Measures of Central Tendency (1)

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Descriptive measures

- In previous chapter, we discussed how a raw data set can be organized and summarized by tables and graphs.

- Another method of summarizing data set precisely is to compute number (a single number).

- Number that can be describe data sets are called descriptive measures

Descriptive measures

- Four types of descriptive measure

1. Measures of central tendency
2. Measures of Location
3. Measures of Dispersion
4. Shape of the distribution

Measures of Central Tendency

- It is a summary measure
- Attempts to describe the whole data set with a single value
- Represents the center of the distribution

Types of C.T.

1. Mean
2. Median
3. Mode

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Types of mean

1. Arithmetic mean
2. Geometric mean
3. Harmonic mean

Arithmetic Mean

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- $\boldsymbol{Mean} = (???)$

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- $\boldsymbol{Mean = \frac{(5+3+9+2+7+5+8)}{7} = 5.57\ Unit}$

Formula: AM

Ungrouped Data	Grouped Data

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Group data: AM

Class	Frequency
5-9	4
9-13	3
13-17	3
17-21	3

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5-9	4
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Calculate mean from this table…

Group data: AM

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Calculate mean or arithmetic mean from this table…

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Calculate mean or arithmetic mean or average from this table…

Group data: AM

Class	Frequency
5-9	4
9-13	3
13-17	3
17-21	3

$f_i = Frequency$
$x_i = Mid\ value$

Calculate mean or arithmetic mean or average from this table…

$$\bar{X}_{AM} = \frac{\sum(\boldsymbol{f_i} \times \boldsymbol{x_i})}{\sum \boldsymbol{f_i}}$$

Group data: AM

Class	f_i	x_i	$f_i \times x_i$
5-9	4	7	28
9-13	3	11	33
13-17	3	15	45
17-21	3	19	57
Total	13		163

Calculate mean or arithmetic mean or average from this table…

$$\bar{X}_{AM} = \frac{\sum(\boldsymbol{f_i} \times \boldsymbol{x_i})}{\sum \boldsymbol{f_i}} = \frac{\mathbf{163}}{\mathbf{13}} = \mathbf{12.53}$$

Group data: AM

Class	f_i	x_i
5-9	4	7
9-13	f_1	11
13-17	3	15
17-21	3	19

Find the value of f_1, when the average in 12.53

Group data: AM

Class	f_i	x_i	$f_i \times x_i$
5-9	4	7	28
9-13	f_1	11	$11f_1$
13-17	3	15	45
17-21	3	19	57

Find the value of f_1, when the average in 12.53

Solution:
We know that,

$$\bar{x} = \frac{\sum(f_i \times x_i)}{\sum f_i}$$

$$12.53 = \frac{28 + 11f_1 + 45 + 57}{4 + f_1 + 3 + 3}$$

$$\therefore f_1 = 3$$

Self Practice

No of observations (x_i)	f_i
0	46
1	f_1
2	f_2
3	25
4	10
5	5

Find the values of f_1 and f_2, when the average is 1.46, and total number of observations is 200

Geometric Mean

- It is useful when dealing with data that exhibits
- Exponential growth, or
- Growth over the year/ change over a period of times, or
- Geometric progression

Geometric Mean

GM for ungrouped data:

$$\bar{x}_{\mathrm{GM}} = (x_1 \times x_2 \times \cdots \times x_n)^{\frac{1}{n}}$$

GM for grouped data:

$$\bar{x}_{\mathrm{GM}} = \left(x_1^{f_1} \times x_2^{f_2} \times \cdots \times x_n^{f_n}\right)^{\frac{1}{\sum f_i}}$$

Geometric Mean

- For example: 5, 3, 9, 2, 7, 5, 8

- $\bar{X}_{GM} = (5 \times 3 \times 9 \times 2 \times 7 \times 5 \times 8)^{\frac{1}{7}} = 4.98$

Geometric Mean

- Find the geometric mean for the following distribution:

Marks	0-10	10-20	20-30	30-40	40-50
f_i	5	7	15	25	8

Geometric Mean

- If x_i are the percent value (rate) for a given time t,

$$Average\ growth\ rate = \{(1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_n)\}^{\frac{1}{n}} - 1$$

$$Average\ depreciation\ rate = \{(1 - r_1) \times (1 - r_2) \times \cdots \times (1 - r_n)\}^{\frac{1}{n}} - 1$$

Geometric Mean

- Suppose you have an investment that grew by 10% in the first year, 5% in the second year, 8% in the third year. What was the average growth rate for these three years.

- Solution: Since, this is a geometric progression rate. So, geometric mean may used here.

Average growth rate, $\bar{X}_{GM} = \{(1+0.10)\times(1+0.05)\times(1+0.08)\}^{\frac{1}{3}} - 1$

$$\bar{X}_{GM} = 0.0748$$

Comment: Thus, the average growth rate is 7.48%

Geometric Mean

- Let's consider an assets that depreciated by 15% in the first year, 8% in the second year, and 12% in the third year. What was the average depreciation rate over these three years.

$$Average\ depreciation\ rate = \{(1 - r_1) \times (1 - r_2) \times \cdots \times (1 - r_n)\}^{\frac{1}{n}} - 1$$

Harmonic Mean

- It is necessary to compute the average of some variables such as the average speed, average velocity, and so on.

Harmonic Mean

HM for ungrouped data:

$$\bar{X} = \frac{\text{n}}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

HM for grouped data:

$$\bar{X} = \frac{\sum f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}}$$

Harmonic Mean

- For example: 5, 3, 9, 2, 7, 5, 8

- $\bar{X}_{HM} = \frac{7}{\frac{1}{5}+\frac{1}{3}+\cdots+\frac{1}{8}} = (???)$

Harmonic Mean

- A car travels 50 miles at 40 mph, 60miles at 50mph and 40 miles at 60mph.What is the average speed of the trip?

- Solution: Here, Distance: 50 miles, 60 miles, 40 miles

Speed: 40 mph, 50 mph, 60 mph

The mean (harmonic mean) can be written as,

$$HM = \frac{\sum f_i}{\sum\left(\frac{f_i}{x_i}\right)} = \frac{50+60+40}{\frac{50}{40}+\frac{60}{50}+\frac{40}{60}} = 48.08\ mph$$

Some points...

- $AM \geq GM \geq HM$

- $GM = \sqrt{AM \times HM}$

Mathematical exercise

To access additional mathematical problems, please refer to the PDF lecture notes.

Thank You