# Measures of Dispersion

Md. Ismail Hossain Riday



• In previous chapter, we learned how to summarize a raw data set using a single value that indicates the center of the distribution.

• It gives only partial information of the data set.



For example, Record the examination results of two students

Student A: 50, 49, 51, 50

Student B: 100, 100, 0, 0

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• Student A: 50, 49, 51, 50 ( $\bar{X}_A = 50$ )

Student B: 100, 100, 0, 0



• For example, Record the examination results of two students

• Student A: 50, 49, 51, 50 ( $\bar{X}_A = 50$ )

• Student B: 100, 100, 0, 0 ( $\bar{X}_B = 50$ )

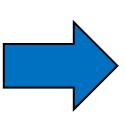
For example, Record the examination results of two students

• Student A: 50, 49, 51, 50 ( $\bar{X}_A = 50$ )

• Student B: 100, 100, 0, 0 ( $\bar{X}_B = 50$ )

On average both students are equal in getting final exam score

So it is also important to describe "How much the observation vary from one to another



**Dispersion/Variation** 



# Measures of Dispersion

 Dispersion may describe how much the observation vary from one to another.

 Descriptive measures that indicate the amount of variation in a data set are called measures of dispersion.



# Why do we need?

To determine the reliability of an average

To compare the variability of two or more data sets.



# Types of Measures of Dispersion

Two types of measures of dispersion

1. Absolute measure

2. Relative measure



### Absolute measures

Only measure the inherent variation of a data set



#### Absolute measures

Only measure the inherent variation of a data set

- Important absolute measures are-
- 1. Range
- 2. Mean Deviation
- 3. Variance
- 4. Quartile Deviation



 $\blacksquare$  Range = Highest value - Lowest value



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For example, weights of 5 newly born babies (in pounds)

7.5, 4.5, 10.1, 9.6, and 5.5



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(Range = ????)



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Range = 10.1 - 4.5 = 5.6 pounds



• Let  $x_1, x_2, ..., x_n$  be n observations, where  $\bar{x}$  is average/ or mean

• 
$$MD = \frac{\sum |x_i - \bar{x}|}{n}$$
 [for ungrouped data]

• 
$$MD = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$
 [for grouped data]



• For example, x = 1,2,4,5.



• For example, x = 1,2,4,5. Calculate the mean deviation.

$x_i$	
1	
2	
4	
5	



• For example, x = 1,2,4,5. Calculate the mean deviation.

$x_i$	$\overline{x}$	$ x_i - \overline{x} $
1		2
2	$\sum x_i$	1
4	$\frac{\sum x_i}{n} = 3$	1
5		2

$$MD = \frac{\sum |x_i - \bar{x}|}{n} = \frac{6}{4} = 1.5$$



Sale (in \$)	20-30	30-40	40-50	50-60	60-70
$f_i$	3	5	12	3	2

Calculate the mean deviation about mean. (Ans: 7.776)



#### Variance

Let  $x_1, x_2, ..., x_n$  be n observations

For grouped, 
$$\sigma^2 = \frac{\sum f_i(x_i - \mu)^2}{N}$$

•  $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$  [Population variance (ungrouped)]

$$\sigma^2 = \frac{\sum x_i^2 - N\mu^2}{N}$$

•  $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$  [Sample variance (ungrouped)]

$$s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{n-1}$$

For grouped, 
$$s^2 = \frac{\sum f_i(x_i - \bar{x})^2}{n-1}$$



Square root of variance

• 
$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$
 [Population standard deviation (ungrouped)]

• 
$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$
 [Sample variance (ungrouped)]



For example: weights of 10 newly born babies (in pounds)

7.5, 4.5, 10.1, 9.6, 5.5, 6.6, 7.8, 5.9, 6.0, 5.5

- a) Calculate the standard deviation

b) Interpretation



$\boldsymbol{x_i}$	$\overline{x}$	$(x_i - \overline{x})^2$

$\boldsymbol{x_i}$	$\overline{\boldsymbol{x}}$		$(x_i - \overline{x})^2$	
7.5	$\frac{\sum x_i}{n} = 6.9$		0.36	
4.5			0.57	
10.1			10.24	
9.6			7.29	
5.5			1.96	
6.6			0.09	
7.8			0.81	
5.9			1	
6.0			0.81	
5.5			1.96	

For example: weights of 10 newly born babies (in pounds)

7.5, 4.5, 10.1, 9.6, 5.5, 6.6, 7.8, 5.9, 6.0, 5.5

a) Calculate the standard deviation

b) Interpretation

For example: weights of 10 newly born babies (in pounds)

a) Calculate the standard deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{30.28}{10 - 1}} = \sqrt{3.36} = 1.83 \ pounds$$

• b) Interpretation

The average variation of the weights of the newly born babies from the mean weights is 1.83 pounds.



# Standard Deviation (Alternative)

$$s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{n-1} = 3.36$$

$\boldsymbol{x_i}$	$\overline{\boldsymbol{x}}$	$x_i^2$
7.5	$\frac{\sum x_i}{n} = 6.9$	56.25
4.5		20.25
10.1		102.01
9.6		92.16
5.5		30.25
6.6		43.56
7.8		60.84
5.9		34.81
6.0		36
5.5		30.25



#### Relative measures

 To compare the variation of two or more data sets having different or same units



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Coefficient of Range

Coefficient of MD

Coefficient of variance

Coefficient of QD



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Coefficient of Range

Coefficient of MD

Coefficient of variance

Coefficient of QD



# Coefficient of variance (CV)

It is a ratio

Between standard deviation (SD) and mean

Expressed as percent

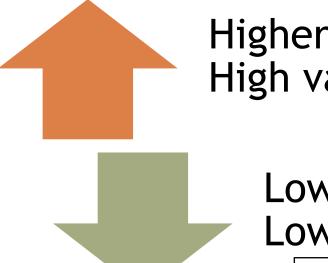
General formula of 
$$CV = \frac{SD}{mean} \times 100$$



# Coefficient of variance (CV)

• 
$$CV = \frac{\sigma}{\mu} \times 100$$
; [For population]

• 
$$CV = \frac{s}{\bar{x}} \times 100$$
; [For sample]



Higher CV = High variation

Lower CV = Low Variation

More consistent

More uniform

More homogeneous

More stable



# Coefficient of variance (CV)

Section A: Mean = 30 and SD = 4

$$CV_A = \frac{4}{30} \times 100 = 13.3\%$$

Section B: Mean = 25 and SD = 6

$$CV_B = \frac{6}{25} \times 100 = 24\%$$

• Which section is more consistent in getting final exam mark?

Since, the CV of A is lower than the CV of B. Thus, section A is more consistent comparatively section B



# OTHANK You