Design Aspects
Turbine
Turbo-machines

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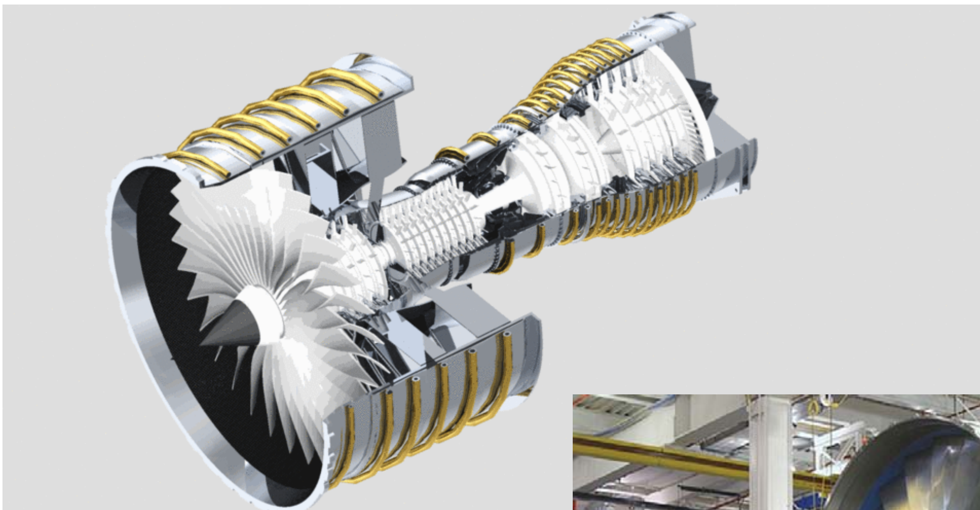
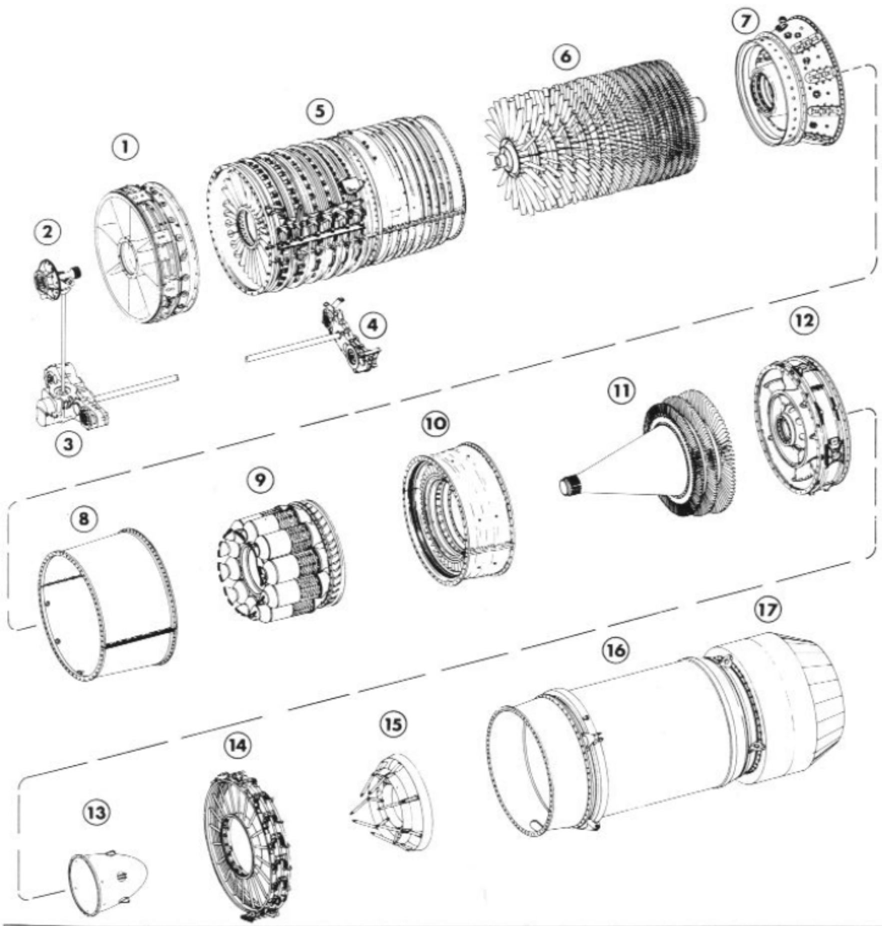
Turbo-machines



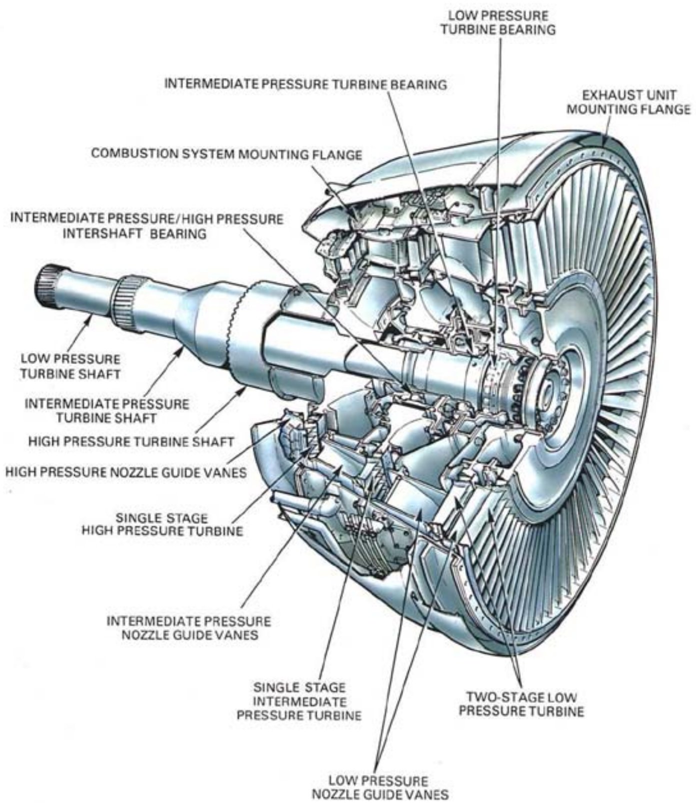
V2500 Turbo Fan Engine

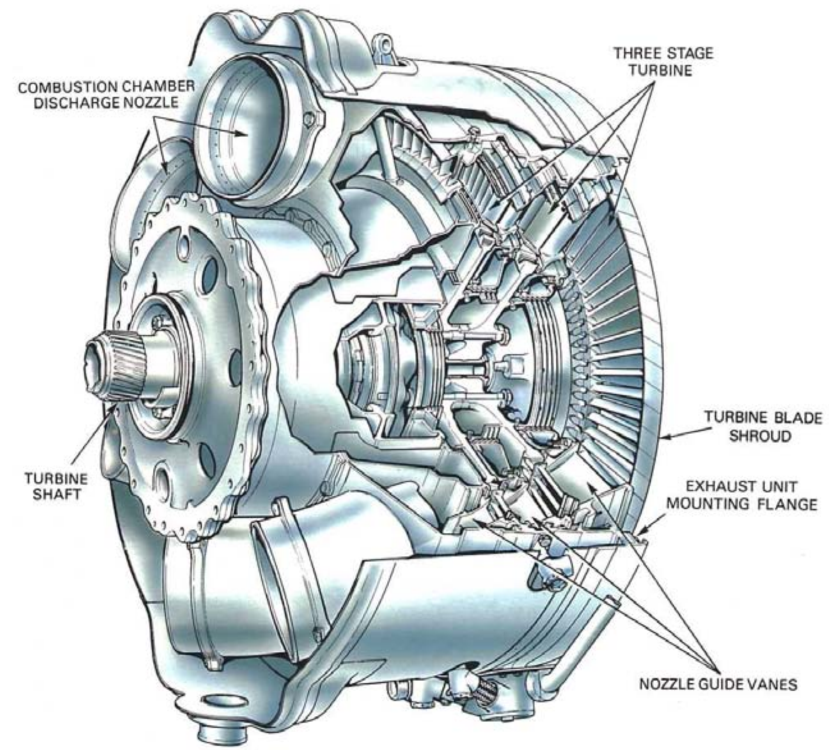
Turbo-machines



1 Compressor front frame
2 Bevel gear
3 Transfer gearbox
4 Accessory gearbox
5 Compressor casing
6 Rotor
7 Compressor rear frame
8 Combustion casing
9 Combustion assembly
10 Turbine casing
11 Turbine rotor
12 Turbine rear frame
13 Rear cone
14 Reheat fuel manifold assembly
15 Flame holder
16 Afterburner
17 Exhaust nozzle

Turbo-machines



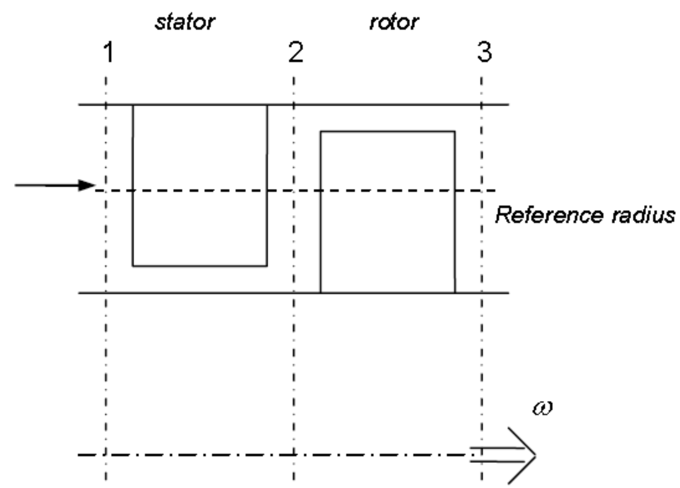


Nomenclature

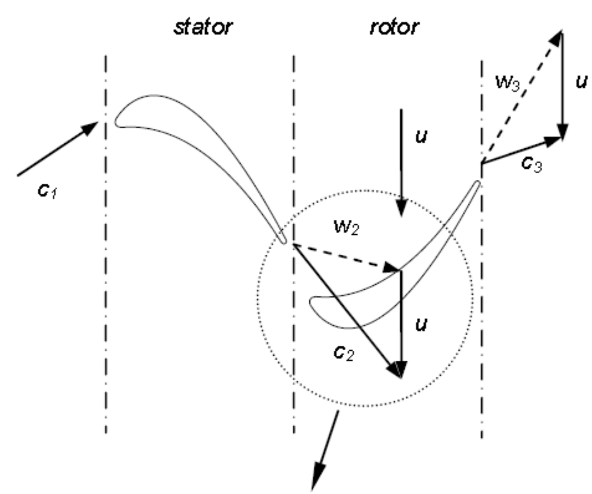
Turbine

Stage denotations

1 stator inlet
2 rotor inlet
3 rotor outlet



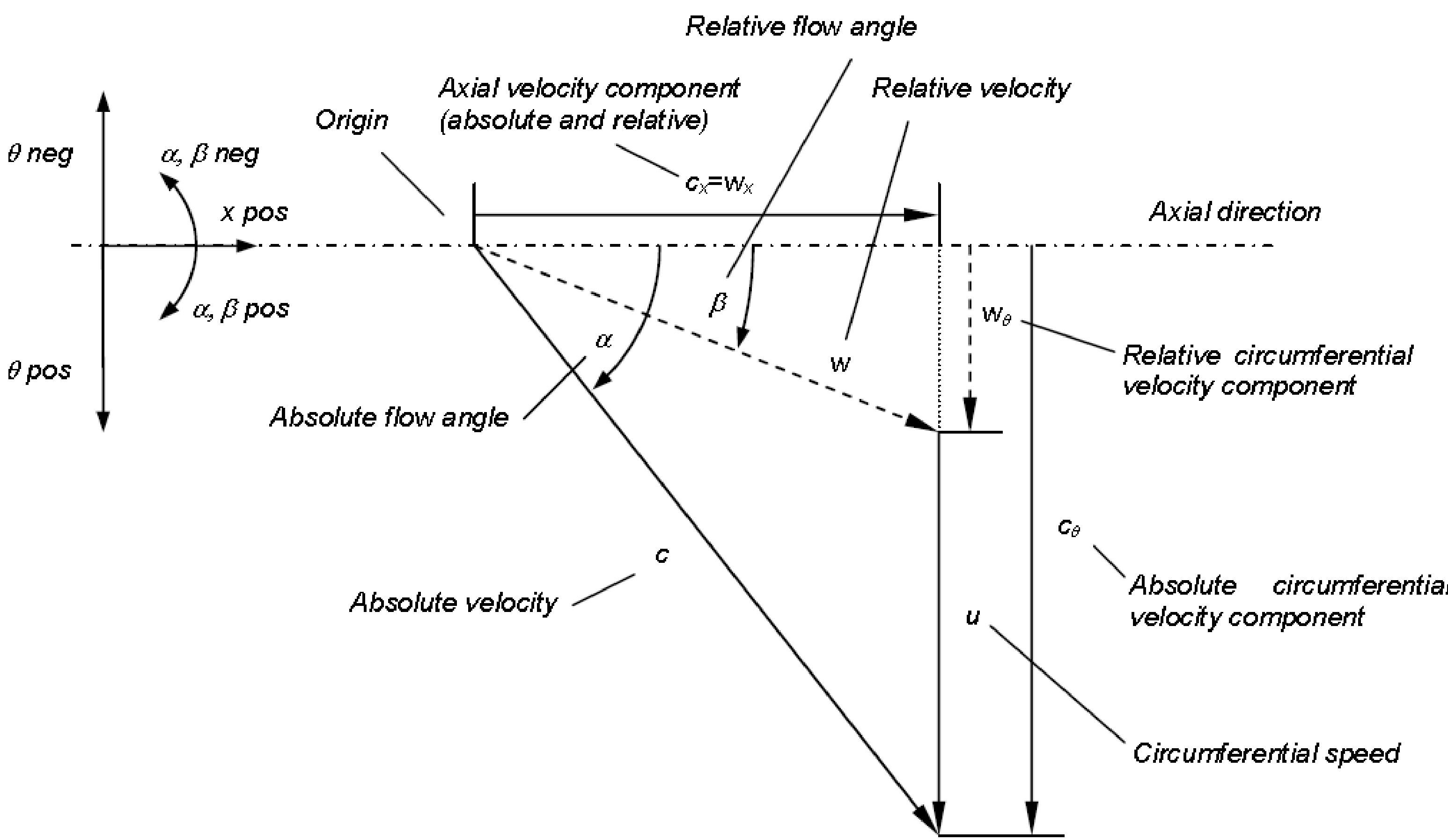
Stage velocity triangles



Velocity triangles denotations and conventions

Nomenclature

Turbine



Stage Velocity Triangle

Reference Position:

Mean Radius: $$r_m = \frac{r_h + r_s}{2}$$ Eq. 1

Euler's Radius: (Radius that splits the annular cross section in half)

$$r_E = \sqrt{\frac{r_h^{\ 2} + r_s^{\ 2}}{2}}$$ Eq. 2

Nomenclature

Nomenclature

Stage Velocity Triangle

Circumferential Speed of the Rotor u at reference radius:

$$u = r_{ref} \cdot \omega \qquad \text{Eq. 3}$$

Relation between absolute velocity ‘c’, relative velocities ‘w’ and circumferential speed ‘u’ are references as follows:

$$w_x = c_x \qquad \text{Eq. 4}$$

$$w_\theta = c_\theta - u \qquad \text{Eq. 5}$$

Where, c_x and c_θ are axial and circumferential components of the respective velocities

Nomenclature

Stage Velocity Triangle

Where cx and cθ are the axial and circumferential components of the respective velocity as follows:

$$c^2 = c_x^{\ 2} + c_\theta^{\ 2} \qquad \text{Eq. 6}$$

$$w^2 = w_x^{\ 2} + w_\theta^{\ 2} \qquad \text{Eq. 7}$$

The flow angles are defined as

$$\tan\alpha = \frac{c_\theta}{c_x} \qquad \text{Eq. 8}$$

$$\tan\beta = \frac{w_\theta}{w_x} \qquad \text{Eq. 9}$$

First: Degree of Reaction 'R'

Degree of Reaction (R) is the change in Enthalpy effectuated in rotor to the change in enthalpy of the stage:

$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} \qquad \text{Eq. 10}$$

It can be written as:

$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{\Delta h_{rotor}}{\Delta h_{stator} + \Delta h_{rotor}} = \frac{h_2 - h_3}{h_1 - h_2 + h_2 - h_3} \qquad \text{Eq. 11}$$

How is enthalpy related to Velocities???

Design Parameter

First: Degree of Reaction 'R'

The change in enthalpies in stator and rotor respectively are related to the velocities as follows:

In the stator the stagnation enthalpy $h_0 = h + \frac{c^2}{2}$ is constant, thus

Thus,

$$h_1 + \frac{c_1^{\ 2}}{2} = h_2 + \frac{c_2^{\ 2}}{2} \qquad \text{Eq. 12}$$

Which leads to:

$$h_1 - h_2 = \frac{1}{2}\left(c_2^{\ 2} - c_1^{\ 2}\right) \qquad \text{Eq. 13}$$

Design Parameter

First: Degree of Reaction 'R'

In rotor the rothalpy is constant

$$I = h + \frac{w^2}{2} - \frac{u^2}{2} \text{ is constant}$$

Thus,

$$h_2 + \frac{w_2{}^2}{2} - \frac{u_2{}^2}{2} = h_3 + \frac{w_3{}^2}{2} - \frac{u_3{}^2}{2} \qquad \text{Eq. 14}$$

Which leads to:

$$h_2 - h_3 = \frac{1}{2}\left(w_3{}^2 - w_2{}^2 - u_3{}^2 + u_2{}^2\right) \qquad \text{Eq. 15}$$

Design Parameter

First: Degree of Reaction 'R'

Substituting all the expression into the equation of stage reaction above leads to the following general expression

$$R = \frac{{w_3}^2 - {w_2}^2 - {u_3}^2 + {u_2}^2}{{c_2}^2 - {c_1}^2 + {w_3}^2 - {w_2}^2 - {u_3}^2 + {u_2}^2} \qquad \text{Eq. 16}$$

For a normal repetition stage with the following restrictions

$$\vec{c}_1 = \vec{c}_3 \qquad \text{Eq. 17}$$

$$c_{x,1} = c_{x,2} = c_{x,3} = const \qquad \text{Eq. 18}$$

$$u_2 = u_3 \qquad \text{Eq. 19}$$

Design Parameter

First: Degree of Reaction 'R'

We get further expressions as:

$$c^2 = c_x^2 + c_\theta^2$$

$$w^2 = w_x^2 + w_\theta^2$$

Thus yielding,

$$R = \frac{w_{\theta,3}^2 - w_{\theta,2}^2}{c_{\theta,2}^2 - c_{\theta,1}^2 + w_{\theta,3}^2 - w_{\theta,2}^2} \qquad \text{Eq. 20}$$

Design Parameter

First: Degree of Reaction 'R'

The relative velocity components in the denominator shall be expressed by the absolute velocity components as $w_\theta = c_\theta - u$ leading to

$$R = \frac{w_{\theta,3}{}^2 - w_{\theta,2}{}^2}{c_{\theta,2}{}^2 - c_{\theta,1}{}^2 + c_{\theta,3}{}^2 - 2c_{\theta,3}u + u^2 - c_{\theta,2}{}^2 + 2c_{\theta,2}u - u^2} \qquad \text{Eq. 21}$$

After canceling out elements the expression can be rewritten as

$$R = \frac{w_{\theta,3}{}^2 - w_{\theta,2}{}^2}{2u \cdot (c_{\theta,2} - c_{\theta,3})} \qquad \text{Eq. 22}$$

First: Degree of Reaction 'R'

At the numerator shall be expressed as

$$w_{\theta,3}{}^2 - w_{\theta,2}{}^2 = \left(w_{\theta,3} - w_{\theta,2}\right)\cdot\left(w_{\theta,3} + w_{\theta,2}\right)$$

Also, $c_\theta = w_\theta + u$

Leads to

$$R = -\frac{\left(w_{\theta,3} - w_{\theta,2}\right)\cdot\left(w_{\theta,3} + w_{\theta,2}\right)}{2u\cdot\left(w_{\theta,2} + u_2 - w_{\theta,3} - u_3\right)} \qquad \text{Eq. 23}$$

Design Parameter

First: Degree of Reaction 'R'

Both the circumferential speeds in the denominator and the relative components $w_{\theta,3} - w_{\theta,2}$ cancel out finally yielding

$$R = -\frac{1}{2u}\left(w_{\theta,3} + w_{\theta,2}\right) \qquad \text{Eq. 24}$$

In the absolute frame of reference as $w_{\theta} = c_{\theta} - u$ yielding

$$R = -\frac{1}{2u}\left(w_{\theta,3} + c_{\theta,2} - u\right) = \frac{1}{2} - \frac{1}{2u}\left(w_{\theta,3} + c_{\theta,2}\right) \qquad \text{Eq. 25}$$

Further expressing R in terms of flow angles $\tan\alpha = \frac{c_{\theta}}{c_x}$

$$R = \frac{1}{2} - \frac{c_x}{2u}\left(\tan\beta_3 + \tan\alpha_2\right) \qquad \text{Eq. 26}$$

First: Degree of Reaction 'R'

$$R = \frac{1}{2} - \frac{c_x}{2u}(\tan\beta_3 + \tan\alpha_2)$$

Eq. 26

- An increase in flow angle α_2 leads to a decrease in degree of reaction ($\alpha_2 \uparrow \Rightarrow R \downarrow$), i.e. the contribution of enthalpy change in the stator to the total change in enthalpy in the stage gets larger
- An increase in flow angle β_3 leads to an increase in degree of reaction ($\beta_3 \uparrow \Rightarrow R \uparrow$), i.e. the contribution of enthalpy change in the rotor to the total change in enthalpy in the stage gets larger
- For turbine stages the degree reaction usually lies in the range [0…1]

Second: Loading Factor ‘ψ ’

The loading factor relates the change in total enthalpy effectuated in the stage to the rotational speed as follows:

$$\psi = \frac{\Delta h_0}{u^2} \qquad \text{Eq. 27}$$

Using Euler’s turbine equation $\Delta h_0 = u_2 c_{\theta,2} - u_3 c_{\theta,3}$ leads to

$$\psi = \frac{u_2 c_{\theta,2} - u_3 c_{\theta,3}}{u^2} \qquad \text{Eq. 28}$$

For a normal repetition stage:

$$\vec{c}_1 = \vec{c}_3 \qquad \text{Eq. 29}$$

$$c_{x,1} = c_{x,2} = c_{x,3} = const \qquad \text{Eq. 30}$$

$$u_2 = u_3 \qquad \text{Eq. 31}$$

Second: Loading Factor ‘ψ ’

The expression of the loading factor can further be simplified to

$$\psi = \frac{c_{\theta,2} - c_{\theta,3}}{u} \qquad \text{Eq. 32}$$

Expressing the absolute flow velocities in the relative frame of reference as $c_{\theta} = w_{\theta} + u$ the loading factor can be expressed as

$$\psi = \frac{w_{\theta,2} - w_{\theta,3}}{u} \qquad \text{Eq. 33}$$

An equivalent expression can be obtained by substituting the relative velocity components at position 1 in the absolute frame of references as $c_{\theta} = w_{\theta} + u$

yielding

$$\psi = -1 + \frac{c_{\theta,2} - w_{\theta,3}}{u} \qquad \text{Eq. 34}$$

Second: Loading Factor 'ψ'

Which also can be expressed in terms of flow angels β_2 and α_1 as follows:

$$\psi = -1 + \frac{c_x}{u}\left(\tan\alpha_2 - \tan\beta_3\right) \qquad \text{Eq. 35}$$

According to the convention of velocity components depicted above flow angle β_3 is negative whilst flow angle α_2 is positive. This leads to the observation:

- Increase in flow angles α_2 and β_3 lead to increase in loading factor ($\alpha_2, \beta_3 \uparrow \Rightarrow \psi \uparrow$)
- To obtain a loading factor smaller than one $\tan\beta_3$ must be greater than $\tan\alpha_2$

Design Parameter

Third: Flow coefficient ' Φ '

The flow coefficient relate the axial velocity component to the circumferential speed as follows:

$$\phi = \frac{c_x}{u} \qquad \text{Eq. 36}$$

The only observation to make for this coefficient is that the higher the axial velocity in the stage the higher the flow coefficient. As can be recognized below the flow coefficient stretches the velocity triangles in the axial direction.

Normalized Velocity Triangle

The normalized velocity components are denoted by the respective capital letters and yield from

$$C = \frac{c}{u_3}$$ Eq. 37 $$W = \frac{w}{u_3}$$ Eq. 38 $$U = \frac{u}{u_3}$$ Eq. 39

The special case of a normal repetition stage shall be regarded here for the sake of simplicity. The applied principle is however valid for all types of turbine stages

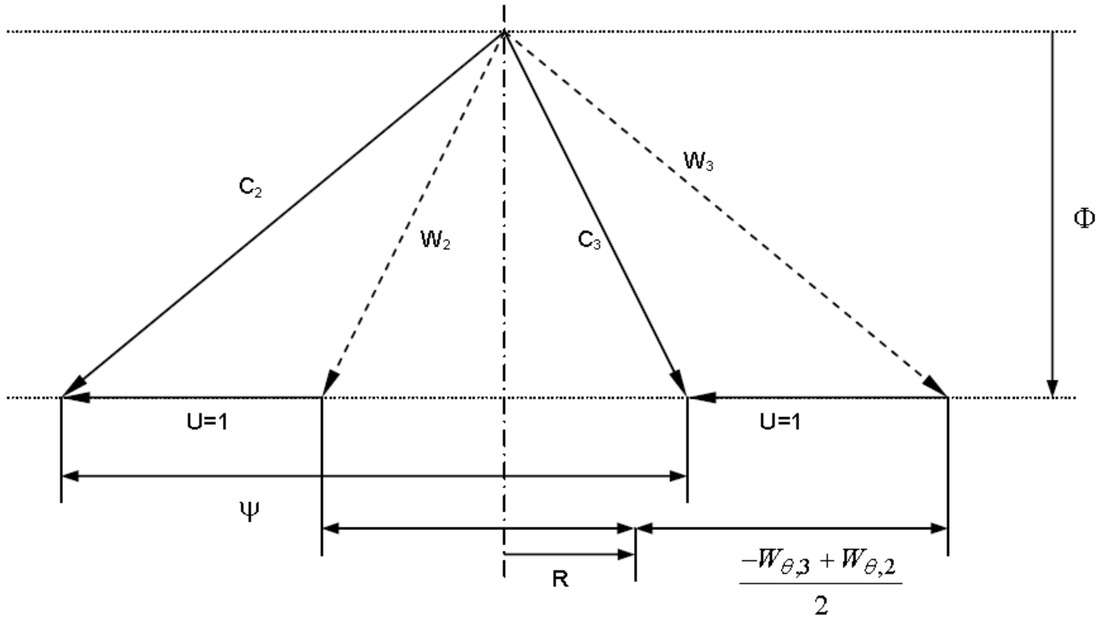
As a normal repetition stage with the condition $c_{x,1} = c_{x,2} = c_{x,3} = const$

Is considered the height of the triangle corresponds to $C_x = \frac{c_x}{u} = \phi$

i.e. the flow coefficient

Design Parameter

Normalized Velocity Triangle



- The height of the velocity triangle corresponds to the flow coefficient Φ
- The loading coefficient corresponds to the circumferential distance between C_2 and C_3. In the case of a repetition stage this equals to the circumferential distance between W_2 and W_3.
- The degree of reaction equals to the distance between axial and half the midpoint between W_2 and W_3.

Normalized Velocity Triangle

Special Cases

Degree of Reaction Equal to Zero (R = 0, Action Turbine)

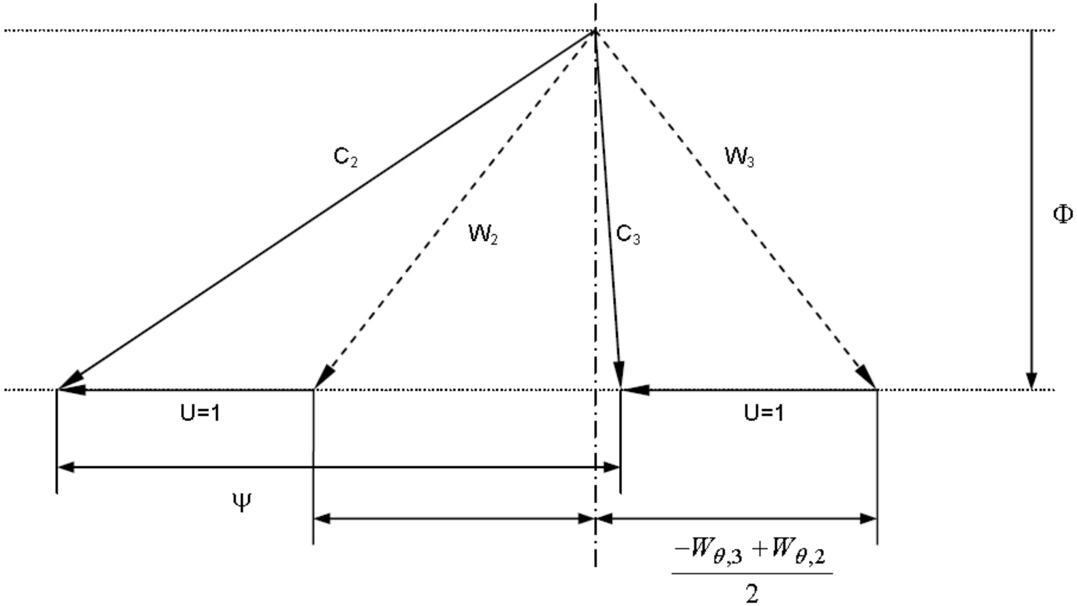
The expression of the degree of reaction yields the following

$$R = 0 = -\frac{1}{2u}\left(w_{\theta,3} + w_{\theta,2}\right) \Rightarrow w_{\theta,2} = -w_{\theta,3} \qquad \text{Eq. 40}$$

Which is equivalent to $w_{\theta,2} = -w_{\theta,3}$ Substituting this expression into the equation of loading coefficient yields

$$\psi = \frac{w_{\theta,2} - w_{\theta,3}}{u} \Rightarrow \psi = \frac{2 \cdot w_{\theta,2}}{u} = \frac{2}{u}\left(c_{\theta,2} - 1\right) \qquad \text{Eq. 41}$$

Normalized Velocity Triangle



- As $w_{\theta,2} = -w_{\theta,3}$ and normal stage it follows that $|w_2| = |w_3|$ and consequently $\Delta h_{rotor} = 0$. The change in enthalpy in an action stage is thus entirely due to change in enthalpy in the stator.
- The forces acting on the rotor are action forces, as the fluid is not accelerated through the rotor. This leads to the denotation of "action stage"
- The rotor only effectuates deflection of the fluid but not expansion as $\beta_2 = -\beta_3$
- As the fluid is not expanded throughout the rotor the pressure up- and downstream of the rotor is (practically) unchanged. In reality a minimum pressure drop is necessary due to losses to drive the fluid, thus $p_3 \approx p_2$
- As a consequence there is little axial force on the rotor in an action turbine

Normalized Velocity Triangle

Special Cases

Degree of Reaction Equal to one half (R = 0.5, Reaction Turbine)

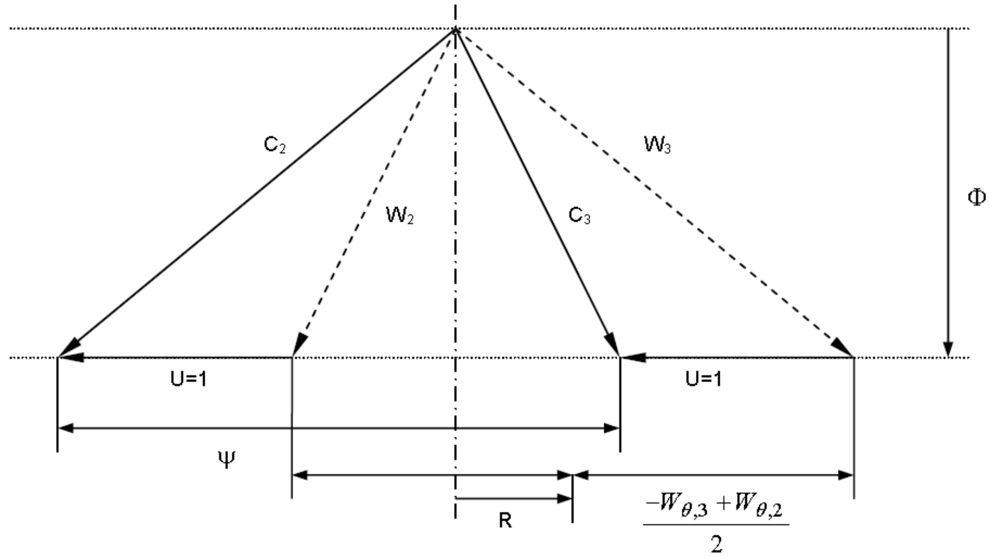
The expression of the degree of reaction yields the following

$$R = \frac{1}{2} = -\frac{1}{2u}\left(w_{\theta,3} + w_{\theta,2}\right) \Rightarrow w_{\theta,2} + u = -w_{\theta,3} \qquad \text{Eq. 42}$$

Which is equivalent to $c_{\theta,2} = -w_{\theta,3}$ Substituting this expression into the equation of loading coefficient yields

$$\psi = \frac{w_{\theta,2} - w_{\theta,3}}{u} \Rightarrow \psi = \frac{2 \cdot w_{\theta,2}}{u} + 1 = \frac{2c_{\theta,2}}{u} - 1 \qquad \text{Eq. 43}$$

Normalized Velocity Triangle



- As $c_{\theta,2} = -w_{\theta,3}$ and normal stage it follows that $|c_2| = |w_3|$ and with the assumption of repetition stage ($c_1 = c_3$) consequently $\Delta h_{rotor} = \Delta h_{stator}$. The change in enthalpy in a reaction stage is thus equally split on stator and rotor.
- The forces acting on the rotor are partially action and partially reaction forces, as the fluid is accelerated through the rotor. This leads to the denotation of “reaction stage”
- Both stator and rotor effectuate expansion of the fluid and thus $p_2 < p_1$ and $p_3 < p_2$
- As a consequence there is a considerable axial force on the rotor in a reaction turbine. In most cases this force is too large to be submitted to an axial bearing and thus must be compensated for. Possible compensations are appropriate arrangement of components such as to cancel out axial forces or application of a thrust compensation devices (e.g. piston), see further below

Normalized Velocity Triangle

Design Parameter

Zero Exit Swirl ($c_{\theta,3} = 0$)

With $c_{\theta,3} = 0 = w_{\theta,3} + u \Rightarrow w_{\theta,3} = -u$ the degree of reaction writes to

$$R = -\frac{1}{2u}\left(w_{\theta,3} + w_{\theta,2}\right) = \frac{1}{2} - \frac{w_{\theta,2}}{2u}$$ Eq. 44

The loading coefficient yields from

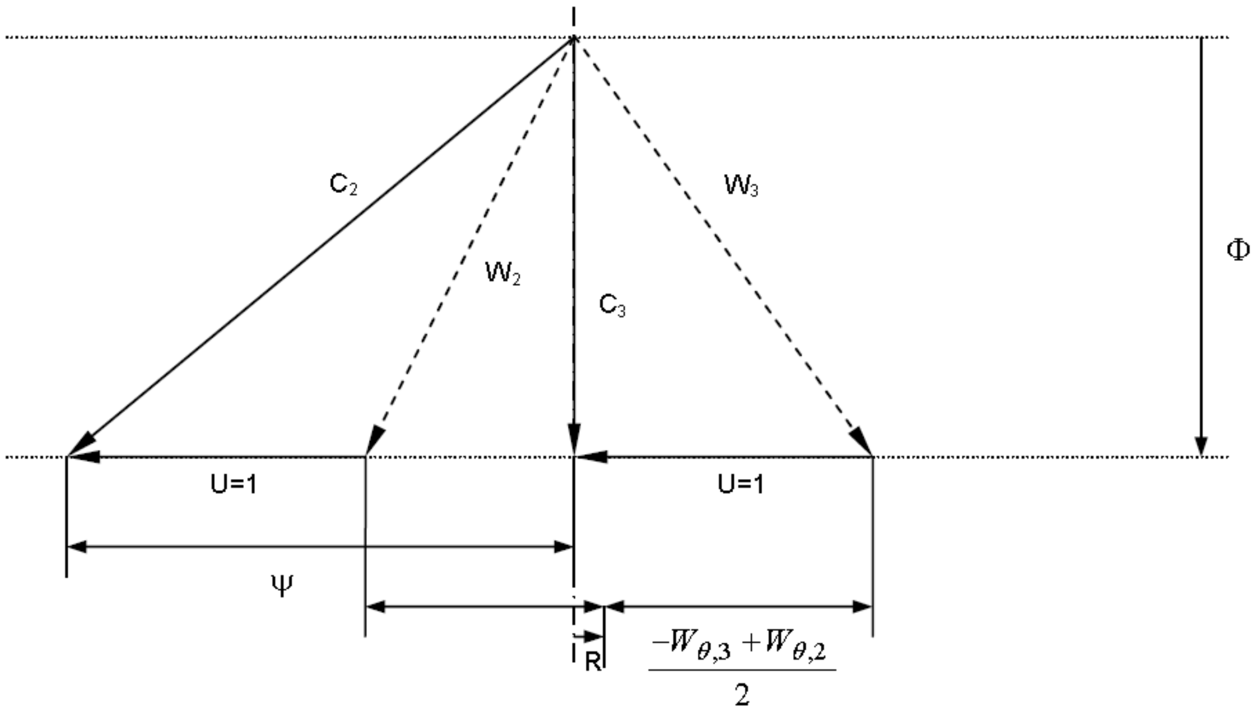
$$\psi = \frac{w_{\theta,2} - w_{\theta,3}}{u} = \frac{w_{\theta,2}}{u} + 1$$ Eq. 43

Combining the two expressions leads after reformulation to a relationship between degree of reaction and loading coefficient as follows:

$$\psi = 2 \cdot (1 - R)$$ Eq. 44

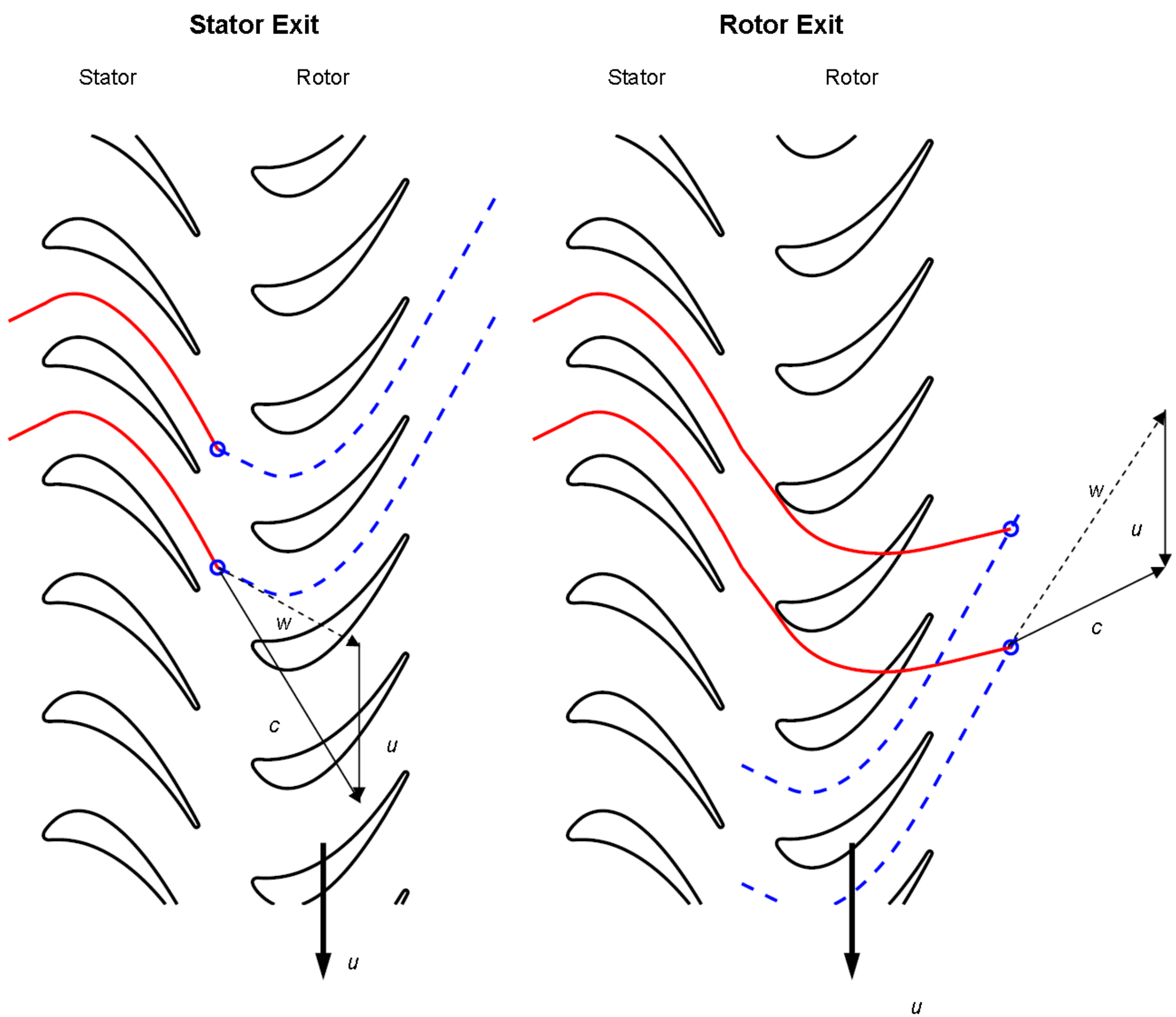
Normalized Velocity Triangle

Velocity Triangle (zero exit swirl)



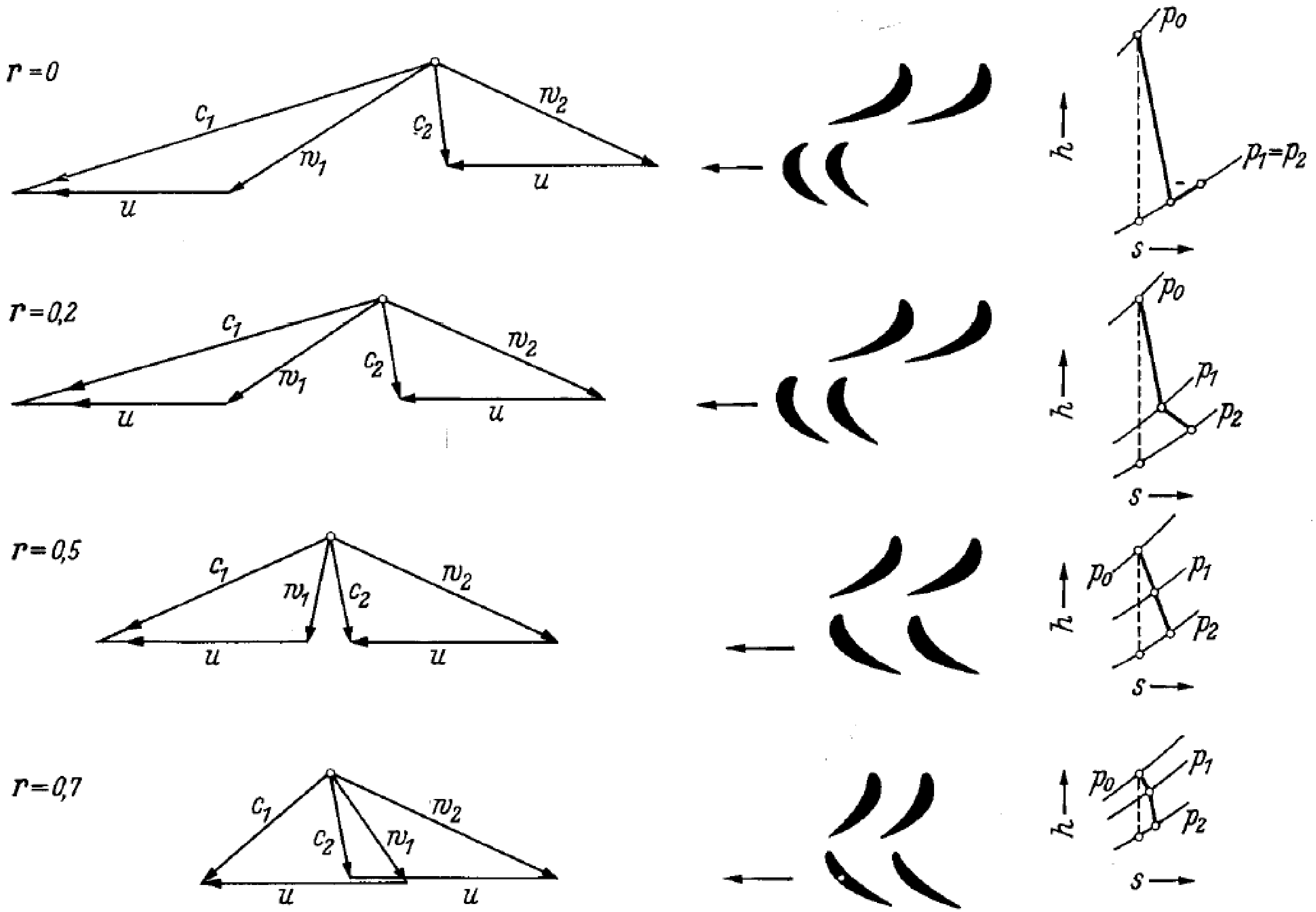
- The flow exists the stage purely axial, i.e. there is no swirl at stage exit
- For a zero exit swirl stage the degree of reaction and the loading factor are dependent
- Example for a zero exit swirl stage are for example last stages of jet engines → thrust maximized when jet flow purely axial

Turbine flow in various frame of Reference



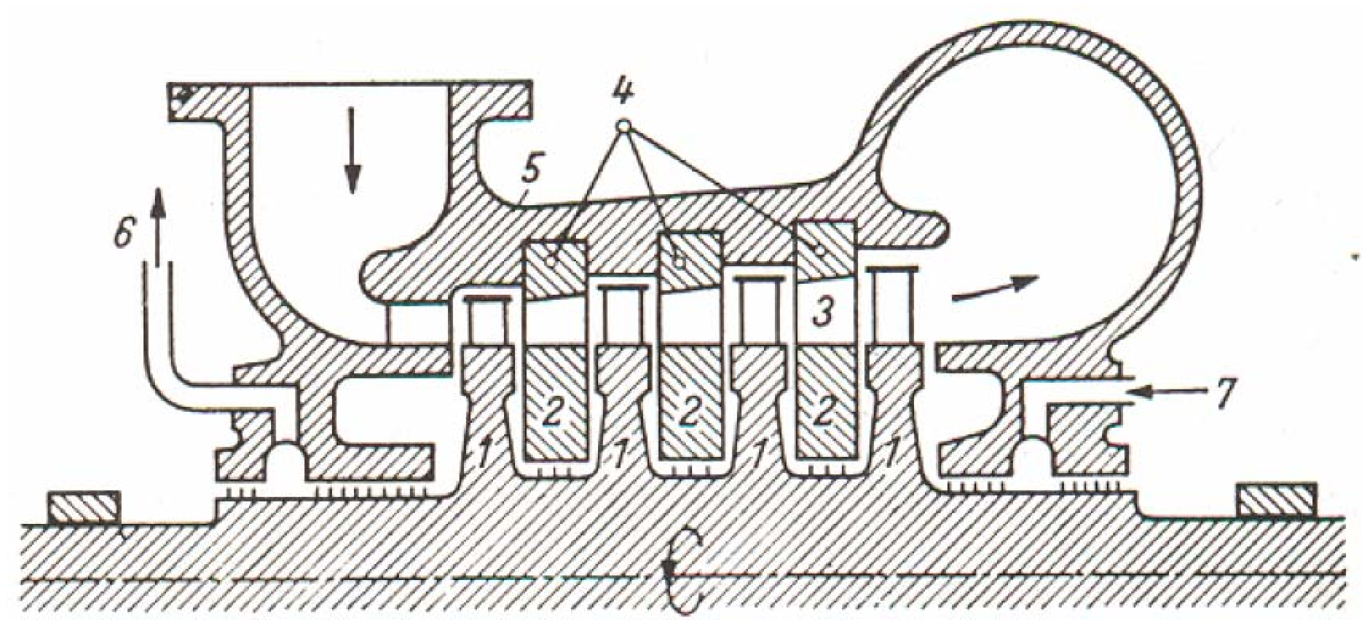
Degree of reaction

Effect of Degree of Reaction ‘R’



Turbine Design Aspects

Disk Rotor

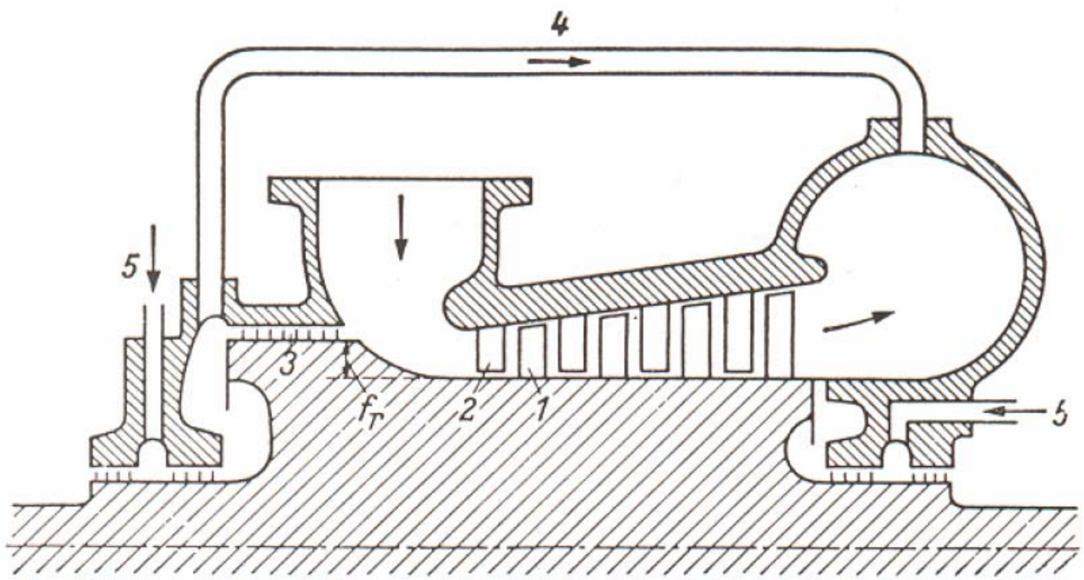


- Used when pressure difference small across rotor (→ action stage)
- Sealing (labyrinth sealing) across stator on small diameter → less leakage
- Little to no axial thrust → often minimized by holes in rotor disks

Turbine Design Aspects

Design Aspects

Drum Rotor

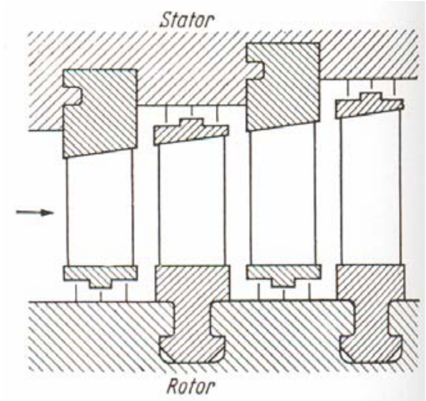


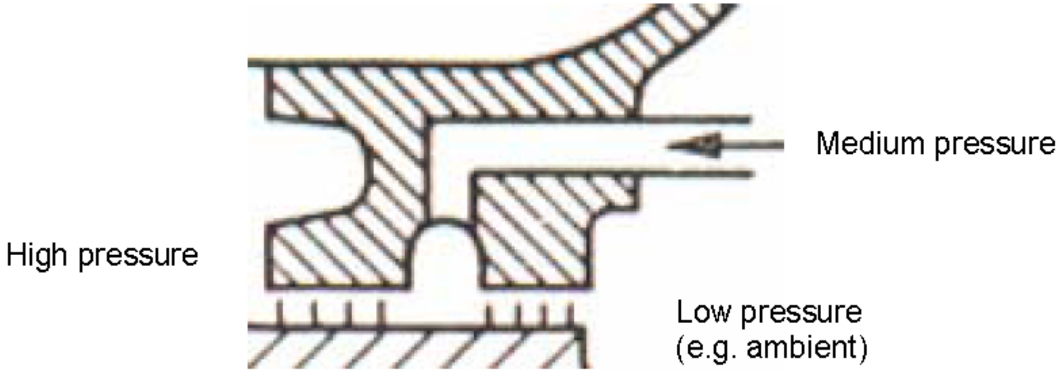
- Used when pressure difference present across rotor (→ reaction stage)
- Considerable axial thrust, which must be balance
- Possible thrust balancing → thrust piston

Turbine Design Aspects

Sealing Aspects

- Commonly labyrinth sealing are used (touch-free) due to high contact speeds as other types of sealing (e.g. O-ring, rubber lip) would fail
- In labyrinth sealing a minimum leakage flow is accepted → note: the higher the leakage flow the lower the stage efficiency
- Both shrouded and shroudless turbines are used (the sketch below depicts a shrouded turbine)
- Shrouded turbines have the general advantage of less leakage however this has to be bought by increased mechanical load on the rotor blades (large mass on large diameter → high centrifugal load → high stresses at blade root → risk for failure)



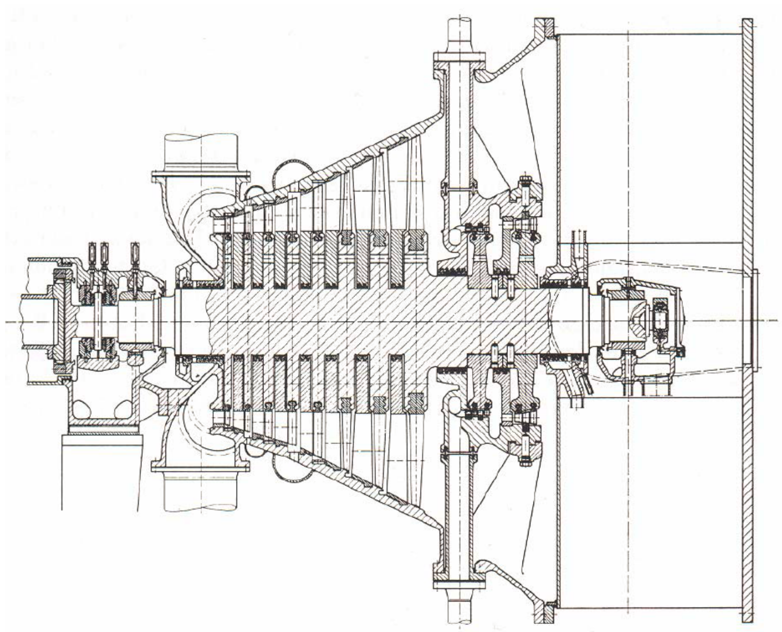


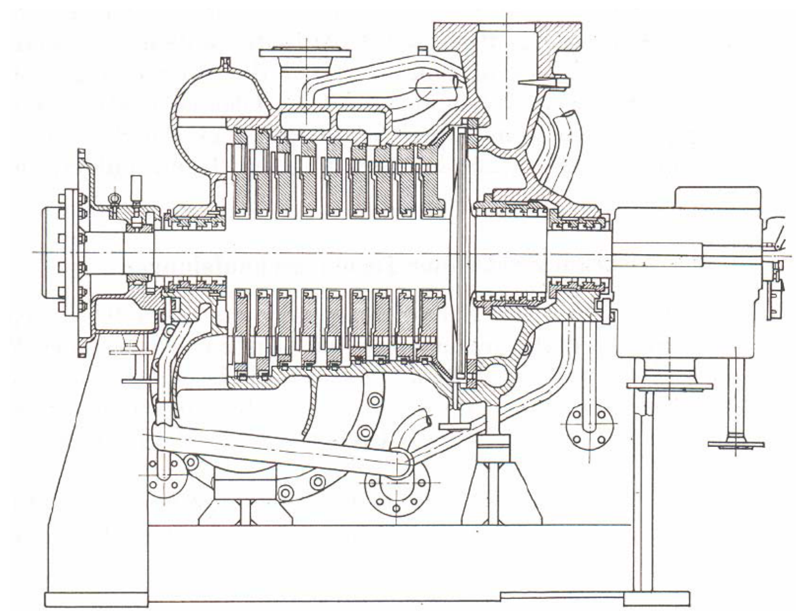
- To reduce leakage flow in labyrinth sealing a increased back-pressure might be applied at one or more axial positions throughout the sealing

Design Aspects

Turbine Design Aspects

Action Turbines

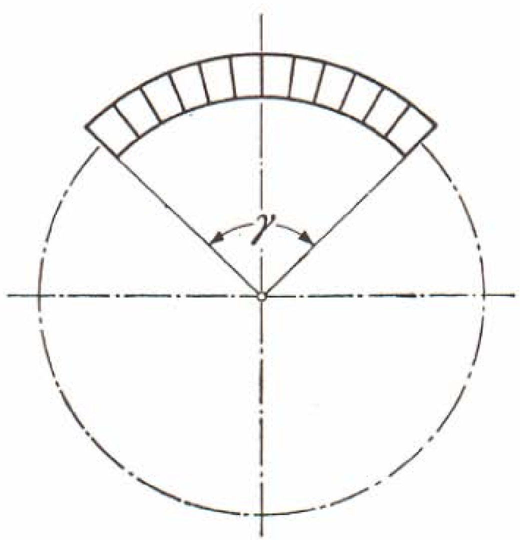


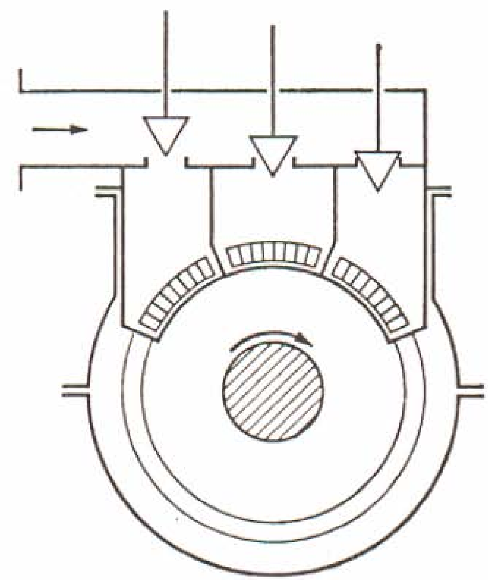


Turbine Design Aspects

Partial Admission

Possibility for partial admission as pressure difference small across rotor (note: partial admission would not work for reaction stage as pressure difference across rotor could not be maintained)

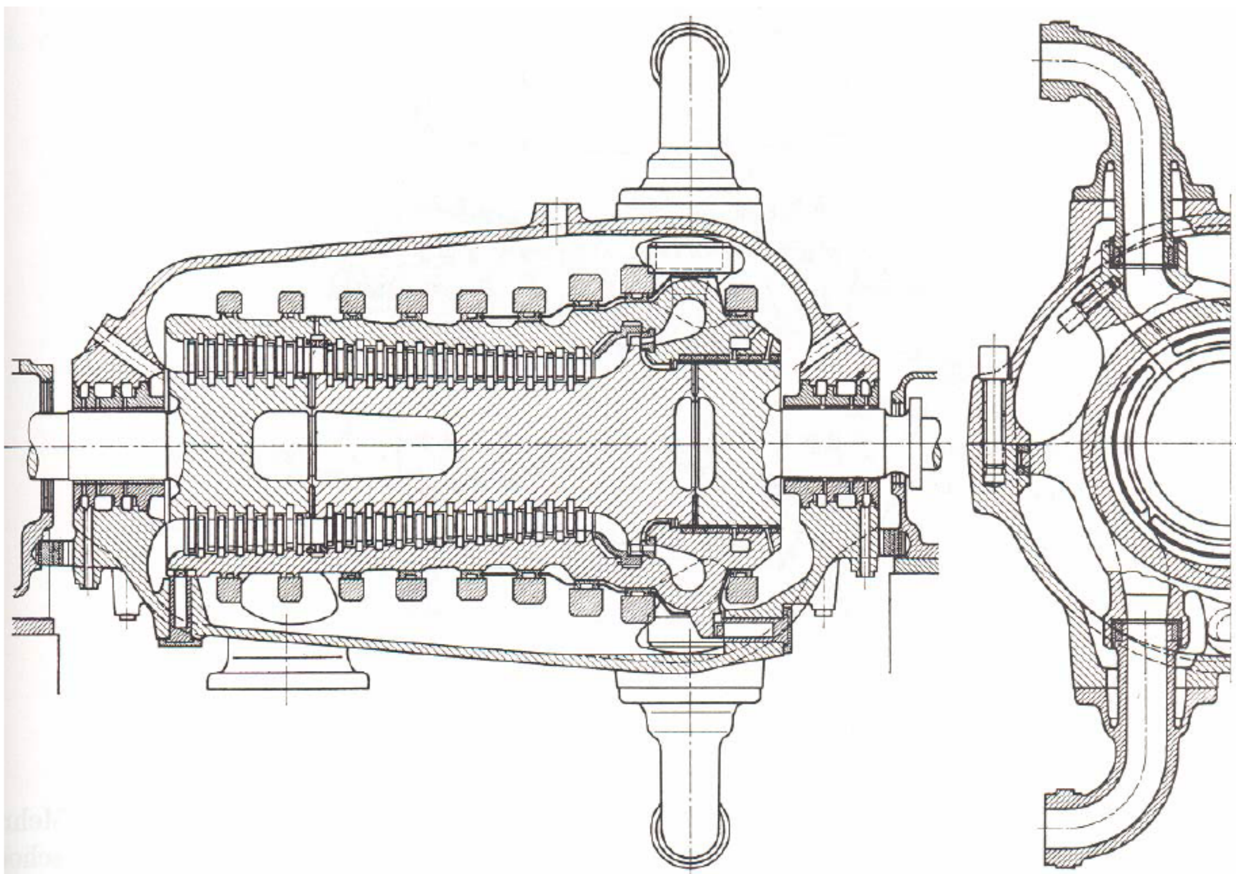




Design Aspects

Turbine Design Aspects

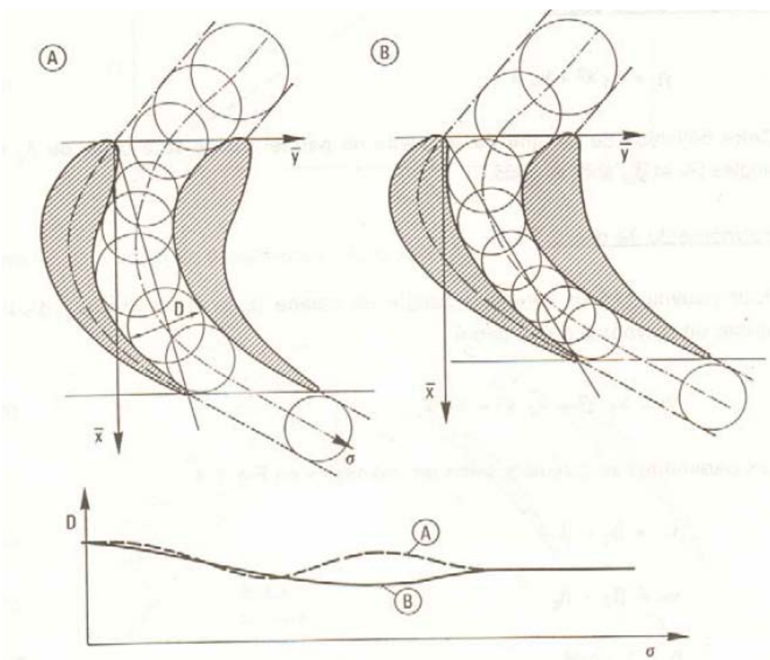
Partial Admission in Steam Turbines



Design Aspects

Turbine Design Aspects

- After determining the velocity triangles the shape of the blade profiles can be determined
- The blade geometry yields the flow passage (e.g. to be determined by tangent circles) and by this the flow velocity distribution around the blade
- Although in- and outflow cross sections are identical the change in cross section throughout the passage might differ → task of the designer to find optimum → nowadays assessed by means of CFD (computational fluid dynamics)

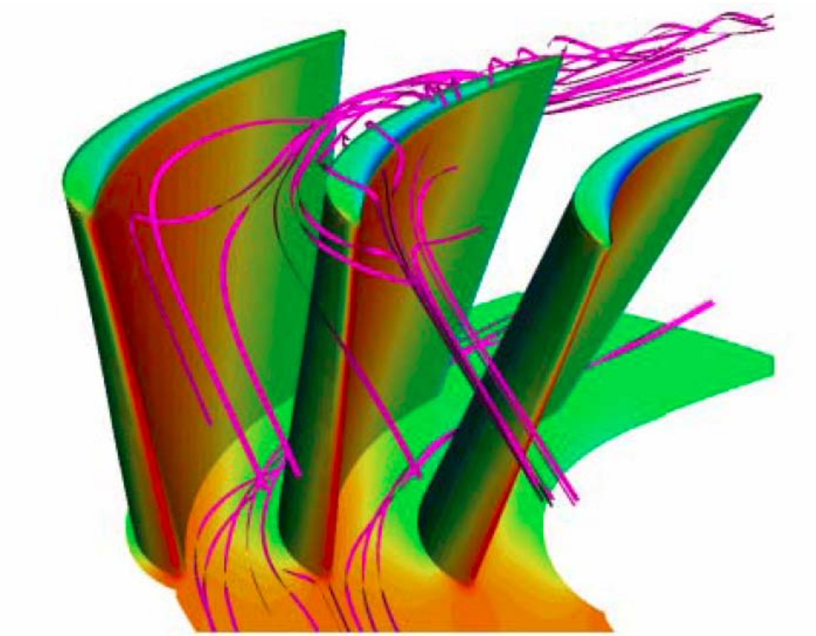


Design Aspects

Turbine Design Aspects

Example of CFD prediction of flow around blade profile

- Colors mark pressure (red high – green medium – blue low)
- Ribbons to visualize secondary flow structure
- CFD results are usually employed to determine efficiency (rather than empirical correlations)

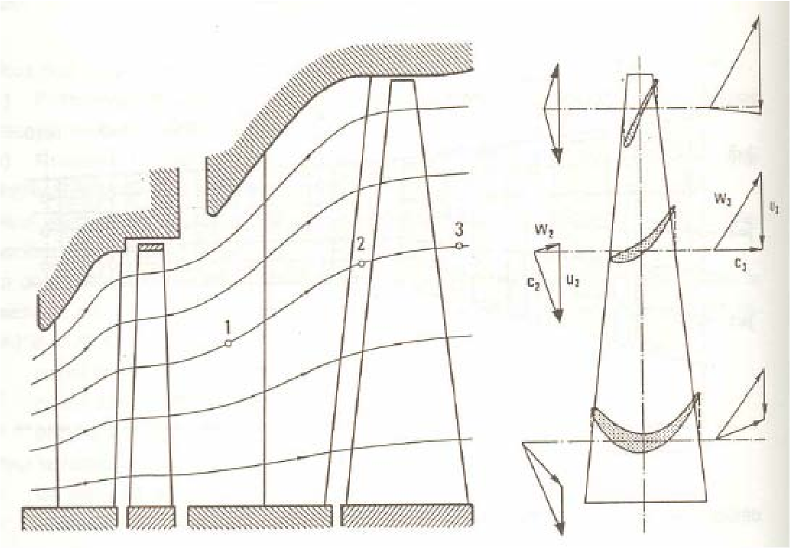


Design Aspects

Turbine Design Aspects

Three Dimensional Effects

- For long blades (e.g. last stage blades) the effects of three-dimensionality cannot be neglected
- Commonly a radial balance between stator and rotor is aimed at, e.g. constant swirl $r \cdot c_{\theta} = const$
- This yields a twisted blade shape



OFF-DESIGN PARAMETERS

If you have any questions regarding off-design parameters

Contact me after class!!!