Sudoku — A Little Lesson in OOP

Axel T. Schreiner
Department of Computer Science
Rochester Institute of Technology
102 Lomb Memorial Drive
Rochester New York 14623-5608 USA
ats@cs.rit.edu

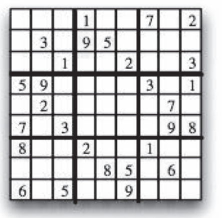
James E. Heliotis
Department of Computer Science
Rochester Institute of Technology
102 Lomb Memorial Drive
Rochester New York 14623-5608 USA
jeh@cs.rit.edu

Abstract: Paying only lip service to the principles of object-oriented programming rarely results in the expected benefits. This paper presents a series of designs for a Sudoku application that will lead introductory students through the all-important process of trial and error. They will see examples of design analysis, criticism, and improvement. The paper concludes with some general pointers why and how the initial mistakes could have been avoided.

Categories and Subject Descriptors: D.2.3 [Coding Tools and Techniques]: Object-oriented programming, D.2.11 [Software Architectures]: Information hiding, Patterns
General Terms: Algorithms, Design, Languages
Keywords: Sudoku, Design, Model-view-controller pattern

1. INTRODUCTION

Sudoku [1], a simple logical puzzle invented in the United States, has turned into a global craze. A 9x9 grid is divided into nine non-overlapping boxes each containing a 3x3 grid of cells. Each row and each column of the original grid as well as each box must contain each of the digits from 1 to 9 exactly once. For a puzzle, a few digits are already entered in the grid and the remaining digits must be deduced.



Sudoku provides an intriguing story line for programming problems. For example, solvers [2,3,4] can serve to discuss approaches to problem solving mandated by different languages. On a less ambitious level, a Sudoku worksheet [5] should be a relatively simple example of a dynamically changing graphical user interface with undo capabilities, combined with a program state modeled by an array with various slices; the state of a grid cell can be an integer or a more sophisticated object controlling data entry.

Adding a solving algorithm might be too difficult for a more introductory programming course because it is likely to require backtracking in a relatively complex data structure, but students might enjoy being asked to implement some machine assistance for the more tedious aspects of Sudoku solving.

This paper chronicles two implementations of a graphical user interface for machine-assisted Sudoku puzzle solving. The first implementation seems to be object-oriented but ends up with rather confusing and inefficient code. The second implementation looks deeper into the philosophy of object orientation and produces a significantly better result. Excerpts from a preliminary version of this paper were presented informally at OOPSLA2006.

2. SUDOKU SOLVING PRIMER

Initially, a cell can contain any digit from 1 to 9. However, the cell belongs to a context of exactly three grid slices: one row, one column, and one box, and the cell may not contain any digit whose value is already in another cell in its context. This observation leads to the notion of a candidate list for each empty cell. Computing the lists is a major benefit of machine-assisted solving — and of course an anathema to all Sudoku aficionados.

There are a couple of small steps that even a limited solver can perform. First, if any candidate list is empty the puzzle has no solution. Second, if a candidate list is a singleton it obviously can decide which digit has to be entered into a cell.

Given a cell with a singleton candidate list, the singleton can be pruned from all candidate lists in the context to which the cell belongs — a more significant step in machine-assisted solving.

Any puzzle worthy of the name will require more sophisticated pruning. For example, if two candidate lists in the same slice are equal pairs, they leave a choice of two digits for two cells in the slice, but the pair can be pruned from the other lists in the slice — often a leap of sophistication in automated solving.

3. IMPLEMENTATIONS

The steps of the programming assignment [5] are intended to encourage an object-oriented solution:

1. Create an observable model (puzzle state and some algorithms, in particular, undo), and test it from the command line.
2. Create a read-only view to experiment with layout and observing.
3. Finally, show the candidate lists in a dynamically changing view, which allows undo and candidate selection based on the lists and which sends requests back to the model.

3.1 Half-Baked Programming

The assignment is structured to include OOP paradigms such as Model-View-Controller and the observer pattern. To make it even simpler, the model is required to have methods such as the following.

```
int[] get (int row, int col)
```

returns candidate digits for the indicated position

```
void set (int row, int col, int digit)
```

enters the digit at the indicated position, remembers the operation for undo, and discards any operations remembered for redo.

The only significant design decision seems to be when to deal with the candidate lists — assume for the moment that each cell is represented as an Integer. set could inform the other cells in its context that a digit has been found or get could compute the lists as needed — following one or more undo operations, get will likely have to ask the cells for current information anyhow. The code for creating or updating a candidate list should look something like the following:

```
// Create canBe candidate list for [row,col]
canBe = new BitSet();
search:
for (int digit=1; digit<=dim; ++digit) {
 // row
 for (int c=0; c<board[row].length; ++c)
  if (c!=col && board[row][c].equals(digit))
   continue search;
 // column
 for (int r=0; r<board.length; ++r)
  if (r!=row && board[r][col].equals(digit))
   continue search;
 // box
 int r = (row/boxDim)*boxDim,
     c = (col/boxDim)*boxDim;
 for (int i = 0; i < boxDim; ++i)
  for (int j=0; j<boxDim; ++j)
   if ((r+i !=row || c+j !=col)
       && board[r+i][c+j].equals(digit))
    continue search;
 // digit is a viable candidate
 canBe.set(digit);
}
```

It looks gruesome, but array slicing has been dealt with. The model could even prune singletons, i.e., if get returns an array of length 1 this could be considered equivalent to a set operation. However, this is likely to confuse the user interface for undo.

It is now time to revisit an earlier assumption as the next design decision: how to model a cell, i.e., what are the elements of each row of board? The idea for now is to make each cell an object that implements an interface Digit containing the following methods.

```
boolean equals (int digit)
```

returns true if digit was the value entered by set,

```
int[] digits ()
```

returns the (possibly cached) result of get, and

```
boolean canBe (int digit)
```

returns true if the digit is a candidate.

There are two implementations of the interface: Move can be the class that represents the effect of set and returns true for the proper argument to equals and false for all calls to `canBe`. On the other hand, before a set operation is successfully performed, there would be a class Digits that holds a candidate list. Digits would return false for equals and true if the argument of `canBe` is a candidate.

Unfortunately, once all the pieces are put together, the result is messy and looks impossible to extend. As discussed above, pruning singleton candidate lists can be done more or less silently, but there is no reasonable way to prune pairs.

If model and view are based on Java's Observable and Observer, the view will use get to inquire about the state of a cell but the result cannot distinguish between a digit entered by set or a singleton candidate list. Depending on the user interaction to be implemented, the view might have to track (and undo) all set operations itself!

3.2 OOP

Where did the approach go wrong — in spite of MVC, the observer pattern, and cell objects with different behaviors? There seem to be three basic mistakes: The model is not informative enough, the code for computing candidate lists exhibits some information leakage from the cell objects to the code in the model, and slice and context iteration should be uniform enough to employ the for-each loop as found in Java since version 5 [7].

3.2.1 Message Architecture

Communication between model and view was based on the Java classes Observable and Observer, i.e., the model sends update to the view and expects the view to use get to acquire the relevant information. This makes the model a rather passive participant in the object conversation and get does not even reveal enough. The following is a problem-specific observer interface that is implemented by the view and used by the model to send information to the view.

```
void move (int row, int col, int digit)
```

announces a users' move.

```
void ok (int row, int col, BitSet digits)
```

describes a candidate list.

```
void queues (int undos, int redos)
```

describes the number of undo and/or redo operations still possible.

Any number of these messages can be sent from the model to each observer as a response to a set, undo or redo operation. This hugely simplifies the implementation of a view because the view now only needs to visualize cell states as indicated by each move and ok it receives. queues messages can be used to control undo and redo button activation.

It turns out that the observer interface does not make the job described by the model interface more complicated. On the contrary it is made simpler. set is still used to enter a digit into a cell, change the undo state, and trigger re-computation of the candidate lists. But now as things change several messages are sent to the observers. An undo operation complements the last set which again results in a series of messages, and a redo operation acts like set but changes the undo state differently.

Table 1: Digit implementations

Digit subclasses:	Move	Digits
private fields	int digit; digit previously set in cell	BitSet digits; candidate list
int digit() returns effect of set	return digit;	throw …
BitSet digits() returns candidate list	throw …	return digits;
boolean isKnown() true if no more choice	return true;	return digits .cardinality() == 1;
prune digit from all related slices		
void infer0()	for (cell: context) cell.infer0(digit);	// no op
void infer0 (int digit)	// skip	digits.clear(digit); ok(. . . digits);
recompute candidates (used after undo())		
void infer1 ()	// no op	digits = 1 … 9; for (cell: context) cell.infer1(digits); ok(. . . digits);
void infer1 (BitSet set)	set.clear(digit);	// no op

Plus, as an unanticipated benefit, machine-assisted solving can now be naturally added to the model interface. Just like set, a request to infer moves from singletons, i.e., to turn singletons into selected digits, can also result in a number of move and ok messages, as can a request to prune singletons, or pairs, etc. The view simply visualizes the resulting changes in the state of the puzzle.

3.2.2 Information Hiding

Turning now to the problem of information leakage, table 1 shows part of what a cell (a Digit) should implement, depending on whether it represents the effect of set or a candidate list. The key aspect is that the representation of the candidate list has been moved into the cell object. While the constituents of a candidate list will have to be disclosed for an ok message to an observer, information hiding etiquette dictates that only a Digits object may modify its own candidate list. Therefore, when a new Move is created to represent a cell in response to a set operation and the candidate lists in the cells' context have to be modified, the algorithm has to be distributed across the two classes.

The last part of the table shows another distributed algorithm — for the benefit of undo, the candidate list can be recomputed from scratch.

Other algorithms are distributed in a similar fashion. For example, a singleton candidate list can broadcast the fact to its context for machine-assisted solving. The following methods are also part of the interface for a cell. They do nothing for a Move but they prune a singleton candidate list recursively(!) from its context:

```
void single () {
 if (digits.cardinality()==1)
  for (cell: context)
   if (cell!=this) cell.single(digits);
}
boolean single (BitSet neighborDigits) {
 if (digits.intersects(neighborDigits)) {
  digits.andNot(neighborDigits);
  ok(… digits);
  single();
  return true;
 } else return false;
}
```

Similar pairs of methods can be implemented to find unique digits in a context or to prune pairs. See [6] for details.

3.2.3 Iteration

Finally on the agenda for improvement is slice iteration. for-each iteration is built into the Java language. Many educators view this as a pure programming concern; however, iterator use is still an important way to illustrate reusability. In our example, array slicing into rows, columns, and boxes seems to require quite a bit of code duplication and therefore introduces a potential for error. In addition, one will note that, thus far, the methods shown were coded with the understanding that they should all benefit from the linguistic simplicity of the for-each loop introduced in Java 5 [7].

An iterator for a single slice can be based on the following abstract class:

```
abstract class Slice implements Iterator<Digit> {
 protected int pos=0; // state
 public boolean hasNext() { // default
  return pos<dim;
 }
 public void remove() {
  throw new UnsupportedOperationException();
 }
}
```

Based on this class, iterators for rows, columns, and boxes can easily be implemented. for-each requires an Iterable or an array but an array cannot be converted to an Iterable. Therefore, even a row has to be wrapped as an Iterable.

Iterators can be constructed from other iterators. Here is how an iterator over the context of a cell is constructed:

```
Iterable<Digit>[] slices (int row, int col) {
 return (Iterable<Digit>[])new Iterable<?>[]{
  row(row), column(col), box(row, col) };
}
Iterable<Digit> context (final int row,
                         final int col) {
 return new Iterable<Digit>() {
  Iterable<Digit>[] slices=slices(row, col);
  public Iterator<Digit> iterator () {
   return new Slice() {
    Iterator<Digit> slice =
                    slices[pos].iterator();
    public boolean hasNext() {
     return slice.hasNext();
    }
    public Digit next() {
     Digit result = slice.next();
     if (!slice.hasNext() && n<slices.length-1)
      slice = slices[++n].iterator();
     return result;
    }
   };
  }
 };
}
```

While this code may look daunting at first, especially in Java which lacks the syntactic sugar that C# provides for generating iterators, it should be noted that the code encapsulates every context traversal in a single place, i.e., when these iterators are used there is no way to mistakenly transpose row and column indexing or select invalid offsets within boxes.

4. CONCLUSIONS

Clearly, the first attempt at a Sudoku worksheet ended up a mess while the second attempt produced a framework where even additional algorithms can be plugged in. Both approaches look object-oriented but there are significant differences:

- The first model relied on existing classes to implement the observer pattern. When it sends an update message it expects the view to find out what it needs to know. Unfortunately, the information function get did not provide sufficient detail.
- The first model used cell objects but did not involve them in algorithms such as candidate list pruning. This resulted in information leakage and complicated algorithms.
- The first model contained a lot of code duplication because there was no uniform approach to array slicing, i.e., there was no systematic use of the iterator pattern.

Through the apparently simple problem of a popular puzzle, we have succeeded in demonstrating in a very practical way the advantages of following basic rules of object-oriented design, and even several of the more advanced design tenets. Here are some examples:

- *When done properly, information hiding expands the usefulness of the objects you design.*
- *The instanceof operator can and should be avoided.*
- *In OOD, algorithms should be distributed over the involved objects through method invocation (object-to-object communication).*
- *Solve large problems by breaking them down into small problems (divide-and-conquer).*
- *As you gain experience with your design, be prepared to refactor it: change the level of an interface, shift responsibilities, etc.* Perhaps more fundamental is the famous rule, “Plan to throw one away; you will anyhow.” [8]

The Sudoku puzzle has shown itself to be an application domain that can be visited once or as many times as appropriate for a laboratory course involving programming and design with objects.

REFERENCES

[1] Mepham M. *Home of the Sudokulist* June 26, 2006 <http://www.sudoku.org.uk/>.

[2] Schreiner, A. *Sudoku Solver (Scheme)* Sep. 21, 2005 <http://www.cs.rit.edu/~ats/plt-2005-1/2/solved.html>.

[3] Alliet, B. *Sudoku Solver (Haskell)* Oct. 12, 2005 <http://darcs.brianweb.net/sudoku/Sudoku.pdf>.

[4] Schreiner, A. *Sudoku Solver (awk/Prolog)* Feb. 9, 2006 <http://www.cs.rit.edu/~ats/plcr-2005-2/ 7/solved.html>.

[5] Schreiner, A. *Sudoku Worksheet* Jan. 19, 2006 <http://www.cs.rit.edu/~ats/java-2005-2/5/problem.html>.

[6] Schreiner, A. *Solution for Sudoku Worksheet* Jan. 26, 2006 <http://www.cs.rit.edu/~ats/java-2005-2/5/solved.html>.

[7] Sun Microsystems *The For-Each Loop* Aug. 25, 2005 <http://java.sun.com/j2se/1.5.0/docs/guide/language/foreach.html>.

[8] Brooks, F. *The Mythical Man Month* (2nd ed) Addison-Wesley 1995.