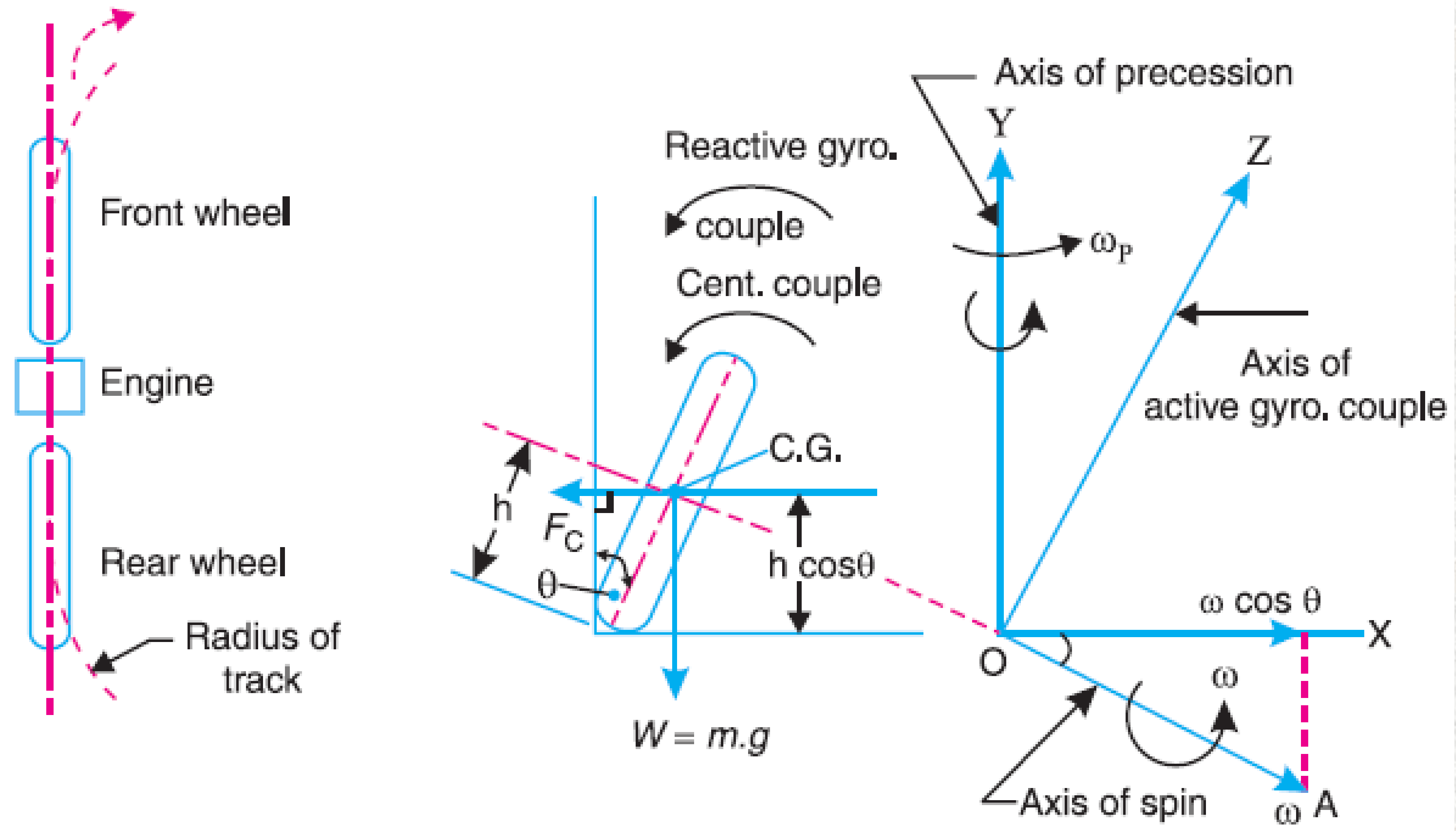
$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

Total vertical reaction at each of the inner wheel,

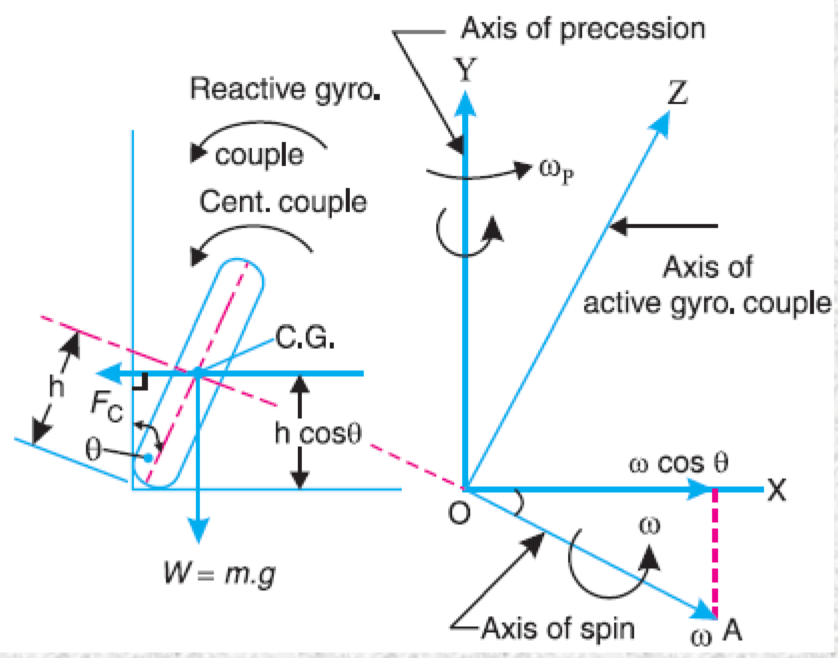
$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$



3.6 Stability of a Two Wheel Vehicle Taking a Turn



Effect of the gyroscopic couple



Total angular momentum

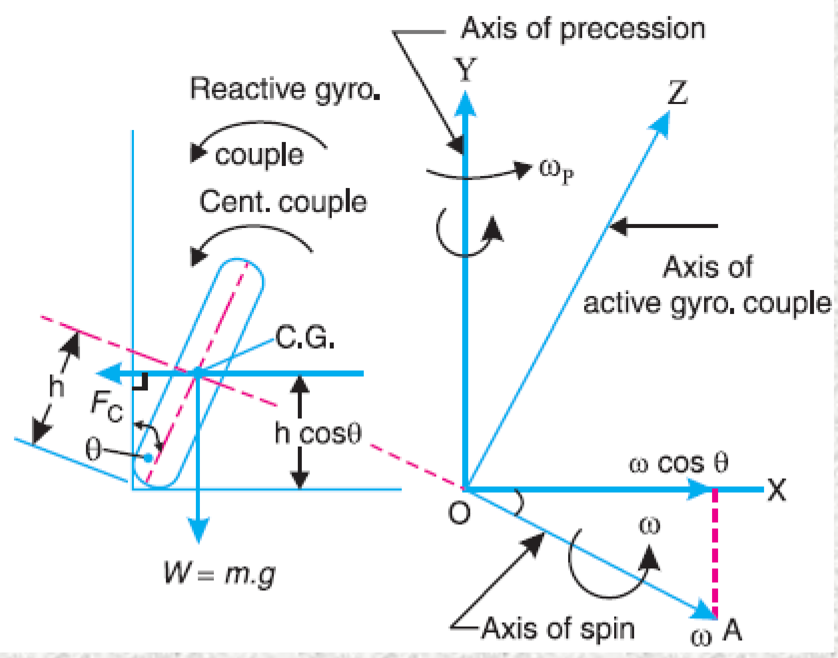
$$(I \times \omega) = 2\, I_W \times \omega_W \pm I_E \times \omega_E$$
$$= 2\, I_W \times \frac{v}{r_W} \pm I_E \times G \times \frac{v}{r_W}$$
$$= \frac{v}{r_W}(2\, I_W \pm G.I_E)$$

The angular momentum vector ***Iω*** due to spin is represented by **OA** inclined to **OX** at an angle **θ**. But the precession axis is vertical. Therefore the spin vector is resolved along **OX**.

Gyroscopic couple

$$C_1 = I.\omega \cos\theta \times \omega_P = \frac{v}{r_W}(2\, I_W \pm G.I_E)\cos\theta \times \frac{v}{R} = \frac{v^2}{R.r_W}(2\, I_W \pm G.I_E)\cos\theta$$

Effect of centrifugal couple



The centrifugal force,

$$F_C = \frac{m.v^2}{R}$$

This force acts horizontally through the centre of gravity (C.G.) along the outward direction

Centrifugal couple,

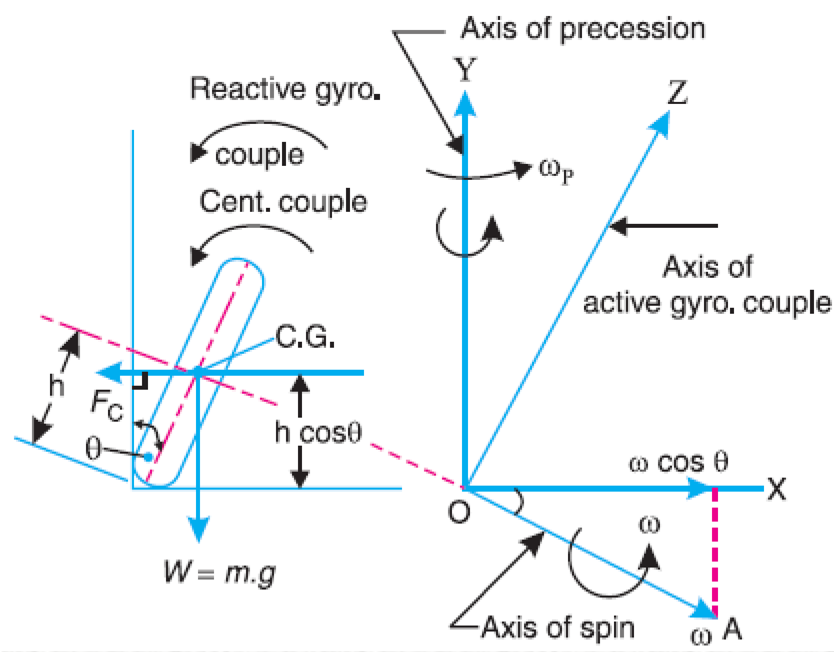
$$C_2 = F_C \times h \cos\theta = \left(\frac{m.v^2}{R}\right) h \cos\theta$$

Total overturning couple,

$$C_O = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$= \frac{v^2}{R.r_W}(2\,I_W + G.I_E)\cos\theta + \frac{m.v^2}{R} \times h\cos\theta$$

$$= \frac{v^2}{R}\left[\frac{2\,I_W + G.I_E}{r_W} + m.h\right]\cos\theta$$



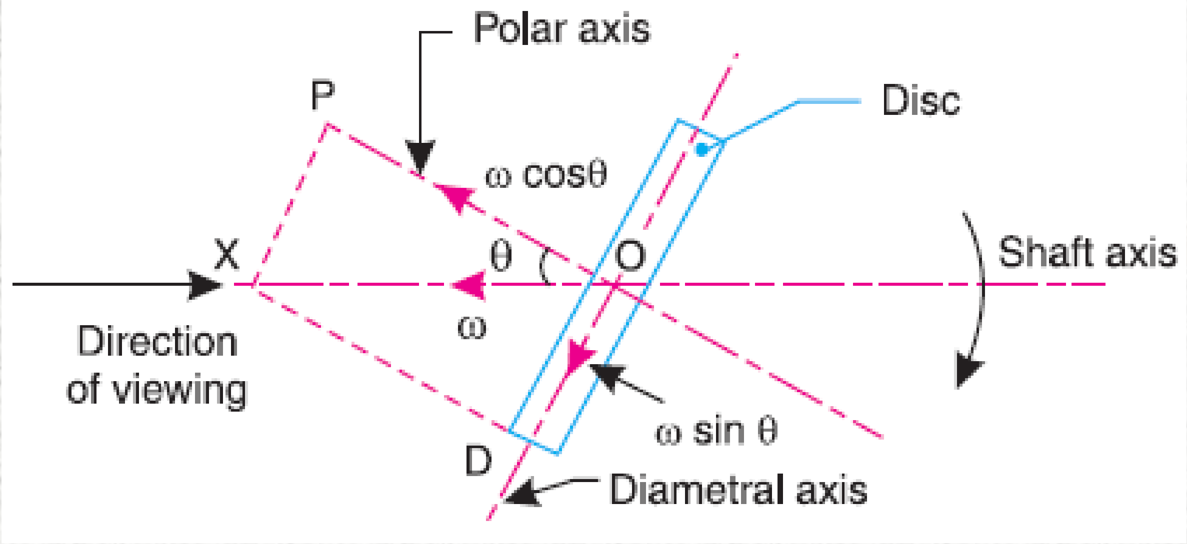
For the stability, the overturning couple must be equal to the balancing couple,

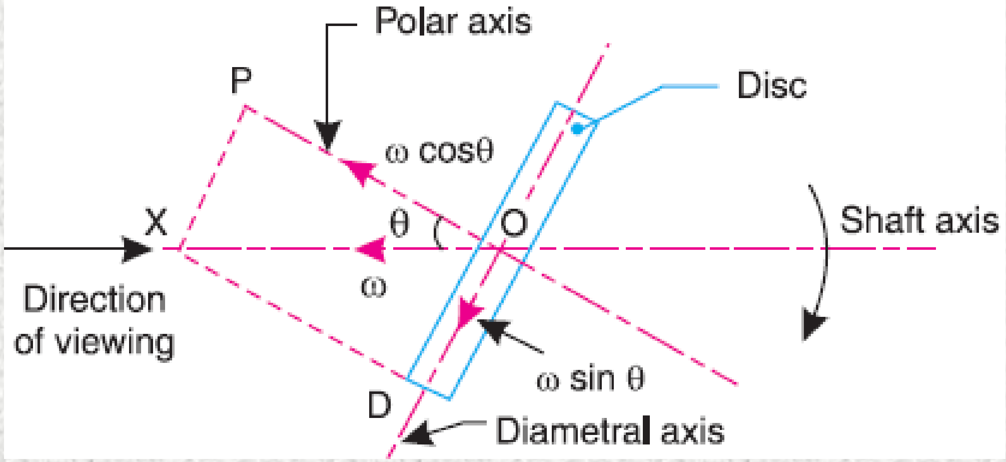
$$\frac{v^2}{R}\left(\frac{2\,I_W + G.I_E}{r_W} + m.h\right)\cos\theta = m.g.h\sin\theta$$

From this expression, the value of the angle of heel (**θ**) may be determined, so that the vehicle does not skid.

3.7 Effect of Gyroscopic Couple on a Disc Fixed Rigidly at a Certain Angle to a Rotating Shaft

Consider a disc fixed rigidly to a rotating shaft such that the polar axis of the disc makes an angle **θ** with the shaft axis, as shown in **Figure**. Let the shaft rotates with an angular velocity **ω** rad/s in the clockwise direction when viewed from the front.





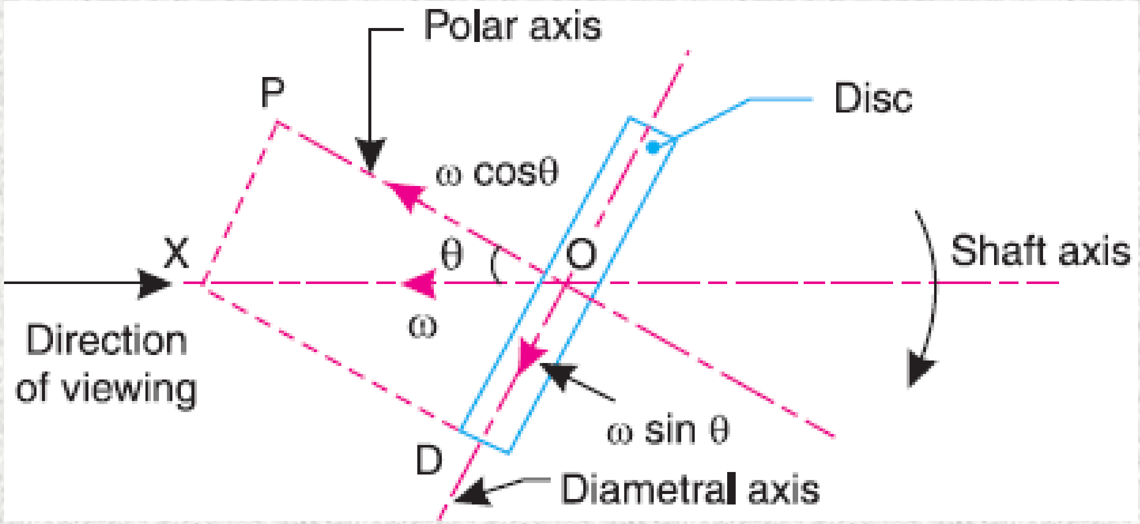
Angular velocity of the disc about the polar axis **OP** or the angular velocity of spin

$$= \omega \cos \theta$$

Since the shaft rotates, therefore the point **P** will move in a plane perpendicular to the plane of paper. In other words, precession is produced about OD.

Angular velocity of the disc about the diametral axis OD or the angular velocity of precession

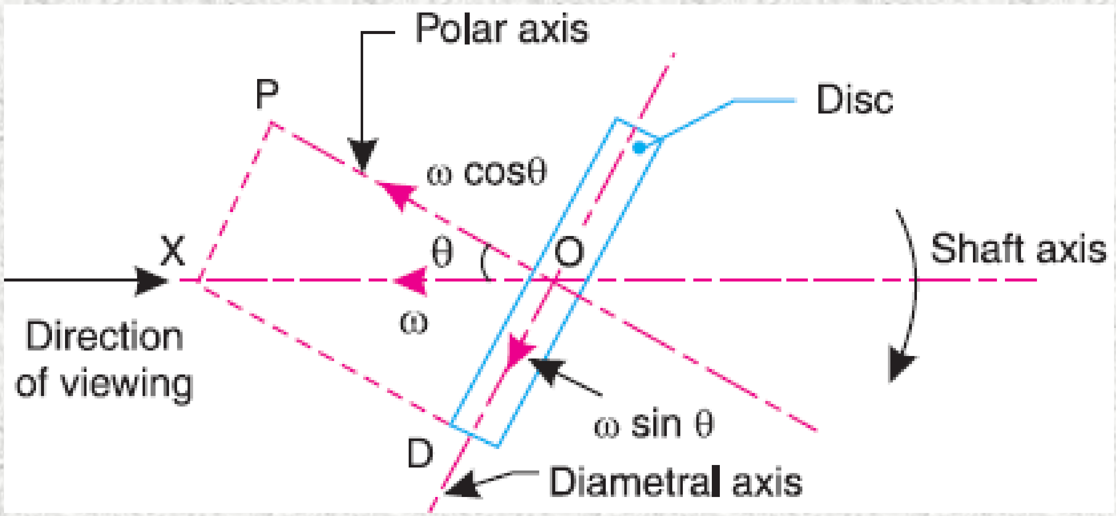
$$= \omega \sin \theta$$



If I_P is the mass moment of inertia of the disc about the polar axis **OP**, then gyroscopic couple acting on the disc,

$$C_P = I_P.\omega \cos\theta.\omega \sin\theta = \frac{1}{2} \times I_P.\omega^2 \sin 2\theta$$

The effect of this gyroscopic couple is to turn the disc in the anticlockwise when viewed from the top, about an axis through **O** in the plane of paper.



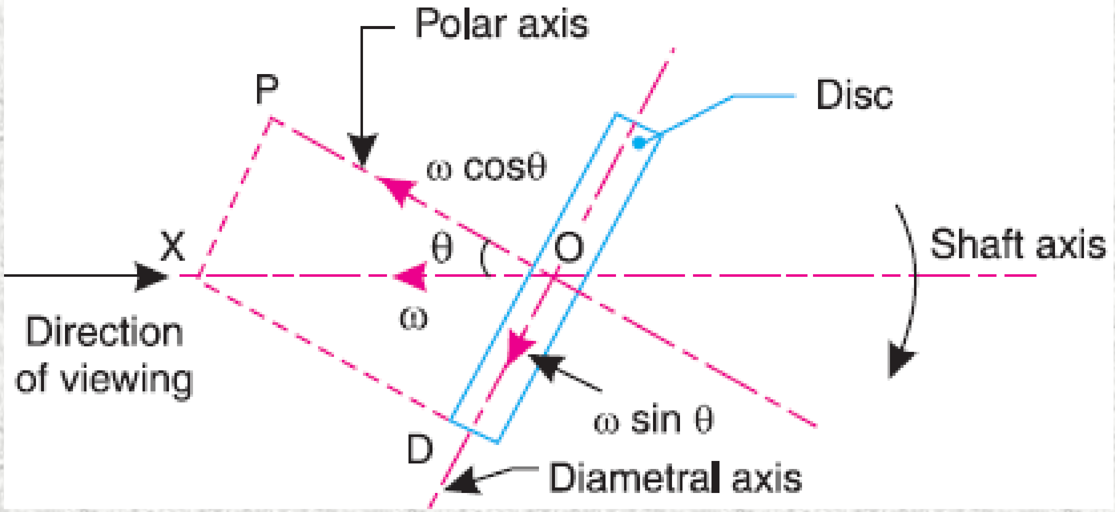
Now consider the movement of point **D** about the polar axis **OP**. In this case, **OD** is axis of spin and **OP** is the axis of precession.

Angular velocity of disc about **OD** or angular velocity of spin

$$= \omega \sin \theta$$

Angular velocity of **D** about **OP** or angular velocity of precession

$$= \omega \cos \theta$$



If I_D is the mass moment of inertia of the disc about the diametral axis **OD**, then gyroscopic couple acting on the disc,

$$C_D = I_D.\omega \sin\theta.\omega\cos\theta = \frac{1}{2}\times I_D.\omega^2\sin 2\theta$$

The effect of this couple will be opposite to that of C_P.

Resultant gyroscopic couple acting on the disc,

$$C = C_P - C_D = \frac{1}{2}\times\omega^2\sin 2\theta\,(I_P - I_D)$$

Example 3.1

A disc with a radius of gyration **60 mm** and a mass of **5 kg** is mounted centrally on a horizontal axle of **80 mm** length, between bearings. It spins about the axle at **800 rpm** anticlockwise when viewed from the right hand side bearing. The axle precesses about a vertical axis at **75 rpm** in an anticlockwise direction when viewed from above. Determine the resultant reaction at each bearing due to the mass and the gyroscopic effect.

R_L = 172.569 N, R_R = -123.519 N

Example 3.2

An airplane is flying at **200 km/h**. It turns towrads the left and completes a quarter circle of **50 m** radius. The mass of the rotary engine and propeller of the plane is **425 kg** and has a radius of gyration of **300 mm**. The engine speed is **2000 rpm** clockwise when seen from the tail. Determine the gyroscopic couple and state its effect.

8901.179 N.m, To raise the nose and dip the tail

Example 3.3

A rear engine automobile is travelling along a curve of **100 m** mean radius. Each of the four wheels has a moment of inertia **2 kg m^2** and an effective diameter of **600 mm**. The rotating parts of the engine have a moment of inertia of **1 kg m^2**. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The gear ratio of engine to the back wheel is **3:1**. The vehicle weighs **205 kg** and has its center of gravity **500 mm** above the road level. The width of the track of the vehicle is **1.5 m**. Determine the limiting speed of the vehicle around the curve all four wheels to maintain contact with road surface, if it is not cambered.

118.516 km/h

Example 3.4

Each road wheel of motor cycle has a moment of inertia of **1.8 kg m^2** and a diameter of **600 mm**. The rotating parts of the engine of the motor cycle have moment of inertia of **0.5 kg m^2**. The speed of the engine is **5** times the speed of the wheels and in the same sense. The mass of the motor cycle along with the rider is **260 kg** and its C. G. is **600 mm** above the ground level when it is standing upright with the rider. Find the angle of heel of the motor cycle if it is travelling at **65 km/h** and taking a turn of **40 m** radius.

43.2^0