Deterministic Learning and Rapid Dynamical Pattern Recognition

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***Abstract*—Recognition of temporal/dynamical patterns is among the most difficult pattern recognition tasks. In this paper, based on a recent result on deterministic learning theory, a deterministic framework is proposed for rapid recognition of dynamical patterns. First, it is shown that a time-varying dynamical pattern can be effectively represented in a time-invariant and spatially distributed manner through deterministic learning. Second, a definition for characterizing similarity of dynamical patterns is given based on system dynamics inherently within dynamical patterns. Third, a mechanism for rapid recognition of dynamical patterns is presented, by which a test dynamical pattern is recognized as similar to a training dynamical pattern if state synchronization is achieved according to a kind of internal and dynamical matching on system dynamics. The synchronization errors can be taken as the measure of similarity between the test and training patterns. The significance of the paper is that a completely dynamical approach is proposed, in which the problem of dynamical pattern recognition is turned into the stability and convergence of a recognition error system. Simulation studies are included to demonstrate the effectiveness of the proposed approach.**

***Index Terms*—Deterministic learning, dynamical pattern recognition, representation, similarity, synchronization.**

I. INTRODUCTION

HUMANS generally excel in dealing with temporal patterns. Human recognition of temporal patterns is an integrated process, in which patterns of information distributed over time can be effectively identified, represented, recognized, and classified. A distinguishing feature of the human recognition process is that it takes place quickly from the beginning of sensing temporal patterns, and temporal patterns are directly processed on the input space for feature extraction and pattern matching [10]. These recognition mechanisms, although not fully understood, are probably quite different from the existing neural network (NN) and statistical approaches for pattern recognition. So far, a great deal of progress has been made for recognition of static patterns (e.g., [7]–[9], and [19]–[21]), however, only limited success has been reported in the literature for rapid recognition of temporal patterns.

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One early result for classification of spatiotemporal patterns is Grossberg's formal avalanche structure [28]. A popular approach for temporal pattern processing is to construct short-term memory (STM) models, such as delay lines [33], decay traces [30], [34], and exponential kernels [31]. These STM models are then embedded into different NN architectures. For example, the time-delay neural network (TDNN) is proposed by combining multilayer perceptrons (MLPs) with the delay line model [33]. With STM models, a temporal pattern is represented as a sequence of pattern states, and recognition of temporal patterns is quite similar to the recognition of static patterns. On the other hand, it has been realized that architectures more natural to deal with temporal patterns are feedback or recurrent NNs [29], [32], [35], [36].

Generally speaking, a difficult problem in temporal pattern recognition is how to appropriately represent the time-varying patterns. The topic of temporal coding, and particularly using neural representations, recently has also become an important topic in neuroscience and related fields (see, e.g., [11]). Among the unresolved problems in this field, one of the most fundamental questions is how temporal patterns can be represented in a time-independent manner [10]. Another important problem currently studied in this area is the definition of similarity between two temporal patterns. This is difficult because, when considering parameter variations, noise, disturbance, etc., it is of course unlikely that two temporal patterns will occur identically. As temporal patterns evolve with time, the existing similarity measures developed for static patterns do not seem appropriate for temporal patterns. For the aforementioned reasons, it appears that in the current literature there are few results on efficient representation, standard similarity definition, and rapid recognition of temporal patterns.

In this paper, we investigate the recognition of a class of temporal patterns, referred to as dynamical patterns, which are generated from a general nonlinear dynamical system

$$\dot{x} = F(x;p) \quad x(t_0) = x_0 \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state of the system, p is a vector of system parameters, and $F(x;p) = [f_1(x;p), \ldots, f_n(x;p)]^T$ represents the system dynamics, in which each $f_i(x;p)$ is an unknown, continuous nonlinear function. A *dynamical pattern* is defined as a recurrent system trajectory generated from the above dynamical system. The class of recurrent trajectories includes periodic, quasi-periodic, almost-periodic, and even some chaotic trajectories, which comprise the most important types (though not all types) of trajectories generated from nonlinear dynamical systems (see [24] for a rigorous definition of recurrent trajectory). Nonlinear dynamical system theory has been found responsible for the formation of numerous dynamical patterns in areas such as hydrodynamics, oceanography, meteorology, biological morphodynamics, and semiconductors [25]–[27]. In other words, nonlinear dynamical systems are capable of exhibiting various types of dynamical patterns. Therefore, the dynamical pattern defined previously covers a wide class of temporal patterns studied in the literature.

The general recognition process for a dynamical pattern usually consists of two phases: the identification phase and the recognition phase. Here, "identification" involves working out the essential features of a pattern one does not recognize, while "recognition" means looking at a pattern and realizing that it is the same or a similar pattern to one seen earlier. For identification of dynamical patterns, we recently proposed a deterministic learning theory for nonlinear dynamical systems [23], [38] by which a locally accurate NN approximation of the underlying system dynamics $F(x;p)$ within a dynamical pattern can be achieved by using localized radial basis function (RBF) NNs. Through deterministic learning, fundamental information of dynamical patterns is obtained and stored as constant RBF neural weights.

In this paper, based on the deterministic learning theory, a unified, deterministic framework is presented for effective representation, similarity definition, and rapid recognition of dynamical patterns. We first propose that, by using the constant RBF networks obtained through deterministic learning, time-varying dynamical patterns can be effectively represented by the locally accurate NN approximations of system dynamics $F(x;p)$. The representation is *time-invariant* in the sense that it is independent of the time attribute. The representation is also *spatially distributed*, since fundamental information is stored in a large number of neurons distributed along the state trajectory of a dynamical pattern. Thus, complete information of both the pattern state and the underlying system dynamics is utilized for appropriate representation of a dynamical pattern. The spatially distributed information implies that a representation using a limited number of extracted features (as in static pattern recognition) is probably incomplete for representation of dynamical patterns. It will be shown that the representation is essential for similarity definition and rapid recognition of dynamical patterns.

Second, a similarity definition for dynamical patterns based on system dynamics is given. From the qualitative analysis of nonlinear dynamical systems, it is understood that the similarity between two dynamical behaviors lies in the *topological equivalence* of two dynamical systems (see [24] for more discussions). This implies that the similarity of dynamical patterns is determined by the similarity of the system dynamics inherently within these dynamical patterns. Thus, in this paper, we propose a similarity definition for dynamical patterns based on information from both system dynamics and pattern states: Dynamical pattern $\mathcal{A}$ is similar to dynamical pattern $\mathcal{B}$ if 1) the state of pattern $\mathcal{A}$ stays within a local region of the state of pattern $\mathcal{B}$ and 2) the difference between the corresponding system dynamics along the state trajectory of pattern $\mathcal{A}$ is small. It is seen that the time attribute of dynamical patterns is excluded from the similarity definition.

Third, we investigate the mechanism for rapid recognition of dynamical patterns. To achieve recognition of a test dynamical pattern from a set of training dynamical patterns, one possible method is to identify the system dynamics of the test dynamical pattern into a constant RBF network (as done for training dynamical patterns through deterministic learning), and then compare the corresponding system dynamics with those of training dynamical patterns. One problem with such a method is that a direct comparison of the NN approximations of system dynamics may be computationally demanding for the time available. For rapid recognition of a test dynamical pattern, it is preferred not to identify the system dynamics again, and complicated computations should be avoided as much as possible for easy and fast recognition.

Based on the time-invariant representation and the similarity definition, we propose an approach for rapid recognition of dynamical patterns. A set of dynamical models are constructed as representatives of the training dynamical patterns, in which the constant RBF networks obtained from the identification phase are embedded. The constant RBF networks can quickly recall the learned knowledge by providing accurate approximations to the previously learned system dynamics of a training dynamical pattern. When a test pattern is presented to one of the dynamical models, a recognition error system is formed, which consists of the system generating the test pattern and the dynamical model corresponding to one of the training patterns. Without identifying the system dynamics of the test pattern, and so without comparing system dynamics of corresponding dynamical patterns via numerical computation, a kind of *internal* and *dynamical* matching of system dynamics of the test and training pattern proceeds in the recognition error system. The state synchronization errors will be proven to be (approximately) proportional to the differences of system dynamics. Thus, the synchronization errors can be taken as similarity measures between the test and the training dynamical patterns. As a result, the test dynamical pattern is being recognized as similar to a training pattern if the state of the dynamical model synchronizes closely with the state of the test pattern.

The significance of the paper is that a novel, completely dynamical framework is presented for temporal/dynamical pattern recognition without using STM models. Compared with other results on stability and convergence analysis of feedback neural networks, e.g., the profound work by Grossberg and his colleagues [29], [32], we utilize more advanced concepts and theories from adaptive control and dynamical systems in this paper, such as the persistence of excitation (PE) condition, recurrent trajectories, and topological equivalence. Specifically, 1) by achieving the satisfaction of the PE condition for a recurrent trajectory, a time-varying dynamical pattern is represented in a time-invariant manner through deterministic learning and 2) by using the concept of topological equivalence, a similarity definition for dynamical patterns is given based on the difference between the system dynamics of dynamical patterns. Further, the problem of rapid recognition of a test dynamical pattern is turned into the stability and synchronization of a recognition error system, in which the representation and similarity definition are involved in a dynamical manner. It is seen that rapid recognition of a test dynamical pattern occurs from the beginning of measuring the state of test pattern, and automatically proceeds with the evolution of the recognition error system. It does not need any numerical computation for pattern matching or feature extraction (which is normally required in many existing approaches for static pattern recognition). This recognition mechanism, with the similarity measures explicitly given, will further facilitate the construction of pattern recognition systems by using the intuitive recognition principle, e.g., the minimal distance or nearest neighbor classification [8]. The constructed recognition system can distinguish and classify dynamical patterns with qualitatively different behaviors, and can assign dynamical patterns based on the similarity of system dynamics to predefined classes.

Throughout the paper, we make the assumption that accurate measurement of all system states are available. In practice, however, it may not be the case. It is, therefore, of interest to extend this work to the cases of partial-state measurements or even just a single output measurement in a noisy environment. It is also necessary to consider the learning and recognition of dynamical patterns using variable neural networks (e.g., as in [37]) for practical implementation. Due to the limitation of space, the investigation along these directions will not be pursued in this paper. The rest of the paper is organized as follows. A brief review on deterministic learning theory is given in Section II. Section III discusses the representation and similarity of dynamical patterns. Section IV investigates the mechanism for rapid recognition of dynamical patterns. Also, simulation results are included in Sections III and IV and Section V concludes the paper.

II. Identification of Dynamical Patterns

In this section, we present a brief review of the deterministic learning theory developed in [23] and [38]. The deterministic learning theory was recently proposed for identification of nonlinear dynamical systems undergoing periodic or recurrent motions. By using the localized RBF neural network, a partial PE condition, i.e., the PE condition of a certain regression subvector constructed out of the RBFs along the recurrent system trajectory, is proven to be satisfied. This partial PE condition leads to exponential stability of the identification error system along the recurrent system orbit. Consequently, accurate NN approximation of the system dynamics is achieved in a local region along the recurrent system trajectory. As this learning mechanism is developed by utilizing concepts and theories of adaptive control and dynamical systems (e.g., [1], [2], and [24]), rather than by introducing algorithms from statistical principles (e.g., [9] and [21]), it is referred to as "deterministic learning" in comparison with the celebrated statistical learning. The deterministic learning theory is essential for the identification of the system dynamics of training dynamical patterns.

Consider nonlinear dynamical system (1). Assume that when started with initial condition x_0, (1) generates a periodic or periodic-like (recurrent) system trajectory. This system trajectory is referred to as a dynamical pattern, and is denoted throughout this paper as $\varphi_\zeta(x_0;p)$ or φ_ζ for conciseness of presentation. The time-varying system state $x(t)$ is referred to as the state of the dynamical pattern $\varphi_\zeta(x_0;p)$. From nonlinear system research (e.g., [22]), it is known that different p, and sometimes different x_0, may yield different dynamical patterns. The objective of deterministic learning [23] is to develop a neural identifier such that the dynamics $F(x;p)$ inherently within dynamical pattern φ_ζ can be accurately identified by the employed neural networks.

A. Dynamical Localized RBF Networks

The following dynamical RBF network is employed for identification of (1):

$$\dot{\hat{x}} = -A(\hat{x} - x) + \widehat{W}^T S(x) \tag{2}$$

where $\hat{x} = [\hat{x}_1, \ldots, \hat{x}_n]^T$ is the state vector of the dynamical RBF network, x is the state of (1), $A = diag\{a_1, \ldots, a_n\}$ is a diagonal matrix, with $a_i > 0$ being design constants, and $\widehat{W}^T S(x) = [\widehat{W}_1^T S_1(x), \ldots, \widehat{W}_n^T S_n(x)]^T$ are localized RBF networks used to approximate the unknown $F(x;p) = [f_1(x;p), \ldots, f_n(x;p)]^T$ in (1). The employment of the localized RBF neural network is due to the following associated properties [23].

- **Linear-in-parameter form**. The RBF networks belong to a class of linearly parameterized networks, and can be described in the following form:

$$f_{nn}(Z) = W^T S(Z) = \sum_{i=1}^{N} w_i s_i(Z) \tag{3}$$

  where $Z \in \Omega_Z \subset R^q$ is the input vector, $W = [w_1, w_2, \cdot, w_N]^T \in R^N$ is the weight vector, $N > 1$ is the NN node number, and $S(Z) = [s_1(Z), \ldots, s_N(Z)]^T$, with $s_i(\cdot)$ being the RBFs. Commonly used RBFs include the Gaussian function and the inverse Hardy's multiquadric function, both of which are localized basis functions in the sense that $s_i(Z) \to 0$ as $\|Z\| \to \infty$ [6].
- **Universal approximation**. It has been shown (e.g., [6]) that for any continuous function $f(Z) : \Omega_Z \to R$, where $\Omega_Z \subset R^q$ is a compact set, and the NN approximator (3) (the node number N is sufficiently large), there exists an ideal constant weight vector W^* such that for each $\epsilon^* > 0$

$$f(Z) = W^{*T} S(Z) + \epsilon(Z) \qquad \forall Z \in \Omega_Z \tag{4}$$

  where $|\epsilon(Z)| < \epsilon^*$ ($\epsilon(Z)$ is denoted as ϵ hereafter to simplify the notation). Moreover, it was shown [13] that Gaussian RBF network $W^T S(Z)$, with a finite (and also large enough) number of Gaussian nodes centered on a regular lattice on Ω_Z, and with fixed variances, can uniformly approximate a smooth function $f(Z) : \Omega_Z \to R$ to any chosen tolerance ϵ^* according to (4).
- **Spatially localized learning**. For localized RBF networks, the spatially localized learning capability implies that for any point Z_ζ, or any bounded trajectory $Z_\zeta(t)$ within the compact set Ω_Z, $f(Z)$ can be approximated by using a limited number of neurons located at the neighborhood of the point, or in a local region along the trajectory

$$f(Z) = W_\zeta^{*T} S_\zeta(Z) + \epsilon_\zeta \tag{5}$$

  where $S_\zeta(Z) = [s_{j_1}(Z), \ldots, s_{j_\zeta}(Z)]^T \in R^{N_\zeta}$, with $N_\zeta < N$, $|s_{j_i}| > \iota$ ($j_i = j_1, \ldots, j_\zeta$), $\iota > 0$ is a small positive constant, $W_\zeta^* = [w_{j_1}^*, \ldots, w_{j_\zeta}^*]^T$, and ϵ_ζ is the approximation error, with $\Big\| \epsilon_\zeta| - |\epsilon| \Big|$ being small. It is seen that $S_\zeta(Z)$ is a subvector of $S(Z)$ with a reduced dimension.

- **Satisfaction of a partial PE condition**. The concept of the PE condition is of great importance in adaptive systems [1], [2]. The definition of the PE condition in a scalar form is restated as follows.

Definition 1: A piecewise-continuous, uniformly bounded vector function $\varpi : R^+ \rightarrow R^m$ is said to satisfy the PE condition, if there exist positive constants α_1, α_2, and T_0 such that

$$\alpha_1 \geq \int_t^{t+T_0} |\varpi(\tau)^T c|^2 d\tau \geq \alpha_2 \qquad \forall t \geq t_0 \tag{6}$$

holds for all unit vectors $c \in R^m$.

For nonlinear system identification, it is very difficult to achieve characterization and *a priori* verification of the PE condition. Based on some recent results on the PE condition [3]–[5], we presented explicitly in [23] that any recurrent trajectory $Z(t)$, as long as it stays within the regular lattice, can lead to PE of a regression subvector $S_\zeta(Z)$ consisting of RBFs with centers located in a small neighborhood of $Z(t)$. Please also refer to [38] for a more detailed analysis.

Lemma 1: [23] Consider a periodic or a periodic-like (recurrent) trajectory $Z(t)$. Assume that $Z(t)$ is a continuous map from $[0, \infty)$ in a compact set $\Omega \subset R^q$, and $\dot{Z}(t)$ is bounded within Ω. Then, for localized RBF network $W^T S(Z)$, with centers placed on a regular lattice, the regressor subvector $S_\zeta(Z)$, as defined in (5), is persistently exciting in the sense of (6).

B. Exponential Stability and Accurate Identification

The weight estimates $\widehat{W}_i$ in (2) are updated by the Lyapunov-based learning law

$$\dot{\widehat{W}}_i = \dot{\widetilde{W}}_i = -\Gamma_i S_i(x) \breve{x}_i - \sigma_i \Gamma_i \widehat{W}_i \tag{7}$$

where $\widetilde{W}_i = \widehat{W}_i - W_i^*$, $\widehat{W}_i$ is the estimate of W_i^*, $\breve{x} = \hat{x} - x$, $\Gamma_i = \Gamma_i^T > 0$, and $\sigma_i > 0$ is a small value.

Along the state trajectory $\varphi_\zeta(x(0))$, the identification error system, which consists of the nonlinear dynamical system (1), the dynamical RBF network (2), and the NN weight learning law (7), is described by

$$\begin{bmatrix} \dot{\breve{x}}_i \\ \dot{\widetilde{W}}_{\zeta i} \end{bmatrix} = \begin{bmatrix} -a_i & S_{\zeta i}(x)^T \\ -\Gamma_{\zeta i} S_{\zeta i}(x) & 0 \end{bmatrix} \begin{bmatrix} \breve{x}_i \\ \widetilde{W}_{\zeta i} \end{bmatrix} + \begin{bmatrix} -\epsilon_{\zeta i} \\ -\sigma_i \Gamma_{\zeta i} \widehat{W}_{\zeta i} \end{bmatrix} \tag{8}$$

and

$$\dot{\widehat{W}}_{\bar{\zeta} i} = \dot{\widetilde{W}}_{\bar{\zeta} i} = -\Gamma_{\bar{\zeta} i} S_{\bar{\zeta} i}(x) \breve{x}_i - \sigma_i \Gamma_{\bar{\zeta} i} \widehat{W}_{\bar{\zeta} i} \tag{9}$$

where subscripts $(\cdot)_{\zeta i}$ and $(\cdot)_{\bar{\zeta} i}$ stand for the regions close to and away from the trajectory, respectively, $S_{\zeta i}(\varphi_\zeta)$ is a subvector of $S_i(x)$, $\widehat{W}_{\zeta i}$ is the corresponding weight subvector, and $|\epsilon_{\zeta i}|$ is close to $|\epsilon_i|$.

Since PE of $S_{\zeta i}(x)$ is guaranteed by Lemma 1, based on the stability results given in [1], the exponential stability of $(\breve{x}_i, \widetilde{W}_{\zeta i}) = 0$ of the nominal part of system (8) can be achieved. Consequently, both the state error $\breve{x}_i(t)$ and the parameter error $\widetilde{W}_{\zeta i}(t)$ can be concluded to converge exponentially to some small neighborhoods of zero, with the sizes of the neighborhoods being determined by $|\epsilon_i^*|$ and $\sigma_i \|\Gamma_{\zeta i} W_{\zeta i}^*\|$. The convergence of $\widetilde{W}_{\zeta i}(t)$ implies that $\widehat{W}_{\zeta i}$ converges to a small neighborhood of $W_{\zeta i}^*$. Then, along the trajectory $\varphi_\zeta(x_0)$, we have

$$f_i(x; p) = \widehat{W}_{\zeta i}^T S_{\zeta i}(x) + \epsilon_{\zeta i_1} \tag{10}$$

where $\epsilon_{\zeta i_1}$ is close to $\epsilon_{\zeta i}$ and ϵ_i. On the other hand, due to the localization property of the employed RBFs, it can be concluded that the entire RBF network $\widehat{W}_i^T S_i(x)$ can approximate the unknown $f_i(x; p)$ along the trajectory $\varphi_\zeta(x_0)$ as

$$f_i(x; p) = \widehat{W}_i^T S_i(x) + \epsilon_{i_1} \tag{11}$$

where $|\epsilon_{i_1}|$ is close to $|\epsilon_{\zeta i_1}|$ and $|\epsilon_i|$.

Due to the convergence of $\widehat{W}_i$, by using

$$\boxed{\overline{W}_i = \mathrm{mean}_{t \in [t_a, t_b]} \widehat{W}_i(t)} \tag{12}$$

where $t_b > t_a > 0$ represent a time segment after the transient process, we can obtain a constant vector of neural weights $\overline{W}_i$. From (10), the system dynamics $f_i(x; p)$ along the orbit $\varphi_\zeta(x_0)$ can be described using constant RBF neural weights

$$f_i(x; p) = \overline{W}_{\zeta i}^T S_{\zeta i}(x) + \epsilon_{\zeta i_2} \tag{13}$$

where $\overline{W}_{\zeta i} = [\bar{w}_{j_1}, \ldots, \bar{w}_{j_\zeta}]^T$ is the subvector of $\overline{W}_i$ and $|\epsilon_{\zeta i_2}|$ is small and is close to $|\epsilon_{\zeta i_1}|$. Moreover, due to the localization property of the employed RBF networks, from (11), we have

$$f_i(x; p) = \overline{W}_i^T S_i(x) + \epsilon_{i_2} \tag{14}$$

where $|\epsilon_{i_2}|$ is small and is close to $|\epsilon_{i_1}|$.

Theorem 1 is concluded.

Theorem 1: [23] Consider the identification error system (8) and (9), which consists of the nonlinear dynamical system (1), the dynamical RBF network (2), and the NN weight updating law (7). For any recurrent trajectory $\varphi_\zeta(x_0)$, starting from an initial condition $x_0 = x(0) \in \Omega$, and with initial values $\widehat{W}_i(0) = 0$, the neural-weight estimates $\widehat{W}_{\zeta i}$ [as given in (8)] converge to small neighborhoods of their optimal values $W_{\zeta i}^*$, and a locally accurate NN approximation for the unknown $F(x; p)$ is obtained along the state trajectory $\varphi_\zeta(x_0)$.

Remark 1: Note that in the literature of nonlinear system identification (e.g., [14], [17], and [18]), accurate state tracking has been achieved, however, the problem of modeling the underlying system dynamics has been less investigated. Although Lyapunov-based learning laws (7) have appeared, e.g., in [5] and [13]–[16], due to the difficulties in satisfying the PE condition, accurate approximation of dynamics $F(x; p)$ has not been achieved so far in the NN identification literature. With the satisfaction of the partial PE condition of $S_{\zeta i}(x)$ (as established in Lemma 1), a locally accurate NN approximation of system dynamics is rigorously proved in Theorem 1 [23]. This locally accurate identification is obtained by making full use of the associated properties of the localized RBF network, especially the one concerning the PE condition.

Remark 2: The recurrent trajectory represents a large class of motions generated from nonlinear dynamical systems, and so corresponds to various dynamical patterns such as periodic, quasi-periodic, almost-periodic, and even some chaotic ones [24]. The satisfaction of the partial PE condition for recurrent trajectories will lead to accurate identification of the underlying system dynamics of many kinds of dynamical patterns.

III. Representation and Similarity

In static pattern recognition, a pattern is usually a set of time-invariant measurements or observations represented in vector or matrix notation [7], [8]. The dimensionality of the vector or matrix representation is generally kept as small as possible by using a limited yet salient feature set for purposes such as removing redundant information and improving classification performance. For example, in statistical pattern recognition, a pattern is represented by a set of d features, or a d-dimensional feature vector which yields a d-dimensional feature space. Subsequently, the task of recognition or classification is accomplished when the d-dimensional feature space is partitioned into compact and disjoint regions, and decision boundaries are constructed in the feature space which separate patterns from different classes into different regions [8], [9].

For dynamical patterns, since the measurements are mostly time-varying in nature, the previous framework for static patterns may not be suitable for representation of dynamical patterns. As indicated in [10], if the time attribute could not be appropriately dealt with, the problem of *time-independent* representation without loss of discrimination power and classification accuracy would be a very difficult task for temporal/dynamical pattern recognition. Further, without a proper representation of dynamical patterns, the problem of how to define the similarity between two dynamical patterns will become another difficulty.

A. Time-Invariant and Spatially Distributed Representation

As introduced in Section II, the system dynamics $F(x;p) = [f_1(x;p), \ldots, f_n(x;p)]^T$ of a dynamical pattern φ_ζ can be accurately approximated by $\overline{W}_i^T S(x)$ $(i = 1, \ldots, n)$ in a local region along the recurrent orbit of the dynamical pattern φ_ζ. The *constant* RBF network $\overline{W}_i^T S(x)$ consists of two types of neural weights: 1) for neurons whose centers are close to the orbit $\varphi_\zeta(x_0)$, their neural weights $\widehat{W}_{\zeta i}$ converge exponentially to a small neighborhood of their optimal values $W^*_{\zeta i}$ and 2) for the neurons with centers far away from the orbit $\varphi_\zeta(x_0)$, the neural weights $\widehat{W}_{\bar{\zeta} i}$ will remain almost unchanged, i.e., remain close to zero. Thus, constant neural weights are obtained for all neurons of the entire RBF network $\overline{W}_i^T S(x)$. Accordingly, from Theorem 1 and (13) and (14), we have the following statements concerning the representation of a dynamical pattern.

1) A dynamical pattern φ_ζ can be represented by using the constant RBF network $\overline{W}_i^T S_i(x)$ $(i = 1, \ldots, n)$, which provides an NN approximation of the *time-invariant* system dynamics $f_i(x;p)$ $(i = 1, \ldots, n)$. This representation, based on the fundamental information extracted from the dynamical pattern φ_ζ, is independent of the time attribute. The NN approximation $\overline{W}_i^T S_i(x)$ is accurate only in a local region (denoted as Ω_{φ_ζ}) along the orbit $\varphi_\zeta(x_0)$. The locally accurate NN approximation provides an efficient solution to the problem of representation of time-varying dynamical patterns.
2) The representation by $\overline{W}_i^T S_i(x)$ is *spatially distributed* in the sense that relevant information is stored in a large number of neurons distributed along the state trajectory of a dynamical pattern. It shows that for appropriate representation of a dynamical pattern, complete information of both the pattern state and the underlying system dynamics is utilized. Specifically, a dynamical pattern is represented by using information of its state trajectory (starting from an initial condition), plus its underlying system dynamics along the state trajectory. Intuitively, the spatially distributed information implies that a representation using a limited number of extracted features (as in static pattern recognition) is probably incomplete for representation of dynamical patterns in many situations.

Concerning the locally accurate NN approximation, the local region Ω_{φ_ζ} is described by

$$\Omega_{\varphi_\zeta} := \Big\{ x \;\Big|\; \mathrm{dist}(x, \varphi_\zeta) < d_\zeta \Rightarrow |\overline{W}_i^T S_i(x) - f_i(x;p)| < \xi_i^*, i = 1, \ldots, n \Big\} \tag{15}$$

where d_ζ and $\xi_i^* > 0$ are constants and ξ_i^* is the approximation error that is close to ϵ_i^* within Ω_{φ_ζ}. This knowledge stored in $\overline{W}_i^T S_i(x)$ can be recalled in a way that whenever the NN input $Z (= x)$ enters the region Ω_{φ_ζ}, the RBF network $\overline{W}_i^T S_i(x)$ will provide accurate approximation to the previously learned dynamics $f_i(x;p)$.

Note that the representation by $\overline{W}_i^T S_i(x)$ will not be used directly for recognition, i.e., recognition by direct comparison of the corresponding neural weights. Instead, for a training dynamical pattern φ_ζ, we construct a dynamical model using $\overline{W}_i^T S_i(x)$ $(i = 1, \ldots, n)$ as

$$\dot{\bar{x}} = -B(\bar{x} - x) + \overline{W}^T S(x) \tag{16}$$

where $\bar{x} = [\bar{x}_1, \ldots, \bar{x}_n]^T$ is the state of the dynamical model, x is the state of an input pattern generated from (1), $\overline{W}^T S(x) = [\overline{W}_1^T S_1(x), \ldots \overline{W}_n^T S_n(x)]^T$ are constant RBF networks obtained through deterministic learning, and $B = \mathrm{diag}\{b_1, \ldots, b_n\}$ is a diagonal matrix, with $b_i > 0$ normally smaller than a_i [a_i is given in (2)].

It is clearly seen that the representation of dynamical patterns is quite different from the representation used in static pattern recognition. As will be detailed in Section IV, the dynamical model (16) will be used as a representative of the training dynamical pattern φ_ζ for rapid recognition of test dynamical patterns.

B. Fundamental Similarity Measure

In the area of pattern recognition, characterizing similarity between temporal or dynamical patterns is another difficult while important problem. It is known that there are many similarity definitions for static patterns, most of which are based on distances, e.g., Euclidean distance, Manhattan distance, and cosine distance [9]. To define the similarity of two dynamical patterns, the existing similarity definitions developed for static patterns might become inappropriate. The dynamical patterns evolve with time and the effects of different initial conditions or system parameters influence the occurrence of dynamical patterns.

To be specific, consider the dynamical pattern φ_ζ [as given by (1)], and another dynamical pattern [denoted as $\varphi_\varsigma(x_{\varsigma 0}, p')$ or φ_ς] generated from the following nonlinear dynamical system:

$$\dot{x} = F'(x; p') \quad x(t_0) = x_{\varsigma 0} \tag{17}$$

where the initial condition $x_{\varsigma 0}$, the system parameter vector p', and the nonlinear vector field $F'(x;p') = [f_1'(x;p'), \ldots, f_n'(x;p')]^T$ are possibly different with those for dynamical pattern φ_ζ. Since small changes in $x(t_0)$ or p' [or p in (1)] may lead to large change of $x(t)$, it is clear that the similarity of dynamical patterns φ_ζ and φ_ς cannot be established by using only the time-varying states $x(t)$ of the patterns, or by certain limited features extracted from $x(t)$.

In the qualitative analysis of nonlinear dynamical systems (e.g., [24]), the contemporary understanding of the similarity between two dynamical behaviors lies in the *topological equivalence* of two dynamical systems. It is understood from studies on nonlinear dynamical systems that the similarity of dynamical patterns is determined by the *similarity* of the system dynamics inherently within these dynamical patterns. Accordingly, in this paper, we propose the following definition of similarity for dynamical patterns.

Definition 2: Dynamical pattern φ_ς [given by (17)] is said to be similar with dynamical pattern φ_ζ [given by (1)], if the state of pattern φ_ς stays within a neighborhood region of the state of pattern φ_ζ, and the differences between the corresponding system dynamics are small along the state of pattern φ_ς, i.e.,

$$\max_{x \in \varphi_\varsigma(x_{\varsigma 0}; p')} \left| f_i'(x;p') - f_i(x;p) \right| < \varepsilon_i^*, \qquad i = 1, \ldots, n \tag{18}$$

where $\varepsilon_i^* > 0$ is the similarity measure between the two dynamical patterns.

Remark 3: It is seen that the previous similarity definition is related to both the states and system dynamics of the two dynamical patterns. It is based on the fundamental information of system dynamics of the two patterns, i.e., $f_i(x;p)$ and $f_i'(x;p')$, which are time-invariant by definition. The state information of the two patterns is also involved; however, it is not required that states of the two patterns match (exactly) in phase space or occur identically. In other words, the time attribute of dynamical patterns is excluded from the similarity definition.

Note that in Definition 2, while the states of the two dynamical patterns are measurable, both $f_i(x;p)$ and $f_i'(x;p')$ are not yet available for characterizing their similarity. Through deterministic learning, system dynamics $f_i(x;p)$ $(i = 1, \ldots, n)$ of pattern φ_ζ has been accurately identified and effectively represented by constant RBF network $\overline{W}_i^T S_i(x)$ $(i = 1, \ldots, n)$. Based on this learning, we further investigate how pattern φ_ς *is recognized* to be similar to pattern φ_ζ.

Combining (15) with (18), when state x of pattern φ_ς stays within the local region Ω_{φ_ζ}, we have

$$\begin{gathered}\operatorname{dist}(x, \varphi_\zeta) < d_\zeta \Rightarrow \\ \max_{x \in \varphi_\varsigma(x_{\varsigma 0}; p')} \left| f_i'(x;p') - \overline{W}_i^T S_i(x) \right| < \varepsilon_i^* + \xi_i^*, \qquad i = 1, \ldots, n\end{gathered} \tag{19}$$

which shows that the difference of system dynamics of patterns φ_ς and φ_ζ is expressed in terms of $f_i'(x;p')$ and $\overline{W}_i^T S(x)$. Thus, we have the following.

Definition 3: Dynamical pattern φ_ς [given by (17)] is recognized to be similar with dynamical pattern φ_ζ (based on the identification of φ_ζ), if the state of pattern φ_ς stays within the local region Ω_{φ_ζ} [as described by (15)], and the differences between the corresponding system dynamics, as expressed in (19), are small along the state of pattern φ_ς.

Remark 4: Note that complete information of both dynamics and states of dynamical patterns are utilized in Definitions 2 and 3. Different from the similarity definitions for static patterns, it is seen that pattern φ_ς being similar (or being recognized as similar) to pattern φ_ζ does not necessarily imply that the reverse is true. On the other hand, initial conditions of dynamical patterns do not play an important role in measuring similarity, since the time attribute of dynamical patterns has been effectively removed.

Note that in (19), system dynamics $f_i'(x;p')$ of pattern φ_ς is still unavailable. As to be shown in Section IV, Definition 3 will be useful in providing an explicit measure of similarity in rapid recognition of pattern φ_ς.

C. Simulations

Consider the two dynamical patterns generated from the Duffing oscillator (see, e.g., [22]), as described by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -p_2 x_1 - p_3 x_1^3 - p_1 x_2 + q\cos(wt)\end{aligned} \tag{20}$$

where $x = [x_1, x_2]^T$ is the state, p_1, p_2, p_3, w, and q are constant parameters, the system dynamics $f_2(x;p) = -p_2 x_1 - p_3 x_1^3 - p_1 x_2$ is an unknown, smooth nonlinear function, and $q\cos(wt)$ is a known periodic term which makes the behaviors of Duffing oscillator more interesting [22]. The Duffing oscillator can generate many types of dynamical behaviors, including periodic, quasi-periodic, and chaotic patterns.

The periodic pattern and the chaotic pattern (shown in Fig. 1, denoted as φ_ζ^1 and φ_ζ^2, respectively), are used to demonstrate the result of this section. The periodic pattern φ_ζ^1 is generated from (20), with initial condition $x(0) = [x_1(0), x_2(0)]^T = [0.0, -1.8]^T$ and system parameters $p_1 = 0.55$, $p_2 = -1.1$, $p_3 = 1.0$, $w = 1.8$, and $q = 1.498$. The chaotic pattern φ_ζ^2 is generated with the same system parameters except $p_1 = 0.35$.

The following dynamical RBF network, which is slightly modified from (2), is employed to identify the unknown dynamics $f_2(x;p)$ of the two training dynamical patterns φ_ζ^1 and φ_ζ^2:

$$\dot{\hat{x}}_2 = -a_2(\hat{x}_2 - x_2) + \widehat{W}_2^T S(x) - q\cos(wt). \tag{21}$$

The RBF network $\widehat{W}_2^T S_2(x)$ is constructed in a regular lattice, with nodes $N = 441$, the centers μ_i evenly spaced on $[-3.0, 3.0] \times [-3.0, 3.0]$, and the widths $\eta_i = 0.3$. The weights

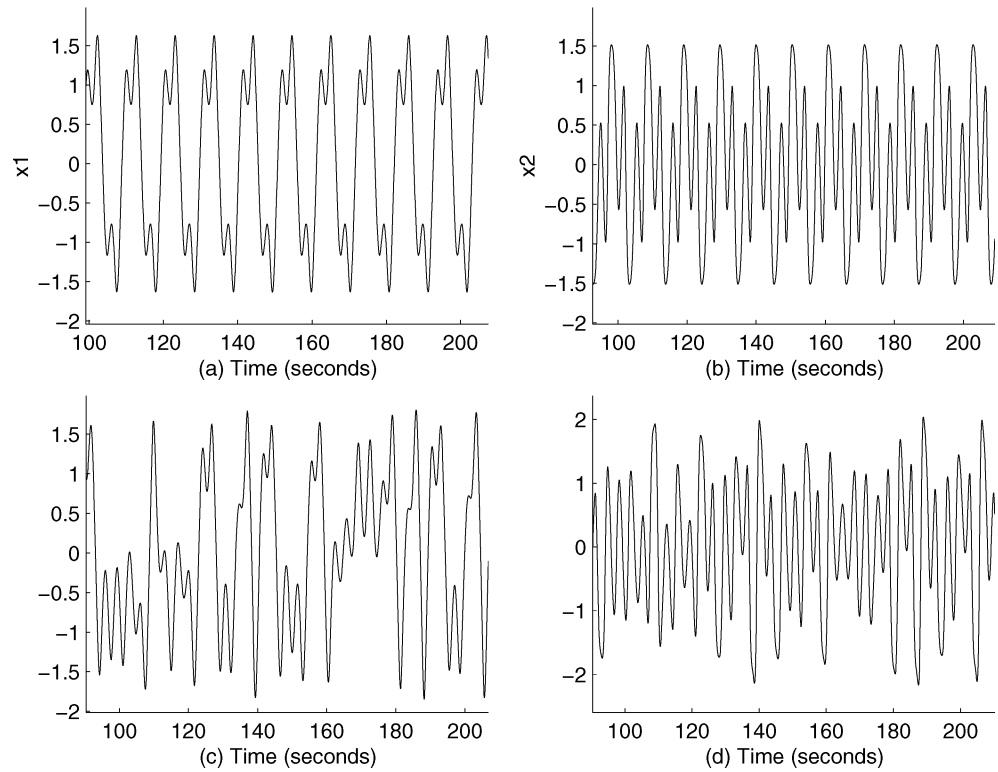

Fig. 1. Periodic and chaotic dynamical patterns.

of the RBF networks are updated according to (7). The design parameters for (21) and (7) are $a_2 = 5, \Gamma_2 = 2$, and $\sigma_2 = 0.001$. The initial weights $\widehat{W}_2(0) = 0.0$.

The phase portrait of dynamical pattern φ_ζ^1 is shown in Fig. 2(a). Its corresponding system dynamics $f_2(x;p)$ is shown in Fig. 2(b). Through deterministic learning, the system dynamics $f_2(x;p)$ of dynamical pattern φ_ζ^1 can be locally accurately identified. According to Theorem 1, exponential convergence of a closed-loop identification system, as well as the convergence of $\widetilde{W}_{\zeta 2}$ (a subvector of $\widetilde{W}_2$) is obtained. In Fig. 2(c), it is seen that some weight estimates (of the neurons whose centers are close to the orbit of the pattern) converge to constant values, while some other weight estimates (of neurons centered far away from the orbit) remain almost zero.

The locally accurate NN approximation of $f_2(x;p)$ along the orbit of the periodic pattern φ_ζ^1 is clearly shown in Fig. 2(d) and (e). In Fig. 2(f), dynamical pattern φ_ζ^1 is represented by the constant RBF network $\overline{W}_2 S(x)$. This representation is definitely time-invariant, based on the fundamental information on system dynamics. It is also spatially distributed, involving a large number of neurons distributed along the orbit of the dynamical pattern. The NN approximation is accurate only in the vicinity of the periodic pattern. For the other region where the orbit of the pattern does not explore, no learning occurs, corresponding to the zero-plane in Fig. 2(f), i.e., the small values of $\overline{W}_2^T S_2(x)$ in the unexplored area.

Similarly, consider the chaotic pattern φ_ζ^2. Pattern φ_ζ^2 is generated from (20), with initial condition $x(0) = [x_1(0), x_2(0)]^T = [0.3, -1.2]^T$ and system parameters $p_1 = 0.35$, $p_2 = -1.1$, $p_3 = 1.0$, $w = 1.8$, and $q = 1.498$. From Fig. 3(a) and (b), we can see the phase portrait and the system dynamics $f_2(x;p)$ of the chaotic pattern φ_ζ^2. Fig. 3(c) shows the partial parameter convergence. The locally accurate NN approximation of system dynamics $f_2(x;p)$ along the orbit of the pattern is shown in Fig. 3(d) and (e). Fig. 3(f) shows the time-invariant representation of chaotic pattern φ_ζ^2. It reveals that although the chaotic pattern φ_ζ^2 looks more complicated than the periodic pattern φ_ζ^1, a chaotic dynamical pattern can be represented in the same way as that of a periodic dynamical pattern.

IV. Rapid Recognition of Dynamical Patterns

In this section, we present a dynamical mechanism by which rapid recognition of dynamical patterns can be implemented by synchronization.

A. Problem Formulation

Consider a training set containing dynamical patterns $\varphi_\zeta^k, k = 1, \ldots, M$, with the kth training pattern φ_ζ^k generated from

$$\dot{x} = F^k(x;p^k) \quad x(t_0) = x_{\zeta 0}^k \tag{22}$$

where p^k is the system parameter vector. As shown in Section II, the system dynamics $F^k(x) = [f_1^k(x;p^k), \ldots, f_n^k(x;p^k)]^T$ can be accurately identified and stored in constant RBF networks $\overline{W}^{k^T} S(x) = [\overline{W}_1^{k^T} S_1(x), \ldots, \overline{W}_n^{k^T} S_n(x)]^T$.

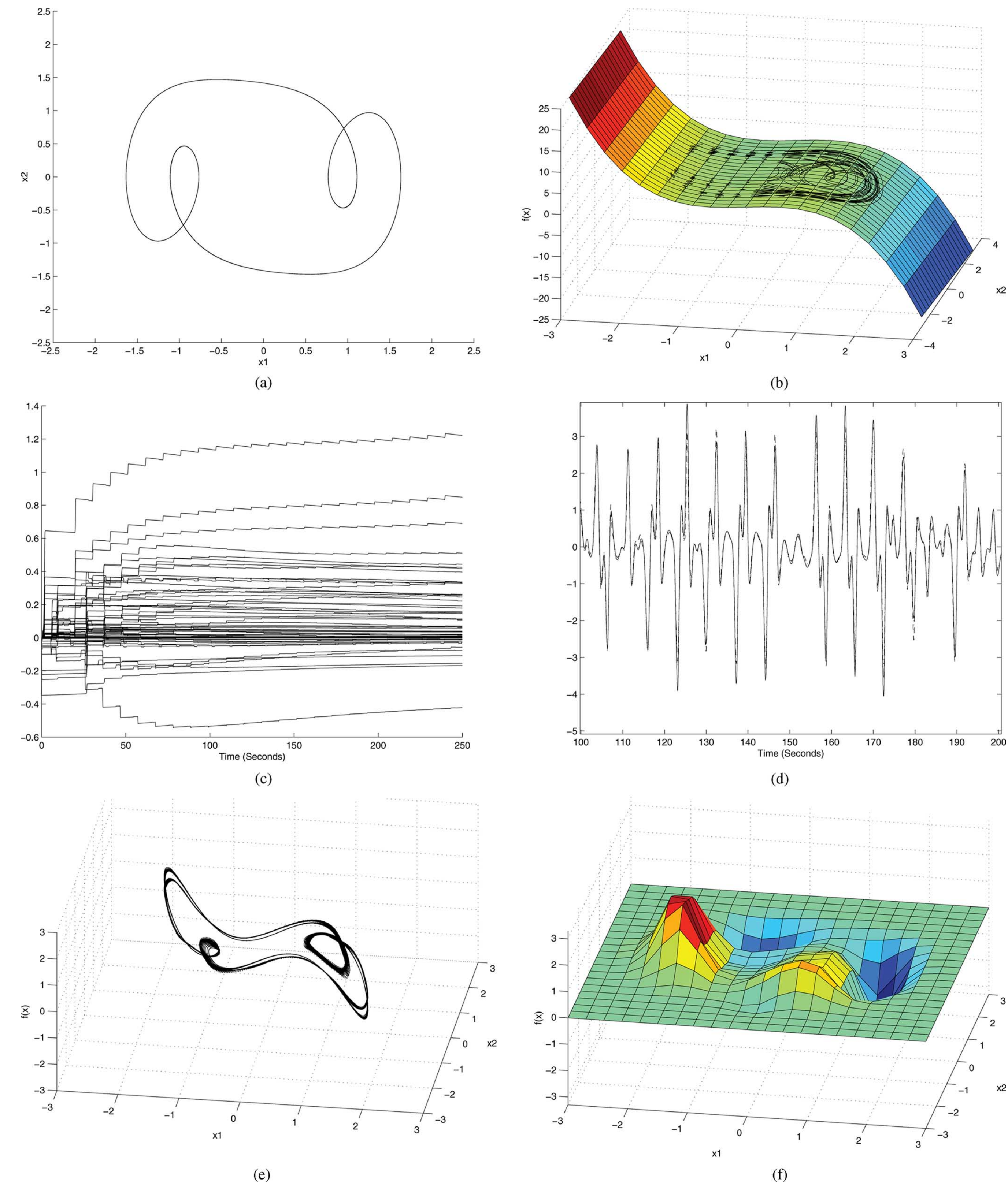

Fig. 2. Deterministic learning of periodic pattern φ_ζ^1. (a) Phase portrait of pattern. (b) System dynamics of pattern φ_ζ^1. (c) Partial parameter convergence. (d) Function approximation: $f_2(x)$ "—," $\widehat{W}_2^T S(x)$ "– –," and $\overline{W}_2^T S(x)$ "…" (e) Approximation along the orbit of pattern φ_ζ^1: $f_2(x)$ "—," $\widehat{W}_2^T S(x)$ "– –," and $\overline{W}_2^T S(x)$ "…" (f) Representation of periodic pattern φ_ζ^1 by $\overline{W}_2^T S(x)$.

Consider dynamical pattern φ_ς [as given by (17)] as a test pattern. Without identifying the system dynamics of the test pattern φ_ς, the recognition problem is to search *rapidly* from the training dynamical patterns φ_ζ^k $(k = 1, \ldots, M)$ for those *similar* to the given test pattern φ_ς in the sense of Definition 3.

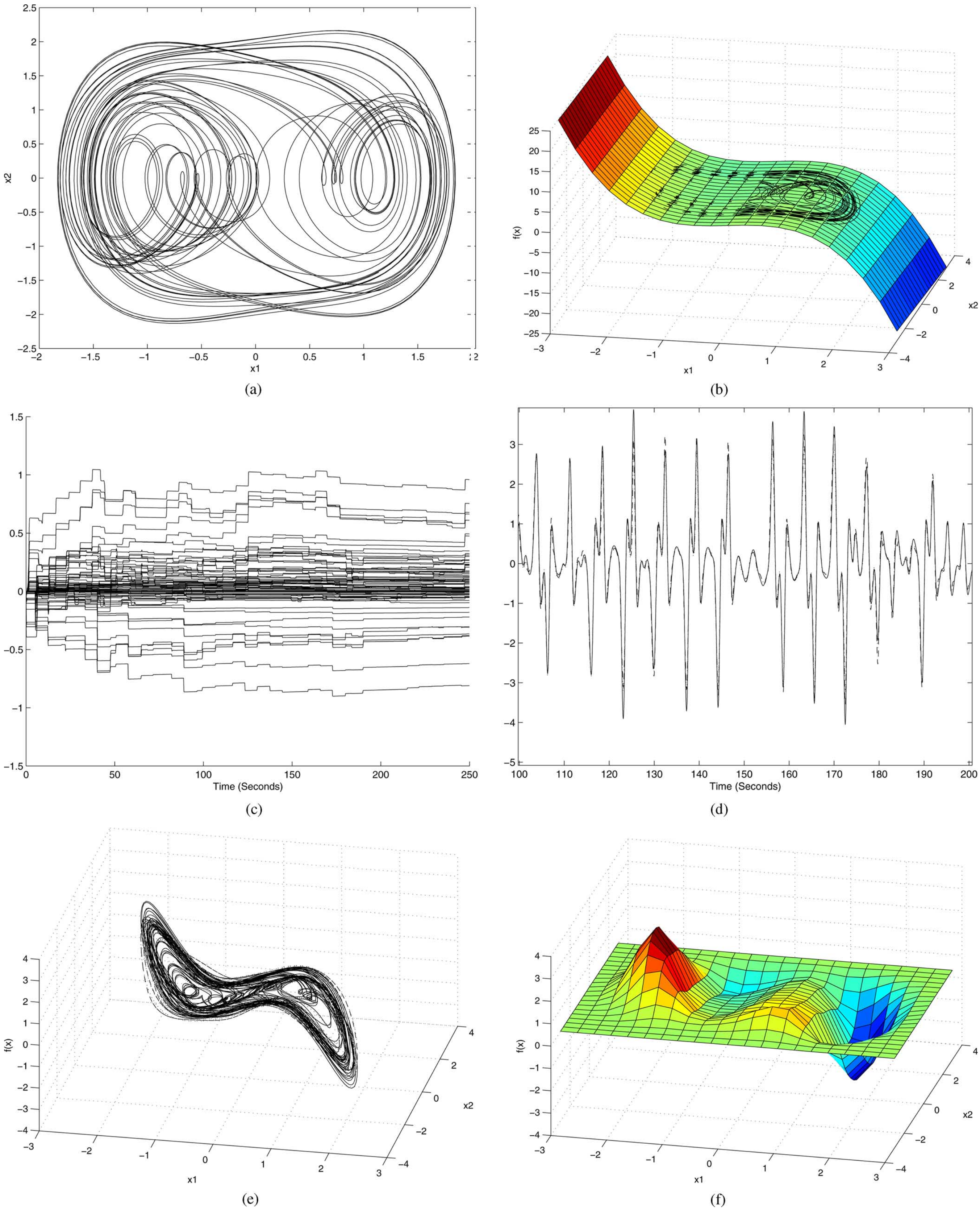

Fig. 3. Deterministic learning of chaotic pattern φ_ζ^2. (a) Phase portrait of pattern φ_ζ^2. (b) System dynamics of pattern φ_ζ^2. (c) Partial parameter convergence. (d) Function approximation: $f_2(x)$ "—," $\widehat{W}_2^T S(x)$ "– –," and $\overline{W}_2^T S(x)$ "…" (e) Approximation along the orbit: $f_2(x)$ "—," $\widehat{W}_2^T S(x)$ "– –," and $\overline{W}_2^T S(x)$ "…" (f) Representation of chaotic pattern φ_ζ^2 by $\overline{W}_2^T S(x)$.

B. Rapid Recognition via Synchronization

In the following, we present how rapid recognition of dynamical patterns is achieved. For the kth ($k = 1, \ldots, M$) training pattern, a dynamical model is constructed (as shown in Section III) by using the time-invariant representation $\overline{W}^{k^T} S(x)$ as

$$\dot{\bar{x}}^k = -B(\bar{x}^k - x) + \overline{W}^{k^T} S(x) \tag{23}$$

where $\bar{x}^k = [\bar{x}_1^k, \ldots, \bar{x}_n^k]^T$ is the state of the dynamical (template) model, x is the state of an input test pattern generated from (17), $B = \mathrm{diag}\{b_1, \ldots, b_n\}$ is a diagonal matrix which is kept the same for all training patterns, and $b_i > 0$ is not chosen as a large value. Then, corresponding to the test pattern φ_ς and the dynamical model (23) (for training pattern φ_ζ^k), we obtain the following recognition error system:

$$\dot{\tilde{x}}_i^k = -b_i \tilde{x}_i^k + \overline{W}_i^{k^T} S_i(x) - f_i'(x; p'), \qquad i = 1, \ldots, n \tag{24}$$

where $\tilde{x}_i^k = \bar{x}_i^k - x_i$ is the state tracking (or synchronization) error.

Note that without identifying the system dynamics of the test pattern φ_ς, the difference on system dynamics of the test and training patterns, i.e., $|\overline{W}_i^{k^T} S_i(x) - f_i'(x; p')|$, is not available from direct computation. Nevertheless, it will be shown that the difference between system dynamics can be explicitly measured by $|\tilde{x}_i^k|$. Thus, if the state $\bar{x}_i^k$ of the dynamical model (23) tracks closely to (or synchronize with) the state x of dynamical pattern φ_ς, i.e., $|\tilde{x}_i^k|$ is small, then the test pattern φ_ς can be recognized as similar to the training pattern φ_ζ^k in the sense of Definition 3. Note that the synchronization is not achieved between the states of dynamical patterns φ_ς and φ_ζ^k.

Theorem 2 describes how a test dynamical pattern is rapidly recognized in a dynamical process by synchronization.

Theorem 2: Consider the recognition error system (24) corresponding to test pattern φ_ς and the dynamical model (23) for training pattern φ_ζ^k. Then, the synchronization error $\tilde{x}^k$ converges exponentially to a neighborhood of zero, with the size of the neighborhood proportional to the difference between system dynamics of test pattern φ_ς and training pattern φ_ζ^k. Further, the test pattern φ_ς is recognized as similar to the training pattern φ_ζ^k if the synchronization error $\|\tilde{x}^k\|$ is small, i.e., the state $\bar{x}^k$ of the dynamical model (23) tracks closely with the state x of test pattern φ_ς.

Proof: To simplify the notion, we remove the superscript $(\cdot)^k$ in the following derivations.

For the recognition error system (24), consider Lyapunov function $V_i = (1/2)\tilde{x}_i^2$. Its derivative is

$$\dot{V}_i = \tilde{x}_i \dot{\tilde{x}}_i = -b_i \tilde{x}_i^2 - \tilde{x}_i(\overline{W}_i^T S(x) - f_i'(x; p')).$$

Note

$$\begin{aligned} -\frac{1}{2} b_i \tilde{x}_i^2 - \tilde{x}_i \Big(\overline{W}_i^T S(x) - f_i'(x; p')\Big) &\le -\frac{1}{2} b_i \tilde{x}_i^2 + |\tilde{x}_i| \Big|\overline{W}_i^T S(x) - f_i'(x; p')\Big| \\ &\le \frac{\Big|\overline{W}_i^T S(x) - f_i'(x; p')\Big|^2}{2b_i}. \end{aligned} \tag{25}$$

Then, we have

$$\dot{V}_i \le -\frac{1}{2} b_i \tilde{x}_i^2 + \frac{(\overline{W}_i^T S_i(x) - f_i'(x; p'))^2}{2b_i}. \tag{26}$$

Denote $\rho_i := \Big|\overline{W}_i^T S(x) - f_i'(x; p')\Big|^2 / 4b_i^2$. Then, (26) gives

$$0 \le V_i(t) < \rho_i + (V_i(0) - \rho_i)\exp(-2b_i t). \tag{27}$$

From (27), we have

$$\tilde{x}_i^2 < 2\rho_i + 2V_i(0)\exp(-2b_i t) \tag{28}$$

which implies that given $\nu_i > \sqrt{2\rho_i}$, there exists a finite time T, such that for all $t \ge T$, the state tracking error $\tilde{x}_i$ will converge exponentially to a neighborhood of zero, i.e., $|\tilde{x}_i| \le \nu_i$, with the size of the neighborhood ν_i proportional to $|\overline{W}_i^T S_i(x) - f_i'(x; p')|$.

When the state of the test pattern stays within the local region Ω_{φ_ζ} corresponding to the training pattern φ_ζ, i.e., $\mathrm{dist}(x, \varphi_\zeta) < d$, $|\overline{W}_i^T S_i(x) - f_i'(x; p')|$ can be expressed in terms of $\xi_i^* + \varepsilon_i^*$, as given by (19). Therefore, $|\tilde{x}_i|$ will converge exponentially to a neighborhood of zero, with the size of the neighborhood expressed by $\nu_i > \sqrt{2\rho_i} = \xi_i^* + \varepsilon_i^* / \sqrt{2} b_i$, i.e., proportional to $\xi_i^* + \varepsilon_i^*$, and inversely proportional to b_i. Since ξ_i^* is small when the training pattern has been accurately identified, b_i is not large, we have that $|\tilde{x}_i|_{t \ge T}$ (for finite T) is proportional to ε_i^*. Thus, it is concluded that the difference between system dynamics of the test and training patterns can be explicitly measured by $|\tilde{x}_i|_{t \ge T}$, $i = 1, \ldots, n$, and, therefore, by $\|\tilde{x}\|_{t \ge T}$. Accordingly, if the state $\bar{x}$ of the dynamical model (23) tracks closely to (or synchronize with) the state x of test pattern φ_ς, i.e., $\|\tilde{x}\|_{t \ge T}$ is small, then the test pattern φ_ς is recognized as similar to the training pattern φ_ζ in the sense of Definition 3. $\diamond$

Remark 5: It is seen that rapid recognition of dynamical patterns is implemented via synchronization due to the internal matching of system dynamics. For recognition of a test dynamical pattern from a set of training dynamical patterns, the synchronization errors can be naturally taken as the measure of similarity between the test and training patterns. The recognition problem is turned into the stability and convergence analysis of the recognition error system (24), the recognition is automatically implemented with the evolvement of the recognition error system (24).

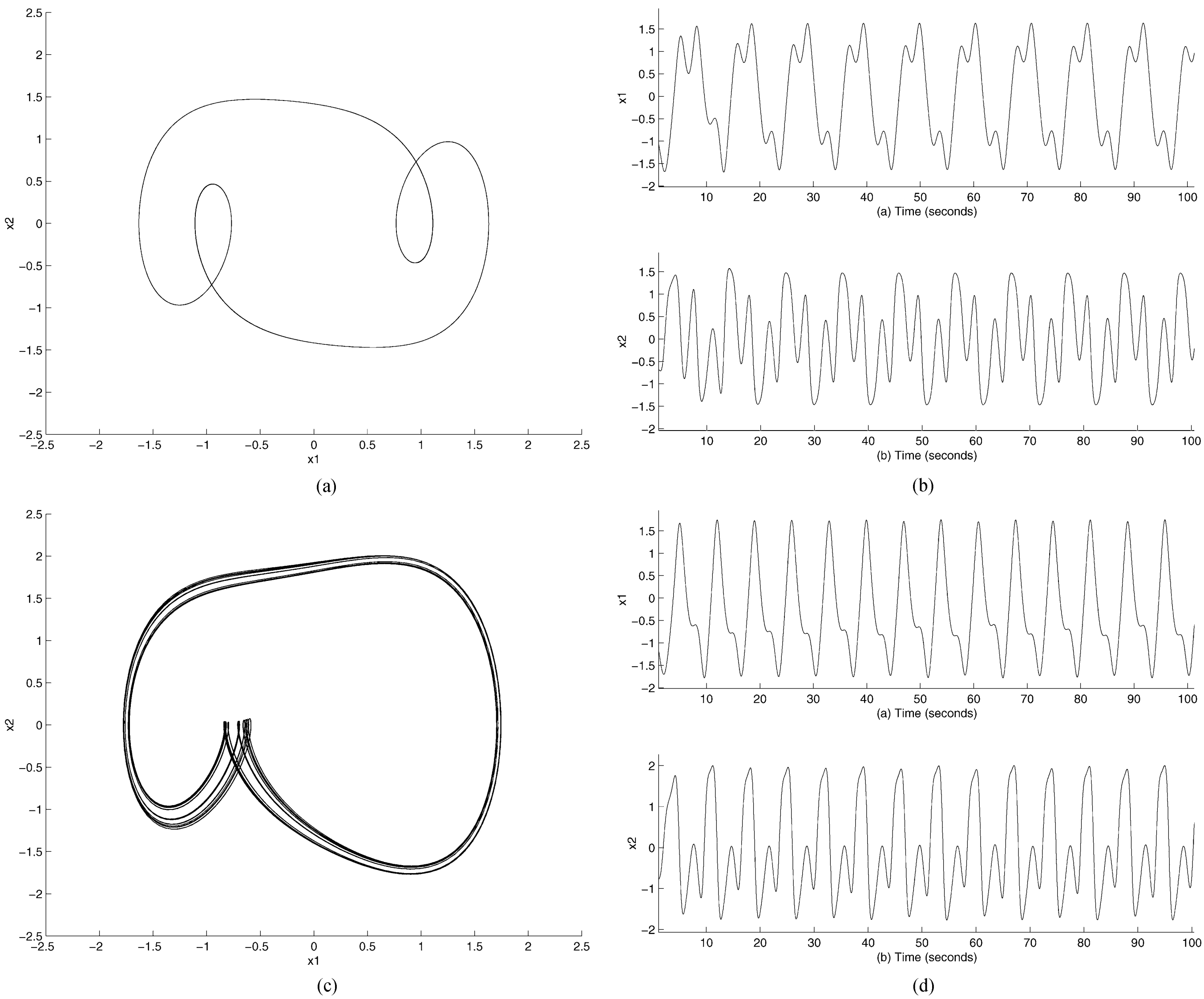

Fig. 4. (a) Test pattern 1: phase portrait. (b) Test pattern 1: time responses. (c) Test pattern 2: phase portrait. (d) Test pattern 2: time responses.

Remark 6: The recognition of a test dynamical pattern φ_ς occurs rapidly, since the recognition process takes place from the beginning of measuring the state x of test pattern φ_ς. In the rapid recognition process, the time-invariant representation and similarity definition are involved in a dynamical manner. The representation, the similarity definition and the recognition mechanism are three important elements of the proposed recognition approach.

C. Simulations

To verify the results of Section IV, dynamical patterns φ_ζ^1 and φ_ζ^2 are taken as two training dynamical patterns. Using the time-invariant representations $\overline{W}_2^{k^T} S_2(x)$ $(k = 1, 2)$ obtained in Section III, two dynamical models are constructed according to (23) for the two training patterns as

$$\dot{\bar{x}}_2^k = -b_2(\bar{x}_2^k - x_2) + \overline{W}_2^{k^T} S_2(x) - q\cos(wt), \qquad k = 1, 2 \tag{29}$$

where $\bar{x}_2^2$ is the state of the dynamical model, x_2 is the state of a test pattern generated from (17) as seen later, and $b_2 > 0$ is a design constant, which should not be a large value ($b_2 = 2$ in this paper).

Two periodic patterns, as shown in Fig. 4, are used as the test patterns 1 and 2. Test pattern 1 is generated from system (20), with initial condition $x(0) = [x_1(0), x_2(0)]^T = [0.0, -1.8]^T$ and system parameters $p_1 = 0.6$, $p_2 = -1.1$, $p_3 = 1.0$, $w = 1.8$, and $q = 1.498$. The initial condition and system parameters of test pattern 2 are the same with those of test pattern 1, except that $p_1 = 0.4$.

First, consider the recognition of test pattern 1 by the training pattern 1. As seen in Fig. 5(a), the state $\bar{x}_2$ of the dynamical model for training pattern 1 tracks closely to the state x_2 of test pattern 1. That is, state synchronization is achieved and the synchronization error is small, as seen in Fig. 5(b). Thus, test pattern 1 is recognized as similar to training pattern 1. Second, for recognition of test pattern 1 by training pattern 2, the state $\bar{x}_2$ of the dynamical model for training pattern 2 can also track to the

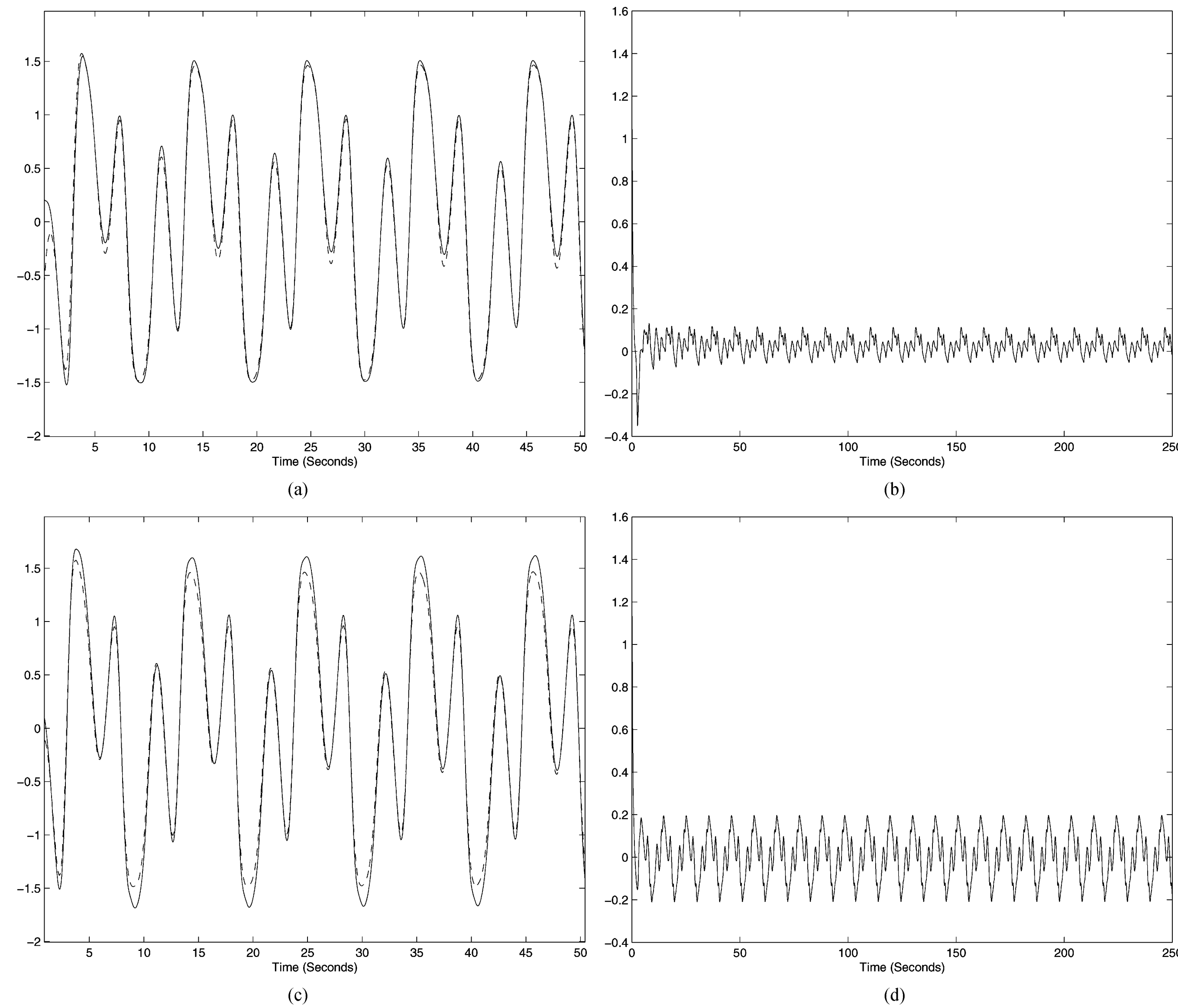

Fig. 5. Recognition of test pattern 1 by training patterns 1 and 2. (a) Dynamical model state $\bar{x}_2$ "—" and test pattern state x_2 "– –." (b) Synchronization error $\tilde{x}_2$. (c) Dynamical model state $\bar{x}_2$ "—" and test pattern state x_2 "– –." (d) Synchronization error $\tilde{x}_2$.

state x_2 of test pattern 1. The state synchronization and the synchronization error are shown in Fig. 5(c) and (d), respectively. It is also seen that the synchronization error in Fig. 5(b) is smaller than that in Fig. 5(d). Since the same b_2 is used in recognition of test pattern 1, this implies that test pattern 1 is more similar to training pattern 1 than to training pattern 2.

Similarly, in recognition of test dynamical pattern 2, it is seen from Fig. 6(a)–(d) that the test pattern 2 is more similar to the chaotic training pattern 2 than to the periodic training pattern 1. It is also seen that rapid state synchronization is achieved within a very short time, which shows that the test patterns are quickly being recognized as similar or dissimilar to the training patterns.

V. Conclusion

In this paper, we have proposed an approach for rapid recognition of dynamical patterns. The elements of the recognition approach include the following: 1) a time-invariant and spatially distributed representation for dynamical patterns, 2) a similarity measure based on system dynamics, and 3) a mechanism in which rapid recognition of dynamical patterns is achieved by state synchronization. It has been shown that a time-varying dynamical pattern can be effectively represented by using complete information of its state trajectory and its underlying system dynamics along the state trajectory. Based on the proposed similarity measure for dynamical patterns, a mechanism for rapid recognition of dynamical patterns has been presented. Rapid recognition can be automatically implemented in a dynamical recognition process without conventional feature extraction. The outcome of the recognition process, i.e., the synchronization error, is naturally taken as the measure of similarity between the test and training patterns. The dynamical recognition process does not need to compare directly the states or system dynamics of the test and training patterns by any form of numerical computation.

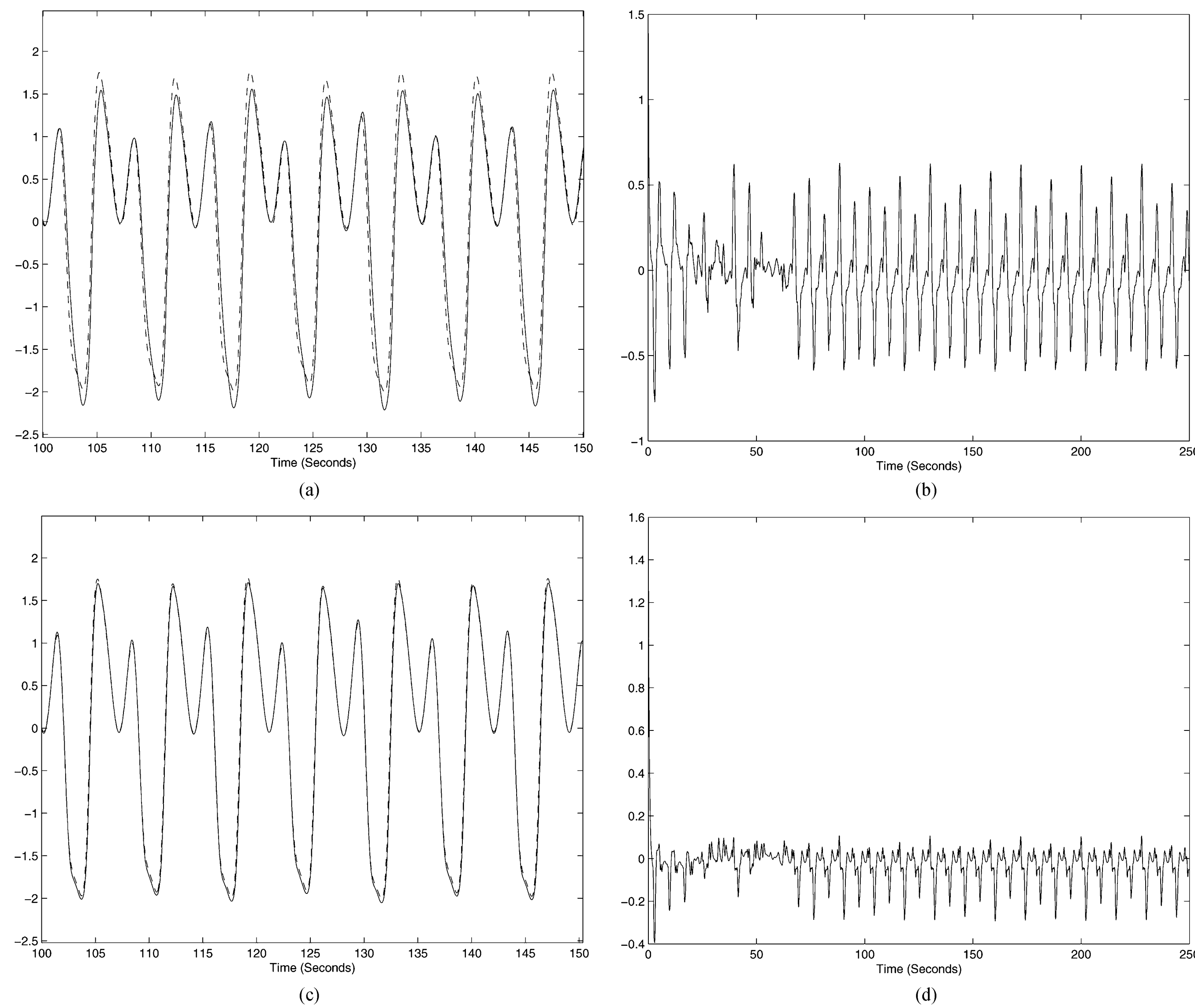

Fig. 6. Recognition of test pattern 2 by training patterns 1 and 2. (a) Dynamical model state $\bar{x}_2$ "—" and test pattern state x_2 "– –." (b) Synchronization error $\tilde{x}_2$. (c) Dynamical model state $\bar{x}_2$ "—" and test pattern state x_2 "– –." (d) Synchronization error $\tilde{x}_2$.

The proposed approach appears to be consistent with mechanisms of human recognition of temporal patterns, and may provide understanding for natural cognitive systems from the perspective of dynamics. It presents a new model for information processing, i.e., dynamical parallel distributed processing (DPDP) in a continuous and analog manner. The results in this paper will facilitate further construction of recognition systems for temporal/dynamical patterns. Specifically, the recognition system can be constructed using many dynamical (template) models [as described in (23)]. Each of the dynamical models represents one training dynamical pattern. Since the similarity between the test and training dynamical patterns can be measured using the synchronization errors, the recognition system can be built up by using the nearest neighbor classification—a commonly used classification algorithm in pattern recognition [8]. The constructed recognition system promises to be able to classify different classes of dynamical patterns, and distinguish a set of dynamical patterns generated from the same class. Extensions of the current work will be further conducted along this direction.

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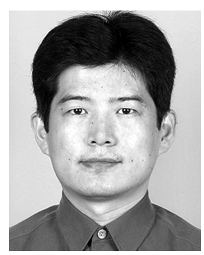

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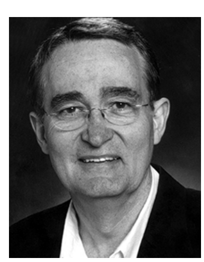

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