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## Chapter 2

# Basic Aerothermodynamics

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## 2. Aerothermodynamics in Gas Turbine Engine

- First law of thermodynamics
- Second law of thermodynamics
- Aerodynamics fundamental equations
  - Continuity equation
  - Energy equation
  - Bernoulli's equation
  - Sound speed and Mach number
  - Stagnation parameters of flow and aerodynamic functions
  - Momentum equation
  - Shockwaves and expansion waves

# 1. Basic Aerodynamics

- The conservation equations for the continuity, momentum and energy of the flow can be expressed in the following form:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} = S(U)$$

- The conservation equations for the continuity, momentum and energy of the flow can be expressed in the following form:

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \\ \rho Y_1 \end{bmatrix}, F(U) = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(\rho E + p) \\ \rho u Y_1 \end{bmatrix}, G(U) = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(\rho E + p) \\ \rho v Y_1 \end{bmatrix}, S(U) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ R_1 \end{bmatrix}$$

Scalar Equation →

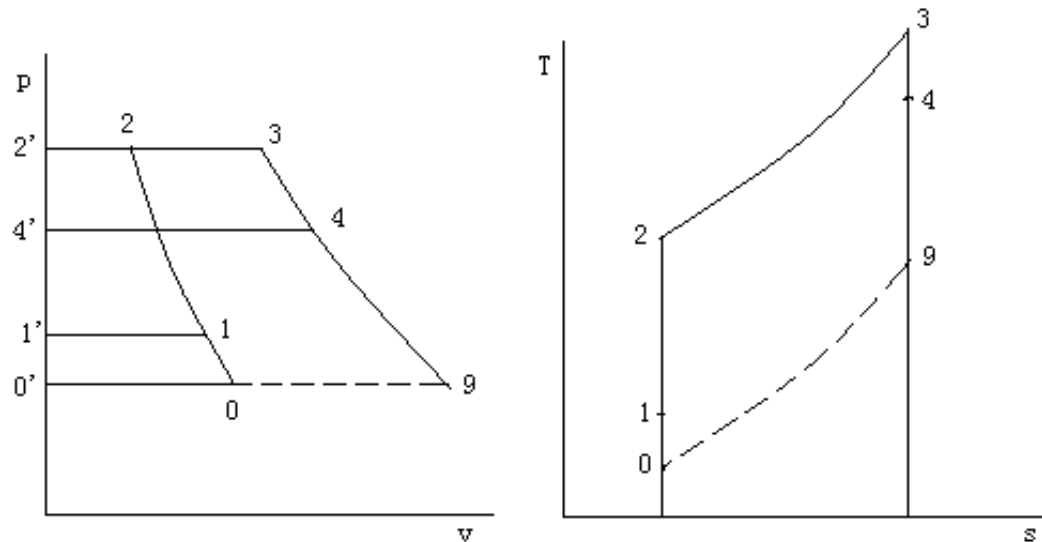
# 1. Basic Aerodynamics

## Solution Methods

- Semi-Implicit Method for Pressure-Linked Equations (**SIMPLE**)
- Semi-Implicit Method for Pressure-Linked Equations-Consistent (**SIMPLEC**)
- Pressure Implicit with Splitting of Operator (**PISO**)
- Approximate Riemann Solvers; e.g. Harten Lax and van Leer- Contact (HLLC), HLL, etc.
- Advection Upwind Splitting Method (**AUSM**)
- Advection Upwind Splitting Method- Plus (**AUSM-plus**)

# 2. Aerothermodynamics in Gas Turbine Engine

## Gas Turbine Cycle



## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

- Equation of state of ideal gas
  - Ideal gas
    - Gas molecule has only a mass but not volume
    - There is no attractive force between the molecules
    - Ideal gas does NOT exist
    - Idealization for real gas

## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

- Equation of state of ideal gas

$$pv = RT, \text{ or } p = \rho RT$$

- $p$  ——— *Pressure,  $N/m^2$*
- $v$  ——— *Specific volume,  $m^3/kg$*
- $T$  ——— *Temperature,  $K$*
- $R$  ——— *Gas constant,  $J/kgK$  (287.1 for air and 288 for gas)*

## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

Heat

➤ Heat is a form of energy exchange

Diagram illustrating the equation for heat exchange:

$$Q = mc(T_2 - T_1)$$

The variables in the equation are defined by the following units:

- $Q$  (Heat) is measured in Joules (J).
- $m$  (Substance mass) is measured in kilograms (kg).
- $c$  (Specific Heat) is measured in Joules per kilogram Kelvin (J/kgK).
- $T_2$  (Temperature after heat exchange) is measured in Kelvin (K).
- $T_1$  (Temperature before heat exchange) is measured in Kelvin (K).



## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

- Specific heat
  - Heat needed to increase  $T$  of 1K for 1kg of substance
  - Dependent on substance, temperature and process
- Specific heat at constant volume

$$Q_v = mc_v(T_2 - T_1)$$

## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

- Specific heat at constant pressure

$$Q_p = mc_p(T_2 - T_1)$$

- **In reality in general, and in particular for certain substances,  $C_v$  and  $C_p$  are functions of temperature.**
- These temperature defined properties are defined in terms of polynomial functions; e.g. **Janaf Table**.

## 2. Aerothermodynamics in Gas Turbine Engine

$$\frac{C_p(T)}{R} = a[1] + a[2]T + a[3]T^2 + a[4]T^3 + a[5]T^4$$

$$\frac{H(T)}{R} = a[1]T + \frac{a[2]T^2}{2} + \frac{a[3]T^3}{3} + \frac{a[4]T^4}{4} + \frac{a[5]T^5}{5} + a[6]$$

$$\frac{S(T)}{R} = a[1]\ln T + a[2]T + \frac{a[3]T^2}{2} + \frac{a[4]T^3}{3} + \frac{a[5]T^4}{4} + a[7]$$

## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

- Ratio of specific heats

$$\gamma = c_p / c_v$$

- For air 1.4;
- For a mixture of gases  $\sim 1.33$ ;
- The value of  $\gamma$  can vary based on composition.
- In addition, the ratio is also temperature dependent when the specific heats are calculated as temperature-dependent functions.
- Hypersonic limit:  $\gamma \rightarrow 1$ .

## 2. Aerothermodynamics in Gas Turbine Engine

- Specific internal energy
  - Summation of molecule average kinetic and potential energy
  - Potential energy for ideal gas = 0 (no attractive forces between the molecules).
  - Specific internal energy of ideal gas is function of temperature

$$u = u(T) \quad \text{J/kg}$$

- Since 'T' represents molecular average kinetic energy of the gas.

## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

➤ Heating gas in container of constant volume

$$q_v = \int_{T_1}^{T_2} c_v(T) dT = c_v(T_2 - T_1)$$

➤  $C_v$  average specific heat (volume)

➤ Internal energy increase

➤  $\Delta u = q_v = c_v(T_2 - T_1)$

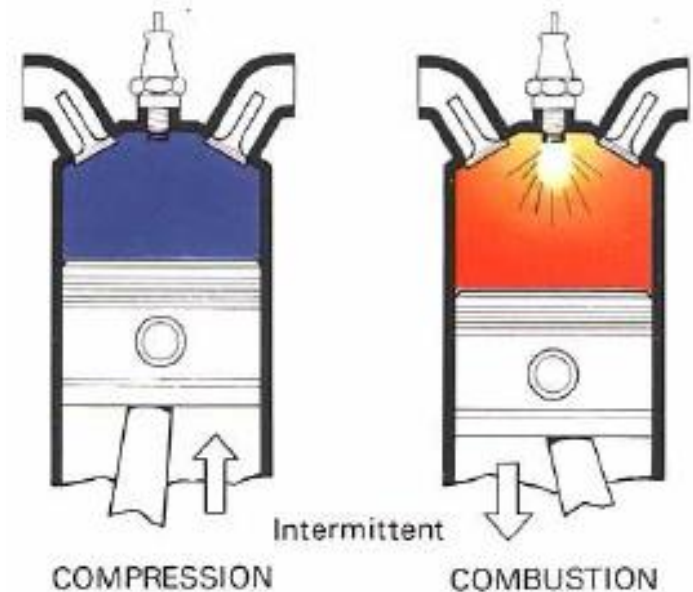
➤ No sense for absolute value of internal energy

## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

#### ➤ Work

- Mechanical work
- In thermal process, gas expansion does work on objects
- Or work can compress gas



## 2. Aerothermodynamics in Gas Turbine Engine

### First law of thermodynamics

➤ Volume change for a unit mass gas  $dv$ , then work:

➤  $dW = p dv$

➤ By integration

$P$  constant: 
$$W = \int_{v_1}^{v_2} p dv$$

$$W = p(v_2 - v_1)$$

➤ If  $V = \text{constant}$ ,  $W = 0$ .



## 2. Aerothermodynamics in Gas Turbine Engine

### Enthalpy

- $h = u + pv = u + p/\rho$  (J/kg)
- $u$  is internal energy
- $pv$  considered as pressure potential energy
- $h(T) = u(T) + p/\rho = u(T) + RT$
- Enthalpy, a state parameter, is a single value function of  $T$ .

$$q_p = \int_{T_1}^{T_2} c_p(T) dT = c_p(T_2 - T_1) = c_v(T_2 - T_1) + p(v_2 - v_1)$$

- Since, Heat = U + W,  
$$= (c_v + R)(T_2 - T_1) = \Delta h$$

$$\therefore \Delta h = c_p(T_2 - T_1)$$

## 2. Aerothermodynamics in Gas Turbine Engine

### Enthalpy

➤  $dq = du + pdv$  = Relation of heat, internal energy and work (transformations and conservaton)

➤  $u = h - pv$

➤  $du = dh - d(pv) = dh - pdv - vdp$

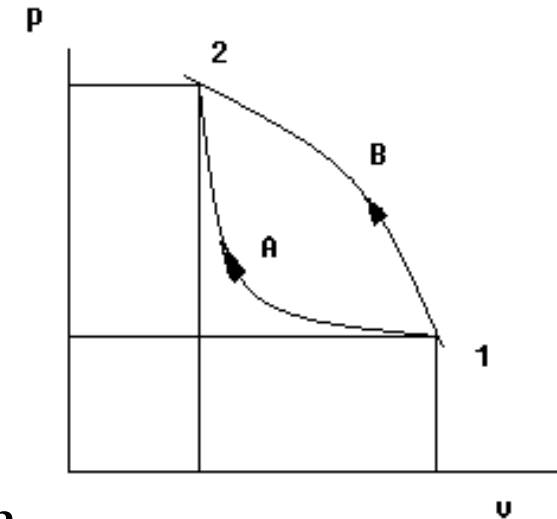
➤  $dq = dh - vdp$  (pressure work)

➤ or,  $dh = dq + vdp$

➤ For no heat addition,  $dh = vdp$  )

$$q = h_2 - h_1 - \int_{p_1}^{p_2} vdp = c_p(T_2 - T_1) - \int_{p_1}^{p_2} vdp$$

➤ For constant pressure combustion,  $dq = dh$



## 2. Aerothermodynamics in Gas Turbine Engine

### Adiabatic Process

$$c_p c_v = \frac{\gamma}{\gamma - 1} (c_p c_v + c_v c_p) \quad p v = R T$$

$$c_v dT + p dv = 0$$

$$\frac{c_v}{c_p - c_v} (v dp + p dv) + p dv = 0$$

$$\frac{dp}{p} + \gamma \frac{dv}{v} = 0$$

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho}$$

$$\left( \frac{dp}{d\rho} \right)_s = a^2$$

## 2. Aerothermodynamics in Gas Turbine Engine

➤  $pv^\gamma = \text{constant}$

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{Adiabatic Process}$$

$$W = -\Delta u = C_v(T_1 - T_2) = \frac{R}{\gamma - 1}(T_1 - T_2) = \frac{RT_1}{\gamma - 1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

➤ I.E. => Work

## 2. Aerothermodynamics in Gas Turbine Engine

- Reversible process
  - Path  $1 \Rightarrow 2$  is the same as  $2 \Rightarrow 1$
  - Uniform, no friction——gas, machine, gas/machine
- Irreversible process
  - Friction exists
- Polytropic process
  - $p v^n = \text{constant}$

### Polytropic Process

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^n = \left( \frac{T_2}{T_1} \right)^{\frac{n}{n-1}}$$

# 2. Aerothermodynamics in Gas Turbine Engine

## Irreversible & Polytropic Process

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^n = \left( \frac{T_2}{T_1} \right)^{\frac{n}{n-1}}$$

$n$  polytropic index

Const P	$n = 0$	$p = \text{const}$
Const V	$n = \infty$	$v = \text{const}$
Const T	$n = 1$	$T = \text{const}$
Adiabatic	$n = \gamma$	$p v^\gamma = \text{const}$
Irreversible	Compression $n > \gamma$ Expansion $1 < n < \gamma$	

# 2. Aerothermodynamics in Gas Turbine Engine

## Irreversible & Polytropic Process

➤  $pv^n = \text{const}$ , intersect at '1'

➤ Compression

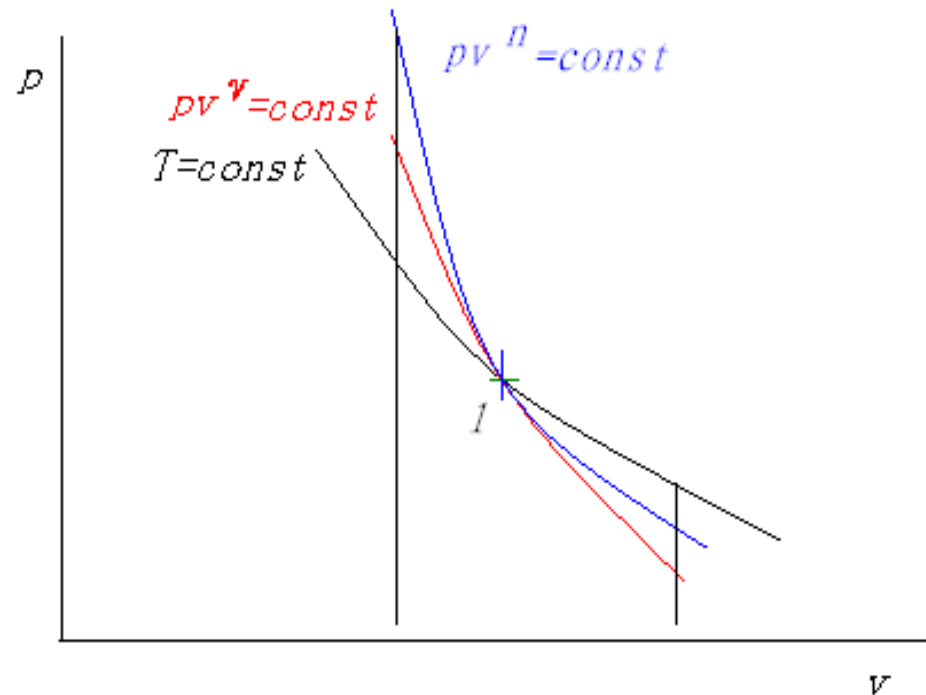
➤  $W = W_n + q_{in}$

➤ *Add more work for compression*

➤ Expansion

➤  $W = W_n - q_{in}$

➤ Get less work done at expansion



## 2. Aerothermodynamics in Gas Turbine Engine

### Irreversible & Polytropic Process

$$W_n = \frac{R}{n-1} (T_1 - T_2) = \frac{RT_1}{n-1} \left( 1 - \frac{1}{\left( \frac{v_2}{v_1} \right)^{n-1}} \right) = \frac{RT_1}{n-1} \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right)$$



## 2. Aerothermodynamics in Gas Turbine Engine

### Irreversible & Polytropic Process

#### ➤ Entropy

$$ds = \frac{dq}{T} \quad \text{J/kg.K}$$

$$\left. \begin{aligned} \Delta s &= C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\ &= C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \\ &= C_p \ln \frac{v_2}{v_1} + C_v \ln \frac{p_2}{p_1} \end{aligned} \right\}$$

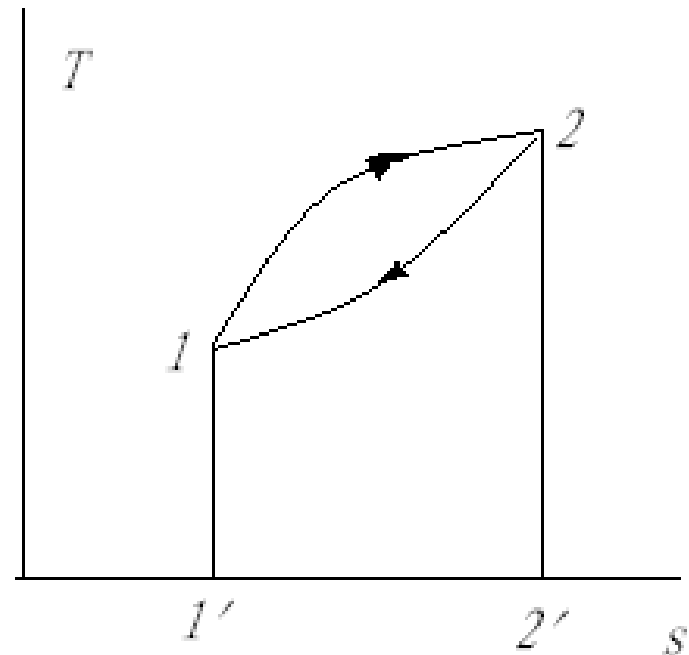
$$q = \int_{s_1}^{s_2} T ds$$

# 2. Aerothermodynamics in Gas Turbine Engine

## Irreversible & Polytropic Process

- Entropy is an abstract parameter for a useful thermodynamic concept ('entity')
- State parameter, that is a function of any two out of  $p$ ,  $v$  and  $T$
- Can be used to get heat
  - $dq = Tds$  J/kg
  - $dq$  and  $ds$  retain same sign
- When irreversible,  $q$  contains  $q_{in}$  (friction heat)

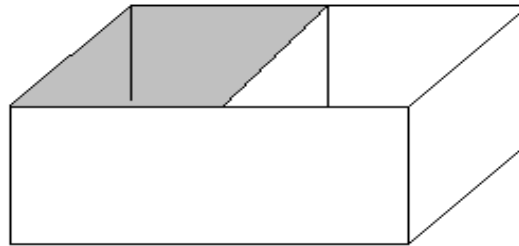
$$q = \int_{s_1}^{s_2} Tds$$



## 2. Aerothermodynamics in Gas Turbine Engine

### Second law of thermodynamics

- Any natural thermal process is irreversible.
- Example:



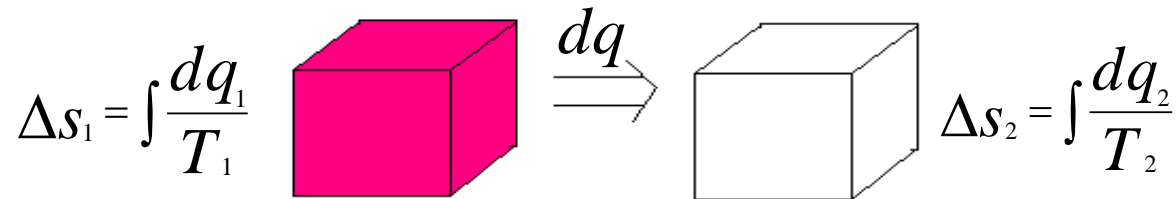
$$\Delta s = C_p \ln \frac{v_2}{v_1} + C_v \ln \frac{p_2}{p_1} = (C_p - C_v) \ln 2$$

$$\Delta s > 0$$

## 2. Aerothermodynamics in Gas Turbine Engine

### Second law of thermodynamics

- Or,
- *Without work, heat can NOT automatically go from LT to HT or it goes always naturally from HT to LT.*



$$\Delta s = \int \left( \frac{1}{T_2} - \frac{1}{T_1} \right) dq \quad T_1 > T_2$$

## 2. Aerothermodynamics in Gas Turbine Engine

### Second law of thermodynamics

- Unique heat source engine can not be made.
- Natural thermal process follows path of entropy increase.
- Naturally go **from ordered to unordered**.
- Notes
  - Naturally
  - Isolated system
  - Can be used in other domain, such as in social sciences

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

- From above equations, flow kinetic energy (speed), enthalpy and pressure potential energy can be converted from one to others.
- If flow stagnates ( $U=0$ ) as isentropic process, the kinetic energy is converted totally to enthalpy. It is called stagnation enthalpy, or total enthalpy:

$$h^* = c_p T + \frac{U^2}{2} \quad U \uparrow \Rightarrow h \downarrow \quad \text{and} \quad p \downarrow$$

- $U$ : ordered movement
- $T$ : disordered movement

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

- Corresponding stagnation temperature or total temperature:

$$T^* = T + \frac{U^2}{2c_p}$$

- Since,

$$c_p = \frac{\gamma}{\gamma - 1} R \quad c = \sqrt{\gamma R T}$$

$$\frac{T^*}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

➤ With the stagnation process under isentropic assumption:

$$\frac{P^*}{p} = \left( \frac{v}{v^*} \right)^\gamma = \left( \frac{T^*}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p^*}{p} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}}$$



## 2. Aerothermodynamics in Gas Turbine Engine

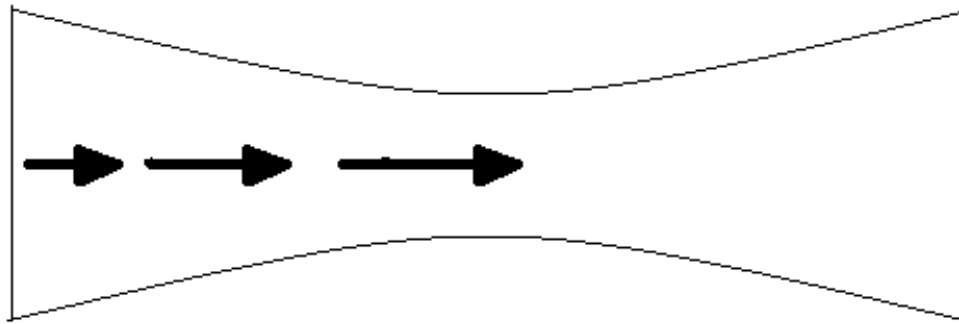
### The Stagnation Process

- According to the 3 equations above, for a given gas flow, the ratios of the total parameters and steady parameters are function of Mach number.
- When air flows **isentropically** in a tube without energy added, **the total parameters** (Enthalpy, temperature, pressure and density) remain **unchanged**.

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

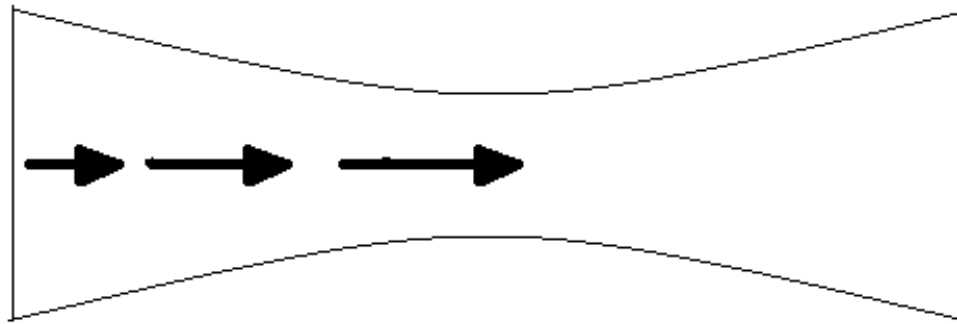
- Critical sound speed  $c_{cr}$  (in tunnel)



- $U$  increases along the tunnel (ex. Laval nozzle)
- When  $U$  increases,  $T$  decreases. Sound speed  $c$  is function of  $T$ , it goes down.

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process



- When  $U = c$ , ie  $M = 1$ , this  $c$  has special meaning, called **critical sound speed  $c_{cr}$** . This section is called critical section and it is the smallest section in the tunnel, also called **throat**.
- $c_{cr}$  does not change, even when  $c$  does.

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

- Definition of **speed coefficient** in a section

$$\lambda = \frac{U}{c_{cr}}$$

- Writing the stagnation state relation  $\frac{T^*}{T} = 1 + \frac{\gamma - 1}{2} M^2$  in

terms of the critical condition, we obtain

$$T_{cr} = \frac{2}{\gamma + 1} T^*$$

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

➤ Thus

$$T_{cr} = \frac{2}{\gamma + 1} T^*$$

$$c_{cr}^2 = \frac{2}{\gamma + 1} \gamma R T^*$$

$$c_{cr} = \sqrt{\frac{2}{\gamma + 1} \gamma R T^*}$$

$$\lambda^2 = \frac{\frac{\gamma + 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M^2}$$

$$M^2 = \frac{\frac{2}{\gamma + 1} \lambda^2}{1 - \frac{\gamma - 1}{\gamma + 1} \lambda^2}$$

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

- Therefore, the stagnation state equations are written in terms of the speed coefficient as,

$$\tau(\lambda) = \frac{T}{T^*} = \left( 1 - \frac{\gamma - 1}{\gamma + 1} \lambda^2 \right)$$

$$\pi(\lambda) = \frac{p}{p^*} = \left( 1 - \frac{\gamma - 1}{\gamma + 1} \lambda^2 \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\varepsilon(\lambda) = \frac{\rho}{\rho^*} = \left( 1 - \frac{\gamma - 1}{\gamma + 1} \lambda^2 \right)^{\frac{1}{\gamma - 1}}$$

## 2. Aerothermodynamics in Gas Turbine Engine

### The Stagnation Process

#### ➤ Flow density function

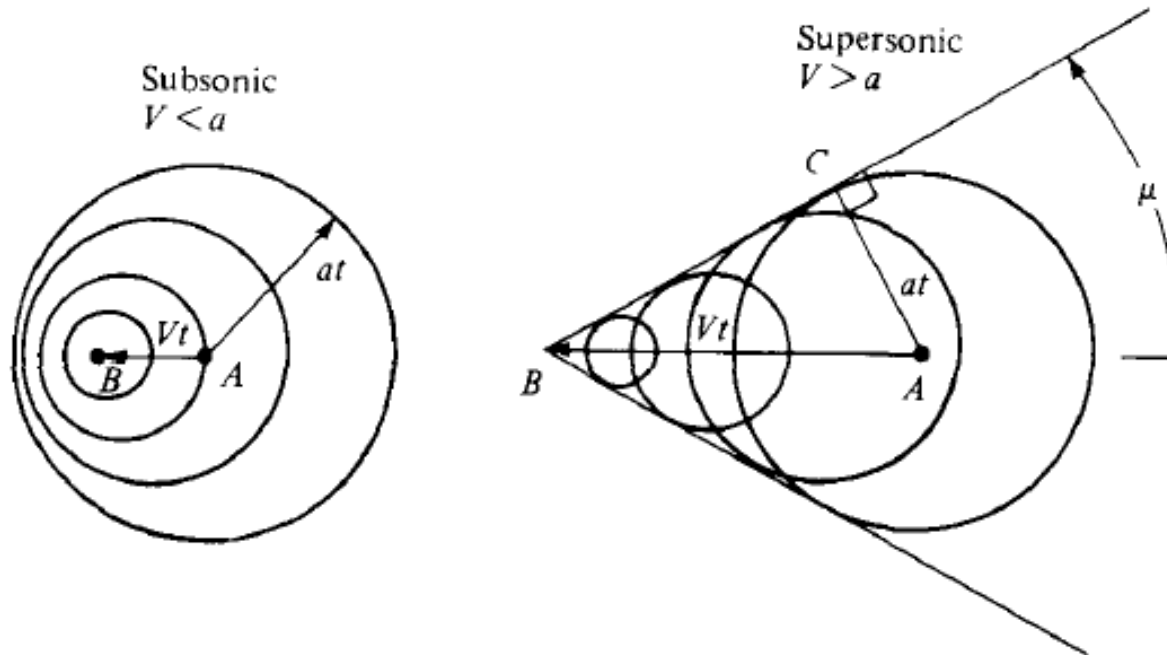
$$q_m = \rho v A$$

$$q(\lambda) = \frac{\rho v}{(\rho v)_{cr}} = \frac{A_{cr}}{A} = \left( \frac{\gamma + 1}{2} \right)^{\frac{1}{\gamma-1}} \lambda \left( 1 - \frac{\gamma-1}{\gamma+1} \lambda^2 \right)^{\frac{1}{\gamma-1}}$$

- $q(\lambda)$  presents relative flow density in section A to the critical section even though the critical section does not exist.
- Ratio of the cross-sections.

## 2. Aerothermodynamics in Gas Turbine Engine

### Shock Waves

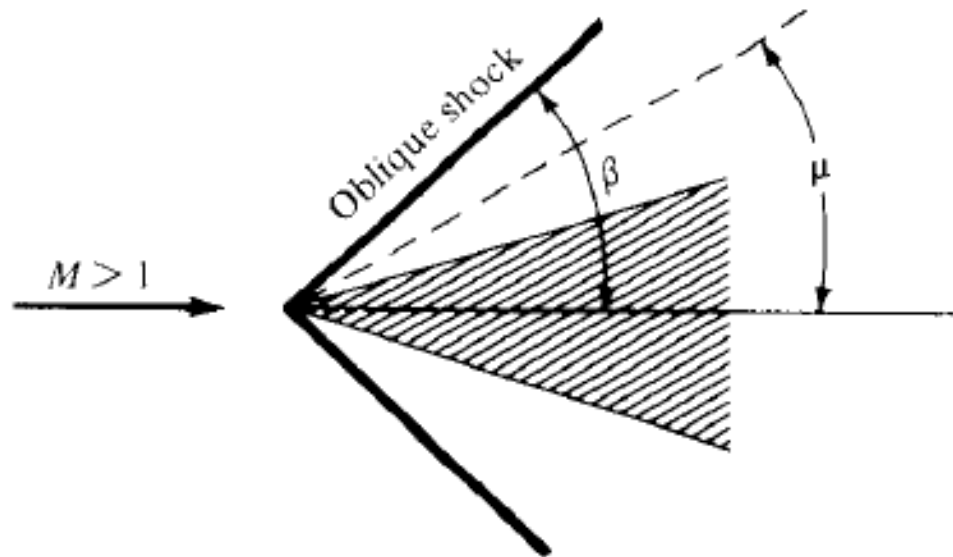


- If the disturbances are stronger than a simple sound wave, then the wave front becomes stronger than a Mach wave, creating an oblique shock wave at an angle  $\beta$  to the freestream, where  $\beta > \mu$ .



## 2. Aerothermodynamics in Gas Turbine Engine

### Shock Waves



- *The Mach wave is a limiting case for an oblique shock wave, i.e. it is an infinitely weak shock wave.*

## 2. Aerothermodynamics in Gas Turbine Engine

### Shock Waves

$$M_{n,1} = M_1 \sin \beta$$

$$M_2 = \frac{M_{n,2}}{\sin (\beta - \theta)}$$

$$M_{n,2}^2 = \frac{1 + [(\gamma - 1)/2]M_{n,1}^2}{\gamma M_{n,1}^2 - (\gamma - 1)/2}$$

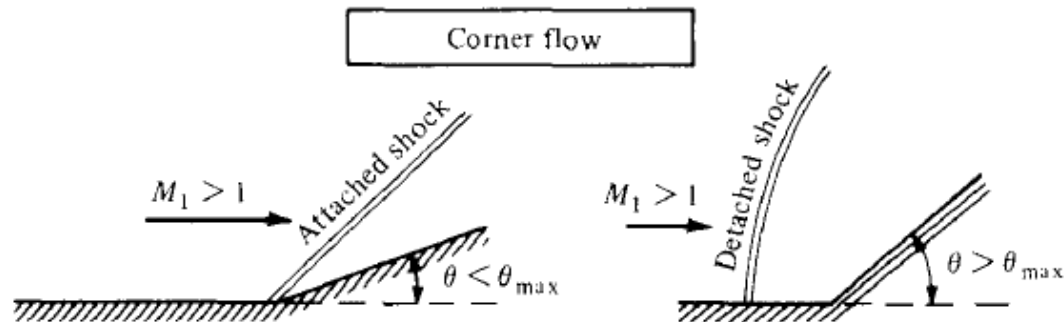
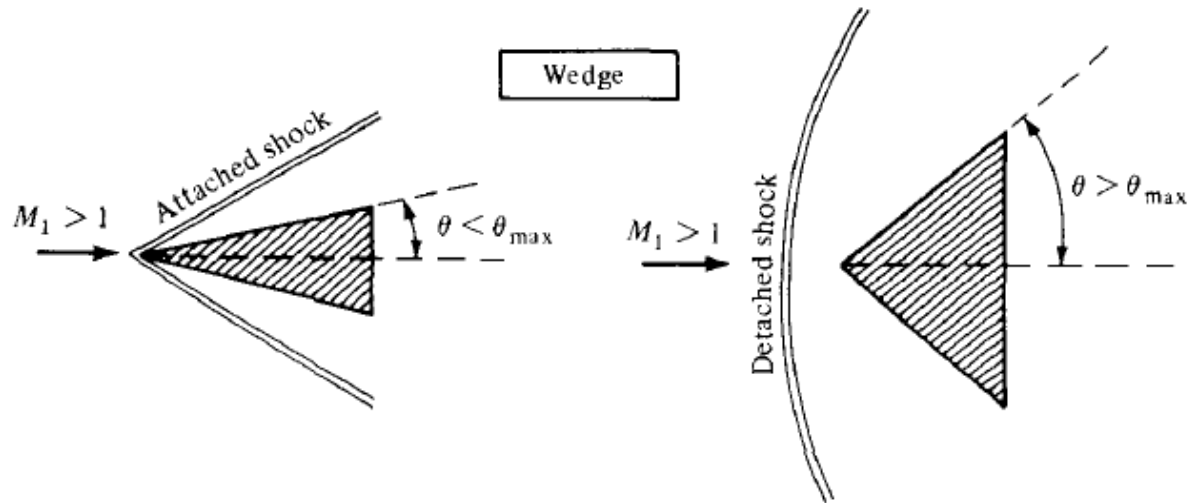
$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_{n,1}^2}{2 + (\gamma - 1)M_{n,1}^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_{n,1}^2 - 1)$$

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}$$

## 2. Aerothermodynamics in Gas Turbine Engine

### Shock Waves



## 2. Aerothermodynamics in Gas Turbine Engine

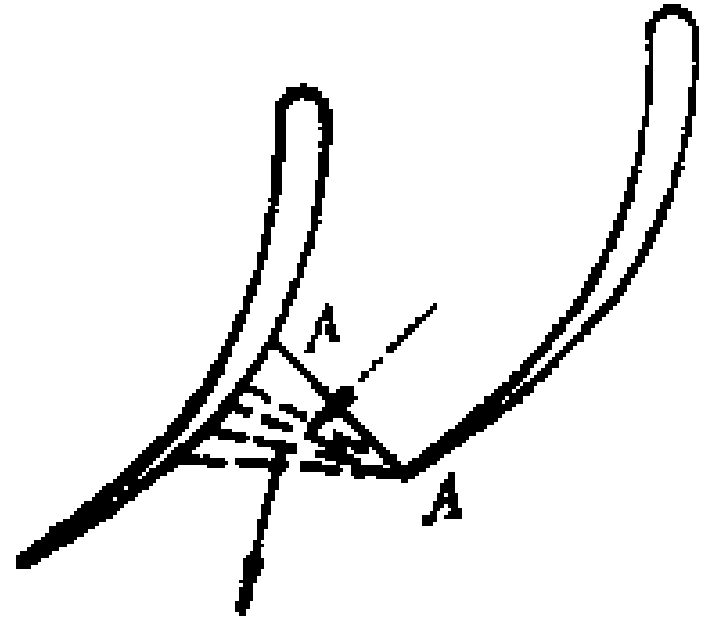
### The Shock and Expansion Waves

- Supersonic flow passing through the shock wave is not isentropic process. Partial mechanical energy Irreversibly changes to heat, and total pressure decreases.
- This is shock wave loss, and usually **total pressure recovery ( $\sigma$ )** is used to present the loss. It is function of the wave strength; the stronger the wave, the greater the loss.

## 2. Aerothermodynamics in Gas Turbine Engine

### The Shock and Expansion Waves

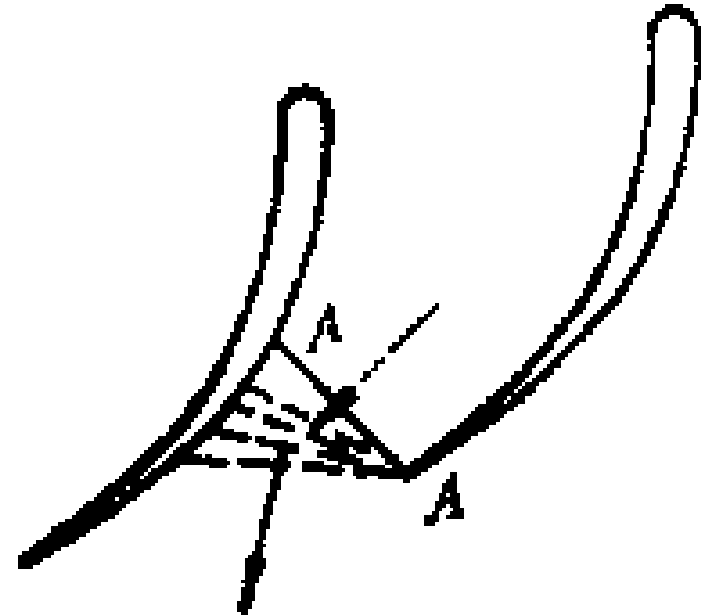
- When a supersonic air flows to a lower pressure zone, there are expansion waves due to air continuous expansion.
- In Fig, turbine cascade passage. In the throat A—A, critical section, flow becomes supersonic.
- In downstream, it is low pressure zone. The flow accelerates, it passes through a series of expansion waves, and speed increases, temperature and pressure decrease.



## 2. Aerothermodynamics in Gas Turbine Engine

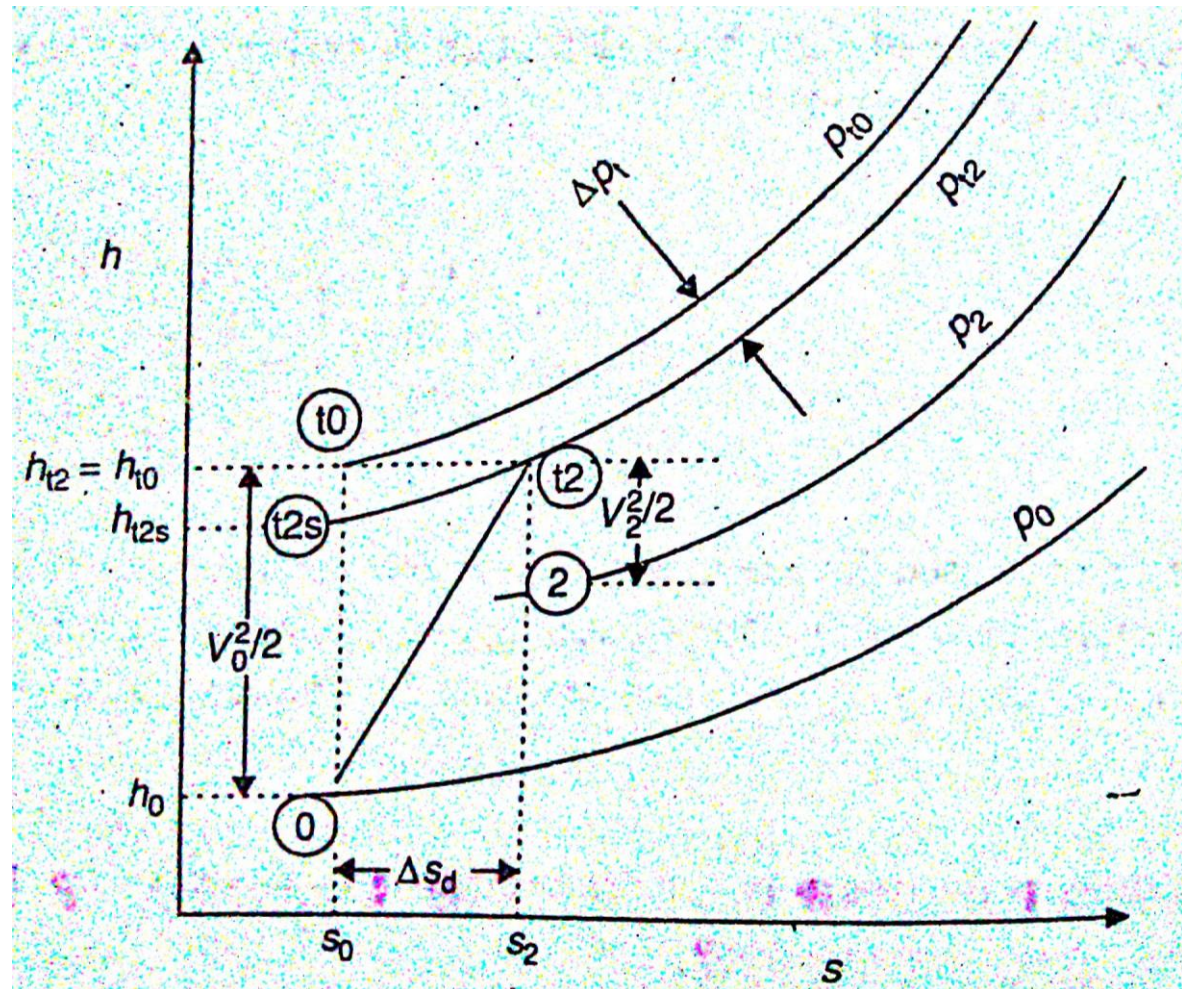
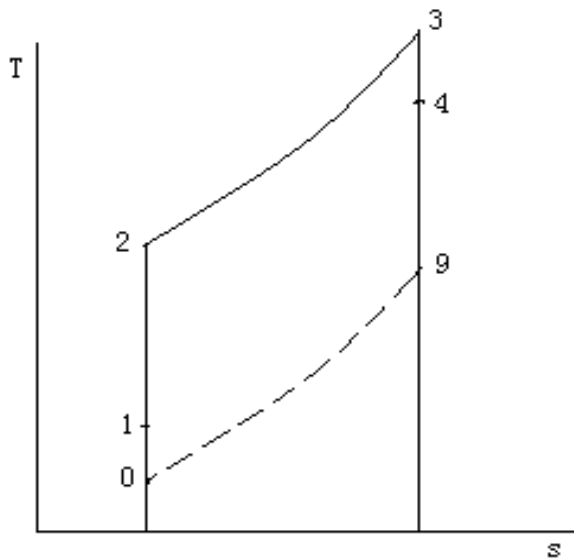
### The Shock and Expansion Waves

- The flow changes also the direction. The bigger the turned angle, the more expansion and flow parameters change more.
- The turned angle depends on exit pressure. The lower the pressure, the bigger the angle.
- If pressure increases, expansion waves may disappear and the flow may be subsonic.



# 2. Aerothermodynamics in Gas Turbine Engine

## Gas Turbine Cycle



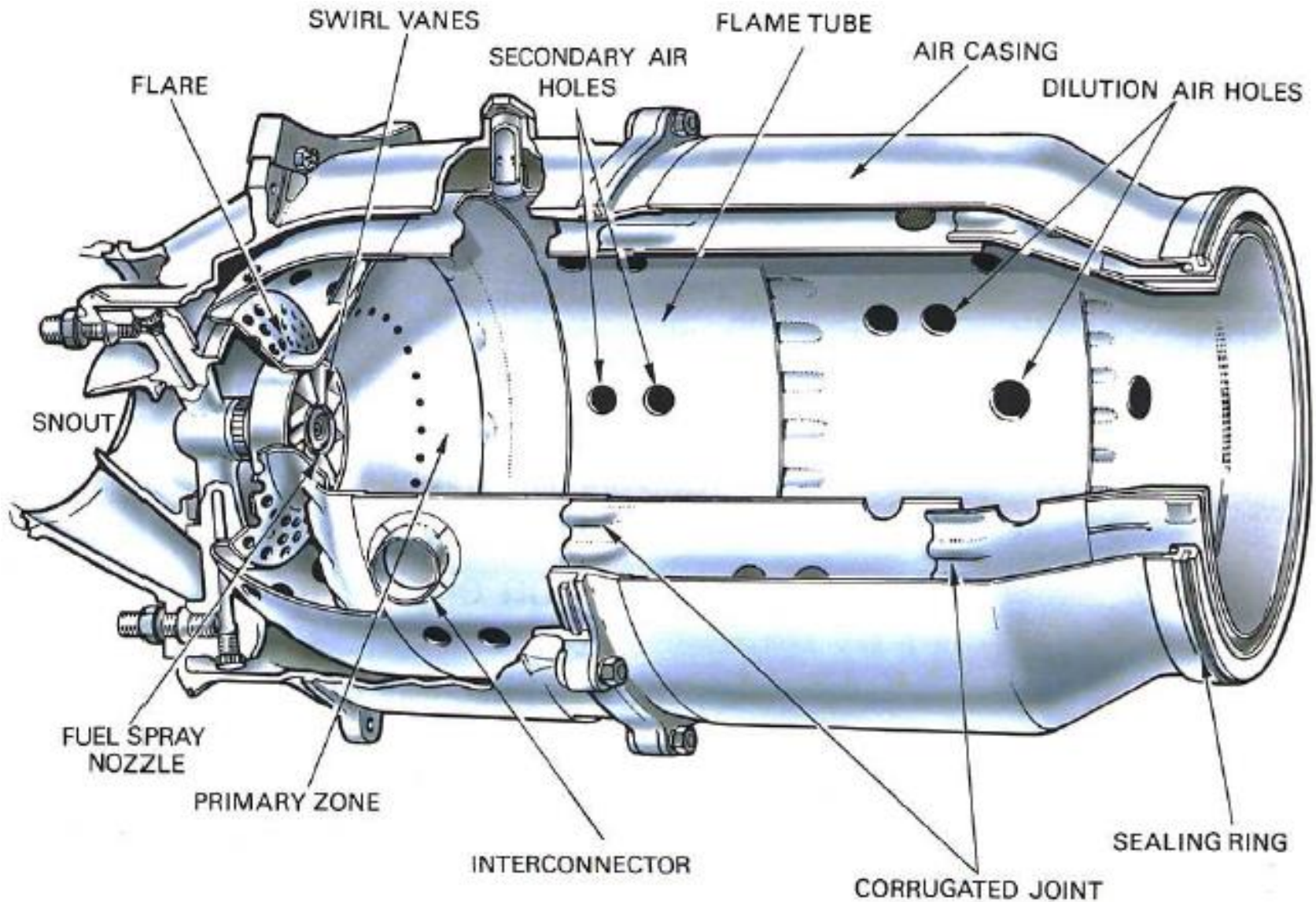
## 2. Aerothermodynamics in Gas Turbine Engine

### The Shock and Expansion Waves

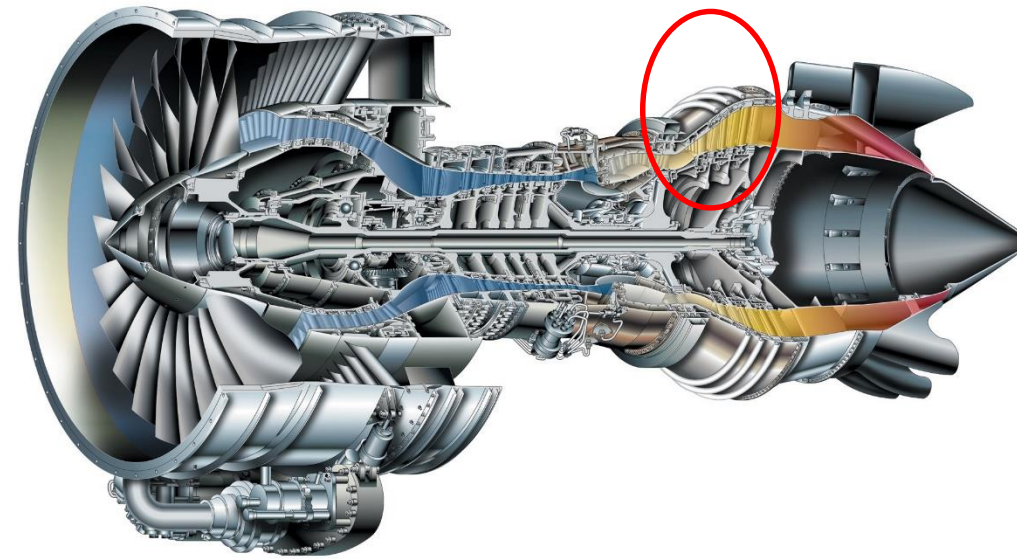
- When supersonic flow passes through the shock wave, sharply speed decreases, pressure and temperature increase.
- After normal wave, the flow is certainly subsonic. But after oblique shock wave, it is still supersonic.
- Strength of the shock wave is described by pressure ratio of after and before. It is only function of  $M$  for normal shock wave, the greater  $M$ , the stronger the wave.
- For oblique shock wave, the greater  $M$  and  $q$ , the stronger the wave.



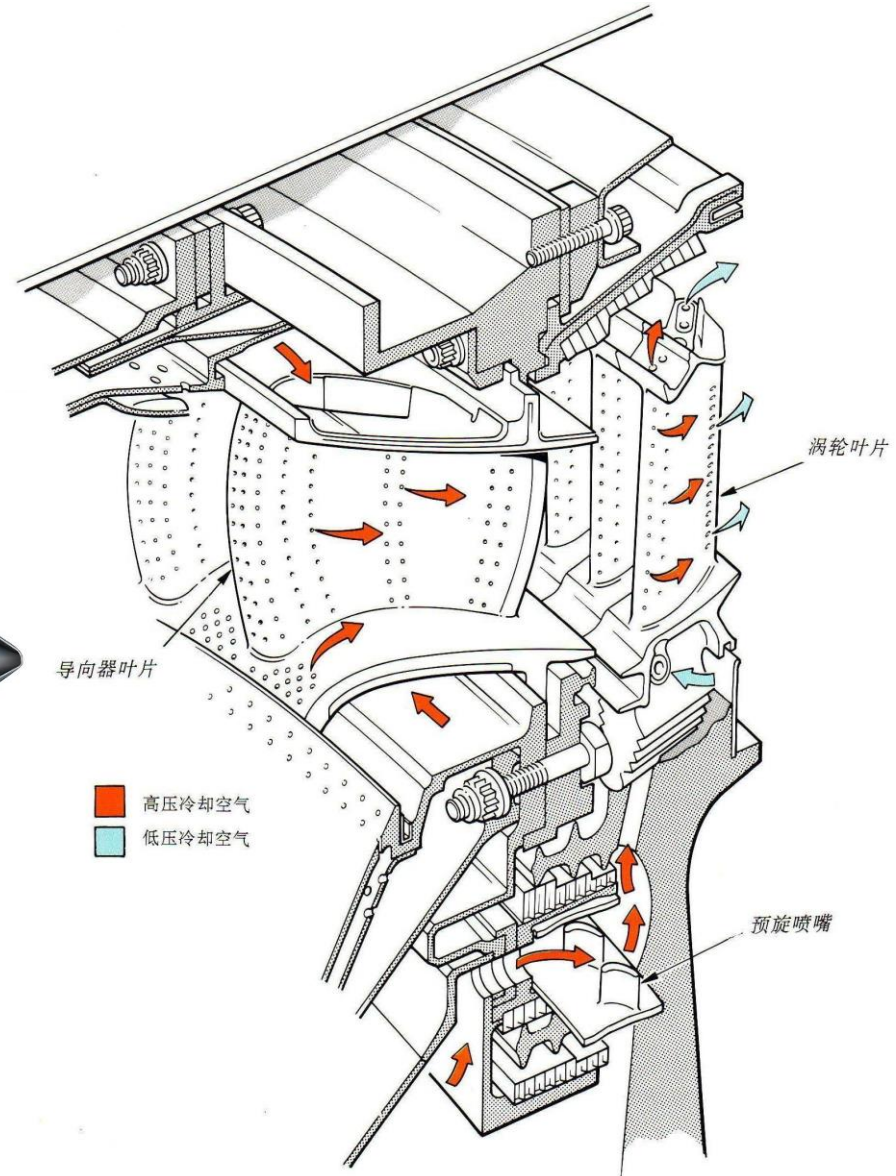
# Combustion chamber



# Turbine



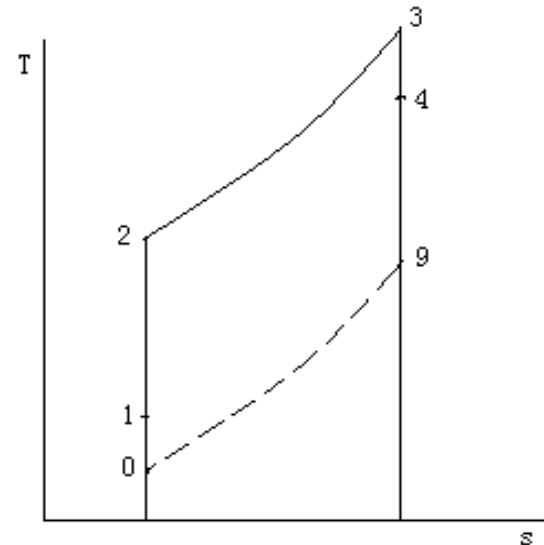
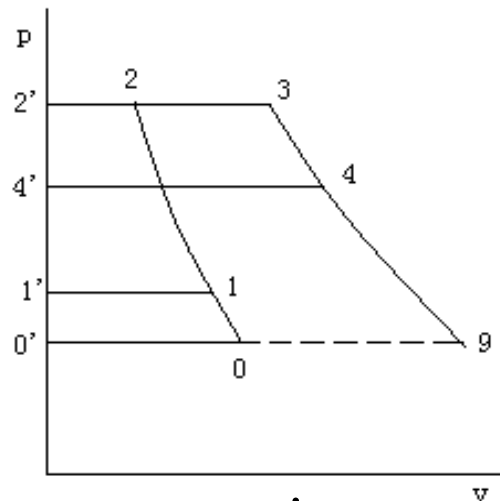
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# 2. Aerothermodynamics in Gas Turbine Engine

## Thermodynamic Cycle

### ➤ 1. Ideal cycle (Brayton cycle)



➤ 0-2 Isentropic compression

➤ 2-3 Isobar heating

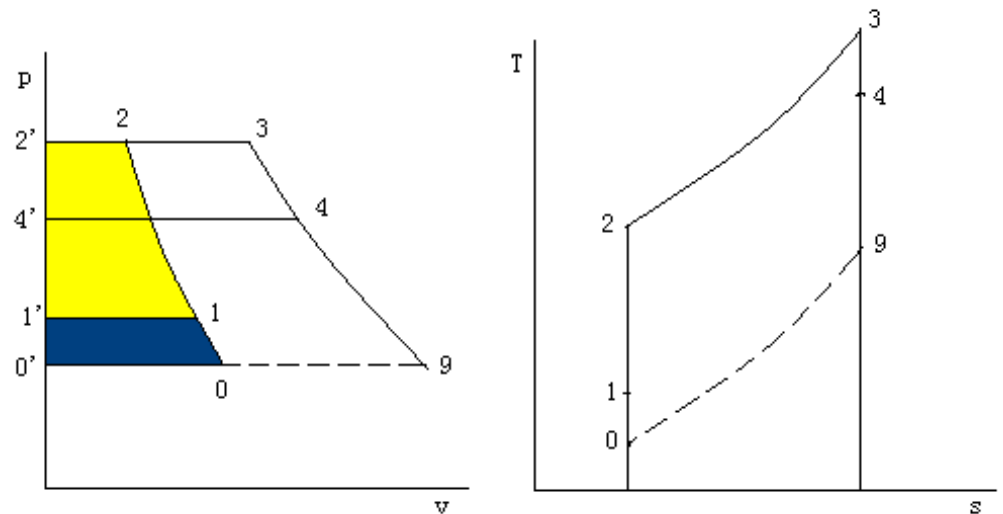
➤ 3-9 Isentropic expansion

➤ 9-0 Isobar cooling

# 2. Aerothermodynamics in Gas Turbine Engine

## Thermodynamic Cycle

### ➤ 0-2 Isentropic compression



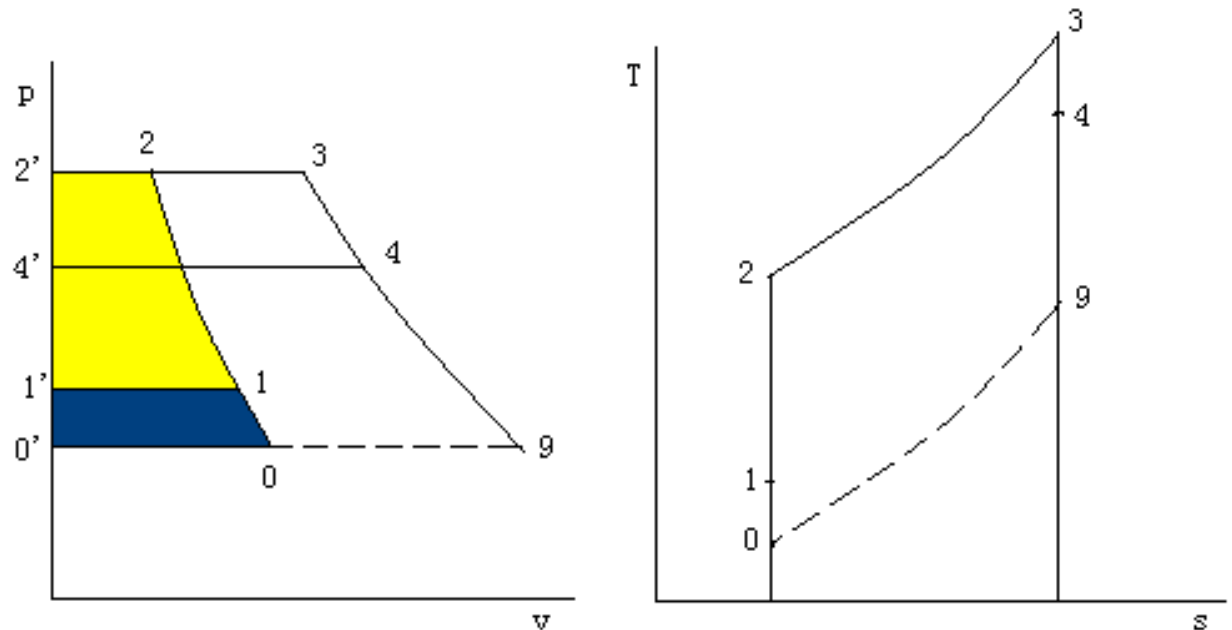
### ➤ Diffuser and compressor

➤ 0-1 speed pressure rise. 0 atmosphere condition. Add dynamic energy to substance and to increase pressure to 1. Area 011'0'0 presents dynamic energy difference.

## 2. Aerothermodynamics in Gas Turbine Engine

### Thermodynamic Cycle

- 0-2 Isentropic compression

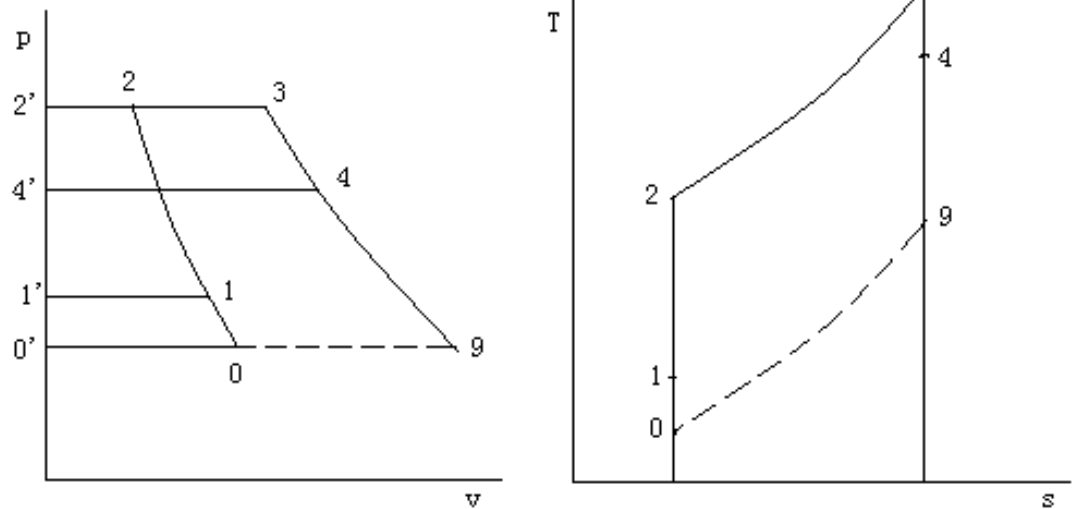


- 1-2: compressor. Pressure from 1 to 2, work added is the area 122'1'1.

# 2. Aerothermodynamics in Gas Turbine Engine

## Thermodynamic Cycle

### ➤ 2-3 Isobar heating



### ➤ Combustion chamber

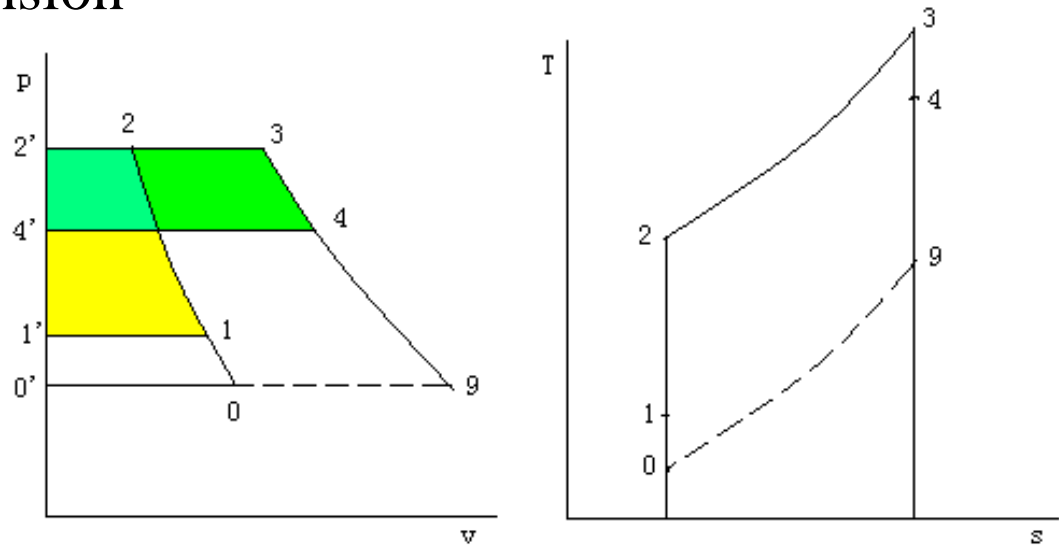
➤ Burn ideally kerosene at constant pressure in combustion chamber and substance properties unchanged.

➤ Total temperature  $T_2^* \Rightarrow T_3^*$  .

# 2. Aerothermodynamics in Gas Turbine Engine

## Thermodynamic Cycle

### ➤ 3-9 Isentropic expansion



### ➤ Turbine and nozzle

➤ 3-4 presents expansion in turbine, heat  $\Rightarrow$  mechanical energy giving to compressor. Area  $344'2'3 = \text{Area } 122'1'1$ , total pressure  $p_3^* \Rightarrow p_4^*$ .



# Thermodynamic Cycle

ension

The left diagram is a  $P-v$  (Pressure-volume) plot. The vertical axis is labeled  $P$  and the horizontal axis is labeled  $v$ . The cycle processes are shown as follows: 1-2 is a vertical line (isobaric compression), 2-3 is a horizontal line (isobaric expansion), 3-4 is a diagonal line (isentropic expansion), and 4-1 is a vertical line (isobaric compression). The cycle is shaded in green. The cycle processes 1'-2'-3'-4'-1' are shown as follows: 1'-2' is a vertical line (isobaric compression), 2'-3' is a horizontal line (isobaric expansion), 3'-4' is a diagonal line (isentropic expansion), and 4'-1' is a vertical line (isobaric compression). The cycle is shaded in yellow. The cycle processes 1-2-3-4-1 and 1'-2'-3'-4'-1' are shown. The cycle processes 1-2-3-4-1 and 1'-2'-3'-4'-1' are shown. The cycle processes 1-2-3-4-1 and 1'-2'-3'-4'-1' are shown.

The right diagram is a  $T-s$  (Temperature-entropy) plot. The vertical axis is labeled  $T$  and the horizontal axis is labeled  $s$ . The cycle processes are shown as follows: 1-2 is a vertical line (isobaric compression), 2-3 is a diagonal line (isentropic expansion), 3-4 is a vertical line (isobaric compression), and 4-1 is a diagonal line (isentropic expansion). The cycle is shaded in green. The cycle processes 1'-2'-3'-4'-1' are shown as follows: 1'-2' is a vertical line (isobaric compression), 2'-3' is a diagonal line (isentropic expansion), 3'-4' is a vertical line (isobaric compression), and 4'-1' is a diagonal line (isentropic expansion). The cycle is shaded in yellow. The cycle processes 1-2-3-4-1 and 1'-2'-3'-4'-1' are shown. The cycle processes 1-2-3-4-1 and 1'-2'-3'-4'-1' are shown. The cycle processes 1-2-3-4-1 and 1'-2'-3'-4'-1' are shown.

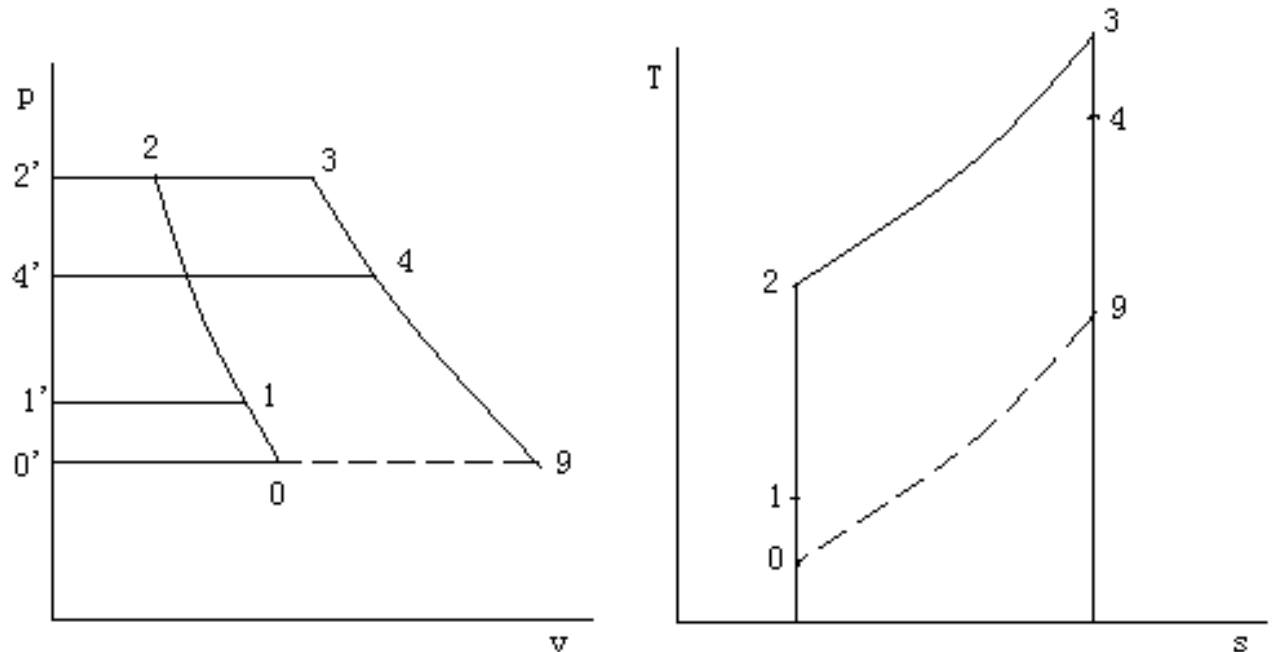
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## 2. Aerothermodynamics in Gas Turbine Engine

### Thermodynamic Cycle

➤ 9-0 Isobar heat release



➤ Dash line, accomplished in atmosphere. This process is unavoidable, The cycle is closed.

## 2. Aerothermodynamics in Gas Turbine Engine

### Thermodynamic Cycle

- Specific heat added in the cycle

$$q_1 = c_p (T_3^* - T_2^*)$$

- Specific heat lost to the surrounding

$$q_2 = c_p (T_9 - T_0)$$

- Specific work in the cycle

$$W = q_1 - q_2$$

## 2. Aerothermodynamics in Gas Turbine Engine

### Thermodynamic Cycle

- Thermal efficiency in the cycle

$$\eta_t = \frac{W}{q_1} = \frac{q_1 - q_2}{q_1} = 1 - \frac{1}{\pi^{\frac{\gamma-1}{\gamma}}}$$

- Where pressure ratio

$$\pi = \frac{p_2^*}{p_0}$$

## 2. Aerothermodynamics in Gas Turbine Engine

### Thermodynamic Cycle

- Cycle work as mechanical energy

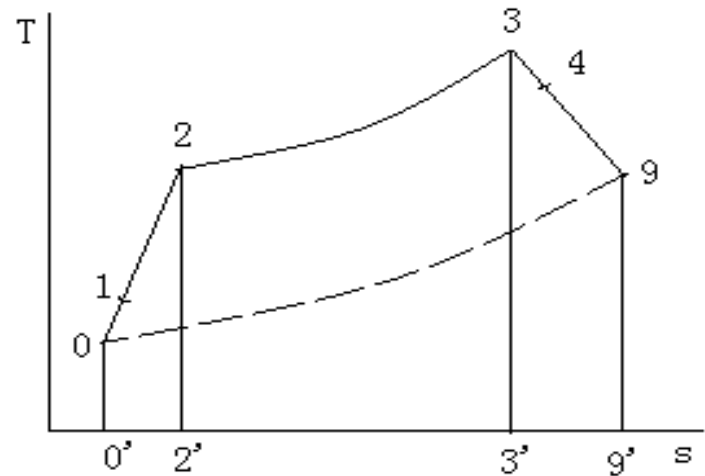
$$W = W_T + \frac{v_9^2}{2} - W_C - \frac{v_0^2}{2}$$

- if  $W_T = W_C$

$$W = \frac{v_9^2 - v_0^2}{2}$$

# 2. Aerothermodynamics in Gas Turbine Engine

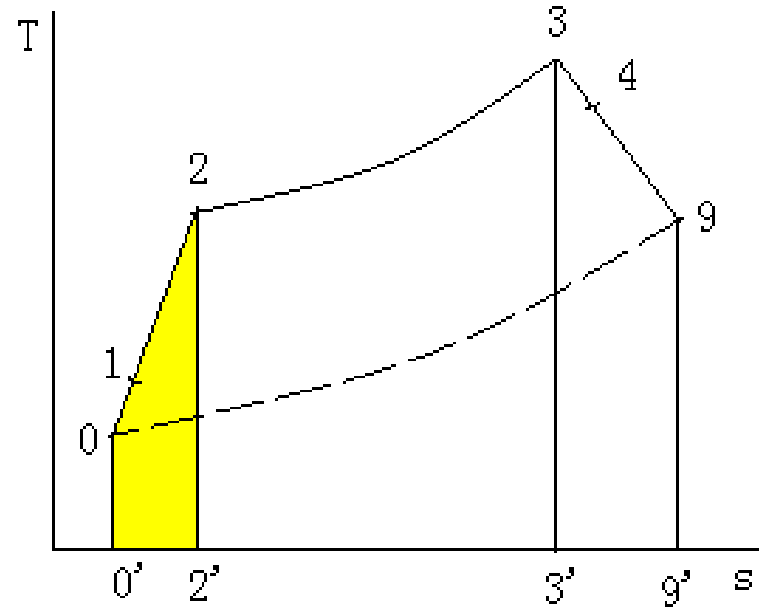
## Actual Thermodynamic Cycle



- 0-2 Compression (non isentropic)
- 2-3 Heating (non isobar)
- 3-9 Expansion (non isentropic)
- 9-0 Isobar heat release

## 2. Aerothermodynamics in Gas Turbine Engine

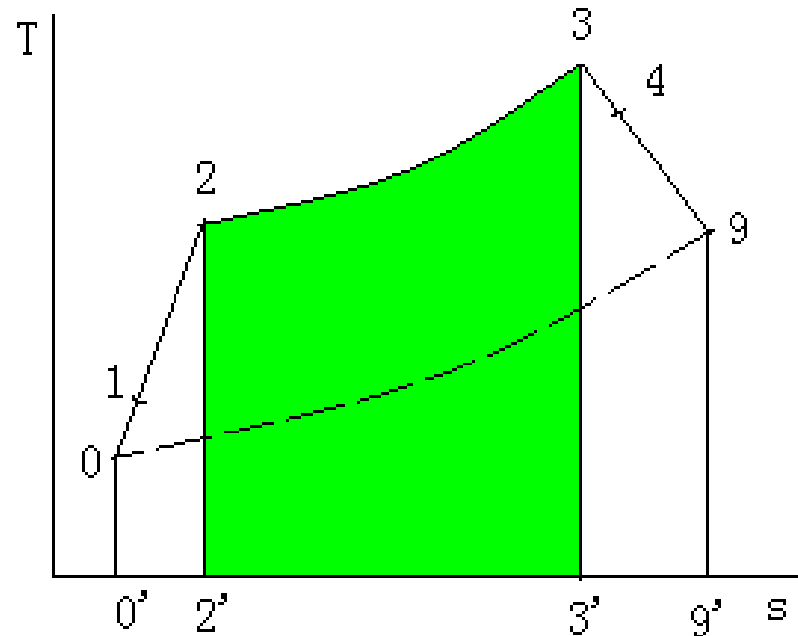
### Actual Thermodynamic Cycle



- 0-2 Compression
  - Stagnation in diffuser and compression in compressor suffer from many types of losses.
  - Non isentropic

## 2. Aerothermodynamics in Gas Turbine Engine

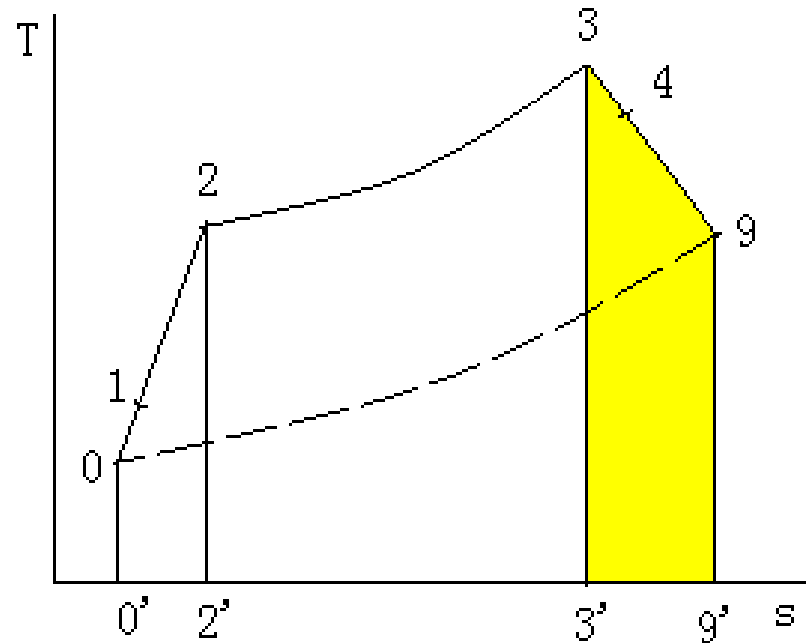
## Actual Thermodynamic Cycle



- 2-3 Non isobar combustion
  - Existing flow losses and thermal resistance losses lowers the pressure in combustion chamber.
  - Composition of substance changes.

## 2. Aerothermodynamics in Gas Turbine Engine

### Actual Thermodynamic Cycle



- 3-9 Expansion
  - There are always losses in turbine and nozzle.
  - Non isentropic



# 2. Aerothermodynamics in Gas Turbine Engine

## Actual Thermodynamic Cycle

➤ Heat added:

➤ Area 2'233'2'

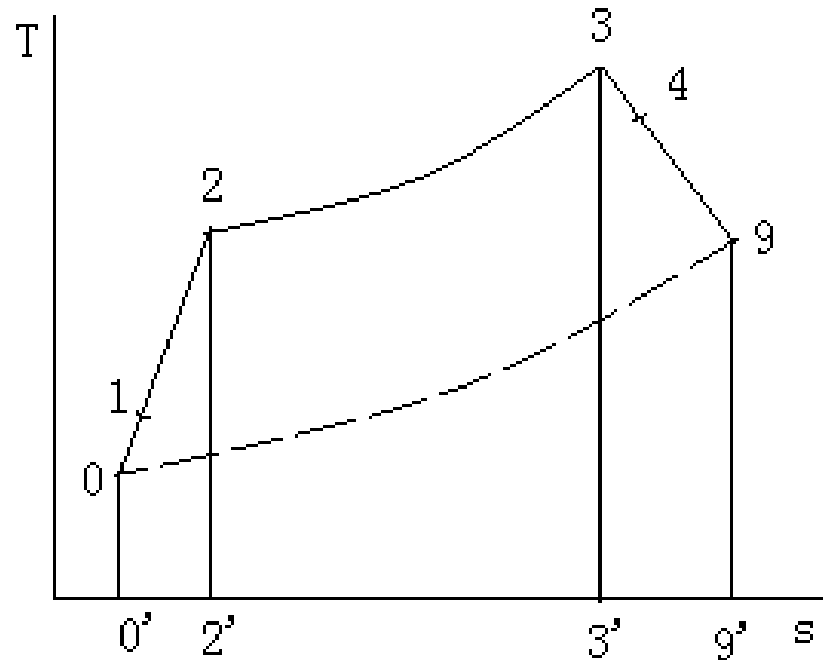
$$q_1 = c_p (T_3^* - T_2^*)$$

➤ Heat released

➤ Area 0'099'0'

$$q_2 = c_p (T_9 - T_0)$$

➤  $c_p$  gas specific heat



## 2. Aerothermodynamics in Gas Turbine Engine

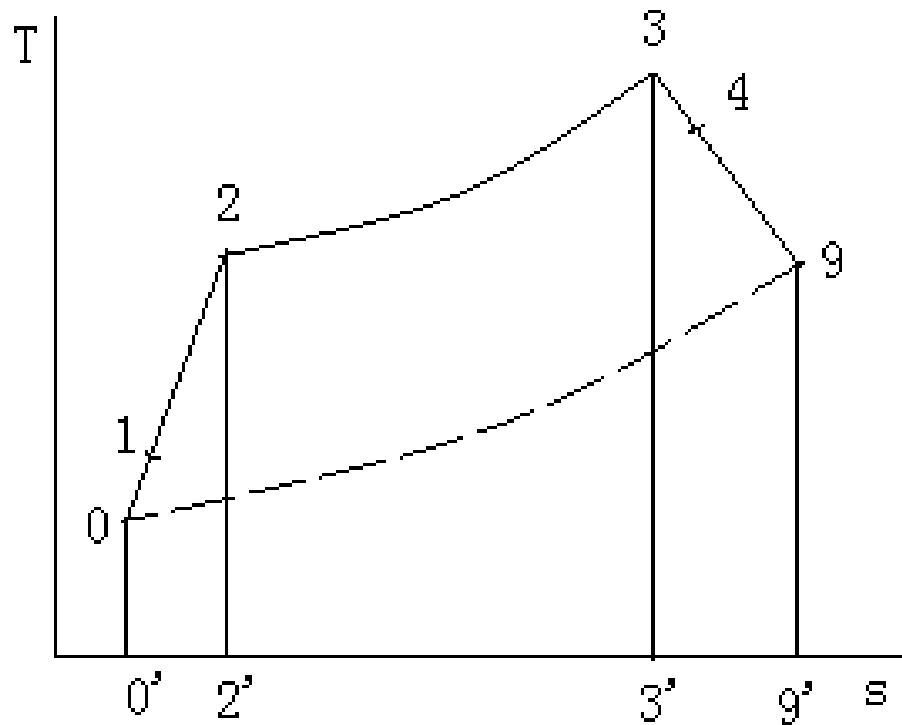
### Actual Thermodynamic Cycle

➤ Efficiency

$$\eta_t = \frac{q_1 - q_2}{q_1}$$

➤ Work

$$W = q_1 - q_2$$

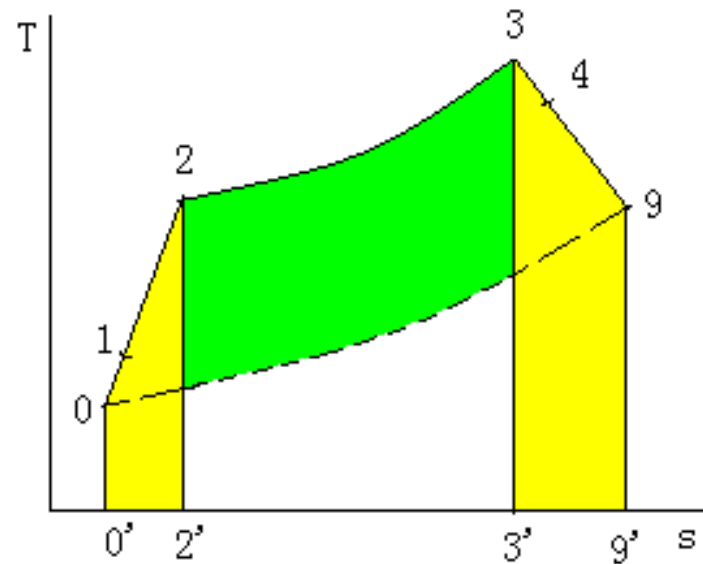


## 2. Aerothermodynamics in Gas Turbine Engine

### Actual Thermodynamic Cycle

- If  $T_3^*$  lower,  $q_1=q_2$ , then  $\eta_t=0$ , no output work.
- Work can be presented by mechanical energy, same as ideal cycle:

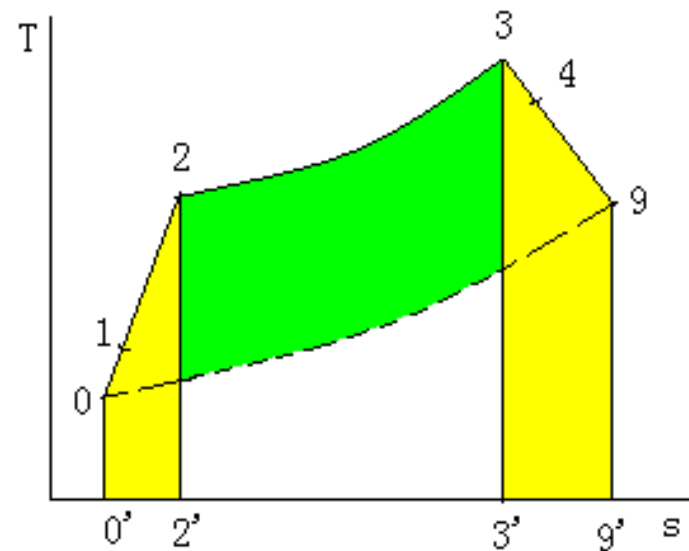
$$W = \frac{v_9^2 - v_0^2}{2}$$



## 2. Aerothermodynamics in Gas Turbine Engine

### Actual Thermodynamic Cycle

- Under the same pressure ratio  $p$  and the same  $T_3^*$ , work is smaller in real cycle than ideal cycle.
- Note that area in diagram T-s is heat, not work.



## 2. Aerothermodynamics in Gas Turbine Engine

### **Actual Thermodynamic Cycle**

- If not take into account the change in and increase in mass flow, differences between real cycle and ideal cycle are:
  - Friction and flow losses
  - Total pressure loss
  - Heating resistance
- If nozzle gas kinetic energy is smaller, velocity of air jet is smaller.
- To improve the engine's efficiency, components with high efficiency and performance should be used.



