Department of Mechanical Engineering,
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ENGINEERING ECONOMICS

# Effective Interest and Cash Flow

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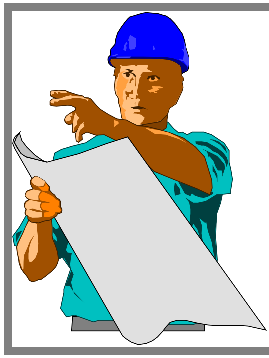
2018

Lecture 5



# Nominal Interest rate

- If a **financial institutions** uses a **unit of time other than a year**, i.e. a **quarter, a month, half-year**, then it quotes interest rate on annual basis such as **r% <u>compounded</u> monthly, quarterly, or half-yearly**

- The interest rate or Annual Percentage Rate (APR) is called the **nominal interest rate**.

# Effective Interest rate

- The effective interest rate is the one rate that **truly represents** the **interest earned in a year**.

$$i_a = (1+r/M)^M - 1$$

*where,*

$i_a$ = *effective annual interest rate*
*M =the number of compounding period per year*
*r/M = the interest rate per compounding period*

# Nominal and Effective Interest Rates with Different Compounding Periods

| | Effective Rates | | | | |
|---|---|---|---|---|---|
| Nominal Rate | Compounding Annually | Compounding Semiannually | Compounding Quarterly | Compounding Monthly | Compounding Daily |
| 4% | 4.00% | 4.04% | 4.06% | 4.07% | 4.08% |
| 5 | 5.00 | 5.06 | 5.09 | 5.12 | 5.13 |
| 6 | 6.00 | 6.09 | 6.14 | 6.17 | 6.18 |
| 7 | 7.00 | 7.12 | 7.19 | 7.23 | 7.25 |
| 8 | 8.00 | 8.16 | 8.24 | 8.30 | 8.33 |
| 9 | 9.00 | 9.20 | 9.31 | 9.38 | 9.42 |
| 10 | 10.00 | 10.25 | 10.38 | 10.47 | 10.52 |
| 11 | 11.00 | 11.30 | 11.46 | 11.57 | 11.62 |
| 12 | 12.00 | 12.36 | 12.55 | 12.68 | 12.74 |

# Effective Interest rate per payment period

- If the **transaction occurs more than one time a year** then, **effective interest rate per payment period** becomes.

$$i = (1+r/(CK))^c - 1$$

*where,*

*$i$ = effective interest rate per payment period*
*$C$ = No. of compound period per payment period*
*$K$ = No. of payment period per year*
*$r/CK$ = the interest rate per compounding period*

# Continuous compounding interest rate

- As the **number of compounding periods 'M' becomes large**, then r/M becomes too small, hence as M approaches infinity, r/M tends to '0', we comes to the situation of continuous compounding.

$$i_c = e^{r/k} - 1$$

*where,*

$i_c$ = *effective interest rate per payment period*

$r$ = *nominal interest rate*

$K$ = *No. of payment period per year*

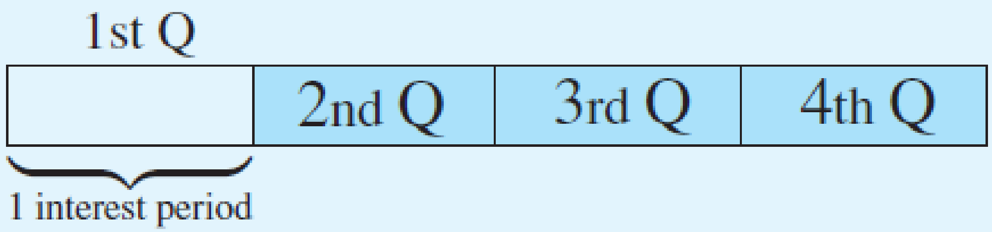
# Calculating an Effective Interest Rate with Quarterly Payment

(a) Quarterly compounding:

$r = 8\%$, $M = 4$, $C = 1$ interest period per quarter, and $K = 4$ payments per year. Then

$$i = \left(1 + \frac{0.08}{4}\right)^1 - 1 = 2.00\%.$$

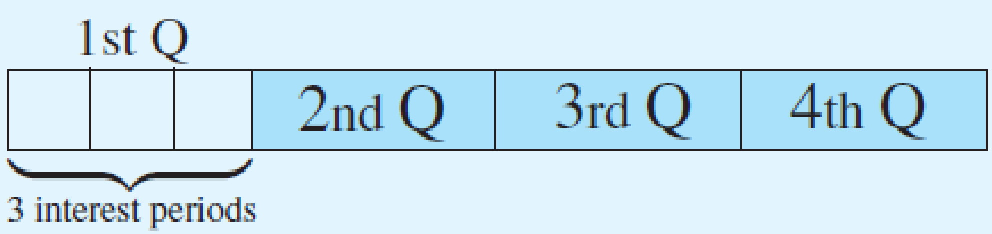
$i = (1+r/(CK))^c - 1$




(b) Monthly compounding:

$r = 8\%$, $M = 12$, $C = 3$ interest periods per quarter, and $K = 4$ payments per year. Then

$$i = \left(1 + \frac{0.08}{12}\right)^3 - 1 = 2.013\%.$$

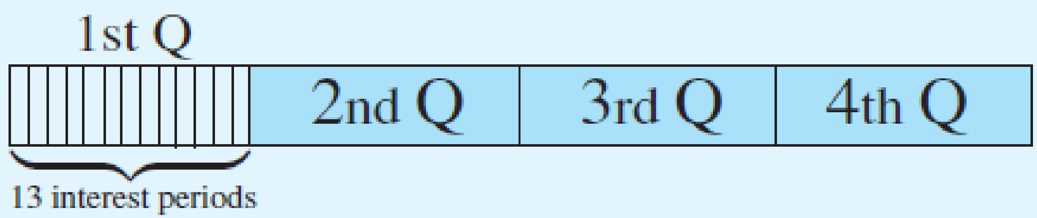


# Calculating an Effective Interest Rate with Quarterly Payment

(c) Weekly compounding:

$r = 8\%$, $M = 52$, $C = 13$ interest periods per quarter, and $K = 4$ payments per year. Then

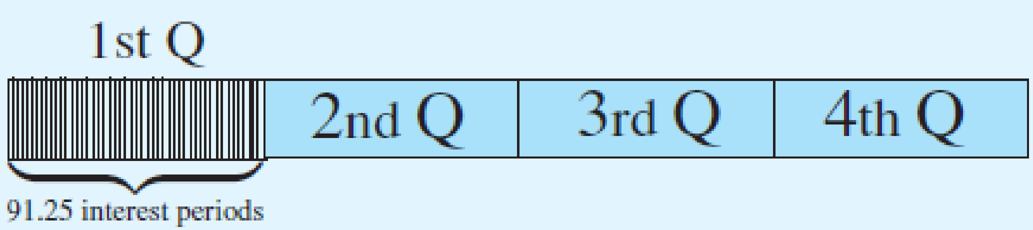
$$i = \left(1 + \frac{0.08}{52}\right)^{13} - 1 = 2.0186\%.$$




(d) Daily compounding:

$r = 8\%$, $M = 365$, $C = 91.25$ days per quarter, and $K = 4$. Then

$$i = \left(1 + \frac{0.08}{365}\right)^{91.25} - 1 = 2.0199\%.$$

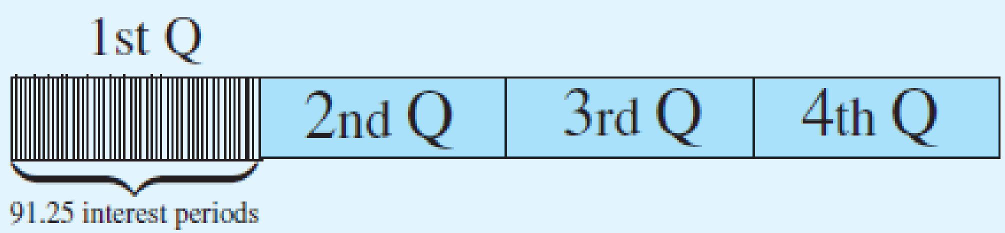


# Calculating an Effective Interest Rate with Quarterly Payment

(d) Daily compounding:

$r = 8\%$, $M = 365$, $C = 91.25$ days per quarter, and $K = 4$. Then

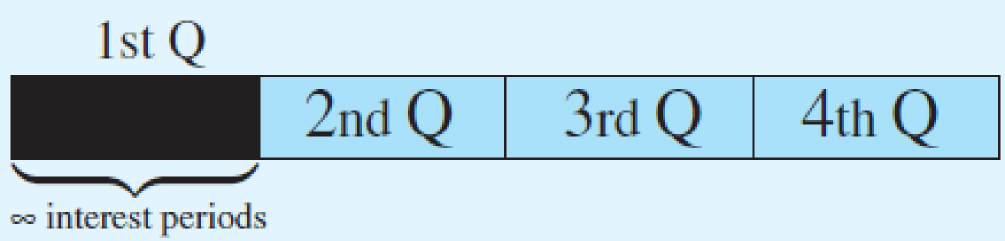
$$i = \left(1 + \frac{0.08}{365}\right)^{91.25} - 1 = 2.0199\%.$$




(e) Continuous compounding:

$r = 8\%$, $M \rightarrow \infty$, $C \rightarrow \infty$, and $K = 4$. Then, from Eq. (4.3),

$$i = e^{0.08/4} - 1 = 2.0201\%.$$



# Examples 1

Suppose you make equal quarterly deposits of Rs 1,000 into a fund that pays interest at a rate of 12% compounded monthly.

Find the balance at the end of year '1'

Quarterly Amount deposited, A = Rs 1,000

Nominal interest rate, r = 12% per year

No. of compounding period per year, M = 12

No. of payment period per year, K = 4

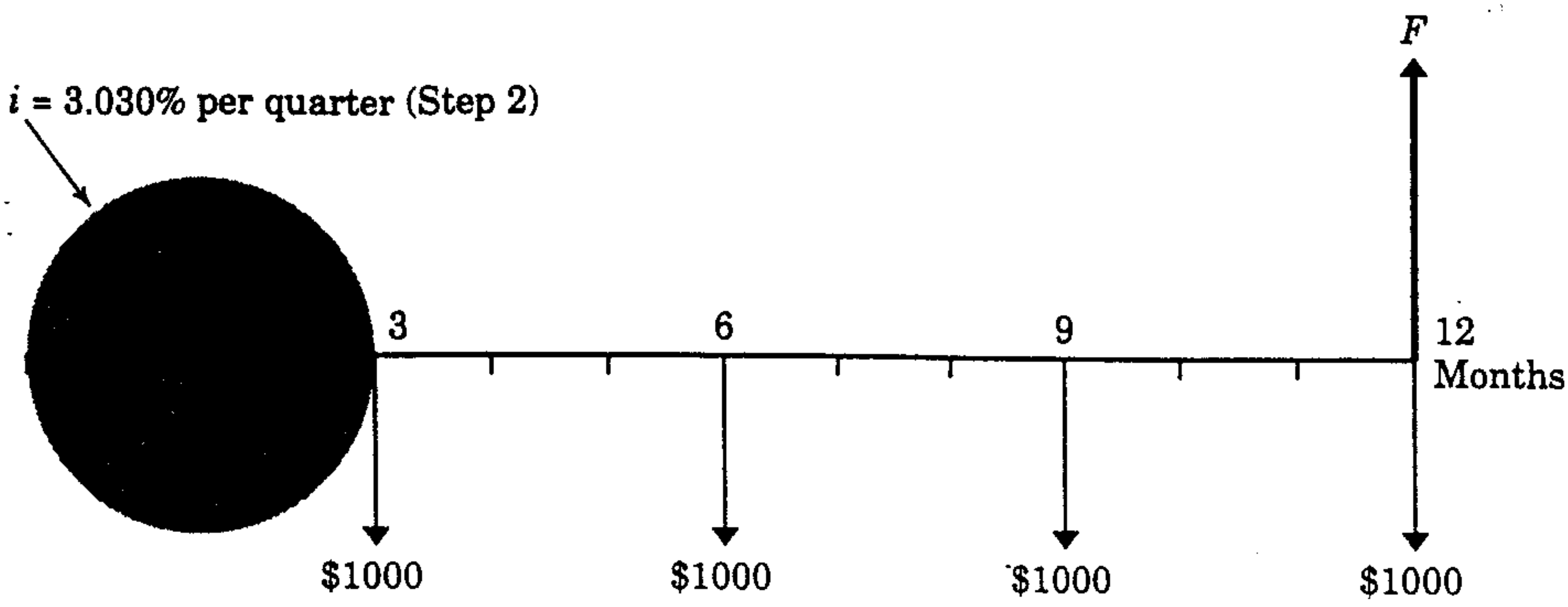
Compounding period per payment period, C = M/K = 3

Effective interest rate per payment period, $i = (1+r/CK)^{C} - 1$

No of payment period, $N = K \times 1 = 4$

$F = A \times (F/A, i, N)$

# Examples 1




F = A x (F/A, i, N)

## APPENDIX A Interest Factors for Discrete Compounding

### 3.0%

| | Single Payment | | Equal Payment Series | | | | Gradient Series | | |
|---|---|---|---|---|---|---|---|---|---|
| *N* | Compound Amount Factor *(F/P,i,N)* | Present Worth Factor *(P/F,i,N)* | Compound Amount Factor *(F/A,i,N)* | Sinking Fund Factor *(A/F,i,N)* | Present Worth Factor *(P/A,i,N)* | Capital Recovery Factor *(A/P,i,N)* | Gradient Uniform Series *(A/G,i,N)* | Gradient Present Worth *(P/G,i,N)* | *N* |
| 1 | 1.0300 | 0.9709 | 1.0000 | 1.0000 | 0.9709 | 1.0300 | 0.0000 | 0.0000 | 1 |
| 2 | 1.0609 | 0.9426 | 2.0300 | 0.4926 | 1.9135 | 0.5226 | 0.4926 | 0.9426 | 2 |
| 3 | 1.0927 | 0.9151 | 3.0909 | 0.3235 | 2.8286 | 0.3535 | 0.9803 | 2.7729 | 3 |
| 4 | 1.1255 | 0.8885 | 4.1836 | 0.2390 | 3.7171 | 0.2690 | 1.4631 | 5.4383 | 4 |
| 5 | 1.1593 | 0.8626 | 5.3091 | 0.1884 | 4.5797 | 0.2184 | 1.9409 | 8.8888 | 5 |
| 6 | 1.1941 | 0.8375 | 6.4684 | 0.1546 | 5.4172 | 0.1846 | 2.4138 | 13.0762 | 6 |
| 7 | 1.2299 | 0.8131 | 7.6625 | 0.1305 | 6.2303 | 0.1605 | 2.8819 | 17.9547 | 7 |
| 8 | 1.2668 | 0.7894 | 8.8923 | 0.1125 | 7.0197 | 0.1425 | 3.3450 | 23.4806 | 8 |
| 9 | 1.3048 | 0.7664 | 10.1591 | 0.0984 | 7.7861 | 0.1284 | 3.8032 | 29.6119 | 9 |
| 10 | 1.3439 | 0.7441 | 11.4639 | 0.0872 | 8.5302 | 0.1172 | 4.2565 | 36.3088 | 110 |
| 11 | 1.3842 | 0.7224 | 12.8078 | 0.0781 | 9.2526 | 0.1081 | 4.7049 | 43.5330 | 11 |
| 12 | 1.4258 | 0.7014 | 14.1920 | 0.0705 | 9.9540 | 0.1005 | 5.1485 | 51.2482 | 12 |
| 13 | 1.4685 | 0.6810 | 15.6178 | 0.0640 | 10.6350 | 0.0940 | 5.5872 | 59.4196 | 13 |
| 14 | 1.5126 | 0.6611 | 17.0863 | 0.0585 | 11.2961 | 0.0885 | 6.0210 | 68.0141 | 14 |
| 15 | 1.5580 | 0.6419 | 18.5989 | 0.0538 | 11.9379 | 0.0838 | 6.4500 | 77.0002 | 15 |

4.0%

| | Single Payment | | Equal Payment Series | | | | Gradient Series | | |
|---|---|---|---|---|---|---|---|---|---|
| **N** | **Compound Amount Factor (F/P,i,N)** | **Present Worth Factor (P/F,i,N)** | **Compound Amount Factor (F/A,i,N)** | **Sinking Fund Factor (A/F,i,N)** | **Present Worth Factor (P/A,i,N)** | **Capital Recovery Factor (A/P,i,N)** | **Gradient Uniform Series (A/G,i,N)** | **Gradient Present Worth (P/G,i,N)** | **N** |
| 1 | 1.0400 | 0.9615 | 1.0000 | 1.0000 | 0.9615 | 1.0400 | 0.0000 | 0.0000 | 1 |
| 2 | 1.0816 | 0.9246 | 2.0400 | 0.4902 | 1.8861 | 0.5302 | 0.4902 | 0.9246 | 2 |
| 3 | 1.1249 | 0.8890 | 3.1216 | 0.3203 | 2.7751 | 0.3603 | 0.9739 | 2.7025 | 3 |
| 4 | 1.1699 | 0.8548 | 4.2465 | 0.2355 | 3.6299 | 0.2755 | 1.4510 | 5.2670 | 4 |
| 5 | 1.2167 | 0.8219 | 5.4163 | 0.1846 | 4.4518 | 0.2246 | 1.9216 | 8.5547 | 5 |
| 6 | 1.2653 | 0.7903 | 6.6330 | 0.1508 | 5.2421 | 0.1908 | 2.3857 | 12.5062 | 6 |
| 7 | 1.3159 | 0.7599 | 7.8983 | 0.1266 | 6.0021 | 0.1666 | 2.8433 | 17.0657 | 7 |
| 8 | 1.3686 | 0.7307 | 9.2142 | 0.1085 | 6.7327 | 0.1485 | 3.2944 | 22.1806 | 8 |
| 9 | 1.4233 | 0.7026 | 10.5828 | 0.0945 | 7.4353 | 0.1345 | 3.7391 | 27.8013 | 9 |
| 10 | 1.4802 | 0.6756 | 12.0061 | 0.0833 | 8.1109 | 0.1233 | 4.1773 | 33.8814 | 10 |
| 11 | 1.5395 | 0.6496 | 13.4864 | 0.0741 | 8.7605 | 0.1141 | 4.6090 | 40.3772 | 11 |
| 12 | 1.6010 | 0.6246 | 15.0258 | 0.0666 | 9.3851 | 0.1066 | 5.0343 | 47.2477 | 12 |
| 13 | 1.6651 | 0.6006 | 16.6268 | 0.0601 | 9.9856 | 0.1001 | 5.4533 | 54.4546 | 13 |
| 14 | 1.7317 | 0.5775 | 18.2919 | 0.0547 | 10.5631 | 0.0947 | 5.8659 | 61.9618 | 14 |
| 15 | 1.8009 | 0.5553 | 20.0236 | 0.0499 | 11.1184 | 0.0899 | 6.2721 | 69.7355 | 15 |

# Interpolate for 3.03%

# Examples 2

A series of equal quarterly receipts of Rs 500 extends over a period of 5 years. What is the present value of this quarterly payment series at 8% interest compounded continuously?

Equal quarterly receipt, A = Rs 500

Nominal interest rate, r = 8%

No. of compounding period per year = continuous

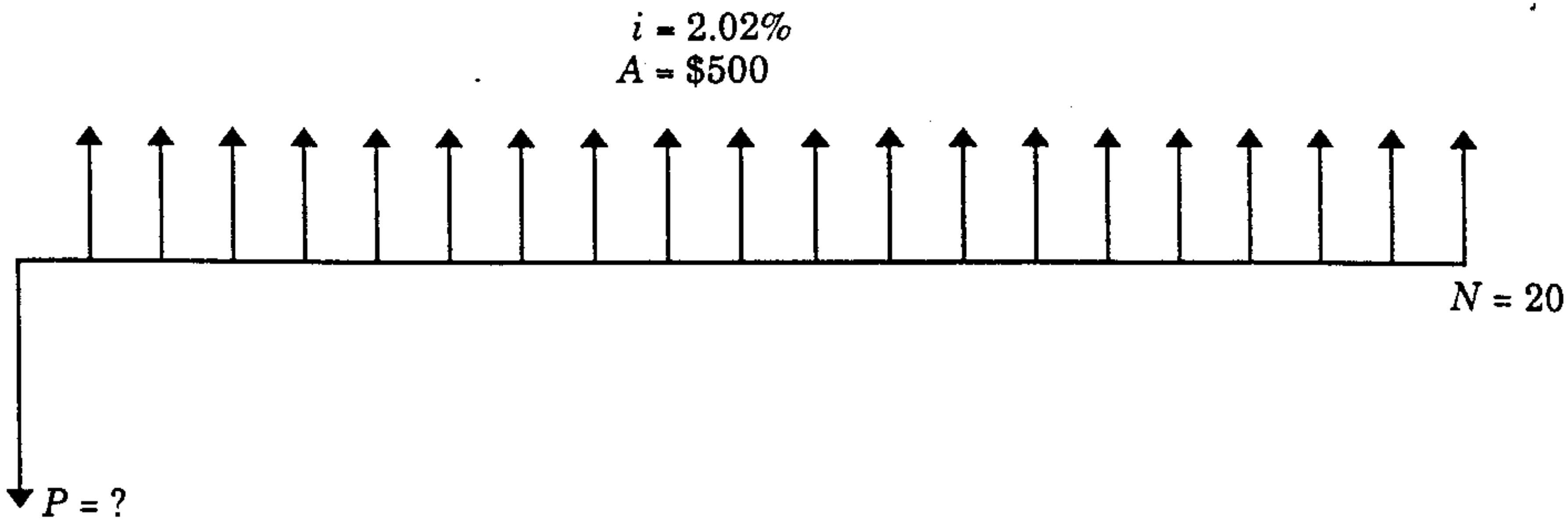
No. of receipts per year, K = 4

Effective interest rate per quarter, $\boldsymbol{I_c = e^{r/k} - 1}$

No of quarterly receipt, N = 4 qtr/yr x 5yrs = 20

$P = A(P/A, i_c, N)$

# Examples 2




$$P = A(P/A, i_c, N)$$

**2.0%**

| | Single Payment | | Equal Payment Series | | | | Gradient Series | | |
|---|---|---|---|---|---|---|---|---|---|
| N | Compound Amount Factor (F/P,i,N) | Present Worth Factor (P/F,i,N) | Compound Amount Factor (F/A,i,N) | Sinking Fund Factor (A/F,i,N) | Present Worth Factor (P/A,i,N) | Capital Recovery Factor (A/P,i,N) | Gradient Uniform Series (A/G,i,N) | Gradient Present Worth (P/G,i,N) | N |
| 1 | 1.0200 | 0.9804 | 1.0000 | 1.0000 | 0.9804 | 1.0200 | 0.0000 | 0.0000 | 1 |
| 2 | 1.0404 | 0.9612 | 2.0200 | 0.4950 | 1.9416 | 0.5150 | 0.4950 | 0.9612 | 2 |
| 3 | 1.0612 | 0.9423 | 3.0604 | 0.3268 | 2.8839 | 0.3468 | 0.9868 | 2.8458 | 3 |
| 4 | 1.0824 | 0.9238 | 4.1216 | 0.2426 | 3.8077 | 0.2626 | 1.4752 | 5.6173 | 4 |
| 5 | 1.1041 | 0.9057 | 5.2040 | 0.1922 | 4.7135 | 0.2122 | 1.9604 | 9.2403 | 5 |
| 6 | 1.1262 | 0.8880 | 6.3081 | 0.1585 | 5.6014 | 0.1785 | 2.4423 | 13.6801 | 6 |
| 7 | 1.1487 | 0.8706 | 7.4343 | 0.1345 | 6.4720 | 0.1545 | 2.9208 | 18.9035 | 7 |
| 8 | 1.1717 | 0.8535 | 8.5830 | 0.1165 | 7.3255 | 0.1365 | 3.3961 | 24.8779 | 8 |
| 9 | 1.1951 | 0.8368 | 9.7546 | 0.1025 | 8.1622 | 0.1225 | 3.8681 | 31.5720 | 9 |
| 10 | 1.2190 | 0.8203 | 10.9497 | 0.0913 | 8.9826 | 0.1113 | 4.3367 | 38.9551 | 10 |
| 11 | 1.2434 | 0.8043 | 12.1687 | 0.0822 | 9.7868 | 0.1022 | 4.8021 | 46.9977 | 11 |
| 12 | 1.2682 | 0.7885 | 13.4121 | 0.0746 | 10.5753 | 0.0946 | 5.2642 | 55.6712 | 12 |
| 13 | 1.2936 | 0.7730 | 14.6803 | 0.0681 | 11.3484 | 0.0881 | 5.7231 | 64.9475 | 13 |
| 14 | 1.3195 | 0.7579 | 15.9739 | 0.0626 | 12.1062 | 0.0826 | 6.1786 | 74.7999 | 14 |
| 15 | 1.3459 | 0.7430 | 17.2934 | 0.0578 | 12.8493 | 0.0778 | 6.6309 | 85.2021 | 15 |
| 16 | 1.3728 | 0.7284 | 18.6393 | 0.0537 | 13.5777 | 0.0737 | 7.0799 | 96.1288 | 16 |
| 17 | 1.4002 | 0.7142 | 20.0121 | 0.0500 | 14.2919 | 0.0700 | 7.5256 | 107.5554 | 17 |
| 18 | 1.4282 | 0.7002 | 21.4123 | 0.0467 | 14.9920 | 0.0667 | 7.9681 | 119.4581 | 18 |
| 19 | 1.4568 | 0.6864 | 22.8406 | 0.0438 | 15.6785 | 0.0638 | 8.4073 | 131.8139 | 19 |
| 20 | 1.4859 | 0.6730 | 24.2974 | 0.0412 | 16.3514 | 0.0612 | 8.8433 | 144.6003 | 20 |
| 21 | 1.5157 | 0.6598 | 25.7833 | 0.0388 | 17.0112 | 0.0588 | 9.2760 | 157.7959 | 21 |

# APPENDIX A Interest Factors for Discrete Compounding

## 3.0%

| | Single Payment | | Equal Payment Series | | | | Gradient Series | | |
|---|---|---|---|---|---|---|---|---|---|
| | Compound Amount Factor | Present Worth Factor | Compound Amount Factor | Sinking Fund Factor | Present Worth Factor | Capital Recovery Factor | Gradient Uniform Series | Gradient Present Worth | |
| *N* | *(F/P,i,N)* | *(P/F,i,N)* | *(F/A,i,N)* | *(A/F,i,N)* | *(P/A,i,N)* | *(A/P,i,N)* | *(A/G,i,N)* | *(P/G,i,N)* | *N* |
| 1 | 1.0300 | 0.9709 | 1.0000 | 1.0000 | 0.9709 | 1.0300 | 0.0000 | 0.0000 | 1 |
| 2 | 1.0609 | 0.9426 | 2.0300 | 0.4926 | 1.9135 | 0.5226 | 0.4926 | 0.9426 | 2 |
| 3 | 1.0927 | 0.9151 | 3.0909 | 0.3235 | 2.8286 | 0.3535 | 0.9803 | 2.7729 | 3 |
| 4 | 1.1255 | 0.8885 | 4.1836 | 0.2390 | 3.7171 | 0.2690 | 1.4631 | 5.4383 | 4 |
| 5 | 1.1593 | 0.8626 | 5.3091 | 0.1884 | 4.5797 | 0.2184 | 1.9409 | 8.8888 | 5 |
| 6 | 1.1941 | 0.8375 | 6.4684 | 0.1546 | 5.4172 | 0.1846 | 2.4138 | 13.0762 | 6 |
| 7 | 1.2299 | 0.8131 | 7.6625 | 0.1305 | 6.2303 | 0.1605 | 2.8819 | 17.9547 | 7 |
| 8 | 1.2668 | 0.7894 | 8.8923 | 0.1125 | 7.0197 | 0.1425 | 3.3450 | 23.4806 | 8 |
| 9 | 1.3048 | 0.7664 | 10.1591 | 0.0984 | 7.7861 | 0.1284 | 3.8032 | 29.6119 | 9 |
| 10 | 1.3439 | 0.7441 | 11.4639 | 0.0872 | 8.5302 | 0.1172 | 4.2565 | 36.3088 | 110 |
| 11 | 1.3842 | 0.7224 | 12.8078 | 0.0781 | 9.2526 | 0.1081 | 4.7049 | 43.5330 | 11 |
| 12 | 1.4258 | 0.7014 | 14.1920 | 0.0705 | 9.9540 | 0.1005 | 5.1485 | 51.2482 | 12 |
| 13 | 1.4685 | 0.6810 | 15.6178 | 0.0640 | 10.6350 | 0.0940 | 5.5872 | 59.4196 | 13 |
| 14 | 1.5126 | 0.6611 | 17.0863 | 0.0585 | 11.2961 | 0.0885 | 6.0210 | 68.0141 | 14 |
| 15 | 1.5580 | 0.6419 | 18.5989 | 0.0538 | 11.9379 | 0.0838 | 6.4500 | 77.0002 | 15 |
| 16 | 1.6047 | 0.6232 | 20.1569 | 0.0496 | 12.5611 | 0.0796 | 6.8742 | 86.3477 | 16 |
| 17 | 1.6528 | 0.6050 | 21.7616 | 0.0460 | 13.1661 | 0.0760 | 7.2936 | 96.0280 | 17 |
| 18 | 1.7024 | 0.5874 | 23.4144 | 0.0427 | 13.7535 | 0.0727 | 7.7081 | 106.0137 | 18 |
| 19 | 1.7535 | 0.5703 | 25.1169 | 0.0398 | 14.3238 | 0.0698 | 8.1179 | 116.2788 | 19 |
| 20 | 1.8061 | 0.5537 | 26.8704 | 0.0372 | 14.8775 | 0.0672 | 8.5229 | 126.7987 | 20 |
| 21 | 1.8603 | 0.5375 | 28.6765 | 0.0349 | 15.4150 | 0.0649 | 8.9231 | 137.5496 | 21 |
| 22 | 1.9161 | 0.5219 | 30.5368 | 0.0327 | 15.9396 | 0.0627 | 9.3186 | 148.5094 | 22 |

# Examples 3

Suppose you make Rs 1,000 monthly deposit to a registered retirement savings plan that pays interest at a rate of 10% compounded quarterly. Compute the balance at the end of 8 years

Monthly Amount deposited, A = Rs 1,000

Nominal interest rate, r = 10%

No. of compounding period per year, M = 4

No. of payment period per year, K = 12

Compounding period per payment period, C = M/K = 1/3

Effective interest rate per payment period, $i = (1+r/CK)^{C} - 1$

No of payment period, $N = K \times 8 = 96$

$F = A \times (F/A, i, N)$

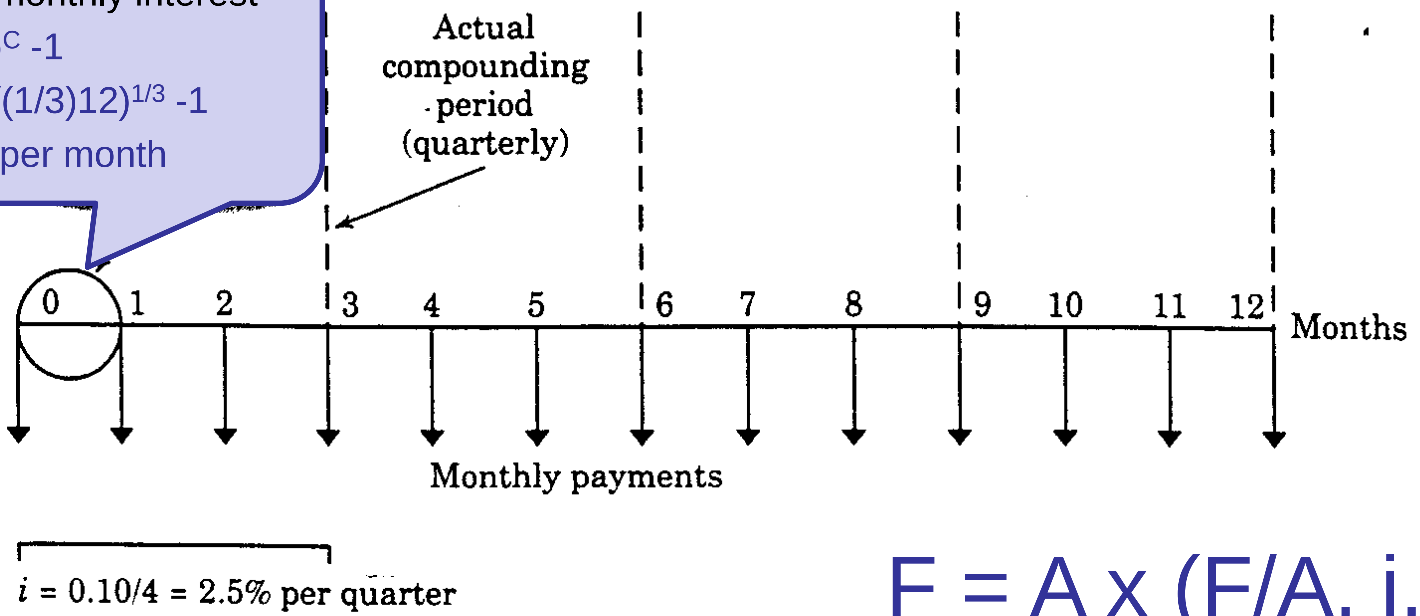
# Examples 3

Equivalent monthly interest

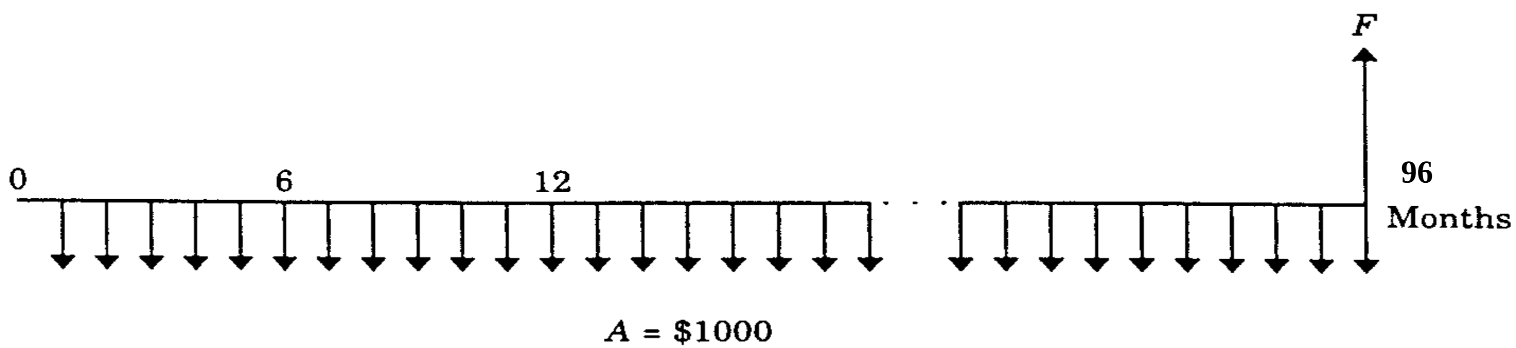
$i = (1+r/CK)^C - 1$

$= (1+0.10/(1/3)12)^{1/3} - 1$

$= 0.826\%$ per month




$$F = A \times (F/A, i, N)$$



## 0.75%

| | Single Payment | | Equal Payment Series | | | | Gradient Series | | |
|---|---|---|---|---|---|---|---|---|---|
| **N** | **Compound Amount Factor (F/P,i,N)** | **Present Worth Factor (P/F,i,N)** | **Compound Amount Factor (F/A,i,N)** | **Sinking Fund Factor (A/F,i,N)** | **Present Worth Factor (P/A,i,N)** | **Capital Recovery Factor (A/P,i,N)** | **Gradient Uniform Series (A/G,i,N)** | **Gradient Present Worth (P/G,i,N)** | **N** |
| 1 | 1.0075 | 0.9926 | 1.0000 | 1.0000 | 0.9926 | 1.0075 | 0.0000 | 0.0000 | 1 |
| 2 | 1.0151 | 0.9852 | 2.0075 | 0.4981 | 1.9777 | 0.5056 | 0.4981 | 0.9852 | 2 |
| 3 | 1.0227 | 0.9778 | 3.0226 | 0.3308 | 2.9556 | 0.3383 | 0.9950 | 2.9408 | 3 |
| 4 | 1.0303 | 0.9706 | 4.0452 | 0.2472 | 3.9261 | 0.2547 | 1.4907 | 5.8525 | 4 |
| 5 | 1.0381 | 0.9633 | 5.0756 | 0.1970 | 4.8894 | 0.2045 | 1.9851 | 9.7058 | 5 |
| 6 | 1.0459 | 0.9562 | 6.1136 | 0.1636 | 5.8456 | 0.1711 | 2.4782 | 14.4866 | 6 |
| 7 | 1.0537 | 0.9490 | 7.1595 | 0.1397 | 6.7946 | 0.1472 | 2.9701 | 20.1808 | 7 |
| 8 | 1.0616 | 0.9420 | 8.2132 | 0.1218 | 7.7366 | 0.1293 | 3.4608 | 26.7747 | 8 |
| 9 | 1.0696 | 0.9350 | 9.2748 | 0.1078 | 8.6716 | 0.1153 | 3.9502 | 34.2544 | 9 |
| 10 | 1.0776 | 0.9280 | 10.3443 | 0.0967 | 9.5996 | 0.1042 | 4.4384 | 42.6064 | 10 |
| 72 | 1.7126 | 0.5839 | 95.0070 | 0.0105 | 55.4768 | 0.0180 | 32.2882 | 1791.2463 | 72 |
| 80 | 1.8180 | 0.5500 | 109.0725 | 0.0092 | 59.9944 | 0.0167 | 35.5391 | 2132.1472 | 80 |
| 84 | 1.8732 | 0.5338 | 116.4269 | 0.0086 | 62.1540 | 0.0161 | 37.1357 | 2308.1283 | 84 |
| 90 | 1.9591 | 0.5104 | 127.8790 | 0.0078 | 65.2746 | 0.0153 | 39.4946 | 2577.9961 | 90 |
| 96 | 2.0489 | 0.4881 | 139.8562 | 0.0072 | 68.2584 | 0.0147 | 41.8107 | 2853.9352 | 96 |
| 100 | 2.1111 | 0.4737 | 148.1445 | 0.0068 | 70.1746 | 0.0143 | 43.3311 | 3040.7453 | 100 |
| 108 | 2.2411 | 0.4462 | 165.4832 | 0.0060 | 73.8394 | 0.0135 | 46.3154 | 3419.9041 | 108 |
| 120 | 2.4514 | 0.4079 | 193.5143 | 0.0052 | 78.9417 | 0.0127 | 50.6521 | 3998.5621 | 120 |
| 240 | 6.0092 | 0.1664 | 667.8869 | 0.0015 | 111.1450 | 0.0090 | 85.4210 | 9494.1162 | 240 |
| 360 | 14.7306 | 0.0679 | 1830.7435 | 0.0005 | 124.2819 | 0.0080 | 107.1145 | 13312.3871 | 360 |

1.0%

| | Single Payment | | Equal Payment Series | | | | Gradient Series | | |
|---|---|---|---|---|---|---|---|---|---|
| N | Compound Amount Factor (F/P,i,N) | Present Worth Factor (P/F,i,N) | Compound Amount Factor (F/A,i,N) | Sinking Fund Factor (A/F,i,N) | Present Worth Factor (P/A,i,N) | Capital Recovery Factor (A/P,i,N) | Gradient Uniform Series (A/G,i,N) | Gradient Present Worth (P/G,i,N) | N |
| 1 | 1.0100 | 0.9901 | 1.0000 | 1.0000 | 0.9901 | 1.0100 | 0.0000 | 0.0000 | 1 |
| 2 | 1.0201 | 0.9803 | 2.0100 | 0.4975 | 1.9704 | 0.5075 | 0.4975 | 0.9803 | 2 |
| 3 | 1.0303 | 0.9706 | 3.0301 | 0.3300 | 2.9410 | 0.3400 | 0.9934 | 2.9215 | 3 |
| 4 | 1.0406 | 0.9610 | 4.0604 | 0.2463 | 3.9020 | 0.2563 | 1.4876 | 5.8044 | 4 |
| 5 | 1.0510 | 0.9515 | 5.1010 | 0.1960 | 4.8534 | 0.2060 | 1.9801 | 9.6103 | 5 |
| 6 | 1.0615 | 0.9420 | 6.1520 | 0.1625 | 5.7955 | 0.1725 | 2.4710 | 14.3205 | 6 |
| 7 | 1.0721 | 0.9327 | 7.2135 | 0.1386 | 6.7282 | 0.1486 | 2.9602 | 19.9168 | 7 |
| 8 | 1.0829 | 0.9235 | 8.2857 | 0.1207 | 7.6517 | 0.1307 | 3.4478 | 26.3812 | 8 |
| 9 | 1.0937 | 0.9143 | 9.3685 | 0.1067 | 8.5660 | 0.1167 | 3.9337 | 33.6959 | 9 |
| 10 | 1.1046 | 0.9053 | 10.4622 | 0.0956 | 9.4713 | 0.1056 | 4.4179 | 41.8435 | 10 |
| 72 | 2.0471 | 0.4885 | 104.7099 | 0.0096 | 51.1504 | 0.0196 | 31.2386 | 1597.8673 | 72 |
| 80 | 2.2167 | 0.4511 | 121.6715 | 0.0082 | 54.8882 | 0.0182 | 34.2492 | 1879.8771 | 80 |
| 84 | 2.3067 | 0.4335 | 130.6723 | 0.0077 | 56.6485 | 0.0177 | 35.7170 | 2023.3153 | 84 |
| 90 | 2.4486 | 0.4084 | 144.8633 | 0.0069 | 59.1609 | 0.0169 | 37.8724 | 2240.5675 | 90 |
| 96 | 2.5993 | 0.3847 | 159.9273 | 0.0063 | 61.5277 | 0.0163 | 39.9727 | 2459.4298 | 96 |
| 100 | 2.7048 | 0.3697 | 170.4814 | 0.0059 | 63.0289 | 0.0159 | 41.3426 | 2605.7758 | 100 |
| 108 | 2.9289 | 0.3414 | 192.8926 | 0.0052 | 65.8578 | 0.0152 | 44.0103 | 2898.4203 | 108 |
| 120 | 3.3004 | 0.3030 | 230.0387 | 0.0043 | 69.7005 | 0.0143 | 47.8349 | 3334.1148 | 120 |
| 240 | 10.8926 | 0.0918 | 989.2554 | 0.0010 | 90.8194 | 0.0110 | 75.7393 | 6878.6016 | 240 |
| 360 | 35.9496 | 0.0278 | 3494.9641 | 0.0003 | 97.2183 | 0.0103 | 89.6995 | 8720.4323 | 360 |

# Cash Flow Example 1

If you win a lottery you have the choice of either accepting

- Rs I million now or
- taking the 20 annual installments (Rs 100,000 each year, a total of Rs 2 million).

A local bank offers an annual interest rate of 10%

**Which option would be more desirable?**

# Cash Flow Example 2

To help you reach Rs 5,000 goal in 5 years from now,

your father 'offers to give you Rs 500 now.

You plan to get a part-time job and make additional deposits at the end of each year. ( The first deposit is made at the end of first year.).

If all your money is deposited in a bank that pays 7% interest,

**how large must your annual deposits be?**

# Cash Flow Example 3

A small firm has borrowed Rs 100,000 to purchase certain equipment.

The loan carries an interest rate of 15% and is to be repaid in equal installments over the next 5 years.

**Compute the amount of this annual installment.**

# Cash Flow Example 4

Ram and Sita just opened two savings accounts at a bank.

The accounts earn 10% annual interest.

Ram wants to deposit Rs 1000 in his account at the end of first year and increase this amount by Rs 300 for each of the following 5 years.

Sita wants to deposit an equal amount each year for next 6 years.

**What should be the size of Sita's annual deposit** so that two accounts would have equal balances at the end of 6 years?

# Cash Flow Example 5

A self-employed individual, Krishna, is opening a retirement account at a bank.

His goal is to accumulate Rs 1,000,000 in the account by the time he retires from work in 20 years.

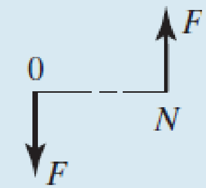
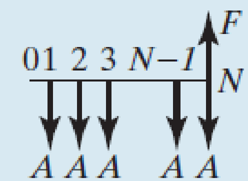
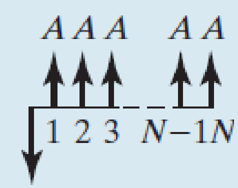
A local bank is willing to open such a retirement account that pays 8% interest compounded annually, throughout the 20 years.

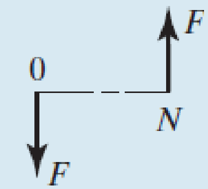
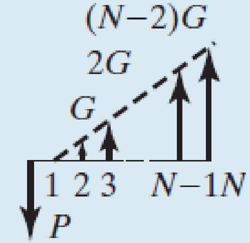
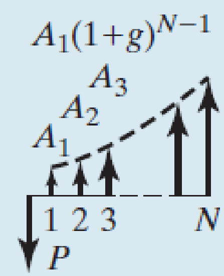
Krishna expects his annual income will increase at a 6% annual rate during his working career.

He wishes to start with a deposit at the end of year '1' of A, and increase the deposit at a rate of 6% each year thereafter.

**What should be the size of his first deposit A?**

The first deposit will occur at the end of year ‘1', and the subsequent deposits will he made at the end of each year. The last deposit will be made at the end of year'20'.

| Flow Type | Factor Notation | Formula | Excel Command | Cash Flow Diagram |
|---|---|---|---|---|
| SINGLE | Compound amount $(F/P, i, N)$ | $F = P(1+i)^N$ | $=\mathrm{FV}(i, N, P,, 0)$ |  |
| | Present worth $(P/F, i, N)$ | $P = F(1+i)^{-N}$ | $=\mathrm{PV}(i, N, F,, 0)$ | |
| EQUAL PAYMENT SERIES | Compound amount $(F/A, i, N)$ | $F = A\left[\frac{(1+i)^N - 1}{i}\right]$ | $=\mathrm{PV}(i, N, A,, 0)$ |  |
| | Sinking fund $(A/F, i, N)$ | $A = F\left[\frac{i}{(1+i)^N - 1}\right]$ | $=\mathrm{PMT}(i, N, P, F, 0)$ | |
| | Present worth $(P/A, i, N)$ | $P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$ | $=\mathrm{PV}(i, N, A,, 0)$ |  |
| | Capital recovery $(A/P, i, N)$ | $A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$ | $=\mathrm{PMT}(i, N,, P)$ | |

| Flow Type | Factor Notation | Formula | Excel Command | Cash Flow Diagram |
|---|---|---|---|---|
| SINGLE | Compound amount $(F/P, i, N)$ | $F = P(1+i)^N$ | =FV($i$, $N$, $P$,, 0) |  |
| | Present worth $(P/F, i, N)$ | $P = F(1+i)^{-N}$ | =PV($i$, $N$, $F$,, 0) | |
| GRADIENT | Linear gradient | | | |
| | Present worth $(P/G, i, N)$ | $P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right]$ | |  |
| | Conversion factor $(A/G, i, N)$ | $A = G\left[\frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]}\right]$ | | |
| SERIES | Geometric gradient | | | |
| | Present worth $(P/A_1, g, i, N)$ | $P = \left[\begin{array}{l} A_1\left[\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\frac{N}{1+i}\right)(\text{if } i = g) \end{array}\right.$ | |  |

# Assignment L5

- Problems from Chan S Park, Contemporary Engineering Economics, Fourth Edition Book
  - 4.1, 4.2,4.3, 4.9, 4.14, 4.16, 4.26, 4.28, 4.31, 4.51,