THEORY OF MACHINES AND MECHANISMS II

Mechanical IV/I

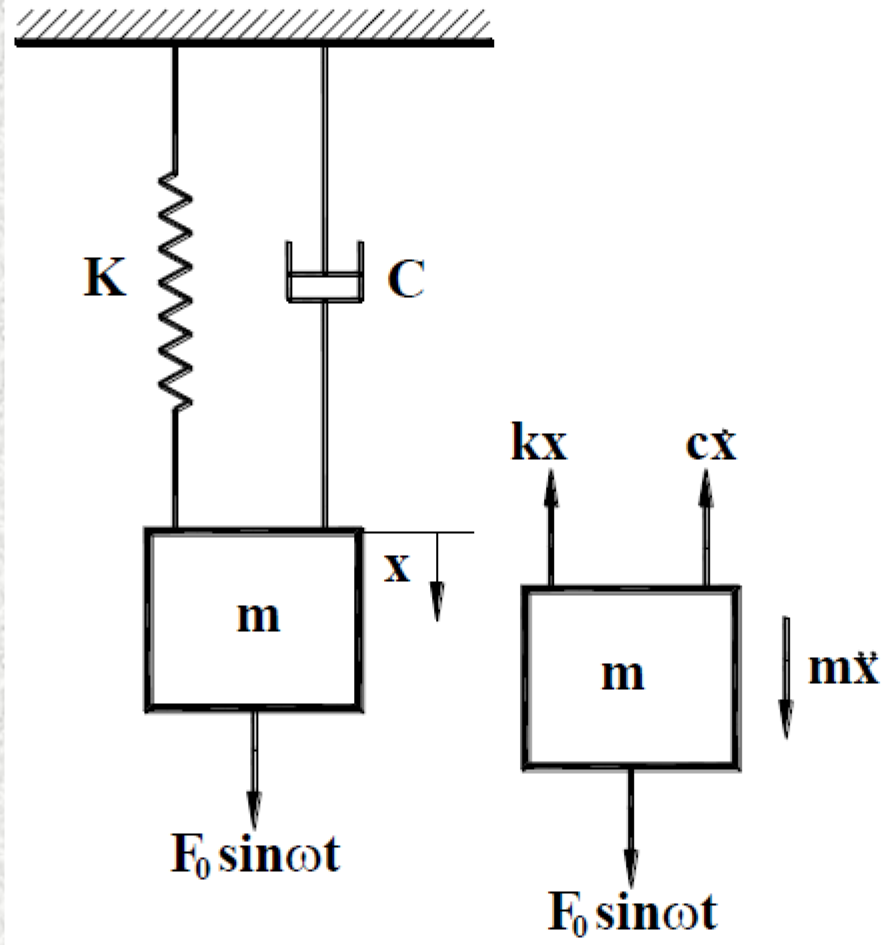
Chapter 6

Vibration of Single Degree of Freedom Systems (Forced)

6.4 Forced Harmonic Vibrations of SDOF Systems

The equation of motion for the damped forced vibration is given as

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t \qquad \text{..........} (1)$$



The solution to Equation (1) consists of two parts, the complementary function, which is the solution of the homogeneous equation, and the particular integral.

$$x = x_c + x_p \qquad \text{..........} (2)$$

The complementary function part is the solution when no input is applied. This gives the natural behavior of the system.

Hence the complementary function, in this case, is a damped free vibration discussed earlier.

$$x_c = A_1 \sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi'\right) e^{-\xi\omega_n t} \quad \text{..........} \ (3)$$

The particular integral part is the solution due to imposed input. The particular solution to the above equation is a steady state oscillation of the same frequency ω as that of the excitation. We can assume the particular solution of the form

$$x_p = X \sin(\omega t - \phi) \quad \text{..........} \ (4)$$

where X is the amplitude of oscillation and ϕ is the phase of the displacement with respect to the exciting force.

The amplitude and phase in the above equation are found by substituting Equation (4) into the Equation (1).

Differentiating Equation (4) twice, we get

$$\dot{x}_p = \omega X \cos(\omega t - \phi) = \omega X \sin(\omega t - \phi + \pi/2) \quad \text{..........} (5)$$

$$\ddot{x}_p = -\omega^2 X \sin(\omega t - \phi) = \omega^2 X \sin(\omega t - \phi + \pi) \quad \text{..........} (6)$$

Substituting Equations (4), (5), and (6) into Equation (1),

$$m\omega^2 X \sin(\omega t - \phi + \pi) + c\omega X \sin(\omega t - \phi + \pi/2) + k X \sin(\omega t - \phi) = F_0 \sin\omega t \quad \text{..........} (7)$$

The above Equation can be rewritten as

Impressed Force

$$m\omega^2 X \sin(\omega t - \phi + \pi) + c\omega X \sin(\omega t - \phi + \pi/2) + k X \sin(\omega t - \phi) - F_0 \sin\omega t = 0 \quad \text{..........} (8)$$

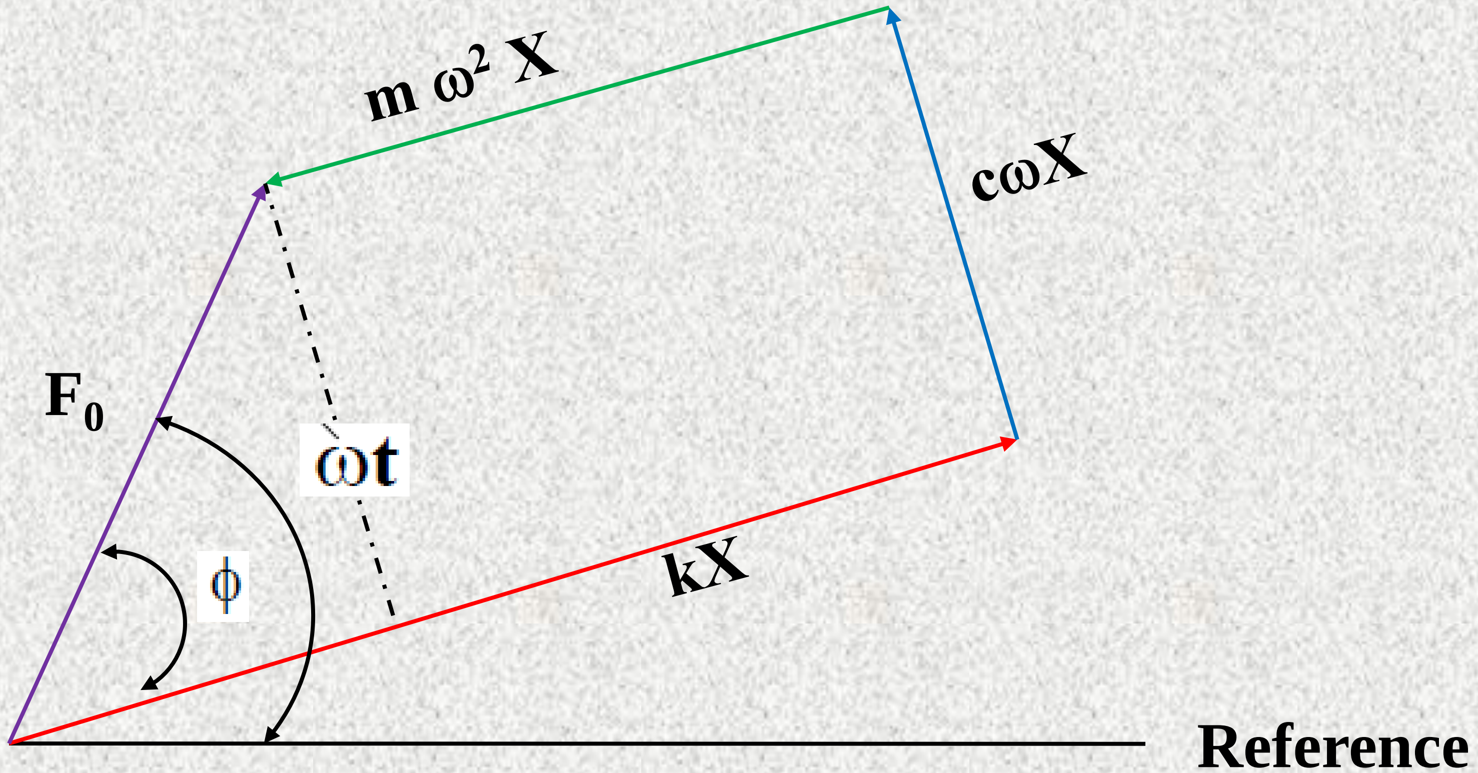
Inertia Force | Damping Force | Spring Force

The vector diagram coming out from the Equation (8) is drawn. Damping force is perpendicular to the spring force and inertia force is perpendicular to damping force.

Impressed Force

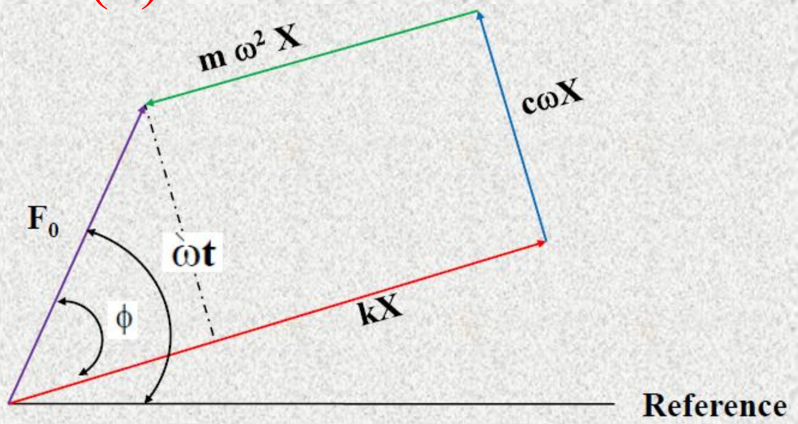
$$m\omega^2 X \sin(\omega t - \phi + \pi) + c\omega X \sin(\omega t - \phi + \pi/2) + k X \sin(\omega t - \phi) - F_0 \sin\omega t = 0$$

Inertia Force | Damping Force | Spring Force ……….. (8)



$$F_0^2 = (kX - m\omega^2 X)^2 + (c\omega X)^2$$
$$= [(k - m\omega^2)^2 + (c\omega)^2]\, X^2$$

$$\therefore \quad X = \frac{F_0}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}} \quad \text{.......... (9)}$$



and

$$\tan\phi = \frac{c\omega}{k - m\omega^2}$$

$$\phi = \tan^{-1}\frac{c\omega}{k - m\omega^2} \quad \text{.......... (10)}$$

Dividing the numerator and denominator of Equations (9) and (10) by *k*, we obtain

$$X = \frac{\frac{F_0}{k}}{\sqrt{\left(1 - \frac{m\omega^2}{k}\right)^2 + \left(\frac{c\omega}{k}\right)^2}} \quad \text{.... (11)} \qquad \text{and} \qquad \phi = \tan^{-1}\frac{\frac{c\omega}{k}}{1 - \frac{m\omega^2}{k}} \quad \text{.... (12)}$$

The above Equations may be further expressed in terms of the following quantities:

$$\omega_n = \sqrt{\frac{k}{m}}$$ = natural frequency of undamped oscillation

$$c_c = 2m\omega_n$$ = critical damping

$$\xi = \frac{c}{c_c}$$ = damping ratio

$$\frac{c\omega}{k} = \frac{c}{c_c} \cdot \frac{c_c\omega}{k} = 2\xi\frac{\omega}{\omega_n}$$

The nondimensional expressions for the amplitude and phase then become

$$\frac{Xk}{F_0} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \quad . (13)$$

and

$$\phi = \tan^{-1}\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2} \quad . (14)$$

So the particular solution of the Equation (1) can be written as, i.e., Equation (4)

$$x_p = \frac{F_0}{k} \cdot \frac{\sin(\omega t - \phi)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \quad \text{..........} (15)$$

Hence, the complete solution is given by substituting x_c and x_p into Equation (2) as

$$x = A_1 \sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi'\right) e^{-\xi\omega_n t} + \frac{F_0}{k} \cdot \frac{\sin(\omega t - \phi)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \quad \text{..........} (16)$$

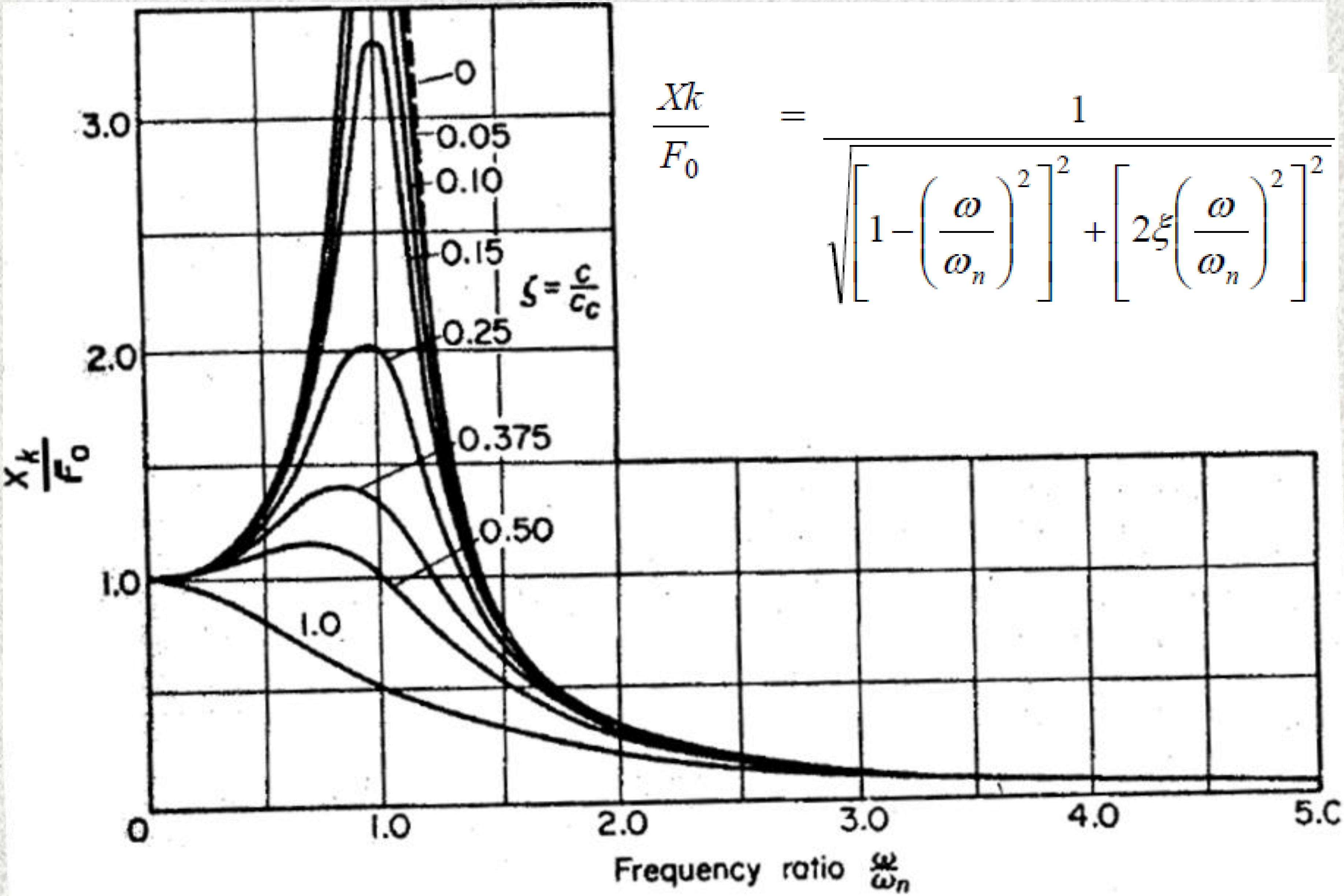
where constant A_1 and ϕ' can be determined form initial conditions.

$$x = A_1 \sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi'\right)e^{-\xi\omega_n t} + \frac{F_0}{k}\cdot\frac{\sin(\omega t - \phi)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}} \quad \text{.......... (16)}$$

The first part of the complete solution, i.e. complementary function, is seen to decay with time, and vanishes ultimately. This part, in engineering practice, is commonly called as transient vibrations.

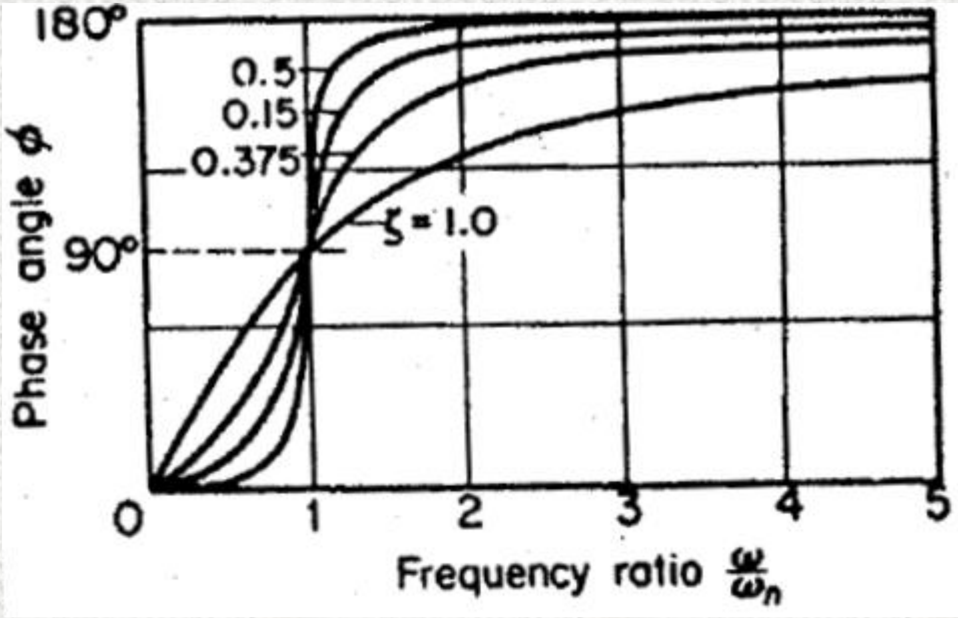
The second part, i.e. the particular solution, is seen to be a sinusoidal vibration with constant amplitude and is called steady state vibrations.

It may also be noted that the transient vibrations take place at the damped natural frequency of the system, whereas the steady state vibration occurs at the frequency of excitation.



$$\frac{Xk}{F_0} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)^2\right]^2}}$$

$$\phi = \tan^{-1}\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

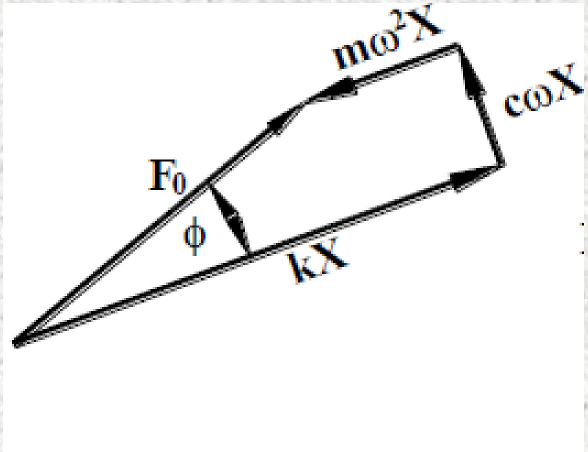


These curves show that the damping factor has a large influence on the amplitude and phase angle in the frequency region near resonance.

Further understanding of the behavior of the system may by obtained by studying the force diagram corresponding to the regions ω/ω_n small, $\omega/\omega_n = 1$, and ω/ω_n large.

When ω/ω_n<<1

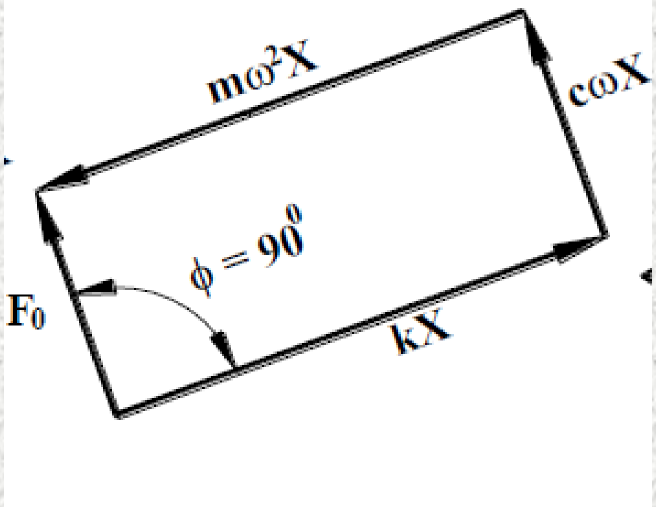
For small values of ω/ω_n<<1, both the inertia and damping force are small, which results in a small phase angle ϕ. The magnitude of the impressed force is nearly equal to the spring force as shown in **Figure**.



(a) ω/ω_n<<1

When ω/ω_n=1

For **ω/ω_n = 1**, the phase angle is 90^0 and the force diagram appears as in **Figure**. The inertia force, which is now larger, is balanced by the spring force; whereas the impressed force overcomes the damping force. The amplitude at resonance can be found, either from Equations (11) and (13) or from **Figure** to be

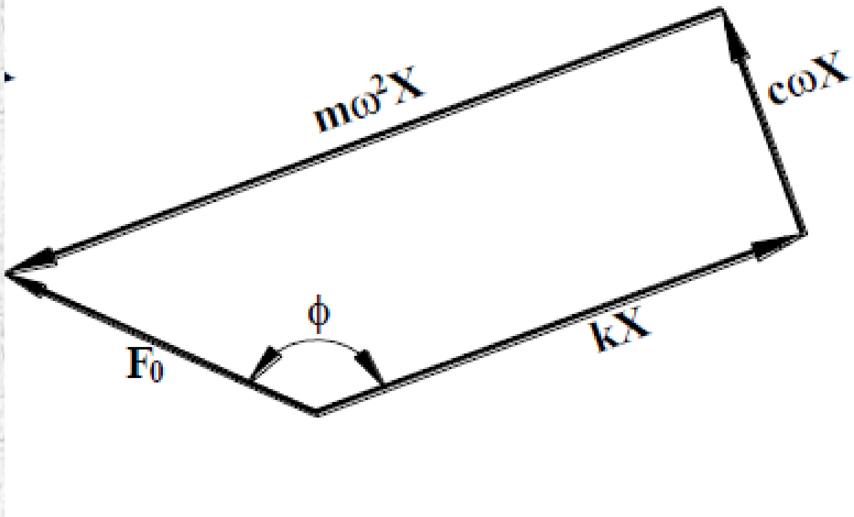


(b) ω/ω_n = 1

$$X = \frac{F_0}{c\omega_n} = \frac{F_0}{2\xi k}$$

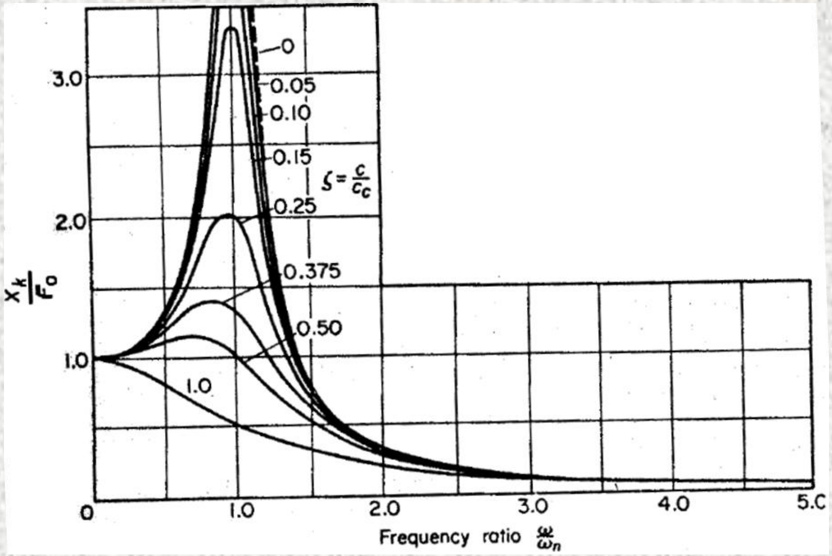
When ω/ω_n>>1

Al large values of $\omega/\omega_n >> 1$, ϕ approaches 180^0, and the impressed force is expended almost entirely in overcoming the large inertia force as shown in **Figure**.



(c) $\omega/\omega_n >> 1$

It is seen that the maximum amplitude occurs not at the resonant frequency but a little towards its left. This shift increases with the increase in damping. For zero damping the maximum amplitude (infinite value), of course, is obtained at the resonant frequency.



The frequency at which the maximum amplitude occurs can be obtained from Equation (13) by differentiating this equation with respect to ω/ω_n and equating this differential to zero.

$$\frac{d(Xk/F_0)}{d(\omega/\omega_n)} = -\frac{1}{2}\left[\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2\right]^{-\frac{3}{2}}\left[-4\frac{\omega}{\omega_n}\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\} + 8\xi^2\left(\frac{\omega}{\omega_n}\right)\right]$$

Then, $\left.\dfrac{d(Xk/F_0)}{d(\omega/\omega_n)}\right|_{\omega=\omega_p} = 0$ gives

$$\left(\frac{\omega_p}{\omega_n}\right)^2 = 1-2\xi^2$$

$$\therefore \quad \frac{\omega_p}{\omega_n} = \sqrt{1-2\xi^2} \quad \ldots. (17)$$

where ω_p means the frequency corresponding to the peak amplitude. No maxima or peak amplitude will occur when the expression within the radical sign is negative, i.e. for $\xi > 0.707$. For value of $\xi > 0.707$, the response curve is always below the unity magnification line.

Then the corresponding peak amplitude is obtained by substituting

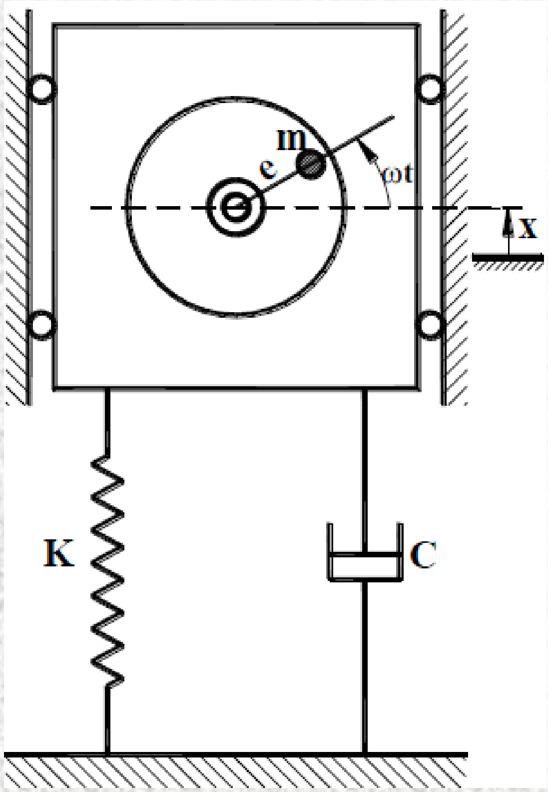
$$\frac{\omega}{\omega_n} = \frac{\omega_p}{\omega_n} = \sqrt{1 - 2\xi^2}$$

into Equation (13) as

$$X_p = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{\omega_p}{\omega_n}\right)^2\right]^2 + \left[2\xi\,\frac{\omega_p}{\omega_n}\right]^2}} = \frac{\frac{F_0}{k}}{\sqrt{[1 - (1 - 2\xi^2)]^2 + 4\xi^2(1 - 2\xi^2)}}$$

$$X_p = \frac{\frac{F_0}{k}}{2\xi\sqrt{1 - \xi^2}} \quad \text{……….} \quad (18)$$

6.5 Rotating Unbalance



Consider a spring mass system constrained to move in the vertical direction excited by a rotating machine that is unbalanced, as shown in **Figure**. The unbalance is represented by an eccentric mass *m* with eccentricity *e* which is rotating with angular velocity *ω*. Letting *x* be the displacement of the nonrotating mass (*M* – *m*) from the static equilibrium position, the displacement of *m* is $x + e \sin\omega t$.

The equation of motion is then

$$(M - m)\,\ddot{x} + m\frac{d^2}{dt^2}(x + e\sin\omega t) = -kx - c\dot{x} \quad \ldots\ldots\ldots\ldots (19)$$

which can be rearranged to

$$M\ddot{x} + c\dot{x} + kx = (me\omega^2)\sin\omega t \quad \ldots\ldots\ldots\ldots (20)$$

The above equation is identical to Equation (1), where F_0 is replaced by $me\omega^2$, and hence the steady state solution of the previous section can be replaced by

$$X = \frac{me\omega^2}{\sqrt{\left(k - M\omega^2\right)^2 + (c\omega)^2}} \quad \text{.......... (21)}$$

and

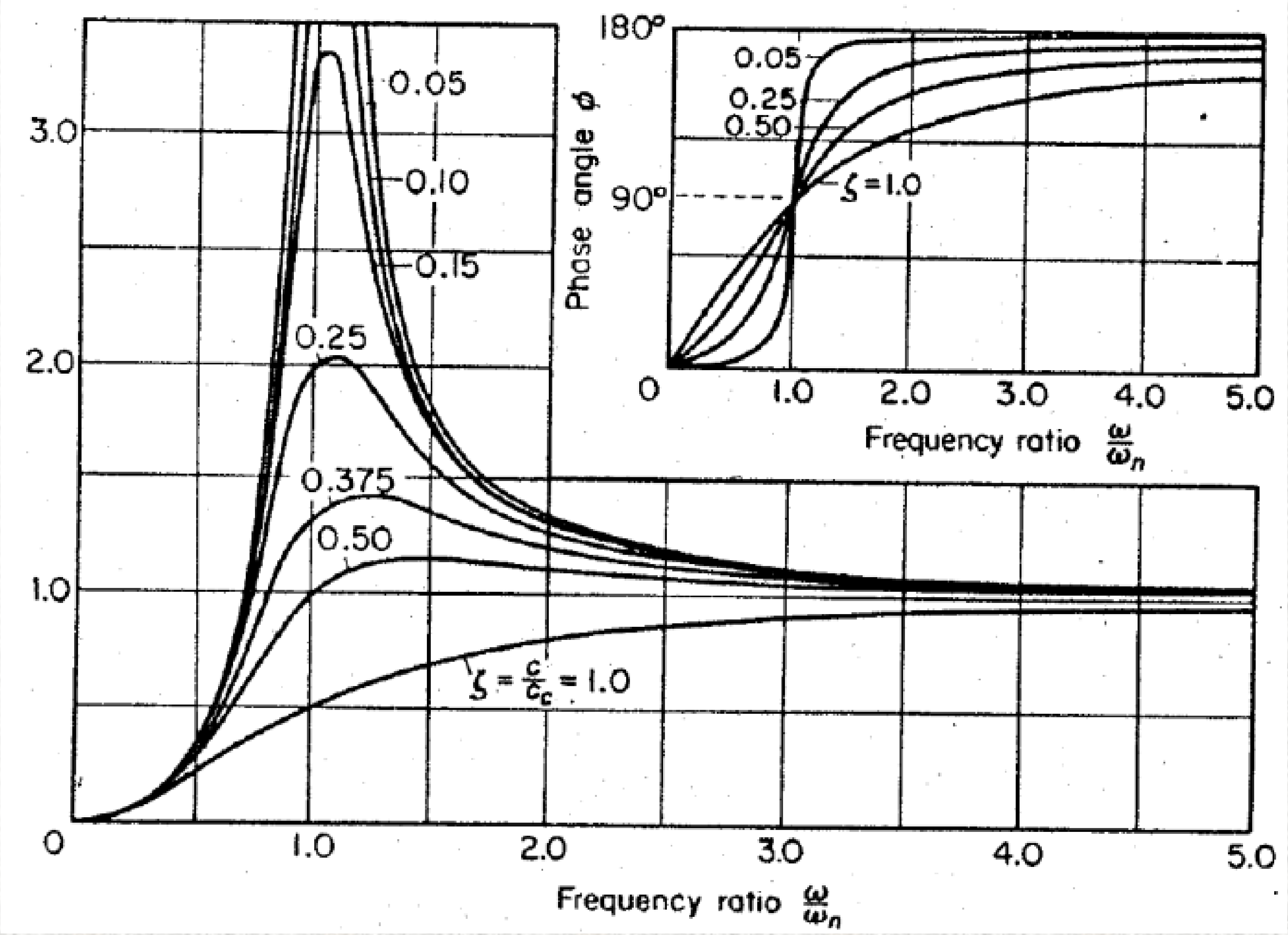
$$\phi = \tan^{-1}\frac{c\omega}{k - M\omega^2} \quad \text{.......... (22)}$$

These can further reduced to nondimensional form

$$\frac{M}{m}\frac{X}{e} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \quad \text{.......... (23)}$$

and

$$\phi = \tan^{-1}\frac{2\xi\left(\frac{\omega}{\omega_n}\right)^2}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \quad \text{.......... (24)}$$



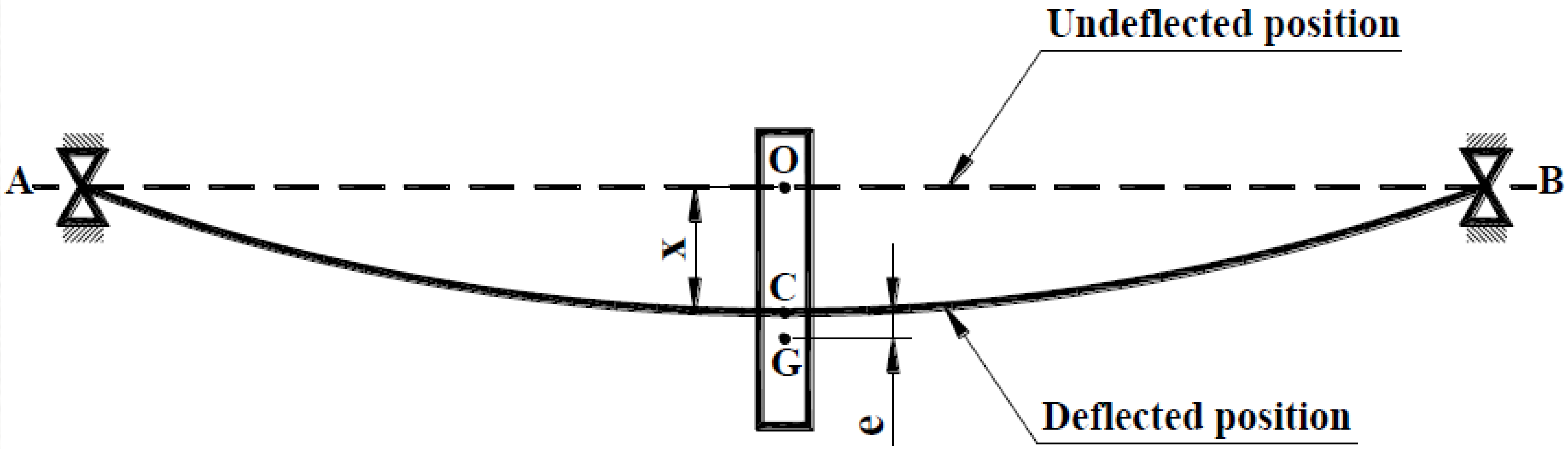
0.05
0.10
0.15
0.25
0.375
0.50
$\zeta = \frac{c}{c_c} = 1.0$
3.0
2.0
1.0
0
1.0
2.0
3.0
4.0
5.0
Frequency ratio $\frac{\omega}{\omega_n}$
180°
90°
Phase angle ϕ
0.05
0.25
0.50
$\zeta = 1.0$
0
1.0
2.0
3.0
4.0
5.0
Frequency ratio $\frac{\omega}{\omega_n}$

6.6 Whirling of Rotating Shafts

We have already discussed that when the natural frequency of the system coincides with the external forcing frequency, it is called resonance. The speeds at which resonance occurs are known as the critical speeds. These speeds are also termed as whirling or whipping speeds.

At these speeds the amplitudes of vibration of rotors is excessively large and the large amount of force is transmitted to the foundations or bearings. In the region of critical speeds the system may fail because of violent nature of vibrations in the transverse direction. Therefore, it is very important to find the natural frequency of the shaft to avoid the occurrence of critical speed which may result in excessive noise and breaking of the shaft.

The critical speed may occur because of eccentric mounting of the rotor, nonuniform distribution of rotor material, bending of shaft; etc.



Considering the equilibrium of the disc, two forces are acting on it. The centrifugal force acts outwards through *G* and restoring force acts through *C* radially inwards.

Thus, restoring force = Centrifugal force

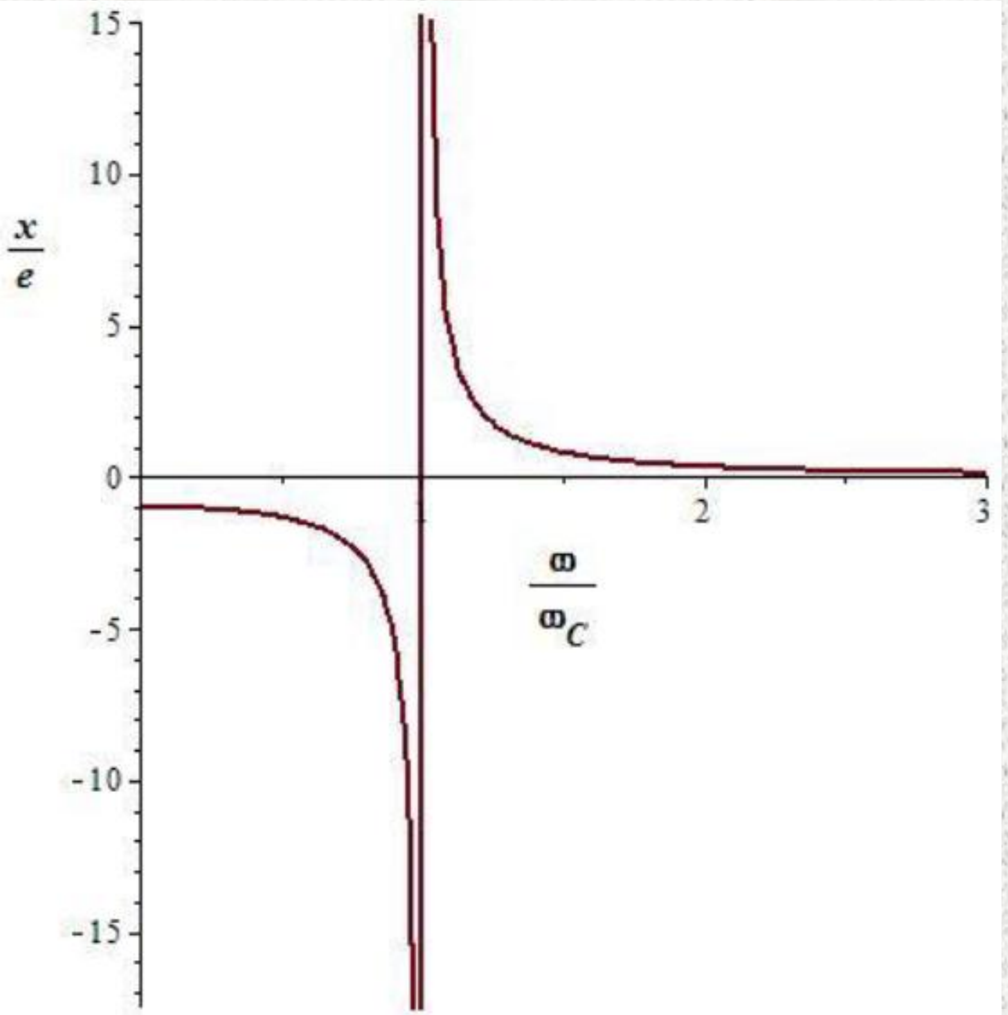
$$kx = m\omega^2(x + e)$$

$$kx - m\omega^2 x = m\omega^2 e$$

$$\frac{x}{e} = \frac{m\omega^2}{k - m\varpi^2} = \frac{1}{\frac{k}{m} - 1} \quad \text{..........} \; (25)$$

Substituting $\omega_c^2 = k/m$,

$$\frac{x}{e} = \frac{1}{\left(\frac{\omega_c}{\omega}\right)^2 - 1} \quad \text{.......... (26)}$$



6.7 Vibration Isolation and Transmissibility

The high speed engines and machines when mounted on foundations and supports cause vibrations of excessive amplitudes because of unbalanced forces set up during their working. These are the disturbing forces which damage the foundation on which the machines are mounted.

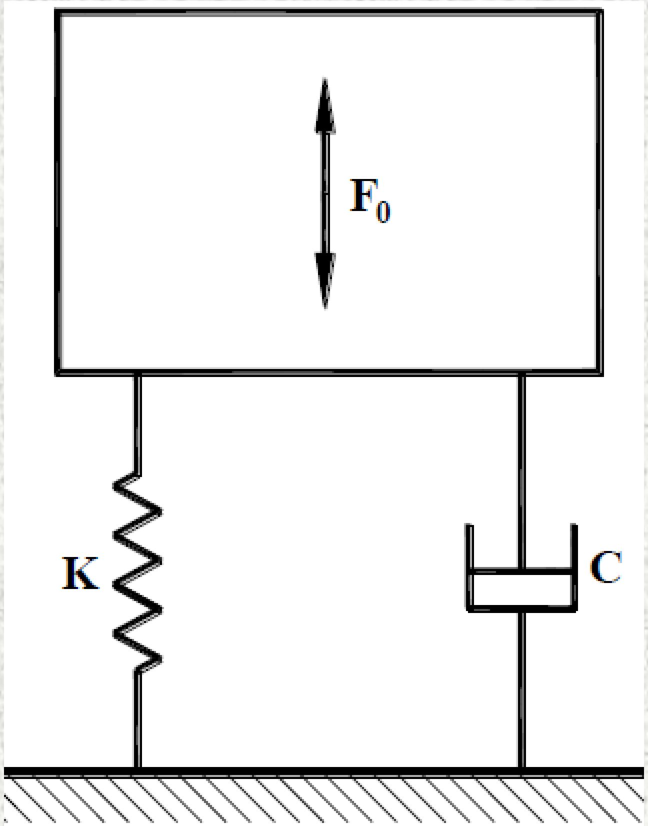
So the vibrations of transmitted to the foundations should be eliminated or reduced considerably by using some devices such as springs, dampers, etc, between the foundation and the machine.

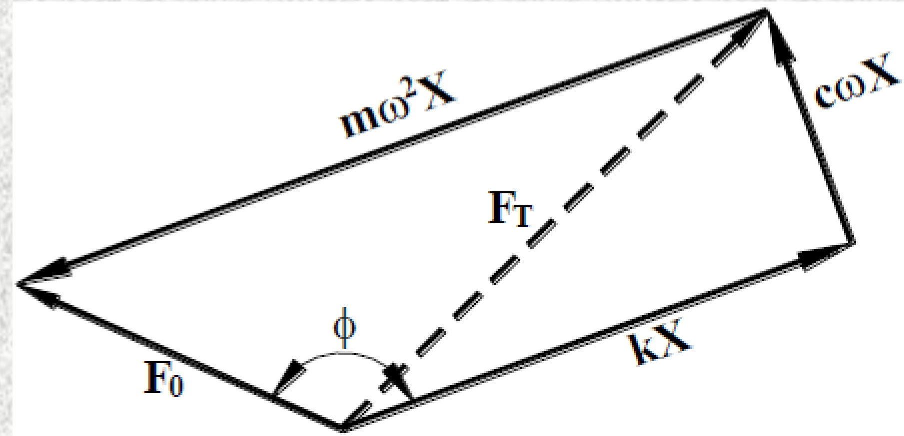
These devices isolate the vibrations by absorbing some of the disturbing energy themselves and allow only a fraction of it to pass through them to the foundation. Thus the amplitude of vibration is minimized and the adjoining structure or foundation is not subjected to heavy disturbances.

The isolation is expressed in terms of force or motion. Lesser the amount of force or motion transmitted to the foundation greater is said to be the isolation. So machines are mounted on isolation mounts.

The materials normally used for vibration isolation are rubber, felt cork, metallic springs, etc. These are placed between the foundation and the vibrating body.

These materials are used for different operating conditions, such as rubber is used for high frequency vibrations, felt for low frequency ratio, springs for high frequency ratio, etc.





Let $F_0 \sin\omega t$ be the exciting force acting on the spring mass damper system as shown in **Figure**. The transmitted force through the spring and damper is

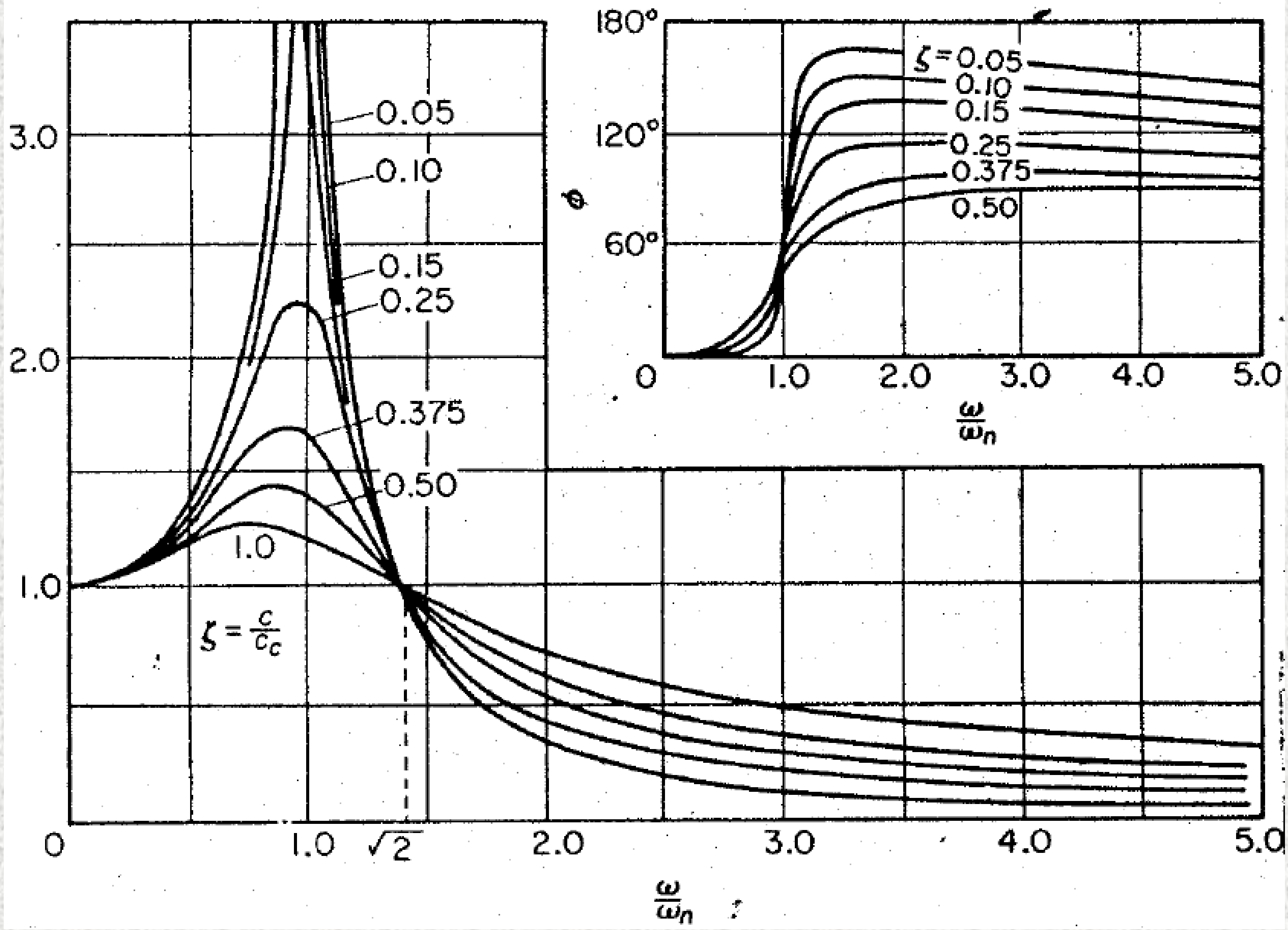
$$F_T = \sqrt{(kX)^2 + (c\omega X)^2} = kX\sqrt{1 + \left(\frac{c\omega}{k}\right)^2} \quad \text{.... (27)}$$

Substituting *kX* from Equation (13),

$$F_T = \frac{F_0\sqrt{1+\left(\frac{c\omega}{k}\right)^2}}{\sqrt{\left[1-\frac{m\omega^2}{k}\right]^2+\left(\frac{c\omega}{k}\right)^2}} = \frac{F_0\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} \quad \text{.. (28)}$$

The term transmissibility (*TR*) in the case of force-excited system is defined as the ratio of the force transmitted to the foundation to that impressed upon system

$$TR = \frac{F_T}{F_0} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} \quad \text{.......... (29)}$$



0.05
0.10
0.15
0.25
0.375
0.50
1.0
$\zeta = \frac{c}{c_c}$
3.0
2.0
1.0
0
1.0
$\sqrt{2}$
2.0
3.0
4.0
5.0
$\frac{\omega}{\omega_n}$
180°
120°
60°
ϕ
ζ=0.05
0.10
0.15
0.25
0.375
0.50

It is clear from the above figure that all the curves start from unity value of transmissibility, pass through the unit transmissibility at $\omega/\omega_n = \sqrt{2}$ and after that they tend to zero as ω/ω_n tends to ∞.

These curves can be divided into three distinct frequency. These regions are respectively controlled by the three parameters of the system: mass, damping and stiffness.

The region where isolation is really effective is when ω/ω_n is large. This is the mass controlled region. Larger mass gives low natural frequency and consequently higher values of ω/ω_n. Damping in this region deteriorates the performance of the system. In the vibration isolation region $\omega/\omega_n > 2$ and for negligible damping Equation (29) can be rewritten as

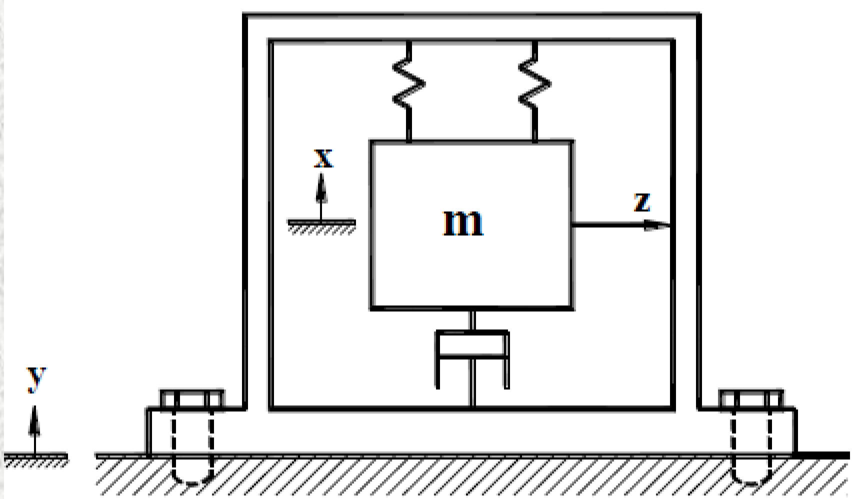
$$TR = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} \quad \ldots\ldots\ldots\ldots (30)$$

The region on the other extreme is when ω/ω_n is small. This is the spring controlled region. Larger stiffness gives high value of natural frequency and consequently low value of frequency ratio. The middle region, always to be avoided, is the damping controlled region.

6.8 Vibration Measuring Instruments

To determine the behavior of such instruments we consider the equation of motion *m*, which is

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \quad \text{.......... (31)}$$



Letting the relative displacement of the mass and case be

$$z = (x - y) \quad \text{.......... (32)}$$

Then the above equation becomes

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \quad \text{..........} (33)$$

Assuming sinusoidal motion $y = Y\sin\omega t$ for the vibrating body, we obtain the equation

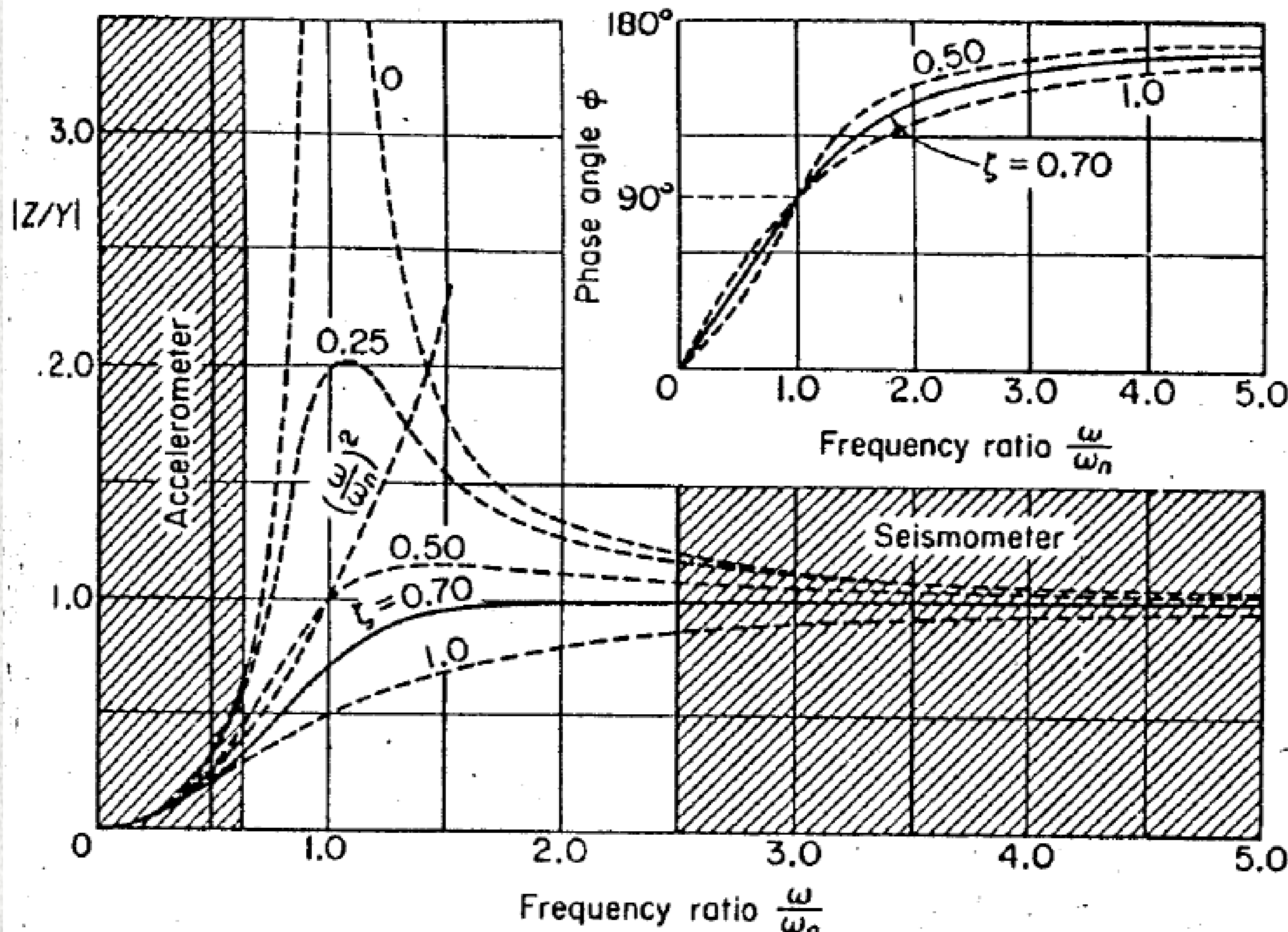
$$m\ddot{z} + c\dot{z} + kz = mY\omega^2 \sin\omega t \quad \text{..........} (34)$$

which is identical Equation (20) with z and $mY\omega^2$ replacing x and $me\omega^2$ respectively. The steady state solution is $z = Z\sin(\omega t - \phi)$ is then available from inspection to be

$$Z = \frac{mY\omega^2}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}} = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}} \quad \text{......} (35)$$

and

$$\tan\phi = \frac{\omega c}{k - m\omega^2} = \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \quad \text{......} (36)$$



180°
0.50
1.0
Phase angle φ
ζ = 0.70
90°
0
1.0
2.0
3.0
4.0
5.0
Frequency ratio $\frac{\omega}{\omega_n}$
3.0
|Z/Y|
2.0
1.0
0
Accelerometer
0.25
$\left(\frac{\omega}{\omega_n}\right)^2$
0.50
ζ = 0.70
1.0
Seismometer
Frequency ratio $\frac{\omega}{\omega_n}$

6.8.1 Seismometer

A *seismometer* is a low natural frequency instrument. Thus the range of frequencies for which it is intended is characterized by a large value of ω/ω_n.

Above Equation shows that as ω/ω_n tends to infinity, the relative displacement Z becomes equal to Y, or $|Z/Y| = 1.0$. The mass m then remains stationary while the supporting case moves with the vibrating body.

One of the disadvantages of the seismometer is its large size. Since $|Z| = |Y|$, the relative motion of the seismic mass must be of the same order of magnitude as the vibration is to be measured.

The relative motion is generally converted to an electrical voltage by making the seismic mass a magnet moving relative to coils fixed in the case. A typical instrument of this kind may have a natural frequency of 2 to 5 Hz and a useful range of 10 to 500 Hz.

6.8.2 Accelerometer

Most vibration measurements today are made with accelerometers. Even earthquakes are recorded by accelerometers, and the velocity and displacement are obtained by integration. Accelerometers are preferred as vibration measuring instruments because of their small size and high sensitivity.

Accelerometers are high natural frequency instruments, and the useful range of their operation is ω/ω_n from zero to about 0.4. Equation (35) for ω/ω_n tends zero leads to

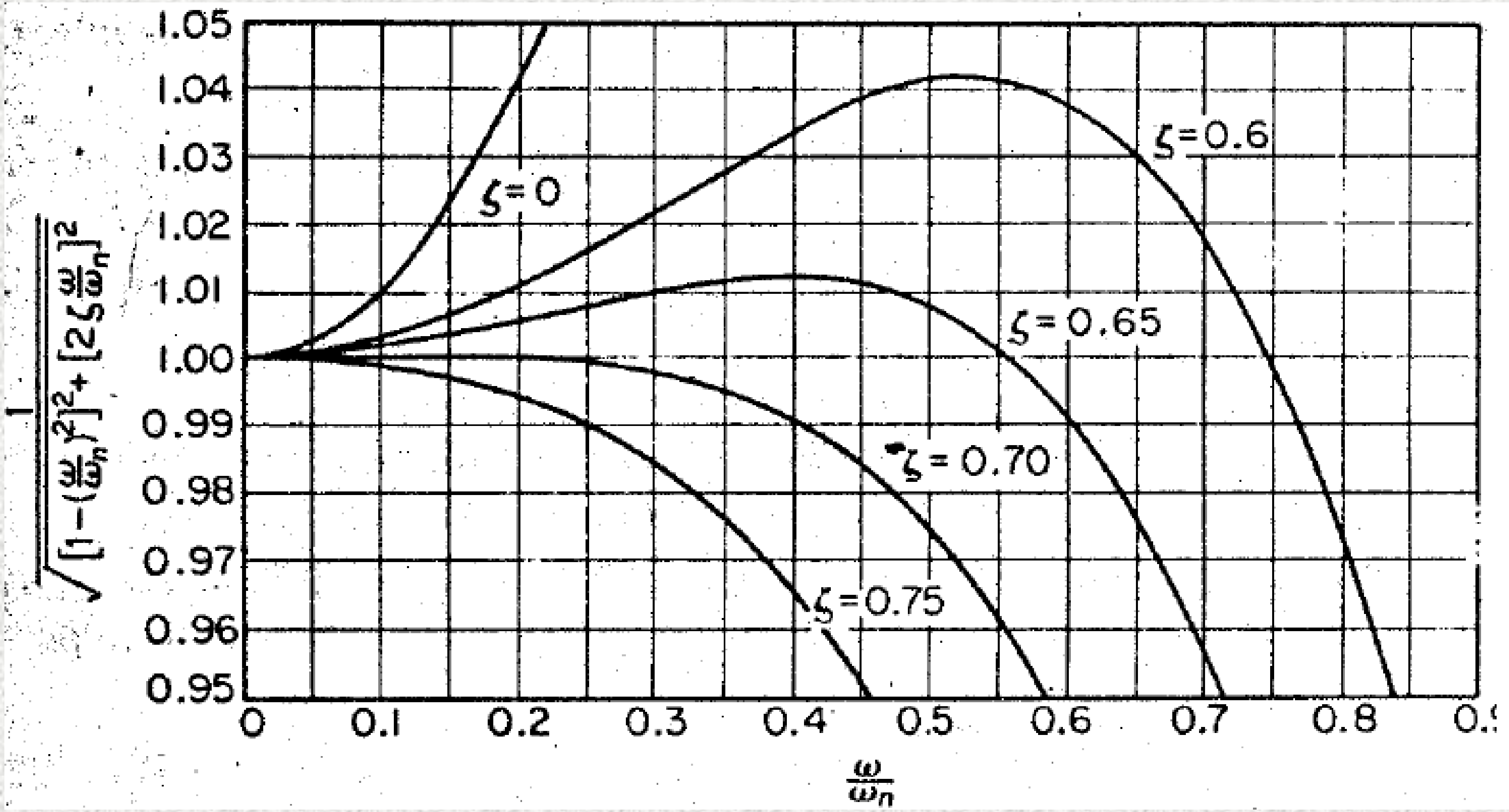
$$Z = \frac{\omega^2 Y}{\omega_n^2} = \frac{acceleration}{\omega_n^2} \quad \text{......(37)}$$

and hence Z becomes proportional to the acceleration of the motion to be measured.

The sensitivity decreases, however, as ω_n increases, so that ω_n should be no higher than necessary. For example, accelerometers used extensively for earthquake measurements have a natural frequency of 20 Hz, which allows ground motions of frequencies less than 8 Hz to be faithfully reproduced. Actually, motions up to 16 Hz can be measured by applying a correction from the instrument calibration.

For a greater range of frequencies, the piezometer crystal accelerometer is extensively used. Its natural frequency is generally very high, and the crystal accelerometer can be used upto frequencies upto 1000 Hz or more.

The useful frequency range of the undamped accelerometer is somewhat limited because the denominator $1 - (\omega/\omega_n)^2$ drops off rapidly as ω increases. However with damping in the range of $\xi = 0.65$ to 0.70, the reduction in the term $1 - (\omega/\omega_n)^2$ is compensated for by the additional term $(2\xi\omega/\omega_n)^2$ to greatly extend the useful of the instrument.

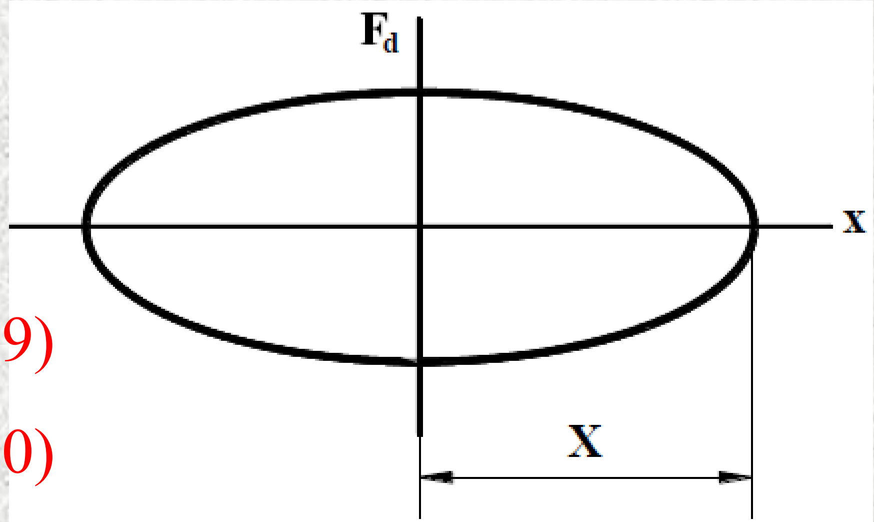


Most accelerometers utilize damping near $\xi = 0.7$, which not only extends the useful frequency range but prevents phase distortion.

6.9 Energy Dissipated by Damping

Energy dissipation is usually determined under conditions of cyclic oscillations. Depending on the type of damping present, the force-displacement relationship when plotted may differ greatly. In all cases, however, the force-displacement curve will enclose an area, referred to as the hysteresis loop, that is proportional to the energy lost per cycle. The energy lost per cycle due to a damping force F_d is computed from the general equation

$$W_d = \oint F_d \, dx \quad \text{..........(38)}$$



With the steady state displacement and velocity

$$x = X \sin(\omega t - \phi) \quad \text{..........(39)}$$

$$\dot{x} = \omega X \cos(\omega t - \phi) \quad \text{..........(40)}$$

For the viscous damping, the damping force is

$$F_d = c\,\dot{x} \quad \text{..........(41)}$$

Equation (38) becomes

$$W_d = \oint c\dot{x}dx = \oint c\dot{x}^2 dt$$

$$\therefore \quad W_d = c\omega^2 X^2 \int_0^{2\pi/\omega} \cos^2(\omega t - \phi)dt = \pi c\omega X^2 \quad \text{..........(42)}$$

Of particular interest is the energy dissipated in forced vibration at resonance.

Substituting $\omega_n = \sqrt{k/m}$ and $c = 2\xi\sqrt{km}$,

$$W_d = 2\xi\pi k X^2 \quad \text{..........(43)}$$

6.10 General Force Excitation

Apply Fourier Series expansion.

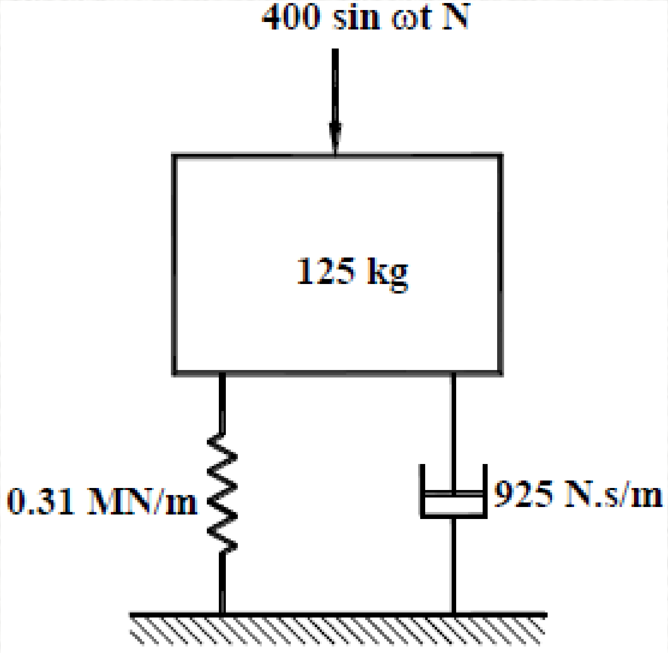
Example 6.10

An 82 kg machine tool is mounted on an elastic foundation. An experiment is run to determine the stiffness and damping properties of the foundation. When the tool is excited with a harmonic force of magnitude 8000 N at a variety of excitation frequencies, the maximum steady-state amplitude obtained is 4.1 mm at a frequency of 40 Hz.

0.1792, 5.535 kN/m

Example 6.11

For what excitation frequencies will the steady state amplitude of the machine of **Figure** be less than 1.5 mm?



18.7 rad/s < ϖ < 67.5 rad/s

Example 6.12

In a resonance test under harmonic excitation, it was noted that the amplitude of motion at resonance was exactly twice the amplitude at an excitation frequency 20 % greater than resonance. Determine the damping ratio of the system.

0.1375

Example 6.13

A 150 kg machine operates at 1200 rpm and has a rotating unbalance of 0.45 kgm. Determine the maximum stiffness of an undamped isolator such that the force transmitted to the machine's foundation is less than 2000 N. Also determine the dynamic amplitude of the machine.

520.24 kN/m, 3.844 mm