THEORY OF MACHINE AND MECHANISM II

TUTORIAL NO: 7

VIBRATION OF TWO DEGREE OF FREEDOM SYSTEMS

1. Write the equations of motion for the system shown in **Figure P7.1** and determine its natural frequencies and mode shapes.

2. Determine the natural frequencies for the two mass system shown in **Figure P7.2**. Both masses move only vertically.

3. Determine the free vibration of the system shown in **Figure P7.3** when the initial conditions are $x_1(0) = 5$, $\dot{x}_1(0) = 0$; $x_2(0) = 0$, $\dot{x}_2(0) = 0$. Take $k_1 = k_3 = k$; $k_2 = 2k$ and $m_1 = m_2 = m$.

4. Determine the natural frequencies for the system shown in **Figure P7.4**, $m_1 = 1$ kg, $m_2 = 2$ kg, $k_1 = 2000$ N/m, $k_2 = 1000$ N/m, $k_3 = 3000$ N/m.

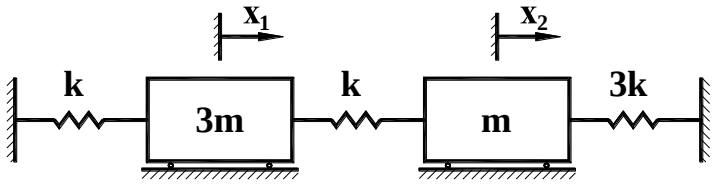

Figure P7.1

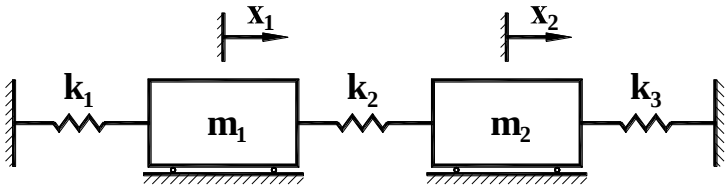

Figure P7.3/P7.4

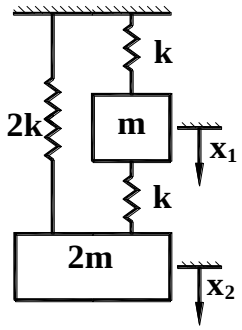

Figure P7.2

5. Determine the natural frequencies of vibrations and the ratio of amplitudes of motion for the system shown in **Figure P7.5**. Given $m_1 = 10$ kg, $m_2 = 15$ kg, and $k = 320$ N/m.

6. Derive the equation of motion of the vibratory system shown in **Figure P7.6**. Determine the natural frequencies for given data $k_1 = 98000$ N/m, $k_2 = 19600$ N/m; $m_1 = 196$ kg, $m_2 = 49$ kg.

7. **Figure P7.7** shows an overhead crane schematically. The cabin is at the center of the beam of length l_1. Reduce the system to an equivalent two degrees of freedom system and determine the natural frequencies. Assume $EI = 21 \times 10^6$ Nm^2, $m_1 = 3000$ kg, $l_1 = 5$ m, $EA = 82.47 \times 10^6$ N, $m_2 = 700$ kg, $l_2 = 6$ m.

8. Determine the two natural frequencies and mode shapes for the system shown in **Figure P7.8**. The string is stretched with a large tension T.

9. In the system shown in **Figure P7.9**, the mass m_1 is excited by a harmonic force having a maximum value of 50 N and a frequency of 2 Hz. Find the forced amplitude of each mass for $m_1 = 10$ kg, $m_2 = 5$ kg, $k_1 = 8000$ N/m, and $k_2 = 2000$ N/m.

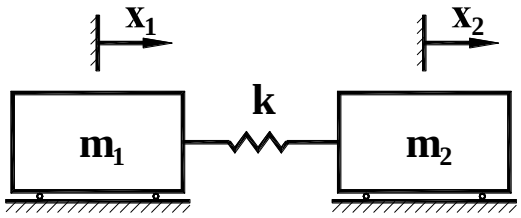

Figure P7.5

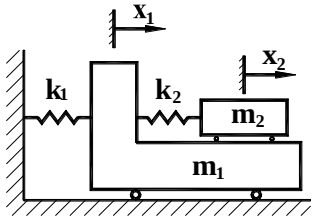

Figure P7.6

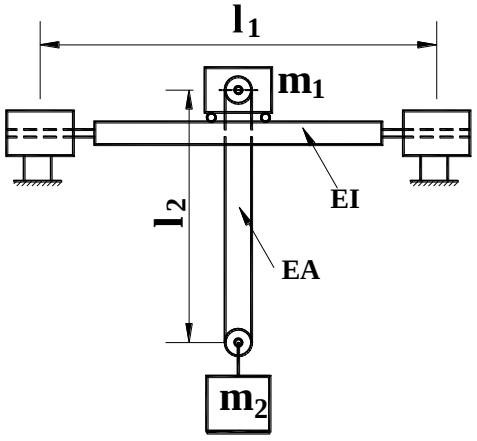

Figure P7.7

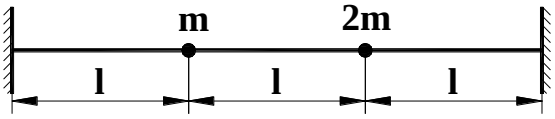

Figure P7.8

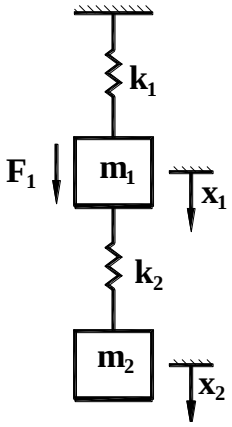

Figure 7.9

10. A reciprocating engine has a mass of 40 kg and runs at a constant speed of 3000 rpm. After it was installed it vibrated with a large amplitude at operating speed. What dynamic vibration absorber should be coupled to the system if the nearest resonant frequency of the combined system has to be at least 25 % away from the operating speed?

11. Write a computer program to take inputs of mass, stiffness, initial velocity and initial displacement for a two degree of freedom system from the user and gives output for the natural frequency and response plot.

ANSWERS

1. $0.5695\frac{k}{m}$, $4.0972\frac{k}{m}$; 3.43, – 0.0972

2. $\frac{k}{m}$, $2.5\frac{k}{m}$

3. $2.5\cos\sqrt{\frac{k}{m}}\,t + 2.5\cos\sqrt{\frac{3k}{m}}\,t$,

$2.5\cos\sqrt{\frac{k}{m}}\,t - 2.5\cos\sqrt{\frac{3k}{m}}\,t$

4. 40.42 rad/s, 58.01 rad/s

5. $\omega_1 = 0$, $\omega_2 = 7.30 rad/s$; 1, – 1.5

6. 16.6 rad/s, 26.9 rad/s

7. 46.42 rad/s, 221.28 rad/s

8. $\frac{T}{ml}\left(\frac{3\pm\sqrt{3}}{2}\right)$; $-1\pm\sqrt{3}$

9. 9.77 mm, 16.15 mm

10. 13.67 kg, 1340 kN/m