

A Jacobian-free Newton-Krylov method for cell-centred finite volume solid mechanics

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Abstract

This work extends the application of Jacobian-free Newton-Krylov (JFNK) methods to higher-order cell-centred finite-volume formulations for solid mechanics. While conventional cell-centred schemes are typically restricted to second-order accuracy, we present third- and fourth-order formulations that employ local least-squares reconstructions for gradient evaluation combined with Gaussian quadrature at cell faces for flux integration. These schemes enable accurate resolution of complex stress and deformation fields in both linear and nonlinear solid mechanics, while retaining the flexibility and geometric generality of finite-volume methods. A novel contribution of this study is the development and assessment of a JFNK solution strategy for these higher-order formulations, which eliminates the need to assemble and store large Jacobian matrices explicitly. Instead, we demonstrate that a compact-stencil approximate Jacobian can be effectively employed as a preconditioner, mirroring the efficiency gains observed in second-order frameworks. The proposed methodology is benchmarked across a suite of two- and three-dimensional test problems involving elastic and nonlinear materials, where key performance metrics, including accuracy, computational cost, memory usage, and robustness, are systematically evaluated. Results confirm that the higher-order formulations deliver substantial accuracy improvements over second-order schemes, while the JFNK approach achieves strong performance and scalability with only minimal modifications to existing segregated frameworks. These findings underscore the potential of combining higher-order finite-volume methods with JFNK solvers to advance the state of the art in computational solid mechanics. The implementations are openly released in the solids4foam toolbox for OpenFOAM, supporting further exploration and adoption by the community.

Keywords: Jacobian-free Newton-Krylov, higher-order, cell-centred finite volume method, GMRES, solids4foam, OpenFOAM

1 Introduction

To be added

2 Mathematical Model and Numerical Methods

3 Test Cases

To be added.

4 Conclusions

Data Availability. **To be updated** The codes presented are publicly available at <https://github.com/solids4foam/solids4foam> on the `feature-petsc-snes` branch, and the cases and plotting scripts are available at <https://github.com/solids4foam/solid-benchmarks>.

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