THEORY OF MACHINES AND MECHANISMS II

Mechanical IV/I

Chapter 5

Balance of Machinery

5.1 Introduction

In the system of rotating masses, the rotating masses have eccentricity due to limited accuracy in manufacturing, fitting tolerances, etc.

A mass attached to a rotating shaft will rotate with the shaft and if the centre of gravity of the rotating mass does not lie on the axis of the shaft then the mass will be effectively rotating about an axis at certain radius equal to the eccentricity.

Since the mass has to remain at that radius, the shaft will be pulled in the direction of the mass by a force equal to the centrifugal force due to inertia of the rotating mass.

The rotating centrifugal force provides harmonic excitation to system which thereby causes forced vibration of the machines.

It is very essential that all the rotating and reciprocating parts should be completely balanced as far as possible.

If these parts are not properly balanced, the dynamic forces are set up.

These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

Balancing of Rotating Masses

Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it.

In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass.

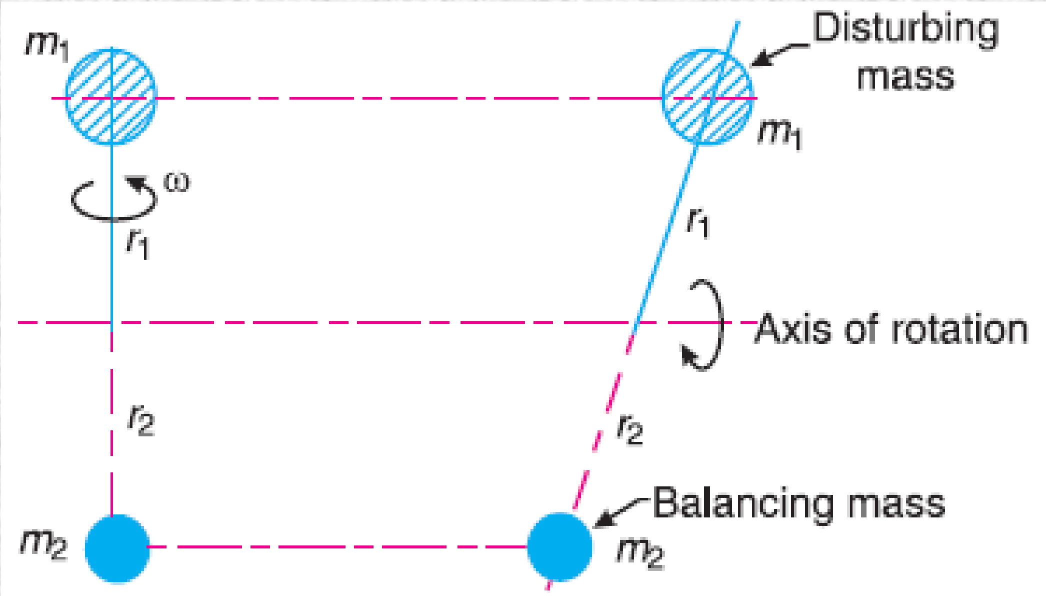
This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called balancing of rotating masses.

5.1 Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass $\mathbf{m_1}$ attached to a shaft rotating at $\boldsymbol{\omega}$ **rad/s**. Let $\mathbf{r_1}$ be the radius of rotation of the mass $\mathbf{m_1}$.

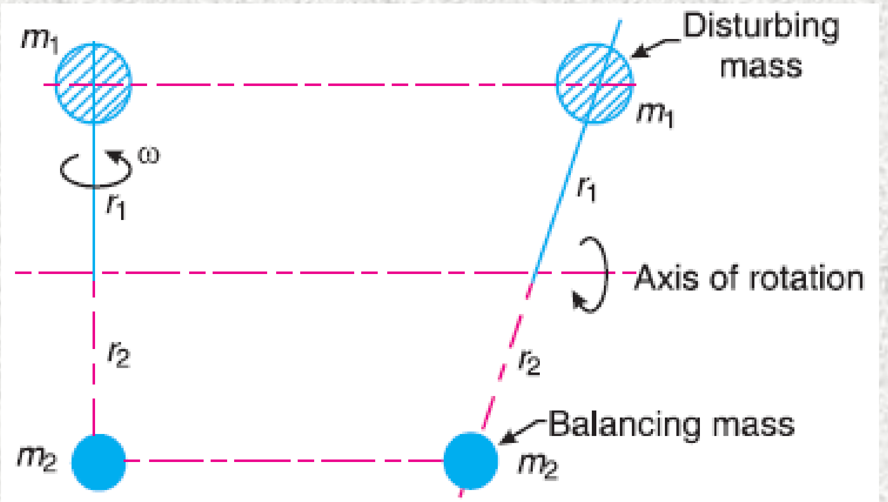
The centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$



This centrifugal force acts radially outwards and thus produces bending moment on the shaft.

In order to counteract the effect of this force, a balancing mass (**m_2**) may be attached in the same plane of rotation as that of disturbing mass (**m_1**) such that the centrifugal forces due to the two masses are equal and opposite.



If the radius of rotation of the balancing mass **m_2** is **r_2**, then centrifugal force due to mass **m_2**,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Then for the balance

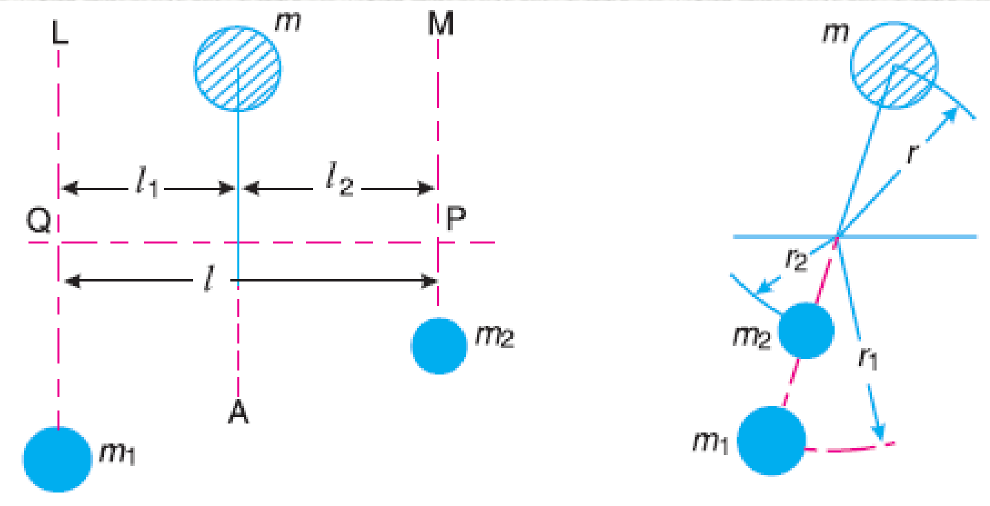
$$m_1.\omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

5.2 Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

In order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same.
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

When the plane of the disturbing mass lies in between the planes of the two balancing masses



Force Balance

$$F_C = F_{C1} + F_{C2}$$

$$m \cdot \omega^2 \cdot r = m_1 . \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2$$

Moment Balance

Moment about Point P

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

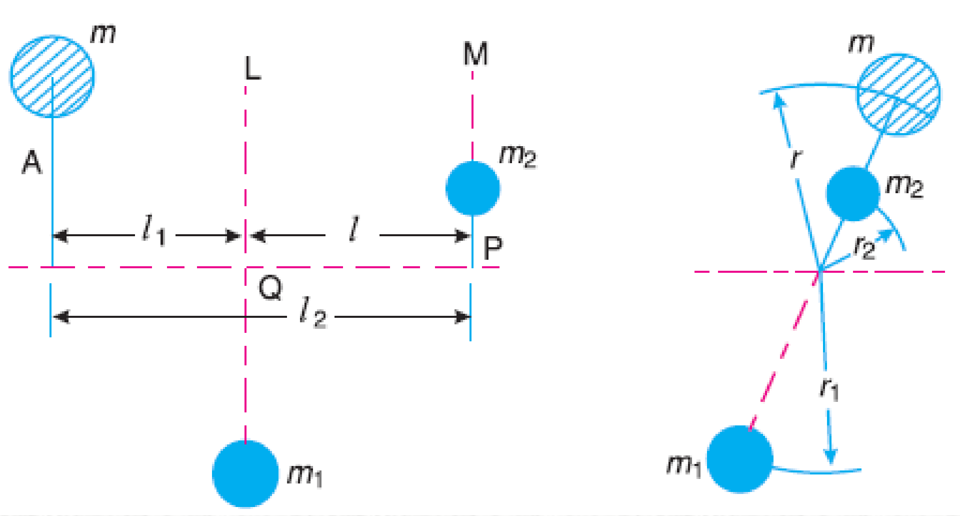
$$m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l}$$

Moment about Point Q

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l}$$

When the plane of the disturbing mass lies on one end of the planes of the balancing masses



Force Balance

$$F_C + F_{C2} = F_{C1}$$

$$m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$m \cdot r + m_2 \cdot r_2 = m_1 . r_1$$

Moment Balance

Moment about Point P

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l}$$

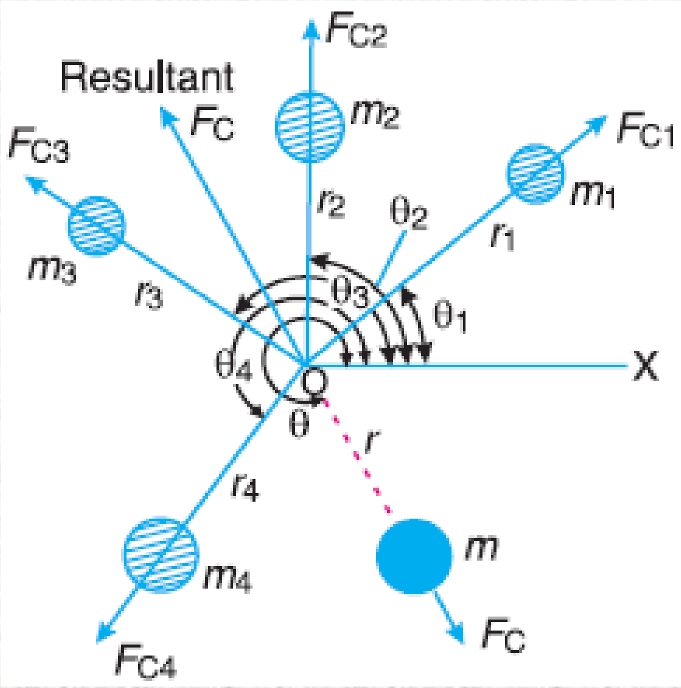
Moment about Point Q

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l}$$

5.3 Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude $\mathbf{m_1}$, $\mathbf{m_2}$, $\mathbf{m_3}$ and $\mathbf{m_4}$ at distances of $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$ and $\mathbf{r_4}$ from the axis of the rotating shaft. Let $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$, $\boldsymbol{\theta_3}$ and $\boldsymbol{\theta_4}$ be the angles of these masses with the horizontal line OX. Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of **ω rad/s**.



Sum of horizontal components of the centrifugal forces,

$$\sum H = m_1\omega^2 r_1 cos\theta_1 + m_2\omega^2 r_2 cos\theta_2 + \ \dots \dots$$

Sum of vertical components of the centrifugal forces,

$$\sum V = m_1\omega^2 r_1 sin\theta_1 + m_2\omega^2 r_2 sin\theta_2 + \ \dots \dots$$

Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

If θ is the angle, which the resultant force makes with the horizontal, then

$$\theta = tan^{-1}\left(\frac{\sum V}{\sum H}\right)$$

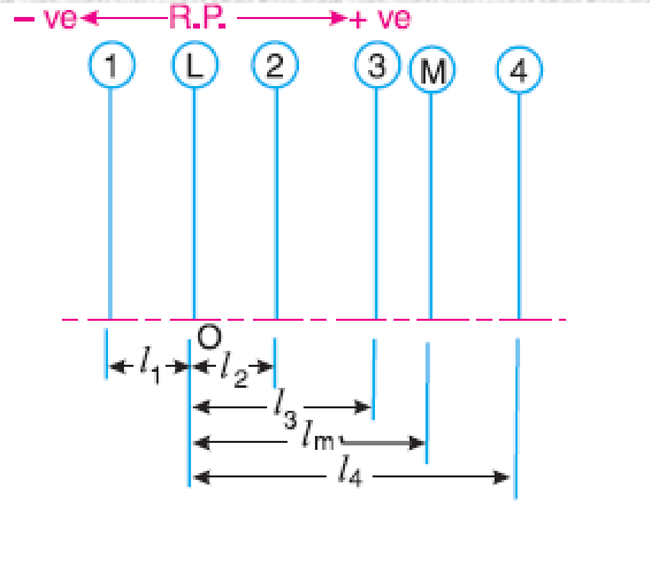
The magnitude of the balancing mass m and its radial position r is then

$$F_C = m\omega^2 r$$

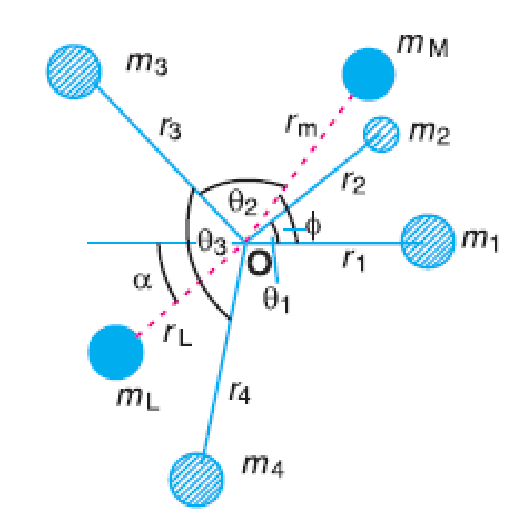
5.4 Balancing of Several Masses Rotating in Different Planes

In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, i.e. the resultant force must be zero.
2. The couples about the reference plane must balance, i.e. the resultant couple must be zero.

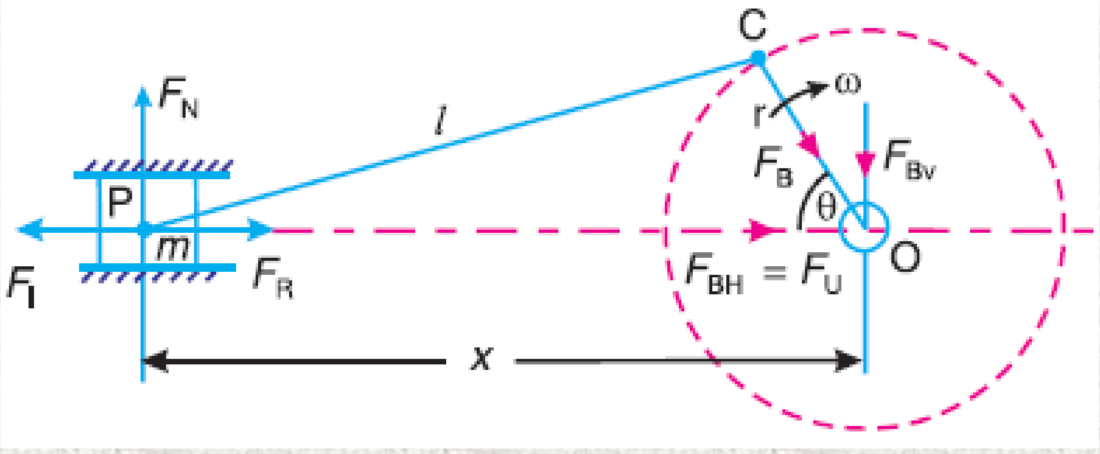


(*a*) Position of planes of the masses

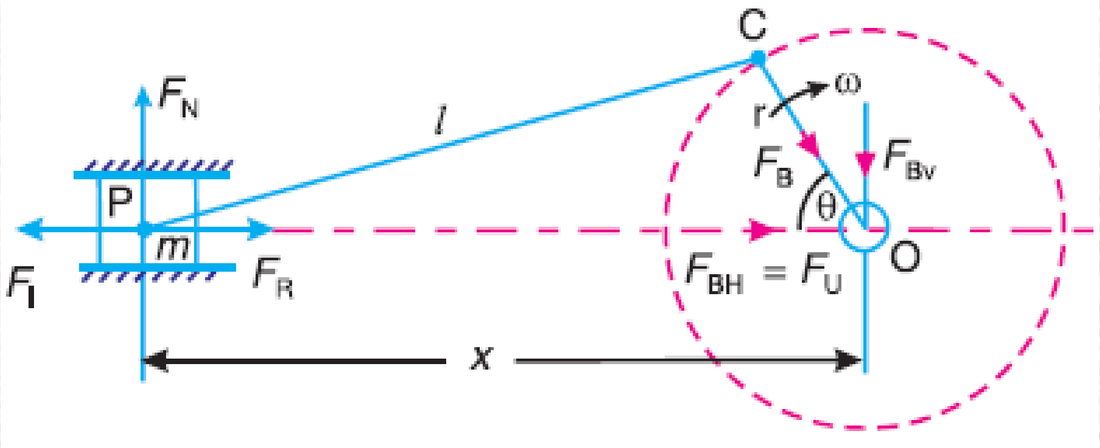


(*b*) Angular position of the masses

Balancing of Reciprocating Masses



The various forces act on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as unbalanced force or shaking force. Thus if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.



Since $\mathbf{F_R}$ and $\mathbf{F_I}$ are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of $\mathbf{F_B}$ (i.e. $\mathbf{F_{BH}}$) acting along the line of reciprocation is also equal and opposite to $\mathbf{F_I}$. This force $\mathbf{F_{BH} = F_U}$ is an unbalanced force or shaking force and required to be properly balanced.