



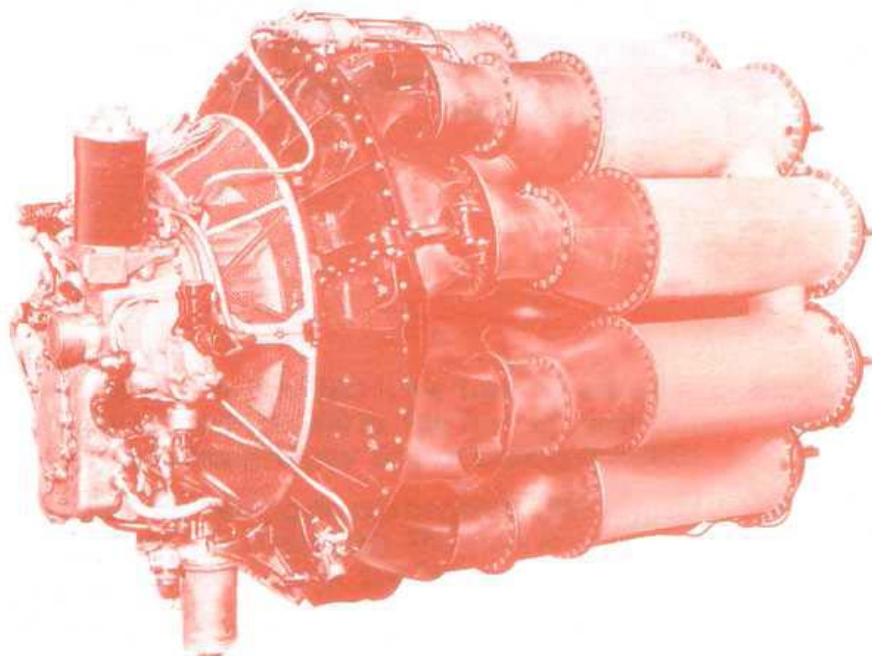
# Design Aspects Compressor Turbo-machines

Hari Bahadur Dura  
Lecturer

Institute of Engineering (IOE)  
Tribhuvan University  
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# Turbo-machines

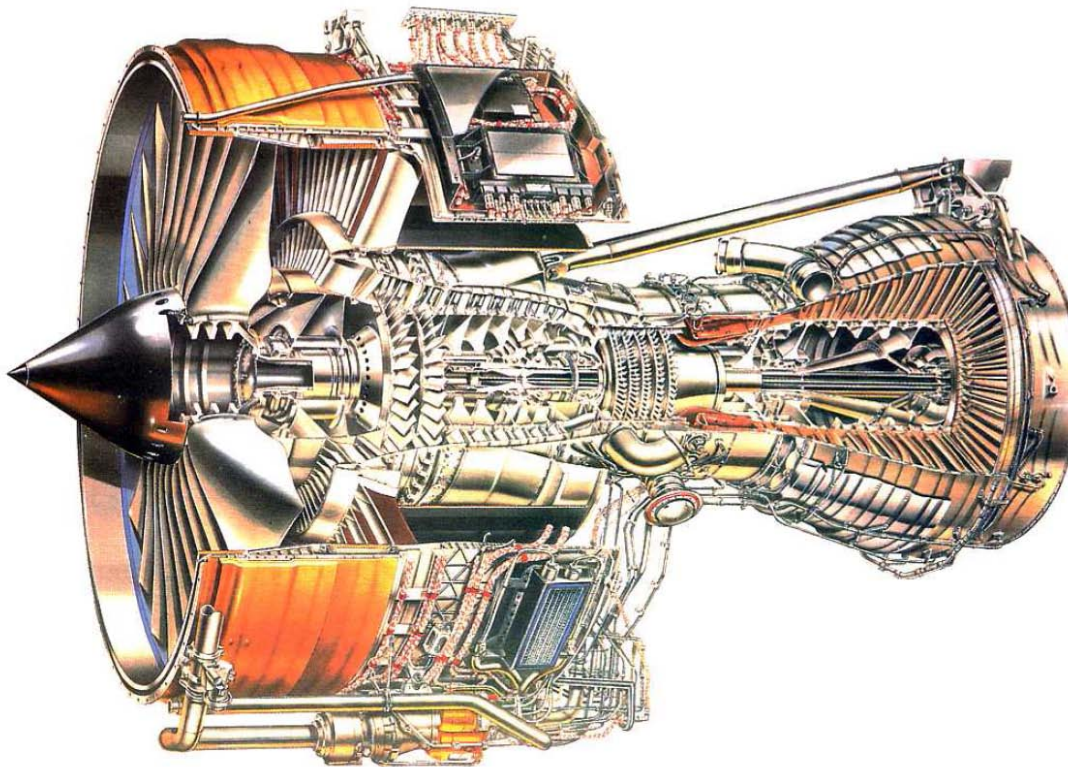


Rolls-Royce B23 Welland

On 1 April, 1943, Rolls-Royce assumed responsibility for the Power Jets W2B which, a month earlier, had made its first flight in the Gloster E28/39 at 1200lb thrust. Later known as the B23 Welland it was, during April, put through a 100 hr test at the design rating of 1600 lb thrust. In June, 1943, it flew in a Gloster Meteor at 1400lb thrust. Production Welland-Meteors were in action against V-1 flying bombs in August 1944.



# Turbo-machines

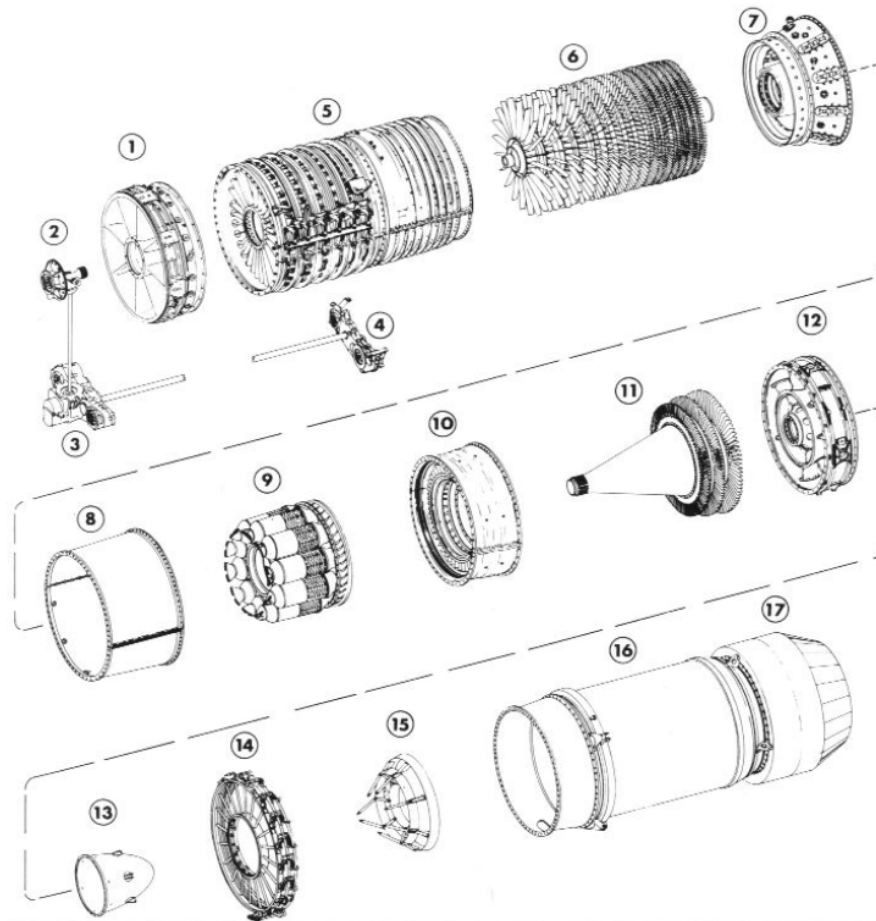


Rolls – Royce Trent 800

Detailed engineering design began in 1988 to meet the propulsion requirements of the Airbus A330 (Trent 700) and Boeing 777 (Trent 800). The Trent first ran in August 1990, and in January 1994 a Trent 800 demonstrated a world record thrust of 106,087 lb.

The engine entered service in March 1995 in the Airbus A330.

# Turbo-machines

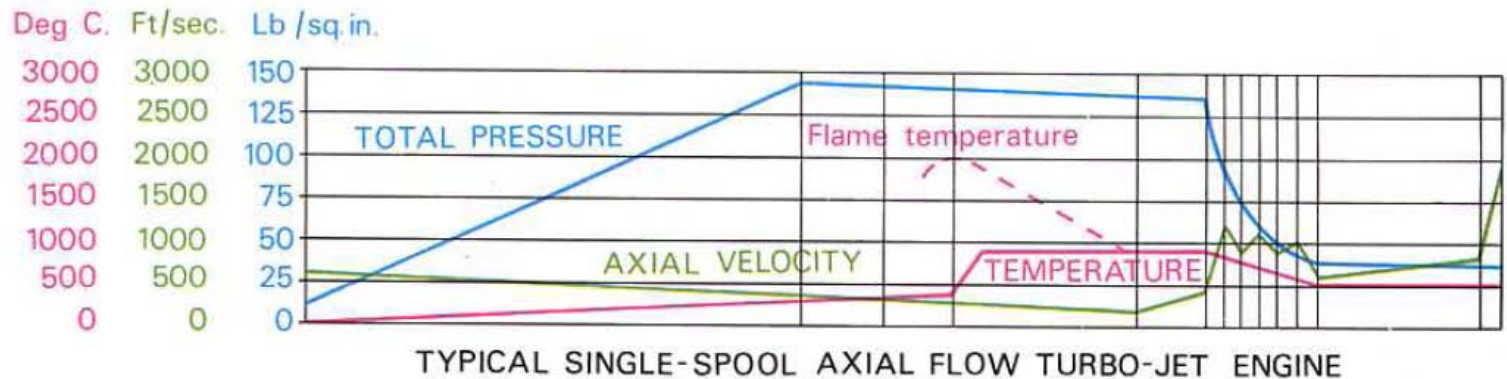
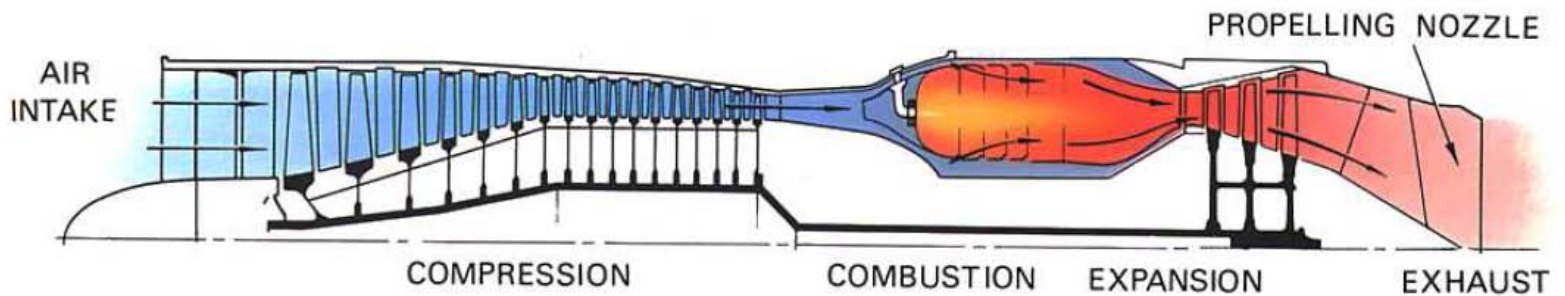


- |                          |                         |                                  |
|--------------------------|-------------------------|----------------------------------|
| 1 Compressor front frame | 7 Compressor rear frame | 13 Rear cone                     |
| 2 Bevel gear             | 8 Combustion casing     | 14 Reheat fuel manifold assembly |
| 3 Transfer gearbox       | 9 Combustion assembly   | 15 Flame holder                  |
| 4 Accessory gearbox      | 10 Turbine casing       | 16 Afterburner                   |
| 5 Compressor casing      | 11 Turbine rotor        | 17 Exhaust nozzle                |
| 6 Rotor                  | 12 Turbine rear frame   |                                  |



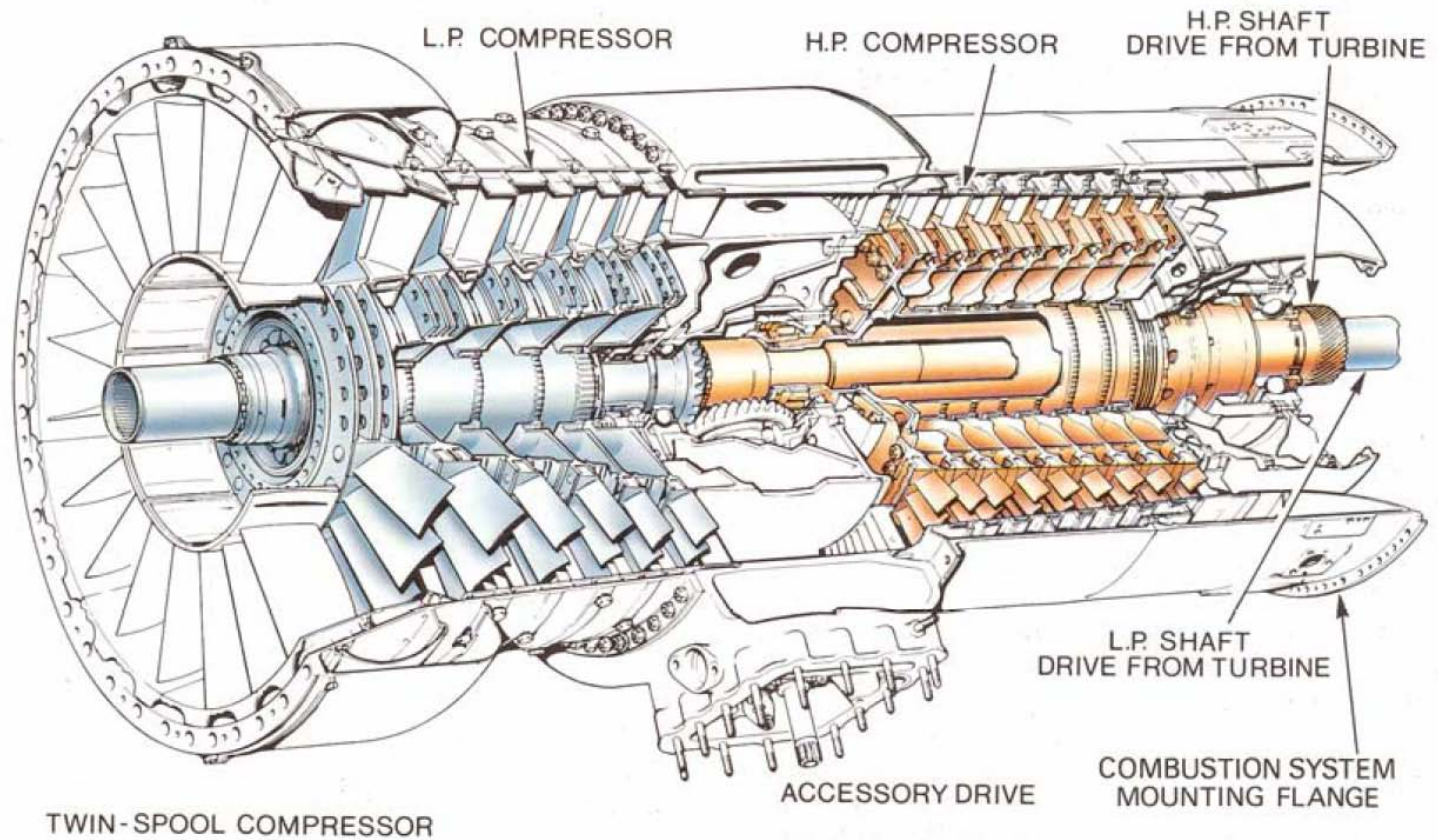


# Turbo-machines



## Axial Flow Turbo-machines

# Compressor Stage and Velocity Triangle

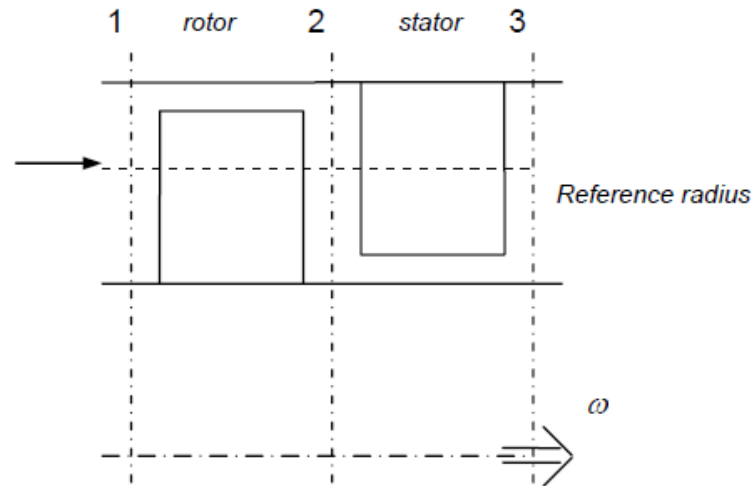




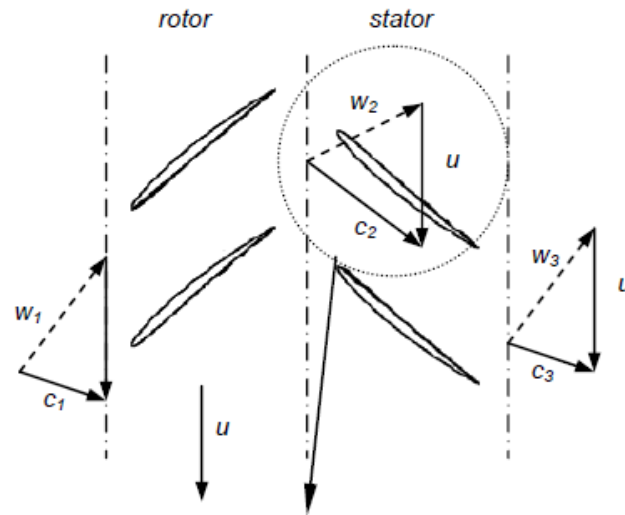
## Compressor Stage and Velocity Triangle

Stage denotations

- 1 rotor inlet
- 2 stator inlet
- 3 stator outlet



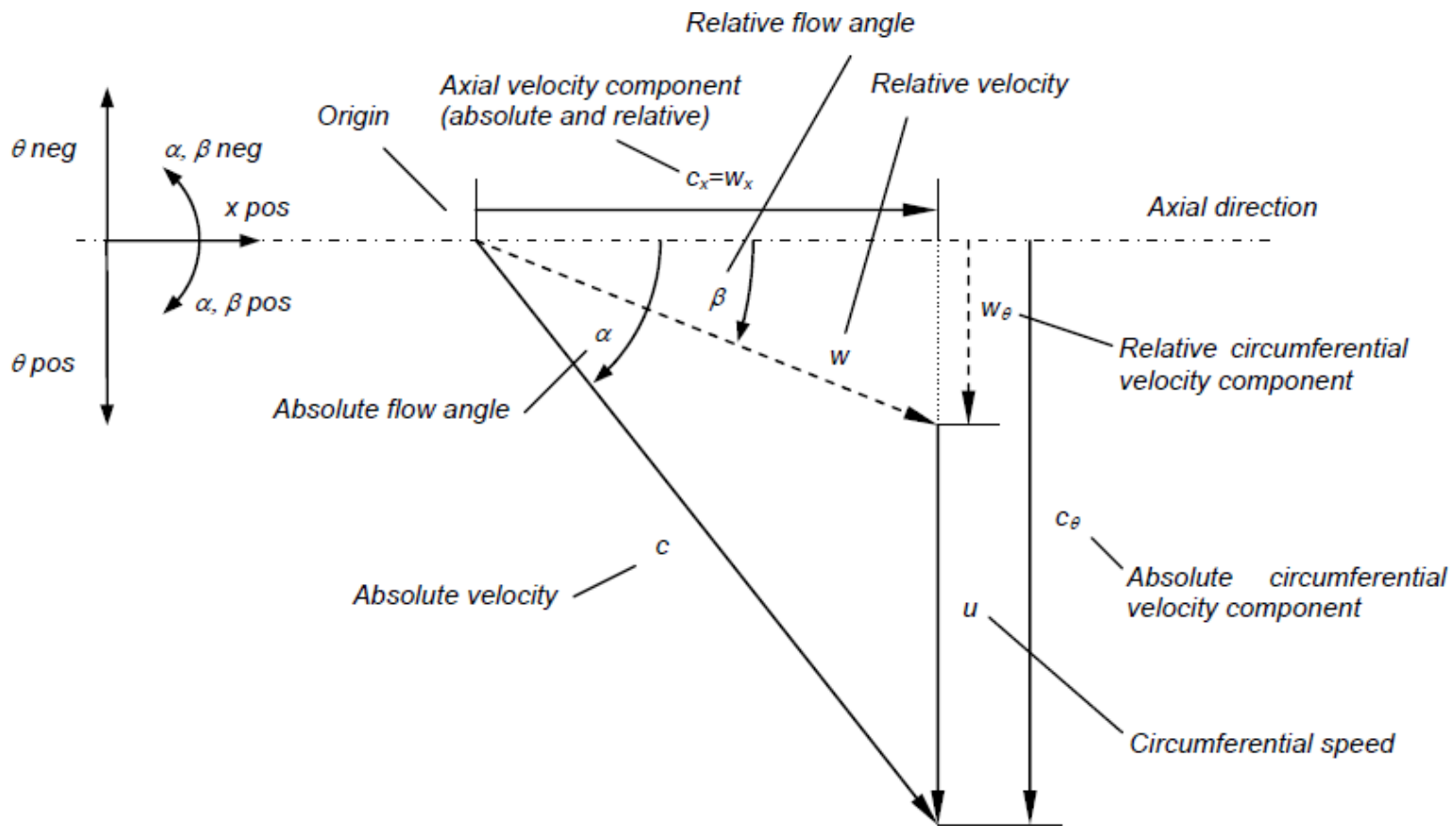
Stage velocity triangles



Velocity triangles denotations and conventions



## Compressor Stage and Velocity Triangle







## Stage Velocity Triangle

Reference Position:

Mean Radius: 
$$r_m = \frac{r_h + r_s}{2} \quad \text{Eq. 1}$$

Euler's Radius: (Radius that splits the annular cross section in half)

$$r_E = \sqrt{\frac{r_h^2 + r_s^2}{2}} \quad \text{Eq. 2}$$



## Stage Velocity Triangle

Circumferential Speed of the Rotor  $u$  at reference radius:

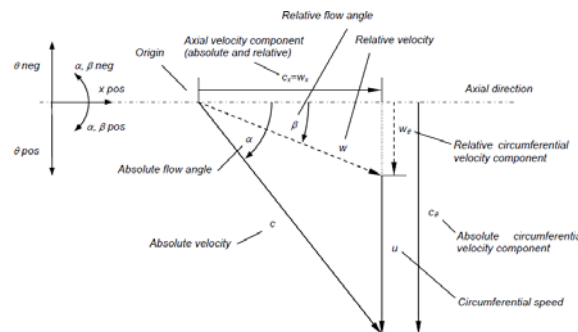
$$u = r_{ref} \cdot \omega \quad \text{Eq. 3}$$

Relation between absolute velocity ' $c$ ', relative velocities ' $w$ ' and circumferential speed ' $u$ ' are references as follows:

$$w_x = c_x \quad \text{Eq. 4}$$

$$w_\theta = c_\theta - u \quad \text{Eq. 5}$$

Where,  $c_x$  and  $c_\theta$  are axial and circumferential components of the respective velocities





## Stage Velocity Triangle

Where  $c_x$  and  $c_\theta$  are the axial and circumferential components of the respective velocity as follows:

$$c^2 = c_x^2 + c_\theta^2 \quad \text{Eq. 6}$$

$$w^2 = w_x^2 + w_\theta^2 \quad \text{Eq. 7}$$

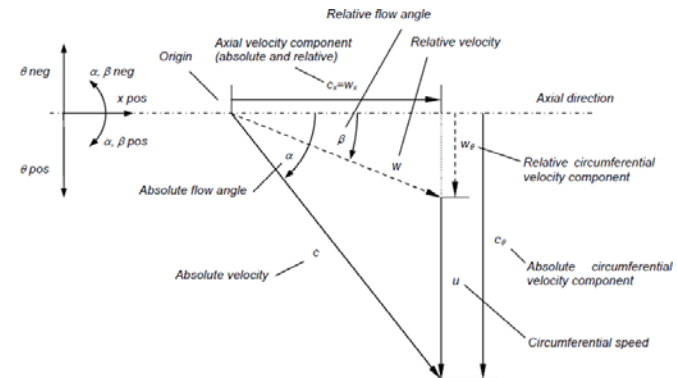
The flow angles are defined as

$$\tan \alpha = \frac{c_\theta}{c_x}$$

Eq. 8

$$\tan \beta = \frac{w_\theta}{w_x}$$

Eq. 9





## First: Degree of Reaction 'R'

Degree of Reaction (R) is the change in Enthalpy effectuated in rotor to the change in enthalpy of the stage:

$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} \quad \text{Eq. 10}$$

It can be written as:

$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{\Delta h_{rotor}}{\Delta h_{rotor} + \Delta h_{stator}} = \frac{h_2 - h_1}{h_2 - h_1 + h_3 - h_2} \quad \text{Eq. 11}$$

How is enthalpy related to Velocities???



## First: Degree of Reaction 'R'

The change in enthalpies in stator and rotor respectively are related to the velocities as follows:

In rotor the rothalpy is constant

$$I = h + \frac{w^2}{2} - \frac{u^2}{2} \text{ is constant}$$

Thus,

$$h_2 + \frac{w_2^2}{2} - \frac{u_2^2}{2} = h_1 + \frac{w_1^2}{2} - \frac{u_1^2}{2} \quad \text{Eq. 12}$$

Which leads to:

$$h_2 - h_1 = \frac{1}{2} (w_1^2 - w_2^2 - u_1^2 + u_2^2) \quad \text{Eq. 13}$$



## First: Degree of Reaction 'R'

$$h_2 - h_1 = \frac{1}{2} \left( w_1^2 - w_2^2 - u_1^2 + u_2^2 \right) \quad \text{Eq. 13}$$

Note:

$$\frac{1}{2} \left( w_1^2 - w_2^2 \right)$$

Is the contribution arising from the deceleration of the relative flow.

$$\frac{1}{2} \left( u_2^2 - u_1^2 \right)$$

Is the contribution arising from the centrifugal effect.



## First: Degree of Reaction 'R'

In stator, the stagnation enthalpy  $h_0 = h + \frac{c^2}{2}$  is constant, thus

$$h_3 + \frac{c_3^2}{2} = h_2 + \frac{c_2^2}{2} \quad \text{Eq. 14}$$

Leading to

$$h_3 - h_2 = \frac{1}{2} (c_2^2 - c_3^2) \quad \text{Eq. 15}$$

Substituting all the parameters,

$$R = \frac{w_1^2 - w_2^2 - u_1^2 + u_2^2}{w_1^2 - w_2^2 - u_1^2 + u_2^2 + c_2^2 - c_3^2} \quad \text{Eq. 16}$$



## First: Degree of Reaction 'R'

For a **normal repetition stage** with the following restrictions:

$$\vec{c}_1 = \vec{c}_3 \quad \text{Eq. 17}$$

$$c_{x,1} = c_{x,2} = c_{x,3} = \text{const} \quad \text{Eq. 18}$$

$$u_2 = u_3 \quad \text{Eq. 19}$$

We get further expressions as:

$$c^2 = c_x^2 + c_\theta^2$$

$$w^2 = w_x^2 + w_\theta^2$$

Thus yielding,

$$R = \frac{w_{\theta,1}^2 - w_{\theta,2}^2}{w_{\theta,1}^2 - w_{\theta,2}^2 + c_{\theta,2}^2 - c_{\theta,3}^2} \quad \text{Eq. 20}$$



## First: Degree of Reaction 'R'

The relative velocity components in the denominator shall be expressed by the absolute velocity components as  $w_{\theta} = c_{\theta} - u$  leading to

$$R = \frac{w_{\theta,1}^2 - w_{\theta,2}^2}{c_{\theta,1}^2 - 2c_{\theta,1}u + u^2 - c_{\theta,2}^2 + 2c_{\theta,2}u - u^2 + c_{\theta,2}^2 - c_{\theta,3}^2} \quad \text{Eq. 21}$$

After canceling out elements the expression can be rewritten as

$$R = \frac{w_{\theta,1}^2 - w_{\theta,2}^2}{2u \cdot (c_{\theta,2} - c_{\theta,3})} \quad \text{Eq. 22}$$



## First: Degree of Reaction 'R'

At the numerator shall be expressed as

$$w_{\theta,3}^2 - w_{\theta,2}^2 = (w_{\theta,3} - w_{\theta,2}) \cdot (w_{\theta,3} + w_{\theta,2})$$

Also,

$$c_{\theta} = w_{\theta} + u$$

Leads to

$$R = \frac{(w_{\theta,1} - w_{\theta,2}) \cdot (w_{\theta,1} + w_{\theta,2})}{2u \cdot (w_{\theta,2} + u_2 - w_{\theta,3} - u_3)} \quad \text{Eq. 23}$$





## First: Degree of Reaction 'R'

Both the circumferential speeds in the denominator and the relative components  $w_{\theta,1} - w_{\theta,2}$  cancel out finally yielding

$$R = -\frac{1}{2u} (w_{\theta,1} + w_{\theta,2})$$

Eq. 24

In the absolute frame of reference as  $w_{\theta} = c_{\theta} - u$  yielding

$$R = -\frac{1}{2u} (c_{\theta,1} - u + w_{\theta,2}) = \frac{1}{2} - \frac{1}{2u} (c_{\theta,1} + w_{\theta,2})$$

Eq. 25

Further expressing R in terms of flow angles  $\tan \alpha = \frac{c_{\theta}}{c_x}$

$$R = \frac{1}{2} - \frac{c_x}{2u} (\tan \alpha_1 + \tan \beta_2)$$

Eq. 26



## First: Degree of Reaction 'R'

$$R = \frac{1}{2} - \frac{c_x}{2u} (\tan \alpha_1 + \tan \beta_2) \quad \text{Eq. 26}$$

- An increase in flow angle  $\beta_2$  leads to an increase in degree of reaction ( $\beta_2 \uparrow \Rightarrow R \uparrow$ ), i.e. the contribution of enthalpy change in the rotor to the total change in enthalpy in the stage gets larger
- An increase in flow angle  $\alpha_1$  leads to an decrease in degree of reaction ( $\alpha_1 \uparrow \Rightarrow R \downarrow$ ), i.e. the contribution of enthalpy change in the stator to the total change in enthalpy in the stage gets larger
- For compressor stages the degree reaction usually lies in the range [0.5...1]



## Second: Loading Factor ' $\psi$ '

The loading factor relates the change in total enthalpy effectuated in the stage to the rotational speed as follows:

$$\psi = \frac{\Delta h_0}{u^2} \quad \text{Eq. 27}$$

Using Euler's turbine equation  $\Delta h_0 = u_2 c_{\theta,2} - u_1 c_{\theta,1}$  leads to

$$\psi = \frac{u_2 c_{\theta,2} - u_1 c_{\theta,1}}{u^2} \quad \text{Eq. 28}$$

For a normal repetition stage:

$$\vec{c}_1 = \vec{c}_3 \quad \text{Eq. 29}$$

$$c_{x,1} = c_{x,2} = c_{x,3} = \text{const} \quad \text{Eq. 30}$$

$$u_2 = u_3 \quad \text{Eq. 31}$$



## Second: Loading Factor ' $\psi$ '

The expression of the loading factor can further be simplified to

$$\psi = \frac{c_{\theta,2} - c_{\theta,1}}{u} \quad \text{Eq. 32}$$

Expressing the absolute flow velocities in the relative frame of reference as  $c_{\theta} = w_{\theta} + u$  the loading factor can be expressed as

$$\psi = \frac{w_{\theta,2} - w_{\theta,1}}{u} \quad \text{Eq. 33}$$

An equivalent expression can be obtained by substituting the relative velocity components at position 1 in the absolute frame of references as

$$c_{\theta} = w_{\theta} + u$$

yielding

$$\psi = 1 + \frac{w_{\theta,2} - c_{\theta,1}}{u} \quad \text{Eq. 34}$$



## Second: Loading Factor ' $\psi$ '

Which also can be expressed in terms of flow angles  $\beta_2$  and  $\alpha_1$  as follows:

$$\psi = 1 + \frac{c_x}{u} (\tan \beta_2 - \tan \alpha_1) \quad \text{Eq. 35}$$

According to the convention of velocity components depicted above flow angle  $\beta_2$  is negative whilst flow angle  $\alpha_1$  is positive. This leads to the observation:

- Decrease in flow angles  $\beta_2$  and  $\alpha_1$  lead to increase in loading factor ( $\beta_2, \alpha_1 \downarrow \Rightarrow \psi \uparrow$ )

$$\psi = 1 + \frac{w_{\theta,2} - c_{\theta,1}}{u}$$

$$\tan \alpha = \frac{c_{\theta}}{c_x}$$

$$\tan \beta = \frac{w_{\theta}}{w_x}$$

$$w_x = c_x$$





## Third: Flow coefficient ' $\phi$ '

The flow coefficient relate the axial velocity component to the circumferential speed as follows:

$$\phi = \frac{c_x}{u} \quad \text{Eq. 36}$$

The only observation to make for this coefficient is that the higher the axial velocity in the stage the higher the flow coefficient. As can be recognized below the flow coefficient stretches the velocity triangles in the axial direction.



# Normalized Velocity Triangle

The normalized velocity components are denoted by the respective capital letters and yield from

$$C = \frac{c}{u_3} \quad \text{Eq. 37}$$

$$W = \frac{w}{u_3} \quad \text{Eq. 38}$$

$$U = \frac{u}{u_3} \quad \text{Eq. 39}$$

The special case of a normal repetition stage shall be regarded here for the sake of simplicity. The applied principle is however valid for all types of turbine stages

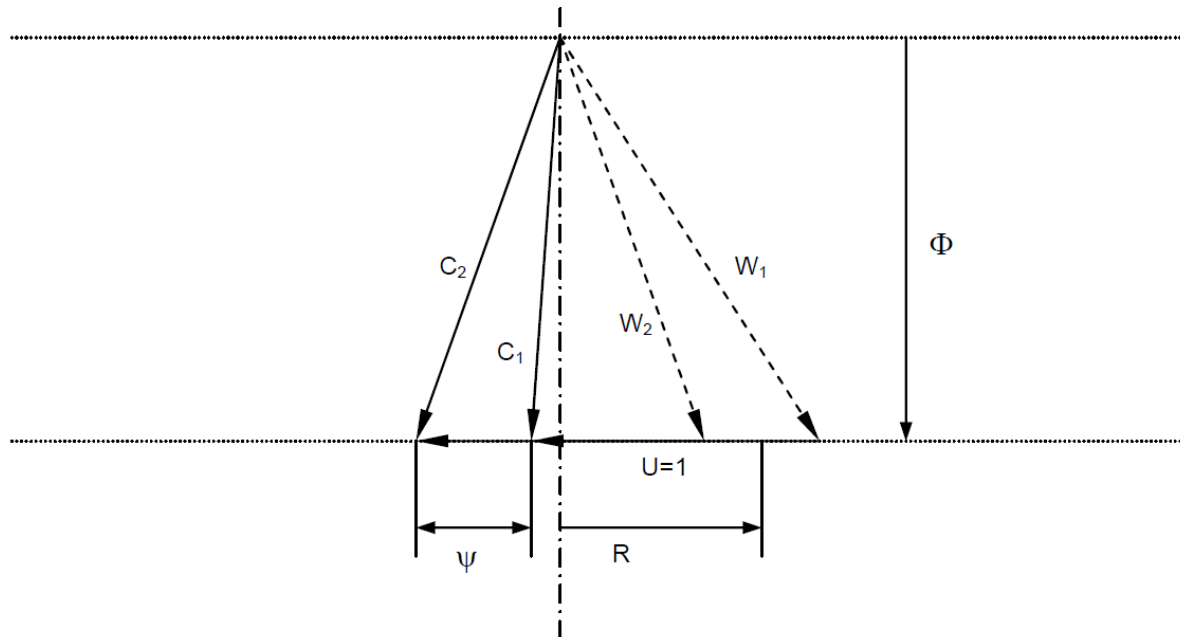
As a normal repetition stage with the condition  $c_{x,1} = c_{x,2} = c_{x,3} = \text{const}$

Is considered the height of the triangle corresponds to  $C_x = \frac{c_x}{u} = \phi$

i.e. the flow coefficient



## Normalized Velocity Triangle



- The height of the velocity triangle corresponds to the **flow coefficient**  $\Phi$
- The **loading coefficient** corresponds to the circumferential distance between  $C_2$  and  $C_1$ . In the case of a repetition stage this equals to the circumferential distance between  $W_2$  and  $W_1$ .
- The **degree of reaction** equals to the distance between axial and half the midpoint between  $W_2$  and  $W_1$ .



# Normalized Velocity Triangle

## Special Cases

### Degree of Reaction Equal to One Half ( $R = 0.5$ )

The expression of the degree of reaction yields the following

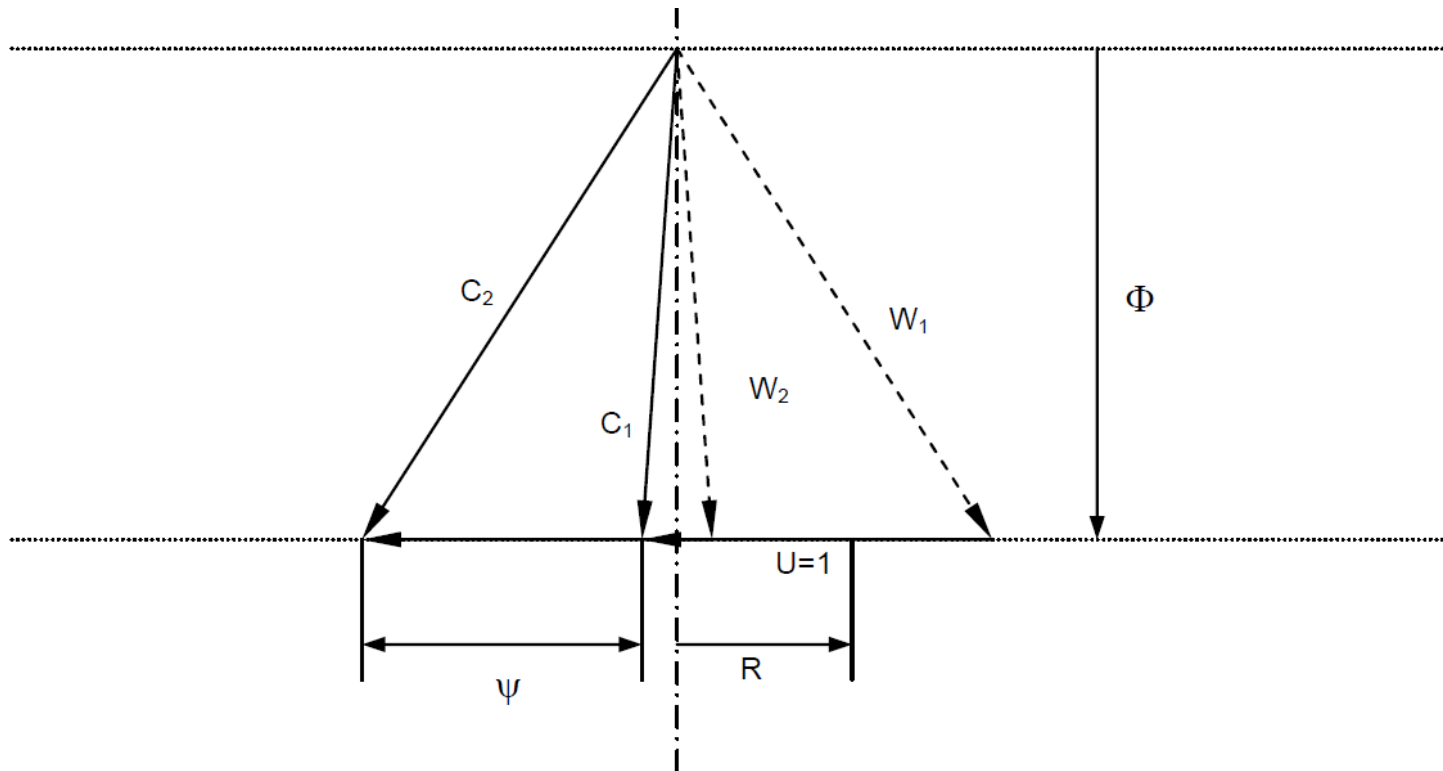
$$R = \frac{1}{2} = -\frac{1}{2u} (w_{\theta,1} + w_{\theta,2}) \Rightarrow w_{\theta,2} + u = -w_{\theta,1} \quad \text{Eq. 40}$$

Which is equivalent to  $c_{\theta,2} = -w_{\theta,1}$  Substituting this expression into the equation of loading coefficient yields

$$\psi = \frac{w_{\theta,2} - w_{\theta,1}}{u} \Rightarrow \psi = \frac{2 \cdot w_{\theta,2}}{u} + 1 = \frac{2c_{\theta,2}}{u} - 1 \quad \text{Eq. 41}$$



## Normalized Velocity Triangle



- As  $c_{\theta,2} = -w_{\theta,1}$  and normal stage it follows that  $|c_2| = |w_1|$  and with the assumption of repetition stage ( $c_1 = c_3$ ) consequently  $\Delta h_{rotor} = \Delta h_{stator}$ . The change in enthalpy in a reaction stage is thus equally split on rotor and stator.
- Both stator and rotor effectuate compression of the fluid and thus  $p_2 > p_1$  and  $p_3 > p_2$





# Normalized Velocity Triangle

Zero Exit Swirl ( $c_{\theta,3} = 0$ )

With  $c_{\theta,3} = 0 = w_{\theta,3} + u \Rightarrow w_{\theta,3} = -u$  and  $w_{\theta,3} = w_{\theta,1}$  the degree of reaction writes to

$$R = -\frac{1}{2u} (w_{\theta,1} + w_{\theta,2}) = \frac{1}{2} - \frac{w_{\theta,2}}{2u} \quad \text{Eq. 42}$$

The loading coefficient yields from

$$\psi = \frac{w_{\theta,2} - w_{\theta,1}}{u} = \frac{w_{\theta,2}}{u} + 1 \quad \text{Eq. 43}$$

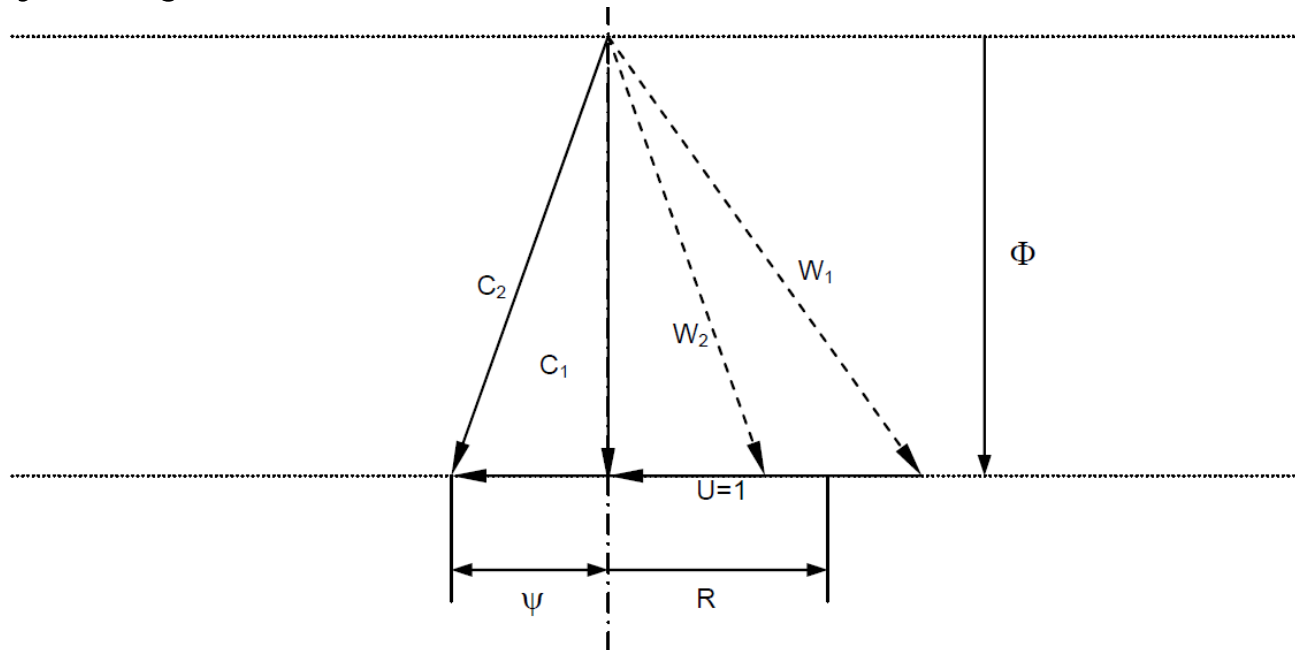
Combining the two expressions leads after reformulation to a relationship between degree of reaction and loading coefficient as follows:

$$\psi = 2 \cdot (1 - R) \quad \text{Eq. 44}$$



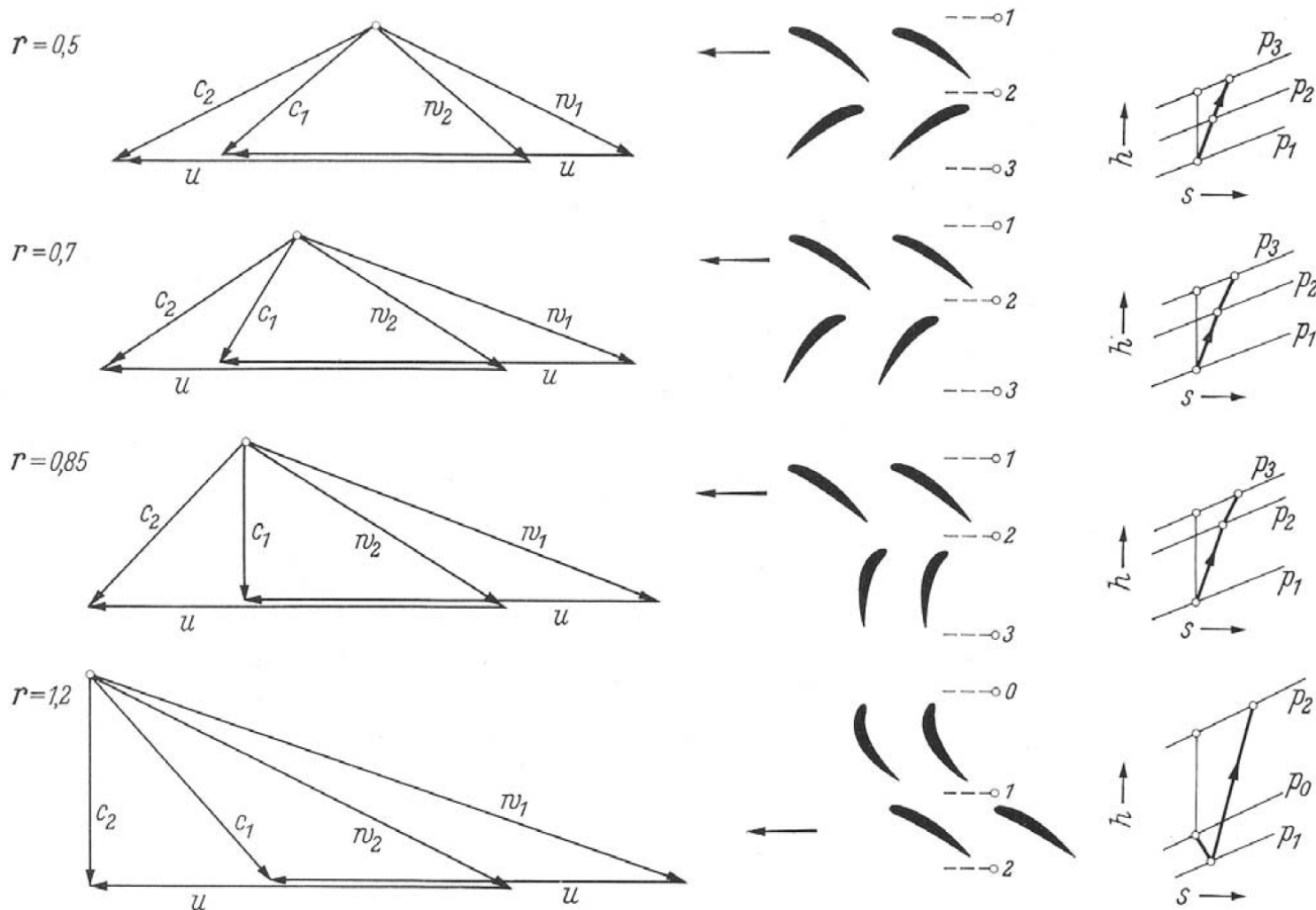
# Normalized Velocity Triangle

Velocity Triangle (zero exit swirl)



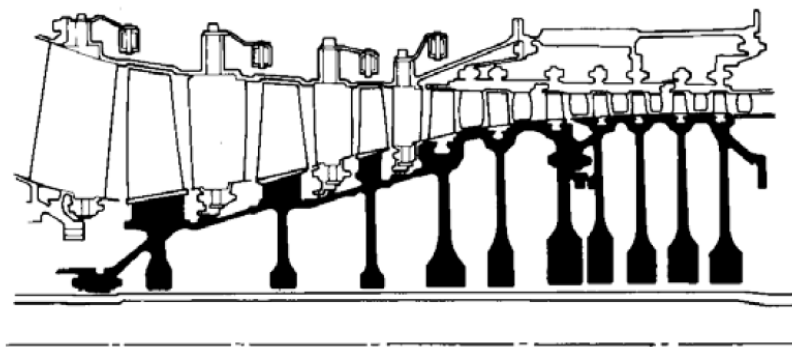
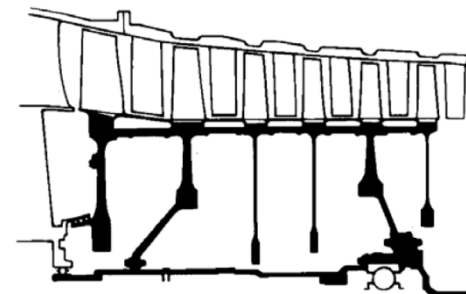
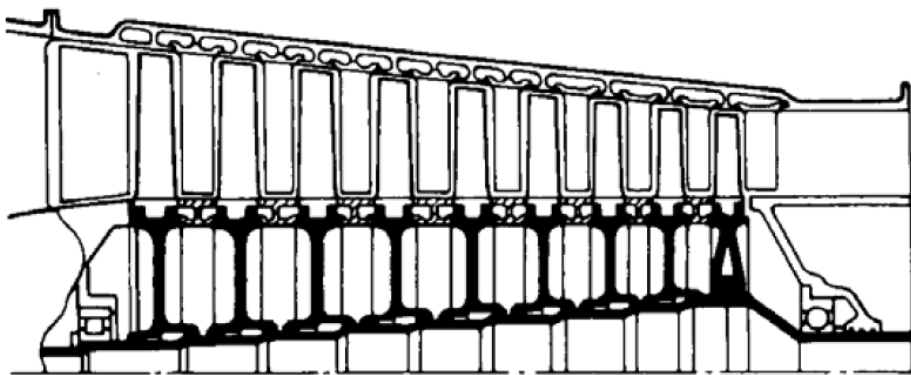
- The flow exists the stage purely axial, i.e. there is no swirl at stage exit
- For a zero exit swirl stage the degree of reaction and the loading factor are dependent

# Axial Compressor – Effect of R





# Compressor Design Aspects



Rotor drum and disk

INTAKE CASING

STATOR VANE

ROTOR BLADE

MAIN SHAFT DRIVE FROM TURBINE

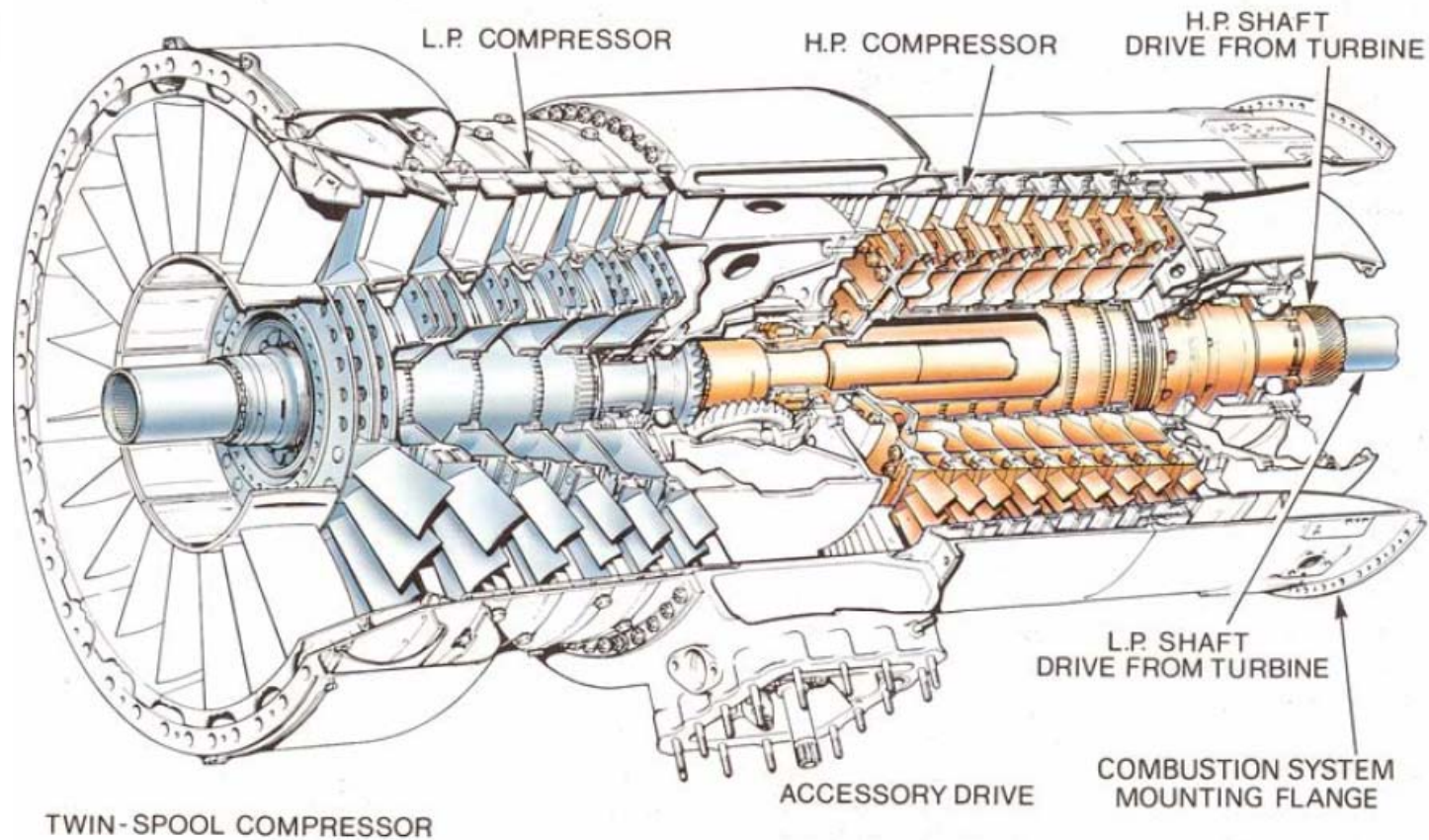
SINGLE-SHAFT COMPRESSOR

ACCESSORY DRIVE

COMBUSTION SYSTEM MOUNTING FLANGE

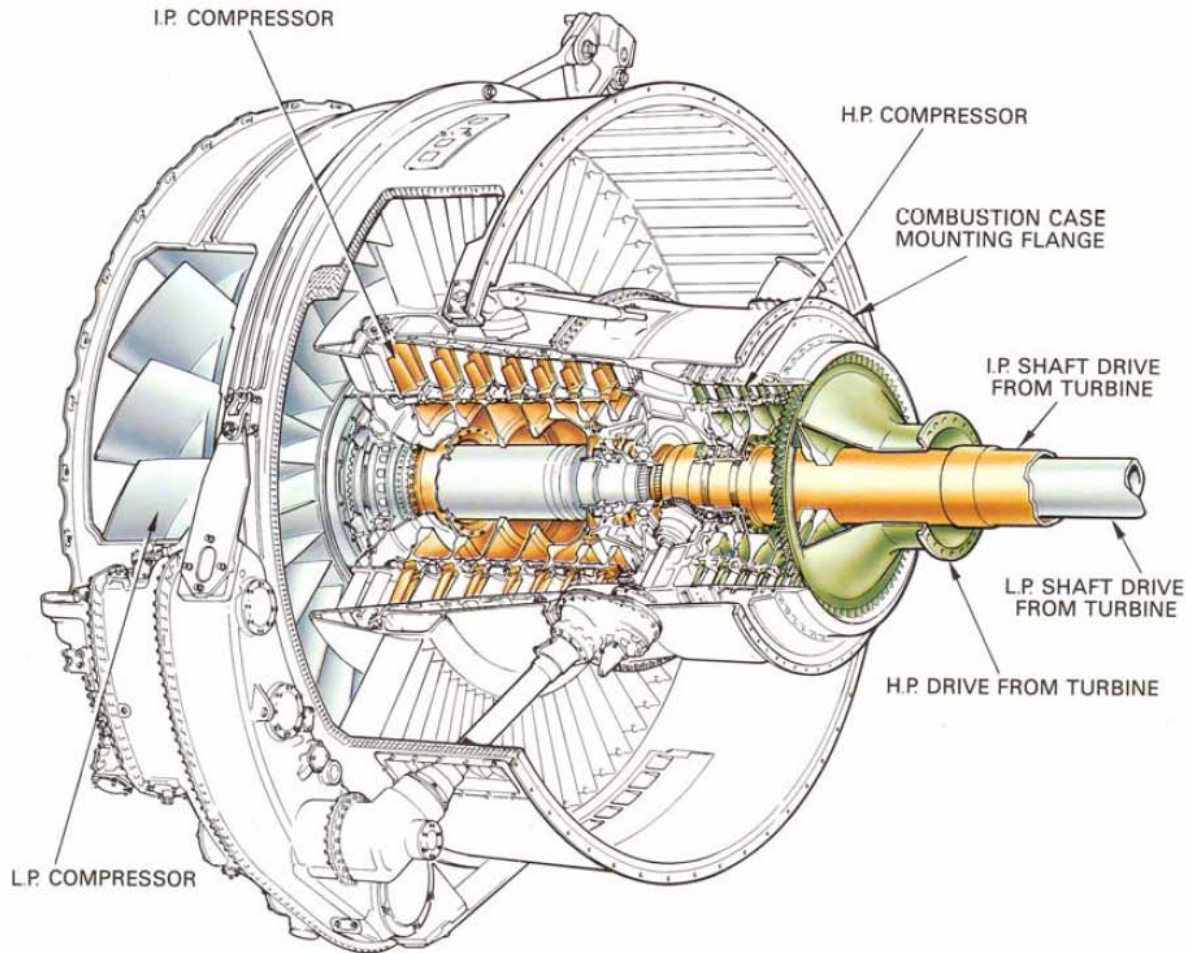


# Multiple Rotors



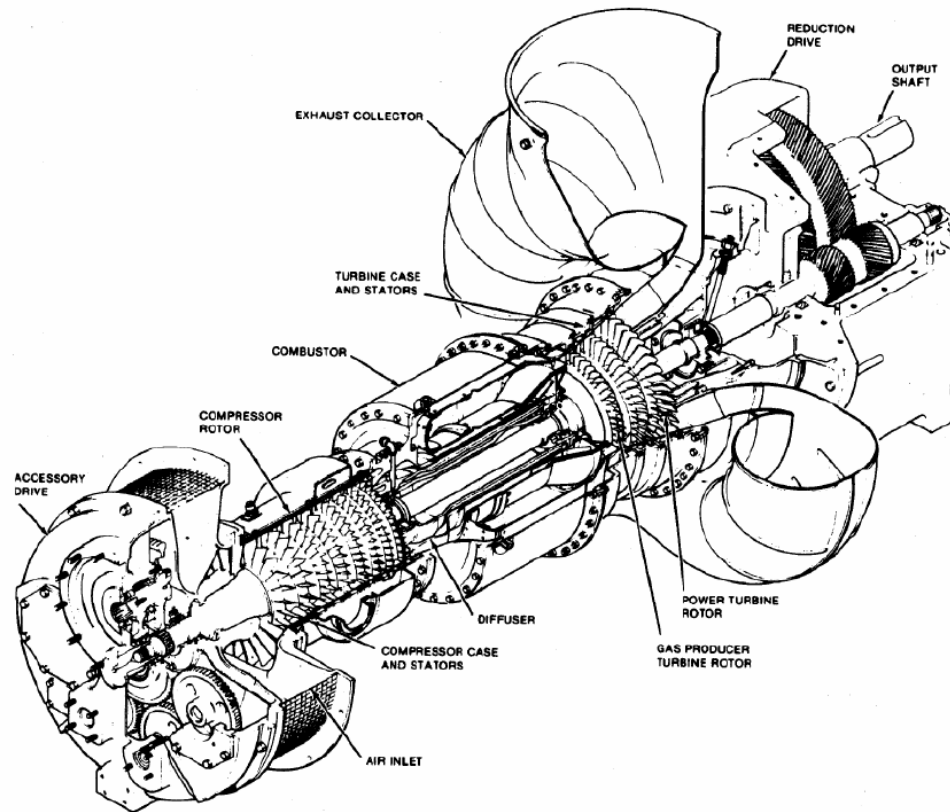
- Single spool simple and robust construction
- Twin spool optimum speed for LP and HP compressor; LP usually rotates slower than HP

# Multiple Rotors



- Optimum fan speed; fan speed is lower than LP and HO compressor (large diameter gives high blade tip speeds → supersonic due to low speed of sound)
- Avoid having a geared fan (a gear box for these power ratings is heavy)

# Solar Turbines Mechanical Drive

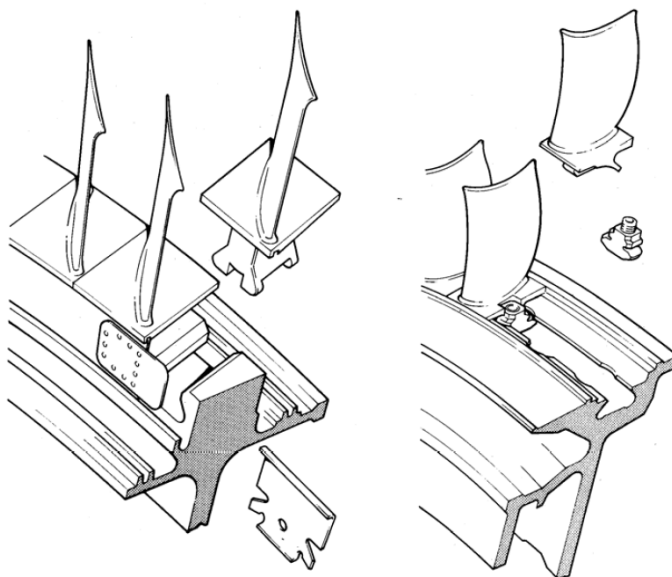


Different problem for power generation turbine:

- Power shaft must rotate at constant speed. At part load behavior the compressor would however work in a better operating point at part speed.
- Solution: free power turbine



# Rotor Blade Mount

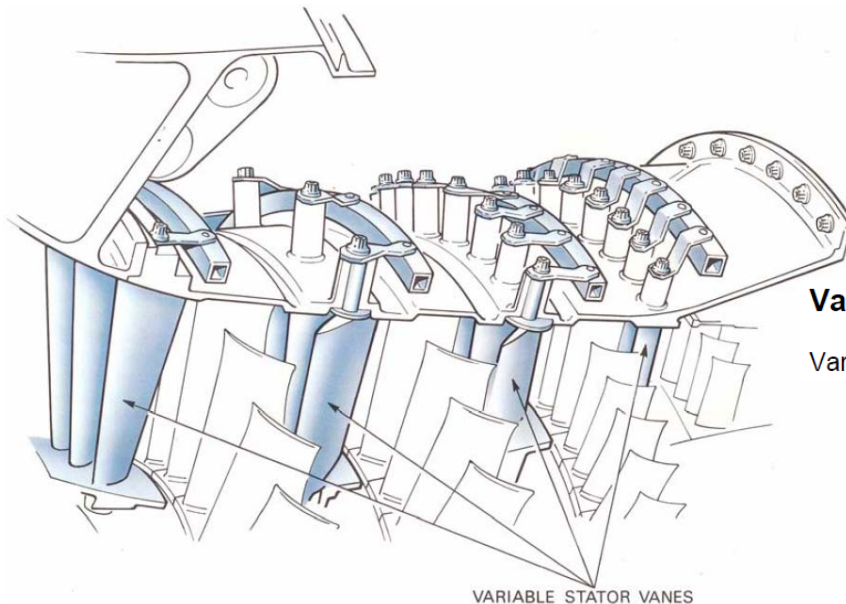
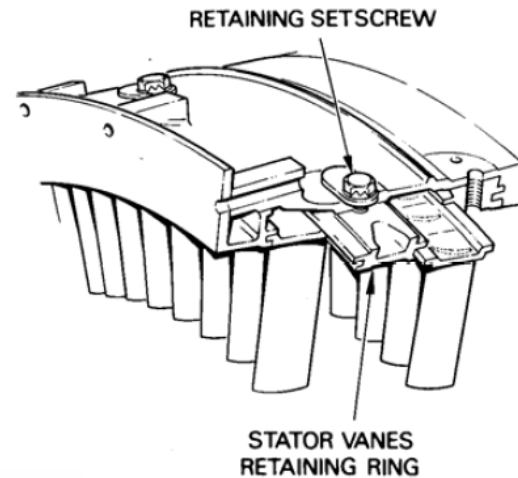
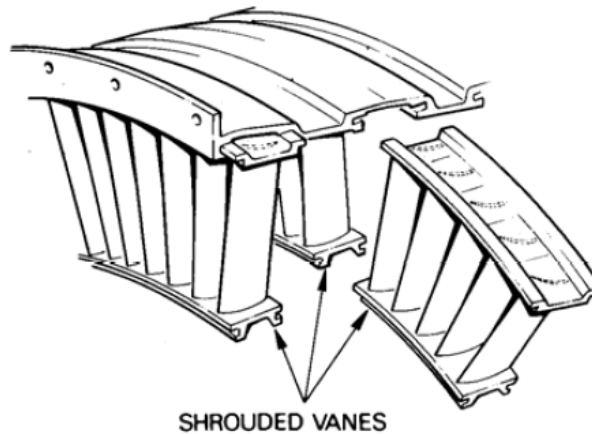


Comparison of compressor and turbine blade fixations:

- Turbine: fir tree
- Compressor: dove tail

Reason: higher load at higher temperatures for turbines, fir tree distributes loads more efficiently but is more costly

# Stator Vane Mount



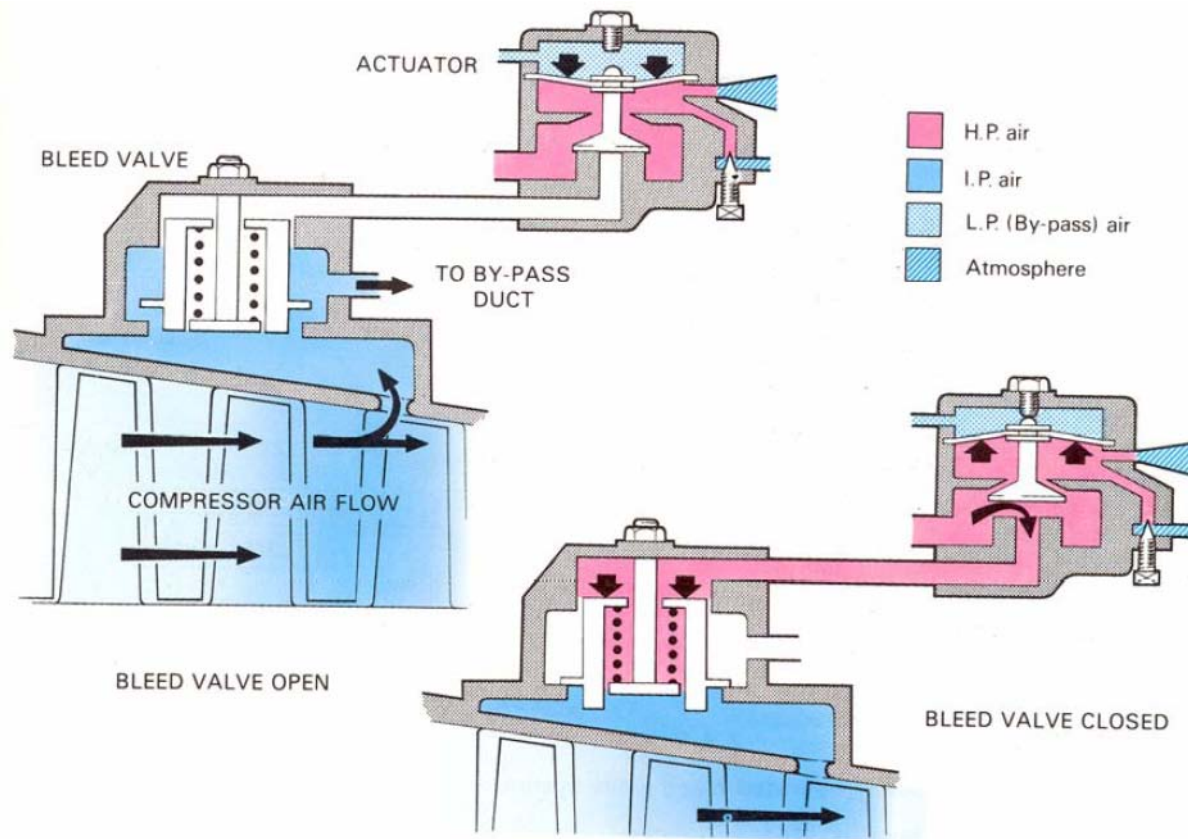
## Variable Stage Geometry

Variable guide vanes: control of stager angle of all blades in a blade row

Purpose:

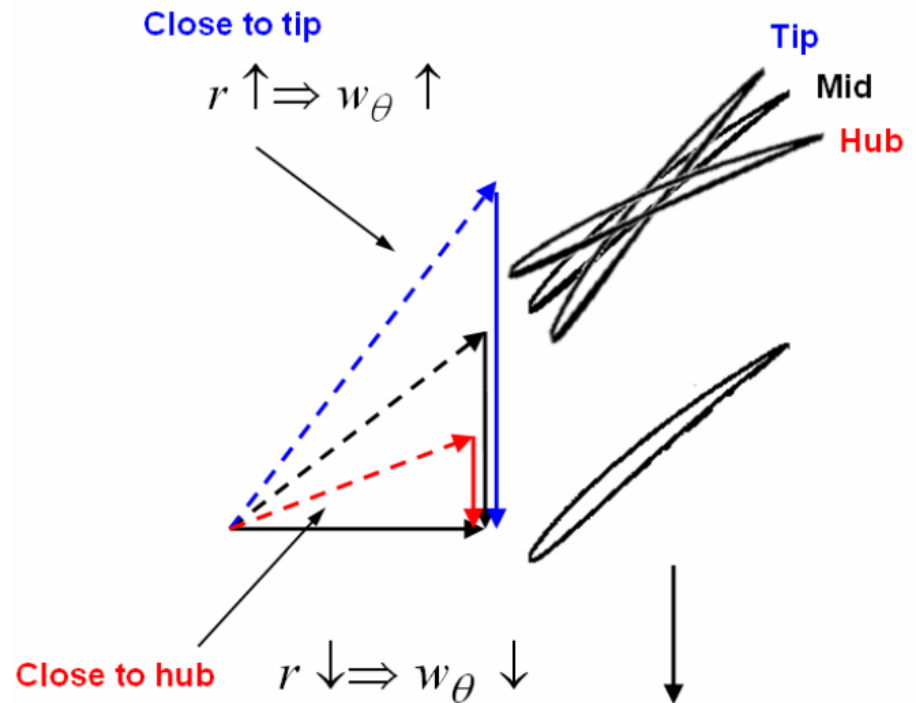
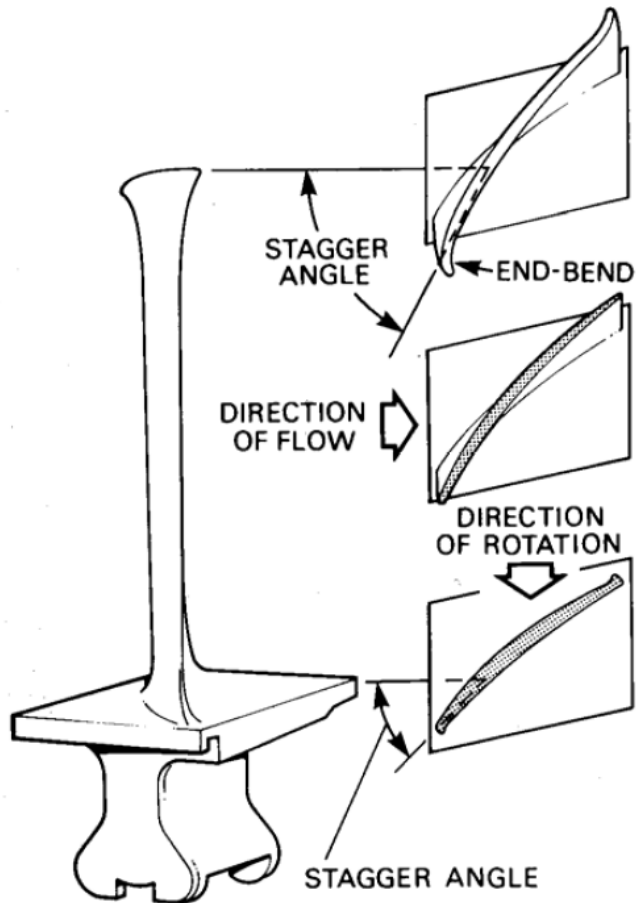
- Minimize incidence at off-design operation
- Pre-swirl for load regulation

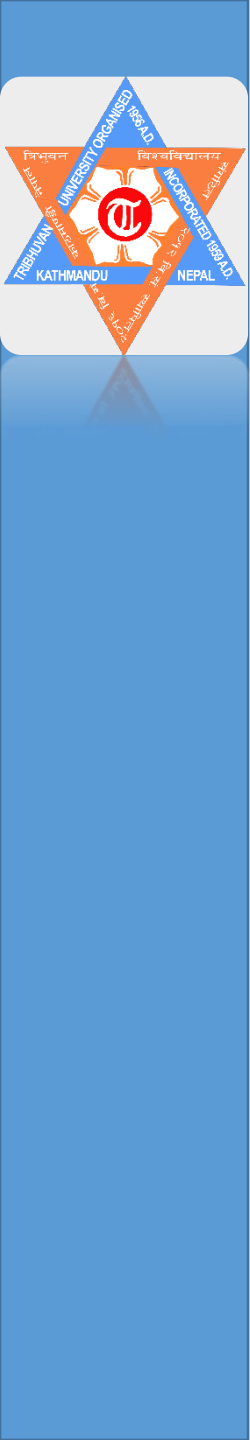
# Surge Control (Bleed Valves)



Bleed valves are used to avoid compressor surge during fast acceleration. Their use during steady operation however would be very wasteful and is therefore not taken into consideration.

# Blade Twist





# OFF-DESIGN PARAMETERS

If you have any questions regarding off-design parameters  
Contact me after class!!!