

## THEORY OF MACHINE AND MECHANISM II

### TUTORIAL NO: 8

#### VIBRATION OF MULTI DEGREE OF FREEDOM SYSTEMS

1. Figure P8.1 shows a three degrees of freedom system. Determine the stiffness matrix.

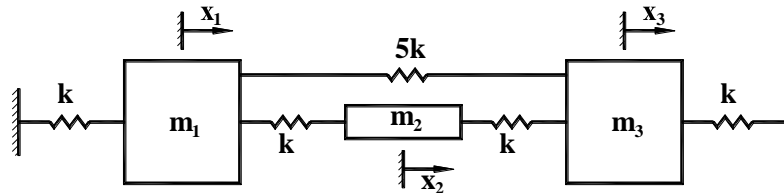


Figure P8.1

2. Determine the influence coefficients and flexibility matrix of the spring-mass system shown in Figure P8.2.

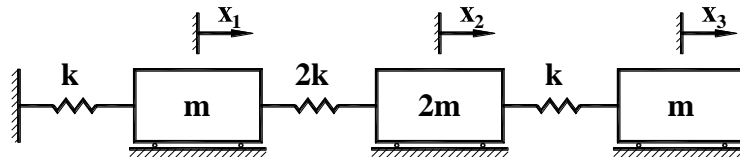


Figure P8.2

3. Determine the natural frequencies and the corresponding mode shapes of the system shown in Figure P8.3.
4. Determine the natural frequencies and the corresponding mode shapes of multi degree of freedom spring mass system shown in Figure P8.4.

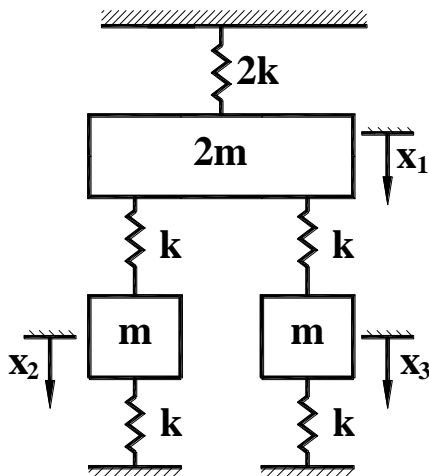


Figure P8.3

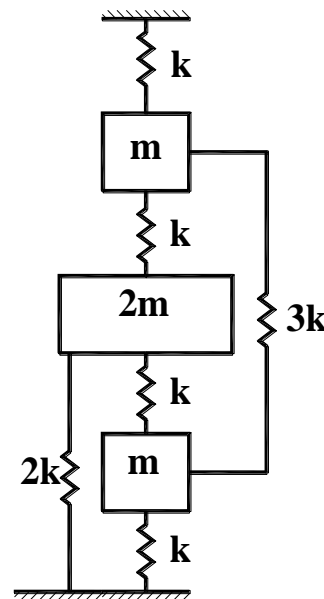
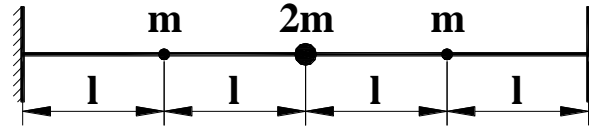


Figure P8.4

5. A string is stretched with a large tension  $T$  between two points and has three point masses fixed along its length as shown in **Figure P8.5**. The masses can vibrate freely in the lateral direction. Determine the three natural frequencies and the corresponding mode shapes.



**Figure P8.5**

**ANSWERS**

1. 
$$\begin{bmatrix} 6k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 6k \end{bmatrix}$$

2. 
$$\begin{bmatrix} 1/k & 1/k & 1/k \\ 1/k & 2/3k & 2/3k \\ 1/3k & 2/3k & 5/3k \end{bmatrix}$$

3.  $\omega_1 = 0.4369 \sqrt{k/m}$ ,  $\omega_2 = 1.1441 \sqrt{k/m}$ ,  $\omega_3 = 1.732 \sqrt{k/m}$ ;  $\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$ ,  $\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$ ,  $\begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix}$

4.  $\omega_1 = 1.1206 \sqrt{k/m}$ ,  $\omega_2 = 1.6311 \sqrt{k/m}$ ,  $\omega_3 = 2.8432 \sqrt{k/m}$

5.  $\omega_1 = 0.766 \sqrt{T/ml}$ ,  $\omega_2 = 1.414 \sqrt{T/ml}$ ,  $\omega_3 = 1.847 \sqrt{T/ml}$ ;  $\begin{Bmatrix} 1 \\ 1.414 \\ 1 \end{Bmatrix}$ ,  $\begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$ ,  $\begin{Bmatrix} 1 \\ -1.414 \\ 1 \end{Bmatrix}$