Growing money on Trees & Graphs

Stock market analysis using Multidimensional Scaling

Ravee Malla

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Motivation

- Economics is too serious a subject to be left to *economists!*
- Identify trends and clusters in the market
- Study time evolution of the market
- Investor risk reduction
- Predict future trends
- Ease of visualization

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- **Portfolio:** A set $\mathcal{P} = \langle \mathcal{B}, p, r, \mathcal{N}, \tau \rangle$ where $\mathcal{B} \subset \mathcal{C}$, $\mathcal{N} : \mathcal{B} \to \mathbb{Z}_+$

Problem Overview

- **Portfolio Optimization:** Design a portfolio $\mathcal{P}$ s.t.

$$\begin{aligned} \underset{\mathcal{B} \subset \mathcal{C}}{\text{maximize}} \quad & \sum_{b \in \mathcal{B}} r(p, b, \tau) * N(b) \\ \text{subject to} \quad & \sum_{b \in \mathcal{B}} p(b, \tau) * N(b) \leq \Gamma \end{aligned} \tag{1}$$

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- **Create a tool to help analyze the market and make better investment decisions**

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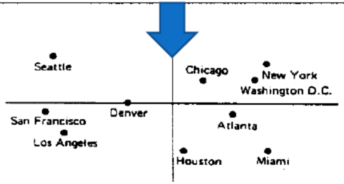
Table 1 Flying Mileages Between 10 American Cities

Atlanta	Chicago	Denver	Houston	Los Angeles	Miami	New York	San Francisco	Seattle	Washington, DC	
0	587	1212	701	1936	604	748	2139	2182	543	Atlanta
587	0	920	940	1745	1188	713	1858	1737	597	Chicago
1212	920	0	879	831	1726	1631	949	1021	1494	Denver
701	940	879	0	1374	968	1420	1645	1891	1220	Houston
1936	1745	831	1374	0	2339	2451	347	959	2300	Los Angeles
604	1188	1726	968	2339	0	1092	2594	2734	923	Miami
748	713	1631	1420	2451	1092	0	2571	2408	205	New York
2139	1858	949	1645	347	2594	2571	0	678	2442	San Francisco
2182	1737	1021	1891	959	2734	2408	678	0	2329	Seattle
543	597	1494	1220	2300	923	205	2442	2329	0	Washington, DC



A map of the 10 cities

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- $\rho : \mathcal{C} \times \mathcal{C} \times T \rightarrow [-1, 1] \equiv \rho^{\tau}_{i,j} \rightarrow \frac{\langle r_i^{\tau} r_j^{\tau} \rangle - \langle r_i^{\tau} \rangle \langle r_j^{\tau} \rangle}{\sqrt{[\langle {r_i^{\tau}}^2 \rangle - \langle r_i^{\tau} \rangle^2][\langle {r_j^{\tau}}^2 \rangle - \langle r_j^{\tau} \rangle^2]}}$

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- Why?
  - Globally high ρ implies that market is generally correlated, so it is either in a recession or boom phase
  - Treasure trove of rich probability theory applicable
  - Easy to compute and analyze

Metric on the market

- Given the correlation measure ρ, define $d_{ij} = \sqrt{2(1 - \rho_{ij})}$
- Properties preserved by $d, \forall i, j, k$
  - $d_{ij} \geq 0$
  - $d_{ii} = 0$
  - $d_{ij} < d_{ik} + d_{kj}$
  - $d_{ij} = d_{ji}$
  - Ultrametricity: Mapping preserves linearity

Using MDS

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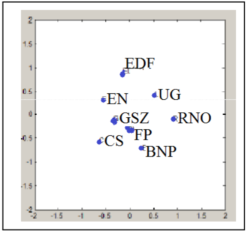

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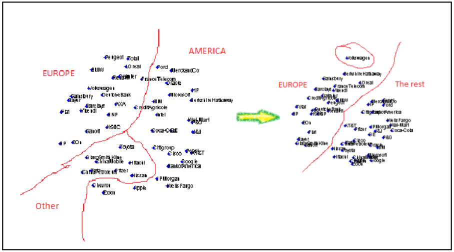
- Easy, apply MDS algorithm to d^τ for successive τ
- **Critical Flaw!** MDS algorithm assumes random initialization, so successive maps will change dramatically.
- Add another component to S penalizing changes in assigned coordinates
- $S^\tau(X_1^\tau, \ldots, X_N^\tau) = \sum_{i \neq j} \sqrt{{d_{i,j}^\tau}^2 - \|X_i^\tau - X_j^\tau\|^2} + \sum_i w_i * \|X_i^\tau - X_i^{\tau-1}\|$

Subprime crisis 2008

- 3 individual markets got correlated & came together due to a recession
- Can define global market score looking at the maps and learn it to predict future recession



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- Center of mass $v_m = \{v | v \in V, l(\tau, v) \text{ is min}\}$

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Conclusions

Thanks

Questions or Comments?