THEORY OF MACHINE AND MECHANISM II

TUTORIAL NO: 6(C)

VIBRATION OF SINGLE DEGREE OF FREEDOM SYSTEMS

(Forced Harmonic Vibration)

1. Consider a spring-mass-damper system with k = 4000 N/m, m = 10 kg, and C = 40 N.s/m. Find the steady-state and total response of the system under the harmonic force F(t) = 200 cos 10t N and the initial conditions $x_0 = 0.1$ m, and $\dot{x}_0 = 0$.

2. A vibrating system having a mass 1 kg is suspended by a spring of stiffness 1000 N/m and it is put to harmonic excitation of 10 N. Assuming viscous damping, determine:
(a) damped frequency
(b) the resonant frequency,
(c) the phase angle at resonance,
(d) the amplitude at resonance,
(e) the frequency corresponding to the peak amplitude, and
(f) the peak amplitude. Take C = 40 Ns/m

3. When a 50 kg machine, placed on an undamped isolator, is subjected to a harmonic excitation at 125 Hz, its steady state amplitude is observed as 1.8 mm. When the machine is attached to two of these isolators in series and subjected to the same excitation, its steady state amplitude is 1.2 mm. What is the stiffness of one of these isolators?

4. A 45 kg machine is placed at the end of a 1.6 m cantilever beam of elastic modulus of 200 GPa and cross sectional moment of inertia 1.6×10^{-5} m^4. As it operates, the machine produces a harmonic force of magnitude 125 N. At what operating speeds will the machine's steady state amplitude be less than 0.2 mm?

5. A 65 kg industrial sewing machine has a rotating unbalance of 0.15 kg m. The machine operates at 125 Hz and is mounted on a foundation of equivalent stiffness 2 MN/m and damping ratio 0.12. What is the machine's steady state amplitude?

6. An electric motor, of mass 60 kg, rated speed 3000 rpm, and an unbalance 0.002 kgm, is to be mounted on an isolator to achieve a force transmissibility of less than 0.25. Determine:
(a) the stiffness of the isolator,
(b) the dynamic amplitude of the motor, and
(c) the force transmitted to the foundation.

7. A machine having a mass of 100 kg and supported on springs of total stiffness 7.84×10^5 N/m has an unbalanced rotating element which results in a disturbing force of 392 N at a speed of 3000 rpm. Assuming a damping factor of $\xi = 0.20$, determine
(a) the amplitude of motion due to unbalance,
(b) the transmissibility ratio, and
(c) the transmitted force.

8. A vibrometer having the amplitude of vibration of the machine part as 4 mm and $\xi = 0.20$, performs harmonic motion. If the difference between the maximum and minimum recorded

value is 10 mm, determine the natural frequency of vibrometer if the frequency of the vibration part is 12 rad/s.

9. Determine the power required to vibrate a spring mass system with an amplitude of 15 cm and at a frequency of 100 Hz. The system has a damping ratio 0.05 and a damped natural frequency of 22 Hz as found out from the vibration record. The mass of the system is 0.5 kg.

10. A rotor of mass 12 kg is mounted in the middle of 25 mm diameter shaft supported between two bearings placed at 900 mm from each other. The rotor is having 0.02 mm eccentricity. If the system rotates at 3000 rpm, determine the amplitude of steady state vibrations and the dynamic force on the bearings. Take $E = 2 \times 10^5$ N/mm^2.

ANSWERS

1. $[0.01 \sin 19.8997\, t + 0.1 \cos 19.8997\, t]\, e^{-2t} + 0.0667 \sin (10t - 0.1326)$
2. 24.495 rad/s, 31.623 rad/s, 90^0, 7.906 mm, 14.142 rad/s, 10.206 mm
3. 15.42 kN/m
4. 195.434rad/s < ϖ < 256.851 rad/s
5. 2.425 mm
6. 1.1844 MN/m, 0.04167 mm, 49.348 N
7. 0.04283 mm, 0.1487, 58.291 N
8. 5.6614 rad/s
9. 307.35 W
10. 0.02542 mm, 3.209 N