THEORY OF MACHINES AND MECHANISMS II

Mechanical IV/I

Chapter 4

Governors

4.1 Introduction

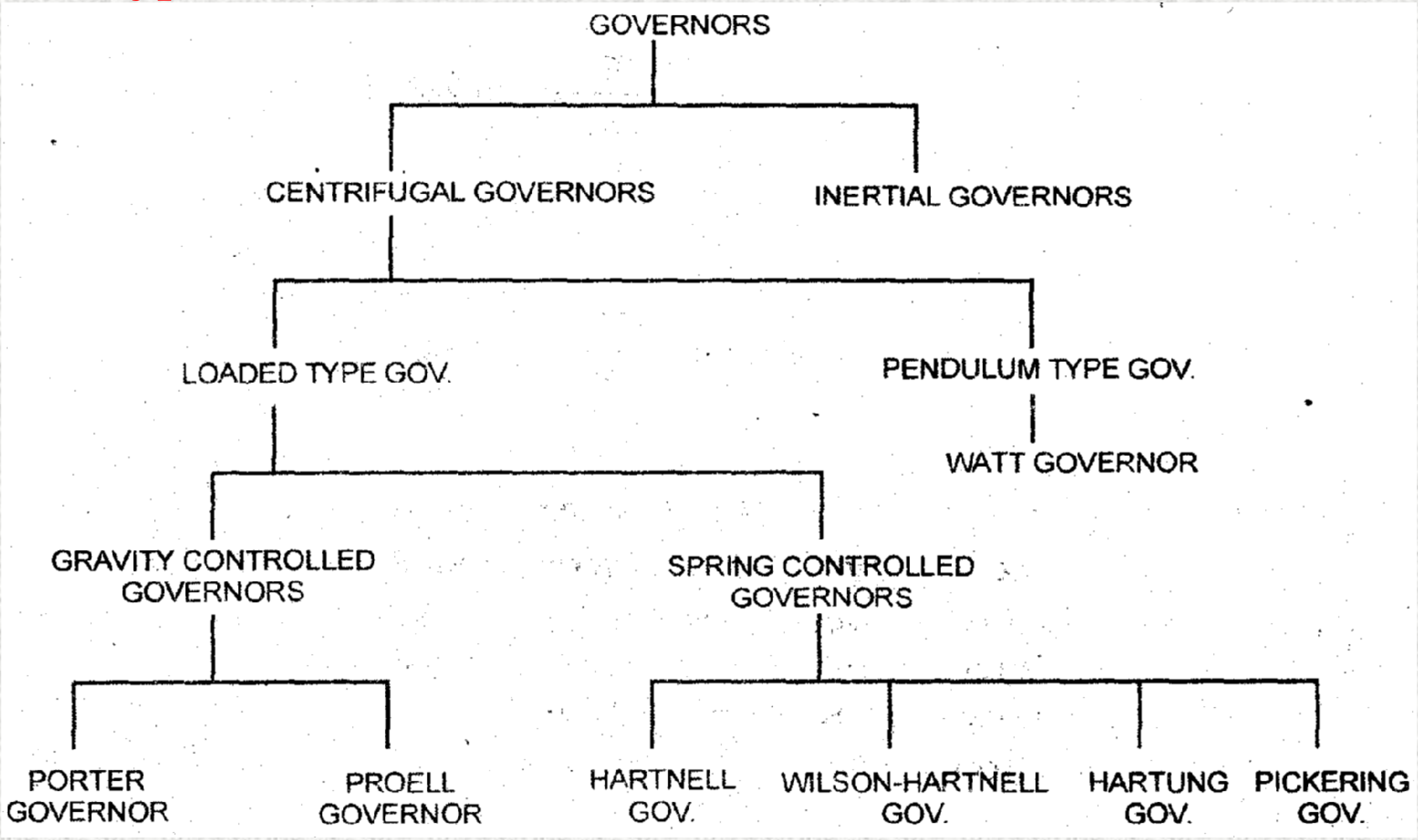
The function of a governor is to regulate the mean speed of an engine, when there are variations in the load .

When the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid.

On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.

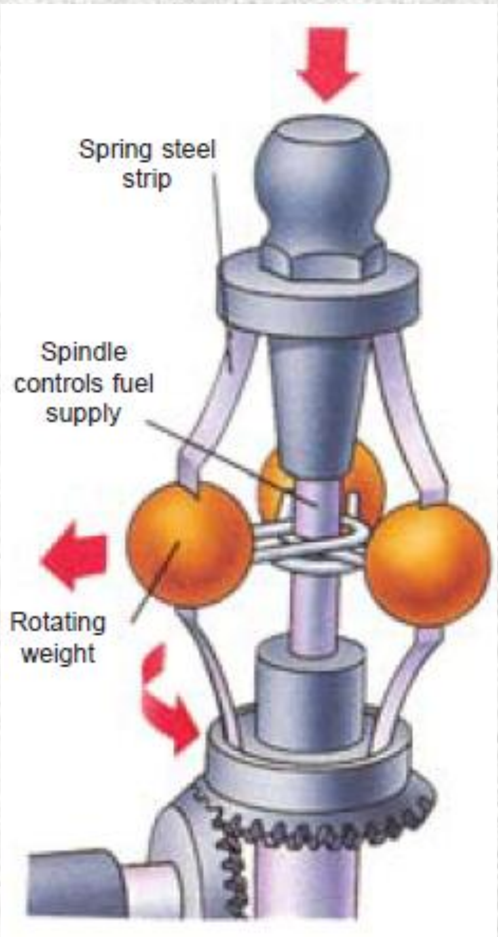
The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

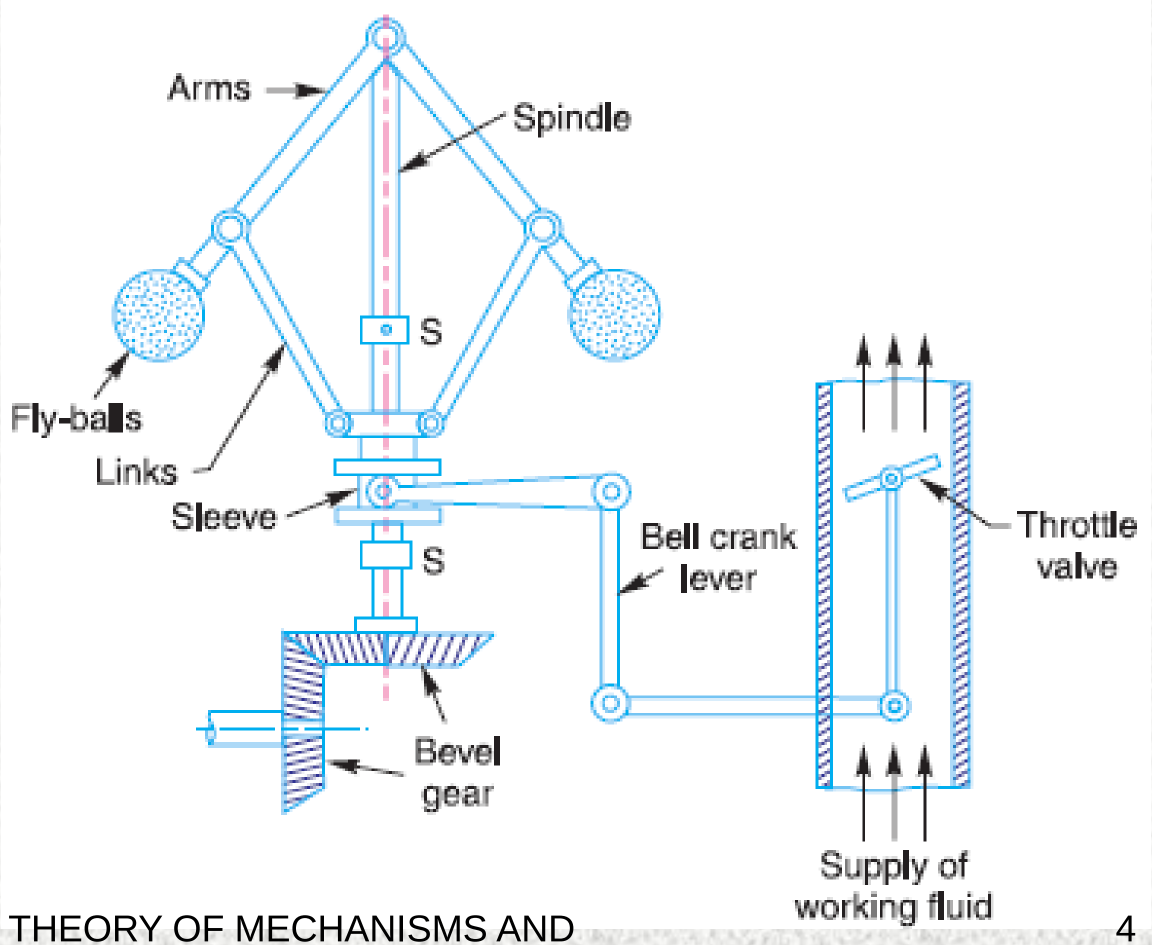
4.2 Types of Governors



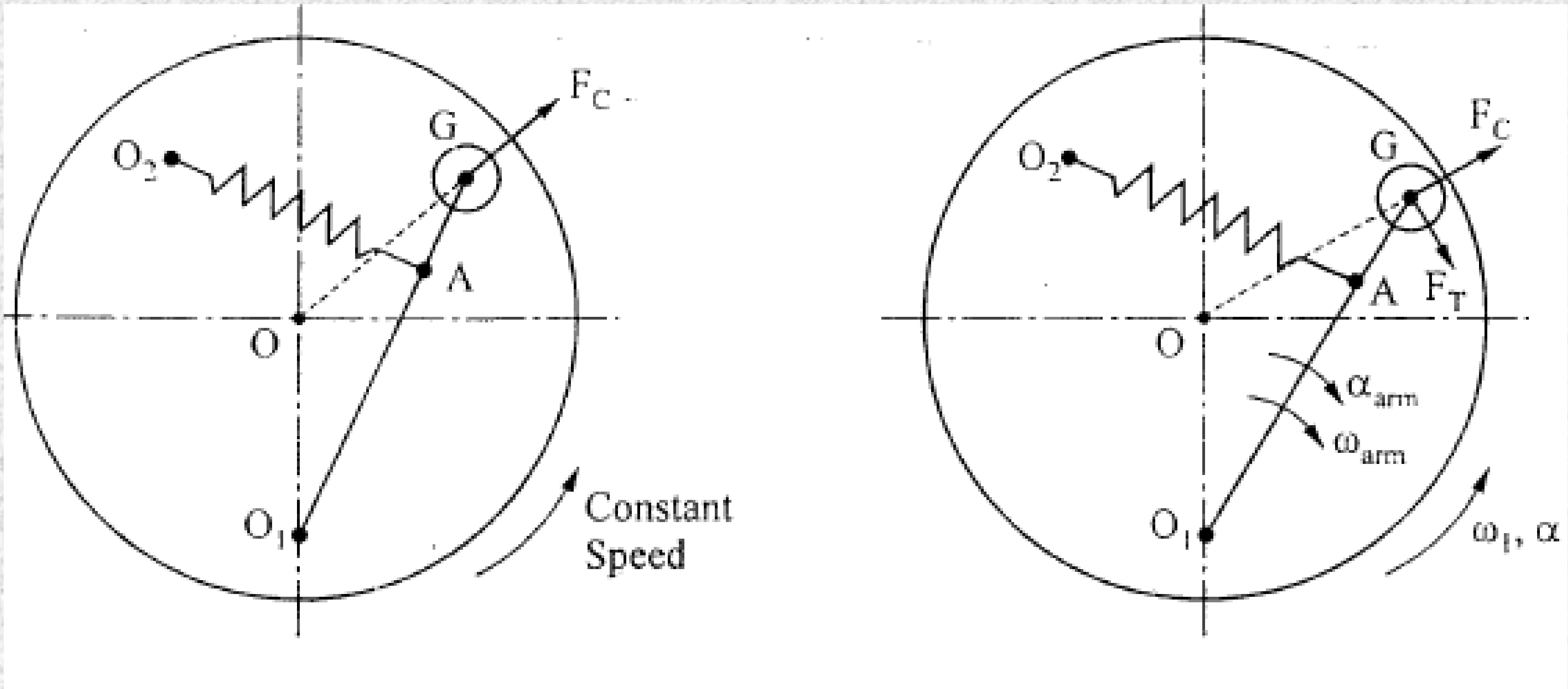
Centrifugal Governors

In these governors, the change in centrifugal forces of the rotating masses due to change in the speed of the engine is utilized for movement of the governor sleeve. One of this type of governors is shown in **Figure**. These governors are commonly used because of simplicity in operation.





Inertia Governor



(a) Constant Speed

(b) Accelerating

If the speed increases, then the mass G has a centripetal as well as tangential component. This will result in an angular motion of G about O_1 with angular velocity ω_{arm} and angular acceleration α_{arm}. It is seen that the directions of rotation of the shaft and the arm are opposite to each other. The movement of the arm is transferred to achieve governing. It has a very quick response.

Terms Used in Governors

Height of a governor

It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by **h**.

Equilibrium speed

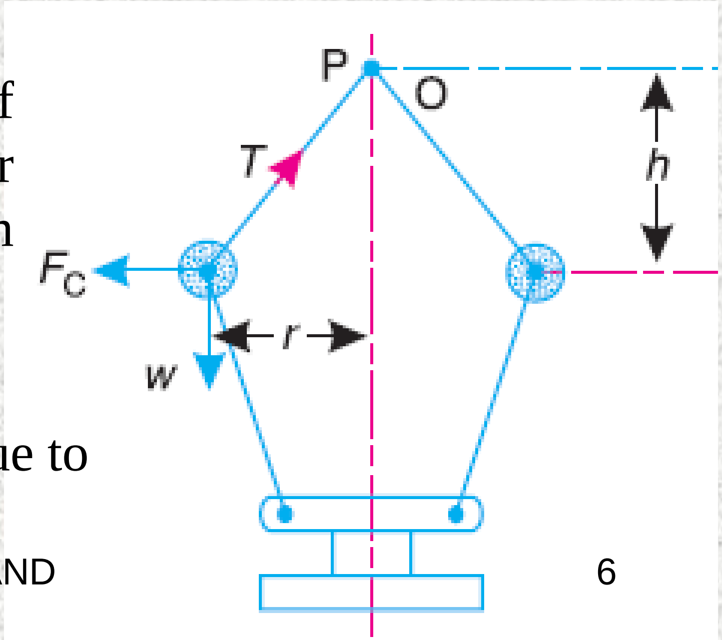
It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

Mean equilibrium speed

It is the speed at the mean position of the balls or the sleeve.

Maximum and minimum equilibrium speeds

The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.



Sleeve lift

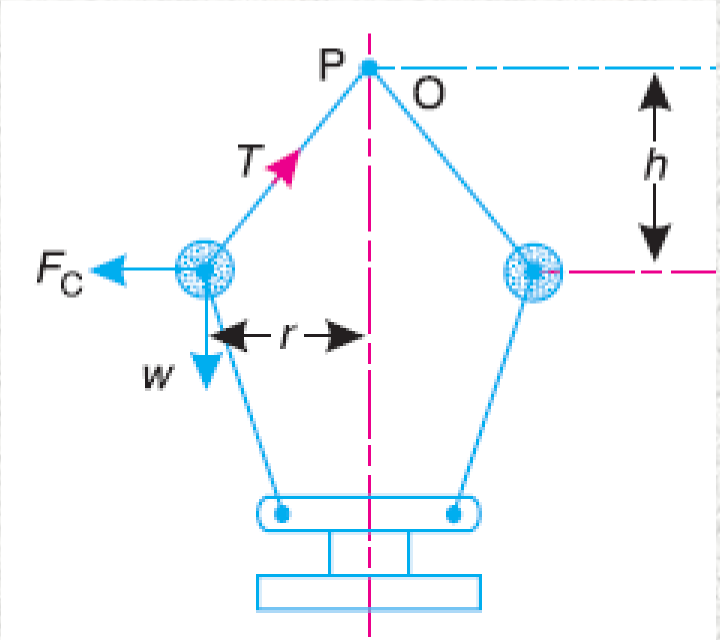
It is the vertical distance which the sleeve travels due to change in equilibrium speed.

4.2.1 Watt Governor

It is the original form of governor as used by Watt on some of his early steam engines.

In this type of governor, each ball is attached to an arm, which is pivoted on the axis of rotation.

The sleeve is attached to the governor balls by arms, pin-jointed at both ends, and is free to slide along the governor shaft.

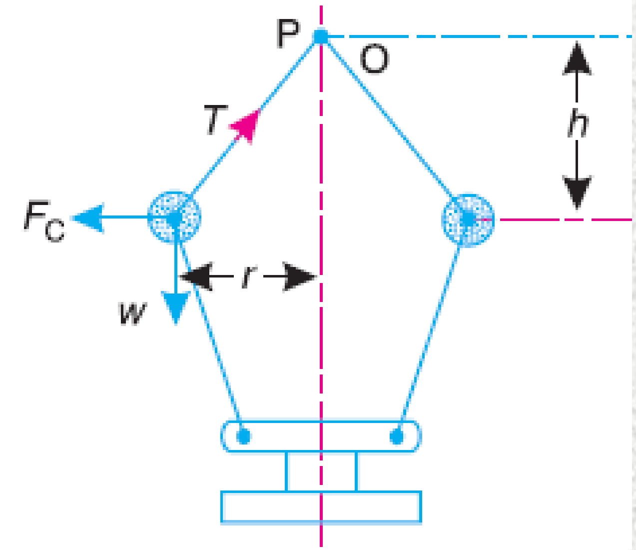


It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls.

Taking moments about point O,

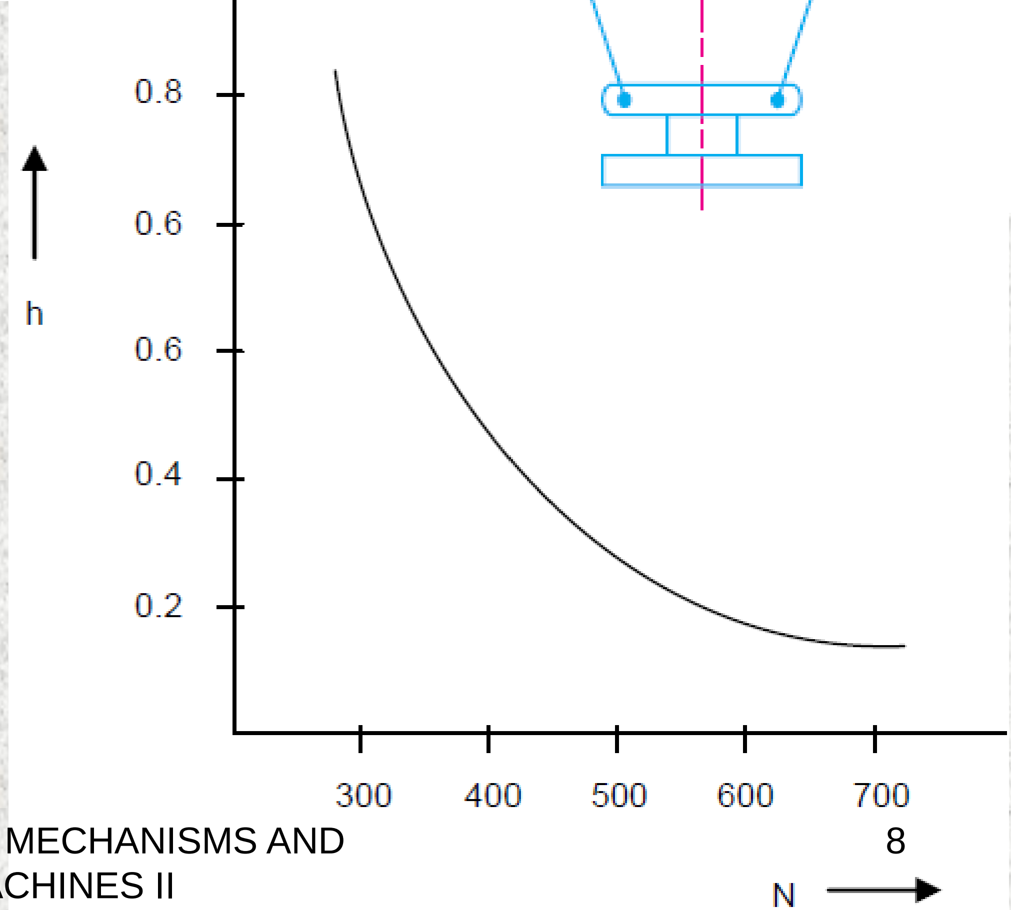
$$F_C \times h = w \times r = m.g.r$$

$$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2$$



If N is the speed in r.p.m.,

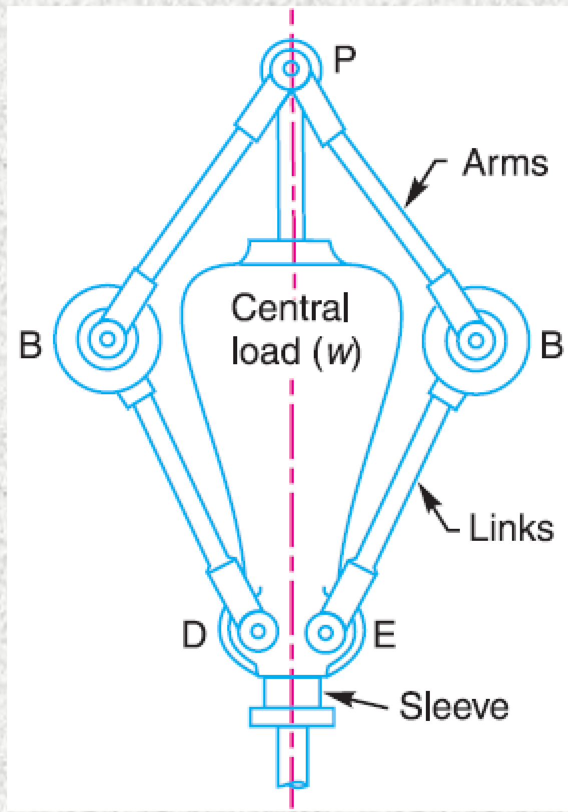
$$h = \frac{g \times 3600}{4\pi^2 N^2} = \frac{894.56}{N^2}$$

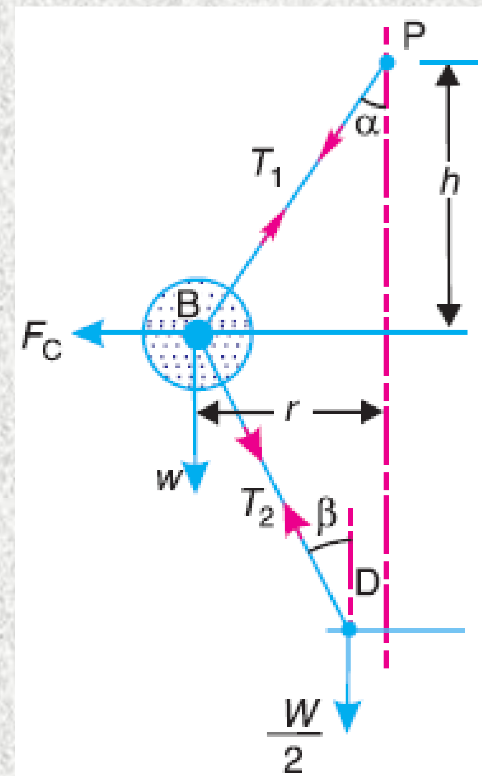


At high speed the change in height **h** is very small which indicates that the sensitiveness of the governor is very poor at high speeds because of flatness of the curve at higher speeds.

4.2.2 Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.





Considering the equilibrium of the forces acting at D,

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2\cos\beta}$$

Resolving the forces acting at B vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots\ldots (1)$$

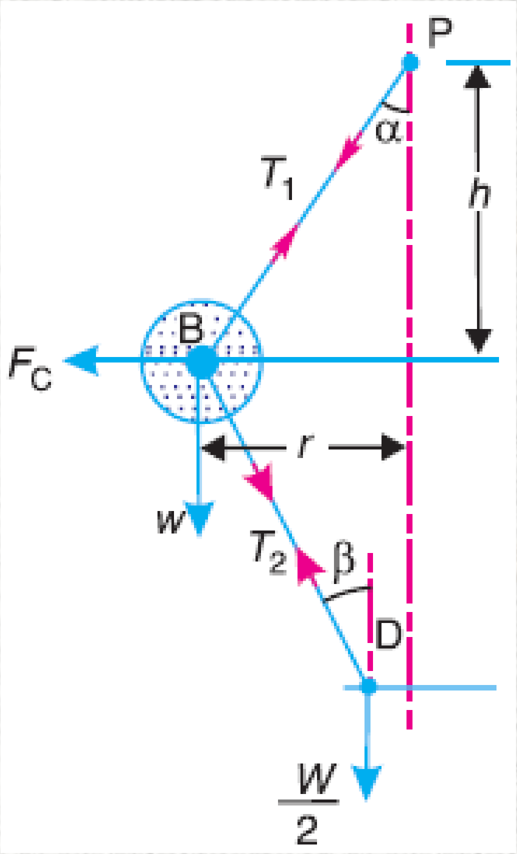
Resolving the forces acting at B horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots\ldots (2)$$



$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots\ldots (1)$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots\ldots (2)$$

Dividing equation (2) by equation (1),

$$\frac{\sin\alpha}{\cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

$$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$,

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q$$

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q$$

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{894.56}{h}$$

When the length of arms are equal to the length of links and the points P and D lie on the same vertical line, then

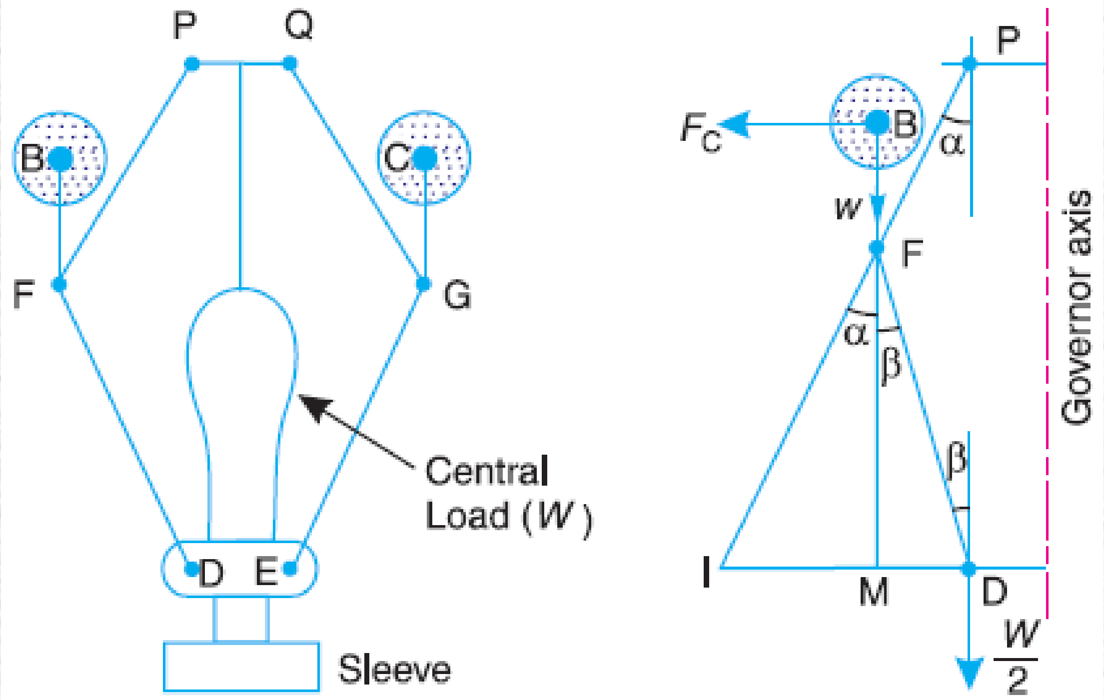
$$\tan\alpha = \tan\beta \quad or\ q = \frac{\tan\alpha}{\tan\beta} = 1$$

$$N^2 = \frac{(m+M)}{m} \times \frac{894.56}{h}$$

On comparing similar expression for the Watt governor, we find that the mass of the central load increase the height of the governor in the ratio of **(1+M/m)**

4.2.3 Proell Governor

The Proell governor has the balls fixed at B and C to the extension of the links DF and EG. The arms FP and GQ are pivoted at P and Q respectively.



The instantaneous centre (I) lies on the intersection of the line PF produced and the line from D drawn perpendicular to the spindle axis. The perpendicular BM is drawn on ID.

Taking moments about I,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID$$

$$F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

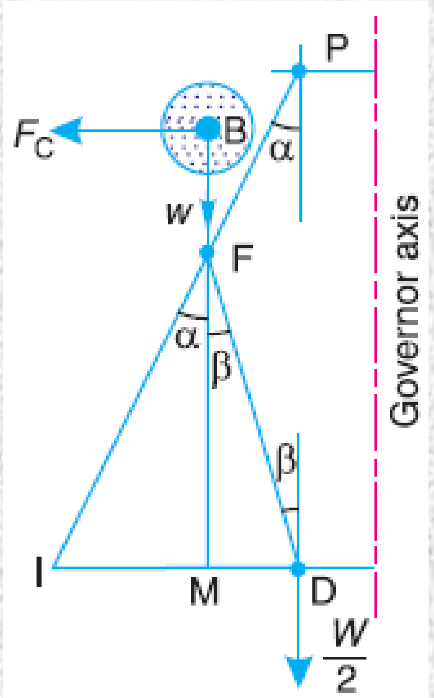
$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

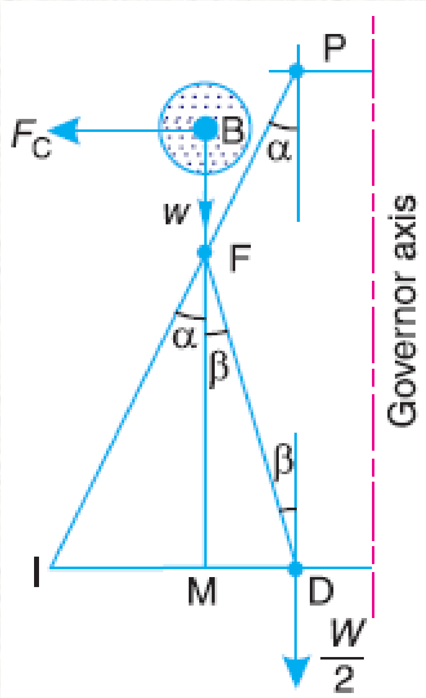
$$m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1 + q)\right]$$

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1 + q)}{m}\right]\frac{g}{h}$$



$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h}$$

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{894.56}{h}$$



In order to have the same equilibrium speed for the given values of m, M and h, balls of smaller masses are used in the Proell governor than in the Porter governor.

When α = β, then $q = 1$.

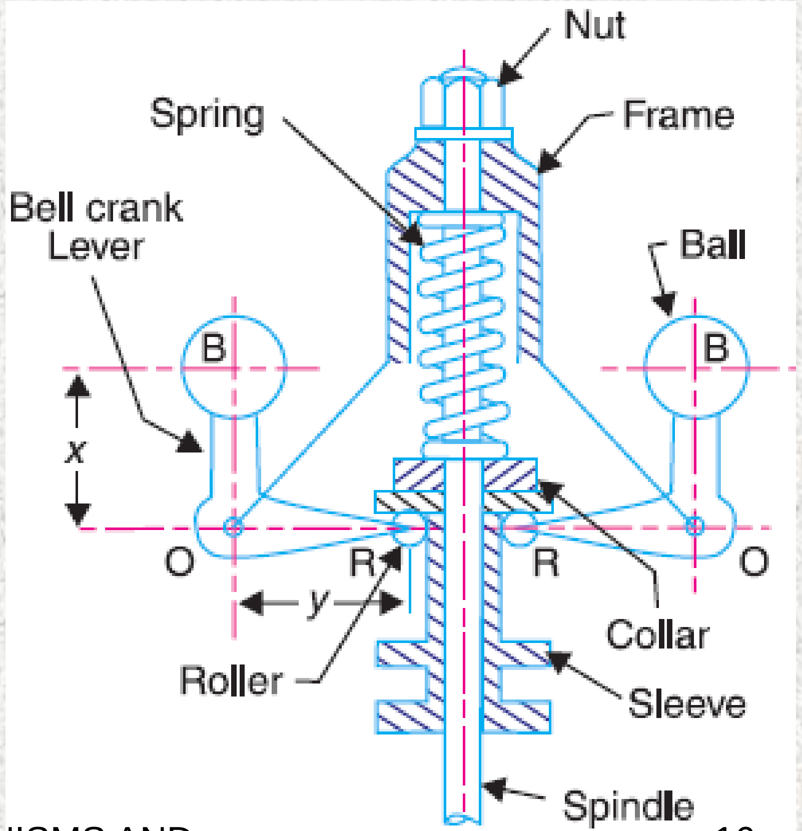
$$N^2 = \frac{FM}{BM}\left[\frac{m + M}{m}\right]\frac{894.56}{h}$$

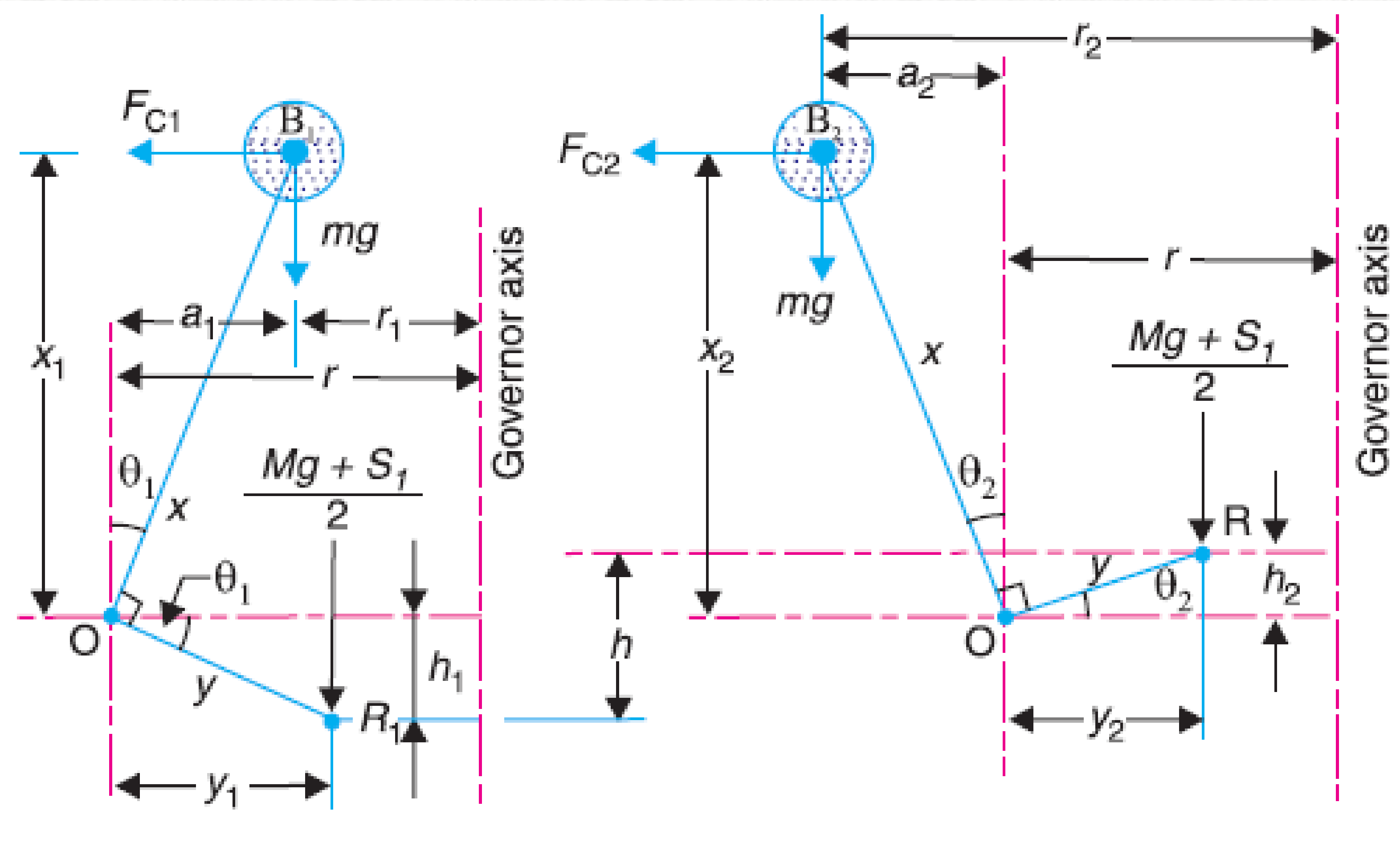
4.2.4 Hartnell Governor

The two bell crank levers have been provided which can have rotating motion about fulcrums O-O. One end of each bell crank lever carries a ball and a roller at the end of other arm. The rollers make contact with the sleeve. The frame is connected to the spindle. A helical spring is mounted around the spindle between frame and sleeve. With the rotation of the spindle, all these parts rotate.

With the increase of speed, the radius of rotation of the balls increases and the rollers lift the sleeve against the spring force. With the decrease in speed, the sleeve moves downwards. The movement of the sleeve are transferred to the throttle of the engine through linkages.

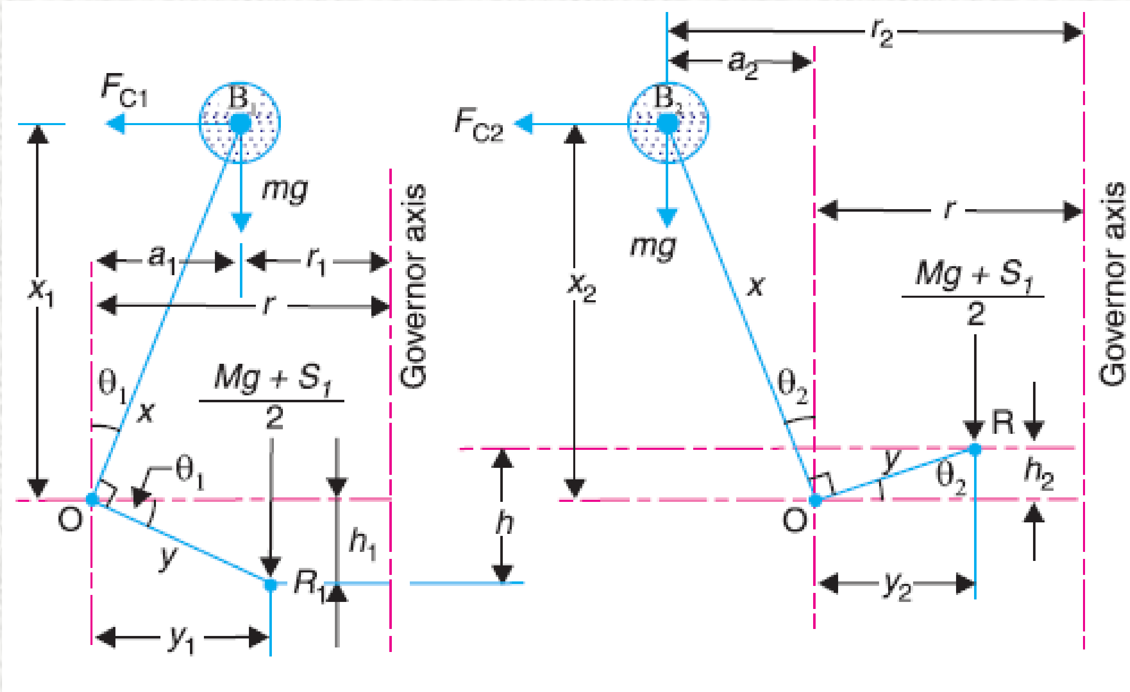
The spring force may be adjusted by screwing a nut up or down on the sleeve.



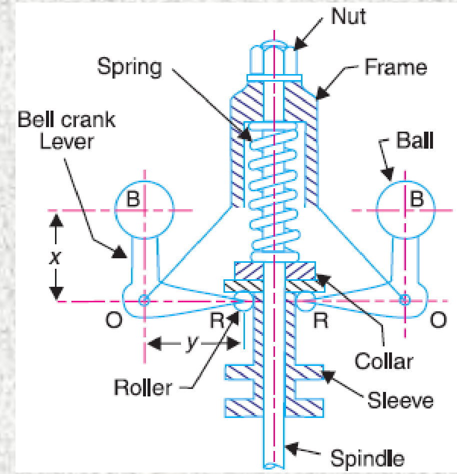


(*a*) Minimum position. (*b*) Maximum position.

The minimum and maximum position is shown in **Figure**. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2.



(*a*) Minimum position. (*b*) Maximum position.



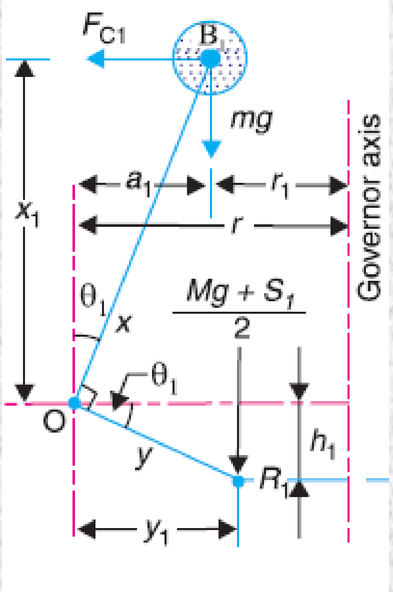
For the minimum position i.e. when the radius of rotation changes from r to r_1, the compression of the spring or the lift of sleeve h_1 is given by

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x}$$

For the maximum position i.e. when the radius of rotation changes from r to r_2, the compression of the spring or lift of sleeve h_2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x}$$

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad \ldots\ldots (1) \qquad \frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad \ldots\ldots (2)$$



(*a*) Minimum position

Adding equations (1) and (2),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x}$$

$$h = (r_2 - r_1)\frac{y}{x} \quad \ldots\ldots (3)$$

Now for minimum position, taking moments about point O,

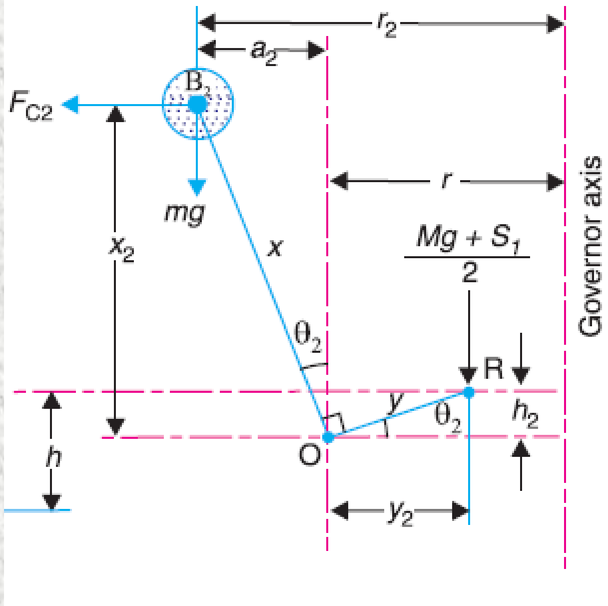
$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

$$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \quad \ldots\ldots (4)$$

Again for maximum position, taking moments about point O,

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

$$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \quad \ldots\ldots (5)$$



(*b*) Maximum position.

Subtracting equation (4) from equation (5),

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

Substituting $S_2 - S_1 = h.s,$ and $h = (r_2 - r_1)\dfrac{y}{x}$

$$s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (i.e. $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (i.e. m.g), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \quad \text{……} (6)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \quad \text{……} (7)$$

Subtracting equation (6) from equation (7),

$$S_2 - S_1 = 2(F_{C2} - F_{C1})\frac{x}{y} \quad \text{……} (8)$$

Substituting $S_2 - S_1 = h.s,$ and $h = (r_2 - r_1)\dfrac{y}{x}$

$$s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \quad \ldots.. (9)$$

Since the stiffness for a given spring is constant for all positions, therefore for minimum and intermediate position,

$$s = 2\left(\frac{F_C - F_{C1}}{r - r_1}\right)\left(\frac{x}{y}\right)^2 \quad \ldots.. (10)$$

For intermediate and maximum position,

$$s = 2\left(\frac{F_{C2} - F_C}{r_2 - r}\right)\left(\frac{x}{y}\right)^2 \quad \ldots.. (11)$$

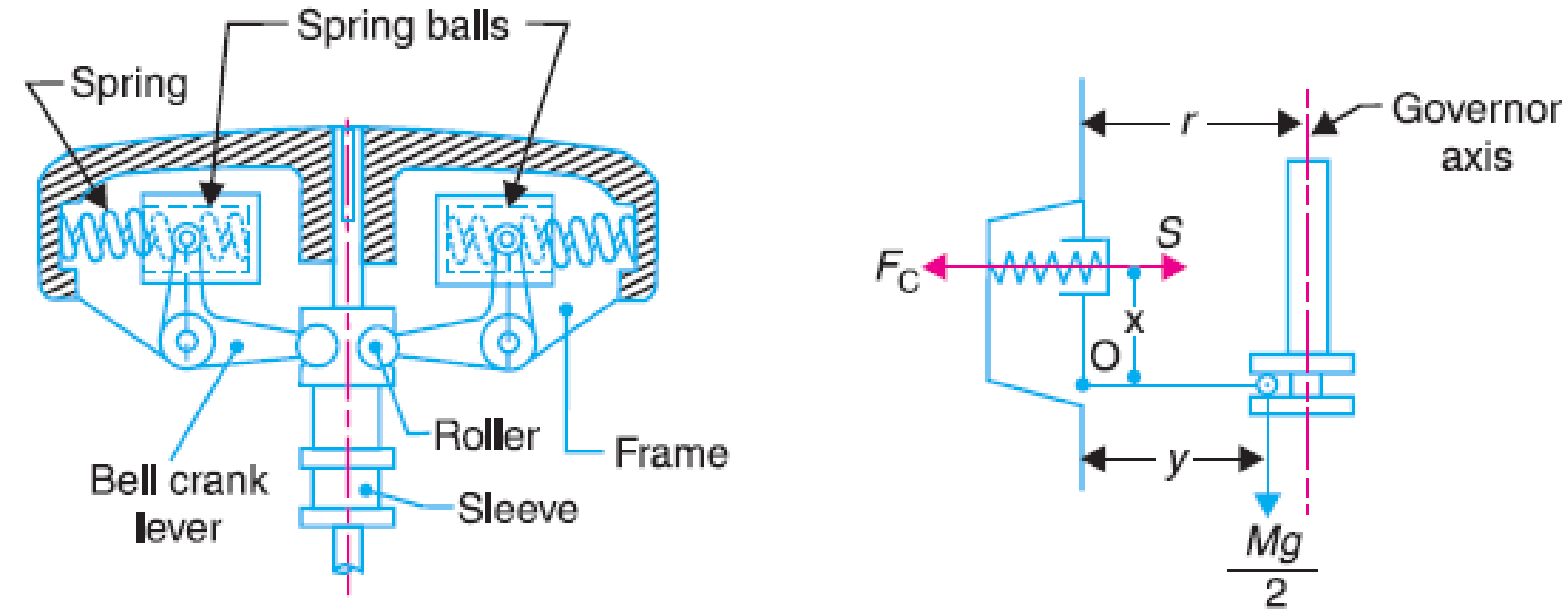
From equations (9), (10) and (11),

$$\frac{F_{C2} - F_{C1}}{r_2 - r_1} = \frac{F_C - F_{C1}}{r - r_1} = \frac{F_{C2} - F_C}{r_2 - r}$$

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{C2} - (F_{C2} - F_{C1})\left(\frac{r_2 - r}{r_2 - r_1}\right)$$

4.2.5 Hartung Governor

A spring controlled governor of the Hartung type is shown in **Figure**. In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

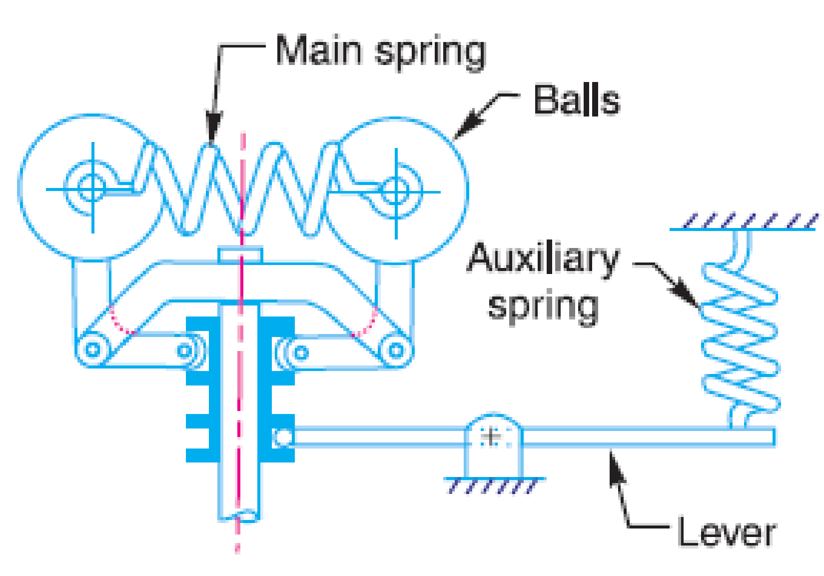


Neglecting the effect of obliquity of the arms, taking moments about the fulcrum O,

$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

4.2.6 Wilson-Hartnell Governor

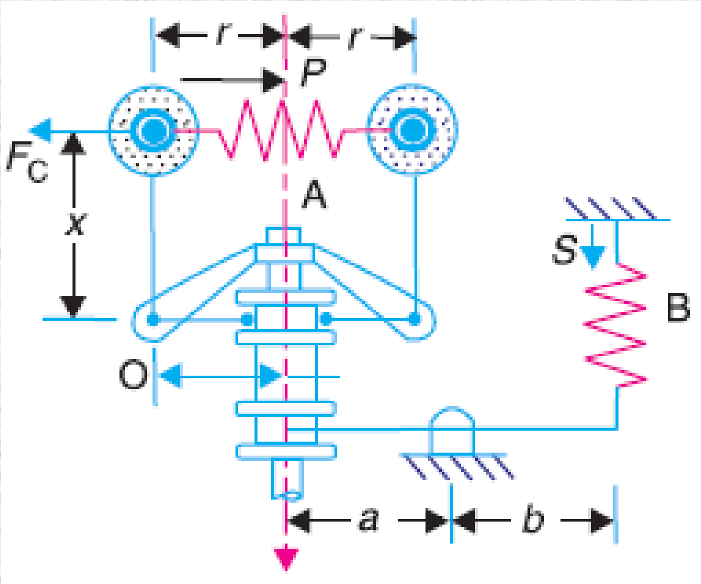
A Wilson-Hartnell governor is a governor in which the balls are connected by a spring in tension as shown in **Figure**. An auxiliary spring is attached to the sleeve mechanism through a lever by means of which the equilibrium speed for a given radius may be adjusted. The main spring may be considered of two equal parts each belonging to both the balls.



Wilson-Hartnell governor.

Now total downward force on the sleeve

$$= M.g + S \times b/a$$



Taking moments about O,

$$(F_C - P)\,x = \frac{M.g + S \times b/a}{2} \times y$$

At minimum equilibrium speed,

$$(F_{C1} - P_1)\,x = \frac{M.g + S_1 \times b/a}{2} \times y$$

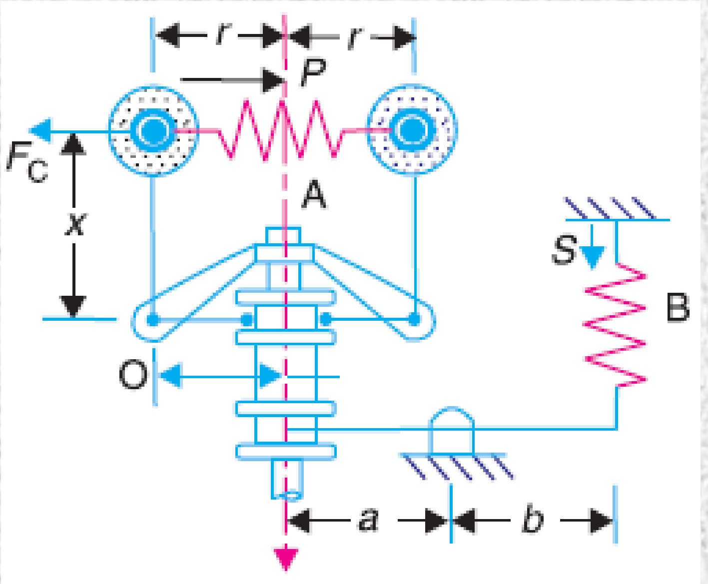
At maximum equilibrium speed,

$$(F_{C2} - P_2)\,x = \frac{M.g + S_2 \times b/a}{2} \times y$$

Subtracting

$$[(F_{C2} - F_{C1}) - (P_2 - P_1)]\,x = (S_2 - S_1)\,\frac{b}{a} \times \frac{y}{2}$$

When the radius increases from r_1 to r_2, the ball springs extend by the amount $2(r_2 - r_1)$ and the auxiliary spring extend by the amount $(r_2 - r_1)\frac{y}{x} \times \frac{b}{a}$.



$$P_2 - P_1 = 2\, s_b \times 2\,(r_2 - r_1) = 4\, s_b\,(r_2 - r_1)$$

$$S_2 - S_1 = s_a\,(r_2 - r_1)\,\frac{y}{x} \times \frac{b}{a}$$

Substituting the values of $(P_2 - P_1)$ and $(S_2 - S_1)$

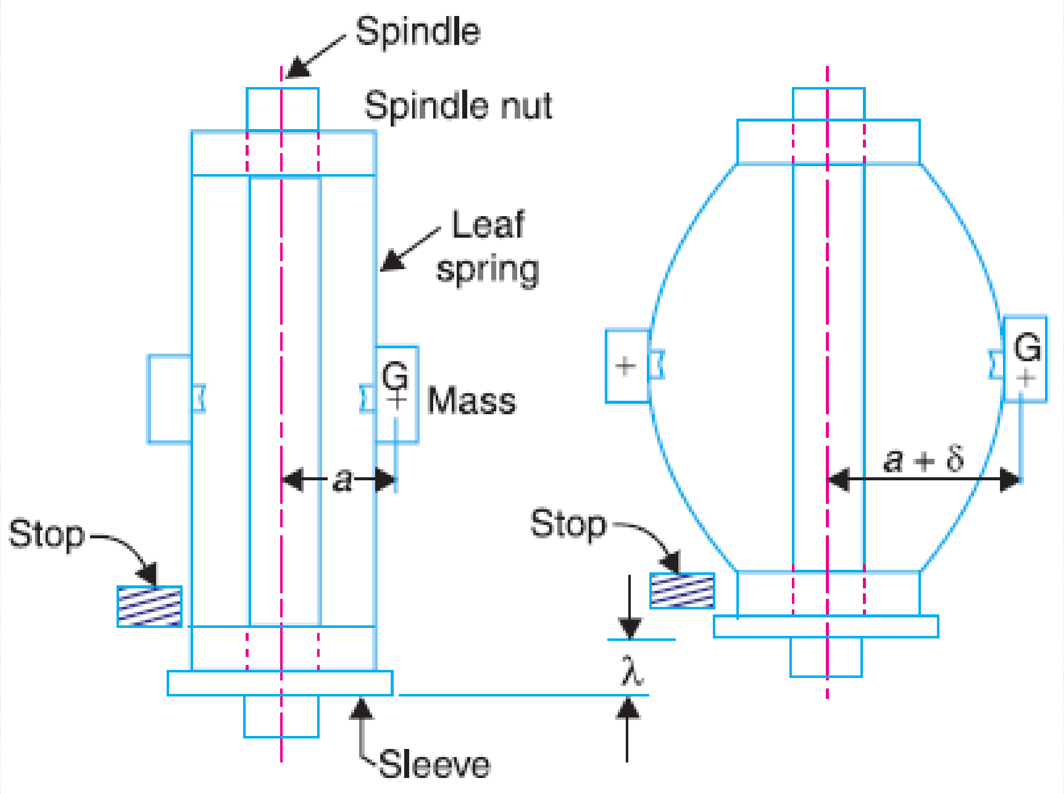
$$[(F_{C2} - F_{C1}) - 4\, s_b\,(r_2 - r_1)]\, x = s_a\,(r_2 - r_1)\,\frac{y}{x} \times \frac{b}{a} \times \frac{b}{a} \times \frac{y}{2}$$

$$(F_{C2} - F_{C1}) - 4\, s_b\,(r_2 - r_1) = \frac{s_a}{2}\,(r_2 - r_1)\left(\frac{y}{x} \times \frac{b}{a}\right)^2$$

$$4\, s_b + \frac{s_a}{2}\left(\frac{y}{x} \times \frac{b}{a}\right)^2 = \frac{F_{C2} - F_{C1}}{r_2 - r_1}$$

4.2.7 Pickering Governor

A Pickering governor is mostly used for driving gramophone. It consists of three straight leaf springs arranged at equal angular intervals round the spindle. Each spring carries a weight at the centre. The weights move outwards and the springs bend as they rotate about the spindle axis with increasing speed.



The maximum deflection of a leaf spring with both ends fixed and carrying a load (W) at the centre is,

$$\delta = \frac{W.l^3}{192\ EI}$$

In case of a Pickering governor, the central load is the centrifugal force

$$W = F_C = m.\omega^2 (a + \delta)$$

Substituting the value of W

$$\delta = \frac{m.\omega^2\ (a + \delta)\ l^3}{192\ E.I}$$

The empirical relation between the lift of the sleeve and the deflection δ is approximately given as

$$\lambda = \frac{2.4\ \delta^2}{l}$$

4.3 Performance of Governors

4.3.1 Sensitiveness of Governors

Sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

Let N_1 = Minimum equilibrium speed,

N_2 = Maximum equilibrium speed, and

N = Mean equilibrium speed $= \dfrac{N_1 + N_2}{2}$

$\therefore$ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

4.3.2 Stability of Governors

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

4.3.3 Isochronous Governors

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

4.3.4 Hunting

A governor is said to be hunt if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place.

Example 4.1

A loaded governor of the Porter type has equal arms and links each **250 mm** long. The mass of each ball is **2 kg** and the central mass is **12 kg**. When the ball radius is **150 mm**, the valve is fully open and when the radius is **185 mm**, the valve is closed. Find the maximum speed and the range of speed. If the maximum speed is to be increased **20%** by an addition of mass to the central load, find what additional mass is required.

192.98 r.p.m. ; 16.03 r.p.m.; 6.16 kg

Example 4.2

A Hartnell governor has a speed range of **290 rpm** to **310 rpm** for a lift of **15 mm**. The sleeve arm and the ball arm **80 mm** and **120 mm** respectively. The radius of rotation of the balls is **120 mm** from the governor axis, when the ball arm is vertical and the speed of the governor is minimum. If the mass of each ball is **2.5 kg**, determine:

a) load on the spring for minimum and maximum speeds, and

b) spring rate.

830.034 N, 1126.307 N, 19.751 kN/m