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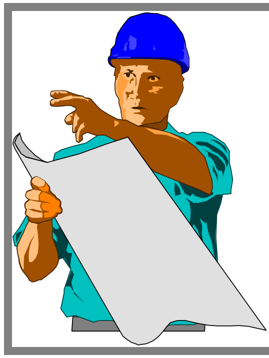
ENGINEERING ECONOMICS

Project Evaluation Techniques

Future Value Analysis and Mutually Exclusive Projects



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2018

Lecture 7

Future Value Analysis

The NPV or NPW measures the surplus in an investment project at time '0'.

Sometimes we might **need to find the equivalent worth or value** of a project **at the end of the investment period**.

Hence, the **Net Future Value (NFV) or Net Future Worth (NFW) measures the surplus at the end** of the **investment period**.

NFV Criterion

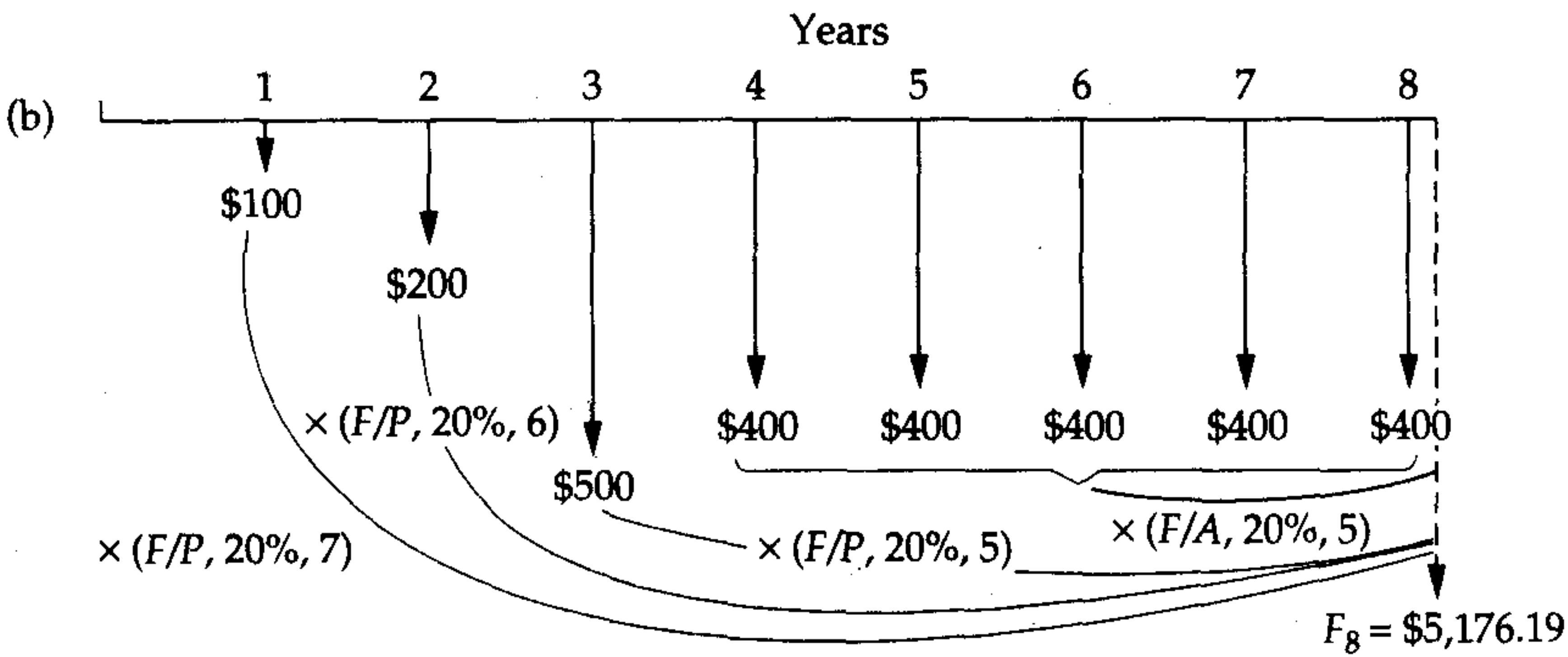
$$NFV = A_0(1+i)^n + A_1(1+i)^{n-1} + A_2/(1+i)^{n-2} + \ldots\ldots + A_N$$

$$= \sum_{n=0}^{N} A_n(1+i)^{N-n}$$

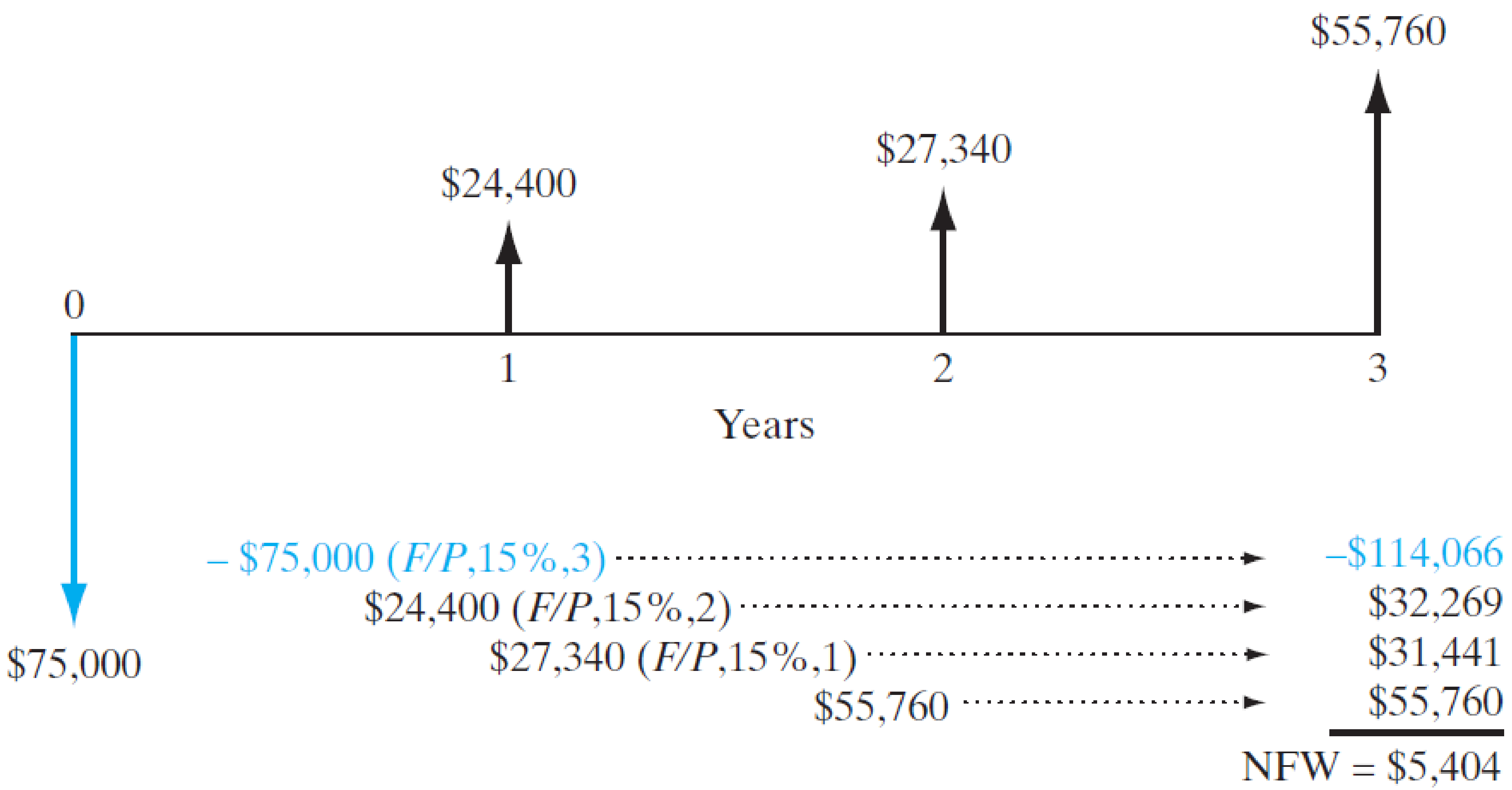
$$NFV = \sum A_n (F/P, i, N-n)$$

If NFV >0, accept the project
If NFV = 0, remain indifferent
If NFV<0, reject the project

NET FUTURE VALUE



NET FUTURE VALUE



Robot manufacturing facility

Compute the **equivalent worth** of this investment **at the start of operation**.
Assume that the company's expected **MARR is 15%.**

Calendar Year End of Year	'06 0	'07 1	'08 2	'09 3	'10 4	'11 5	'12 6	'13 7	'14 8
After-tax cash flows									
A. Operating revenue				\$6	\$8	\$13	\$18	\$14	\$8
B. Investment									
Land	−1.5								+2
Building		−4	−6						+3
Equipment			−13						+3
Net cash flow	−\$1.5	−\$4	−\$19	\$6	\$8	\$13	\$18	\$14	\$16

Method 1

$$PW(15\%) = -\$1.5 - \$4(P/F, 15\%, 1) - \$19(P/F, 15\%, 2)$$
$$+ \$6(P/F, 15\%, 3) + \$8(P/F, 15\%, 4) + \$13(P/F, 15\%, 5)$$
$$+ \$18(P/F, 15\%, 6) + \$14(P/F, 15\%, 7) + \$16(P/F, 15\%, 8)$$
$$= \$13.91 \text{ million.}$$

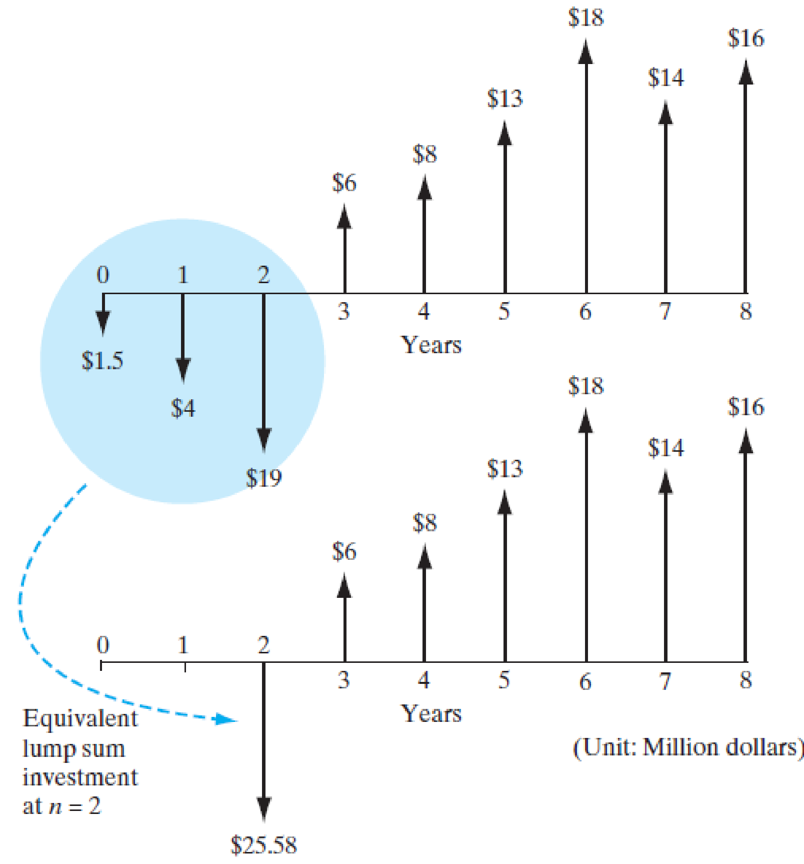
Then,

$$FW(15\%) = PW(15\%)\,(F/P, 15\%, 2)$$
$$= \$18.40 \text{ million.}$$

Method 2

$$-\$1.5(F/P, 15\%, 2) - \$4(F/P, 15\%, 1) - \$19 = -\$25.58 \text{ million,}$$

$$FW(15\%) = -\$25.58 + \$6(P/F, 15\%, 1) + \$8(P/F, 15\%, 2) + \cdots$$
$$+ \$16(P/F, 15\%, 6)$$
$$= \$18.40 \text{ million.}$$



Horton Corporation would set the price of the plant at **$43.98 million** ($18.40 + $25.58) at a minimum.

Example 1

An investment company is considering two **independent investment proposals**. Their expected cash flow streams are given as follows:

Year	Proj. A	Proj. B	Year	Proj. A	Proj. B
0	-250,000	-350,000	6	72,500	70,000
1	72,500	60,000	7		70,000
2	72,500	60,000	8		70,000
3	72,500	60,000	9		70,000
4	72,500	70,000	10		70,000
5	72,500	70,000			

If the company wants **MARR is 13%**, which proposals should be accepted to the company? Perform **NFV analysis**

APPENDIX A Interest Factors for Discrete Compounding

13.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
1	1.1300	0.8850	1.0000	1.0000	0.8850	1.1300	0.0000	0.0000	1
2	1.2769	0.7831	2.1300	0.4695	1.6681	0.5995	0.4695	0.7831	2
3	1.4429	0.6931	3.4069	0.2935	2.3612	0.4235	0.9187	2.1692	3
4	1.6305	0.6133	4.8498	0.2062	2.9745	0.3362	1.3479	4.0092	4
5	1.8424	0.5428	6.4803	0.1543	3.5172	0.2843	1.7571	6.1802	5
6	2.0820	0.4803	8.3227	0.1202	3.9975	0.2502	2.1468	8.5818	6
7	2.3526	0.4251	10.4047	0.0961	4.4226	0.2261	2.5171	11.1322	7
8	2.6584	0.3762	12.7573	0.0784	4.7988	0.2084	2.8685	13.7653	8
9	3.0040	0.3329	15.4157	0.0649	5.1317	0.1949	3.2014	16.4284	9
10	3.3946	0.2946	18.4197	0.0543	5.4262	0.1843	3.5162	19.0797	10
11	3.8359	0.2607	21.8143	0.0458	5.6869	0.1758	3.8134	21.6867	11
12	4.3345	0.2307	25.6502	0.0390	5.9176	0.1690	4.0936	24.2244	12
13	4.8980	0.2042	29.9847	0.0334	6.1218	0.1634	4.3573	26.6744	13
14	5.5348	0.1807	34.8827	0.0287	6.3025	0.1587	4.6050	29.0232	14
15	6.2543	0.1599	40.4175	0.0247	6.4624	0.1547	4.8375	31.2617	15

Capitalized Equivalent Method

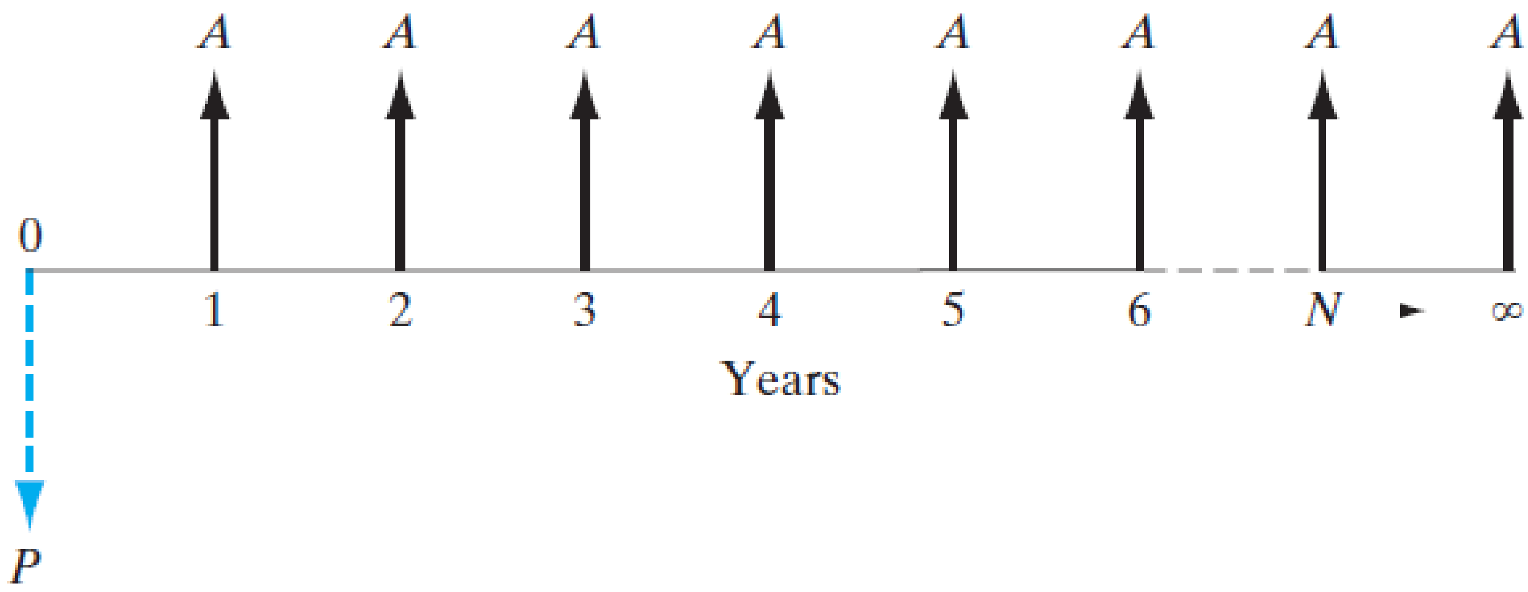
Another method of PV criterion is useful **when the life of project is perpetual** or **planning horizon is very long (say, 40 years or more).**

Capitalized Equivalent (CE) for Perpetual Service life Project

$$CE(i) = A / i$$

The process of **calculating PV cost for infinite period** is called **capitalization of project cost.**

The **cost** is **known as** the **Capitalized cost** i.e. the **amount of money** to be **invested now** to **get** a **certain return 'A'** at the **end of each and every year forever**.



$$\lim_{N\to\infty}(P/A, i, N) = \lim_{N\to\infty}\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = \frac{1}{i}.$$

Thus,

$$PW(i) = A(P/A, i, N \to \infty) = \frac{A}{i}.$$

Given: $A = \$2$ million, $i = 8\%$ per year, $N = \infty$

Find: $CE(8\%)$

The capitalized cost equation is

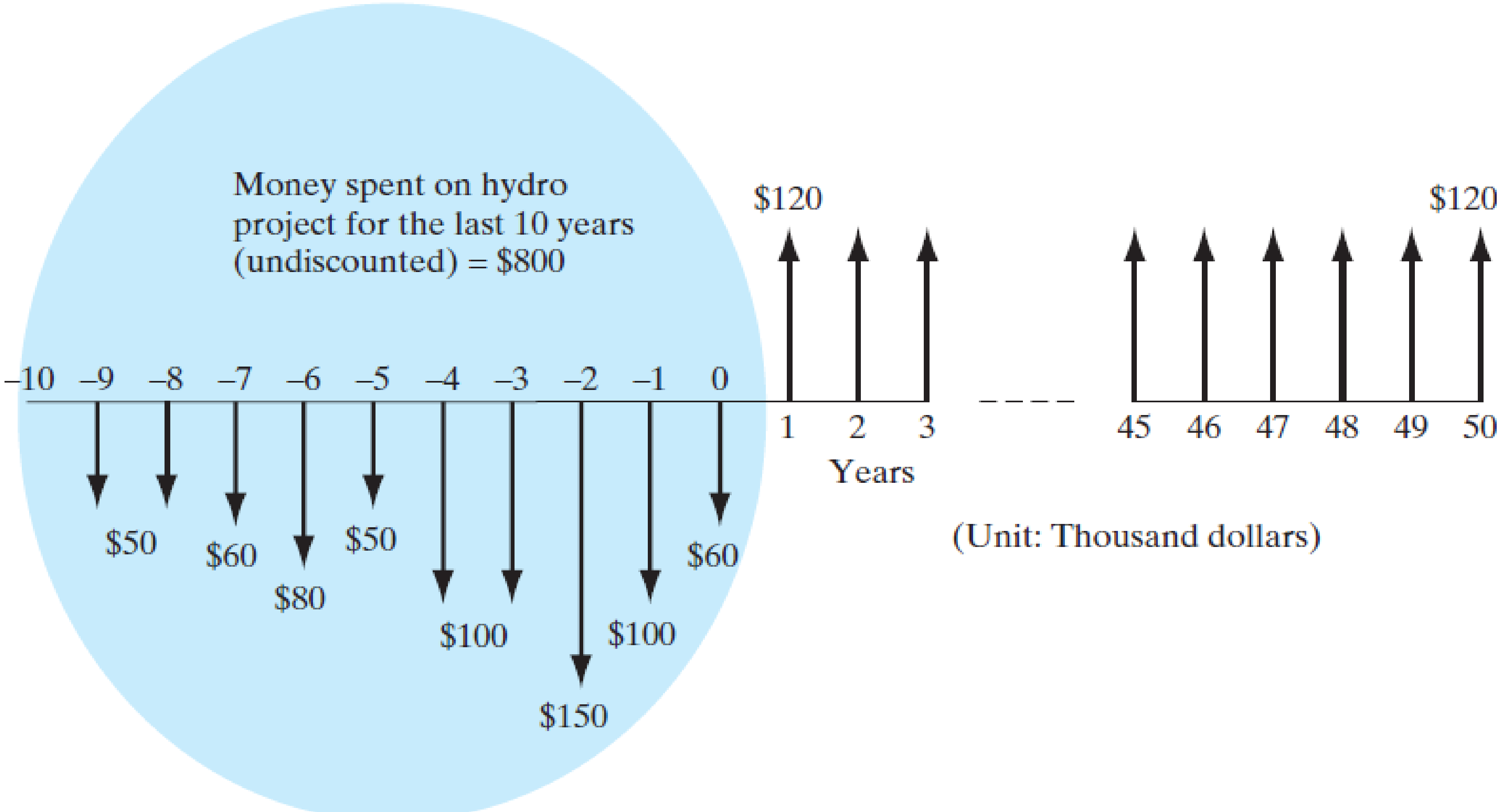
$$CE(i) = \frac{A}{i}$$

$$CE(8\%) = \$2{,}000{,}000/0.08$$

$$= \$25{,}000{,}000.$$

Hydro Power Plant

Service life = 50, MARR = 8%



(a) At $i = 8\%$ and with a service life of 50 years:
We can make use of two uniform series elements in the invested ca
flow to help us find the equivalent total investment at the start of power generation. Using "K" to indicate thousand,

$$F_1 = -\$50K(F/A, 8\%, 10) - \$10K(F/P, 8\%, 7) - \$30K(F/P, 8\%, 6)$$

$$- \$50K(F/A, 8\%, 4)(F/P, 8\%, 1) - \$50K(F/P, 8\%, 2) - \$10K$$

$$= -\$1101K.$$

The equivalent total benefits at the start of generation is

$$F_2 = \$120K(P/A, 8\%, 50) = \$1468.$$

Summing, we find the net equivalent worth at the start of power generation:

$$F_1 + F_2 = -\$1101K + \$1468K$$

$$= \$367K.$$

$$CE(8\%) = -\$1101K + \$120K/(0.08)$$

$$= \$399K.$$

Mutually Exclusive Projects

Mutually exclusive means

that **any one of several alternatives** will **fulfill the same need** and

that **selecting one alternative means** that **others will be excluded**.

Revenue Projects and Service Projects

Revenue projects are those projects **whose revenues depend on the choice of the alternative**.

Service projects are those projects **whose revenues do not depend on the choice of the project**.

For **revenue projects**, we use NPV of revenues and **choose** the project which has the **highest NPV**.

For **service projects**, we use the NPV of costs and **choose** the project which has the **least negative NPV**.

Analysis Period

It is the time-span over which the economic effects of an investment will be evaluated. It is also called as ***study period*** or ***planning horizon***. It may be taken as the required service period.

Situations when analysis period and project life differ

1. **Equal lives**
2. **Project life longer than analysis period**
3. **Project life shorter than analysis period**
4. **Analysis period not specified**

1. Equal Project Lives

Automation option

	Machining-Center Methods		
	Option 1	*Option 2*	*Option 3*
Investment	\$319,000	\$369,800	\$369,800
Proceeds from sales of broken machines	50,000	50,000	50,000
Net investment cost	\$269,000	\$319,800	\$319,800
Annual Net Savings:			
Direct labor	\$ 61,800	\$ 68,800	\$ 92,800
Set-up	19,700	19,700	25,500
Total savings	\$ 81,500	\$ 88,500	\$118,300
Service life	5 years	5 years	5 years

Given: **Cash flows for three projects, i = 12% per year**

Find: **The NPW of each project, select best option**

- For Option 1:

$$PW(12\%)_{\text{Option 1}} = -\$269{,}000 + \$81{,}500(P/A, 12\%, 5)$$
$$= \$24{,}789$$

- For Option 2:

$$PW(12\%)_{\text{Option 2}} = -\$319{,}800 + \$88{,}500(P/A, 12\%, 5)$$
$$= -\$777$$

- For Option 3:

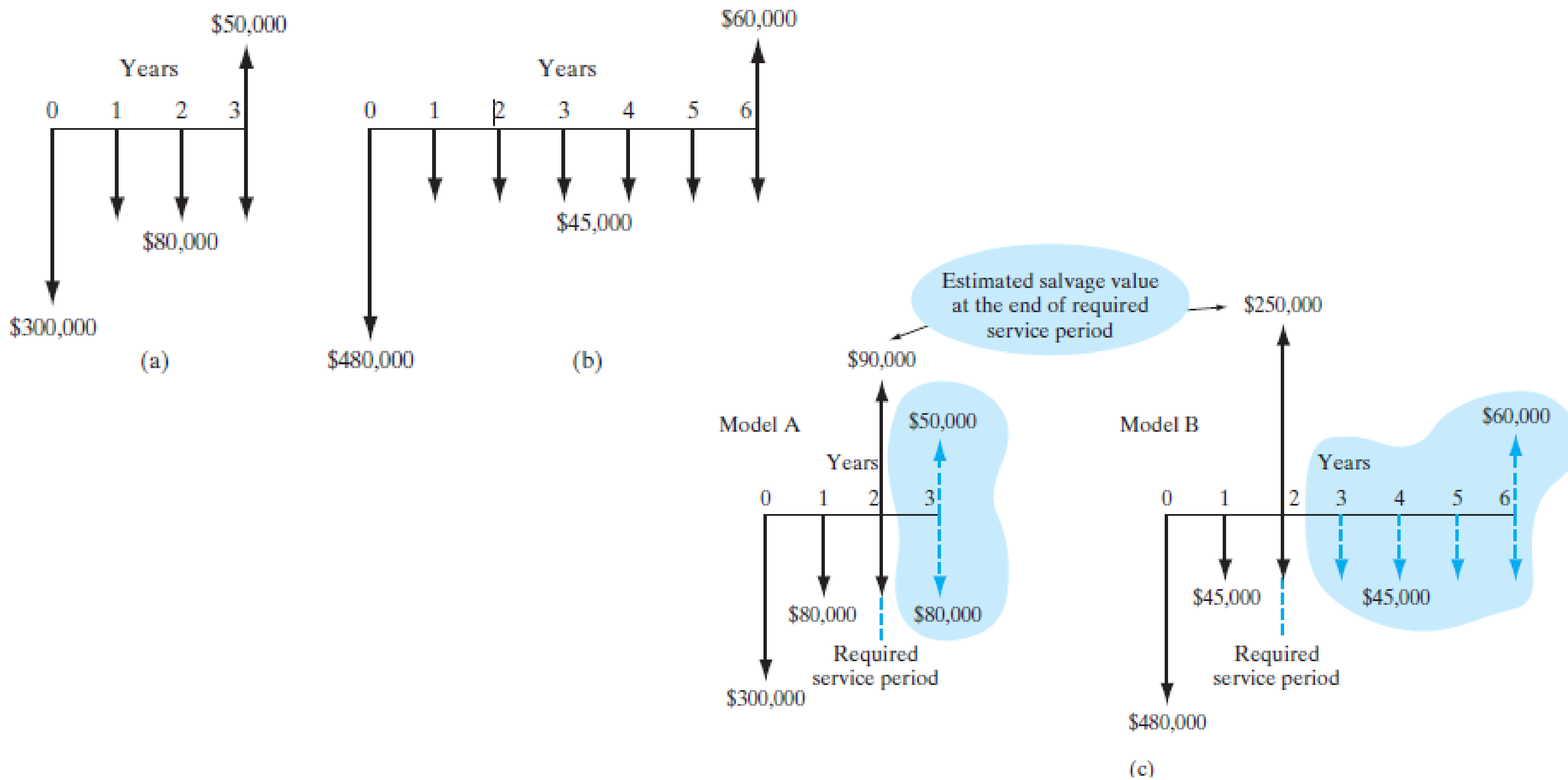
$$PW(12\%)_{\text{Option 3}} = -\$319{,}800 + \$118{,}300(P/A, 12\%, 5)$$
$$= \$106{,}645$$

Option 3 has the greater PW and thus would be preferred

2. Project life longer than analysis period

Ripper-bulldozer to dig and load radio active material within <u>two year</u>

Period	Model A		Model B	
0	−$300		−$480	
1	−80		−45	
2	−80	+ $\boxed{90}$	−45	+ $\boxed{250}$
3	−80	+50	−45	
4			−45	
5			−45	
6			−45	+60



$$PW(15\%)_A = -\$300 - \$80(P/F, 15\%, 1) + \$10(P/F, 15\%, 2)$$

$$= -\$362$$

$$PW(15\%)_B = -\$480 - \$45(P/F, 15\%, 1) + \$205(P/F, 15\%, 2)$$

$$= -\$364$$

Model A has the greater PW **(least negative)** and thus would be **preferred**

3. Project life shorter than analysis period

Installation of an automatic mailing system to handle product announcements and invoices

n	Semi-Automatic Model A	Full-Automatic Model B
0	−$12,500	−$15,000
1	−5,000	−4,000
2	−5,000	−4,000
3	−5,000 + 2,000	−4,000
4		−4,000 + 1,500
$\boxed{5}$		

Given: Cash flows for two alternatives as shown, analysis period of 5 years, i = 15%
Find: The NPW of each alternative, select which option

n	Model A		Model B	
0	$-\$12{,}500$		$-\$15{,}000$	
1	$-5{,}000$		$-4{,}000$	
2	$-5{,}000$		$-4{,}000$	
3	$-5{,}000 +$	$2{,}000$	$-4{,}000$	
4	$-5{,}000 -$	$\boxed{6{,}000}$	$-4{,}000 +$	$1{,}500$
5	$-5{,}000 -$	$\boxed{6{,}000}$	$-5{,}000 -$	$\boxed{6{,}000}$

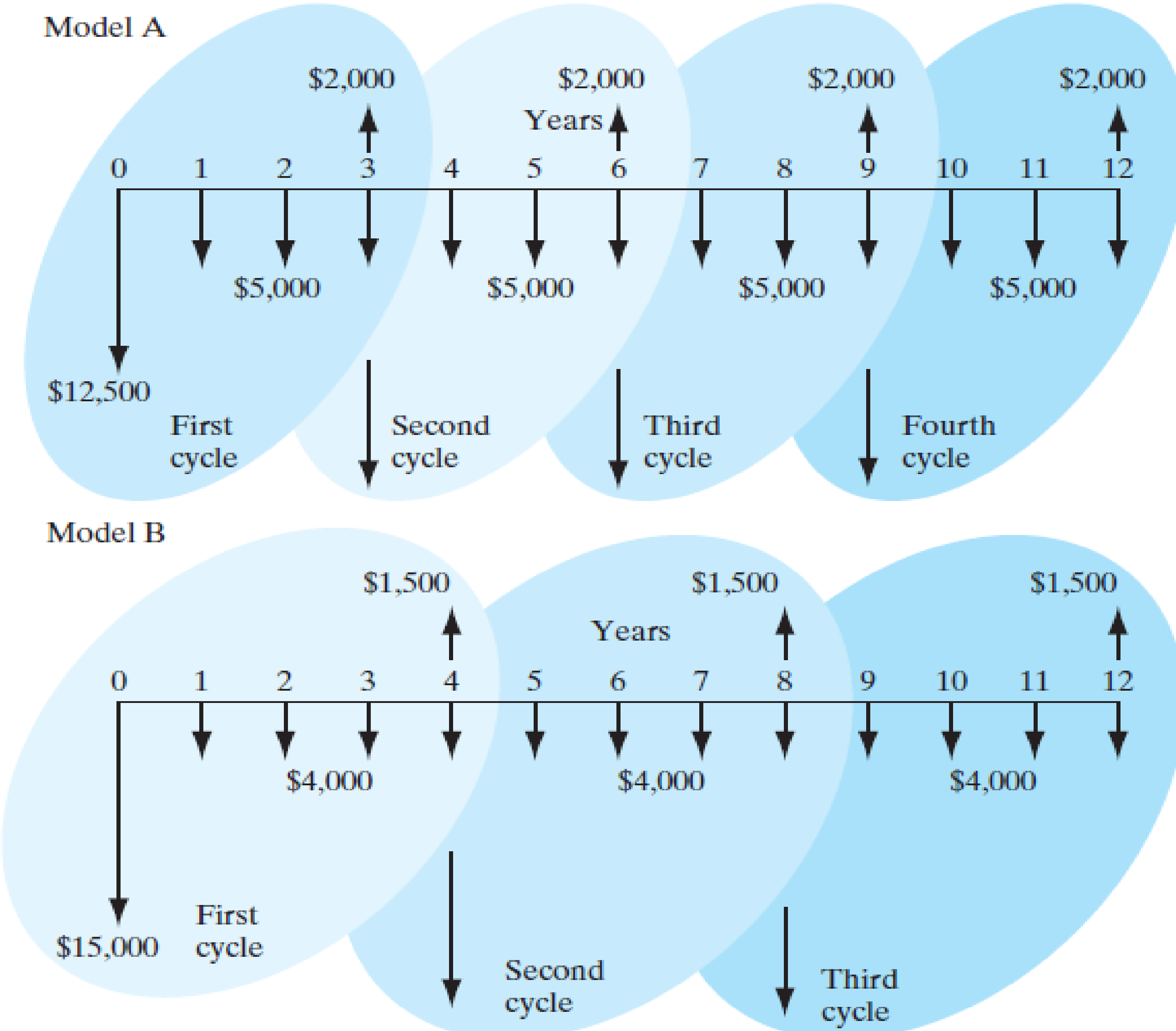
Operating Cost

Leasing Cost

$$PW(15\%)_A = -\$12{,}500 - \$5000(P/A, 15\%, 2) - \$3000(P/F, 15\%, 3)$$
$$- \$11{,}000(P/A, 15\%, 2)\,(P/F, 15\%, 3)$$
$$= -\$34{,}359,$$
$$PW(15\%)_B = -\$15{,}000 - \$4000(P/A, 15\%, 3) - \$2500(P/F, 15\%, 4)$$
$$- \$11{,}000(P/F, 15\%, 5)$$
$$= -\$31{,}031.$$

Since these are service projects, model B is the better choice.

4. Analysis period not specified



Lowest common multiple criteria

(3 x 4)

Model A: There are 4 replacements in a 12-year period. The PW for the first investment cycle is

$$PW(15\%) = -\$12{,}500 - \$5000(P/A, 15\%, 2) - \$3000(P/F, 15\%, 3) = -\$22.601.$$

With 4 replacement cycles, the total PW is

$$PW(15\%) = -\$22{,}601[1 + (P/F, 15\%, 3) + (P/F, 15\%, 6) + (P/F, 15\%, 9)] = -\$53.657.$$

Model B: The PW for the first investment cycle is

Model B is a better choice

$$PW(15\%) = -\$15{,}000 - \$4000(P/A, 15\%, 3) - \$2500(P/F, 15\%, 4) = -\$25{,}562.$$

With 3 replacement cycles in 12 years, the total PW is

$$PW(15\%) = -\$25{,}562[1 + (P/F, 15\%, 4) + (P/F, 15\%, 8)] = -\$48{,}534.$$

Exercise 1

Consider the following two **mutually exclusive investment projects**, each with **MARR = 15%**:

	Project's Cash Flow	
n	A	B
0	−$6,000	−$8,000
1	800	11,500
2	14,000	400

(a) On the basis of the **NPW criterion**, which **project** would be **selected?**

(b) Study for **MARR = 7%, 30% and 50%.**

APPENDIX A Interest Factors for Discrete Compounding

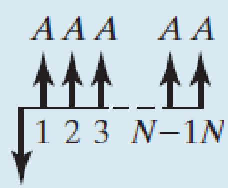
15.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
1	1.1500	0.8696	1.0000	1.0000	0.8696	1.1500	0.0000	0.0000	1
2	1.3225	0.7561	2.1500	0.4651	1.6257	0.6151	0.4651	0.7561	2
3	1.5209	0.6575	3.4725	0.2880	2.2832	0.4380	0.9071	2.0712	3
4	1.7490	0.5718	4.9934	0.2003	2.8550	0.3503	1.3263	3.7864	4
5	2.0114	0.4972	6.7424	0.1483	3.3522	0.2983	1.7228	5.7751	5
6	2.3131	0.4323	8.7537	0.1142	3.7845	0.2642	2.0972	7.9368	6
7	2.6600	0.3759	11.0668	0.0904	4.1604	0.2404	2.4498	10.1924	7
8	3.0590	0.3269	13.7268	0.0729	4.4873	0.2229	2.7813	12.4807	8
9	3.5179	0.2843	16.7858	0.0596	4.7716	0.2096	3.0922	14.7548	9
10	4.0456	0.2472	20.3037	0.0493	5.0188	0.1993	3.3832	16.9795	10

Present worth $(P/A, i, N)$

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$=PV(i, N, A,, 0)$



Capital recovery $(A/P, i, N)$

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$=PMT(i, N,, P)$

APPENDIX A Interest Factors for Discrete Compounding

7.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
1	1.0700	0.9346	1.0000	1.0000	0.9346	1.0700	0.0000	0.0000	1
2	1.1449	0.8734	2.0700	0.4831	1.8080	0.5531	0.4831	0.8734	2
3	1.2250	0.8163	3.2149	0.3111	2.6243	0.3811	0.9549	2.5060	3
4	1.3108	0.7629	4.4399	0.2252	3.3872	0.2952	1.4155	4.7947	4
5	1.4026	0.7130	5.7507	0.1739	4.1002	0.2439	1.8650	7.6467	5
6	1.5007	0.6663	7.1533	0.1398	4.7665	0.2098	2.3032	10.9784	6
7	1.6058	0.6227	8.6540	0.1156	5.3893	0.1856	2.7304	14.7149	7
8	1.7182	0.5820	10.2598	0.0975	5.9713	0.1675	3.1465	18.7889	8
9	1.8385	0.5439	11.9780	0.0835	6.5152	0.1535	3.5517	23.1404	9
10	1.9672	0.5083	13.8164	0.0724	7.0236	0.1424	3.9461	27.7156	10

30.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
1	1.3000	0.7692	1.0000	1.0000	0.7692	1.3000	0.0000	0.0000	1
2	1.6900	0.5917	2.3000	0.4348	1.3609	0.7348	0.4348	0.5917	2
3	2.1970	0.4552	3.9900	0.2506	1.8161	0.5506	0.8271	1.5020	3
4	2.8561	0.3501	6.1870	0.1616	2.1662	0.4616	1.1783	2.5524	4
5	3.7129	0.2693	9.0431	0.1106	2.4356	0.4106	1.4903	3.6297	5
6	4.8268	0.2072	12.7560	0.0784	2.6427	0.3784	1.7654	4.6656	6
7	6.2749	0.1594	17.5828	0.0569	2.8021	0.3569	2.0063	5.6218	7
8	8.1573	0.1226	23.8577	0.0419	2.9247	0.3419	2.2156	6.4800	8
9	10.6045	0.0943	32.0150	0.0312	3.0190	0.3312	2.3963	7.2343	9
10	13.7858	0.0725	42.6195	0.0235	3.0915	0.3235	2.5512	7.8872	10
11	17.9216	0.0558	56.4053	0.0177	3.1473	0.3177	2.6833	8.4452	11
12	23.2981	0.0429	74.3270	0.0135	3.1903	0.3135	2.7952	8.9173	12
13	30.2875	0.0330	97.6250	0.0102	3.2233	0.3102	2.8895	9.3135	13
14	39.3738	0.0254	127.9125	0.0078	3.2487	0.3078	2.9685	9.6437	14
15	51.1859	0.0195	167.2863	0.0060	3.2682	0.3060	3.0344	9.9172	15

APPENDIX A Interest Factors for Discrete Compounding

50.0%

	Single Payment		Equal Payment Series				Gradient Series		
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	N
1	1.5000	0.6667	1.0000	1.0000	0.6667	1.5000	0.0000	0.0000	1
2	2.2500	0.4444	2.5000	0.4000	1.1111	0.9000	0.4000	0.4444	2
3	3.3750	0.2963	4.7500	0.2105	1.4074	0.7105	0.7368	1.0370	3
4	5.0625	0.1975	8.1250	0.1231	1.6049	0.6231	1.0154	1.6296	4
5	7.5938	0.1317	13.1875	0.0758	1.7366	0.5758	1.2417	2.1564	5
6	11.3906	0.0878	20.7813	0.0481	1.8244	0.5481	1.4226	2.5953	6
7	17.0859	0.0585	32.1719	0.0311	1.8829	0.5311	1.5648	2.9465	7
8	25.6289	0.0390	49.2578	0.0203	1.9220	0.5203	1.6752	3.2196	8
9	38.4434	0.0260	74.8867	0.0134	1.9480	0.5134	1.7596	3.4277	9
10	57.6650	0.0173	113.3301	0.0088	1.9653	0.5088	1.8235	3.5838	10
11	86.4976	0.0116	170.9951	0.0058	1.9769	0.5058	1.8713	3.6994	11
12	129.7463	0.0077	257.4927	0.0039	1.9846	0.5039	1.9068	3.7842	12
13	194.6195	0.0051	387.2390	0.0026	1.9897	0.5026	1.9329	3.8459	13
14	291.9293	0.0034	581.8585	0.0017	1.9931	0.5017	1.9519	3.8904	14
15	437.8939	0.0023	873.7878	0.0011	1.9954	0.5011	1.9657	3.9224	15

Exercise 2

Consider the following **two mutually exclusive investment projects**

	Project's Cash Flow	
n	A	B
0	−$20,000	−$25,000
1	17,500	25,500
2	17,000	18,000
3	15,000	

On the basis of the **NPW criterion**, which project would be selected if you **use an infinite planning horizon** with **project repeatability** (the same costs and benefits) likely? Assume that **i = 12%**.

12.0%

N	Single Payment: Compound Amount Factor (F/P,i,N)	Single Payment: Present Worth Factor (P/F,i,N)	Equal Payment Series: Compound Amount Factor (F/A,i,N)	Equal Payment Series: Sinking Fund Factor (A/F,i,N)	Equal Payment Series: Present Worth Factor (P/A,i,N)	Equal Payment Series: Capital Recovery Factor (A/P,i,N)	Gradient Series: Gradient Uniform Series (A/G,i,N)	Gradient Series: Gradient Present Worth (P/G,i,N)	N
1	1.1200	0.8929	1.0000	1.0000	0.8929	1.1200	0.0000	0.0000	1
2	1.2544	0.7972	2.1200	0.4717	1.6901	0.5917	0.4717	0.7972	2
3	1.4049	0.7118	3.3744	0.2963	2.4018	0.4163	0.9246	2.2208	3
4	1.5735	0.6355	4.7793	0.2092	3.0373	0.3292	1.3589	4.1273	4
5	1.7623	0.5674	6.3528	0.1574	3.6048	0.2774	1.7746	6.3970	5
6	1.9738	0.5066	8.1152	0.1232	4.1114	0.2432	2.1720	8.9302	6
7	2.2107	0.4523	10.0890	0.0991	4.5638	0.2191	2.5515	11.6443	7
8	2.4760	0.4039	12.2997	0.0813	4.9676	0.2013	2.9131	14.4714	8
9	2.7731	0.3606	14.7757	0.0677	5.3282	0.1877	3.2574	17.3563	9
10	3.1058	0.3220	17.5487	0.0570	5.6502	0.1770	3.5847	20.2541	10
11	3.4785	0.2875	20.6546	0.0484	5.9377	0.1684	3.8953	23.1288	11
12	3.8960	0.2567	24.1331	0.0414	6.1944	0.1614	4.1897	25.9523	12
13	4.3635	0.2292	28.0291	0.0357	6.4235	0.1557	4.4683	28.7024	13
14	4.8871	0.2046	32.3926	0.0309	6.6282	0.1509	4.7317	31.3624	14
15	5.4736	0.1827	37.2797	0.0268	6.8109	0.1468	4.9803	33.9202	15

Practice

- 5.10, 5.11, 5.14, 5.17, 5.27, 5.29, 5.30, 5.31, 5.36, 5.38, 5.40, 5.46, ST5.1.