OPTIMAL CAPACITOR PLACEMENT ON RADIAL DISTRIBUTION SYSTEMS

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Abstract - Capacitor placement problem on radial distribution systems is formulated and a solution algorithm is proposed. The location, type, and size of capacitors, voltage constraints, and load variations are considered in the problem. The objective of capacitor placement is peak power and energy loss reduction by taking into account the cost of capacitors. The problem is formulated as a mixed integer programming problem. The power flows in the system are explicitly represented and the voltage constraints are incorporated. The proposed solution methodology decomposes the problem into a master problem and a slave problem. The master problem is used to determine the location of the capacitors. The slave problem is used by the master problem to determine the type and size of the capacitors placed on the system. In solving the slave problem, an efficient phase I - phase II algorithm is used. Proposed solution methodology has been implemented and the test results are included in this paper.

I. INTRODUCTION

The general capacitor placement problem consists of determining the location, type, and the size of capacitors to be installed in the nodes of a radial distribution system such that the economic benefits due to peak power and energy loss reduction be weighted against the cost of installment of such capacitors while keeping the voltage profile of the system within defined limits.

The optimal capacitor placement problem as defined above has many parameters, such as; the location, type, and cost of capacitors, voltage constraints, and load variations on the system. These parameters determine the complexity of the problem.

Conventionally, the problem has been formulated by using a voltage independent reactive current model and solved by fixing some of the parameters and using analytical methods [1-3]. Recently there has been some studies to solve the problem in its general form. There are basically three approaches. The first one is the dynamic programming type approach by treating the sizes of capacitors as discrete variables, [4-5]. The second approach is to combine the conventional analytical methods with heuristics [6-7]. Third approach, pioneered by Grainger et. al., is to formulate the problem as a nonlinear programming problem by treating the capacitor sizes and the locations as continuous variables [8-13]. The application of this approach to general problem with the voltage regulator problem is given in [13].

In this paper, a formulation for the general capacitor placement problem as a mixed integer programming problem will be given first. The formulation considers all the parameters of the problem stated above and also the voltage constraints. Furthermore, the ac power flow formulation is used to represent the power flows and the voltage profile in the radial distribution systems.

A solution methodology for the general problem is proposed in this paper. The solution is based on the decomposition of the problem into hierarchical levels. The problem at the top level, called the *master problem*, is an integer programming problem and is used to *place the capacitors* (i.e. to determine the number and the location of the capacitors). A search algorithm has been developed for the master problem. The problem at the bottom level is called the *slave problem* and is used by the master problem to determine the types and the settings of the capacitors placed. Decomposition schemes are also used to further decompose the slave problems into *base problems*. Base problems are solved by using the efficient solution algorithm developed for a special capacitor placement problem called the *sizing problem*. The sizing problem and the associated solution algorithm are presented in another paper [14].

This paper consists of seven sections. In section II, the general capacitor placement problem is formulated and its complexity is discussed. In section III, it is shown that the problem can be decomposed into a *master problem* and a *slave problem*. Section IV and V are devoted to the development of solution methodologies for the slave and master problems respectively. Section VI contains the test studies of the solution method applied to two different systems. Conclusions are given in section VII.

II. FORMULATION OF THE PROBLEM

We consider the general capacitor placement problem as determining the places (number and location), types, and settings (capacities) of the capacitors to be placed on a radial distribution system. The objectives are to reduce the power and energy losses on the system and to maintain the voltage regulation while keeping the cost of capacitor addition at a minimum.

Since we are interested in energy loss in the system, it is necessary to take into account the load variations for a given period of time, T. We assume that the load variations can be approximated in discrete levels. Furthermore, the loads are assumed to vary in a conforming way (i.e., all the loads enjoy the same pattern of variations). We let $S(\tau)$ be the common *Load Duration Curve* as shown in Fig.1. Then a load, say load Q_L, can be represented as

$$Q_L(\tau) = Q_L^0 S(\tau) \tag{1}$$

Where, Q_L^0 represents the peak value.

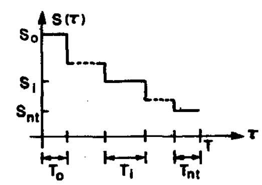

Figure 1 : Load Duration Curve

Under these assumptions, the time period, T can be divided into intervals during which the load profile of the system is assumed to be constant. Let there be nt such *load levels (load profiles)*.

Then for each load level, we have: (i) power flow equations, (ii) voltage constraints as bounds on the magnitude of the system node voltages. (iii) capacity and control constraints on the control variables (capacitors).

We will represent the constraints imposed by power flows on a radial distribution system by a new set of ac power flow equations, called *DistFlow* equations. They substitute for the conventional ac power flow equations. To summarize the idea, consider a 3–Ø, balanced radial distribution feeder with n branches/nodes, l laterals, and nc shunt

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capacitors placed at the nodes of the system. In Fig.2, one-line diagram of such a network is shown.

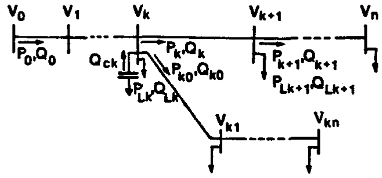

Fig.2 : One line diagram of a Distribution Feeder

It can be shown that power flow through each branch in the lateral can be described by the following recursive equations.

$$P_{k+1} = P_k - r_{k+1}P_k^2{+}Q_k^2/V_k^2 - P_{Lk+1} \tag{2.i}$$

$$Q_{k+1} = Q_k - x_{k+1}P_k^2{+}Q_k^2/V_k^2 - Q_{Lk+1} + Q_{ck+1} \tag{2.ii}$$

$$V_{k+1}^2 = V_k^2 - 2(r_{k+1}P_k + x_{k+1}Q_k) + (r_{k+1}^2 {+} x_{k+1}^2)P_k^2{+}Q_k^2/V_k^2 \tag{2.iii}$$

Where,

P_k, Q_k : real and reactive power flows into the receiving end of branch $k+1$ connecting node k and node $k+1$,

V_k : bus voltage magnitude at node k,

Q_{ck} : reactive power injection from capacitor at node k.

Eq.(2), called the *branch flow equation*, has the following form

$$\mathbf{x}_{k+1} = \mathbf{f}_{k+1}(\mathbf{x}_k, u_{k+1}) \tag{3.i}$$

Where, $\mathbf{x}_k = [P_k\ Q_k\ V_k^2]^T$ and $u_{k+1} = Q_{ck+1}$.

Note that if there is no capacitor at node k, then u_k does not appear in Eq.(3.i). By abusing notation, we will simply use **u** as an nc dimensional vector containing the nc capacitors i.e.,

$$\mathbf{u}^T = [u_1 \cdots u_{nc}] = [Q_{cn_1} \cdots Q_{cn_{nc}}]$$

In addition to the branch flow equations of (3.i), there are terminal conditions to be satisfied for each lateral (counting the main feeder as the 0'th lateral). For example, for lateral k shown in the figure, we have the following terminal conditions:

(i) at the branching node k where the lateral is connected to the main feeder, we define a dummy variable V_{k0} and let

$$\mathbf{x}_{k0_3} = V_{k0}^2 = V_k^2 = \mathbf{x}_{0k_3} \tag{3.ii}$$

(ii) at the end of the lateral, there is no power sent to the other branches, i.e.,

$$\mathbf{x}_{kn_1} = P_{kn} = 0 \quad ; \quad \mathbf{x}_{kn_2} = Q_{kn} = 0 \tag{3.iii}$$

Hence, for the general feeder considered, there are $3(n+l+1)$ DistFlow equations corresponding to Eq.(2) and Eq.(3). They will be represented by the following equations.

$$\mathbf{G}(\mathbf{x},\mathbf{u}) = \mathbf{0} \tag{4}$$

Where, $\mathbf{x} = [\mathbf{x}_1^T \cdots \mathbf{x}_l^T\, \mathbf{x}_0^T]^T$, $\mathbf{x}_k = [\mathbf{x}_{k0}^T \cdots \mathbf{x}_{kn}^T]^T$.

DistFlow equations can be used to determine the *operating point*, **x** of the system for a given *load profile*, P_{Li}, Q_{Li} $i = 1, \ldots, n$ and the capacitor settings, **u** . We prefer to use DistFlow equations over conventional ac power flow equations because the special structure of the DistFlow equations can be utilized to develop a computationally efficient and numerically robust solution algorithm. The details of such a solution algorithm are presented in [14].

For the capacitor placement problem since there are *nt* different load profiles to be considered, the overall DistFlow equations are

$$\mathbf{G}^i(\mathbf{x}^i, \mathbf{u}^i) = 0 \qquad i = 0,1 \cdots nt \tag{5}$$

Where, $\mathbf{x}^i$, $\mathbf{u}^i$ represent the state and the control variables corresponding to the load profile i respectively.

The voltage constraints can be taken into account by specifying upper and lower bounds on the magnitude of the node voltages as follows,

$$V^{min2} \le V_j^{i2} = v_j^i(\mathbf{x}^i) \le V^{max2} \qquad j = 1 \cdots n \quad i = 0,1 \cdots nt \tag{6}$$

These bounds constitute a set of functional inequality constraints of the form,

$$\mathbf{H}^i(\mathbf{x}^i) \le 0 \qquad i = 0,1, \ldots, nt \tag{7}$$

We will consider two different types of capacitors and represent them as follows:

i) **Fixed Capacitors** : They will be treated as reactive power sources with the constant magnitude at al load levels, i.e.,

$$\mathbf{u}^0 = \mathbf{u}^1 = \cdots = \mathbf{u}^{nt} \tag{8.i}$$

ii) **Switched Capacitors**: It will be assumed that the *settings (capacities)* of a switched capacitor, u_k^i can be changed/controlled at every load level considered. Therefore, for each capacitor, there are $nt+1$ settings, u_k^i $i = 0,1, \ldots, nt$ to be determined. We will also assume that the setting of a capacitor for the peak load, u_k^0 will be bigger than the ones for other lower load levels u_k^i. Hence, the sizes (nominal capacities) of capacitors will be determined by $\mathbf{u}^0$ and the relationship between the size and and the settings of a capacitor will be as follows.

$$0 \le u_k^i \le u_k^0 \tag{8.ii}$$

The objective terms, namely the real power and energy loss and the capacitor cost, can be formulated as follows. For each load level i , let the power loss in the system be $p_i(\mathbf{x}^i)$. Then the total cost of energy loss can be written as

$$k_e \sum_{i=0}^{nt} T_i p_i(\mathbf{x}^i) \tag{9}$$

where, T_i is the duration of the load for load level i and the constant k_e is the energy cost per unit. The cost for the real power loss at peak load level can be added to this sum by modifying T_0 accordingly.

The capacitor cost function is usually step like as shown in Fig.3 since in practice capacitors are grouped in banks of standard discrete capacities (usually 300 kvar sizes at 23 kV level).

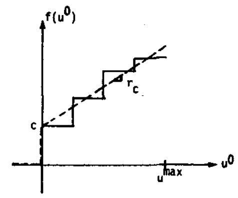

Figure 3 : Capacitor cost functions

Such a function is not easy to handle within this formulation framework; therefore, it will be approximated by a linear function with a fixed charge as shown in Fig.3 by a dotted line. This function can be formulated by using a decision variable $e \in \{0,1\}$ as

$$f(u^o) = c.e + r_c.u^o \qquad 0 \le u^o \le u^{\max}.e \tag{10}$$

Where, u^o and r_c represent the size and the marginal cost of the capacitor respectively. Note that $e = 0$ corresponds to the decision that the capacitor not to be placed.

To summarize, let the types and places of *nc* "candidate" capacitors, initially considered for installment, be given and let the sets C_1, C_2 contain the switched and the fixed capacitors respectively. Then we can write the general capacitor placement problem as a standard optimization problem as follows.

$$
\begin{array}{lll}
P & \min\ f_o = k_e \sum_{i=0}^{nt} T_i p_i(\mathbf{x}^i) + \sum_{k=1}^{nc} (r_{ck} u_k^0 + c_k.e_k) & \\
 & s.t. \quad \mathbf{G}^i(\mathbf{x}^i, \mathbf{u}^i) = \mathbf{0} & \\
 & \qquad \mathbf{H}^i(\mathbf{x}^i) \le \mathbf{0} & i{=}0,1, \ldots, nt \\
 & \qquad \mathbf{0} \le \mathbf{u}^0 \le u^{\max}.\mathbf{e} & \\
 & \qquad 0 \le u_k^i \le u_k^0 & k \in C_1 = \{sw.\ cap.\} \\
 & \qquad u_k^i = u_k^0 & k \in C_2 = \{fixed\ cap.\} \\
 & \qquad e_k \in \{0,1\} &
\end{array}
$$

This is a *non-linear, mixed integer programming problem*. Decision variables, $\mathbf{e} = [e_1 \cdots e_{nc}]^T$ are to be used to choose the capacitors among initially designated ones and continuous variables $\mathbf{u}^i$, $i = 0,1,\ldots,nt$ will be used to determine the optimal settings of the capacitors.

Voltage regulators are not explicitly represented in the capacitor placement problem presented here. However, the formulation and the solution algorithm introduced in this paper can be generalized to include the voltage regulators in the following fashion:

- Voltage regulators, such as regulating transformers, can be represented by their equivalent circuits in the DistFlow equations.
- The solution algorithm for sizing problem, proposed in [14] for determining the optimal sizes of capacitors placed on the system, can be generalized to obtain the optimal settings of the voltage regulators placed on the system,
- Voltage regulation for a given set of capacitors and voltage regulators can be obtained again by using the solution algorithm developed for the sizing problem. This is demonstrated in [14] by using only the capacitors.
- It seems appropriate to put the voltage regulator placement problem (to find the locations for the voltage regulators) at the top level of hierarchical decomposition scheme proposed in this paper in solving the overall problem.

Further investigation is required to complete the generalization.

III. DECOMPOSITION OF THE PROBLEM

The problem formulated in the previous section is a non-linear, mixed integer programming problem of the following form,

$$\min_{\mathbf{e},\mathbf{u}} \{ f_o(\mathbf{e},\mathbf{u}) \mid \mathbf{e} \in E \ ; \ \mathbf{u} \in U \} \tag{11}$$

where, $\mathbf{e}$ and $\mathbf{u}$ correspond to the decision vector, and the control vector, respectively, and E and U represent the constraint sets imposed on these vectors. We adapt a general solution approach which decomposes the problem by making use of the following property of the optimization

$$\min_{\mathbf{e}\in E,\mathbf{u}\in U} f_o(\mathbf{e},\mathbf{u}) = \min_{\mathbf{e}\in E} \{ \inf_{\mathbf{u}\in U} f_o(\mathbf{e},\mathbf{u}) \} \tag{12}$$

assuming that for each decision, $\mathbf{e}$ the problem in braces, called the *slave problem*,

$$g(\mathbf{e}) = \inf_{\mathbf{u}\in U} f_o(\mathbf{e},\mathbf{u}) \tag{13}$$

has a solution and it is "easy" to find it. Then the main problem becomes,

$$\min_{\mathbf{e}\in E} g(\mathbf{e}) \tag{14}$$

and is called the *master problem.* These problems can be characterized as,
Master Problem (MP) : Integer Programming Problem
Slave Problem (SP) : Non-linear Differentiable Optimization Problem

The solution for this decomposed problem requires an efficient solution algorithm for the slave problem and a search procedure over E, the set defined by all the possible decisions, for the master problem. In the next two sections, we'll discuss and develop such solution schemes.

IV. SLAVE PROBLEM

As indicated in the previous section, the slave problem assumes that the capacitors are placed and it is a special case of the capacitor placement problem. The problem is a non-linear differentiable optimization problem with quite a large number of equality and inequality constraints. It is still not easy to solve the slave problem. Our aim here is to study the special features and the structure of the problem and to develop an efficient solution methodology by exploiting these features.

For the slave problem, the type of capacitors (fixed or switched), and the number and diversity of load levels are important parameters that determine the structure and the size of the problem. Therefore, we first relax the parameter "type" by assuming that they are given. Then the problem becomes a sizing problem (i.e., determining the capacitor settings) and we need to consider two cases; one with fixed type capacitors and one with switched type capacitors. In the following first two subsections, we will show that for these two cases the problem is either of the *base type problem* or it can be decomposed into the base type subproblems. The base problems can be solved by the efficient solution method developed for the sizing problem, *PS* in [14]. Finally, in the last subsection, we will come back to the general slave problem and consider the types as well as the settings of the capacitors.

4.1 Fixed Capacitor Problem

When all the capacitors to be placed are of fixed type, the problem becomes

$$Pfx \qquad \min \ f_o = k_e \sum_{i=0}^{nt} T_i p_i(\mathbf{x}^i) + \sum_{k\in C_2} r_{ck} u_k$$
$$s.t. \qquad \mathbf{G}^i(\mathbf{x}^i,\mathbf{u}) = \mathbf{0} \qquad i=0,1,\ldots,nt$$
$$\mathbf{H}^i(\mathbf{x}^i) \le \mathbf{0}$$
$$\mathbf{0} \le \mathbf{u} \le \mathbf{u}^{\max}$$

This general form of fixed capacitor problem, *Pfx* is a base problem because it is essentially the same as the sizing problem, *PS*. Here, because of the consideration of more load levels, the objective function comprises more power loss terms and the constraint set is bigger due to extra DistFlow equations and the corresponding voltage regulation constraints. But this does not change the structure of the problem very much. Therefore, the solution methodology developed for the sizing problem can readily be applied for this problem with small modification.

Note that for this general case, existence of different load levels makes it possible to have both lower bound voltage violation at peak load level and upper bound voltage violation at light load level for a given control $\mathbf{u}$. This is especially true when the load levels are diverse. Such cases are handled in the Phase I - Phase II type solution algorithm of the sizing problem by considering both the lower and the upper voltage constraints in calculating the search direction (for details, see the solution algorithm for the base problem in [14]).

4.2. Switched Capacitor Problem

When all the capacitors to be placed are of the switched type, the slave problem can be re-written as follows.

$$Psw \qquad \min \ f_o = k_e T_0 p_0(\mathbf{x}^0) + \sum_{k\in C_1} r_{ck} u_k^0 + k_e \sum_{i=1}^{nt} T_i p_i(\mathbf{x}^i)$$

$$s.t. \qquad \mathbf{G}^0(\mathbf{x}^0,\mathbf{u}^0) = \mathbf{0}$$
$$\mathbf{H}^0(\mathbf{x}^0) \le \mathbf{0} \tag{15}$$
$$0 \le u_k^0 \le u_k^{\max} \qquad k \in C_1$$

$$\mathbf{G}^i(\mathbf{x}^i,\mathbf{u}^i) = \mathbf{0} \quad i = 1,\ldots,nt$$
$$\mathbf{H}^i(\mathbf{x}^i) \le \mathbf{0} \tag{16}$$
$$\mathbf{0} \le \mathbf{u}^i$$

$$\mathbf{u}^i - \mathbf{u}^0 \le \mathbf{0} \qquad i = 1,\ldots,nt \tag{17}$$

Note that the two constraint sets (15) and (16) are coupled through (17), to indicate that the capacitor sizes, $\mathbf{u}^0$ are the upper bound constraints for capacitor settings at the other load levels, $\mathbf{u}^i$. This weak coupling between $\mathbf{u}^0$ and $\mathbf{u}^i$'s can be exploited to decompose the problem into smaller subproblems. In appendix, it is shown that Psw can be decomposed into the following $nt+1$ base problems.

The main problem, SW_o

$$SW_0 \qquad \min_{\mathbf{u}^0} f_o = k_e T_0 p_0(\mathbf{x}^0) + \sum_{k\in C_1} r_{ck} u_k^0 + \sum_{i=1}^{nt} f_i(\mathbf{u}^0)$$
$$s.t. \qquad \mathbf{G}^0(\mathbf{x}^0,\mathbf{u}^0) = \mathbf{0}$$
$$\mathbf{H}^0(\mathbf{x}^0) \le \mathbf{0}$$
$$0 \le u_k^0 \le u_k^{\max} \qquad k \in C_1$$

Subproblems, $SW_i \quad i = 1 \cdots nt$

$$SW_i \qquad f_i(\mathbf{u}^0) = \min_{\mathbf{u}^i} k_e T_i p_i(\mathbf{x}^i)$$

$$s.t. \qquad \mathbf{G}^i(\mathbf{x}^i, \mathbf{u}^i) = \mathbf{0}$$

$$\mathbf{H}^i(\mathbf{x}^i) \le \mathbf{0}$$

$$\mathbf{0} \le \mathbf{u}^i \le \mathbf{u}^0$$

The subproblems, SW_i involve only one load level and therefore, they can be solved by the algorithm developed for the sizing problem for given capacitor sizes, $\mathbf{u}^0$. The solutions will correspond to the *optimal capacitor settings*, $\hat{\mathbf{u}}^i$ for the off-peak load levels considered.

The *main problem*, SW_0, is also a sizing problem; but computationally it is not of the easy type due to existence of $f_i(\mathbf{u}^0)$'s - the extra terms coupled to the subproblems - in the objective function. Updating these terms at each iteration during the solution of SW_0 requires solution of the subproblems. However, we can use a simple, heuristic scheme to update $f_i(\mathbf{u}^0)$ and $\nabla f_i(\mathbf{u}^0)$ in the main problem SW_0. To begin with, let the solutions of the subproblems SW_i for a given $\mathbf{u}^0$ be denoted $(\mathbf{u}^i)^*$. As we move $\mathbf{u}^0$ from iteration to iteration in SW_0, $f_i(\mathbf{u}^0)$ and $\nabla f_i(\mathbf{u}^o)$ should be calculated by solving SW_i with the new $\bar{\mathbf{u}}^0$ for a new $(\mathbf{u}^i)^*$, say $(\bar{\mathbf{u}}^i)^*$. We propose, however, instead of solving SW_i for a new $(\bar{\mathbf{u}}^i)^*$, to use the old $(\mathbf{u}^i)^*$ unless the constraint (17) is violated, i.e., we set the k'th component of $\tilde{\mathbf{u}}^i$

$$\tilde{u}_k^i = \begin{cases} \bar{u}_k^0 & \text{if } \bar{u}_k^0 \le (u_k^i)^* \\ (u_k^i)^* & \textit{otherwise} \end{cases} \tag{18}$$

We simply use $\tilde{\mathbf{u}}^i$ in evaluating $f_i(\bar{\mathbf{u}}^0)$ and $\nabla f_i(\bar{\mathbf{u}}^0)$ as follows:

$$f_i(\bar{\mathbf{u}}^0) \approx k_e T_i p_i(\tilde{\mathbf{x}}^i, \tilde{\mathbf{u}}^i) \tag{19}$$

$$\nabla_k f_i(\bar{\mathbf{u}}^0) \approx \begin{cases} \nabla_k f_i(\tilde{\mathbf{u}}^i) & \text{if } \bar{u}_k^0 \le (u_k^i)^* \\ 0 & \textit{otherwise} \end{cases} \quad k = 1, \ldots, nc \tag{20}$$

Note that if $\mathbf{u}^0$ and $\bar{\mathbf{u}}^0$ do not differ very much, the approximation will be good. To assure this, we start the procedure by first solving the subproblems with the capacitor sizes set to their maximum, $\mathbf{u}^{\max}$.

The overall iterative algorithm is shown in Fig.4. In the algorithm, an iteration comprises the solution of the subproblems, SW_i first and then the main problem, SW_0. Convergence check at end of an iteration involves checking if there is a status change in the constraint set of Eq.(17). (i.e., a non-binding constraint becomes binding or vice versa). If there is such a status change, then we go back and update the slave problems; otherwise we stop iterating since the solution is converged.

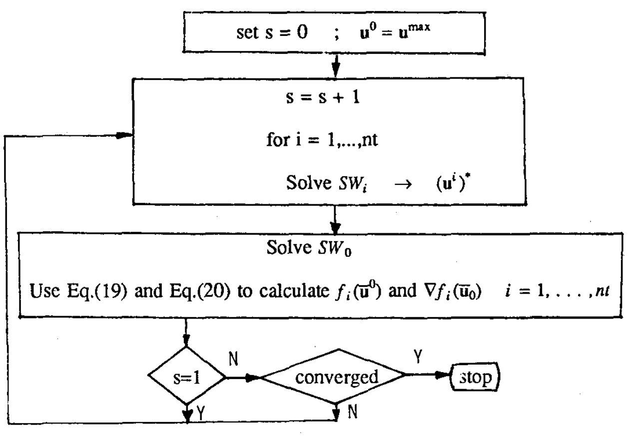

Figure 4 : Block diagram of SW Capacitor Algorithm

4.3. Mixed Type Capacitor Problem

In this section, we go back to consider the general slave problem in which it is not known *a priori* which capacitor is of fixed and which one is of switched type and one has to determine the types of capacitors in addition to their settings. Note that, in general, the switching capacitors are more expensive in both fixed cost, c and the marginal cost, r_c than the fixed capacitors. Therefore, we propose the following heuristic selection scheme by using the solution methodology developed for the switched capacitor problem.

Step 0 : Assume all the capacitors are of the switched type.
Step 1 : Solve the problem considering only the switched capacitors and keeping the fixed capacitor power injections as constant at their nominal settings, i.e., as loads.
Step 2 : Using the results of step 1, check the settings of switched capacitors at light load level (level nt). For the ones with nonzero light load settings, (i.e., $u_k^{nt} \neq 0$) assign that portion of capacitor as fixed type.
Step 3 : If any fixing occurred, then go to step 1; otherwise, stop.

V. MASTER PROBLEM

We follow a general approach in solving the master problem, which is an integer programming problem, and first construct a decision graph and then develop an efficient search scheme to place the capacitors (i.e., to determine their numbers and places).

Let there be nc initially given candidate capacitors. Then, all possible decisions about choosing the capacitors for placement among the candidate capacitors can be arranged as a *decision graph* assuming one decision is made at a time. Such a graph is shown in Fig.5 for $nc = 3$.

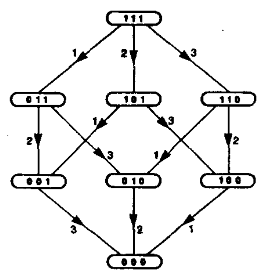

Figure 5 : Decision graph for 3 variable case

In the figure, each node represents a particular decision, $\mathbf{e} = [e_1\, e_2\, e_3]^T$; $e_i \in \{0,1\}$. Where, $e_i = 0$ means that the capacitor is not chosen, and $e_i = 1$ means it is still a candidate. A branch from a node to another indicates how the transition can be achieved: taking out the candidate capacitor whose number is shown on the branch. Such a relationship is indicated in graph theory by calling a node and all the nodes emanating from it, a *parent* and its *children*, respectively.

Search starts from the root which corresponds to the case where all candidate capacitors are chosen to be placed at the designated nodes of the system. Then there are two possible search techniques, *depth-first search* and *breadth-first search*, that can be employed to get a local optimum [17]. Here, rather than employing these general search techniques directly, the children of a given node(parent) are *sorted* first according to their contribution to the objective. We propose a *sorting procedure* which works sort of like a "steepest descent" approach in discrete case. The procedure is

- given a node which is identified by its decision vector, $\mathbf{e}$ and the solution of the corresponding *Slave Problem*, (i.e., the control vector $\hat{\mathbf{u}} = [\hat{u}_1 \cdots \hat{u}_{nc}]^T$ and the objective $f_o(\hat{\mathbf{u}})$).
- for all existing capacitors, $k = 1, \ldots, nc \;\; s.t. \;\; e_k \neq 0$
  - construct $\mathbf{u}$ by removing the capacitor k and keeping the rest, i.e.,

$$u_j = \begin{cases} 0 & \text{if } j = k \\ \hat{u}_j & \textit{otherwise} \end{cases}$$

- calculate the new objective, $f_o^k(\mathrm{u})$ (This requires only DistFlow solutions).
- let the contribution of the capacitor k to the objective be

$$\Delta f_o^k = f_o^k - f_o .$$

- sort the existing capacitors by using the Δf_o^k's

The Δf_o^k's can used to choose the child to branch out (i.e., capacitor to take out) since the child with the smallest Δf_o^k is more likely the one who contributes to the objective the least.

The first procedure, depth-first search, *visits* the children of a parent node (i.e., solves the associated slave problems) according to the order determined by the sorting procedure and branches out on the first one which gives a lower objective than that of the parent. This search is of order nc and therefore computationally attractive. However the branching criteria is appropriate only for unconstrainted case. When the voltage constraints are active, a better criteria would be to check how well the deficiency created by a capacitor removal be compensated by the rest of the capacitors by running the corresponding slave problems. This corresponds to the breath-first type search which is conducted by visiting all the children first and then branching out on the one with the smallest objective. This search is of order nc^2 and requires at most $nc(nc+1)/2$ slave problem solutions. However, the search can be made much faster by noting that the objectives, f_o^k calculated by sorting procedure will be bigger than the ones calculated by the search. Therefore, in practice, the search can be conducted only on the children with negative Δf_o^k's.

VI. TEST STUDIES

The proposed solution methodology has been implemented in Fortran 77 on both VAX 11/750 and IBM AT. The program uses the algorithm developed for the sizing problem in [14] as a subroutine to solve the base problems. We present the test results of two systems in this section to illustrate the performance of the proposed solution scheme.

The first test system, TS1 is a 9-branch main feeder test system developed by Grainger et. al., [9]. The second test system, TS2 is a 69-branch, 9-lateral test system derived from a portion of the PG&E distribution system. The network data of this system is given in Table 1.

We adopted the following cost figures : energy cost, $k_e = 0.06$ *\$/kWh* ; capacitor fixed cost, $c = 1000\,\$$; capacitor marginal cost, $r_c = 3$ *\$/kvar*. The load duration data assumed for the systems is given in Table 2. It is also assumed that substations have regulating transformers which are tapped to +5% during peak load and set to nominal otherwise. The substation voltage without tap is taken as the base kV and the lower and upper voltage limits are assumed to be 0.9 and 1.1 p.u. respectively.

1. Fixed Capacitor Placement

We first present two test results that correspond to the solution of the general fixed capacitor placement problem for TS1 and TS2.

Test System 1

The test results of the capacitor placement with fixed capacitors is summarized in Table 3. They include the search for the master problem and the slave problems solved during the search. In the table, each box corresponds to a node of the search tree and it contains the pertinent information about the solution of the associated slave problem. The first column of a box contains: (i) the net revenue due to the energy loss reduction, *rev* in dollars, (ii) energy loss reduction, ΔE in MWh, (iii) the minimum bus voltage in the system, $V_{\min}$ in p.u., and (iv) the total number of iterations, *itr* . The second column contains the optimal settings of the capacitors, $\mathbf{Q}_c$. The number on a branch of the trees identifies the capacitor taken out for branching. The breadth-fist type search is used for the solution of the master problem in both tests. Childrn of a parent are *visited* (i.e. associated slave problem is solved) in an order determined by the sorting procedure introduced in section 5. In the table, this corresponds to visiting the children from left to right.

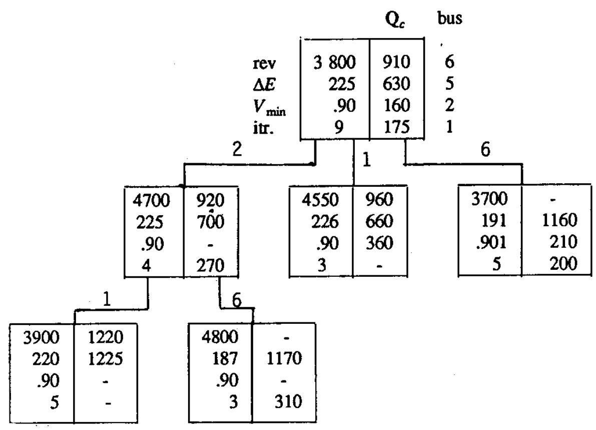

Table 3: Test run results for TS1 - fixed capacitor placement

System	Load Levels S_0	S_1	S_2	Time Intervals T_0	T_1	T_2
TS1.	1.1	0.6	0.3	1000.	6760.	1000.
TS2	1.8	1.	0.5	1000.	6760.	1000.

Table 2 : Load Duration data for the test systems.

Table 1 : Network Data of TS2

Br. No	Sd. Nd.	Rv. Nd.	Br. Par. r(ohm)	x(ohm)	Rv. Nd. Load P(KW)	Q(KVAR)
1	0	1	0.0005	0.0012	0.	0.
2	1	2	0.0005	0.0012	0.	0.
3	2	2e	0.	0.	0.	0.
4	2e	3	0.0015	0.0036	0.	0.
5	3	4	0.0251	0.0294	0.	0.
6	4	5	0.3660	0.1864	2.60	2.20
7	5	6	0.3811	0.1941	40.40	30.00
8	6	7	0.0922	0.0470	75.00	54.00
9	7	8	0.0493	0.0251	30.00	22.00
10	8	9	0.8190	0.2707	28.00	19.00
11	9	10	0.1872	0.0619	145.00	104.00
12	10	11	0.7114	0.2351	145.00	104.00
13	11	12	1.0300	0.3400	8.00	5.50
14	12	13	1.0440	0.3450	8.00	5.50
15	13	14	1.0580	0.3496	0.	0.
16	14	15	0.1966	0.0650	45.50	30.00
17	15	16	0.3744	0.1238	60.00	35.00
18	16	17	0.0047	0.0016	60.00	35.00
19	17	18	0.3276	0.1083	0.	0.
20	18	19	0.2106	0.0696	1.00	0.60
21	19	20	0.3416	0.1129	114.00	81.00
22	20	21	0.0140	0.0046	5.30	3.50
23	21	22	0.1591	0.0526	0.	0.
24	22	23	0.3463	0.1145	28.00	20.00
25	23	24	0.7488	0.2475	0.	0.
26	24	25	0.3089	0.1021	14.00	10.00
27	25	26	0.1732	0.0572	14.00	10.00
28	2	27	0.0044	0.0108	26.00	18.60
29	27	28	0.0640	0.1565	26.00	18.60
30	28	29	0.3978	0.1315	0.	0.
31	29	30	0.0702	0.0232	0.	0.
32	30	31	0.3510	0.1160	0.	0.
33	31	32	0.8390	0.2816	14.00	10.00
34	32	33	1.7080	0.5646	19.50	14.00
35	33	34	1.4740	0.4873	6.00	4.00
36	2e	27e	0.0044	0.0108	26.00	18.55
37	27e	28e	0.0640	0.1565	26.00	18.55
38	28e	65	0.1053	0.1230	0.	0.
39	65	66	0.0304	0.0355	24.00	17.00
40	66	67	0.0018	0.0021	24.00	17.00
41	67	68	0.7283	0.8509	1.20	1.00
42	68	69	0.3100	0.3623	0.	0.
43	69	70	0.0410	0.0478	6.00	4.30
44	70	88	0.0092	0.0116	0.	0.
45	88	89	0.1089	0.1373	39.22	26.30
46	89	90	0.0009	0.0012	39.22	26.30
47	3	35	0.0034	0.0084	0.	0.
48	35	36	0.0851	0.2083	79.00	56.40
49	36	37	0.2898	0.7091	384.70	274.50
50	37	38	0.0822	0.2011	384.70	274.50
51	7	40	0.0928	0.0473	40.50	28.30
52	40	41	0.3319	0.1114	3.60	2.70
53	8	42	0.1740	0.0886	4.35	3.50
54	42	43	0.2030	0.1034	26.40	19.00
55	43	44	0.2842	0.1447	24.00	17.20
56	44	45	0.2813	0.1433	0.	0.
57	45	46	1.5900	0.5337	0.	0.
58	46	47	0.7837	0.2630	0.	0.
59	47	48	0.3042	0.1006	100.00	72.00
60	48	49	0.3861	0.1172	0.	0.
61	49	50	0.5075	0.2585	1244.00	888.00
62	50	51	0.0974	0.0496	32.00	23.00
63	51	52	0.1450	0.0738	0.	0.
64	52	53	0.7105	0.3619	227.00	162.00
65	53	54	1.0410	0.5302	59.00	42.00
66	10	55	0.2012	0.0611	18.00	13.00
67	55	56	0.0047	0.0014	18.00	13.00
68	11	57	0.7394	0.2444	28.00	20.00
69	57	58	0.0047	0.0016	28.00	20.00

Substation Voltage (kv) = 12.66
Base KVA = 10.000
Base Voltage (kV) = 12.660

total load: P(KW) = 3802.19 , Q(KVAR) = 2694.60

Test run for TS1, summarized in Table 3, starts with 4 capacitors. The solution of the slave problem for the root (initial one) gives the optimal settings for these 4 capacitors. Then the capacitor's contributions to the objective, Δf_o^k's are calculated by using the sorting procedure. The results indicate that three of the capacitors - Q_{c1} , Q_{c2} , Q_{c6} - have $\Delta f_o^k < 0$; which implies that they are not *economically feasible* (i.e., their economic contribution due to energy loss reduction is less than their cost). Therefore, the search is conducted only on these three capacitors at the second search level. As a result, Q_{c2} is found to be the least economical and hence is taken out. This corresponds to the branching out on the first child of the root in the table. Then a new search process resumes from this new node; first sorting out its children by using the sorting procedure and then visiting the ones that are economically infeasible. The solution is obtained at the third level of search when the evaluation of capacitors Q_{c6} and Q_{c1} in the last node by the sorting procedure indicated that they are economically feasible.

The total run time for this test on VAX is about 45 sec. of CPU and 8 sec. of I/O.

Test System 2

Test run for TS2 is summarized in Table 4. The test starts with 5 capacitors. After visiting the root, the capacitor with the zero setting, Q_{c69} is taken out and the other two Q_{c11} and Q_{c47} are found to be economically infeasible by the sorting procedure. The search therefore is conducted only on these two capacitors and as a result the last node containing the capacitors Q_{c18} and Q_{c52} is identified as the solution node. The total run time for this test on VAX is recorded as 165 sec. of CPU and 8 sec. of I/O.

Root:

		Q_c	bus
rev	36 000	170	11
ΔE	750	230	18
V_{min}	.907	0	69
itr.	8	210	47
		1 040	52

11:

36 800	-
742	300
.907	-
3	260
	1 020

47:

36 700	220
745	220
.907	-
3	-
	1 200

47:

37 500	-
734	300
.906	-
2	-
	1 200

Table 4 : Test run results for TS2 - Fixed Capacitor placement

We have the following comments/observations about these test results.

1. Search Procedure :
   - The order of search is about nc (number of capacitors placed); which is much less than the worst case bound of nc^2.
   - The search converges to the global optimal point in both tests; although in test 1, for example, there are some other suboptimal solutions with revenues close to each other.
   - As search goes further down to higher levels, more capacitors are taken out to increase the revenues. This causes less energy loss reduction and lower voltage profile. This also shows the sensitivity of the optimal point with respect to the cost figures k_e , r_c , c.
2. The slave problem - fixed capacitor problem:
   - The convergence of slave problem at the root (the initial one) gets slower as the number of load levels increase (the number of iterations is in the order of 2-3 nc). But the other slave problems converge much faster (in the order of 1-2 nc); mainly because it is easier to find a good initial point for them by simply using the results of the parent node.
   - Although there are regulators at the substations, voltage profiles of both systems are below lower voltage limits at the peak load level before the capacitor placement. The solution for TS1 indicates that the capacitors are needed to be used for raising the voltage profile of the system at the peak load level ($V_{min} = 0.9$ p.u.) as well as for loss reduction. The solution for TS2 corresponds to the unconstrainted optimal point ($V_{min} = 0.907$ p.u.); indicating that maximum loss reduction is achieved.

2. Switched Capacitor Case

We now present the test runs for the switched capacitor problem on the same test systems. The optimal places obtained from the general fixed capacitor problem tests are used in these tests also to avoid the search.

Test System 1

The first step of the test runs for TS1 is the solution of the corresponding slave problems SW_i , $i = 1,2$ to get the optimal settings for the off-peak load levels assuming no limit on the size of the capacitors. Each of such solution is obtained in about less than 2 nc iterations by calling the sizing problem subroutine. In the second step of the test, the main problem, SW_o is solved in about 3 nc iterations by using the solution procedure described in section 5. In the third step, convergence checks indicate that the setting of Q_{c5} for the first load level is binding, i.e., $Q_{c5}^1 = Q_{c5}^0$. This constitutes the end of the first iteration. Another iteration is performed to see if the binding status of the capacitors will change. The convergence is obtained at the second iteration when the capacitor settings, Q_c^1 , Q_c^2, are updated and it is found that there is no binding status change.

The solution is summarized in Table 5. The box in the table is similar to that of fixed capacitor case; except here, in addition to the capacitor sizes, $\hat{Q}_c^0$, capacitor settings at the off-peak load levels, $\hat{Q}_c^1$, $\hat{Q}_c^2$, are given also. The total run takes about 12 sec. of CPU and 2 sec. of I/O on VAX.

Table 5 : Test run results for TS1 - Switched Capacitor case

		Q_c^2	Q_c^1	Q_c^0	bus
rev.	5250	600	1170	1170	5
ΔE	195	120	230	300	1
V_{min}	.90				

Test System 2

Test run for TS2 is similar to that of TS1. The solution is obtained in two iterations and found out that only the setting of Q_{c18} for the first load level is binding, i.e., $Q_{c18}^1 = Q_{c18}^0$. The solution is summarized in Table 6. The total run takes about 82 sec. of CPU and 4 sec. of I/O on VAX.

Table 6 : Test run results for TS2 - Switched Capacitor case

		Q_c^2	Q_c^1	Q_c^0	bus
rev.	39 180	197	330	330	18
ΔE	785	620	1 240	1700	52
V_{min}	.909				

We have the following comments about the test results:

- The performed test runs indicate very good convergence characteristics; the number of iterations between the main problem, SW_o and the subproblems, SW_i is usually one or two.
- When the fixed capacitor and the switched capacitor test results are compared, it is seen that: (i) switched capacitor placement yields higher revenues and higher capacitor sizes, especially when the cost data is the same for both cases and the load variations are diverse, (ii) the voltage profile is higher (V_{min} is higher) and voltage regulation is better for the switched capacitor case. This is because of the fact that for the switched capacitor case better compensation is achieved by adjusting the value of capacitors as the load changes.

The final point to be noted about the overall test results is about the effect of the regulators on the solution; (i) feasibility becomes less of a problem, (ii) no upper limit voltage violation has been observed for the given test systems, although the load levels were quite diverse. (iii) as exemplified here by test run for TS1, capacitors can be used together with voltage regulators to keep the voltage profile of the system within defined limits.

Analysis of the Results

Consider the starting point of TS2 in Table 4, where the fixed cost of capacitors are not included. The solution of capacitor sizing gives 1040 kvar and 210 kvar on the nodes 52 and 47 of a lateral respectively, and 170 kvar and 230 kvar on the nodes 11 and 18 of the main feeder respectively. In the system, the loads in the laterals are more concentrated whereas the loads in the main feeder are more evenly distributed. The result is that the size of the capacitors are also more concentrated in the lateral and more evenly distributed in the main feeder. This further reaffirms the fact that the nature of reactive power compensation is rather local.

VII. CONCLUSIONS

In this paper, a general formulation and an efficient solution methodology have been developed for general capacitor placement problem on radial distribution systems.

The general capacitor placement problem consists of placing the capacitors (determining their number and the locations) and determining their types and sizes. The objective is peak power and energy loss reduction while keeping the cost of capacitors at a minimum.

The proposed formulation is comprehensive in the sense that: (i) it considers all the variables of the problem stated above, (ii) it uses the ac power flow equations to represent the system, (iii) voltage constraints are taken into account.

A solution method has been developed for this general problem by decomposing the problem into two hierarchical levels. The top level problem, called the master problem, is an integer programming problem and is used to place the capacitors (determine their number and locations). An efficient search scheme has been developed for the master problem. The second level problem, called the slave problem, is used by the master problem as a subroutine. This problem is further decomposed into two levels: at the top level, the problem consists of determining the type of capacitors and at the bottom level, the problem is to determine the capacitor sizes once the capacitors are placed with their types assigned. These slave problems, called the fixed and switched capacitor problems, are shown to be either the base type or can be decomposed into base type problems. The base type problem is a capacitor sizing problem and is solved by using an efficient phase I - Phase II type solution method presented in [14].

Test results are presented for the proposed solution scheme. They indicate that the method is computationally efficient and the decomposition scheme performs well.

Although not implemented, it is also shown that the formulation and the solution methodology presented in this paper can be generalized to include the voltage regulators in the problem.

Acknowledgements

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Appendix: Decomposition of the Switched Capacitor Problem

We shall apply decomposition techniques to the switched capacitor problem introduced in section 4.2. We present first a general scheme, which leads to the Benders Decomposition, and then a simpler, heuristic based decomposition scheme for the special switched capacitor problem.

Decomposition

A general decomposition scheme is given in [15]. To adopt the derivation for the switched capacitor problem, Psw, we first re-group the variables and constraints as follows,

$$\mathbf{u} = [\mathbf{u}^1 \cdots \mathbf{u}^{nt}]^T$$

$$U = \{ \mathbf{u} \mid \text{Eq.(16) is satisfied} \} \quad ; \quad U_0 = \{ \mathbf{u}^0 \mid \text{Eq.(15) is satisfied} \}$$

Similarly, we partition the objective function as,

$$f_m(\mathbf{u}^0) = k_e T_0 p_0(\mathbf{x}^0(\mathbf{u}^0)) + \sum_{k \varepsilon C_1} r_{ck} u_k^0 \quad ; \quad f_r(\mathbf{u}) = \sum f_{ri} = k_e \sum_{i=1}^{nt} T_i p_i(\mathbf{x}^i(\mathbf{u}^i))$$

Then Psw can be re-written as

$$\min_{u^0,u} \{ f_o = f_m(u^0) + f_r(u) \mid u^0 \varepsilon U_0 \; ; \; u \varepsilon U \; ; \; F(u^0,u) \le 0 \} \quad (a.1)$$

Where, $F(u^0,u)$ corresponds to the coupling constraints of Eq.(17).

We start the decomposition by *projecting (partitioning)* the problem onto space of u^0 alone as follows.

$$\min_{u^0 \varepsilon U_0} \{ \inf_{u \varepsilon U} [f_m(u^0) + f_r(u) \mid F(u^0,u) \le 0] \} \quad (a.2)$$

We assume that the *subproblem* in the braces,

$$f_s(u^0) = \inf_{u \varepsilon U} \{ f_r(u)^{-} F(u^0,u) \le 0 \}$$

can be evaluated for a given u^0 as an optimization problem with respect to the variable u. Then the main problem becomes

$$SW_o \qquad \min_{u^0 \varepsilon U_0} f_o(u^0) = f_m(u^0) + f_s(u^0) \quad (a.3)$$

To assure that the above assumption holds, we must avoid the values of u^0 such that $f_s(u^0)$ does not have a feasible solution. For this, we define a new set, V as follows.

$$V = \{ u^0 \mid \exists u \varepsilon U \;\; s.t. \;\; F(u^0,u) \le 0 \}$$

The set V can be thought of as the projection of the constraint set defined by $F(u^0,u)$ and U onto the space defined by u^0 alone as illustrated in Fig.a.1.

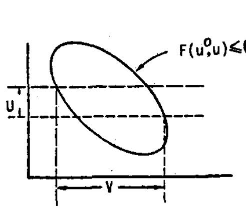

Figure a.1 : Set Projection

Now we generalize the projection by rewriting the problem (a.3) as follows.

$$SW_o \qquad \min_{u^0} \{ f_m(u^0) + f_s(u^0) \mid u^0 \varepsilon U_0 \cap V \} \quad (a.4)$$

For the switched capacitor problem, we have a conjecture that

$$U_0 \subset V.$$

Justification of this conjecture will be discussed later.

Assuming that this conjecture holds for the general case, we can use the partitioned problem (a.3) rather than (a.4) for the main problem, SW_o. The subproblem $f_s(u^0)$ can further be decomposed into nt subproblems of the following form.

$$SW_i \qquad f_{si}(u^0) = \inf_{u^i} f_{ri}(u^i)$$

$$s.t. \quad u^i - u^0 \le 0$$

$$u^i \varepsilon U_i$$

Where, $U_i = \{ u_i \mid Eq.(16) \}$ and $f_s(u^0) = \sum f_{si}(u^0)$.

The explicit form of the main problem SW_0 and the subproblems SW_i $i = 1, \ldots, nt$ are given in sec.4.2.

Benders Decomposition

Assuming that the duality conditions holds for the switched capacitor problem of (a.2) [16], the main problem can be transcribed into the following form.

$$SW_{ob} \quad \min \; f_o(u^0,y) = f_d(u^0) + \sum y_i$$

$$s.t. \quad y_i + \lambda^{ij^T}(u^0 - u^{0j}) \ge f_{si}(u^{0j}) \quad j = 1, \ldots, p \quad (a.5.i)$$

$$u^o \in U_0 \qquad i = 1, \ldots, nt \quad (a.5.ii)$$

Where, u^{ij} and λ^{ij} corresponds to the solution of the subproblem SW_i at iteration j.

The solution algorithm involves the iterative solution of the main problem SW_{ob} and the subproblems, SW_i. After each iteration, a new set of constraints of the type (a.5.i) is added to SW_{ob} until the solution converges.

It is noted that the Benders decomposition has the following features.
(i) The main problem, SW_{ob} is not a base type problem. Additional constraints of (a.5.i), called the cuts, are difficult to handle with the solution algorithm used by the base problem; one has to identify which one these constraints will be binding during the solution of SW_{ob}.
(ii) The contribution of the subproblems to the objective of the main problem, $f_{si}(u^0)$ is approximated by linearizing this term around the previously calculated points, u^{oj}. To see this, let the binding constraint for the subproblem i be the k'th one in Eq.(a.5.i). Then the solution for the corresponding y_i will be

$$y_i = f_{si}(u^{ok}) - \lambda^{ik^T}(u^0 - u^{0k})$$

Therefore, the approximation is good only if the actual solution point, u^0 is close to the calculated point u^{0k} in solving SW_{ob}.

A Heuristic Based Decomposition Scheme

Note that the main problem of (a.3) need not be transcribed into Benders form, SW_{ob} if one can estimate the binding constraints in the sets $u^i - u^0 \le 0$. We develop a solution algorithm based on this principle. The solution scheme uses the sizing algorithm to solve both the main problem and the subproblems and it also uses a better estimate for the contribution of the subproblems to the objective of the main problem. The details of the algorithm is given in section 4.2.

Justification of the Conjecture

$$U_0 \subset V$$

The idea behind this conjecture is as follows. Let $u^0 \varepsilon U_0$. This means that u^0 amount of reactive power compensation from the capacitors is enough to satisfy the voltage constraints for the peak load. But this amount of compensation must suffice to have a feasible point for lower load levels too because the lower the load the higher the voltage profile will be. This observation is due to the strong coupling between the reactive power flow and the voltage profile of the system.

Note that this conjecture was also the underlining idea behind the assumption made when formulating the problem in section 2; where, it was assumed that the capacitor size will be the capacitor setting for the peak load level, u^0 and capacitor settings for all the other lower load levels, u^i will be smaller than u^0.

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Discussion

Roger C. Dugan (McGraw-Edison Power Systems, Cooper Industries Inc., Canonsburg, PA): The reader will discover in a few moments that I have several objections to the methods presented in this paper. However, I do not wish for my objections to reflect poorly upon the efforts of the authors. It would appear that a great deal of good work has been done and I suspect that this paper was intended to emphasize the application of more sophisticated techniques to the problem. Therefore, it may be unfair to expect the authors to respond to all of my objections because they are based on more practical aspects and also apply to previous investigators in this field who have made similar assumptions.

First, one would hope that a method employing nonlinear programming techniques would be faster than simple exhaustive searches. It is not apparent from the paper that this would be the case, and the times quoted during the paper's presentation and discussion lead me to believe otherwise. The number of discrete solutions to this problem are not necessarily large due to the practical constraints that I will mention below. Therefore, an intelligent exhaustive search method that automatically discards many cases due to its knowledge of the way the feeder operates can be reasonably fast. I have investigated a number of search techniques that give very good, but perhaps not optimal, solutions, and the execution times grow approximately linearly with problem size rather than geometrically.

The method proposed in the paper is based on the assumption that the feeder is discrete and the capacitor size is continuous. Perhaps, a more realistic choice would have been the opposite. There are many nodes in a circuit, but utilities generally wish to consider only two or three different capacitor sizes, for example 600- and 1200-kvar banks. I have generally approached the problem by assuming that both the feeder and capacitor sizes are discrete. When considering such a small number of sizes, a simple search can often be done quickly.

Another assumption I would question is that the capacitor is a source of reactive power (Q). (I assume that this implies a constant source because I am not able to ascertain otherwise from the paper.) Of course, this will introduce inaccuracies because a capacitor is a constant impedance element. Since capacitor placement on a feeder significantly affects the voltage, this assumption weakens any claim that the method results in an optimal location. It would seem to be a simple matter to correctly represent capacitors and avoid this difficulty.

My examination of this problem has also indicated that line regulator tap position and control characteristics affect the "optimal" solution. It is not clear how the method presented in the paper properly accounts for these effects.

The constant P-Q load model employed in the proposed method also leads to inaccuracies. The P-Q load model is a peculiar bias of transmission analysts, and it should not be employed on distribution systems without question. Frantz *et al.* [1] have clearly shown that distribution system loads are sensitive to voltage. My experimentation with different load models has shown that one will usually get a different "optimal" solution for each load model. The P-Q load model is best employed to establish the base case voltages from known load conditions. Then one should switch to a more realistic model when studying capacitor additions. In the absence of better information, I will typically use a load model in which the P varies linearly with voltage and the Q varies by the square of the voltage. Lacking this capability, I suspect that a simple constant impedance load model would be better than a constant P-Q model.

Differing economic evaluation criteria among utilities require different approaches to the optimization problem. Utilities using very high values for released substation and generation capacity savings may achieve a more economical solution by optimizing the peak load condition first, although I must admit to being skeptical of this. Then the load is decremented in steps to determine when switched banks should be turned off. For utilities where the costs of losses is more important, a more economical solution can generally be achieved by first optimizing the location of fixed banks at minimum load and then incrementing the load in steps to determine the location, size, and switching levels of the switched banks.

I believe that the latter approach is more practical for most utilities. One reason is that most feeders operate near minimum or average load levels much more than they operate near peak loads. Another reason is that I question whether values for substation and generation costs are based on assumptions compatible with assumptions made for capacitor economic evaluations. This approach can be easily programmed using an intuitive algorithm that recognizes how a feeder typically operates. It yields a near-optimal solution that is difficult to improve upon significantly. I will state the algorithm in words, giving the reader the freedom of choice in selecting techniques for solving the load flow and making decisions.

1) Select the fixed capacitors. This frequently is simply the selection of the optimal location of the one capacitor bank needed at minimum load.
2) Increment the load in steps. At each load level ask the question, "Is more capacitance needed?" If so, then determine optimal switched capacitor locations and sizes based primarily on loss and voltage profile improvement.
3) Once peak load is reached, decrement the load to determine switch-off load levels.

This very simplistic method of locating and sizing capacitors incrementally tends to distribute smaller size banks over the feeder where they are needed and when they are needed. Generally, no more than one capacitor is added at a load level and the search for optimum location is trivial. My simulations of the feeder over a year's time considering daily and seasonal load cycles indicate that it is difficult to significantly improve upon this method economically. One can always find a solution that appears to be a few percentage points better, but I do not think that the basic data are known with sufficient accuracy to quibble over a small difference in an off-line analysis. It would require on-line control to take advantage of the small gains possible. We have implemented this method in an interactive program that uses the above algorithm to get close to the most economical solution. Then the user can tweek the solution interactively taking into account practical considerations. The whole process takes but a few minutes using a personal computer (PC).

Differing approaches to the problem of capacitor size and placement will yield differing "optimal" solutions. I suspect that none are truly optimal and most approaches that consider load variation are generally adequate (methods that optimize only peak load sometimes give poor results when the entire load cycle is considered). I would hesitate to defend one approach over another too strongly because feeder load varies somewhat randomly and it would be difficult to prove which is more optimal. However, I think that methods like I have described, which are based on how the feeder operates, are apt to be more optimal more of the time. They are also simple to program and the programs execute quickly. Therefore, I question the practicality of abandoning the simpler approach in favor of a more sophisticated method like the authors have presented.

Reference

[1] T. Frantz *et al.*, "Load behavior observed in LILCO and RG&E systems," *IEEE Trans. Power App. Syst.*, April 1984, pp. 819–831.

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M. E. Baran and F. F. Wu : We would like to thank Mr. Dugan for his interest in the paper and his insightful questions about the capacitor placement problem.

The algorithm outlined by Mr. Dugan is a special case of switched capacitor problem introduced in this paper. In Sec. 4.2 it is shown that assuming all the capacitors are of switched type, and for a given set of capacitors placed on the system, the problem can be decomposed into smaller subproblems each of which corresponds to minimization of losses at each of the load levels considered. However, these subproblems are coupled to each other due to the cost of capacitors. It is easy to show that when the cost of capacitors are neglected the subproblems become independent and hence the optimization for each load level can be carried out independently. Mr. Dugan's algorithm incorporates both placement and sizing problems into this decomposition scheme. It is indeed a good idea especially since he considers the capacitor sizes as discrete. Therefore, his algorithm will work, as he points out, when the objective function is power loss minimization or voltage regulation.

However, for the general case, where the cost of capacitor is important and it effects the number of capacitors to be placed, the method may not give good answers because of the coupling between the subproblems. Hence, in this case, it is not easy to answer the question how capacitors should be added as the load level increases. It seems that the best aid to answer this question would be the use of switched capacitor problem. Starting from a candidate set of capacitors and assuming them all switchable the problem can be solved by using the switched capacitor algorithm introduced in this paper. The solution will give the capacities of the capacitors, $\mathbf{u}^0$ and their settings at other load levels, $\mathbf{u}^i$, $i = 1, \ldots, nt$ assuming the capacities are continuous variables. These results then can be used in answering the question mentioned above and hence the capacitors can be placed by the method proposed by Mr. Dugan. This way the search introduced in this paper may not need to be carried out any further than the root.

Another point raised by Mr. Dugan is the assumption made in modeling the capacitors. It involves approximating the reactive power injected by a capacitor as constant, independent of the voltage. This assumption is justified based on the fact that $V_c \approx 1$ p.u. and sizes of capacitors determined by the solution need to be rounded off to get the practical size of capacitors. However, the exact model can be incorporated in the method if needed. This is explained in the closure of [14]. Note that this approximation will most likely affect the sizes, not the location, of capacitors.

The power flow model used in this paper (DistFlow equations) can handle voltage dependent loads and the solution algorithm can be generalized to take into account such loads, as explained again in the closure of [14].

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