Department of Mechanical Engineering, Pulchowk campus, Institute of Engineering, Tribhuvan University

ENGINEERING ECONOMICS

Project Evaluation Techniques

Internal Rate of Return (IRR)

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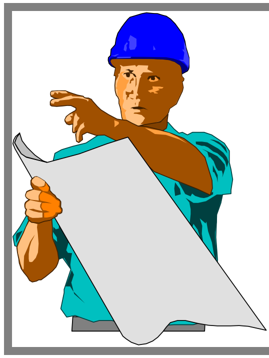
Lecture 9



Limitation of NPV only

	A1	A2
Initial investment	- 1,00,000	- 5,00,000
n	5	5
NPV(10%)	50,000	1,00,000

NPV high so we select

What about high Rate of return?

Internal Rate of Return (IRR)

IRR is the interest rate earned on the unrecovered project balance of investment such that, when the project terminates, the unrecovered project balance will be zero.

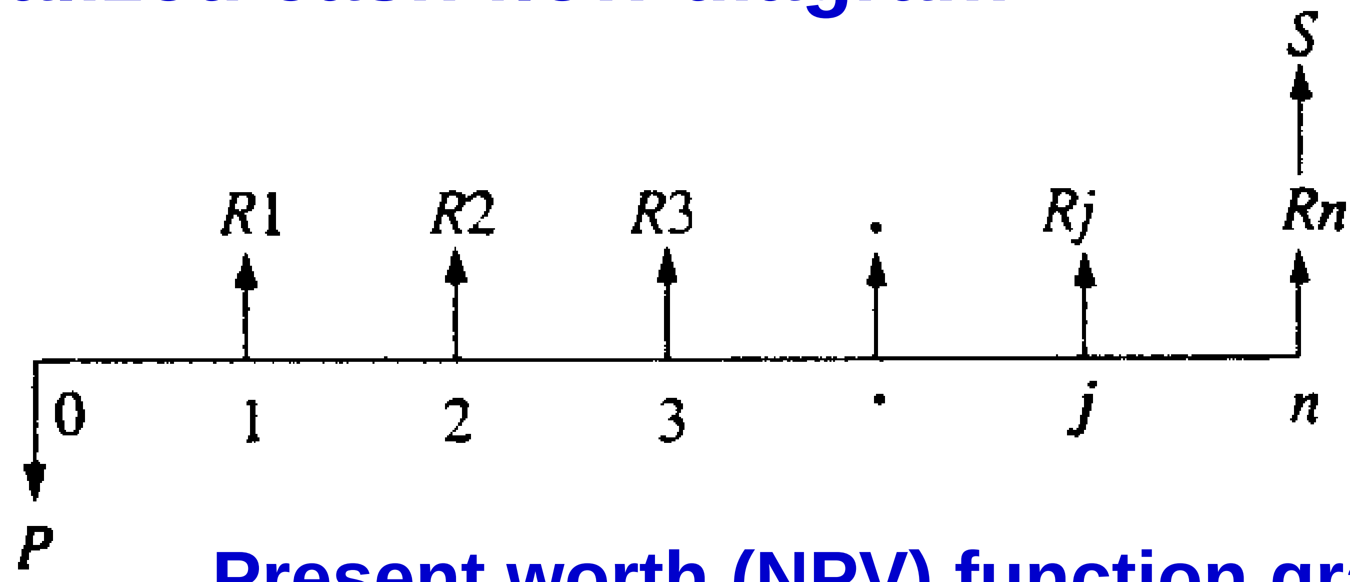
$$NPV = A_0/(1+i^*)^0 + A_1/(1+i^*)^1 + A_2/(1+i^*)^2 \ldots\ldots + A_n/(1+i^*)^n = 0$$

If IRR>MARR, accept the project

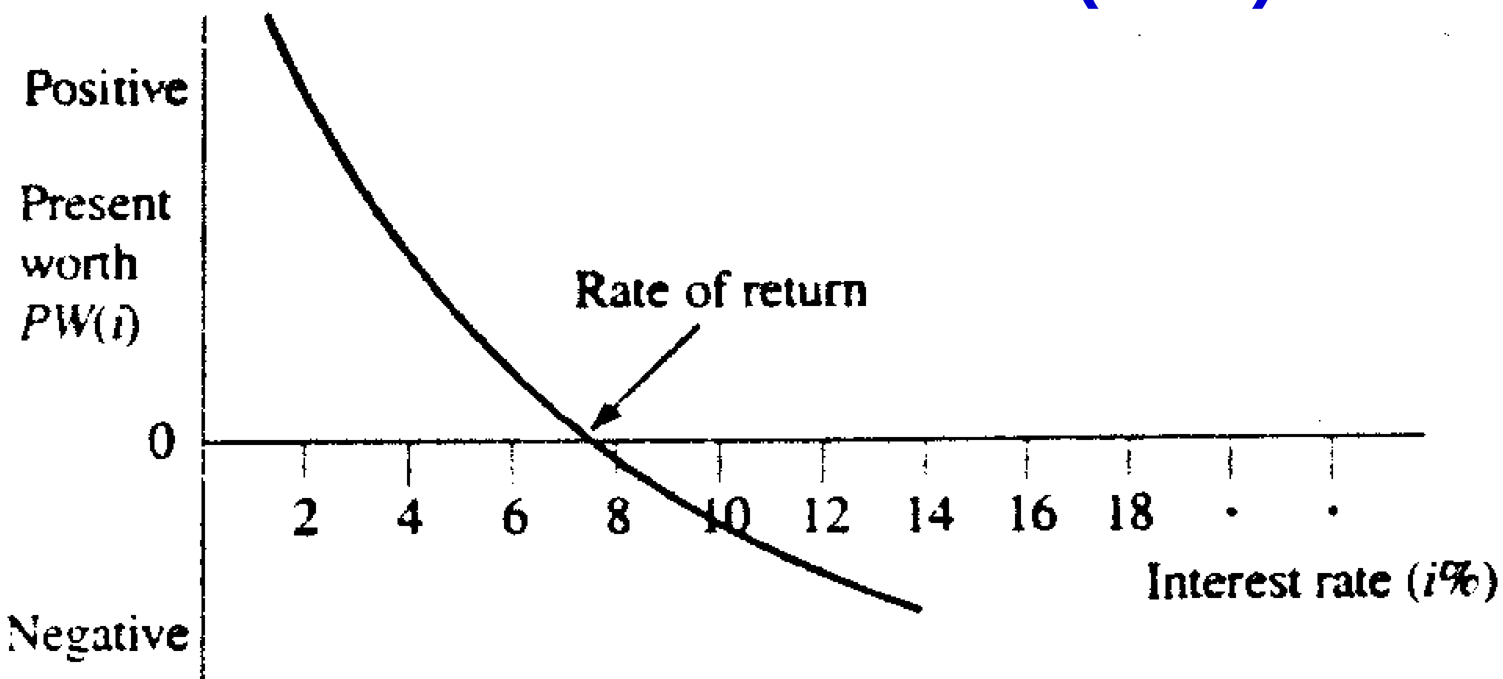
If IRR = MARR, remain indifferent

If IRR < MARR, reject the project

Generalized cash flow diagram



Present worth (NPV) function graph



Simple Investments and Non-simple investment

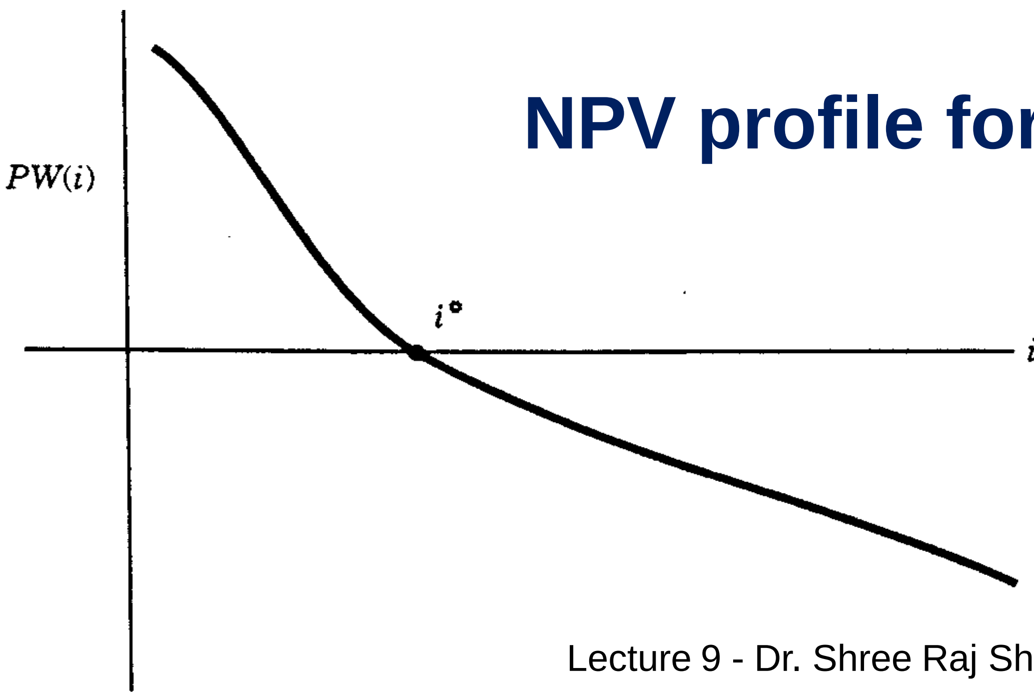
Simple Investments

A simple investment is defined as that investment, when the sign change in the project cash flow occurs only once.

Non-simple Investments

A non-simple investment is that investment where the sign change in the project cash flow occurs more than once

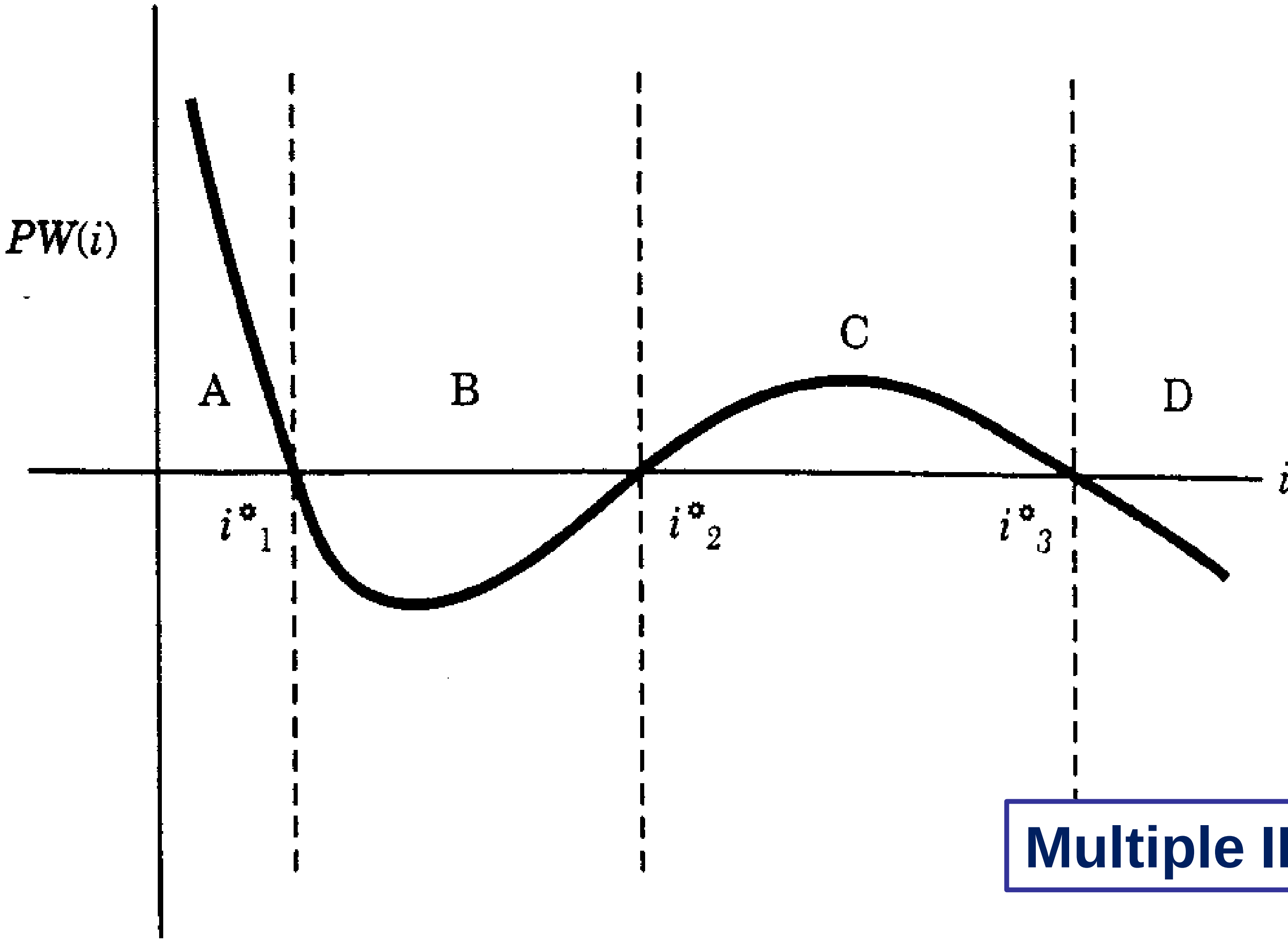
Investment	Cash Flow Sign at Period					
Type	*0*	*1*	*2*	*3*	···	*N*
Simple	−	+	+	+	···	+
Simple	−	−	+	+	···	+
Nonsimple	−	+	−	+	···	−
Nonsimple	−	+	+	−	···	+



NPV profile for a simple investment

Single IRR

NPW profile for a typical non-simple investment



Multiple IRR

Multiple IRR

When there are **multiple values of IRR**, we can **predict unique value** of **IRR** by **examining its cash flows**.

1. Net cash flow rule of sign

2. Accumulated net cash flow rule of sign

Net Cash flow rule of sign

The **number of real i*** that are greater than -100% for a project with 'n' periods is **never greater than the number of sign changes** in the **sequence of the A_n values**.

Period	Net Cash Flow	Sign Change
0	−$100	
1	−20	
2	50	$\boxed{1}$
3	0	
4	60	
5	−30	$\boxed{1}$
6	100	$\boxed{1}$

There are **3 sign changes** in the cash flow sequence, **so** there are **3 or fewer real positive i*s**.

Accumulated Cash flow rule of sign

If the net cash flow sign test shows multiple values of i*, then we should proceed to this sign test.

If the series of cumulative cash flows start negatively and changes the sign only once, then there exists a unique positive i*.

Period (n)	Cash Flow (A_n)	Accumulated Cash Flow (S_n)
0	A_0	$S_0 = A_0$
1	A_1	$S_1 = A_0 + A_1$
2	A_2	$S_2 = A_0 + A_1 + A_2$
$\vdots$	$\vdots$	$\vdots$
N	A_N	$S_N = A_0 + A_1 + A_2 + \cdots + A_N$

Predict the number of real positive rate(s) of return (IRR) for each cash flow series:

Period	A	B	C	D
0	−$100	−$100	$ 0	−$100
1	−200	+50	−50	+50
2	+200	−100	+115	0
3	+200	+60	−66	+200
4	+200	−100		−50

The **cash flow rule of signs** **indicates** the following **possibilities for** the **positive values of IRR (i*)**

Project	Number of Sign Changes in Net Cash Flows	Possible Number of Positive Values of i^*
A	1	1 or 0
B	4	4, 3, 2, 1 or 0
C	2	2, 1, or 0
D	2	2, 1, or 0

For **project A** there is only **once sign change** therefore there is only **one real IRR value**

For cash flows **B, C, and D**, we would like to **apply** the more discriminating **cumulative cash flow test** to see if we can **specify** a **smaller number of possible values of real IRR (i*):**

Project B		Project C		Project D	
Net Cash Flow	*Cumulative Cash Flow*	*Net Cash Flow*	*Cumulative Cash Flow*	*Net Cash Flow*	*Cumulative Cash Flow*
−$100	−$100	$ 0	$ 0	−$100	−$100
+50	−50	−50	−50	+50	−50
−100	−150	+115	+65	0	−50
+60	−90	−66	−1	+200	+150
−100	−190			−50	+100

Only project D begins negatively and passes the test; we may **predict a unique i* value**, rather than 2, 1, or 0 as predicted by the cash flow rule of signs. **Projects B and C fail the test** and we **cannot eliminate the possibility of multiple i*s.**

Methods for determining IRR

1. **The direct-solution method**
2. **The trial-and-error method, and**
3. **The graphic method**

1. The direct-solution method

For **project with two-flow transaction** (Investment with single future payment) **or project with a service life of 2 years of return**.

Use direct Mathematical Solution

n	Project 1	Project 2
0	−$1000	−$2000
1	0	1300
2	0	1500
3	0	
4	+1500	

For Project 1

If **PW(i*) = 0** then **FW(i*) = 0**

$$PW(i^*) = -\$1000 + \$1500\,(P/F, i^*, 4)$$

$$= 0$$

$$\$1000 = \$1500 / (1 + i^*)^4$$

$$\$1000\,(1 + i^*)^4 = \$1500$$

$$(1 + i^*)^4 = 1.5$$

$$1.5 = (1 + i^*)^4$$

Taking a natural log on both sides, we obtain

$$ln(1.5) = 4ln(1 + i^*)$$

$$ln(1 + i^*) = 0.1014.$$

Solving for i^* yields

$$i^* = e^{0.1014} - 1$$

$$= 0.1067, \text{ or } 10.67\%.$$

For Project 2:

$$PW(i) = -\$2000 + \frac{\$1300}{(1+i)} + \frac{\$1500}{(1+i)^2} = 0.$$

Let $X = \frac{1}{(1+i)}$. We may then rewrite the $PW(i)$ as a function of X.

$$PW(i) = -\$2000 + \$1300X + \$1500X^2 = 0$$

This is a quadratic equation having the following solution[2]

$$X = \frac{-\$1300 \pm \sqrt{\$1300^2 - 4(\$1500)(-\$2000)}}{2(\$1500)}$$

$$= \frac{-\$1300 \pm \$3700}{\$3000}$$

$$= 0.8 \text{ or, } -1.667.$$

[2] Given $aX^2 + bX + c = 0$, the solution of the quadratic equation is

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Replacing X-values and solving for i gives us

$$0.8 = \frac{1}{(1+i)}, \quad i = 25\%$$

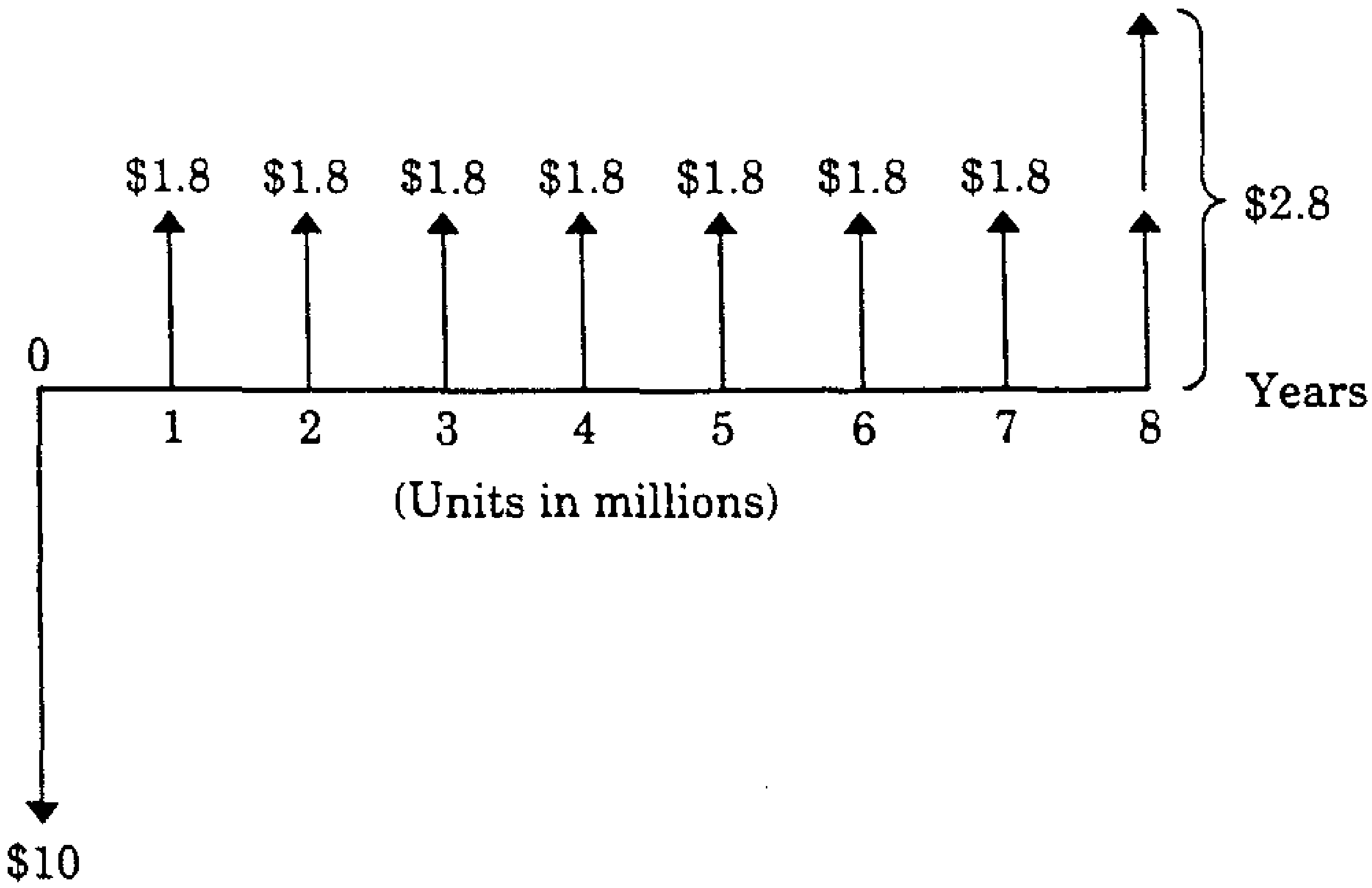
$$-1.667 = \frac{1}{(1+i)}, \quad i = -160\%$$

Since an interest rate less than -100% has no economic significance, we find that the project's i^* is 25%.

2. The trial-and-error method

Agrotech Corporation is considering a proposal to establish a facility to manufacture an electronically controlled "intelligent" crop sprayer. This project would **require** an **investment of $10 million in assets** and would **produce** an **annual after-tax net benefit of $1.8 million** over a **service life of 8 years**. When the project terminates, the **net salvage value** of the assets would be **$1 million**. **Compute** the **rate of return** of this project.

Given: Initial investment (I) = $10 million,
A = $1.8 million, S = 1 million, N = 8 years
Find: Internal Rate of Return (i*)



We **start with** a **guessed interest rate of 8%.**
The **present worth of the cash flows** in millions of dollars is

$$PW(8\%) = -\$10 + \$1.8(P/A, 8\%, 8) + \$1(P/F, 8\%, 8) = \$0.88.$$

Since this **NPV is positive**, we must **raise the interest rate** to **bring** this **value** toward **zero**.
When we use an interest rate of 12%, we find that

$$PW(12\%) = -\$10 + \$1.8(P/A, 12\%, 8) + \$1(P/F, 12\%, 8) = -\$0.65.$$

PW(i) will be **zero** at **i** somewhere **between 8% and 12%.**
Using straight-line interpolation, we approximate

$$i^* \cong 8\% + (12\% - 8\%)\left[\frac{0.88 - 0}{0.88 - (-0.65)}\right]$$

$$= 8\% + 4\%(0.5752)$$

$$= 10.30\%.$$

If we compute the **present worth at** this **interpolated value**, we obtain

$$PW(10.30\%) = -\$10 + \$1.8(P/A, 10.30\%, 8) + \$1(P/F, 10.30\%, 8)$$

$$= -\ \$0.045.$$

As this is **not zero**, we may **recompute the i* at a lower interest rate**, say **10%.**

$$PW(10\%) = -\$10 + \$1.8(P/A, 10\%, 8) + \$1(P/F, 10\%, 8) = \$0.069$$

With **another round of linear interpolation**, we approximate

$$i^* \cong 10\% + (10.30\% - 10\%)\left[\frac{0.069 - 0}{0.069 - (-0.045)}\right]$$

$$= 10\% + 0.30\%(0.6053)$$

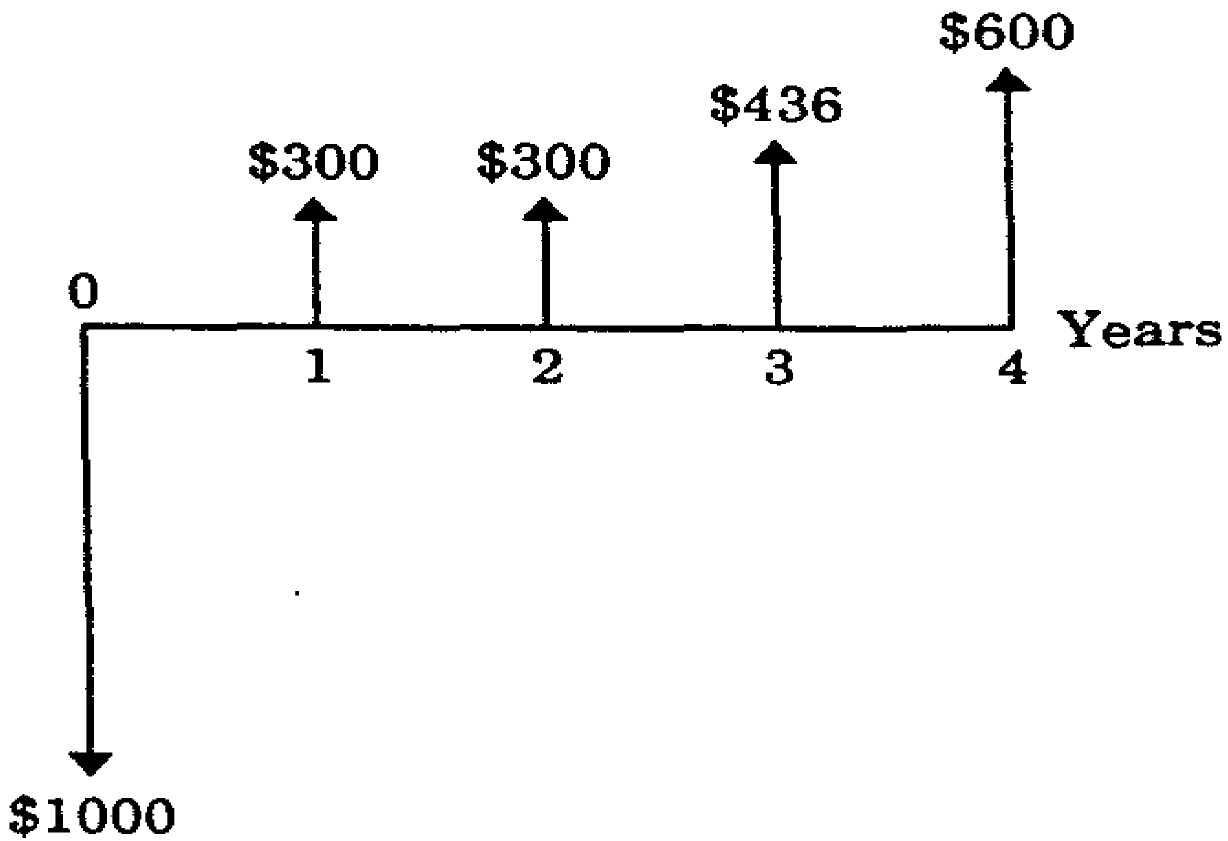
$$= 10.18\%.$$

At this interest rate.

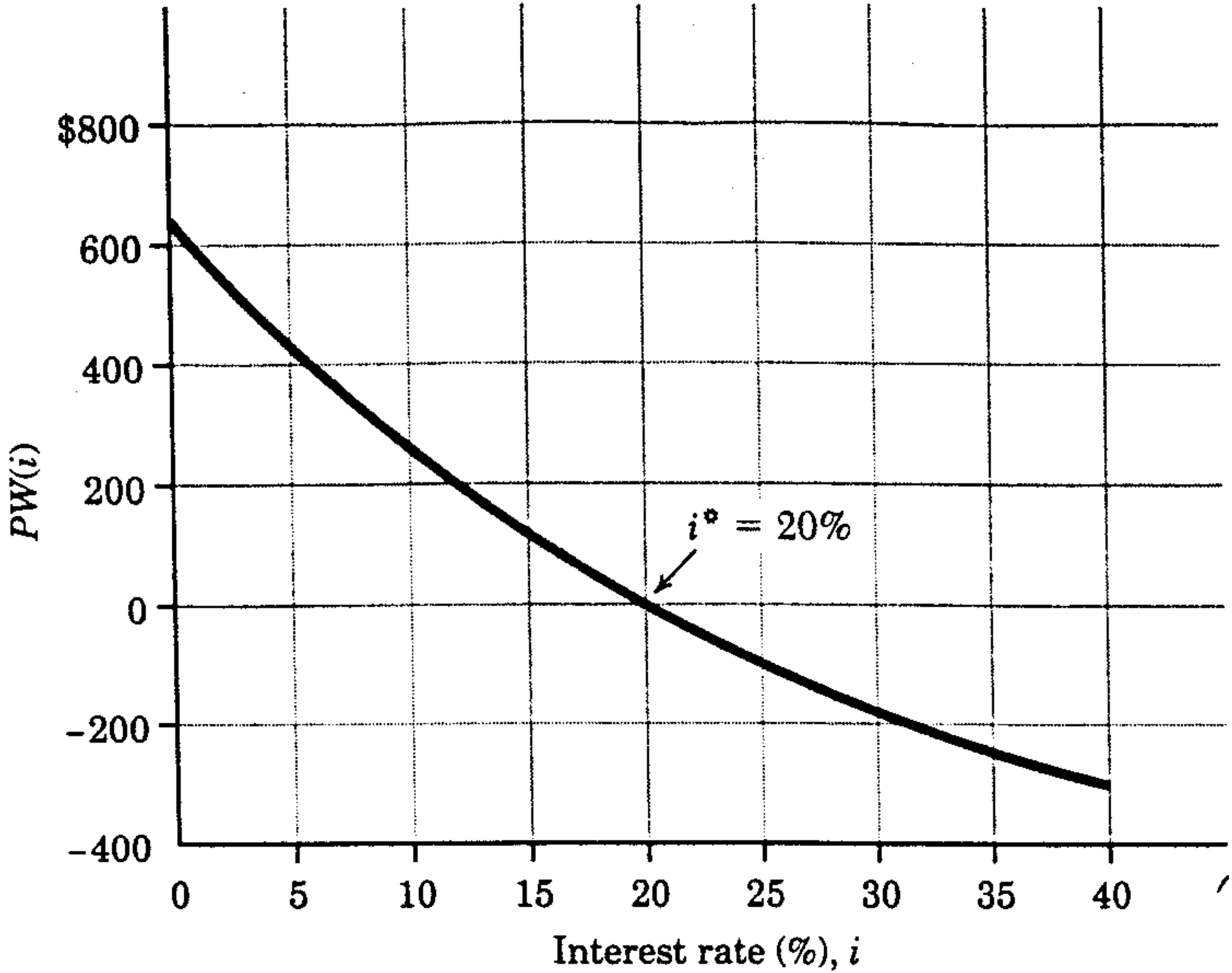
$$PW(10.18\%) = -\$10 + \$1.8(P/A, 10.18\%, 8) + \$1(P/F, 10.18\%, 8)$$

$$= \$0.0007,$$

3. The graphic method



1. We first use $i = 0$ in this equation, to obtain $PW(0\%) = \$636$, which is the vertical axis intercept.
2. Substituting in several other interest rates—5%, 10%, 20%, and 30%—we plot these values of $PW(i)$ as well.


$800
600
400
200
0
−200
−400
PW(i)
i* = 20%
0
5
10
15
20
25
30
35
40
Interest rate (%), i

Solve

Consider two <u>independent</u> investments, A and B, with the following sequences of cash f lows:
Compute the IRR (i*) for each investment.

Net Cash Flow

n	*Project A*	*Project B*
0	−$379	−$379
1	20	100
2	60	100
3	60	100
4	100	100
5	280	100

Incremental IRR Analysis

- General IRR Ranking ignores the scale of the investment.
- Incremental IRR Method address this

n	A1	A2
0	- 1,000	- 5,000
1	2,000	7,000
IRR	100%	40%
PW(10%)	$818	$1364

Initial Investment high for A2

A1 is better with respect to IRR

A2 is better with respect to NPV or PW

MARR = 10%

Incremental IRR Analysis

- General IRR Ranking ignores the scale of the investment.
- Incremental IRR Method address this

n	A1	A2	A2 - A1
0	- 1,000	- 5,000	-4,000
1	2,000	7,000	5,000
IRR	100%	40%	25%
PW(10%)	$818	$1364	$545

MARR = 10%

Incremental IRR Method **measures** the **rate of return** value **for difference in** the **investment amount**

Incremental IRR Testing Procedure

- Generate the **Incremental cash flow** by **subtracting higher** initial investment **project cash flow** (**B**) **with smaller one's** (**A**) so that the initial investment cash flow becomes negative **(B-A).**

- Calculate the IRR_{B-A}

- Selection Criteria

If $IRR_{B-A} > MARR$, select B
If $IRR_{B-A} = MARR$, select either one
If $IRR_{B-A} < MARR$, select A

Example

n	B1	B2	B2 – B1
0	- 3,000	- 12,000	- 9,000
1	1,350	4,200	2,850
2	1,800	6,225	4,425
3	1,500	6,330	4,830

MARR = 10%

NPV (i) = - 9000 + 2850(P/F, I, 1) + 4425(P/F, I, 2) + 4830(P/F, I, 3)

$$0 = -9000 + 2850/(1+i^*) + 4425/(1+i^*)^2 + 4830/(1+i^*)^3$$

Solving, $i^*_{B-A} = 15\%$

Select project B

Perform Incremental IRR Analysis and Select the best option

n	D1	D2	D3
0	-2000	-1000	-3000
1	1500	800	1500
2	1000	500	2000
3	800	500	1000
IRR	34.37%	47.76%	24.81%

Practice

- 7.6, 7.8, 7.11, 7.20, 7.27, 7.29, 7.37, 7.39, 7.41, 7.45, 7.49