Department of Automobile and Mechanical Engineering
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TURBO MACHINE

Chapter 4
Gas Turbine Nozzles

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Gas Turbine Exhaust /Nozzle

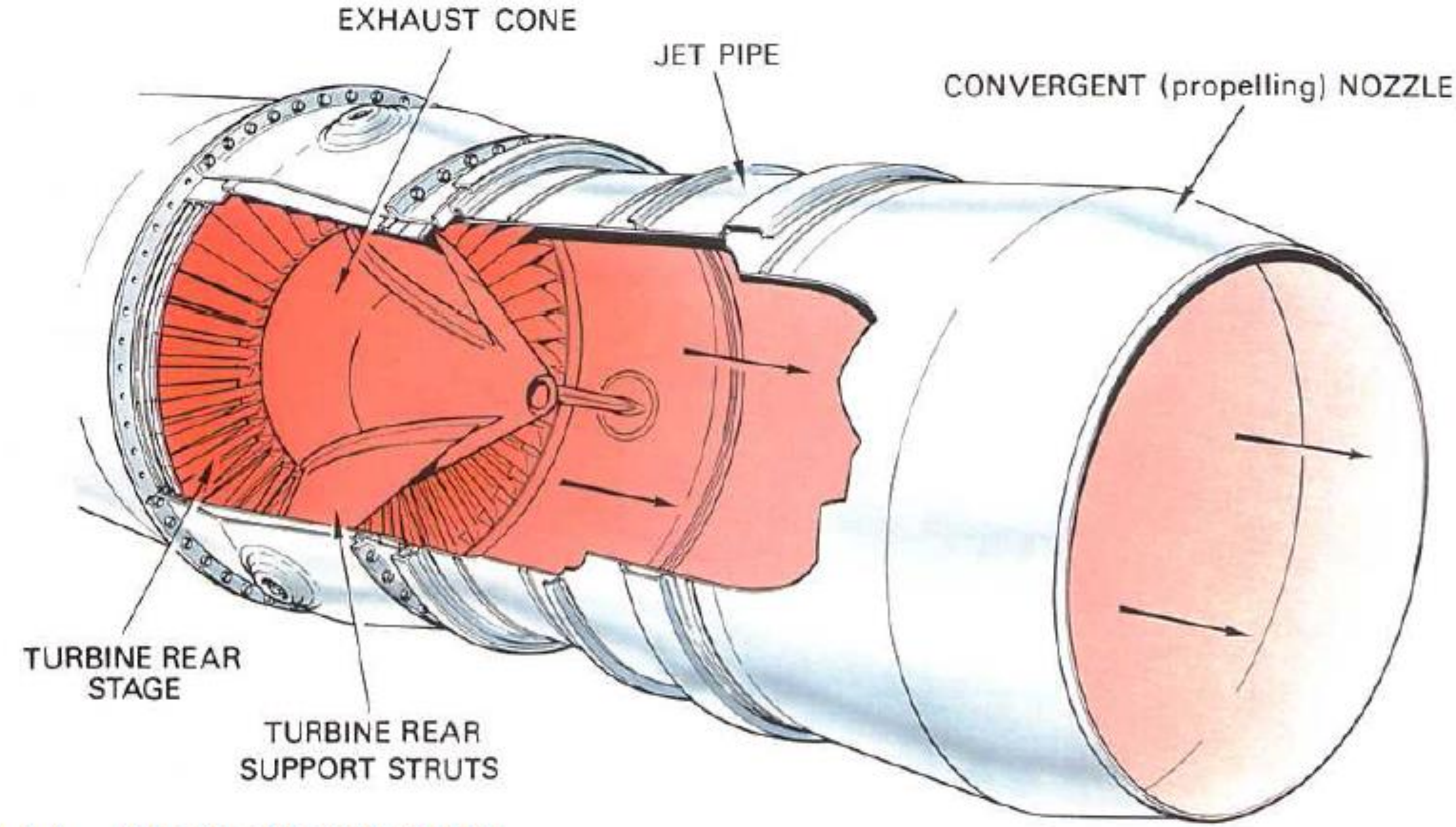


Fig. 6-1 A basic exhaust system.

F-15 Eagle Nozzle

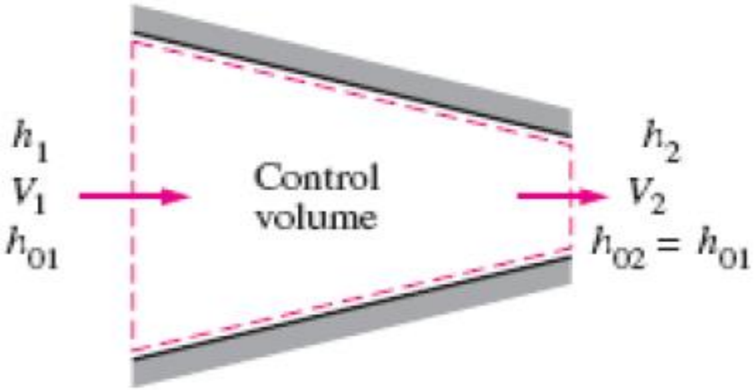
Nozzle Energy Equation

- One dimensional steady state energy eqn:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$q_{in} + w_{in} + (h_{01} + gz_1) = q_{out} + w_{out} + (h_{02} + gz_2)$$

- For isentropic conditions, assuming inlet and exit to be at same datum;



$$h_1 + \frac{\bar{V}_1{}^2}{2} = h_2 + \frac{\bar{V}_2{}^2}{2}$$

$$\text{or, } h_{01} = h_{02}$$

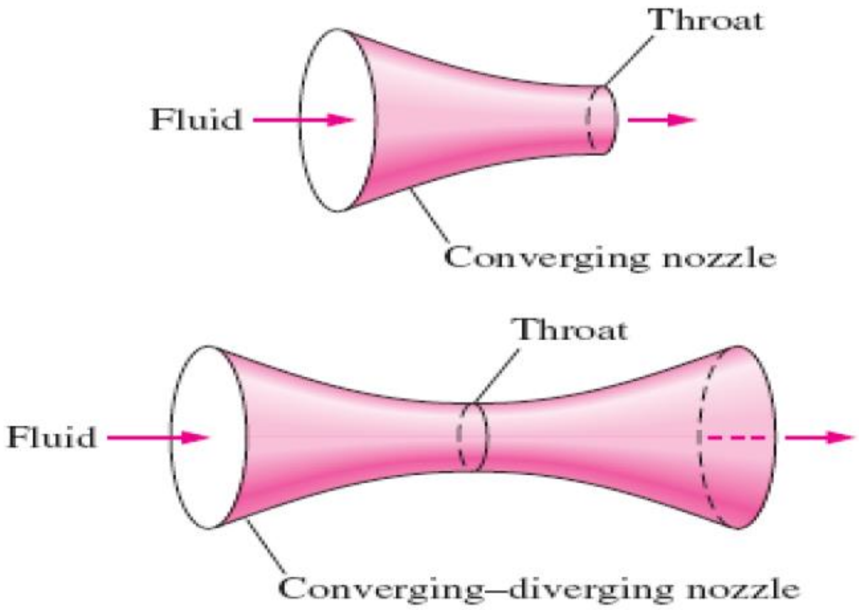
- Inlet stagnation temperature

$$C_p T_0 = C_p T + \frac{\bar{V}^2}{2}$$

$$\text{or, } T_0 = T + \frac{\bar{V}^2}{2C_p}$$

Nozzle Types

- Convergent
- Divergent
- Convergent- Divergent (CD Nozzles/Laval Nozzle)



Basic Relations Review

- C= Local speed of sound, M= local Mach No

$$c = \sqrt{\gamma RT} \qquad M = \frac{\bar{V}}{c}$$

- Fluid flow regimes are often described in terms of the flow Mach Number. The flow is called sonic when M=1, subsonic when M<1 , supersonic when M>1, hypersonic when M>>1 and transonic when M nearly 1.

Flow velocity vs. Flow Area

$$\frac{dA}{A} = \frac{dP}{\rho\bar{V}^2}(1 - M^2)$$

$$\frac{dA}{A} = -\frac{d\bar{V}}{\bar{V}}(1 - M^2)$$

- Subsonic condition: M<1, dA and dP same sign, area decreases and pressure decreases (velocity increases)
- Supersonic Condition: M>1, dA and dP opposite sign, area increases and pressure decreases(velocity increases)
- Hence in case of a CD nozzle, throat area is so designed that M=1 at throat and the flow is said to be **choked.** Pressure does not reduce further until temperature is increased.
- Addition of a divergent part accelerates the gases to M>1

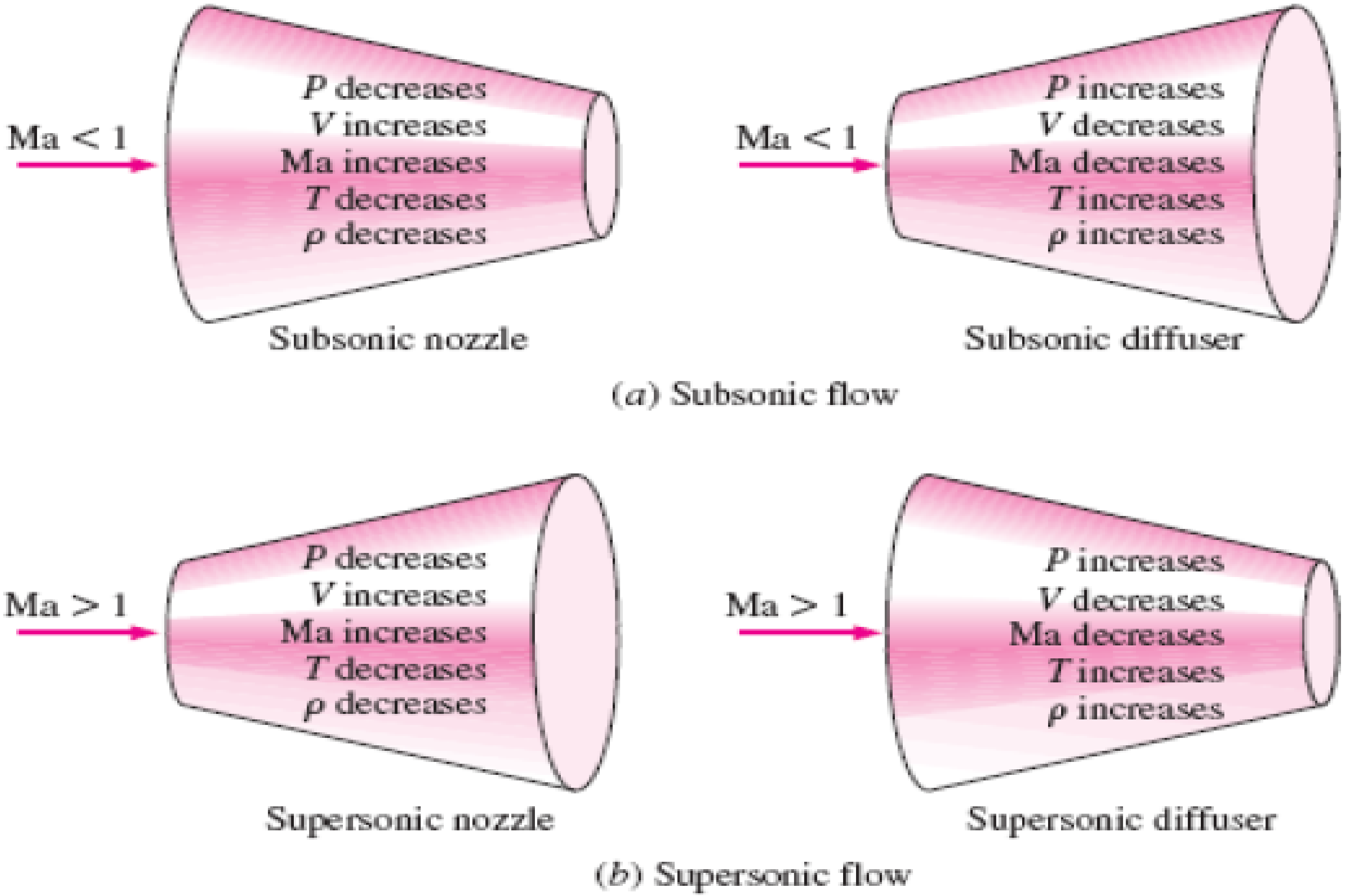


Fig. 7.9: Variation of flow properties in subsonic and supersonic nozzles and diffusers.

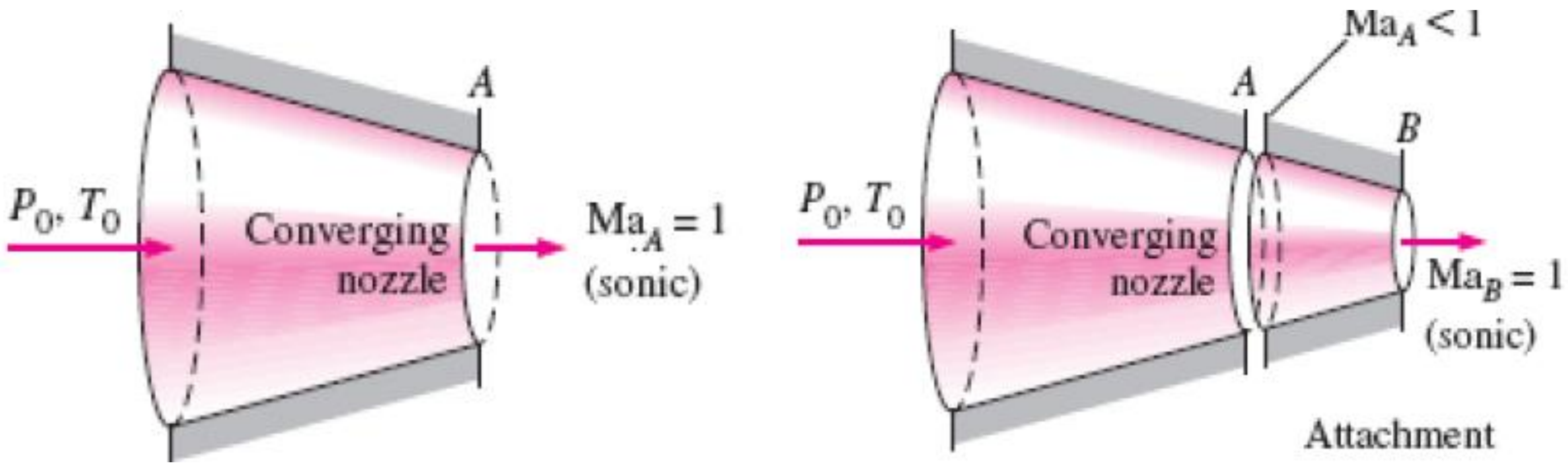


Fig. 7.8: We cannot obtain supersonic velocities by attaching a converging section to a converging nozzle. Doing so will only move the sonic cross section farther downstream and decrease the mass flow rate.

Isentropic Flow Property Relations

- Subscript o represents stagnation properties, M= Mach number, γ= sp. heat ratio

$$\frac{P_0}{P} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)}$$

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right) M^2$$

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{\left(\frac{1}{\gamma-1}\right)}$$

Critical properties

- The properties of a fluid at a location where the Mach number is unity (the throat) are called **critical properties,** and the ratios are called **critical ratios**. It is common practice in the analysis of compressible flow to let the superscript asterisk (*) represent the critical values.
- Putting M=1 in above equations:

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$

$$\frac{P^*}{P_0} = \left[\frac{2}{\gamma+1}\right]^{\left(\frac{\gamma}{\gamma-1}\right)}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\left(\frac{1}{\gamma-1}\right)}$$

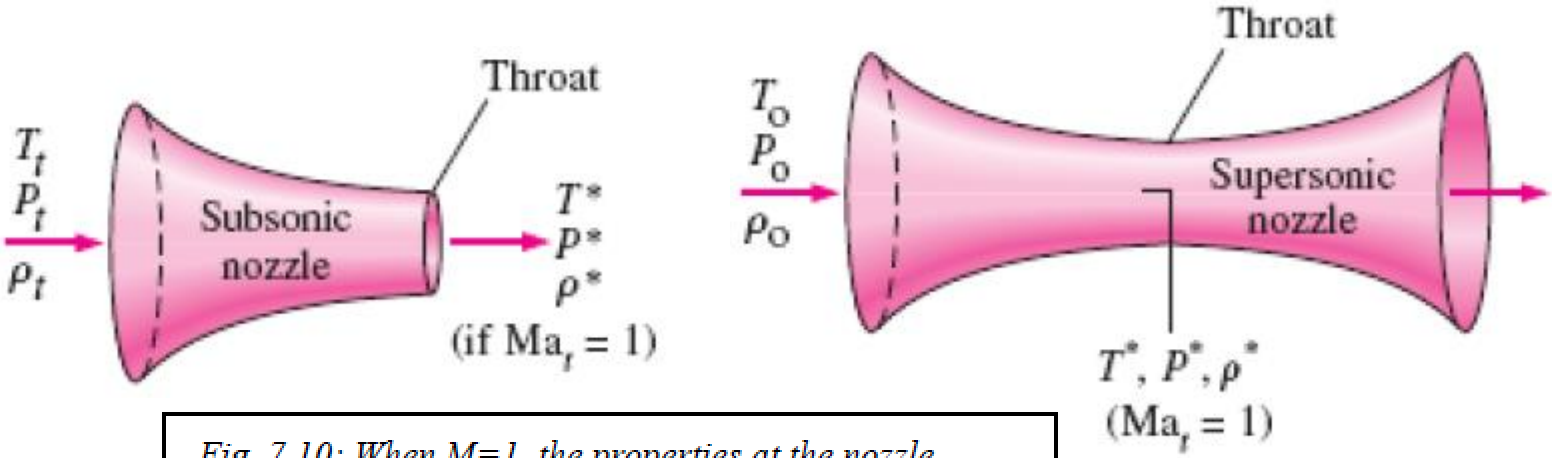


Fig. 7.10: When M=1, the properties at the nozzle throat become the critical properties

	Superheated steam, $k = 1.3$	Hot products of combustion, $k = 1.33$	Air, $k = 1.4$	Monatomic gases, $k = 1.667$
$\frac{P^*}{P_0}$	0.5457	0.5404	0.5283	0.4871
$\frac{T^*}{T_0}$	0.8696	0.8584	0.8333	0.7499
$\frac{\rho^*}{\rho_0}$	0.6276	0.6295	0.6340	0.6495

Isentropic flow through Nozzles

- Convergent Nozzle

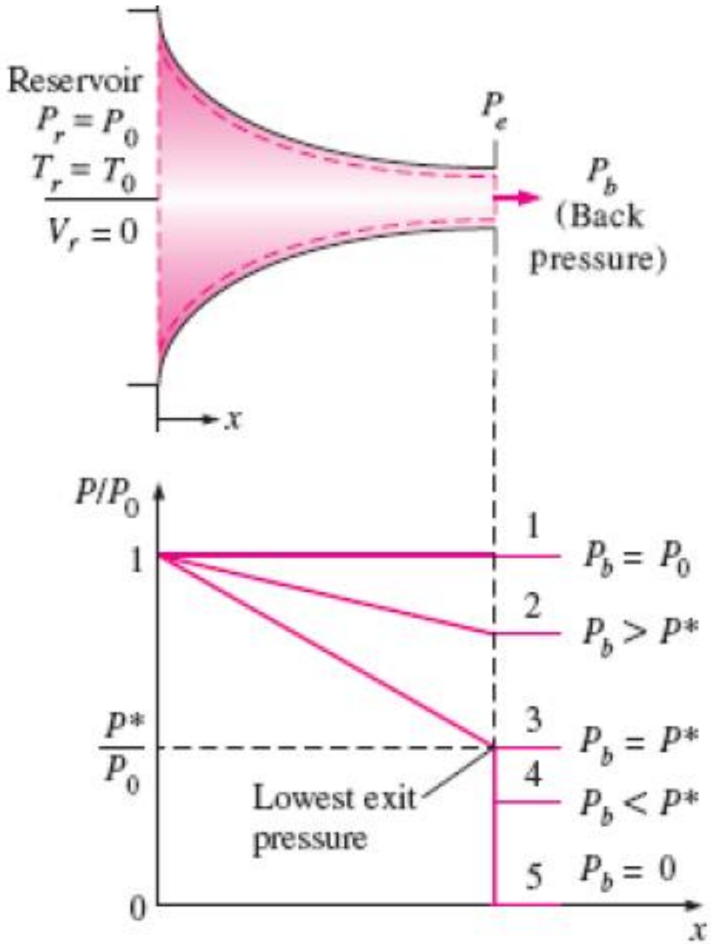


Fig 7.11: The effect of back pressure on the pressure distribution along a converging nozzle

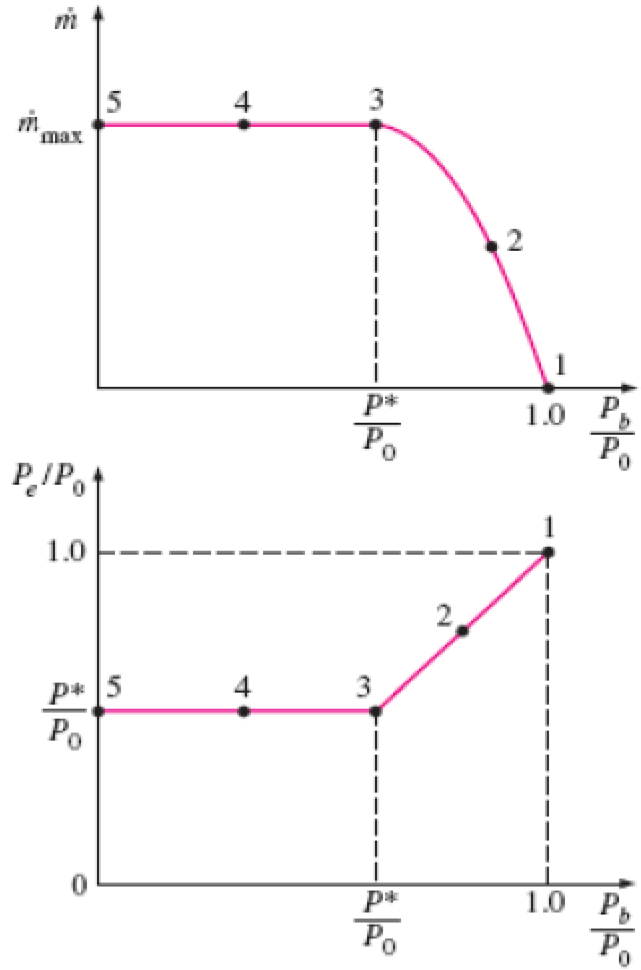


Fig 7.12: The effect of back pressure P_b on the mass flow rate $\dot{m}$ and the exit pressure P_e of a converging nozzle.

Mass flow Rate

- Steady- flow mass flow rate:

$$\dot{m} = \rho A \bar{V} = \frac{P}{RT} \times A\left(M\sqrt{\gamma Rt}\right) = PAM\sqrt{\frac{\gamma}{RT}}$$

$$\dot{m} = \frac{AMP_0\sqrt{\gamma/RT_0}}{[1+(\gamma-1)M^2/2]^{(\gamma+1)/2(\gamma-1)}}$$

- Maximum mass flow rate at M=1

$$\dot{m} = A^* \times P_0 \times \left[\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}\right] \times \sqrt{\frac{\gamma}{RT_0}}$$

- Expression for throat area in terms of M:

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

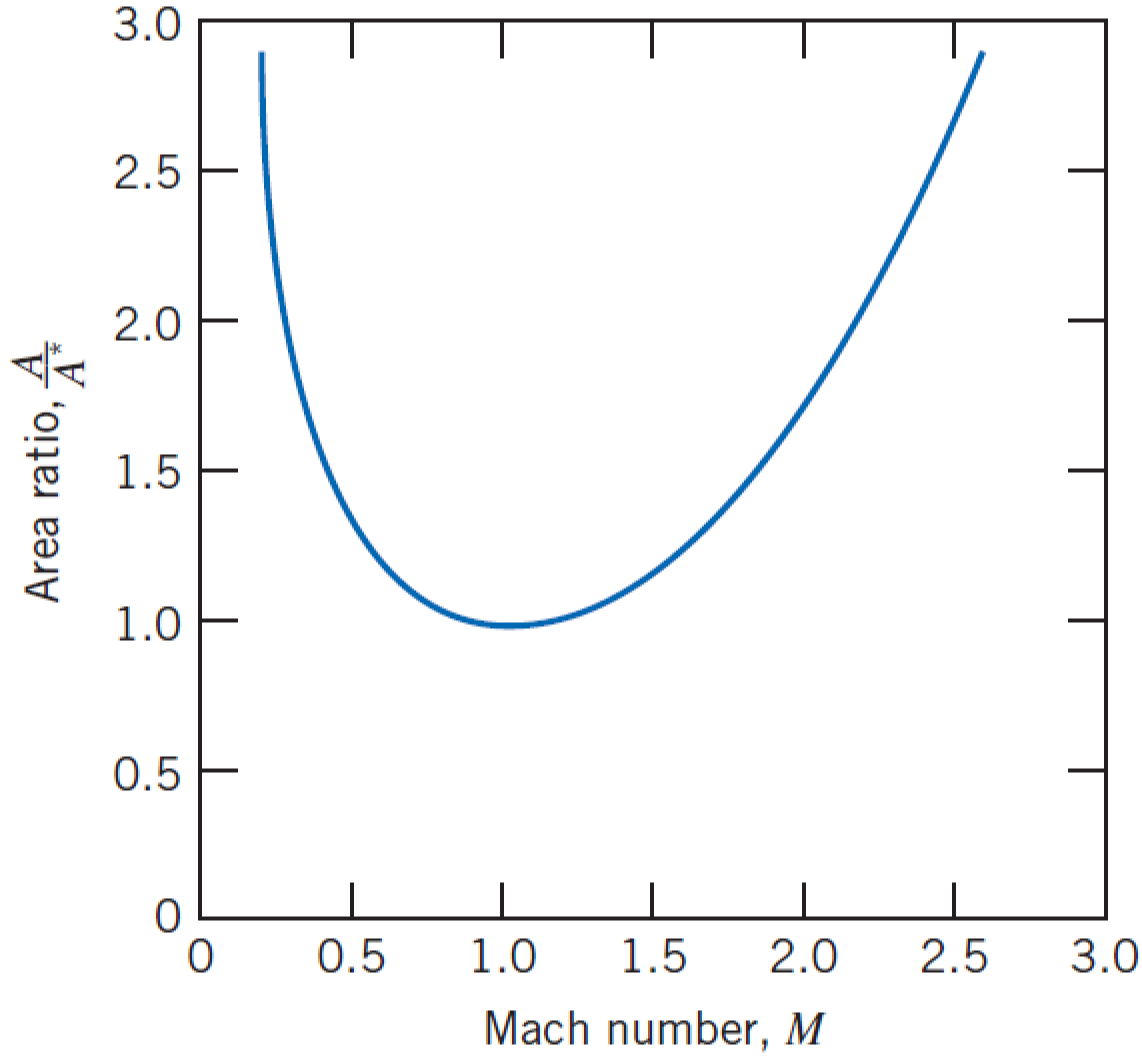


Fig. 13.5 Variation of A/A* with Mach number for isentropic flow of an ideal gas with γ= 1.4.

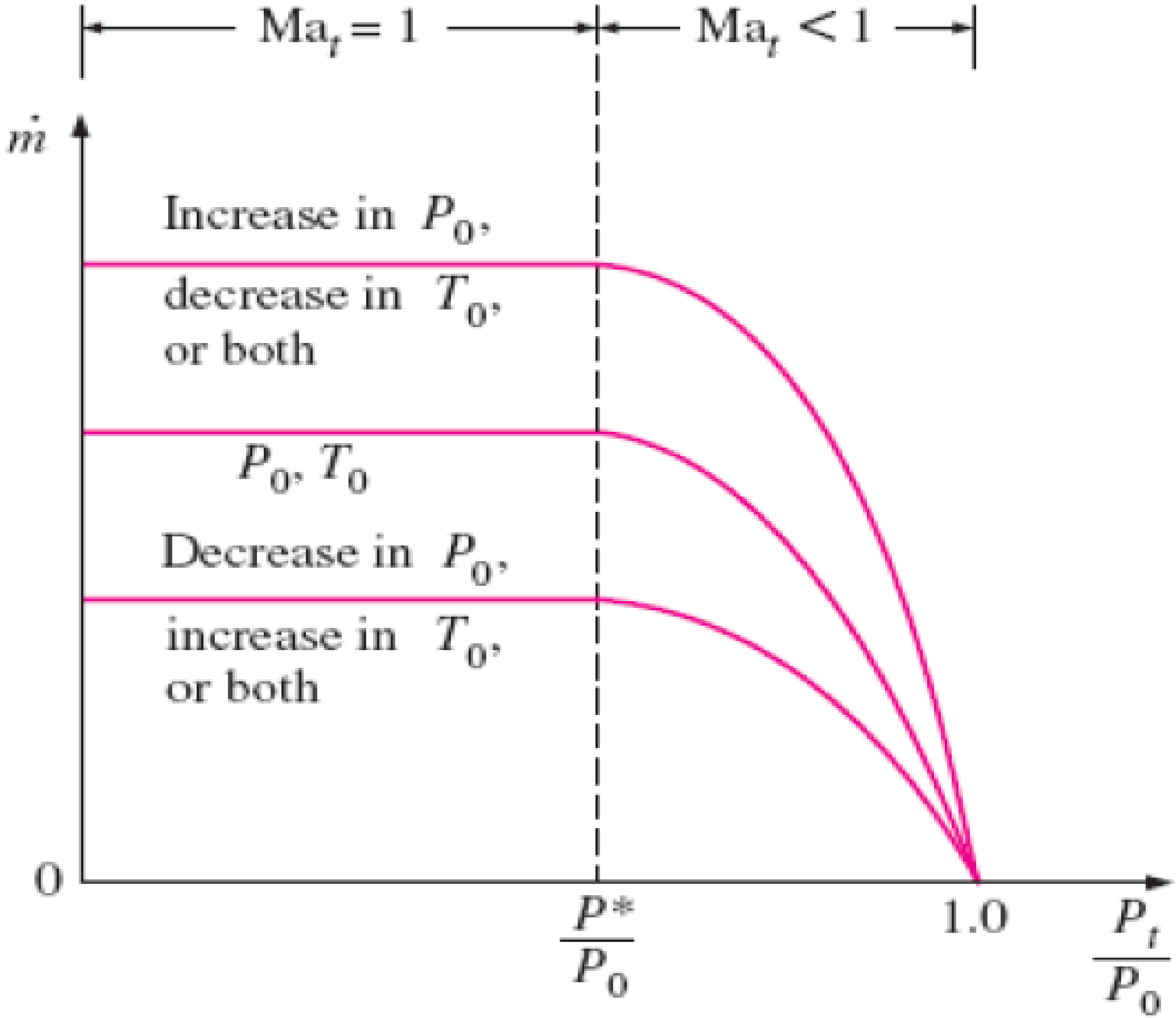


Fig. 7.13: The variation of the mass flow rate through a nozzle with inlet stagnation properties.

NUMERICAL EXAMPLE:

- A converging nozzle, with a throat area of 0.001 m^2, is operated with air at a back pressure of 591 kPa (abs). The nozzle is fed from a large plenum chamber where the absolute stagnation pressure and temperature are 1.0 MPa and 60 ^{0}C. The exit Mach number and mass flow rate are to be determined.

Ans:

M= 0.9

m = 2.2 kg/s

Solution:

The first step is to check for choking. The pressure ratio is

$$\frac{p_b}{p_0} = \frac{5.91 \times 10^5}{1.0 \times 10^6} = 0.591 > 0.528$$

so the flow is *not* choked. Thus $p_b = p_e$, and the flow is isentropic, as sketched on the *Ts* diagram.

Since p_0 = constant, M_e may be found from the pressure ratio,

$$\frac{p_0}{p_e} = \left[1 + \frac{k-1}{2} M_e^2\right]^{k/(k-1)}$$

Solving for M_e, since $p_e = p_b$, we obtain

$$1 + \frac{k-1}{2} M_e^2 = \left(\frac{p_0}{p_b}\right)^{(k-1)/k}$$

$$M_e = \left\{\left[\left(\frac{p_0}{p_b}\right)^{(k-1)/k} - 1\right]\frac{2}{k-1}\right\}^{1/2} = \left\{\left[\left(\frac{1.0 \times 10^6}{5.91 \times 10^5}\right)^{0.286} - 1\right]\frac{2}{1.4-1}\right\}^{1/2} = 0.90$$

The mass flow rate is

$$\dot{m} = \rho_e V_e A_e = \rho_e M_e c_e A_e$$

We need T_e to find ρ_e and c_e. Since T_0 = constant,

$$\frac{T_0}{T_e} = 1 + \frac{k-1}{2} M_e^2$$

or

$$T_e = \frac{T_0}{1 + \dfrac{k-1}{2} M_e^2} = \frac{(273+60)\text{ K}}{1+0.2(0.9)^2} = 287\text{ K}$$

$$c_e = \sqrt{kRT_e} = \left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 287\text{ K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 340\text{ m/s}$$

and

$$\rho_e = \frac{p_e}{RT_e} = 5.91 \times 10^5 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\text{ N}\cdot\text{m}} \times \frac{1}{287\text{ K}} = 7.18\text{ kg/m}^3$$

Finally,

$$\dot{m} = \rho_e M_e c_e A_e = 7.18 \frac{\text{kg}}{\text{m}^3} \times 0.9 \times 340 \frac{\text{m}}{\text{s}} \times 0.001\text{ m}^2$$

$$= 2.20\text{ kg/s} \xleftarrow{\qquad\qquad\qquad \dot{m} \qquad\qquad\qquad}$$

Flow through C-D nozzle

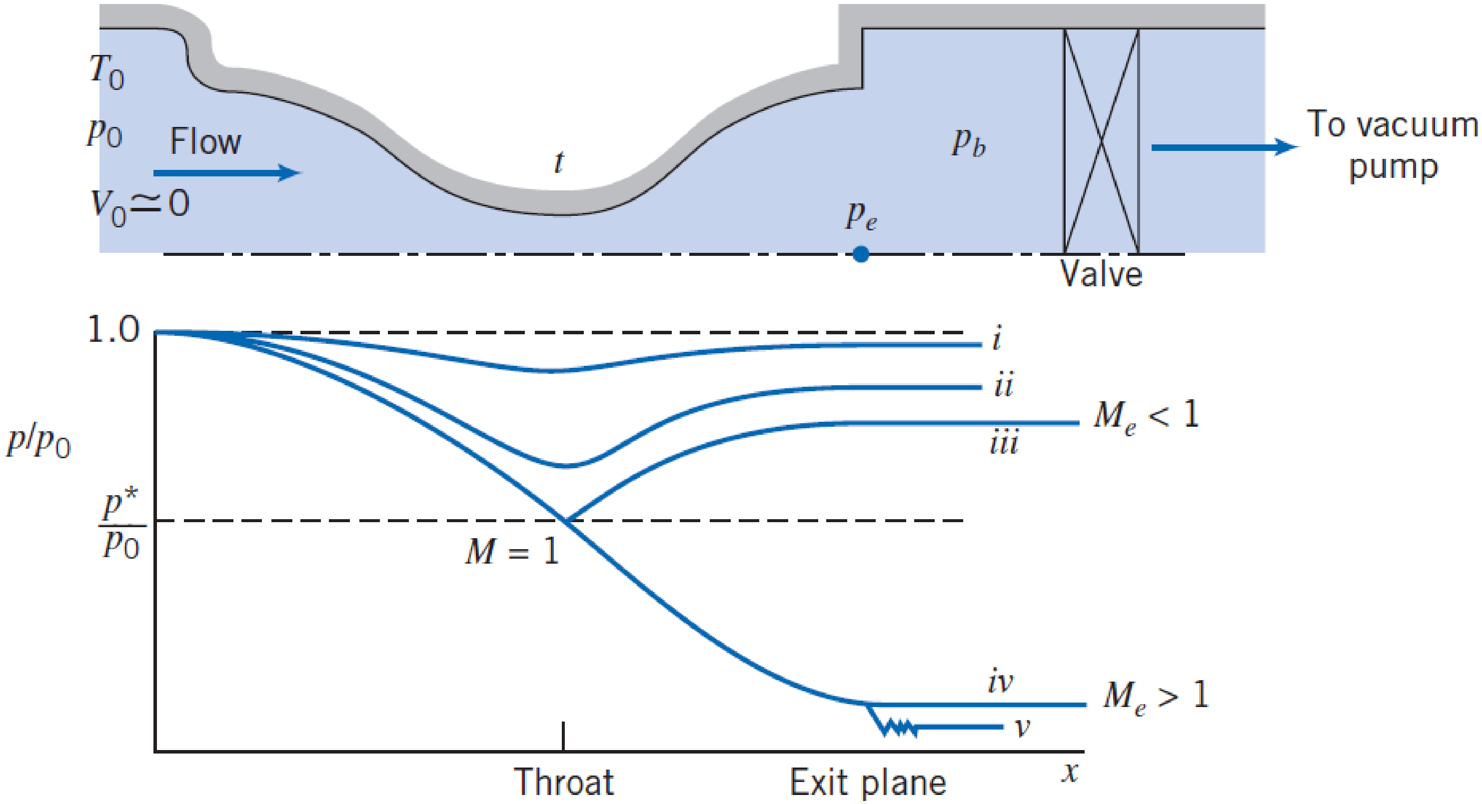


Fig. 13.8 Pressure distributions for isentropic flow in a converging-diverging nozzle.

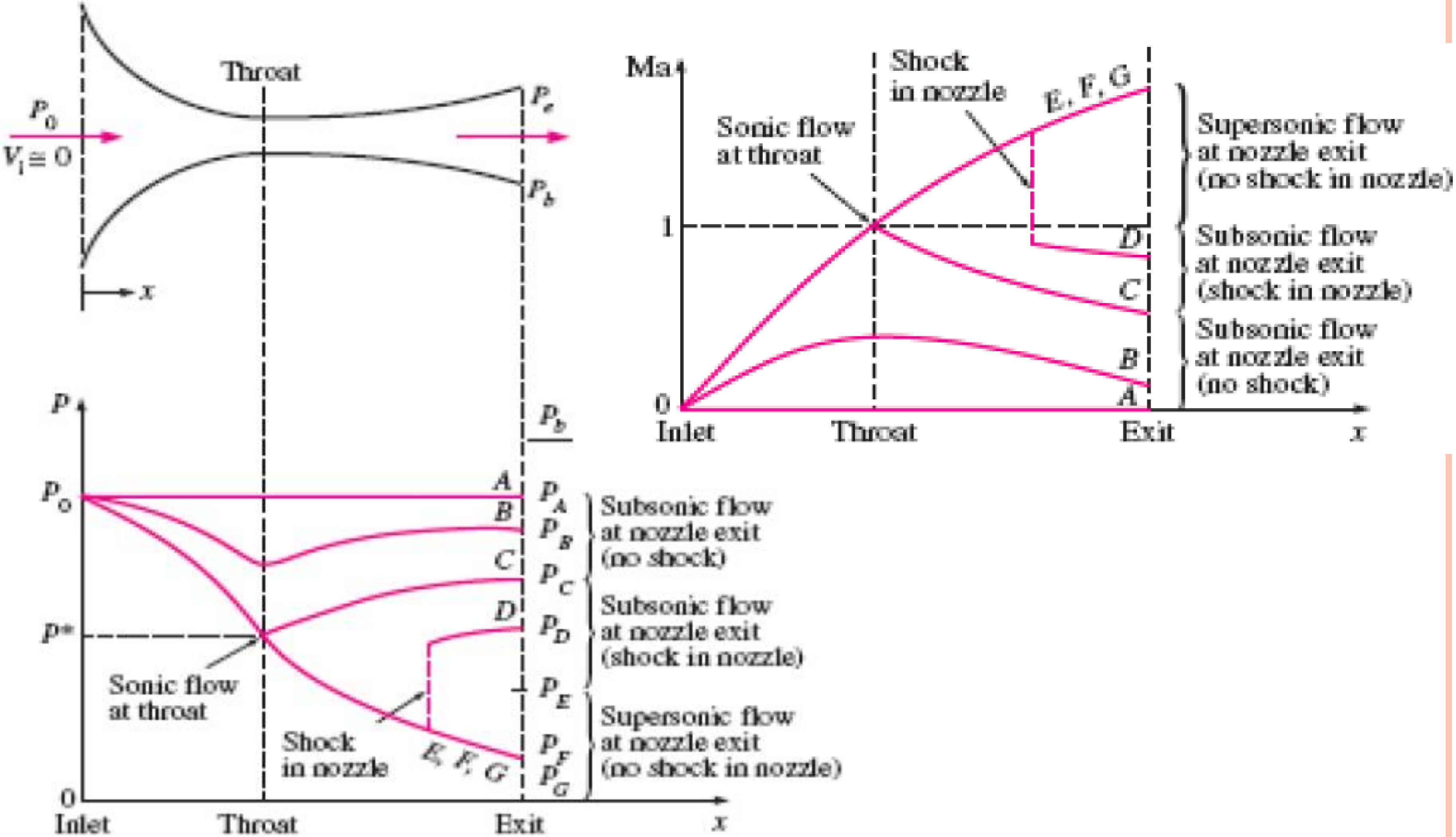
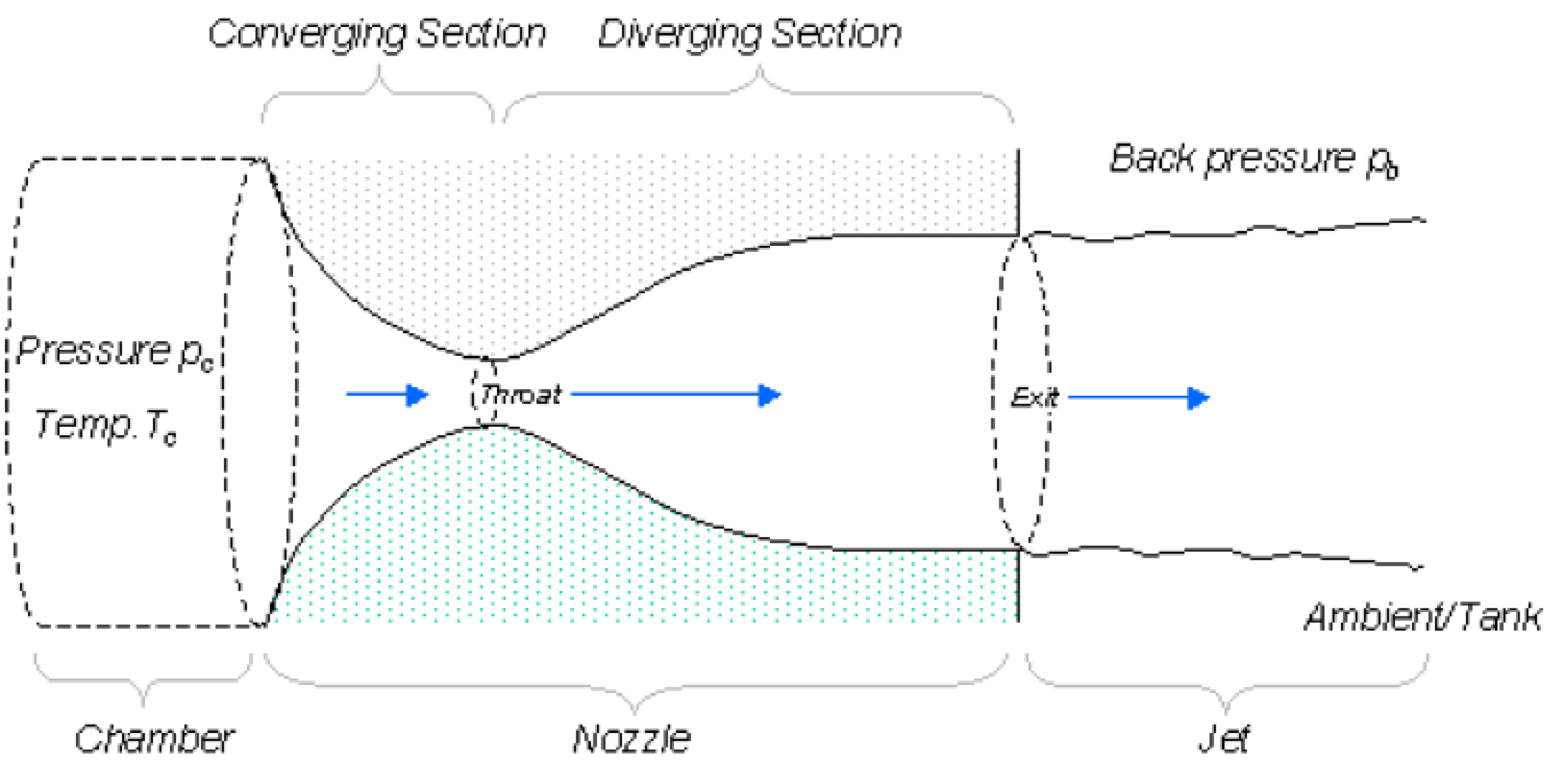


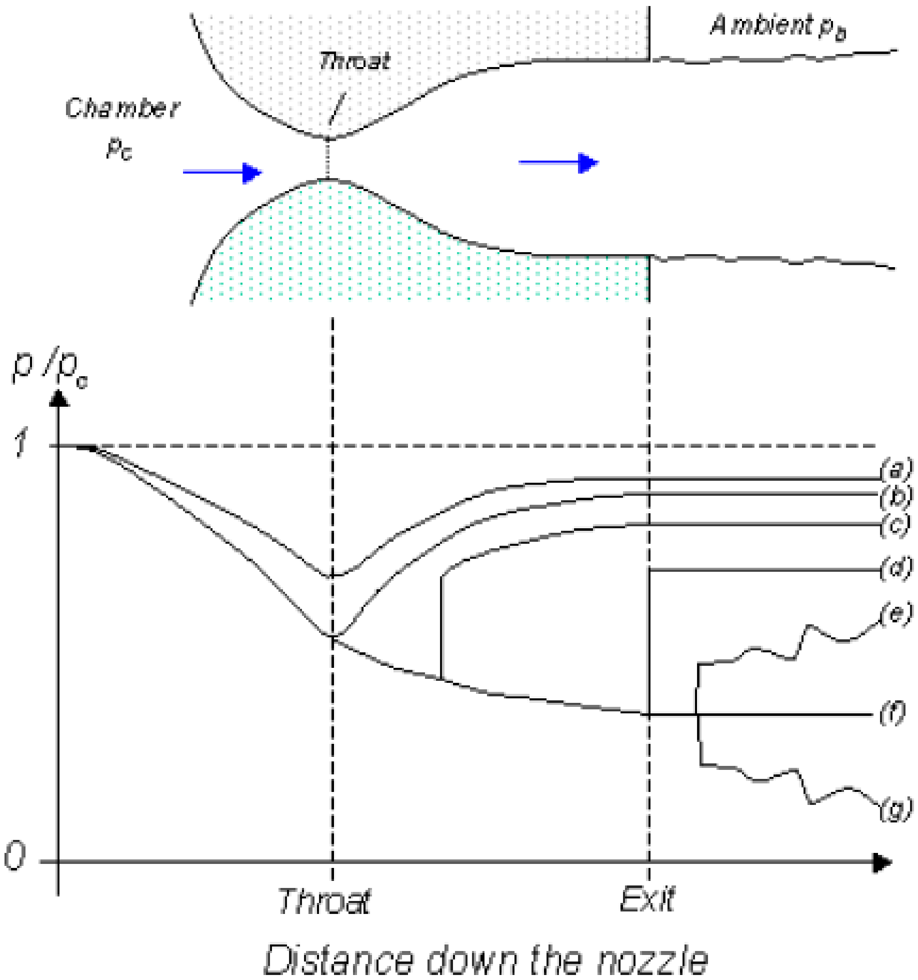
Fig. 7.14: The effects of back pressure on the flow through a converging–diverging nozzle.

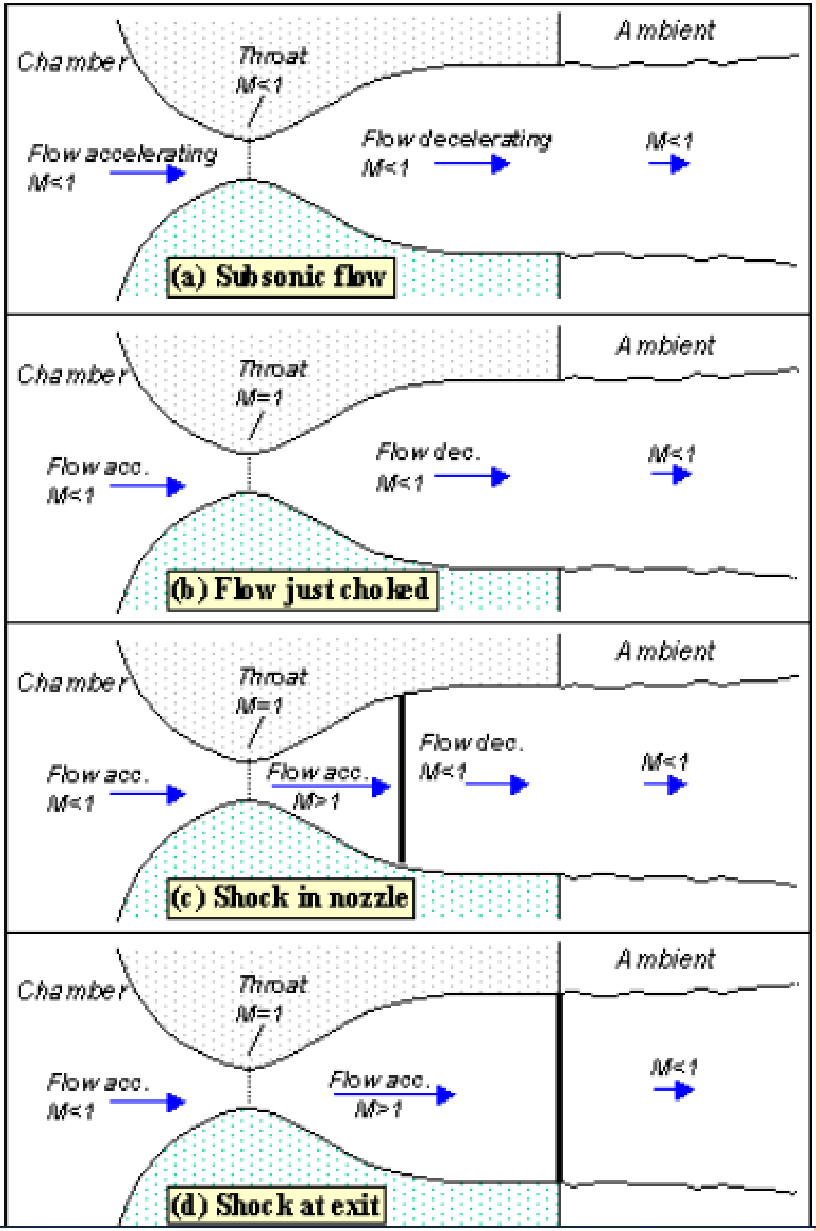
OPERATION OF CD NOZZLES

- Configuration for converging-diverging (CD) nozzle is shown below
- Gas flows through nozzle from region of high pressure (chamber) to low pressure (ambient)
- The chamber is taken as big enough so that any flow velocities are negligible
- Gas flows from chamber into converging portion of nozzle, past the throat, through the diverging portion and then exhausts into the ambient as a jet
- Pressure of ambient is referred to as back pressure

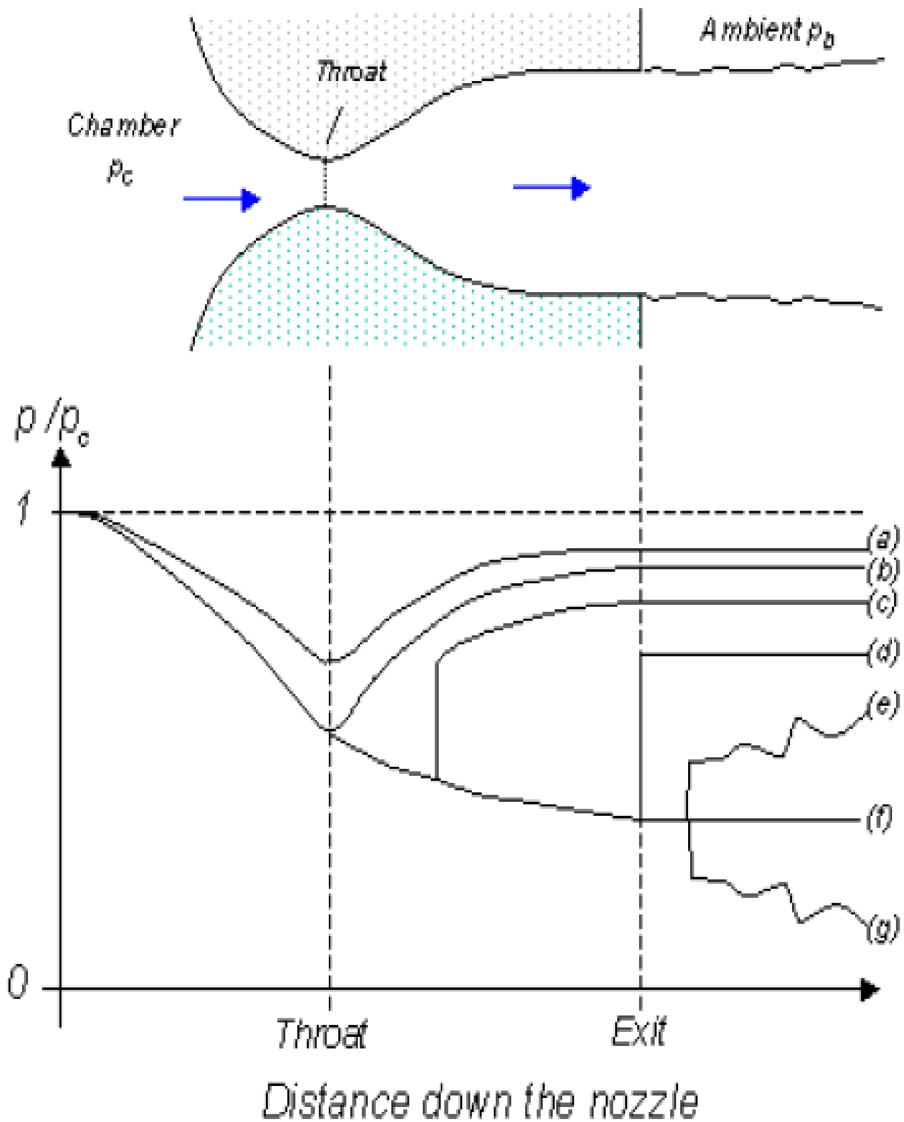


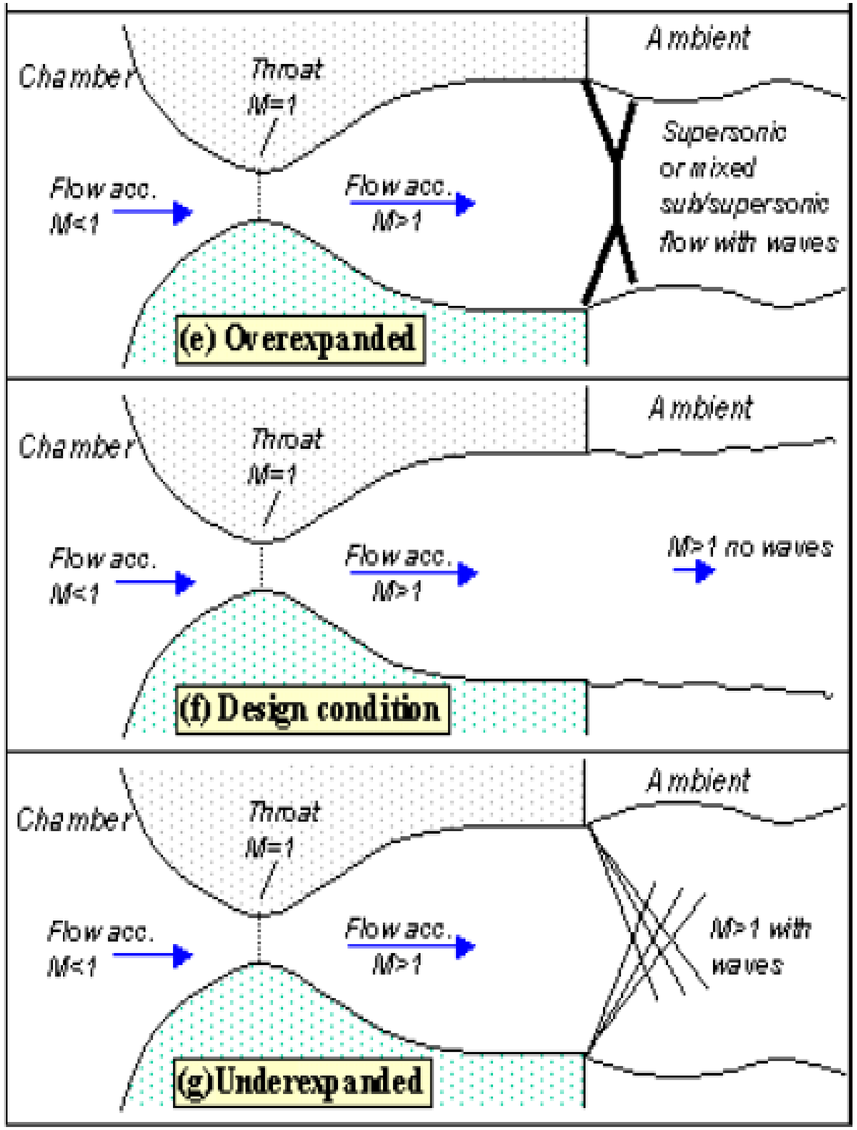
OPERATION OF CD NOZZLES





OPERATION OF CD NOZZLES





- Figure (a) shows the flow through the nozzle when it is completely subsonic (i.e. nozzle isn't choked). The flow accelerates out of the chamber through the converging section, reaching its maximum (subsonic) speed at the throat. The flow then decelerates through the diverging section and exhausts into the ambient as a subsonic jet. Lowering the back pressure in this state increases the flow speed everywhere in the nozzle.

- Further lowering p_b results in figure (b). The flow pattern is exactly the same as in subsonic flow, except that the flow speed at the throat has just reached Mach 1. Flow through the nozzle is now choked since further reductions in the back pressure can't move the point of M=1 away from the throat. However, the flow pattern in the diverging section does change as the back pressure is lowered further.

- As p_b is lowered below that needed to just choke the flow a region of supersonic flow forms just downstream of the throat. Unlike a subsonic flow, the supersonic flow accelerates as the area gets bigger. This region of supersonic acceleration is terminated by a normal shock wave. The shock wave produces a near-instantaneous deceleration of the flow to subsonic speed. This subsonic flow then decelerates through the remainder of the diverging section and exhausts as a subsonic jet. In this regime if the back pressure is lowered or raised the length of supersonic flow in the diverging section before the shock wave increases or decreases, respectively.

- If p_b is lowered enough the supersonic region may be extended all the way down the nozzle until the shock is sitting at the nozzle exit, figure (d). Because of the very long region of acceleration (the entire nozzle length) the flow speed just before the shock will be very large. However, after the shock the flow in the jet will still be subsonic.
- Lowering the back pressure further causes the shock to bend out into the jet, figure (e), and a complex pattern of shocks and reflections is set up in the jet which will now involve a mixture of subsonic and supersonic flow, or (if the back pressure is low enough) just supersonic flow. Because the shock is no longer perpendicular to the flow near the nozzle walls, it deflects it inward as it leaves the exit producing an initially contracting jet. We refer to this as **over-expanded** flow because in this case the pressure at the nozzle exit is lower than that in the ambient (the back pressure)-i.e. the flow has been expanded by the nozzle too much.
- A further lowering of the back pressure changes and weakens the wave pattern in the jet. Eventually, the back pressure will be lowered enough so that it is now equal to the pressure at the nozzle exit. In this case, the waves in the jet disappear altogether, figure (f), and the jet will be uniformly supersonic. This situation, since it is often desirable, is referred to as the 'design condition‘, Pe=Pa.

- Finally, if the back pressure is lowered even further we will create a new imbalance between the exit and back pressures (exit pressure greater than back pressure), figure (g). In this situation, called **under-expanded**, expansion waves that produce gradual turning and acceleration in the jet form at the nozzle exit, initially turning the flow at the jet edges outward in a plume and setting up a different type of complex wave pattern.

Summary Points to Remember:

- When the flow accelerates (sub or supersonically) the pressure drops
- The pressure rises instantaneously across a shock
- The pressure throughout the jet is always the same as the ambient (i.e. the back pressure) unless the jet is supersonic and there are shocks or expansion waves in the jet to produce pressure differences
- The pressure falls across an expansion wave

Nozzle Isentropic Efficiency

- The nozzle isentropic efficiency is defined as:

$$\eta N = \frac{Actual \quad KE \quad at \quad nozzle \quad exit}{Isentropic \quad KE \quad at \quad nozzle \quad exit} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

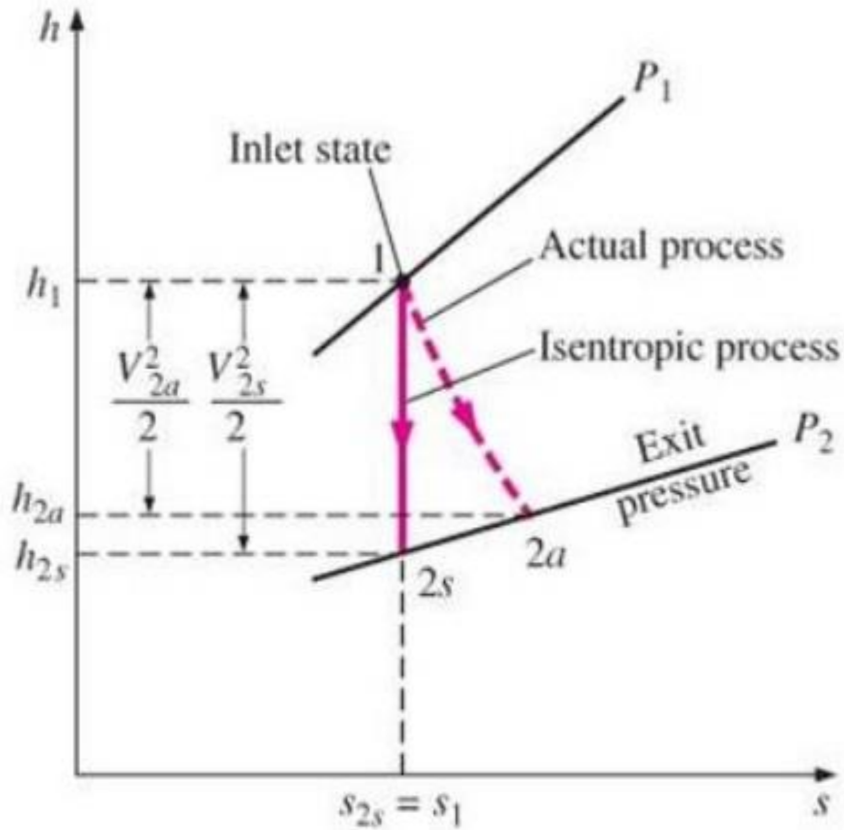


FIGURE 7–54

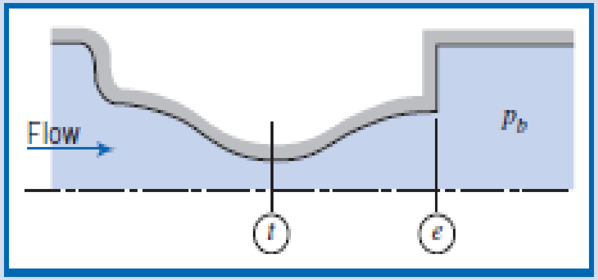
The *h-s* diagram of the actual and isentropic processes of an adiabatic nozzle.

Numerical Example

- Air flows isentropically in a converging-diverging nozzle, with exit area of 0.001 m2. The nozzle is fed from a large plenum where the stagnation conditions are 350K and 1.0 MPa (abs). The exit pressure is 954 kPa (abs) and the Mach number at the throat is 0.68. Fluid properties and area at the nozzle throat and the exit Mach number are to be determined.

Given: Isentropic flow of air in C-D nozzle as shown:

$$T_0 = 350\,\text{K}$$
$$p_0 = 1.0\,\text{MPa (abs)}$$
$$p_b = 954\,\text{kPa (abs)}$$
$$M_t = 0.68 \quad A_e = 0.001\,\text{m}^2$$



Find: (a) Properties and area at nozzle throat.
(b) M_e.

Solution:

Stagnation temperature is constant for isentropic flow. Thus, since

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

then

$$T_t = \frac{T_0}{1 + \frac{k-1}{2}M_t^2} = \frac{350\,\text{K}}{1 + 0.2(0.68)^2} = 320\,\text{K} \longleftarrow T_t$$

Also, since p_0 is constant for isentropic flow, then

$$p_t = p_0\left(\frac{T_t}{T_0}\right)^{k/(k-1)} = p_0\left[\frac{1}{1 + \frac{k-1}{2}M_t^2}\right]^{k/(k-1)}$$

$$p_t = 1.0 \times 10^6\,\text{Pa}\left[\frac{1}{1 + 0.2(0.68)^2}\right]^{3.5} = 734\,\text{kPa (abs)} \longleftarrow p_t$$

so

$$\rho_t = \frac{p_t}{RT_t} = 7.34 \times 10^5 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\ \text{N}\cdot\text{m}} \times \frac{1}{320\ \text{K}} = 7.99\ \text{kg/m}^3 \qquad \rho_t$$

and

$$V_t = M_t c_t = M_t\sqrt{kRT_t}$$

$$V_t = 0.68\left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 320\ \text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 244\ \text{m/s} \qquad V_t$$

From Eq. 13.7d we can obtain a value of A_t/A^*

$$\frac{A_t}{A^*} = \frac{1}{M_t}\left[\frac{1+\frac{k-1}{2}M_t^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} = \frac{1}{0.68}\left[\frac{1+0.2\,(0.68)^2}{1.2}\right]^{3.00} = 1.11$$

but at this point A^* is not known.

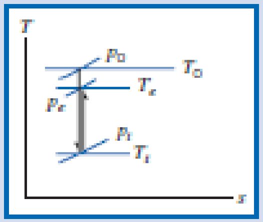
Since $M_t < 1$, flow at the exit must be subsonic. Therefore, $p_e = p_b$. Stagnation properties are constant, so

$$\frac{p_0}{p_e} = \left[1+\frac{k-1}{2}M_e^2\right]^{k/(k-1)}$$

Solving for M_e gives

$$M_e = \left\{\left[\left(\frac{p_0}{p_e}\right)^{(k-1)/k} - 1\right]\frac{2}{k-1}\right\}^{1/2} = \left\{\left[\left(\frac{1.0\times 10^6}{9.54\times 10^5}\right)^{0.286} - 1\right](5)\right\}^{1/2} = 0.26 \qquad M_e$$

The Ts diagram for this flow is



Since A_e and M_e are known, we can compute A^*. From Eq. 13.7d

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{1+\frac{k-1}{2}M_e^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} = \frac{1}{0.26}\left[\frac{1+0.2\,(0.26)^2}{1.2}\right]^{3.00} = 2.317$$

Thus,

$$A^* = \frac{A_e}{2.317} = \frac{0.001\ \text{m}^2}{2.317} = 4.32 \times 10^{-4}\ \text{m}^2$$

and

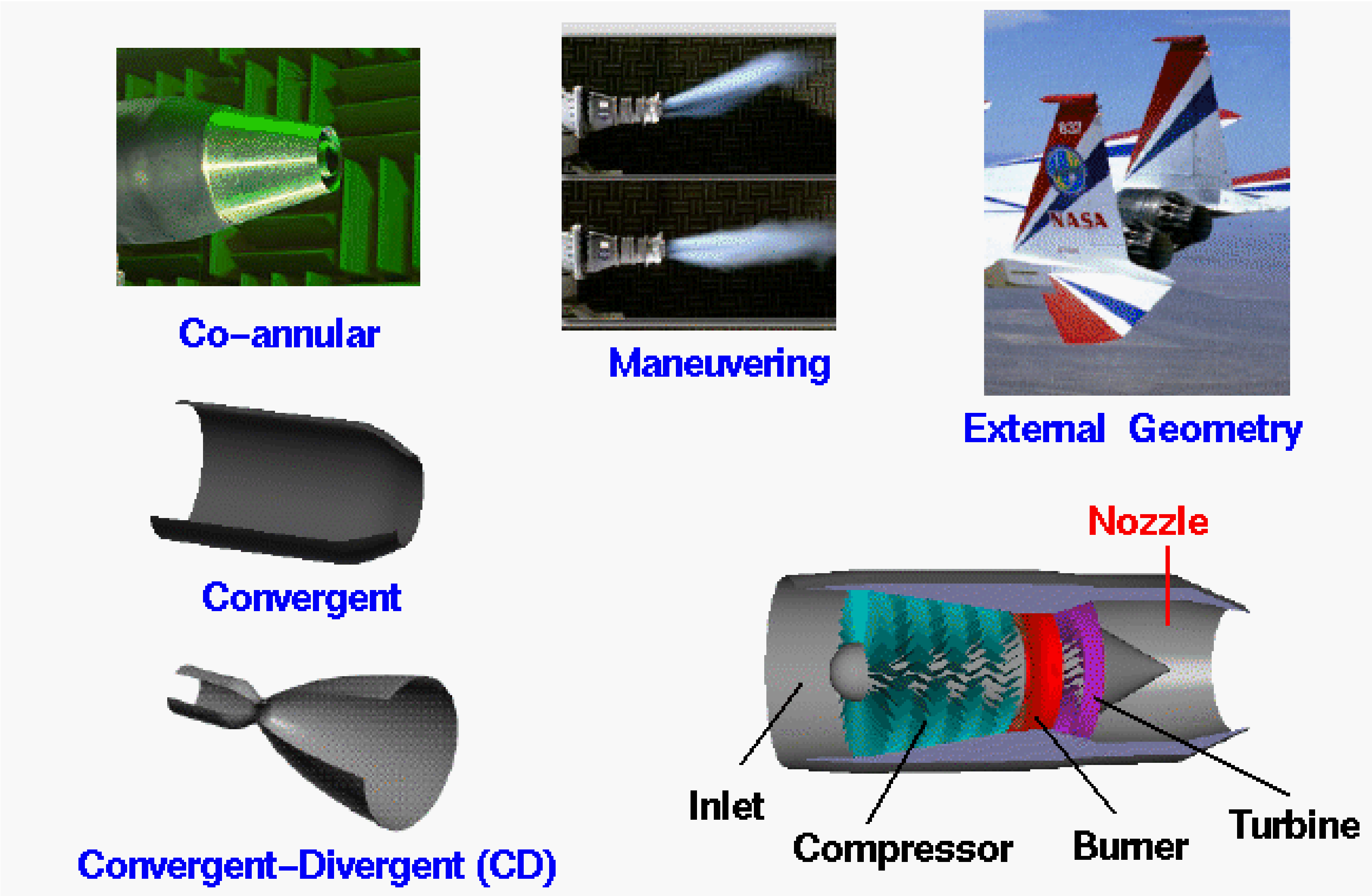
$$A_t = 1.110A^* = (1.110)(4.32\times 10^{-4}\ \text{m}^2)$$
$$= 4.80 \times 10^{-4}\ \text{m}^2 \qquad A_t$$

This problem illustrates use of the isentropic equations, Eqs. 13.7, for flow in a C-D nozzle that is not choked.

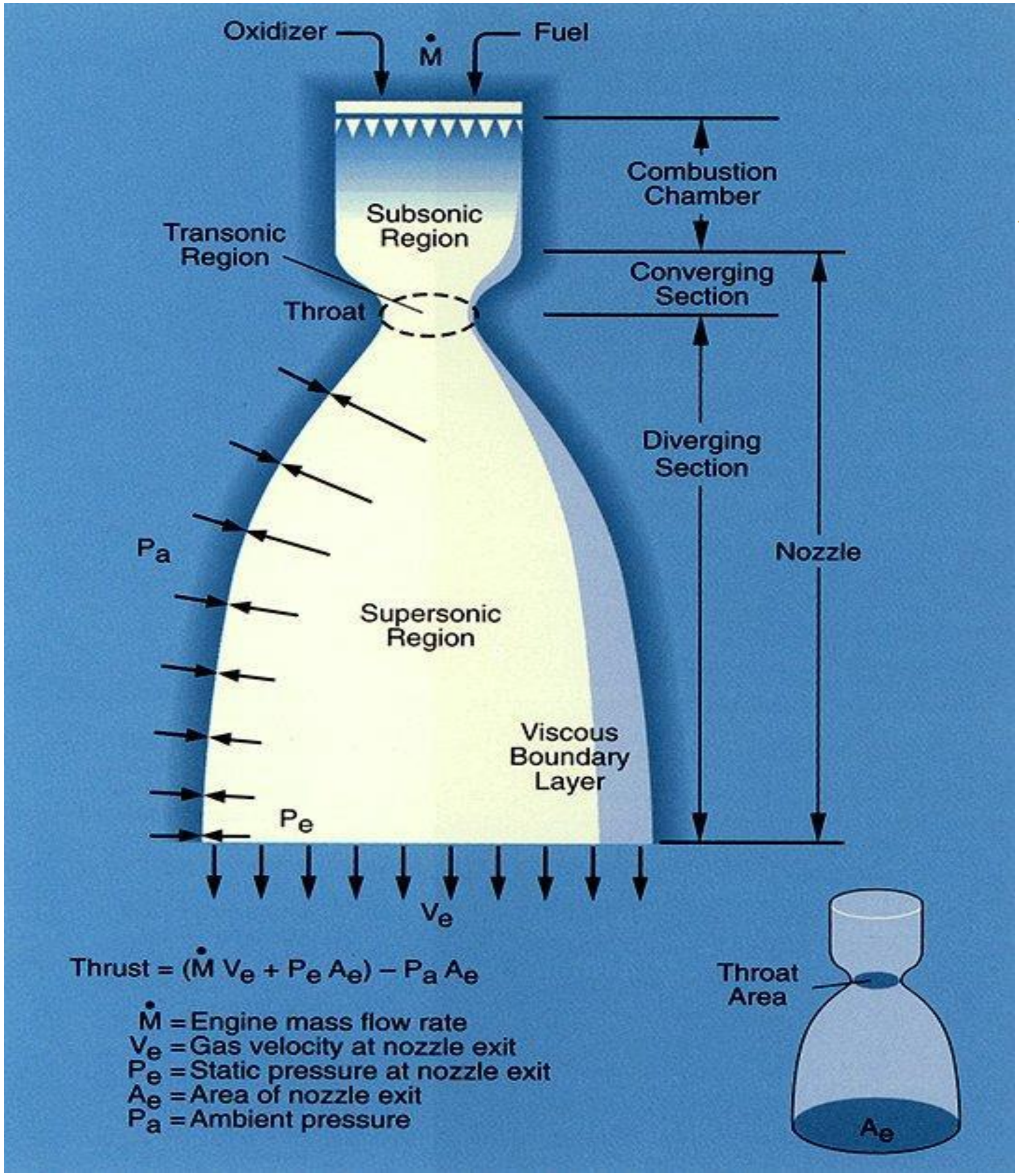
✓ Note that use of Eq. 13.7d allowed us to obtain the throat area without needing to first compute other properties.

The *Excel* workbook for this Example is convenient for performing the calculations (using either the isentropic equations or the basic equations). (The *Excel* add-ins for isentropic flow, on the Web site, also make calculations much easier.)

Nozzle Shape Variations



C-D NOZZLE FOR ROCKET PROPULSION



$$F = \dot{m}_e V_e + (P_e - P_a) A_e$$

$$F = \dot{m}_e V_e$$

- Where F is the thrust developed.
- In case the exit pressure is equal to ambient pressure, pressure thrust equals zero.

Nozzle Design Parameters

- Expansion Area Ratio: Most important parameter in nozzle design is expansion area ratio,

$$\varepsilon = \frac{A_{exit}}{A_{throat}} = \frac{A_e}{A^*}$$

- Fixing other variables (primarily chamber pressure) → only one ratio that optimizes performance for a given altitude (or ambient pressure)
- However, rocket does not travel at only one altitude
- Select expansion ratio that maximizes performance over a range of ambient pressures
- Other factors must also be considered: Nozzle weight, length, manufacturability, cooling (heat transfer), and aerodynamic characteristics.

Rocket Propulsion Nozzle Shape Variation

- Cone
- Bell
- Annular