THEORY OF MACHINE AND MECHANISM II

TUTORIAL NO: 6(A)

VIBRATION OF SINGLE DEGREE OF FREEDOM SYSTEMS

(Undamped Free Vibration)

1. A heavy table is supported by flat steel legs. Its natural period in horizontal motion is 0.4 s. When a 30 kg plate is clamped to its surface, the natural period in horizontal motion is increased to 0.5 s. Determine the effective spring constant and the mass of the table.

2. The period of vibration of the system shown in **Figure P6(A).2** is observed to be 0.7 s. After the spring of constant k_2 = 1.2 kN/m is removed from the system, the period is observed to be 0.9 s. Determine
   (a) the constant k_1 of the remaining spring and
   (b) the mass of block A.

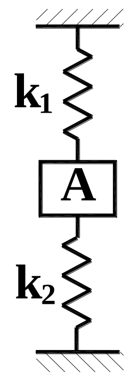

Figure P6(A).2

3. A helical spring, when fixed at one end and loaded at the other, requires a force of 100 N to produce an elongation of 10 mm. The ends of the spring are now rigidly fixed, one end vertically above the other, and a mass of 10 kg is attached at the middle point of its length. Determine the time taken to complete one vibration cycle when the mass is set vibrating in the vertical direction.

4. A light cantilever of rectangular section (5 cm deep and 2.5 cm wide) has a mass fixed at its free end. Find the ratio of the frequency of the lateral vibration in vertical plane to that in the horizontal plane.

5. Determine the natural frequency of the system of **Figure P6(A).5**.

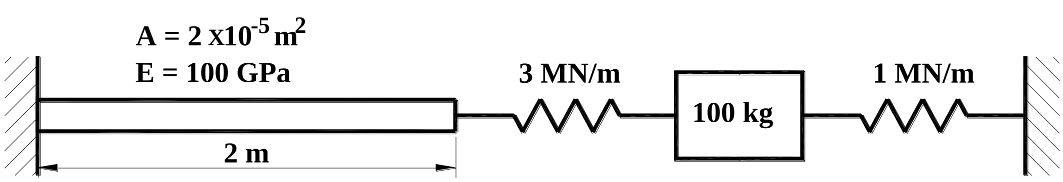

Figure P6(A).5

6. Determine the natural frequency of the mass M for the system shown in **Figure P6(A).6**. Neglect the mass of the cantilever beam.

7. A machine of mass m = 500 kg is mounted on a simply supported steel beam of length l = 2 m having a rectangular cross section (depth = 0.1 m, width = 1.2 m) and Young's modulus E = 2.06×10^{11} N/m^2. To reduce the vertical deflection of the beam, a spring of stiffness k is attached at the mid span as shown in **Figure P6(A).7**. Determine the value of k needed to reduce the deflection of the beam by 25 percent of its original value.

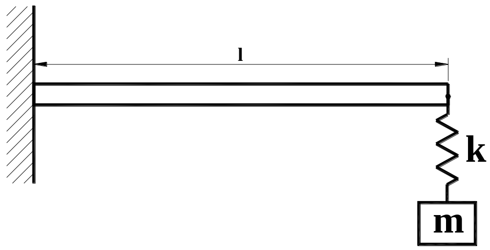

Figure P6(A).6

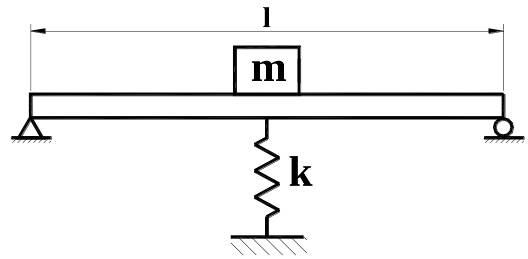

Figure P6(A).7

8. Determine the natural frequency of the system shown in **Figure P6(A).8**. The bar is assumed mass less and the lower spring is attached to the bar midway between the points of attachment of the upper springs. The spring constants are as shown.

9. A small mass m is fastened to a vertical wire which is under tension T as shown in **Figure P6(A).9**. What will be the natural frequency of vibration of the mass if it is displaced laterally a slight distance and then released?

10. A cantilever beam carries a mass M at the free end as shown in **Figure P6(A).10**. A mass m falls from a height h on to the mass M to it without rebounding. Determine the resulting transverse vibration of the beam.

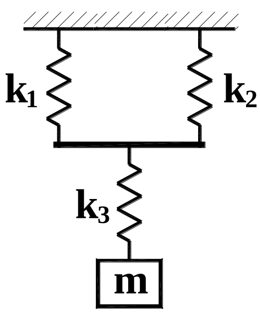

Figure P6(A).8

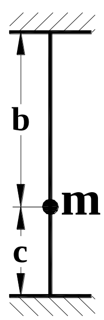

Figure P6(A).9

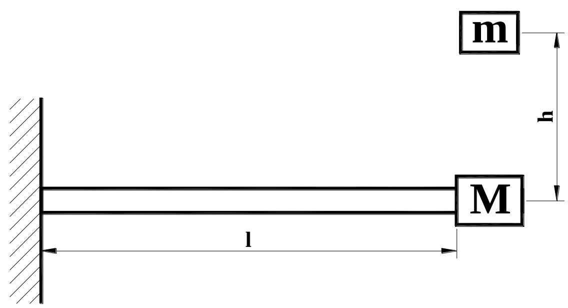

Figure P6(A).10

ANSWERS

1. 53.33 kg, 13159.5 N/m
2. 1.838 kN/m, 37.7 kg
3. 0.0993 s
4. 2:1
5. 132.3 rad/s
6. $\sqrt{\dfrac{3EIk}{(3EI + kl^3)m}}$
7. 41.2 MN/m
8. $\sqrt{\dfrac{4k_1k_2k_3}{m(4k_1k_2 + k_1k_3 + k_2k_3)}}$

9. $\sqrt{\dfrac{T}{m}\left(\dfrac{b+c}{bc}\right)}$

10. x = $\dfrac{\dot{x}(0)}{\omega_n}$ sin ω_nt + x(0) cos ω_nt

where, x(0) = $-\dfrac{mgl^3}{3EI}$, $\dot{x}(0)$ = $\left(\dfrac{m}{M+m}\right)\sqrt{2gh}$, ω_n = $\sqrt{\dfrac{3EI}{l^3(M+m)}}$