When presenting these examples we stressed the fact that each problem can be formalised as a CSP in a number of different ways. In general, it is difficult to find which representation is better and good insights into the principles of constraint programming are of help. We shall return to this matter in Chapter 9. When discussing various alternative representations of a problem as a CSP we focused here on simple examples. Less contrived examples will be mentioned at the end of the book, in Chapter 9.

2.8 Exercises

Exercise 2.1 Consider the following multiplication problem:

```
    P P P
      P P
---------
  P P P P
P P P P
---------
P P P P P
```

where each `P` stands for a (possibly different) prime digit (so 2, 3, 5 or 7). Formulate this problem as a CSP.

For your information: this problem has a unique solution, namely:

```
    7 7 5
      3 3
---------
  2 3 2 5
2 3 2 5
---------
2 5 5 7 5
```

Exercise 2.2 Consider the following puzzle.

Ten cells numbered 0,..,9 inscribe a 10-digit number such that each cell, say i, indicates the total number of occurrences of the digit i in this number. Find this number.

For your information: the answer is 6210001000. Indeed, cell 0 is filled with 6 and there are 6 zeros in this number, etc.

Formulate this problem as a CSP problem.

Exercise 2.3 *Magic Squares.*

A ***magic square of order*** n is defined to be an $n \times n$ matrix made out of the integers from $[1..n^2]$ arranged in such a way that the sum of every row, column, and the two main diagonals is the same. For example

```
 1 15 24  8 17
23  7 16  5 14
20  4 13 22  6
12 21 10 19  3
 9 18  2 11 25
```

is a magic square of order 5, because each row, column and main diagonal sums up to 65. Formulate the problem of finding a magic square of order n as a task of finding a solution to a CSP.

Exercise 2.4 *Latin Squares.*

A ***Latin square of order*** n is defined to be an $n \times n$ matrix made out of the integers in $[1..n]$ with the property that each of the these n integers occurs exactly once in each row and exactly once in each column of the array. For example

```
1 2 3 4 5
2 3 4 5 1
3 4 5 1 2
4 5 1 2 3
5 1 2 3 4
```

is a Latin square of order 5. Formulate the problem of finding a Latin square of order n as a CSP.

Exercise 2.5 *The Graph Colouring Problem.*

Consider the task of assigning to each node of a finite graph a colour in such a way that no two adjacent nodes have the same colour. Such an assignment is called a ***colouring*** of the graph. A colouring of the graph involving the minimal number of colours is called the ***chromatic number*** of the graph. The problem of finding the chromatic number of a graph has several applications, in particular in the fields of scheduling and compiler optimization.

Formulate this problem as a constrained optimization problem.

Exercise 2.6 *The Frequency Assignment Problem.*

Given is a set of n ***cells***, $C := \{c_1, c_2, \ldots, c_n\}$ and a set of m ***frequencies*** (or ***channels***) $F := \{f_1, f_2, \ldots, f_m\}$. An ***assignment*** is a function which associates with each cell c_i a frequency $x_i \in F$. The problem consists in finding an assignment that satisfies the following constraints:

Separations: Given h and k we call the value $d(f_h, f_k) = \mid h - k \mid$ the ***distance*** between two channels f_h and f_k. (The assumption is that consecutive frequencies lie one unit apart.) Given is an $n \times n$ non-negative integer symmetric matrix S, called a ***separation matrix***, such that each s_{ij} represents the minimum distance between the frequencies assigned to the cells c_i and c_j. That is, for all $i \in [1..n]$ and $j \in [1..n]$ it holds that $d(x_i, x_j) \geq s_{ij}$.

Illegal channels: Given is an $n \times m$ boolean matrix F such that if $F_{ij} = \mathit{true}$, then the frequency f_j cannot be assigned to the cell i, i.e., $x_i \neq f_j$.

Separation constraints prevent interference between cells that are located geographically close and that broadcast in each other's area of service. Illegal channels account for channels reserved for external uses (e.g., for military bases). Formalise this problem as a CSP.

Exercise 2.7 Consider the following variant of the Eight Queens Problem. One is asked for a given integer k to place on the (8×8) chess board a maximum number of queens so that each of them attacks exactly k other queens. Formulate this problem as a constrained optimization problem.
Hint. See the alternative formalisation of the n Queens Problem given in Example 2.7.

Exercise 2.8 *Peaceable Coexisting Armies of Queens.*

Two armies of queens (black and white) peacefully coexist on a chessboard when they are placed upon the board in such a way that no two queens from opposing armies can attack each other. The problem is to find the maximum two equal-sized armies. Formulate this problem as a constrained optimization problem.

Exercise 2.9 Formulate the following problem as a constrained optimization problem:

Place a minimum number of queens on the chess board so that each unoccupied field comes under attack.