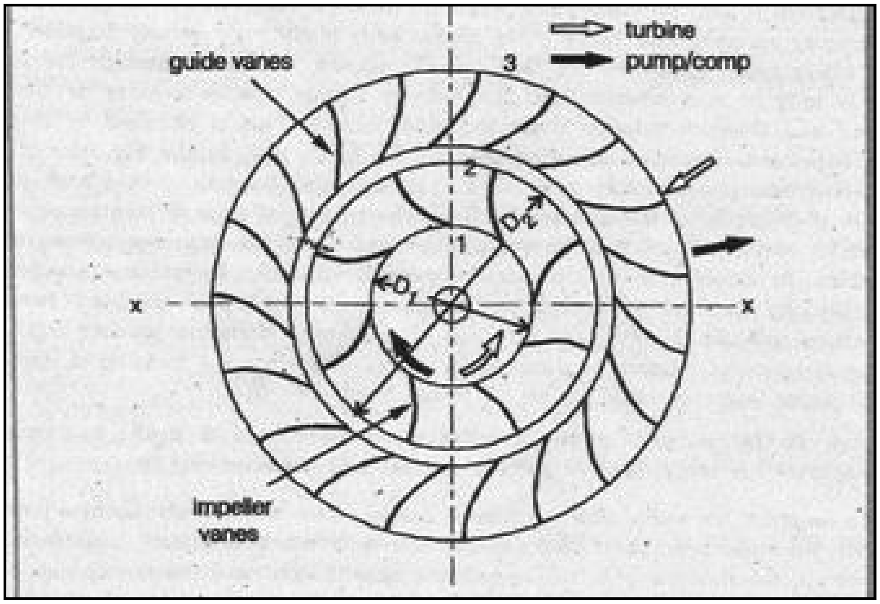
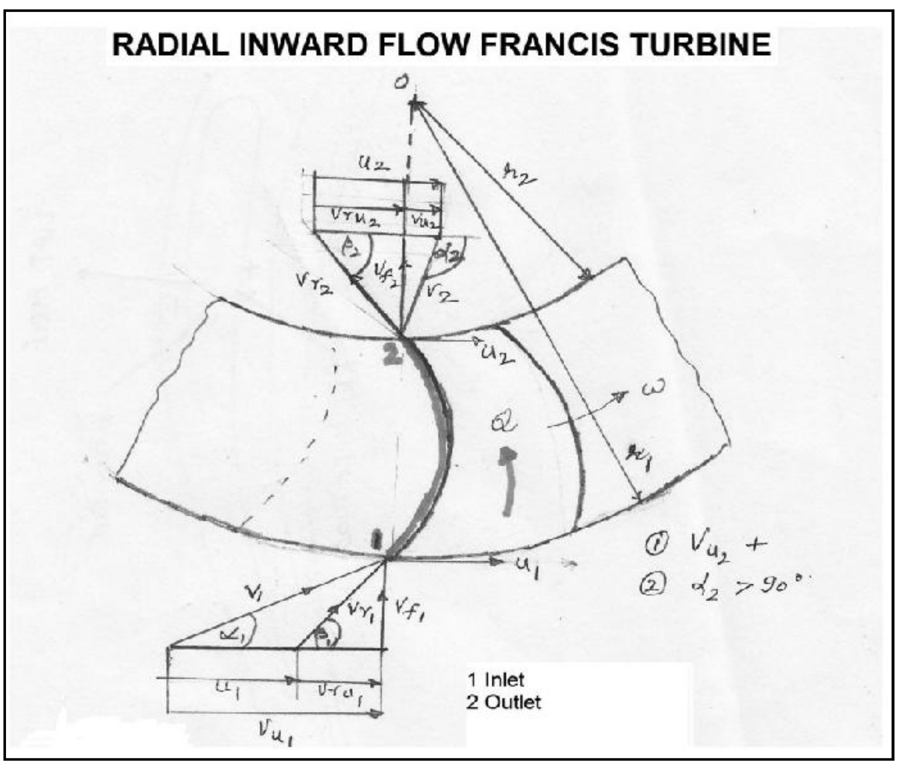
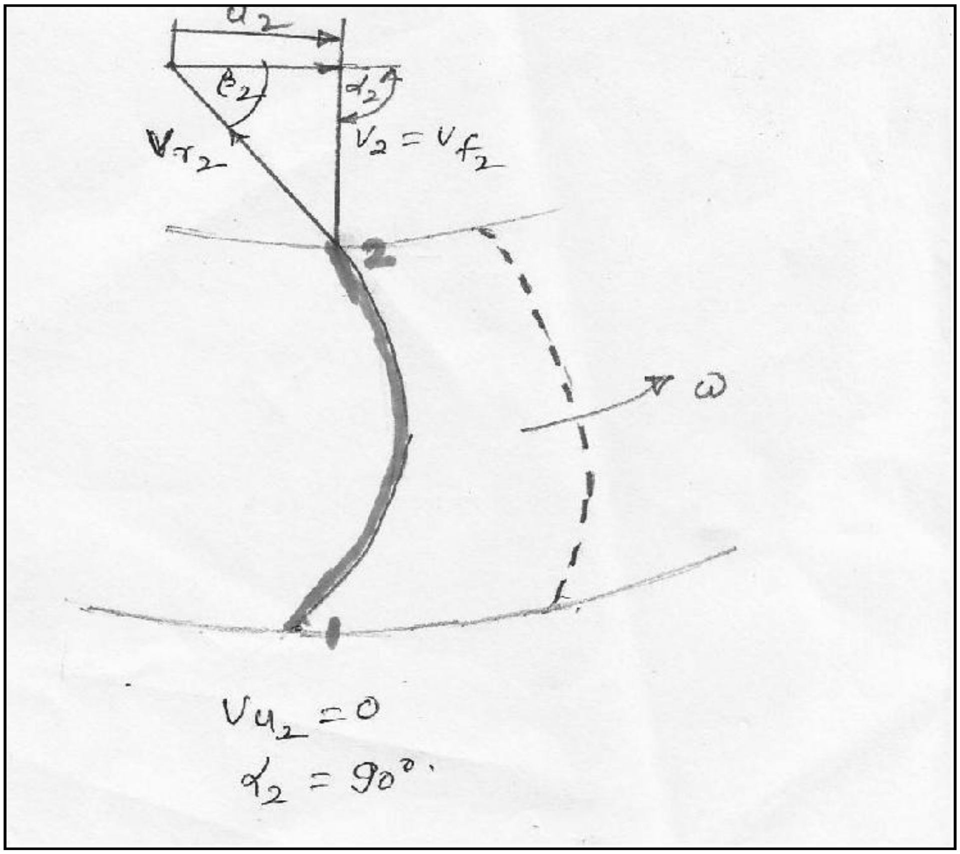
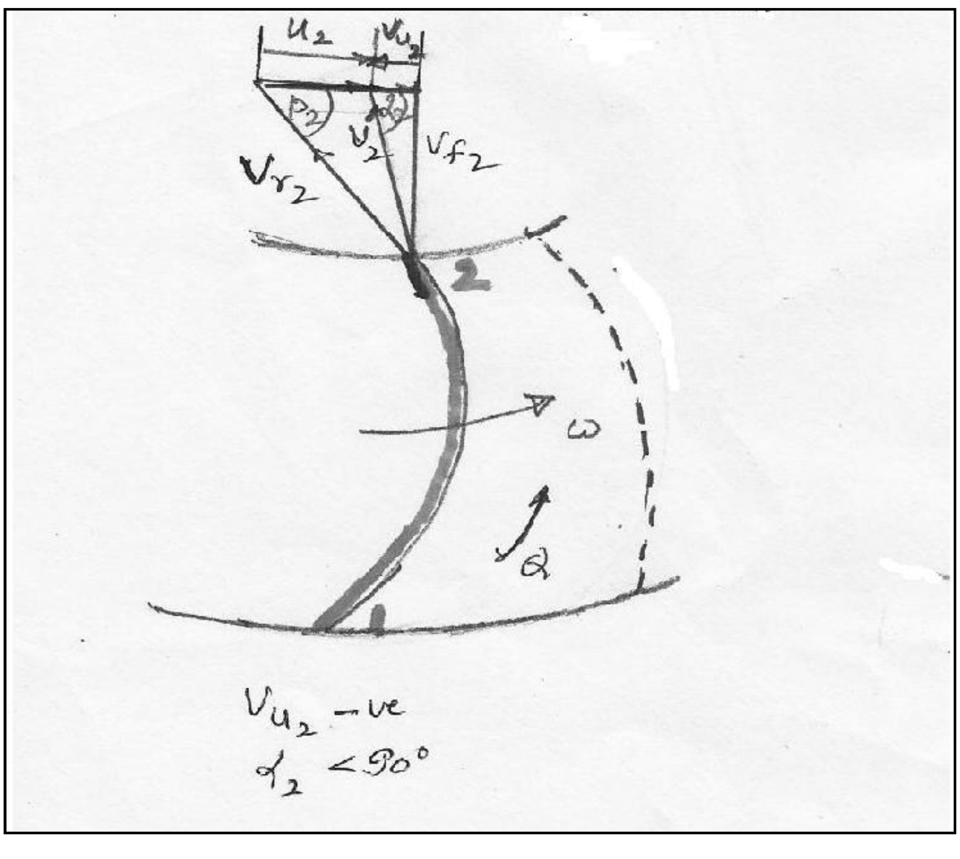
UNIT – 2

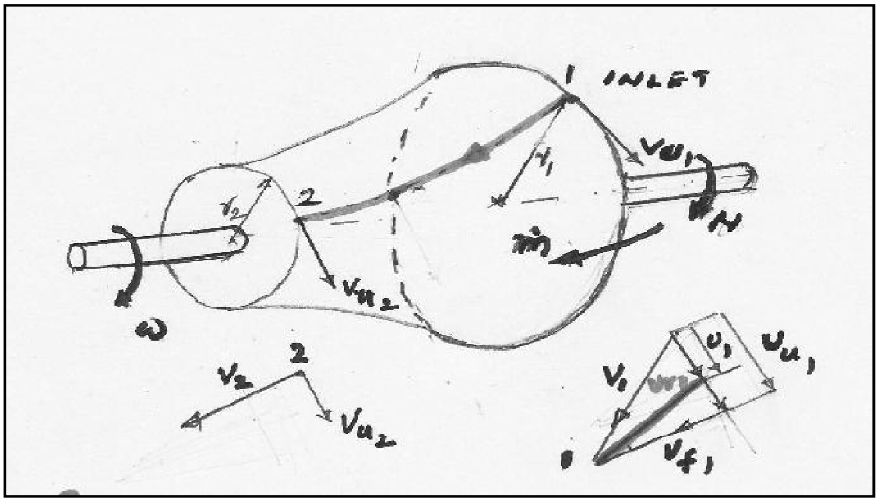
ENERGY TRANSFER IN TURBO MACHINES



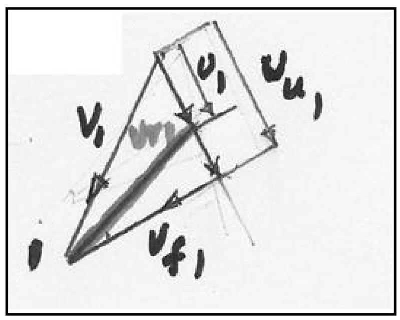


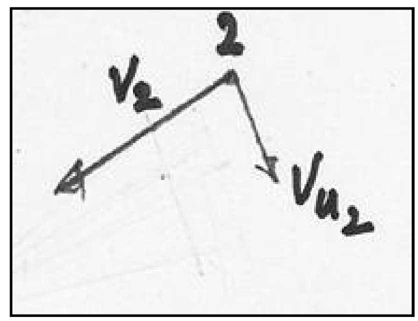

u_2
β_2
α_2
V_{r_2}
$V_2 = V_{f_2}$
2
ω
1
$V_{u_2} = 0$
$\alpha_2 = 90°$


u_2
V_{u_2}
β_2
α_2
V_2
V_{f_2}
V_{r_2}
2
ω
Q
1
V_{u_2} -ve
$\alpha_2 < 90°$

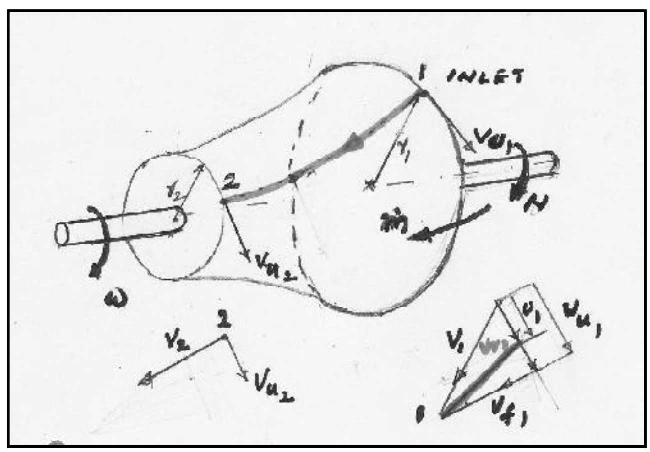


Referring to figure,
1 is inlet; 2 is outlet of rotor





V = Absolute velocity of fluid (m/s)
R = Radius of the wheel (m)
ω = Angular velocity of rotor (rad/s)
N = Speed of rotor (rpm)
U = Linear velocity of vane (m/s)
$\dot{m}$ = Mass flow rate of fluid (kg/s)



Tangential momentum of fluid at inlet

$\dot{m}\ V_{u1}$ (N)

Momentum of momentum OR
Angular momentum of fluid at inlet

$\dot{m}\ .\ V_{u1}\ .\ r_1$ (Nm)

Angular momentum of fluid at outlet

$\dot{m}\ .\ V_{u2}\ .\ r_2$ (Nm)

Torque on the wheel
= Change in angular momentum

$\therefore\ T = \dot{m}\ (V_{u1}\, r_1 - V_{u2}\, r_2)$ (Nm)

$\therefore$ Work done/sec = Torque x angular velocity = T x ω

Taking $\omega\, r_1 = 2\pi\, r_1/N = U_1$

$\omega\, r_2 = 2\pi\, r_2/N = U_2$

Work done/sec = $\dot{m}\ [V_{u1}U_1 - V_{u2}U_2]$ Nm/s or Watts

Work done/Unit mass when m = 1kg

WD/kg = Energy transfer

$= [V_{u1}U_1 - V_{u2}U_2]$ ...[Nm/kg = m^2/sec^2]

The above equation is known as **EULER'S TURBINE EQUATION.**

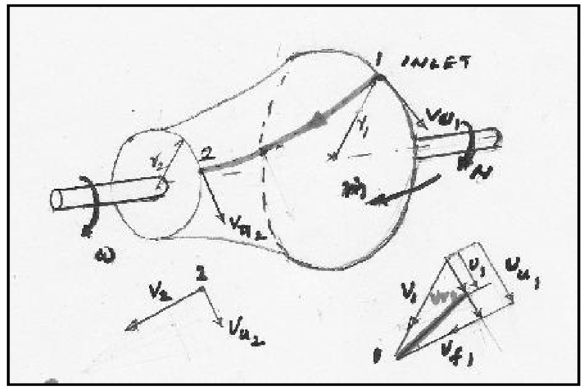
If $V_{u1}U_1 >> V_{u2}U_2$

Then, $[V_{u1}U_1 - V_{u2}U_2]$ is +ve

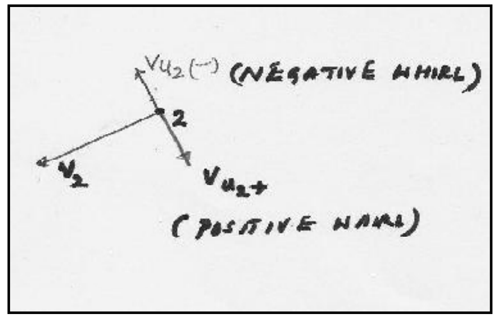
It is applicable to Power Generating Turbo Machines or Turbines.

If $V_{u1}U_1 << V_{u2}U_2$

Then, $[V_{u1}U_1 - V_{u2}U_2]$ is –ve
It is applicable to Power Absorbing Turbo Machines like pump, fans, blowers and compressors.



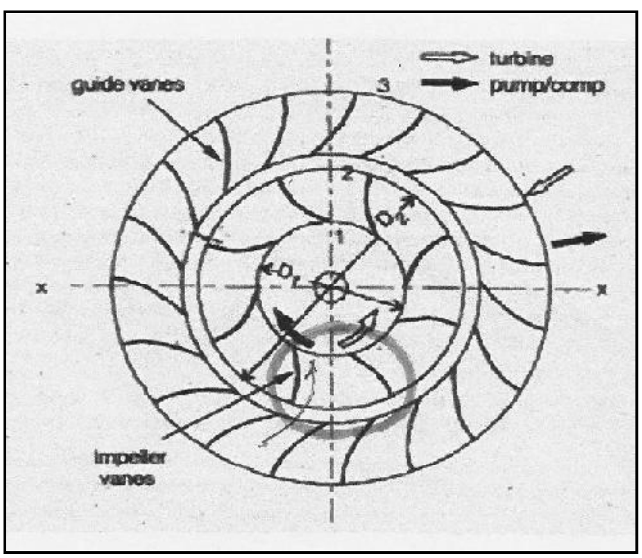
In a turbine if $V_{u1}U_1 >> V_{u2}U_2$ and
V_{u2} is in opposite direction to rotation of wheel, then work done will be greater.



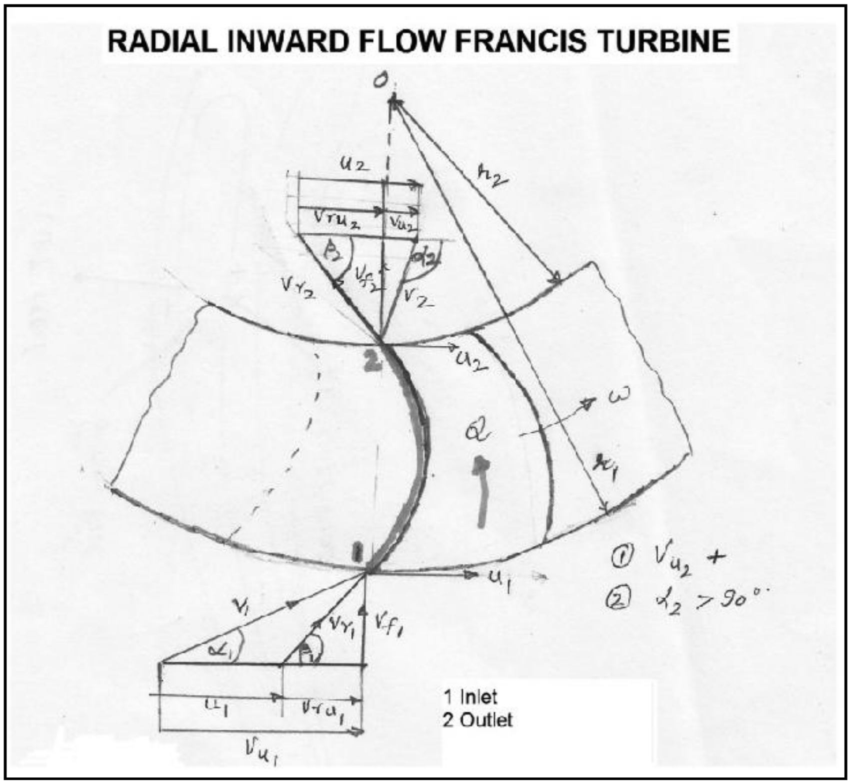
Work done/kg
WD/kg = $[V_{u1}U_1 - (-V_{u2})\ U_2]$
$= V_{u1}U_1 + V_{u2}U_2$ (Nm/kg)
Generally, for a turbine,
work done/kg
WD/kg = $[V_{u1}U_1 \pm V_{u2}U_2]$ (Nm/kg)
where, $V_{u1}U_1 > V_{u2}U_2$

For pumps, fans, blowers and compressors
Work done/kg = $[V_{u2}U_2 - V_{u1}U_1]$
where $V_{u2}U_2 > V_{u1}U_1$
If □ = mass rate of flow in kgs/s
Power developed in a turbine
P = □ $[V_{u1}U_1 \pm V_{u2}U_2]$
Watts or Nm/s or J/s
Power given to fluid in pumps, fans, blowers and compressors
P = □ $[V_{u2}U_2 - V_{u1}U_1]$
Watts or Nm/s or J/s

ALTERNATE FORMS OF EULER'S TURBINE EQUATION



Radial inward flow Francis Water Turbine



Inlet and Outlet Velocity Triangles

Referring to velocity triangles
1 – inlet , 2 – outlet
V_1 = Absolute velocity of the fluid at inlet (before entering the rotor vanes)
V_{r1} = Relative velocity of the fluid at rotor inlet
V_{u1} = Tangential component of absolute velocity

OR

Whirl component of velocity at inlet
V_{f1} = Flow component of absolute velocity at inlet
V_{ru1} = Whirl component of relative velocity at inlet
U_1 = Linear rotor vane velocity at inlet
α_1 = Absolute jet angle at inlet
β_1 = Vane (blade) angle at inlet

Referring to outlet velocity triangle
2 – outlet
V_2 = Absolute velocity of the fluid at outlet after leaving the rotor vanes.
V_{r2} = Relative velocity of the fluid rotor outlet (Just about to leave the rotor)
V_{u2} = Whirl component of absolute velocity at outlet
V_{f2} = Flow component of absolute velocity at outlet

V_{ru2} = Whirl component of relative velocity at outlet
U_2 = Linear rotor velocity at outlet
α_2 = Fluid or jet angle at outlet (To the direction of wheel rotation)
β_2 = Vane (blade) angle at outlet (To the direction of wheel rotation)

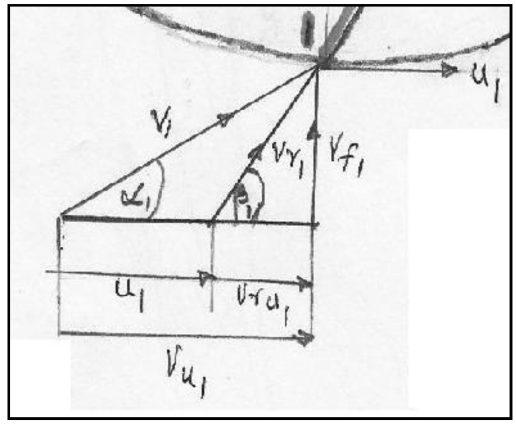
From inlet velocity triangle

$V_{f1}^2 = V_1^2 - V_{u1}^2$

$V_{r1}^2 = V_{f1}^2 + V_{ru1}^2$

$V_{r1}^2 = V_1^2 - V_{u1}^2 + (V_{u1} - U_1)^2$

$\quad = V_1^2 - \cancel{V_{u1}^2} + \cancel{V_{u1}^2} - 2V_{u1}U_1 + U_1^2$



Rearranging

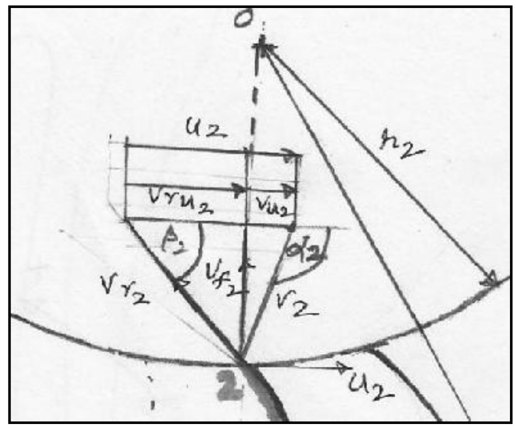
$2V_{u1}U_1 = V_1^2 + U_1^2 - V_{r1}^2$

$$\boxed{V_{u1}U_1 = \frac{V_1^2 + U_1^2 - V_{r1}^2}{2}} \quad \textbf{m}^2\textbf{/s}^2 \textbf{ OR Nm/kg... (1)}$$

From outlet velocity triangle

$V_{r2}^2 = V_{ru2}^2 + V_{f2}^2$

$\quad = (U_2 - V_{u2})^2 + (V_2^2 - V_{u2}^2)$



Taking $V_{ru2} = (U_2 - V_{u2})$ in magnitude only and not in directions

$V_{r2}^2 = U_2^2 - 2V_{u2}U_2 + \cancel{V_{u2}^2} + V_2^2 - \cancel{V_{u2}^2}$

$$\therefore \quad \boxed{V_{u2}U_2 = \frac{V_2^2 + U_2^2 - V_{r2}^2}{2}} \quad \textbf{m}^2\textbf{/s}^2 \textbf{ OR Nm/kg... (2)}$$

CASE 1:

Taking direction of rotation as positive

V_{u1} +ve and V_{u2} also +ve.

Work done/kg or Energy transfer in Turbine

Work done/kg $\quad = \quad (V_{u1}U_1 - V_{u2}U_2)$

$$\text{Energy Transfer (E)} = \left(\frac{V_1^2 + U_1^2 - V_{r1}^2}{2}\right) - \left(\frac{V_2^2 + U_2^2 - V_{r2}^2}{2}\right)$$

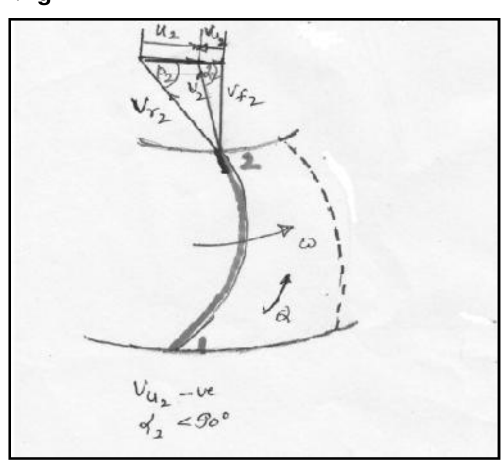
$$= \left(\frac{V_1^2 - V_2^2}{2}\right) + \left(\frac{U_1^2 - U_2^2}{2}\right) + \left(\frac{V_{r2}^2 - V_{r1}^2}{2}\right)$$

COMPONENTS OF ENERGY TRANSFER

1) $\dfrac{V_1^2 - V_2^2}{2}$ is change in absolute kinetic energy in m^2/s^2 or Nm/kg

2) $\dfrac{U_1^2 - U_2^2}{2}$ is change in centrifugal energy of fluid felt as static pressure change in rotor blades in m^2/s^2 or Nm/kg

3) $\dfrac{V_{r2}^2 - V_{r1}^2}{2}$ is change in relative velocity energy felt as static pressure change in rotor blades in m^2/s^2 or Nm/kg



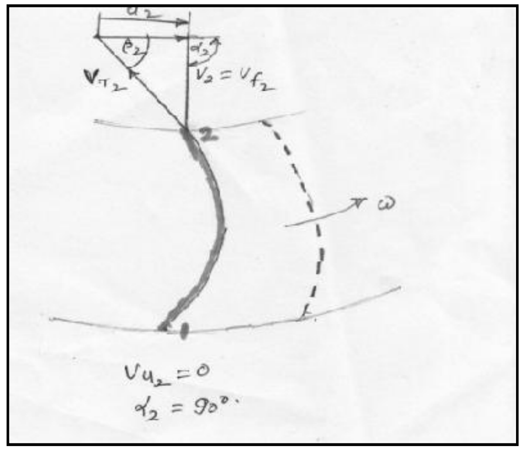
CASE 2:

If V_{u2} is – ve

E = WD/kg = $V_{u1}U_1 + V_{u2}U_2$ (Work done will be more)

$$= \left[\frac{V_1^2 - V_2^2}{2}\right] + \left[\frac{U_1^2 - U_2^2}{2}\right] + \left[\frac{V_{r2}^2 - V_{r1}^2}{2}\right]$$

CASE 3:



If $V_{u2} = 0$ No whirl at outlet commonly used in high capacity turbines

$E = WD/kg = (V_{u1}U_1)$ Nm/kg only

Degree of Reaction R

Degree of Reaction R is the ratio of Energy Transfer due to Static Enthalpy change to Total Energy Transfer due to Total Enthalpy change in a rotor.

$$\mathbf{R} = \frac{\text{Static head}}{\text{Total head}} = \frac{\text{Static enthalpy change}}{\text{Total enthalpy change}} = \frac{\Delta h}{\Delta h_0}$$

$$\Delta h = \frac{(U_1^2 - U_2^2)}{2} + \frac{(V_{r2}^2 - V_{r1}^2)}{2}$$

$$\Delta h_0 = \left[\frac{(V_1^2 - V_2^2)}{2}\right] + \left[\frac{(U_1^2 - U_2^2)}{2}\right] + \left[\frac{(V_{r2}^2 - V_{r1}^2)}{2}\right]$$

$$R = \left[\frac{(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}\right] \quad \text{... (1)}$$

Taking $\frac{(U_1^2 - U_2^2)}{2} + \frac{(V_{r2}^2 - V_{r1}^2)}{2}$ = 'S' as static component

and $\frac{(V_1^2 - V_2^2)}{2}$ = 'KE' Kinetic Energy Component (Absolute velocity change energies)

$$R = \frac{S}{KE + S} = \frac{1}{1 + KE/S}$$

$$\left[\frac{KE}{S}\right] + 1 = \left[\frac{1}{R}\right]$$

$$\frac{KE}{S} = \frac{1}{R} - 1 = \left[\frac{1 - R}{R}\right]$$

$$\boxed{\mathbf{S} = \left[\frac{\mathbf{R}}{\mathbf{1 - R}}\right] \mathbf{KE}}$$

when S = Static energy felt by rotor
KE = Kinetic energy change in rotor (in terms of V_1 and V_2, Absolute velocities)

Examples

1. For axial flow turbo machines, centrifugal forces can be neglected as $U_1 = U_2$

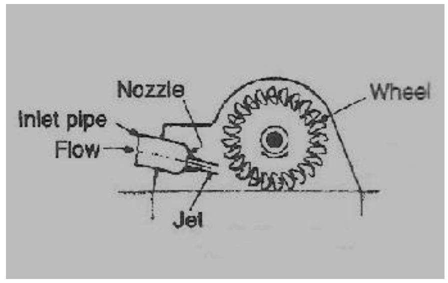
$$R = \frac{(V_{r2}^2 - V_{r1}^2)}{(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)}$$

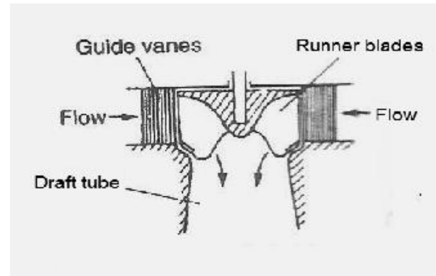
(blade shapes become very important for energy transfer)

2. If $U_1 = U_2$ and there is no relative velocity energy change in the rotor, then Static Pressure (S) is 0.

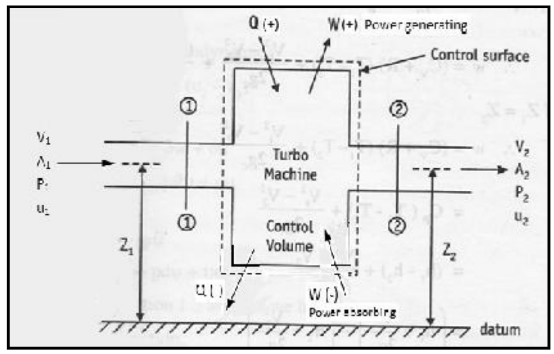
$\therefore$ R = 0 which is known as Impulse Turbo machine.

If R ≠ 0, then the machine is a Reaction Turbo machine. The turbo machine must be running under controlled flow conditions inside the casing and flow passages for pressure changes.





STEADY FLOW ENERGY EQUATION



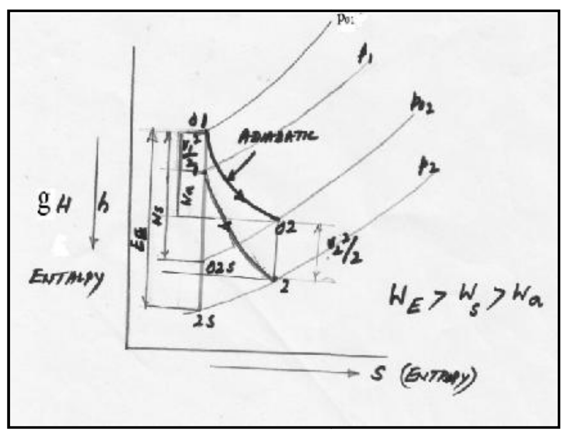
I Law of Thermodynamics

q + $\square$ [$h_1+V_1^2/2+gz_1$] = WD + $\square$ [$h_2+V_2^2/2+gz_2$]

Taking 1 as inlet and 2 as outlet conditions

q	=	Rate of heat transfer
WD	=	Work done
$V_2^2/2$	=	Kinetic Energy
gz	=	Potential Energy
h_0	=	Stagnation Enthalpy
Δh_0	=	Total enthalpy change

Where $h_0 = h + V^2/2 + gz$



When q = 0 for isentropic flow

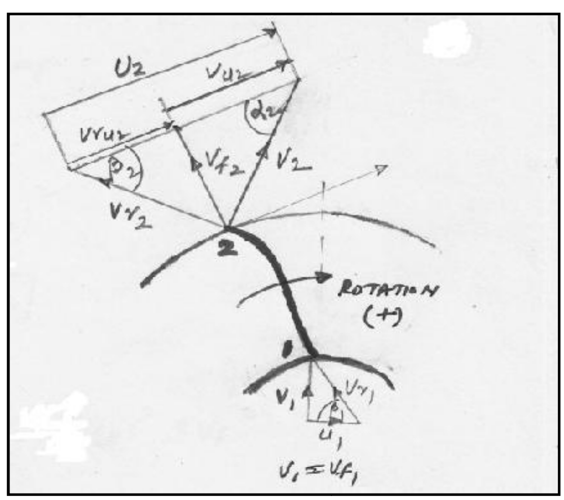
$WD = -(h_{01} - h_{02}) = -\Delta h_0$

In differential form, $-dh_0 = +dw$

$$\therefore \frac{-J}{kg} = \left[\frac{V_1^2 - V_2^2}{2}\right] + \left[\frac{U_1^2 - U_2^2}{2}\right] + \left[\frac{V_{r2}^2 - V_{r1}^2}{2}\right] = dw \text{ (Nm/kg)}$$

is known as **Ideal Euler's Work.**

Effect of Blade Discharge Angle β_2 on Energy Transfer E and Degree of Reaction R



Consider an outward radial flow turbo machine as shown in figure, where 1 is inlet, 2 is outlet

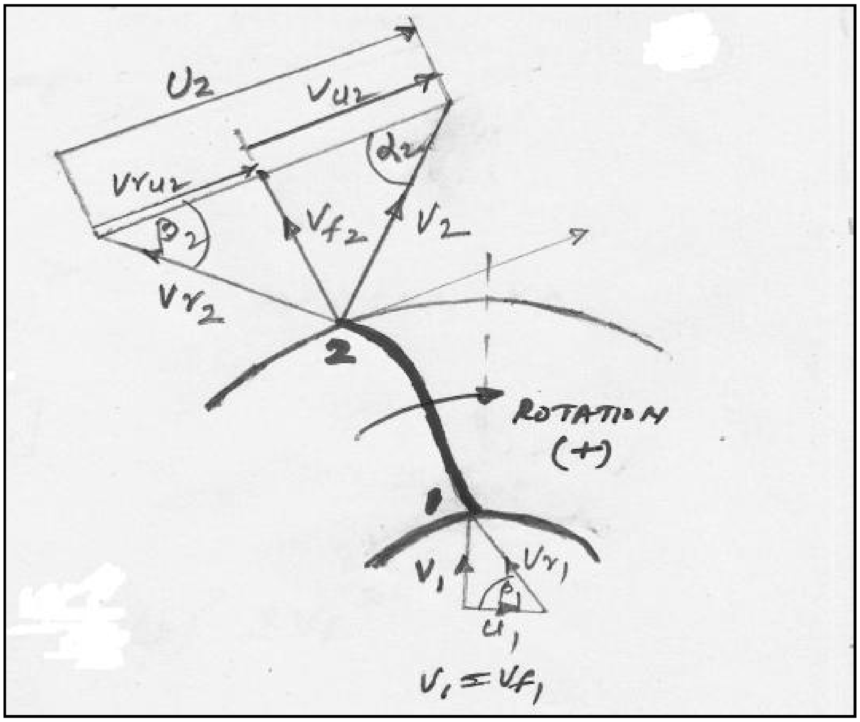
Assumptions

1) Radial velocity of flow is constant, i.e. $V_{f1} = V_{f2} = V_f$

2) No whirl component at inlet $V_{u1} = 0$

3) Diameter at outlet is twice as at inlet, i.e.
$D_2 = 2D_1$ or $U_2 = 2U_1$

4) Blade angle at inlet $\beta_1 = 45^0$, $V_1 = V_{f1} = U_1$



Assuming Turbine Equation
$E = WD/kg = (V_{u1}U_1 - V_{u2}U_2)$
$V_{u1} = 0$ as there is no whirl at inlet
$\therefore V_{u1}U_1 = 0$
$E = [- V_{u2}U_2]$ ... Nm/kg or J/kg or m^2/sec^2

Considering outlet velocity triangle,
$E = - U_2 [U_2 - V_{ru2}]$ as $\text{Cot}\ \beta_2 = \frac{V_{ru2}}{V_{f2}}$

$= - U_2 [U_2 - V_{f2} \text{Cot}\ \beta_2]$

From assumptions
$V_{f1} = V_{f2} = V_f$
$U_2 = 2U_1 = 2V_f$
$E = - 2V_f [2V_f - V_f \text{Cot}\ \beta_2]$
$= - 2V_f^2 [2 - \text{Cot}\ \beta_2]$
$E = 2V_f^2 [\text{Cot}\ \beta_2 - 2]$ taking $V_f = 1$ (unity) for all β_2
$E = 2 [\text{Cot}\ \beta_2 - 2]$ Nm/kg or J/kg

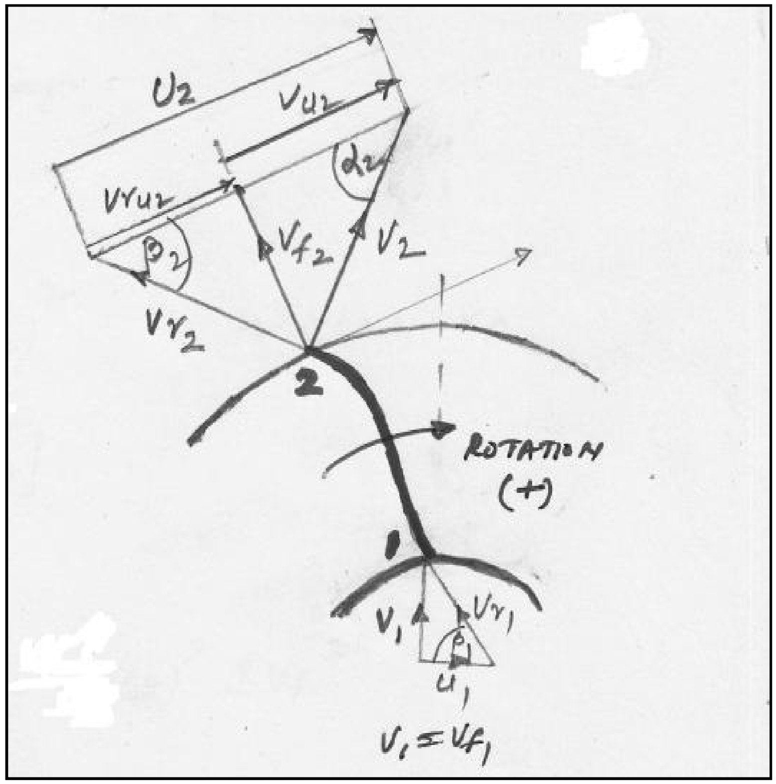
Considering outlet velocity triangle
$V_{r2}^2 = V_{f2}^2 + V_{ru2}^2$
$= V_{f2}^2 + (V_{f2} \text{Cot}\ \beta_2)^2$
$= V_{f2}^2 [1 + \text{Cot}^2\ \beta_2]$

From inlet velocity triangle

$V_{r1}^2 = V_{f1}^2 + V_{f1}^2 = 2\,V_f^2$

Degree of reaction R is given by

$$\boxed{R = \frac{(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}{2 \times E_{Transfer}}}$$



Substituting for V_f

$$R = \frac{[(V_f^2 - 4V_f^2) + V_f^2\,(1 + Cot^2\beta_2) - 2V_f^2]}{2 \times 2V_f\,(Cot\beta_2 - 2)}$$

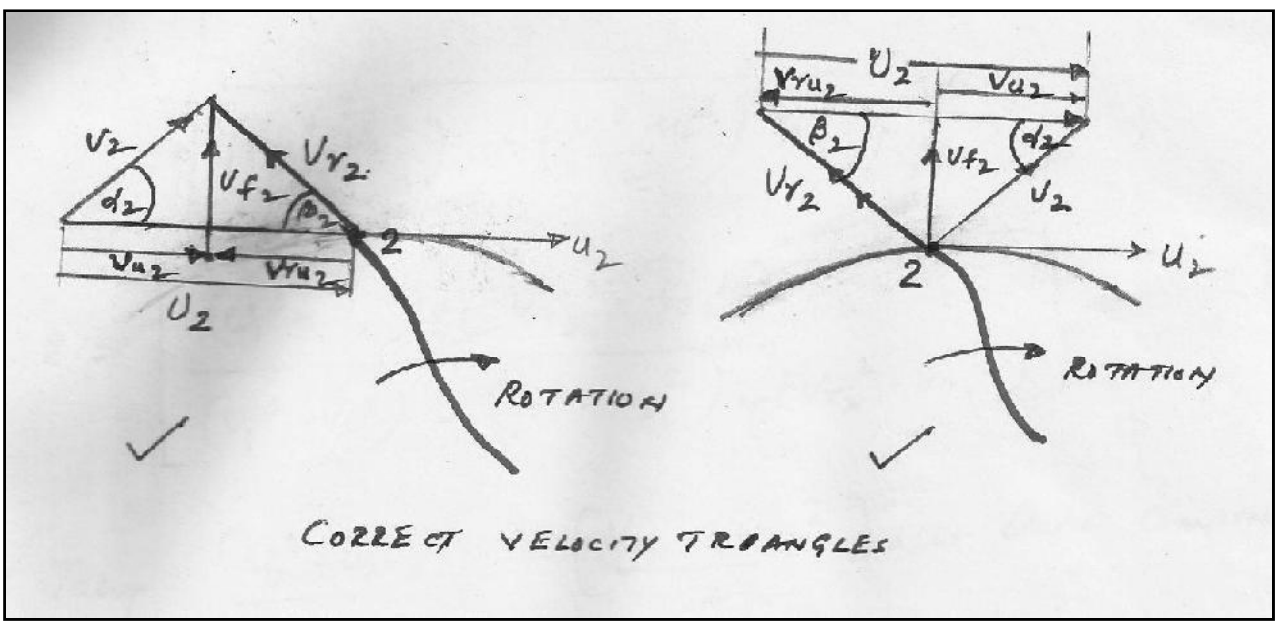
$$= \frac{-5V_f^2 + V_f^2 + V_f^2 Cot^2\beta_2}{4V_f^2\,(Cot\beta_2 - 2)}$$

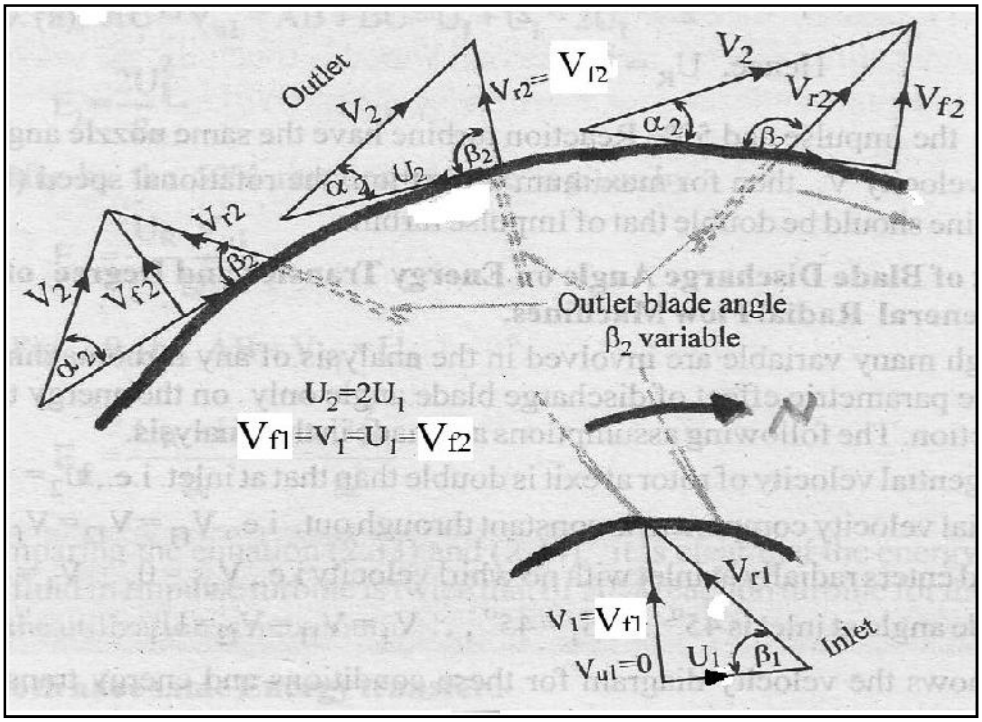
$$= \frac{V_f^2\,[Cot^2\beta_2 - 4]}{4V_f^2\,(Cot\beta_2 - 2)} \qquad \text{Taking } V_f = \text{Unity}$$

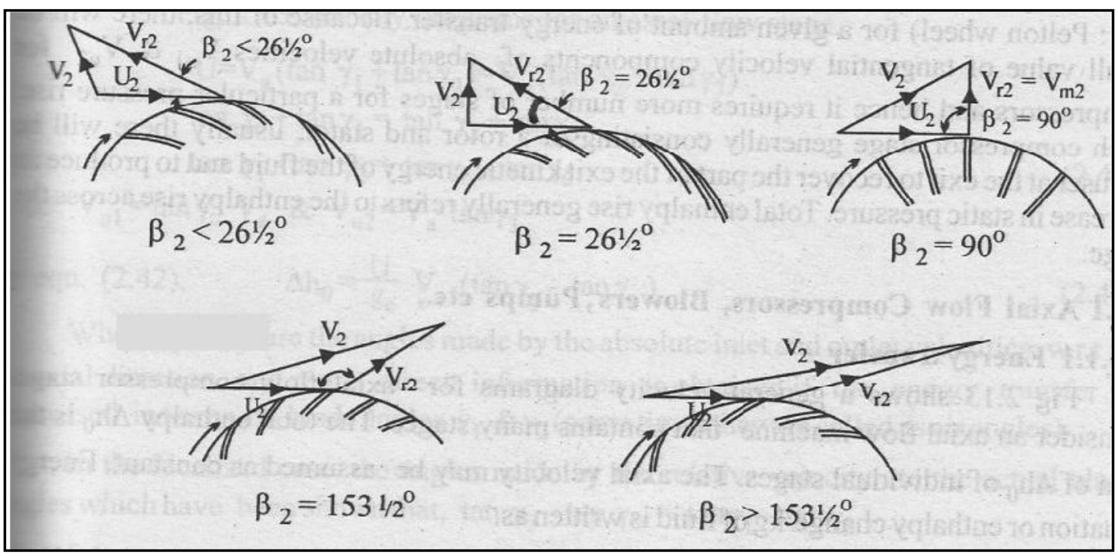
$$= \frac{Cot^2\beta_2 - 4}{4\,(Cot\beta_2 - 2)}$$

$$= \frac{(Cot\,\beta_2 - 2)\,(Cot\,\beta_2 + 2)}{4\,(Cot\,\beta_2 - 2)}$$

$$\boxed{R = \frac{Cot\,\beta_2 + 2}{4}}$$



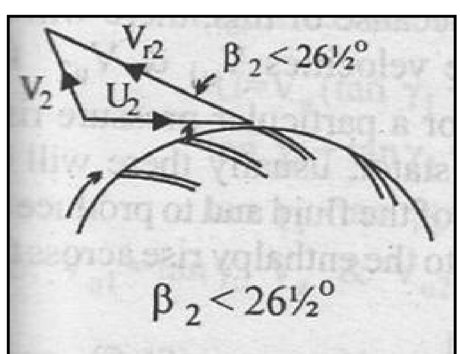




INFERERNCE

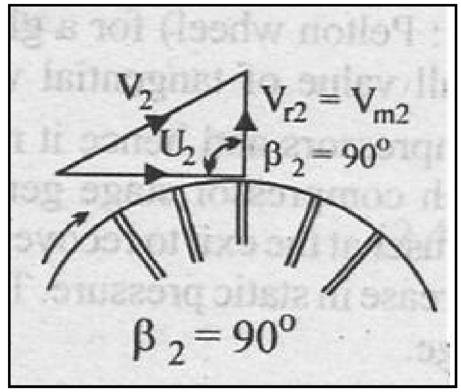
WD/kg or E = 2 (Cot β_2 – 2)

$$\mathbf{R} = \left[\frac{2 + \text{Cot}\,\beta_2}{4}\right]$$



For $\beta_2 = 10^0$ $E = +\ 7.343$
$R > 1\ \ 1.918$

The machine is a Reaction Turbine and V_{u2} is to direction of rotation



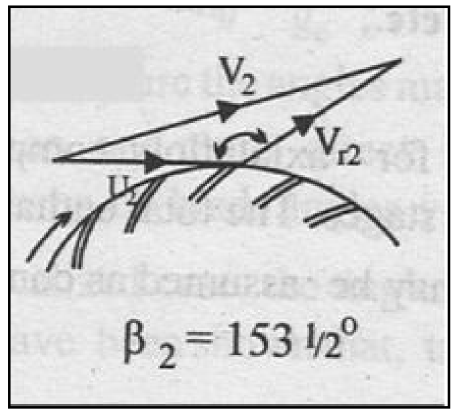
For $\beta_2 = 26.5^0$ $E = 0$
$R = 1$
The machine is rotating and Transferring no energy as $V_{u2} = 0$

For $\beta_2 > 26.5^0$ and $< 153.5^0$ $E = -$ ve
$R = +$ ve

The machine is Power Absorbing like pump or compressor, V_{u2} in same Direction of rotation.

For $\beta_2 = 153.5^0$ $E = -$ ve

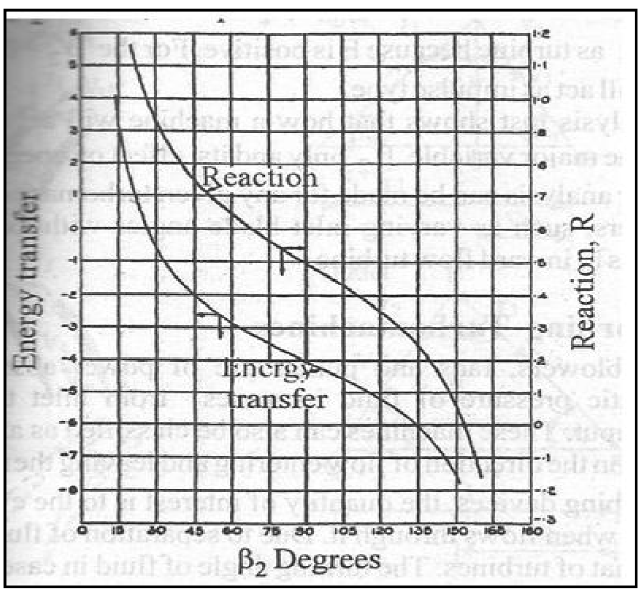
The machine works as R = 0 power absorbing impulse type, V_{u2} and rotation of wheel are in same direction.



For $\beta_2 > 153.5^0$ $E = -$ ve
$R = -$ ve
The machine is power absorbing reaction type, V_{u2} is very high. Static head is less at outlet than at inlet.

WD/kg or $E = 2\ (\text{Cot}\ \beta_2 - 2)$

$$R = \left[\frac{2 + \text{Cot}\ \beta_2}{4}\right]$$



Problem 1

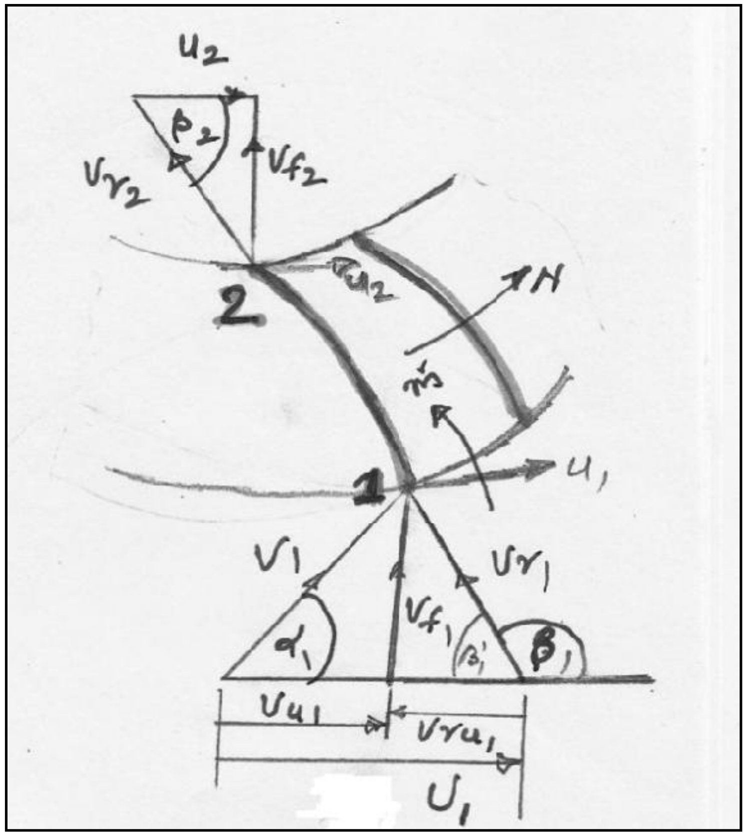
In an inward flow radial turbine, water enters at an angel of 22^0 to the direction of rotation and leaves axially without whirl at outlet. The inlet and exit diameters are 0.6 m and 0.3 m respectively. The rotor speed is 300 rpm. The flow velocity is 3 m/s and constant throughout. The width of the wheel at inlet is 15cms. Neglecting thickness of blades, calculate:

(1) Rotor blade angles at inlet and outlet
(2) Power developed

Solution

Data:

$d_1 = 0.6$ m ; $d_2 = 0.3$ m ; $b_1 = 15$ cm; $N = 300$ rpm; $\alpha_1 = 22^0$; $V_{f1} = V_{f2} = 3$ m/s; $\alpha_2 = 90^0$; $V_{u2} = 0$; $V_2 = V_{f2}$;
$K_1 = 1$ (blockage by blades neglected).



$$U_1 = \frac{\pi\, d_1\, N}{60} = \frac{\pi\ x0.6x300}{60} = 9.425 \text{ m/s}$$

$$U_2 = \frac{\pi\, d_2\, N}{60} = \frac{\pi\ x0.3x300}{60} = 4.713 \text{ m/s}$$

$$V_1 = \frac{V_{f1}}{\sin \alpha_1} = \frac{3}{\sin 22^0} = 8.008 \text{ m/s}$$

$$V_{u1} = \frac{V_{f1}}{\tan \alpha_1} = \frac{3}{\tan 22^0} = 7.425 \text{ m/s}$$

$U_1 > V_{u1}$ and hence the velocity triangles at inlet is as indicated

$V_{ru1} = U_1 - V_{u1}$
$= 9.425 - 7.425$
$= 2$ m/s

$\tan (180 - \beta_1) = V_{f1} / V_{ru1}$

$$\tan \beta_1' = \frac{V_{f1}}{(U_1 - V_{u1})} = \frac{3}{2}$$

$\tan \beta_1' = 56.13^0$

Blade angle at inlet
$\beta_1 = 180 - 56.13 = 123.87^0$

From outlet velocity triangle,
$\tan \beta_2 = V_{f2} / U_2$

$$\beta_2 = \tan^{-1}\left[\frac{V_{f2}}{U_2}\right]$$
$= \tan^{-1} [3/4.713]$
$= 32.478^0$

Mass flow rate $\dot{m} = \rho\, Q$
$= K_1\, \rho\, \pi\, d_1\, b_1\, V_{f1}$
Taking $K_1 = 1$ (no blockage by blades)
$\dot{m} = 1000 \text{ x } \pi \text{ x } 0.6 \text{ x } 0.15 \text{ x } 3$
$= 848.23$ kgs/sec of water
$P = \dot{m}\, [V_{u1}U_1 \pm V_{u2}U_2]$ Nm/s or Watts

but $V_{u2} = 0$ for no whirl at outlet

$P = \square\ [V_{u1}U_1]$
$= 848.23 \times 7.42 \times 9.425$
$= 59319.693$ watts
$= 59.32$ Kw

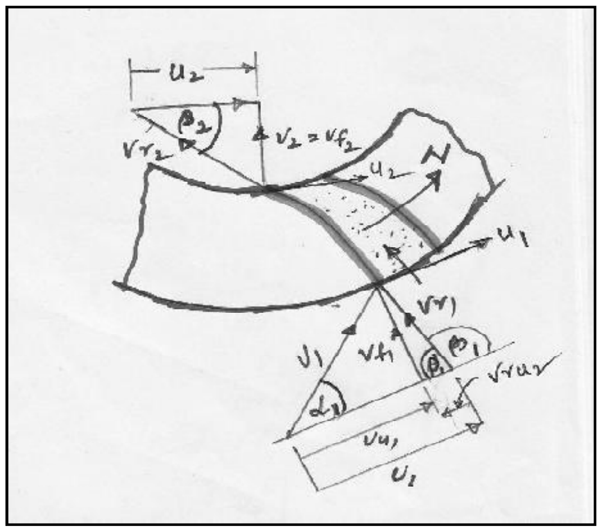
Problem

In a certain turbo machine, the inlet whirl velocity is 15 m/s, inlet flow velocity is 10 m/s, blade speeds are 30 m/s and 8 m/s at inlet and outlet respectively. Discharge is radial with absolute velocity of 15 m/s. If water is the working fluid flowing at a rate of 1500 liters/sec, calculate:

(1) Power in KW
(2) Change in total pressure in bar
(3) Degree of reaction
(4) Utilization factor

(VTU Dec, 2010)

Solution



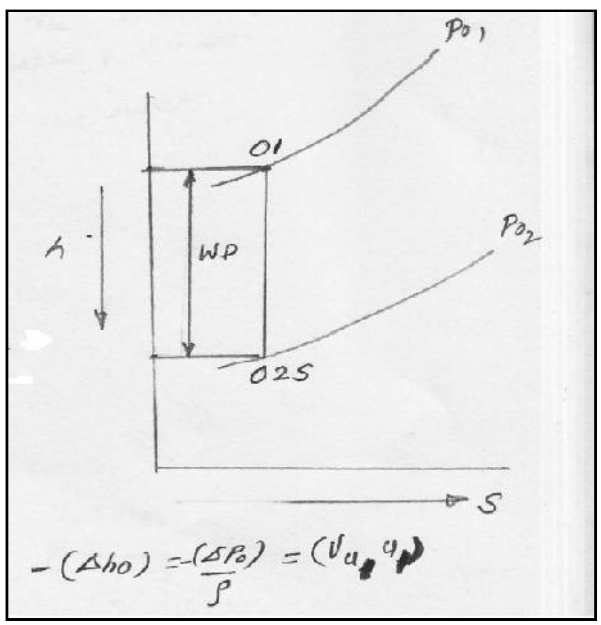
Data:

$V_{u1} = 15$ m/s	$V_2 = 15$ m/s $= V_{f2}$
$V_{f1} = 10$ m/s	$Q = 1000$ liters/sec
$U_1 = 30$ m/s	$P = ?$ Kw
$U_2 = 8$ m/s	$R = ?$
$V_{f1} = V_{f2}$ $\varepsilon = ?$	

$U_1 > V_{u1}$
$\therefore V_{ru1} = U_1 - V_{u1}$
$= 30 - 15 = 15$ m/s

$\beta_1' = \tan^{-1} V_{f1} / V_{ru1}$
$= \tan^{-1} 10/15$
$\beta_1' = 33.69^0$

Blade angle at inlet $\beta_1 = 180 - \beta_1'$
$\beta_1 = 188 - 33.69$
$= 146.31^0$



$V_{r1} = V_{f1} / \beta_1'$
$= 10 / \sin 33.69$
$V_{r1} = 18.028$ m/s

$V_1 = \sqrt{(V_{u1}^2 + V_{f1}^2)}$
$= \sqrt{(15^2 + 10^2)}$
$= 18.028$ m/s

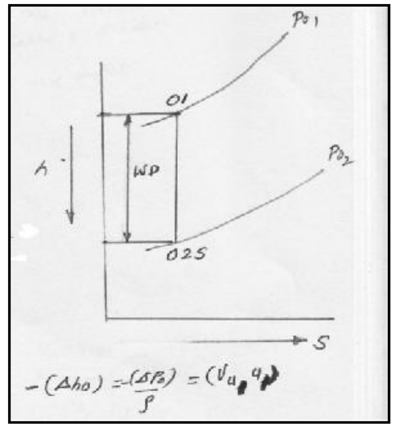
Taking discharge as radial, $V_{f2} = V_2$
$\tan \beta_2 = (15/8)$
$\beta_2 = \tan^{-1} (15/8)$
$= 61.928^0$

$-\ \Delta p_0 = \rho(U_1 V_{u1}) \times 10^{-5}$ bar

$-\ \Delta p_0 = \rho/2\ [U_1^2 - U_2^2 + V_{r2}^2 - V_{r1}^2 + V_1^2 - V_2^2] \times [1/10^5]$

$= 1000/2\ [30^2 - 8^2 + 17^2 - \cancel{18^2} + \cancel{18^2} - 15^2] \times [1/10^5]$

$= -4.5$ bar $\qquad$ (1 bar = 10^5 N/m^2)



Work done = $V_{u1}U_1$ $\qquad$ Nm/kg

P = □ $V_{u1}U_1 = \rho Q V_{u1}U_1$

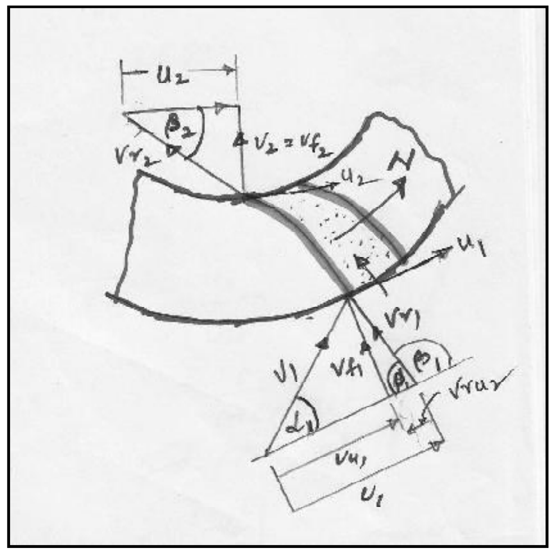
= 1000 x (1500/1000) x 15 x 30

= 675000 watts = 675 Kw

$$R = \frac{(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}{2 \times V_{u1}U_1}$$

$$= \frac{30^2 - 8^2 + 17^2 - 18.028}{2 \times [15 \times 30]}$$

= 0.889



Utilization factor ε

$$\varepsilon = \frac{WD}{WD + V_2^2/2}$$

$$= \frac{V_{u1}U_1}{V_{u1}U_1 + (V_2^2/2)}$$

$$= \frac{15 \times 30}{(15 \times 30) + 15^2/2}$$

$$= \frac{460}{562.5}$$

ε = 0.80

Problem on Water Turbine

A hydraulic reaction turbine of radial inward flow type, works under a head of 160 m of water. At a point at entry, the rotor blade angles are 119^0 and the diameter of the runner 3.65 m. At the exit the diameter is 2.45 m. The absolute velocity of the flow is radially directed with a magnitude of 15.5 m/s and the radial component of velocity at inlet is 10.3 m/s. Determine:

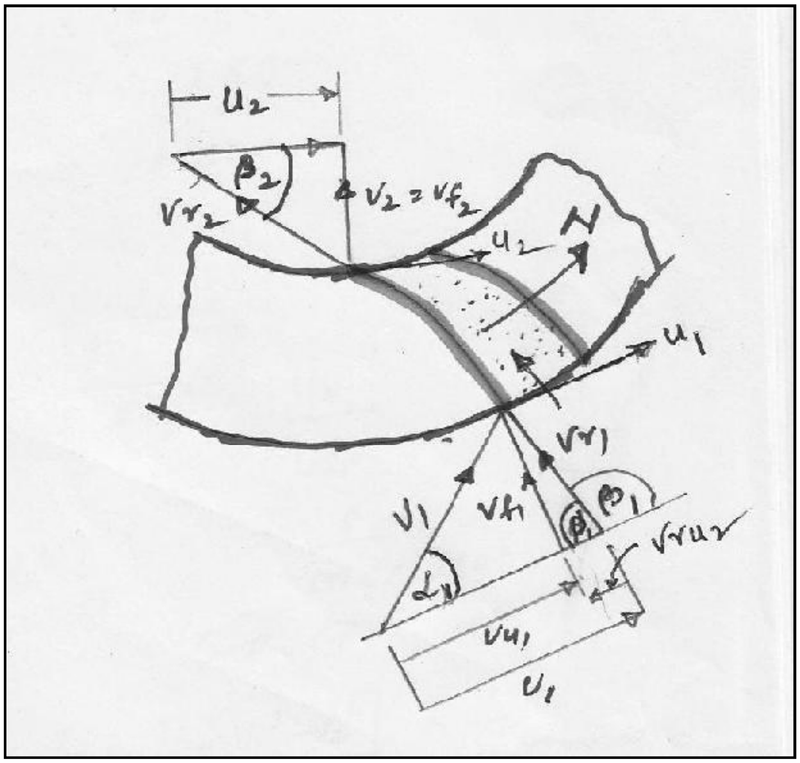
(1) Power developed by the machine for a flow rate of 110 m^3/s
(2) Degree of reaction
(3) Utilization factor

Solution

Inward flow

1 – inlet;	2 – outlet;	H = 160 m;	$\beta_1 = 119^0$	d_1 = 3.65 m
d_2 = 2.45 m	$V_2 = V_{f2}$ = 15.5 m/s	V_{f1} = 10.3 m/s	P =?	Q = 110 m2/s
R = ?	ε = ?			

$V_1 = \sqrt{2gH}$
$= \sqrt{2 x 9.81 x 160}$
$= 56.029$ m/s



$V_{u1} = \sqrt{(V_1^2 - V_{f1}^2)}$
$= \sqrt{(56.029^2 - 10.3^2)}$
$= 55.074$ m/s

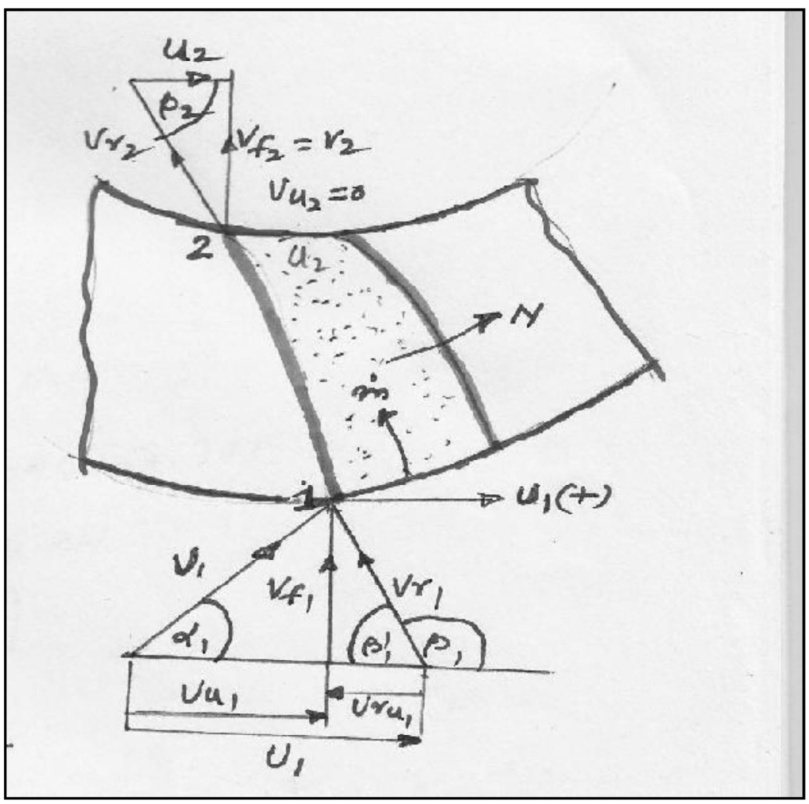
$\alpha_1 = \tan^{-1}(V_{f1} / V_{u1})$
$= \tan^{-1}(10.3/55.074)$
$= 10.593^0$

$\beta_1 = 119^0$ indicates
$U_1 > V_{u1}$
$\beta_1' = 180^0 - 119^0 = 61^0$

$\tan \beta_1' = V_{f1} / V_{ru1}$
$\therefore V_{ru1} = V_{f1} / \tan \beta_1'$
$= 10.3 / \tan 60^0$
$= 5.709$ m/s

$V_{r1} = \sqrt{(V_{f1}^2 + V_{ru1}^2)}$
$= \sqrt{(10.3^2 + 5.709^2)}$
$= 11.775$ m/s

$U_1 = [V_{u1} + V_{ru1}]$
$= [55.074 + 5.709]$
$= 60.783$ m/s



$U_1 = \pi d_1 N / 60$

$\therefore N = (60 \times U_1) / \pi d_1$

$$N = \frac{60 \times 60.783}{\pi \times 3.65}$$

$= 318.047$ rpm

$U_2 = \pi d_2 N / 60$

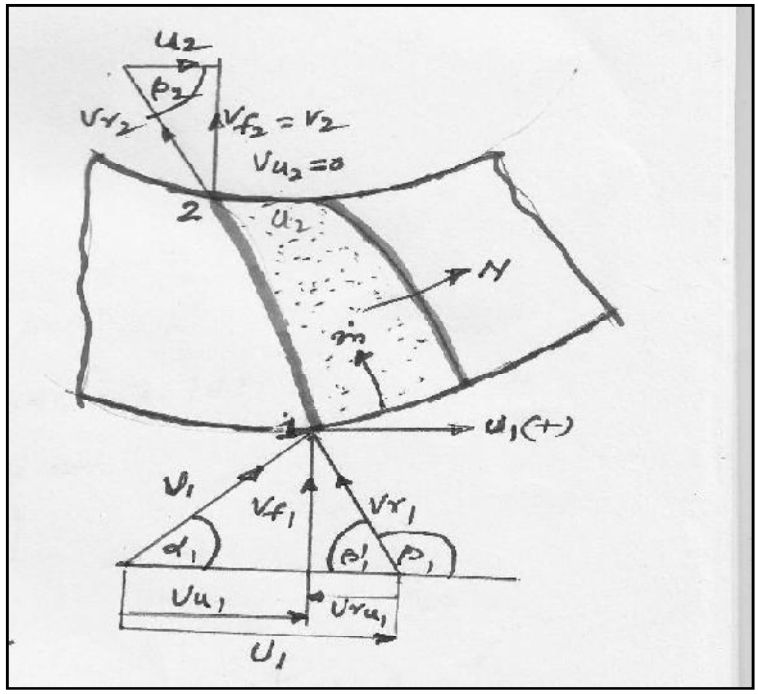
$$= \frac{\pi \times 2.45 \times 318.047}{60}$$

$U_2 = 40.8$ m/s

For radial discharge $V_{u2} = 0$, $d_2 = 90^0$
$V_2 = V_{f2} = 10.5$ m/s

$\tan \beta_2 = V_{f2} / U_2$
$\therefore \beta_2 = \tan^{-1}(10.5 / 40.8)$
$= 14.432^0$

$V_{r2} = \sqrt{(V_{f2}^2 + U_2^2)}$
$= \sqrt{(10.5^2 + 40.8^2)}$
$= 42.129$ m/s



Given
$Q = 110$ m^3/s and $\rho_w = 1000$ kg/m^3

Power developed
$P = \rho\, Q\, V_{u1} U_1$ watts
$= [1000 \times 110 \times 55.074 \times 60.783]$ watts
$= 368231.924$ Kw
or $P = 368.232$ MW

$$R = \frac{(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}{2 \times V_{u1} U_1}$$

$$= \frac{(60.783^2 - 40.8^2 + 42.129^2 - 11.775^2)}{2 \times 55.074 \times 60.783}$$

$= 0.548$

Utilization Factor

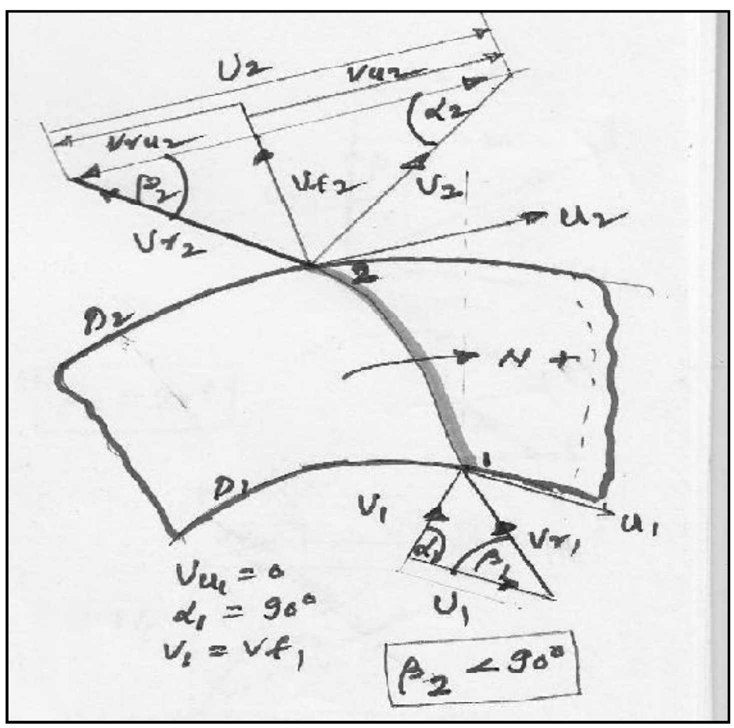
$$\varepsilon = \frac{WD}{[WD + V_2^2/2]}$$

$$= \frac{V_{u1} U_1}{V_{u1} U_1 + (V_2^2/2)}$$

$$= \frac{3347.563}{3347.563 + 55.125}$$

$\varepsilon = 0.984$

GENERAL ANALYSIS OF PUMPS & COMPRESSORS



In compressors and pumps work is done on the fluid
$V_{u2} U_2 > V_{u1} U_1$
$\therefore$ Work done $= [V_{u2}U_2 - V_{u1}U_1]$ Nm/kg or m^2/sec^2

$$= \frac{(V_2^2 - V_1^2)}{2} + \frac{(U_2^2 - U_1^2)}{2} + \frac{(V_{r1}^2 - V_{r2}^2)}{2}$$

Assuming no whirl at inlet
$\alpha_1 = 90^0$, $V_{u1} = 0$
Head developed (m)

$$H = \frac{V_{u2}U_2}{g}$$

$H = U_2 [U_2 - V_{ru2}] / g$

$$= \frac{U_2 [U_2 - V_{f2} \cot \beta_2]}{g}$$

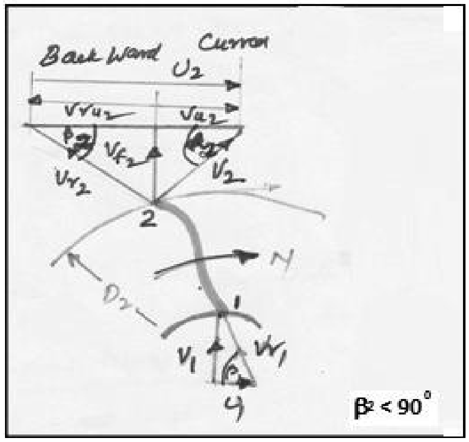
For flow rate in m^3/s

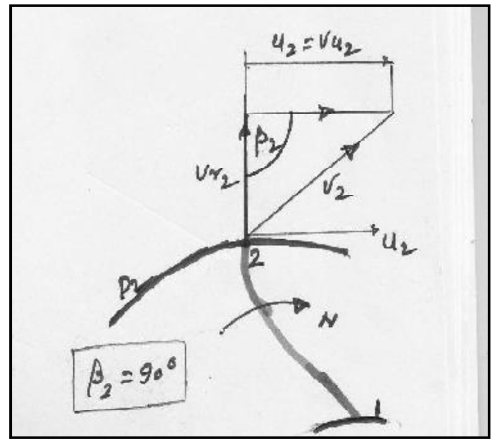
$Q = A_2 V_{f2}$ $\quad\therefore\ V_{f2} = Q / A_2$

$$H = \frac{U_2}{g}\left[U_2 - \frac{Q \cot \beta_2}{2}\right]$$

For a given pump or compressor D_2 , N , A_2 and β_2 are fixed, H and Q are variable.

$$H = \left[\frac{U_2^2}{g}\right]\left[U_2 - \frac{Q \cot \beta_2}{A_2}\right]$$





$$H = \frac{U_2^2}{g} - \frac{(U_2 \cot \beta_2)\, Q}{g A_2}$$

$H = K_1 - K_2 Q$

where $K_1 = U_2^2/g$

$K_2 = U_2 \cot \beta_2 / g A_2$

For backward curved vanes

$\beta_2 < 90^0$

K_2 is +ve

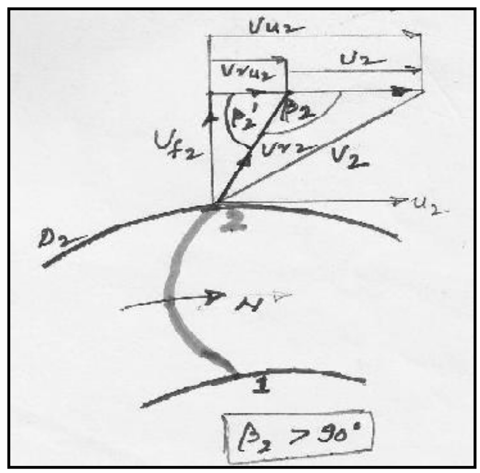
[HQ] curve has –ve slope

For Radial vanes

$\beta_2 = 90^0$

$K_2 = 0$

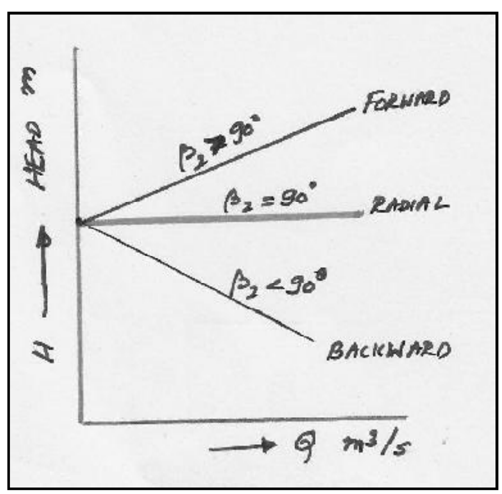
$H = K_1 = U_2^2/g$ = Constant for all Q



For Forward curves vanes

$K_2 = -$ ve

[HQ] curve has +ve slope



Majority of centrifugal pumps will have $\beta_2 \approx 25^0$ to 45^0

(Backward curved vanes)

Radial flow compressors will have $\beta_2 = 90^0$

CENTRIFUGAL PUMP & COMPRESSORS

Basic Analysis

□ = $\rho_1 Q_1 = \rho_2 Q_2$

$Q_1 = A_1 V_{f1}$

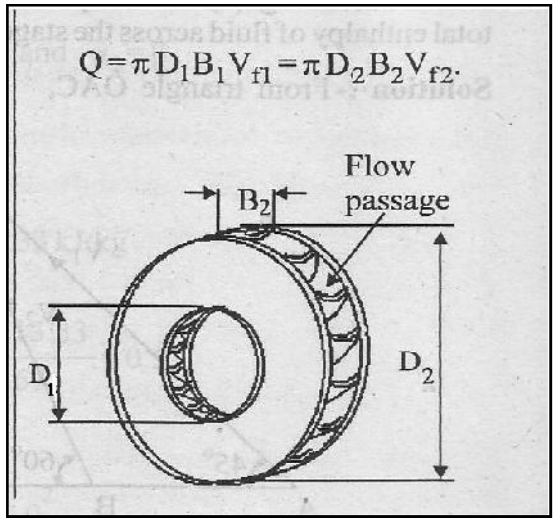
$Q_2 = A_2 V_{f2}$

$A_1 = \pi D_1 B_1$

$A_2 = \pi D_2 B_2$

When Velocity of flow in constant i.e.

$$V_{f1} = V_{f2} = V_f$$



For given mass rate of flow

□ = $\rho\,\pi D_1 B_1 V_{f1} = \rho\pi\, D_2 B_2 V_{f2}$

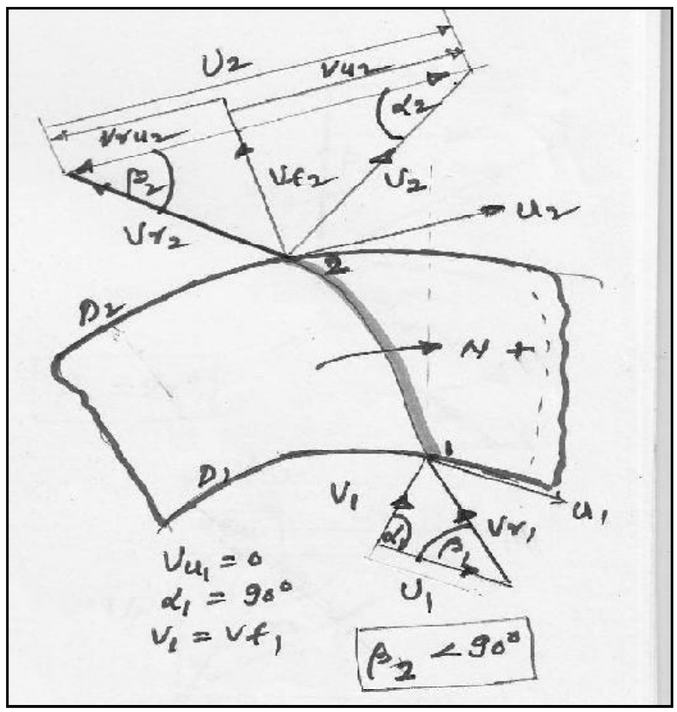
or $D_1 B_1 = D_2 B_2$

or $\dfrac{D_2}{D_1} = \dfrac{B_1}{B_2}$

If $V_{u1} = 0$ i.e. No whirl at inlet

Theoretical work done = WD/kg = $U_2 V_{u2}$

$$= U_2^2 \frac{V_{u2}}{U_2} \quad \ldots (1)$$



For constant velocity of flow

$V_1 = V_{f1} = V_{f2} = U_1 \tan\beta_1$

From exit velocity triangle for $\beta_2 < 90^0$

$V_{u2} = U_2 - V_{ru2}$

$= U_2 - V_{f2} \text{ Cot } \beta_2$

$\therefore \dfrac{V_{u2}}{U_2} = 1 - \dfrac{(V_{f2} \text{ Cot } \beta_2)}{U_2}$

$\therefore$ Putting in theoretical work done equation

$$WD/kg = U_2^2 \left[1 - \frac{(V_{f2} \text{ Cot } \beta_2)}{U_2}\right]$$

From sine rule in outlet velocity triangle

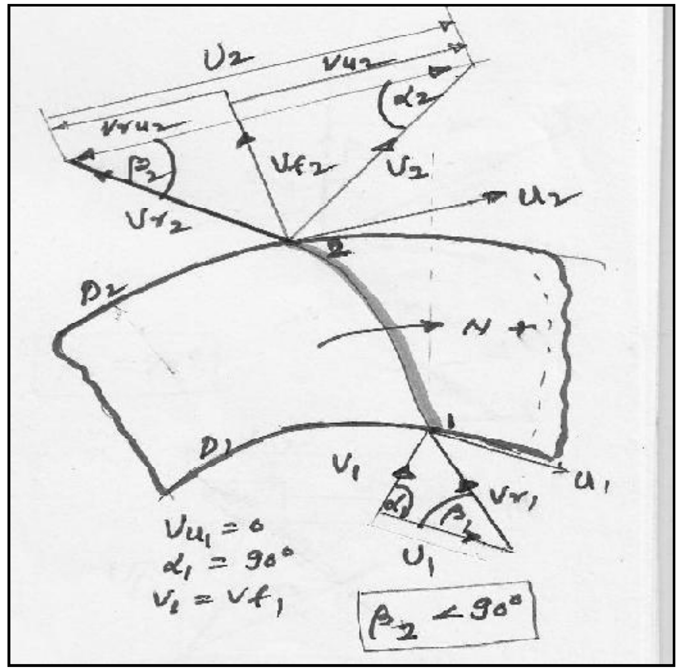
$$\frac{V_2}{\sin\beta_2} = \frac{U_2}{\sin(\alpha_2 + \beta_2)}$$

$$V_{u2} = V_2 \text{ Cos}\alpha_2 = \frac{U_2 \sin\beta_2 \,.\, \text{Cos } \alpha_2}{\sin(\alpha_2 + \beta_2)}$$

$$\frac{V_{u2}}{U_2} = \frac{\sin\beta_2 \,.\, \text{Cos } \alpha_2}{(\sin\beta_2 + \text{Cos } \alpha_2) + (\text{Cos } \alpha_2 + \sin\beta_2)}$$

$$\boxed{\frac{V_{u2}}{U_2} = \frac{\tan\beta_2}{\tan\alpha_2 + \tan\beta_2}}$$

Power developed – Theoretical



$P = \square\ WD/kg = \square\ U_2 V_{u2}$ Nm/s or J/s or Watts

$= \square\ \Delta h_0$

$= \square\ C_p\, \Delta T_0$

For isentropic process, Total Energy or Stagnation Energy Transfer

$$\Delta h_0 = \frac{(\Delta p)_0}{\rho}$$

or $\Delta p_0 = (\Delta h_0)\, \rho$

$$= \rho\, U_2^2 \left[1 - \frac{(V_{f2} \cot \beta_2)}{U_2}\right]$$

Taking static pressure rise due to centrifugal forces and relative velocity of flow known as diffusion effect.

$(p_2 - p_1) = (\Delta p)_{static}$

$$= \rho\, \frac{[U_2^2 - U_1^2]}{2} + \rho\, \frac{[V_{r1}^2 - V_{r2}^2]}{2}$$

$$(p_{02} - p_{01}) = (\Delta p)_0 = \rho\left[\frac{(U_2^2 - U_1^2)}{2} + \frac{(V_{r1}^2 - V_{r2}^2)}{2} + \frac{(V_2^2 - V_1^2)}{2}\right]$$

$$\therefore\ \Delta p_0 = (p_2 - p_1) + \rho\, \frac{(V_2^2 - V_1^2)}{2} \qquad \left(\frac{N}{m^2} = \frac{kg}{m^3} \cdot \frac{m}{sec^2} \cdot m\right)$$

Degree of Reaction R

$$R = \frac{(\Delta p)_{\,static}}{(\Delta p)_{\,stagnation}}$$

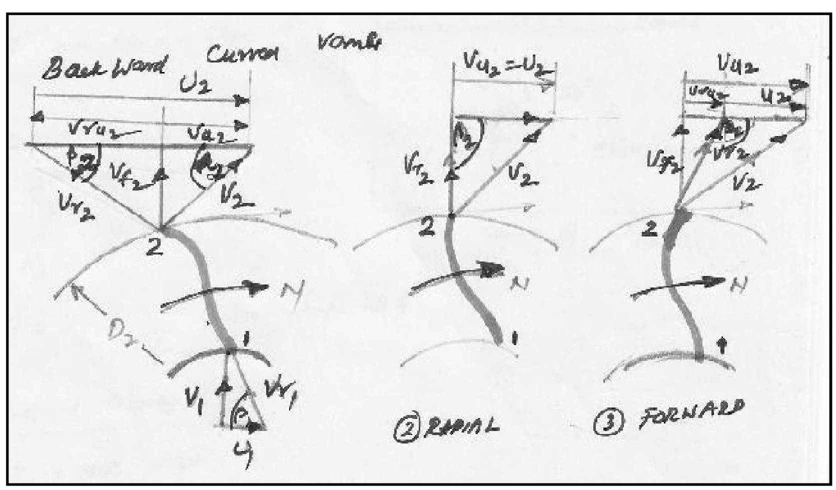
$$R = \frac{V_{u2}U_2 - \mathbf{\frac{(V_2^2 - V_1^2)}{2}}}{V_{u2}U_2}$$

$$R = \frac{V_{u2}U_2 - \left(\mathbf{\frac{[(V_{f2}^2 + V_{u2}^2) - V_{f1}^2]}{2}}\right)}{V_{u2}U_2}$$

$$= 1 - \left(\frac{V_{u2}^2}{2V_{u2}U_2}\right)$$

$$\boxed{\mathbf{R = 1 - \frac{V_{u2}}{2U_2}}}$$

From velocity triangle at outlet for various β_2



1. When $\beta_2 < 90^0$ backward curves vane

$V_{u2} < U_2$

$\therefore\ R < 1 > 0.5$

2. For Radial blades $\beta_2 = 90^0$

$V_{u2} = U_2$

$\therefore$ R = 1 – ½ = 0.5

3. For forward curved vanes $\beta_2 > 90^0$

$V_{u2} > U_2$

$\therefore$ R < 0.5

Problem on Centrifugal Pump

The internal and external diameters of a Centrifugal pump are 20 cm and 40 cm respectively. The pump is running at 1200 rpm. The vane angles at inlet is 20^0. Water enters the impeller radially and velocity of flow is constant. Find the work done by the impeller per kg of water for the following conditions

(a) $\beta_2 = 30^0$ (b) $\beta_2 = 90^0$ (c) $\beta_2 = 100^0$

Solution

Data

Internal diameter $d_1 = 0.2$m

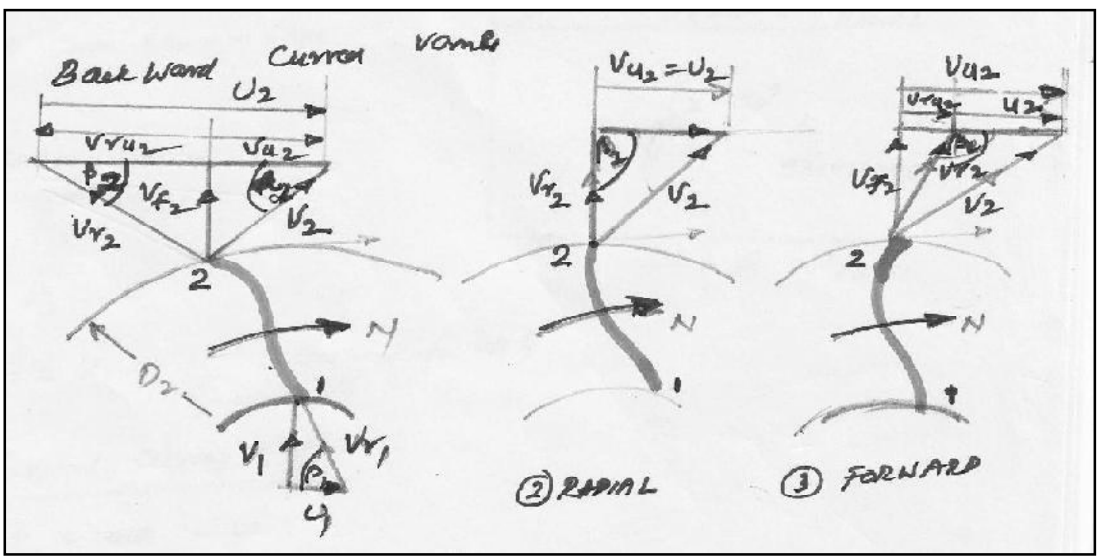
Outer diameter $d_2 = 0.4$ m

Speed N = 1200 rpm

Vane angle at inlet $\beta_1 = 20^0$

Water enters radially $Vu_1 = 0$, $\alpha_1 = 90^0$, $V_1 = V_{f1}$

Flow velocity constant $V_{f1} = V_{f2} = V_f$



$$U_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.2 \times 1200}{60} = 12.5666 \text{ m/s}$$

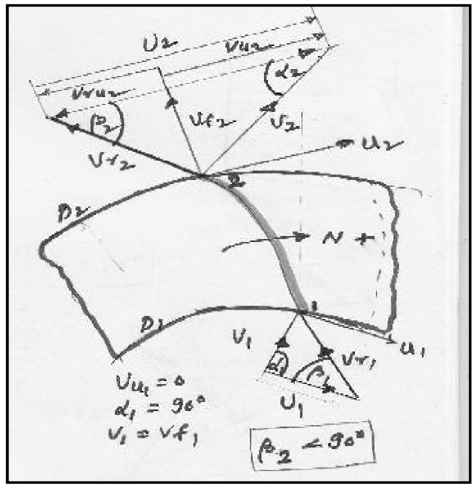
$$U_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.133 \text{ m/s}$$

From inlet velocity triangle and constant velocity of flow

$V_1 = V_{f1} = V_{f2} = V_f = U_1 \tan\beta_1$

$V_f = 12.566 \times \tan 20^0$

$= 4.574$ m/s



(a) $\beta_2 = 30^0$

$$\tan \beta_2 = \frac{V_{f2}}{V_{ru2}} = \frac{V_{f2}}{(U_2 - V_{u2})}$$

$$\therefore V_{u2} = U_2 - \frac{V_{f2}}{\tan\beta_2}$$

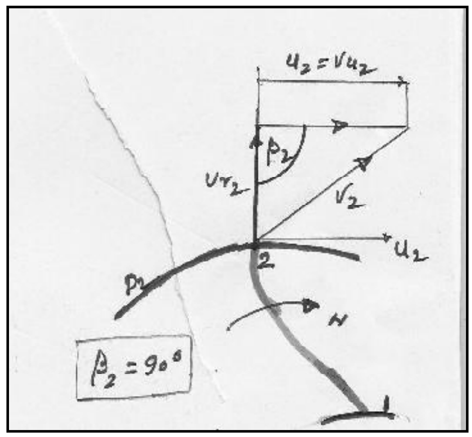
$$= 25.133 - \frac{4.574}{\tan 30}$$

$$= 17.211 \text{ m/s}$$

WD/kg = $[V_{u2}U_2]$

$= 17.211 \times 25.133$

$= 432.554$ Nm/kg



(b) $\beta_2 = 90^0$

$U_2 = V_{u2} = 25.133$ m/s

$$\text{WD/kg} = \frac{U_2^2}{2} = \frac{(25.133)^2}{2}$$

$$= 631.666 \text{ Nm/kg}$$

From velocity triangle at outlet

(c) $\beta_2 > 90^0 = 100^0$

$$\tan\beta_2' = \frac{V_{f2}}{V_{ru2}}$$

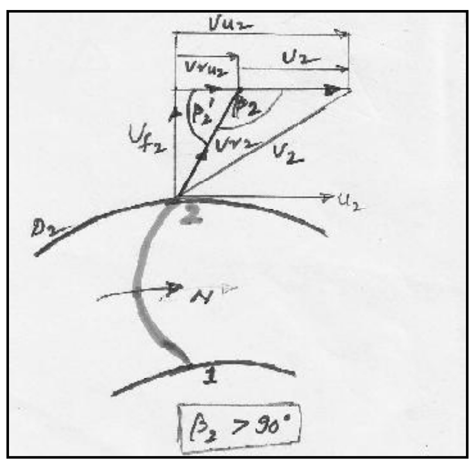
$$\tan\beta_2' = \frac{V_{f2}}{V_{ru2}}$$

$$\text{or } \tan(180 - \beta_2) = \frac{V_{f2}}{(V_{u2} - U_2)}$$

$$\tan 80^0 = \frac{4.574}{[V_{u2} - 25.133]}$$

$$\therefore V_{u2} = 25.133 + \frac{4.574}{\tan 80^0}$$

$$= 25.921 \text{ m/s}$$



WD/kg $= V_{u2}U_2$

$= 25.921 \times 25.133$

E $= 651.48$ Nm/kg (J/kg)

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