



Report Assignment 1

Counting Sort Algorithm

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Group:

| | | | |
|---------------|----------|------------|--|
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Problem description

Parallelize and Evaluate Performances of "Counting Sort" Algorithm, by using **OpenMP**.

Experimental Setup 1

Hardware

CPU

```
processor      : 0
vendor_id     : GenuineIntel
cpu family    : 6
model         : 78
model name    : Intel(R) Core(TM) i7-6600U CPU @ 2.60GHz
stepping      : 3
microcode     : 0xea
cpu MHz       : 800.109
cache size    : 4096 KB
physical id   : 0
siblings      : 4
core id       : 0
cpu cores     : 2
apicid        : 0
initial apicid : 0
fpu           : yes
fpu_exception : yes
cpuid level   : 22
wp            : yes
flags         : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe syscall nx pdpe1gb
rdtscp lm constant_tsc art arch_perfmon pebs bts rep_good nopl xtopology
nonstop_tsc cpuid aperfmperf pni pclmulqdq dtes64 monitor ds_cpl vmx smx est
tm2 ssse3 sdbg fma cx16 xtpr pdcm pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand lahf_lm abm 3dnowprefetch
cpuid_fault epb invpcid_single pti ssbd ibrs ibpb stibp tpr_shadow vnmi
flexpriority ept vpid ept_ad fsgsbase tsc_adjust bmi1 hle avx2 smep bmi2 erms
invpcid rtm mpx rdseed adx smap clflushopt intel_pt xsaveopt xsavec xgetbv1
xsaves dtherm ida arat pln pts hwp hwp_notify hwp_act_window hwp_epp md_clear
flush_l1d
bugs          : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swaps taa itlb_multihit srbds
bogomips      : 5599.85
clflush size   : 64
cache_alignment : 64
address sizes  : 39 bits physical, 48 bits virtual
power management:
```

```

processor      : 1
vendor_id     : GenuineIntel
cpu family    : 6
model         : 78
model name    : Intel(R) Core(TM) i7-6600U CPU @ 2.60GHz
stepping      : 3
microcode     : 0xea
cpu MHz       : 800.044
cache size    : 4096 KB
physical id   : 0
siblings      : 4
core id       : 1
cpu cores     : 2
apicid        : 2
initial apicid : 2
fpu           : yes
fpu_exception : yes
cpuid level   : 22
wp            : yes
flags         : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe syscall nx pdpe1gb
rdtscp lm constant_tsc art arch_perfmon pebs bts rep_good nopl xtopology
nonstop_tsc cpuid aperfmperf pni pclmulqdq dtes64 monitor ds_cpl vmx smx est
tm2 ssse3 sdbg fma cx16 xtpr pdcm pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand lahf_lm abm 3dnowprefetch
cpuid_fault epb invpcid_single pti ssbd ibrs ibpb stibp tpr_shadow vnmi
flexpriority ept vpid ept_ad fsgsbase tsc_adjust bmi1 hle avx2 smep bmi2 erms
invpcid rtm mpx rdseed adx smap clflushopt intel_pt xsaveopt xsavec xgetbv1
xsaves dtherm ida arat pln pts hwp hwp_notify hwp_act_window hwp_epp md_clear
flush_l1d
bugs          : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swapgs taa itlb_multihit srbds
bogomips      : 5599.85
clflush size   : 64
cache_alignment : 64
address sizes  : 39 bits physical, 48 bits virtual
power management:

```

```

processor      : 2
vendor_id     : GenuineIntel
cpu family    : 6
model         : 78
model name    : Intel(R) Core(TM) i7-6600U CPU @ 2.60GHz
stepping      : 3
microcode     : 0xea
cpu MHz       : 800.030
cache size    : 4096 KB
physical id   : 0
siblings      : 4
core id       : 0

```

```

cpu cores    : 2
apicid       : 1
initial apicid : 1
fpu          : yes
fpu_exception : yes
cpuid level : 22
wp           : yes
flags        : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe syscall nx pdpe1gb
rdtscp lm constant_tsc art arch_perfmon pebs bts rep_good nopl xtopology
nonstop_tsc cpuid aperfmperf pni pclmulqdq dtes64 monitor ds_cpl vmx smx est
tm2 ssse3 sdbg fma cx16 xtpr pdcm pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand lahf_lm abm 3dnowprefetch
cpuid_fault epb invpcid_single pti ssbd ibrs ibpb stibp tpr_shadow vnmi
flexpriority ept vpid ept_ad fsgsbase tsc_adjust bmi1 hle avx2 smep bmi2 erms
invpcid rtm mpx rdseed adx smap clflushopt intel_pt xsaveopt xsavec xgetbv1
xsaves dtherm ida arat pln pts hwp hwp_notify hwp_act_window hwp_epp md_clear
flush_l1d
bugs         : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swaps taa itlb_multihit srbds
bogomips     : 5599.85
clflush size : 64
cache_alignment : 64
address sizes : 39 bits physical, 48 bits virtual
power management:

```

```

processor    : 3
vendor_id    : GenuineIntel
cpu family   : 6
model        : 78
model name   : Intel(R) Core(TM) i7-6600U CPU @ 2.60GHz
stepping     : 3
microcode    : 0xea
cpu MHz      : 800.071
cache size   : 4096 KB
physical id   : 0
siblings     : 4
core id      : 1
cpu cores    : 2
apicid       : 3
initial apicid : 3
fpu          : yes
fpu_exception : yes
cpuid level : 22
wp           : yes
flags        : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe syscall nx pdpe1gb
rdtscp lm constant_tsc art arch_perfmon pebs bts rep_good nopl xtopology
nonstop_tsc cpuid aperfmperf pni pclmulqdq dtes64 monitor ds_cpl vmx smx est
tm2 ssse3 sdbg fma cx16 xtpr pdcm pcid sse4_1 sse4_2 x2apic movbe popcnt

```

```

tsc_deadline_timer aes xsave avx f16c rdrand lahf_lm abm 3dnowprefetch
cpuid_fault epb invpcid_single pti ssbd ibrs ibpb stibp tpr_shadow vnmi
flexpriority ept vpid ept_ad fsgsbase tsc_adjust bmi1 hle avx2 smep bmi2 erms
invpcid rtm mpx rdseed adx smap clflushopt intel_pt xsaveopt xsavec xgetbv1
xsaves dtherm ida arat pln pts hwp hwp_notify hwp_act_window hwp_epp md_clear
flush_l1d
bugs          : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swaps taa itlb_multihit srbds
bogomips      : 5599.85
clflush size   : 64
cache_alignment : 64
address sizes  : 39 bits physical, 48 bits virtual
power management:

```

RAM

```

MemTotal:      7544828 kB
MemFree:       5851384 kB
MemAvailable:  6426580 kB
Buffers:       91836 kB
Cached:        782772 kB
SwapCached:    0 kB
Active:        792856 kB
Inactive:      569204 kB
Active(anon):  580584 kB
Inactive(anon): 18820 kB
Active(file):  212272 kB
Inactive(file): 550384 kB
Unevictable:   88420 kB
Mlocked:      0 kB
SwapTotal:     2097148 kB
SwapFree:      2097148 kB
Dirty:         6924 kB
Writeback:     0 kB
AnonPages:     575980 kB
Mapped:        362432 kB
Shmem:         111956 kB
KReclaimable:  63832 kB
Slab:          170452 kB
SReclaimable:  63832 kB
SUnreclaim:    106620 kB
KernelStack:   8624 kB
PageTables:    13808 kB
NFS_Unstable:  0 kB
Bounce:        0 kB
WritebackTmp:  0 kB
CommitLimit:   5869560 kB
Committed_AS:  4190096 kB
VmallocTotal:  34359738367 kB

```

```
VmallocUsed:      29252 kB
VmallocChunk:      0 kB
Percpu:           3056 kB
HardwareCorrupted: 0 kB
AnonHugePages:     0 kB
ShmemHugePages:    0 kB
ShmemPmdMapped:    0 kB
FileHugePages:     0 kB
FilePmdMapped:     0 kB
CmaTotal:          0 kB
CmaFree:           0 kB
HugePages_Total:   0
HugePages_Free:    0
HugePages_Rsvd:    0
HugePages_Surp:    0
Hugepagesize:      2048 kB
Hugetlb:           0 kB
DirectMap4k:       200928 kB
DirectMap2M:       2330624 kB
DirectMap1G:       5242880 kB
```

Software

On this Setup Linux is not Virtualized, and it runs with the following software:

```
Distributor ID: Linuxmint
Description:    Linux Mint 20.2
Release:        20.2
Codename:       uma
GCC:            9.3.0
```

Experimental Setup 2

Hardware

CPU

```
processor       : 0
vendor_id      : GenuineIntel
cpu family     : 6
model          : 142
model name     : Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
stepping       : 10
microcode      : 0xffffffff
cpu MHz        : 1991.998
cache size     : 8192 KB
physical id    : 0
siblings       : 1
core id        : 0
```

```

cpu cores    : 1
apicid       : 0
initial apicid : 0
fpu          : yes
fpu_exception : yes
cpuid level  : 22
wp           : yes
flags        : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush mmx fxsr sse sse2 ss syscall nx pdpe1gb rdtscp lm
constant_tsc arch_perfmon nopl xtopology tsc_reliable nonstop_tsc cpuid pni
pclmulqdq ssse3 fma cx16 pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand hypervisor lahf_lm abm
3dnowprefetch invpcid_single pti ssbd ibrs ibpb stibp fsgsbase tsc_adjust
bmi1 avx2 smep bmi2 invpcid rdseed adx smap clflushopt xsaveopt xsavec
xgetbv1 xsaves arat flush_l1d arch_capabilities
bugs         : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swapgs itlb_multihit srbds
bogomips     : 3983.99
clflush size  : 64
cache_alignment : 64
address sizes : 45 bits physical, 48 bits virtual
power management:

```

```

processor     : 1
vendor_id    : GenuineIntel
cpu family   : 6
model        : 142
model name    : Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
stepping     : 10
microcode    : 0xffffffff
cpu MHz      : 1991.998
cache size   : 8192 KB
physical id   : 2
siblings     : 1
core id      : 0
cpu cores    : 1
apicid       : 2
initial apicid : 2
fpu          : yes
fpu_exception : yes
cpuid level  : 22
wp           : yes
flags        : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush mmx fxsr sse sse2 ss syscall nx pdpe1gb rdtscp lm
constant_tsc arch_perfmon nopl xtopology tsc_reliable nonstop_tsc cpuid pni
pclmulqdq ssse3 fma cx16 pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand hypervisor lahf_lm abm
3dnowprefetch invpcid_single pti ssbd ibrs ibpb stibp fsgsbase tsc_adjust
bmi1 avx2 smep bmi2 invpcid rdseed adx smap clflushopt xsaveopt xsavec
xgetbv1 xsaves arat flush_l1d arch_capabilities

```

bugs : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swapgs itlb_multihit srbds
bogomips : 3983.99
clflush size : 64
cache_alignment : 64
address sizes : 45 bits physical, 48 bits virtual
power management:

processor : 2
vendor_id : GenuineIntel
cpu family : 6
model : 142
model name : Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
stepping : 10
microcode : 0xffffffff
cpu MHz : 1991.998
cache size : 8192 KB
physical id : 4
siblings : 1
core id : 0
cpu cores : 1
apicid : 4
initial apicid : 4
fpu : yes
fpu_exception : yes
cpuid level : 22
wp : yes
flags : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush mmx fxsr sse sse2 ss syscall nx pdpe1gb rdtscp lm
constant_tsc arch_perfmon nopl xtopology tsc_reliable nonstop_tsc cpuid pni
pclmulqdq ssse3 fma cx16 pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand hypervisor lahf_lm abm
3dnowprefetch invpcid_single pti ssbd ibrs ibpb stibp fsgsbase tsc_adjust
bmi1 avx2 smep bmi2 invpcid rdseed adx smap clflushopt xsaveopt xsavec
xgetbv1 xsaves arat flush_l1d arch_capabilities
bugs : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swapgs itlb_multihit srbds
bogomips : 3983.99
clflush size : 64
cache_alignment : 64
address sizes : 45 bits physical, 48 bits virtual
power management:

processor : 3
vendor_id : GenuineIntel
cpu family : 6
model : 142
model name : Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
stepping : 10
microcode : 0xffffffff


```

cpu MHz          : 1991.998
cache size      : 8192 KB
physical id     : 6
siblings        : 1
core id         : 0
cpu cores       : 1
apicid          : 6
initial apicid  : 6
fpu             : yes
fpu_exception   : yes
cpuid level     : 22
wp              : yes
flags           : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush mmx fxsr sse sse2 ss syscall nx pdpe1gb rdtscp lm
constant_tsc arch_perfmon nopl xtopology tsc_reliable nonstop_tsc cpuid pni
pclmulqdq ssse3 fma cx16 pcid sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand hypervisor lahf_lm abm
3dnowprefetch invpcid_single pti ssbd ibrs ibpb stibp fsgsbase tsc_adjust
bmi1 avx2 smep bmi2 invpcid rdseed adx smap clflushopt xsaveopt xsavec
xgetbv1 xsaves arat flush_l1d arch_capabilities
bugs            : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds
swaps itlb_multihit srbds
bogomips        : 3983.99
clflush size     : 64
cache_alignment  : 64
address sizes    : 45 bits physical, 48 bits virtual
power management:

```

RAM

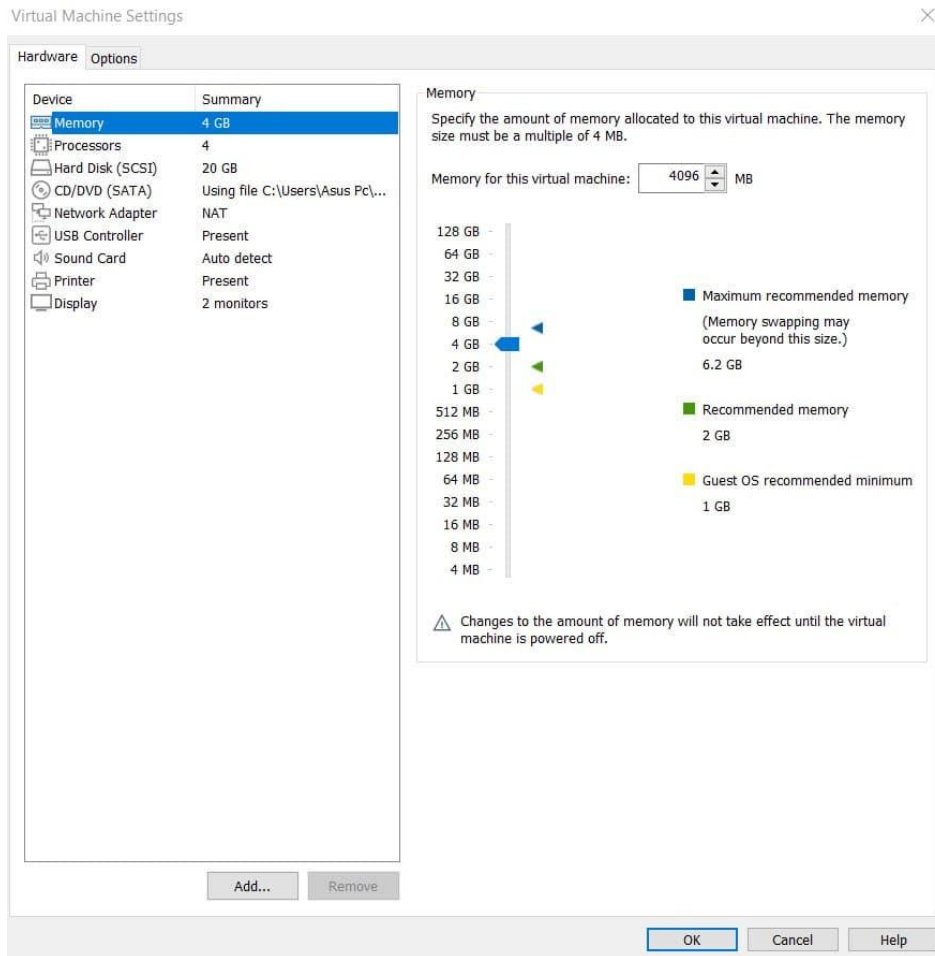
```

MemTotal:    3988332 kB
MemFree:     218908 kB
MemAvailable: 785536 kB
Buffers:     71236 kB
Cached:      826824 kB
SwapCached:  28332 kB
Active:      470672 kB
Inactive:    1510256 kB
Active(anon): 135884 kB
Inactive(anon): 1106584 kB
Active(file): 334788 kB
Inactive(file): 403672 kB
Unevictable: 16 kB
Mlocked:     16 kB
SwapTotal:   945368 kB
SwapFree:    654024 kB
Dirty:       616 kB
Writeback:   0 kB

```

AnonPages: 1057748 kB
Mapped: 456652 kB
Shmem: 159564 kB
KReclaimable: 65872 kB
Slab: 164200 kB
SReclaimable: 65872 kB
SUnreclaim: 98328 kB
KernelStack: 14576 kB
PageTables: 23504 kB
NFS_Unstable: 0 kB
Bounce: 0 kB
WritebackTmp: 0 kB
CommitLimit: 2939532 kB
Committed_AS: 5390960 kB
VmallocTotal: 34359738367 kB
VmallocUsed: 63260 kB
VmallocChunk: 0 kB
Percpu: 94208 kB
HardwareCorrupted: 0 kB
AnonHugePages: 0 kB
ShmemHugePages: 0 kB
ShmemPmdMapped: 0 kB
FileHugePages: 0 kB
FilePmdMapped: 0 kB
HugePages_Total: 0
HugePages_Free: 0
HugePages_Rsvd: 0
HugePages_Surp: 0
Hugepagesize: 2048 kB
Hugetlb: 0 kB
DirectMap4k: 352064 kB
DirectMap2M: 3842048 kB
DirectMap1G: 2097152 kB

VIRTUAL MACHINE SETTINGS



Software

On this Setup Linux is Virtualized, and it runs with the following software:

- Ubuntu 21.04
- GCC 10.3.0

Performance, Speedup & Efficiency

Case study

In this case study, the main purpose was to analyze the performance of our program in the following build setup:

- The sequential program is compiled with gcc optimization O0
- The parallel programs are compiled with the gcc optimization -Ox where $x = 1, 2, 3$. So here we want to highlight the difference between a simple sequential program compared to a parallel one, furthermore the case study is done on a non-random array of multiple size that are 2500000, 5000000, 10000000, 15000000 and with different number of threads (0, 1, 2, 4, 8, 16, 32).

SIZE-2500000-O1

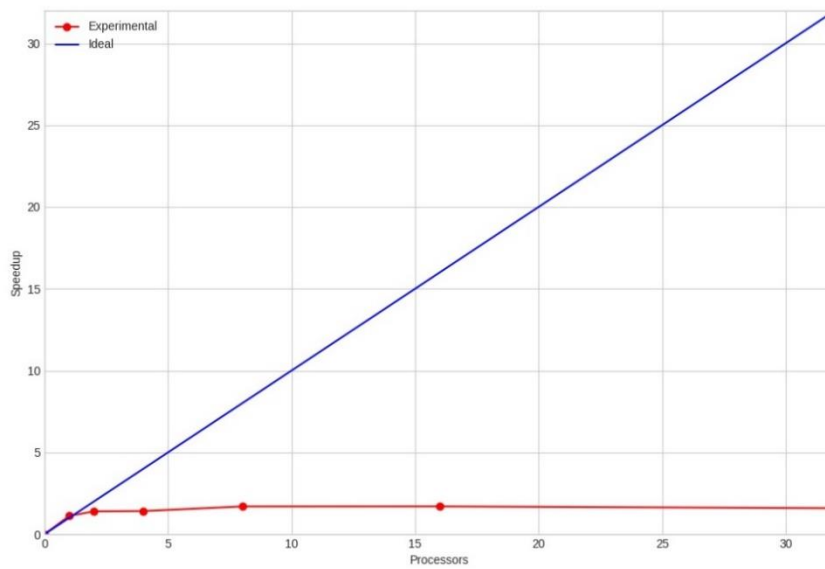
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Serial | 1 | 0.004983412429379 | 0.018338890625000 | 0.017992857142857 | 0.006436090225564 | 0.024628272251309 | 1.000000000000000 | 1.000000000000000 |
| Parallel | 1 | 0.004922877906977 | 0.015099664634146 | 0.014569536423841 | 0.006993377483444 | 0.022000000000000 | 1.11946692051404 | 1.119466920514040 |
| Parallel | 2 | 0.004547000000000 | 0.023641151162791 | 0.026212389380531 | 0.008260273972603 | 0.017623529411765 | 1.39746538141606 | 0.698732690708030 |
| Parallel | 4 | 0.006175442424242 | 0.045796382716049 | 0.053703703703704 | 0.008612676056338 | 0.017391608391608 | 1.41610089744157 | 0.354025224360391 |
| Parallel | 8 | 0.007061601626016 | 0.022161070422535 | 0.021347457627119 | 0.009264462809917 | 0.014508982035928 | 1.69745004786157 | 0.212181255982696 |
| Parallel | 16 | 0.007551925925926 | 0.022761125874126 | 0.020446280991736 | 0.011309734513274 | 0.014478260869565 | 1.70105183717749 | 0.106315739823593 |
| Parallel | 32 | 0.008641230769231 | 0.023601315068493 | 0.024737226277372 | 0.010352459016394 | 0.015485207100592 | 1.59043867423432 | 0.049701208569822 |

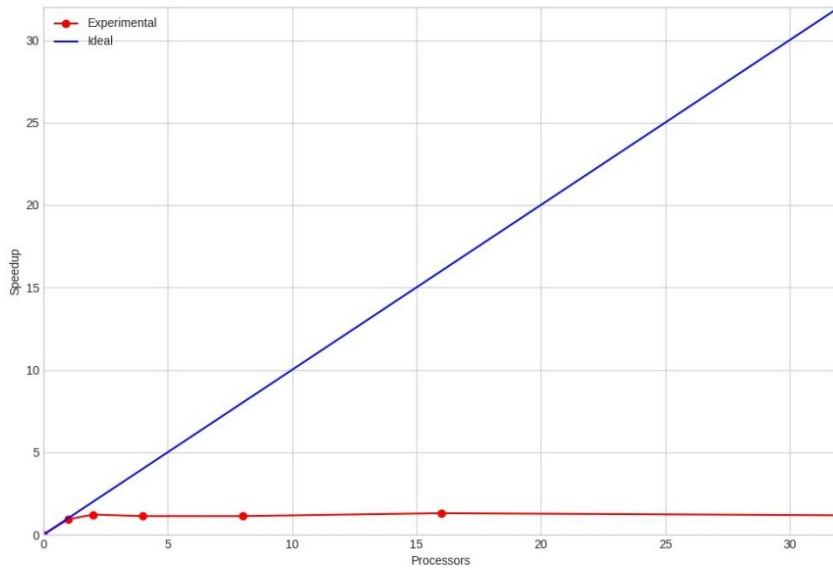
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Serial | 1 | 0.006032372781065 | 0.018376388535032 | 0.016773972602740 | 0.011993421052632 | 0.028574074074074 | 1.000000000000000 | 1.000000000000000 |
| Parallel | 1 | 0.007129634517767 | 0.018108757575758 | 0.019171428571429 | 0.011283422459893 | 0.030547486033520 | 0.935398547779674 | 0.935398547779674 |
| Parallel | 2 | 0.006391076923077 | 0.026217890322581 | 0.025881118881119 | 0.014488888888889 | 0.023450617283951 | 1.218478546986050 | 0.609239273493025 |
| Parallel | 4 | 0.010787779411765 | 0.045288800000000 | 0.046048387096774 | 0.017556451612903 | 0.025347826086957 | 1.127279080109270 | 0.281819770027317 |
| Parallel | 8 | 0.010086389534884 | 0.028864458064516 | 0.028935251798561 | 0.020349315068493 | 0.025357615894040 | 1.126843871816450 | 0.140855483977056 |
| Parallel | 16 | 0.010535715909091 | 0.026291983050848 | 0.024743055555556 | 0.019778571428572 | 0.021926553672316 | 1.303172149216980 | 0.081448259326062 |
| Parallel | 32 | 0.013170913043478 | 0.027333571428572 | 0.025642857142857 | 0.022816793893130 | 0.024311377245509 | 1.175337529647880 | 0.036729297801496 |

Setup 1



Setup 2



SIZE-2500000-O2

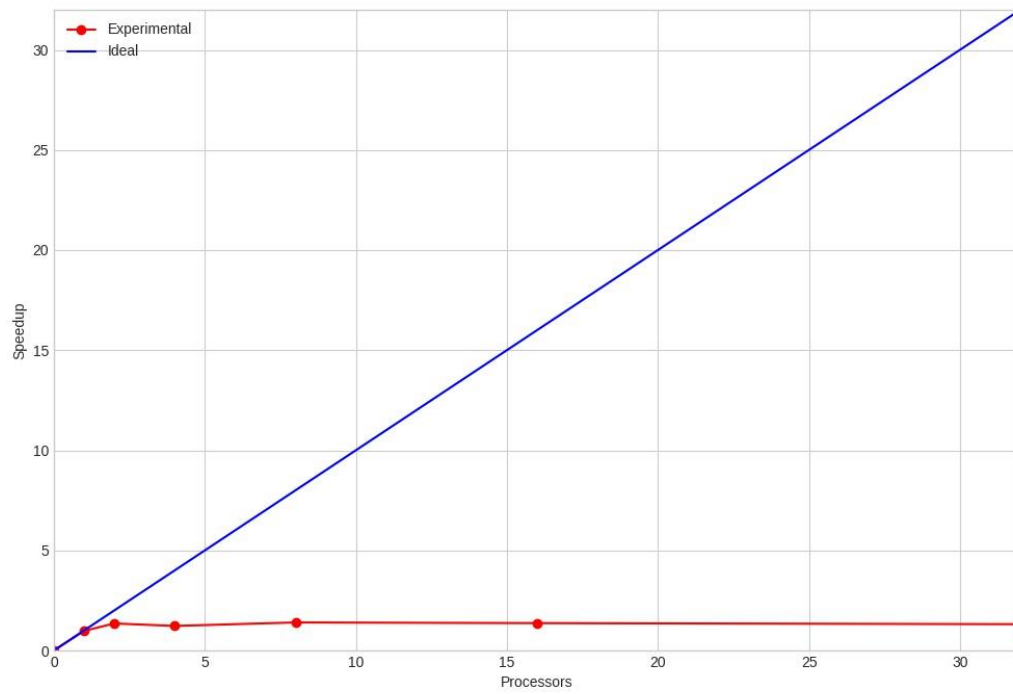
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|----------------------|----------------------|-----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.004985172619047619 | 0.014880198717948715 | 0.014806451612903225 | 0.006104166666666667 | 0.020999999999999994 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.004916365269461078 | 0.014954616883116884 | 0.014843137254901960 | 0.0061798561151079146 | 0.021497461928934002 | 0.9768595041322315 | 0.9768595041322315 |
| Parallel | 2 | 0.004443410714285715 | 0.022344022727272726 | 0.026358208955223877 | 0.0072442748091603058 | 0.015560606060606059 | 1.3495618305744885 | 0.67478091528724427 |
| Parallel | 4 | 0.006110678787878786 | 0.045756445121951220 | 0.053307142857142858 | 0.0091428571428571435 | 0.017132867132867134 | 1.2257142857142853 | 0.30642857142857133 |
| Parallel | 8 | 0.007107145833333334 | 0.022005401459854013 | 0.020327731092436973 | 0.010239999999999991 | 0.015000000000000005 | 1.3999999999999992 | 0.17499999999999991 |
| Parallel | 16 | 0.007665773722627737 | 0.022665627450980393 | 0.020991999999999997 | 0.0113129770992366416 | 0.015374100719424461 | 1.3659335517080016 | 0.08537084698175010 |
| Parallel | 32 | 0.008660166666666667 | 0.023310758865248228 | 0.021114035087719293 | 0.0131101694915254224 | 0.016000000000000007 | 1.3124999999999991 | 0.04101562499999997 |

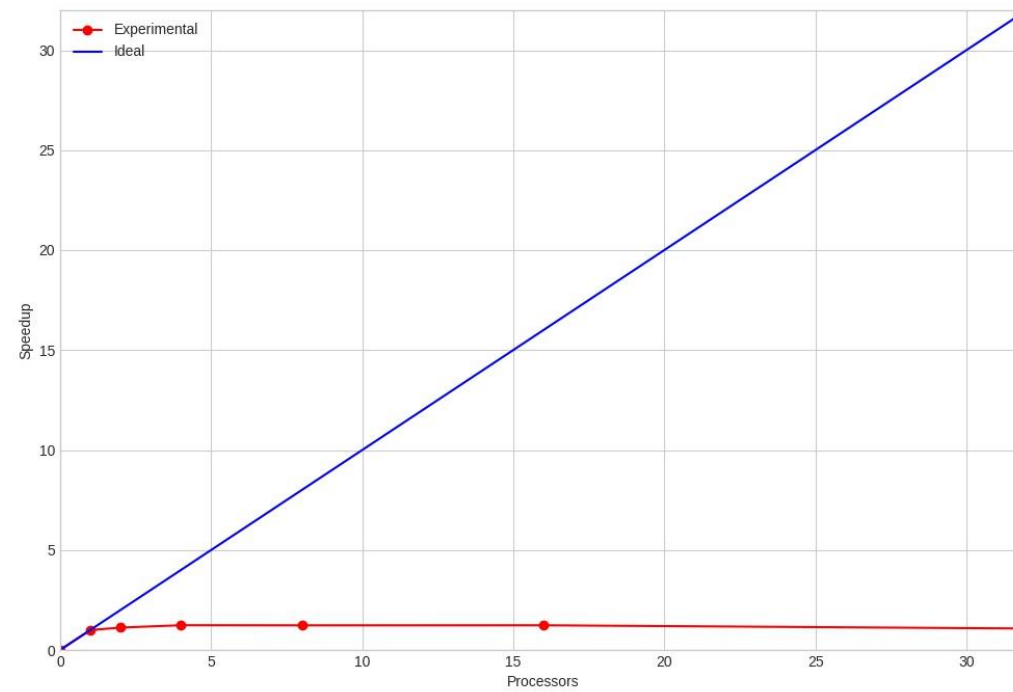
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|--------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.006579062857143 | 0.019566476744186 | 0.018467105263158 | 0.013487179487180 | 0.030629411764706 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.007155246231156 | 0.018731066666667 | 0.018506250000000 | 0.011345177664975 | 0.030796954314721 | 0.994559768855622 | 0.994559768855622 |
| Parallel | 2 | 0.0079180555555556 | 0.030701266187050 | 0.029405594405594 | 0.015719178082192 | 0.027373134328358 | 1.118958881262430 | 0.559479440631214 |
| Parallel | 4 | 0.010692265151515 | 0.047059032679739 | 0.045948905109489 | 0.019666666666667 | 0.024751515151515 | 1.237476234372300 | 0.309369058593075 |
| Parallel | 8 | 0.011486467336684 | 0.029719607142857 | 0.027484848484849 | 0.018141414141414 | 0.024870466321244 | 1.231557598039220 | 0.153944699754902 |
| Parallel | 16 | 0.011636342857143 | 0.028528459770115 | 0.027109489051095 | 0.021492307692308 | 0.024815028901734 | 1.234308929721430 | 0.077144308107590 |
| Parallel | 32 | 0.014531715151515 | 0.031392593750000 | 0.031321428571429 | 0.024443609022556 | 0.028553459119497 | 1.072704068411510 | 0.033522002137860 |

Setup 1



Setup 2



SIZE-2500000-O3

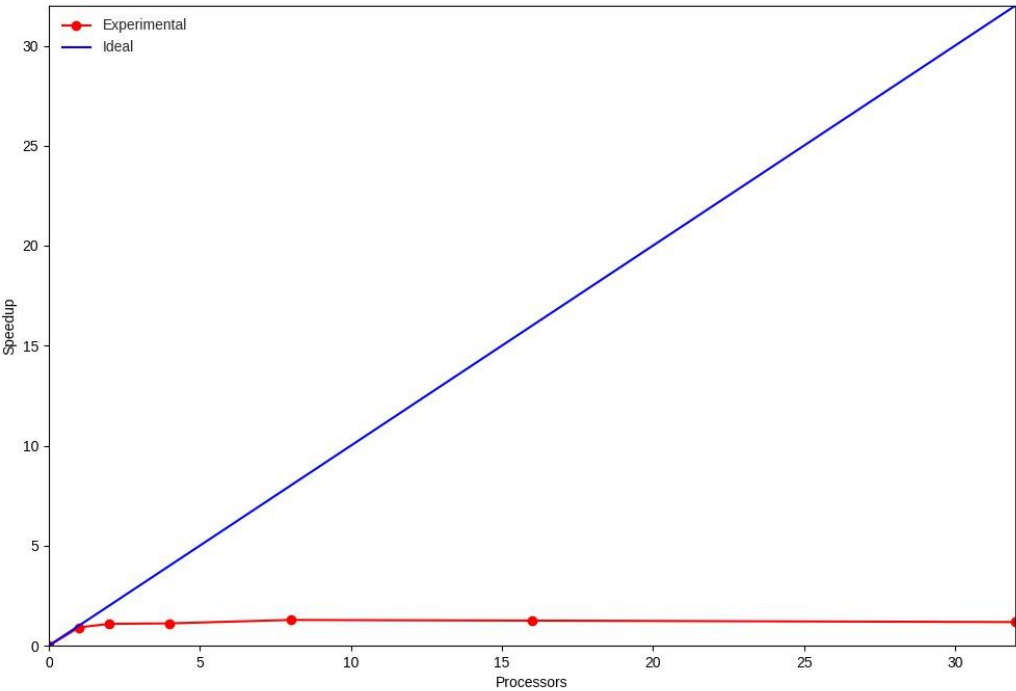
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-----------------------|----------------------|----------------------|----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.0044155581395348836 | 0.014545490196078431 | 0.013023809523809528 | 0.008191489361702130 | 0.020000000000000000 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.0044012690058479525 | 0.016243919117647058 | 0.014159574468085103 | 0.008896296296296296 | 0.021999999999999995 | 0.9090909090909093 | 0.90909090909090928 |
| Parallel | 2 | 0.0038945431034482763 | 0.027027067669172934 | 0.026641791044776115 | 0.009941666666666666 | 0.018389830508474574 | 1.0875576036866361 | 0.54377880184331806 |
| Parallel | 4 | 0.0052870059880239517 | 0.048034598591549293 | 0.054412213740458001 | 0.010651162790697674 | 0.018000000000000002 | 1.1111111111111109 | 0.27777777777777773 |
| Parallel | 8 | 0.0062723759398496245 | 0.021967486111111115 | 0.018280991735537190 | 0.011698412698412698 | 0.015594771241830067 | 1.2824811399832354 | 0.16031014249790443 |
| Parallel | 16 | 0.0069025928571428566 | 0.022744806896551725 | 0.017891472868217052 | 0.013775193798449610 | 0.016000000000000007 | 1.2499999999999996 | 0.07812499999999997 |
| Parallel | 32 | 0.0078903581081081076 | 0.023354952702702703 | 0.018952755905511810 | 0.013692913385826771 | 0.017000000000000001 | 1.1764705882352942 | 0.03676470588235294 |

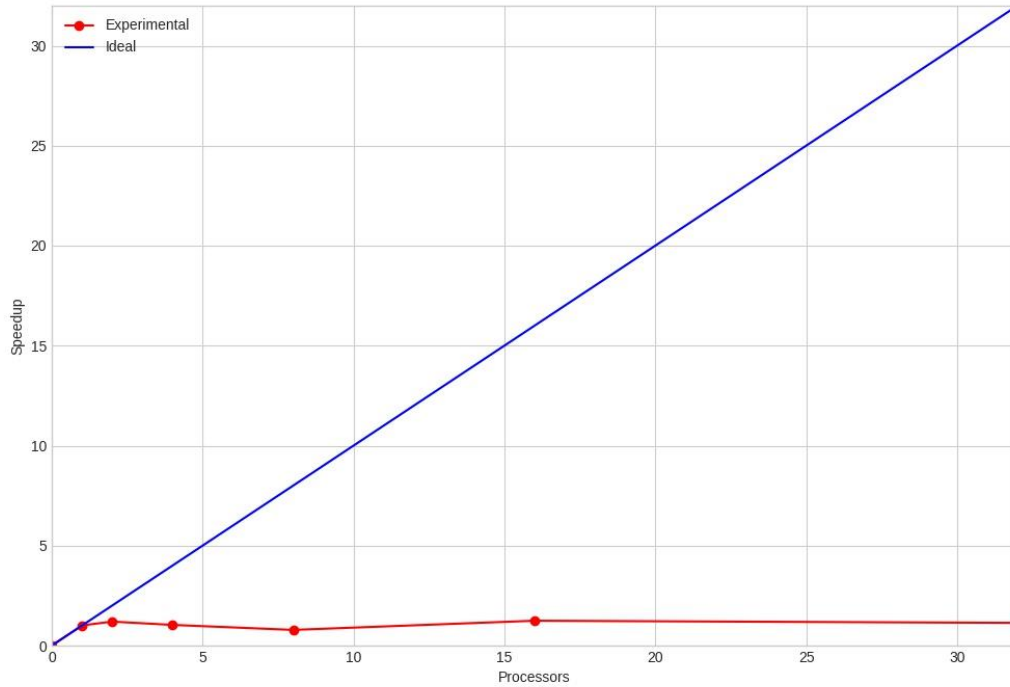
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.005734435754190 | 0.018684898876405 | 0.016455172413793 | 0.011696132596685 | 0.029372881355932 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.005807459302326 | 0.018729712500000 | 0.017540540540541 | 0.012380645161290 | 0.029141025641026 | 1.007956333375590 | 1.007956333375590 |
| Parallel | 2 | 0.006891304568528 | 0.025720010989011 | 0.022804054054054 | 0.015056410256410 | 0.024475935828877 | 1.200071840410600 | 0.600035920205301 |
| Parallel | 4 | 0.009615192982456 | 0.050287314285714 | 0.046275590551181 | 0.021972413793104 | 0.028403726708075 | 1.034120686268330 | 0.258530171567083 |
| Parallel | 8 | 0.012266305699482 | 0.042380010362694 | 0.034591240875912 | 0.026098445595855 | 0.037463917525773 | 0.784031230469297 | 0.098003903808662 |
| Parallel | 16 | 0.009867690476190 | 0.026917442176871 | 0.024533834586466 | 0.018784615384615 | 0.023623188405797 | 1.243391910159090 | 0.077711994384943 |
| Parallel | 32 | 0.012504337349398 | 0.029470591463415 | 0.027452554744526 | 0.023784172661871 | 0.025849056603774 | 1.136323147346280 | 0.035510098354571 |

Setup 1



Setup 2



SIZE-5000000-O1

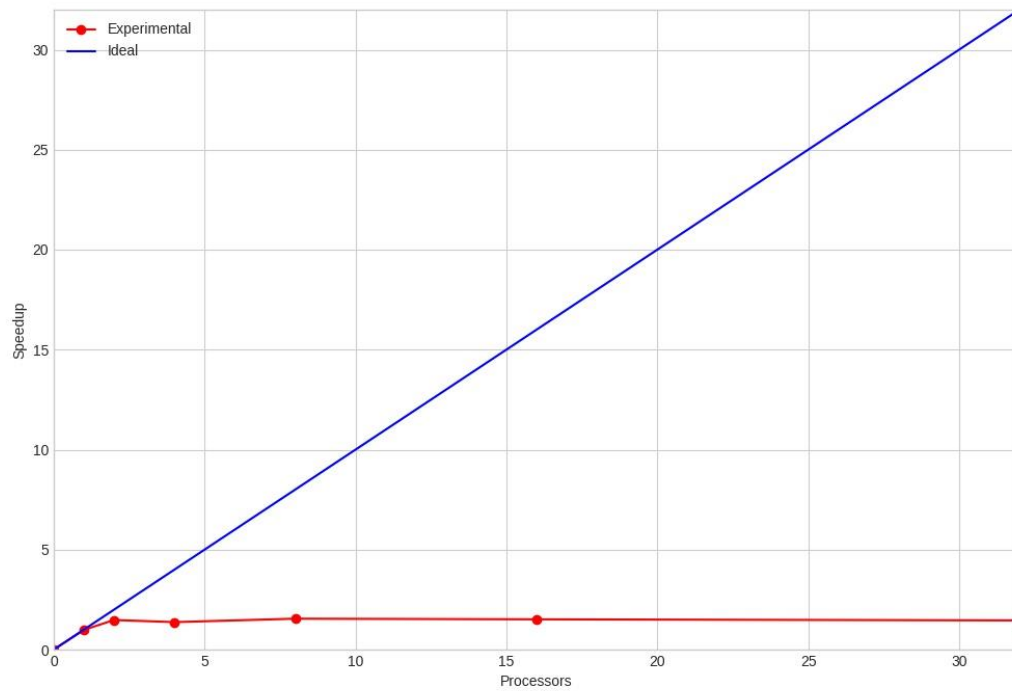
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|----------------------|----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.010020970588235296 | 0.02980183146067416 | 0.028788321167883213 | 0.013814814814814814 | 0.041475675675675665 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.009963071942446045 | 0.02992207534246575 | 0.028284482758620694 | 0.013827272727272728 | 0.041999999999999999 | 0.9875160875160875 | 0.9875160875160875 |
| Parallel | 2 | 0.010407783333333333 | 0.04250705042016807 | 0.046086614173228360 | 0.016779411764705883 | 0.028033333333333341 | 1.4795128065044822 | 0.73975640325224112 |
| Parallel | 4 | 0.014266171428571426 | 0.09310208148148148 | 0.094571428571428556 | 0.017204545454545455 | 0.030192982456140351 | 1.3736859462600308 | 0.34342148656500771 |
| Parallel | 8 | 0.014170200000000001 | 0.04292522535211267 | 0.040175572519083970 | 0.019007462686567167 | 0.026831325301204818 | 1.5457930314688282 | 0.19322412893360352 |
| Parallel | 16 | 0.014989246153846156 | 0.04484670253164557 | 0.041897260273972593 | 0.019944444444444445 | 0.027392592592592590 | 1.5141201233683654 | 0.09463250771052284 |
| Parallel | 32 | 0.016380801242236027 | 0.04570845512820513 | 0.043595744680851062 | 0.020954545454545451 | 0.028424050632911389 | 1.4591754078728025 | 0.04559923149602508 |

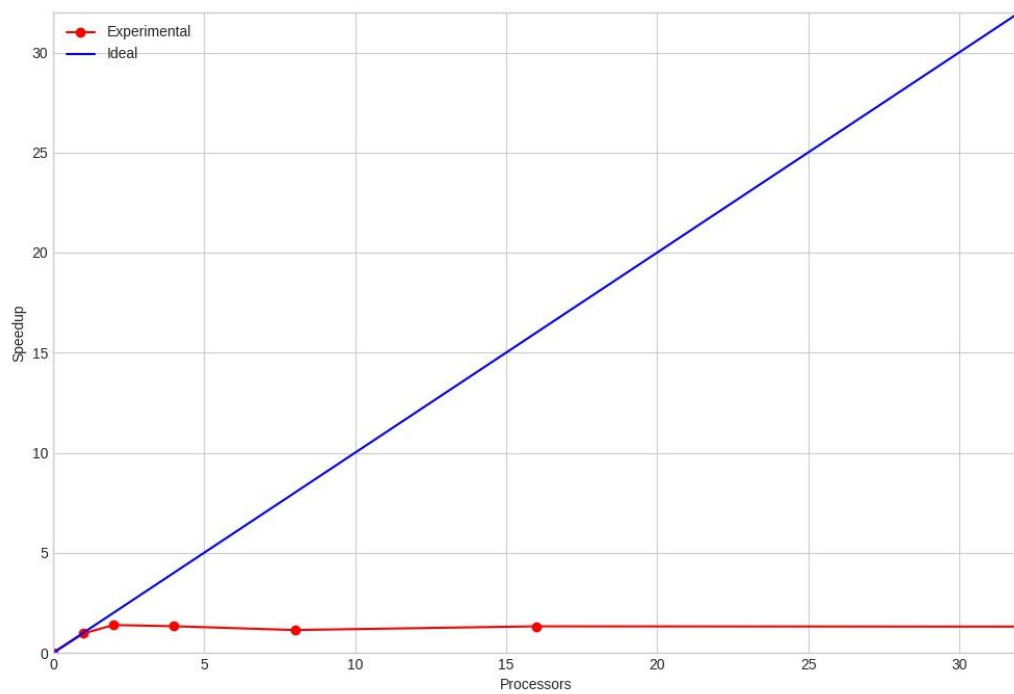
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|--------------------|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Serial | 1 | 0.0111769546153846 | 0.033225733727811 | 0.029020689655173 | 0.019950704225352 | 0.049393548387097 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.0119891000000000 | 0.034190378378378 | 0.0305347222222222 | 0.0203712121212121 | 0.051248648648649 | 0.963801967262198 | 0.963801967262198 |
| Parallel | 2 | 0.011102898876405 | 0.041372624365482 | 0.035220689655173 | 0.021406091370558 | 0.035583756345178 | 1.388092586627400 | 0.694046293313699 |
| Parallel | 4 | 0.019820916666667 | 0.074900787356322 | 0.069642424242424 | 0.030626760563380 | 0.037214689265537 | 1.327259460227130 | 0.331814865056783 |
| Parallel | 8 | 0.020271719512195 | 0.060151853503185 | 0.053150684931507 | 0.036763888888889 | 0.0434444444444445 | 1.136935896378190 | 0.142116987047273 |
| Parallel | 16 | 0.019531618750000 | 0.053584896341464 | 0.048145833333333 | 0.033240259740260 | 0.037335164835165 | 1.322976571957560 | 0.082686035747348 |
| Parallel | 32 | 0.021167853658537 | 0.056034441176471 | 0.049815602836880 | 0.034082089552239 | 0.037739393939394 | 1.308806083807770 | 0.040900190118993 |

Setup 1



Setup 2



SIZE-5000000-O2

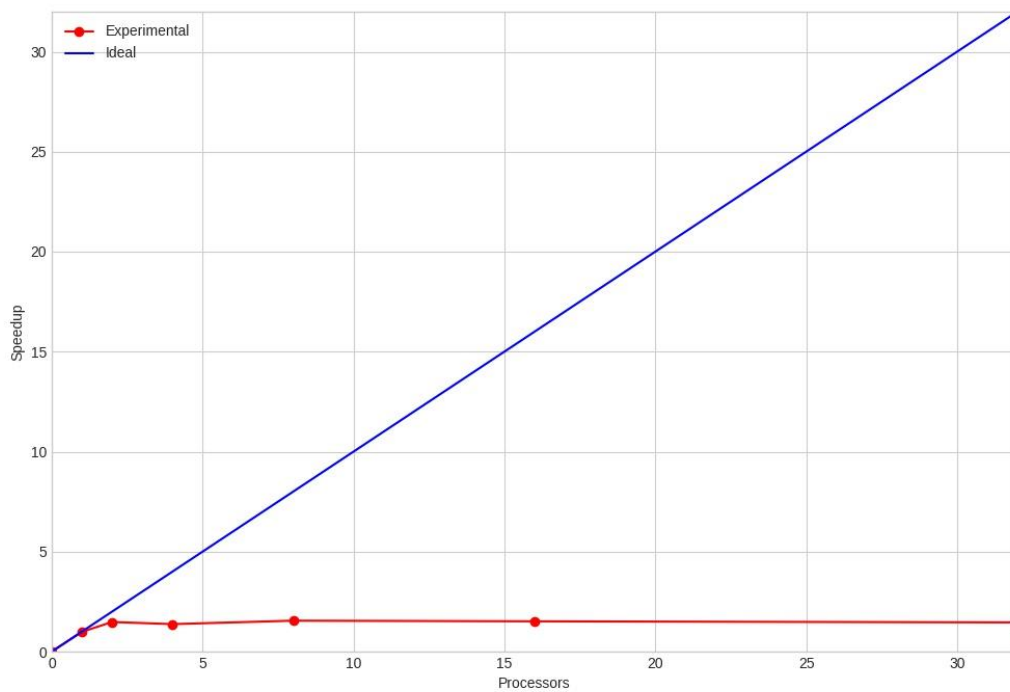
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|----------------------|----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.010020970588235296 | 0.02980183146067416 | 0.028788321167883213 | 0.013814814814814814 | 0.041475675675675665 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.009963071942446045 | 0.02992207534246575 | 0.028284482758620694 | 0.013827272727272728 | 0.041999999999999999 | 0.9875160875160875 | 0.9875160875160875 |
| Parallel | 2 | 0.010407783333333333 | 0.04250705042016807 | 0.046086614173228360 | 0.016779411764705883 | 0.028033333333333341 | 1.4795128065044822 | 0.73975640325224112 |
| Parallel | 4 | 0.014266171428571426 | 0.09310208148148148 | 0.094571428571428556 | 0.017204545454545455 | 0.030192982456140351 | 1.3736859462600308 | 0.34342148656500771 |
| Parallel | 8 | 0.014170200000000001 | 0.04292522535211267 | 0.040175572519083970 | 0.019007462686567167 | 0.026831325301204818 | 1.5457930314688282 | 0.19322412893360352 |
| Parallel | 16 | 0.014989246153846156 | 0.04484670253164557 | 0.041897260273972593 | 0.019944444444444445 | 0.027392592592592590 | 1.5141201233683654 | 0.09463250771052284 |
| Parallel | 32 | 0.016380801242236027 | 0.04570845512820513 | 0.043595744680851062 | 0.020954545454545451 | 0.028424050632911389 | 1.4591754078728025 | 0.04559923149602508 |

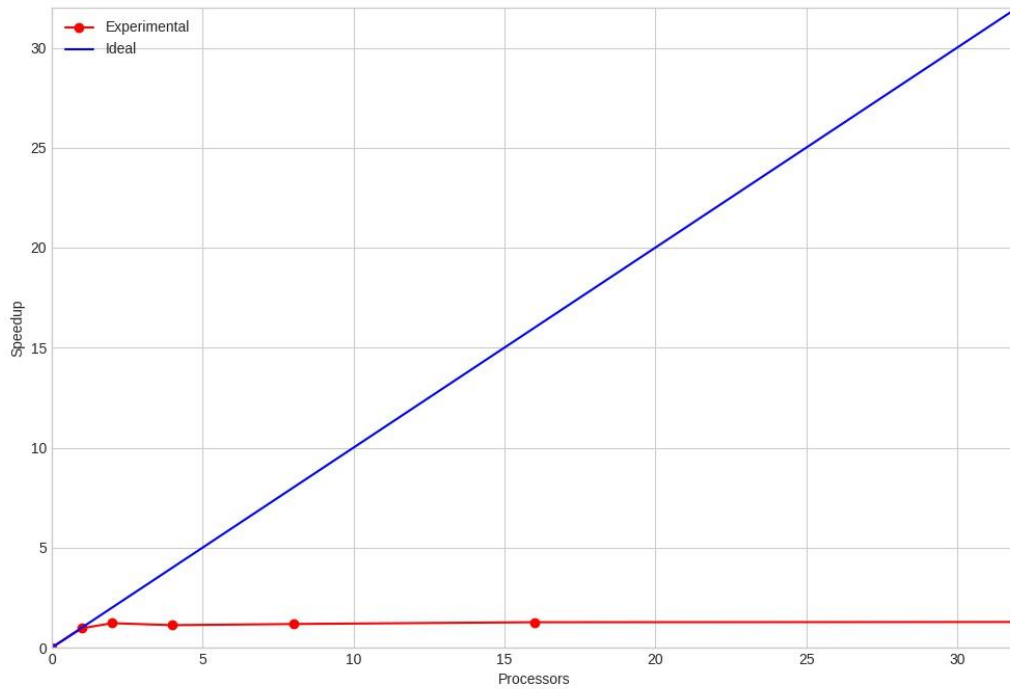
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|--------------------|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Serial | 1 | 0.011761904191617 | 0.035223089887641 | 0.031174825174825 | 0.0214444444444445 | 0.051447058823530 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.012992788819876 | 0.035249136612022 | 0.0325000000000000 | 0.0214545454545456 | 0.053225988700565 | 0.966577795538128 | 0.966577795538128 |
| Parallel | 2 | 0.014988029239766 | 0.047909866310161 | 0.043067073170732 | 0.0250454545454555 | 0.042134408602151 | 1.221022450067180 | 0.610511225033589 |
| Parallel | 4 | 0.022085261146497 | 0.079167502732241 | 0.072197183098592 | 0.034640211640212 | 0.045851063829787 | 1.122047222601340 | 0.280511805650334 |
| Parallel | 8 | 0.0191455600000000 | 0.055540077419355 | 0.050276119402985 | 0.032816176470588 | 0.043620512820513 | 1.179423521113120 | 0.147427940139141 |
| Parallel | 16 | 0.020867895953757 | 0.057045748427673 | 0.050821917808219 | 0.035021428571429 | 0.040556886227545 | 1.268515993434140 | 0.079282249589634 |
| Parallel | 32 | 0.022830391608392 | 0.055451430555556 | 0.049331034482759 | 0.035328125000000 | 0.0401866666666667 | 1.280202193684380 | 0.040006318552637 |

Setup 1



Setup 2



SIZE-5000000-O3

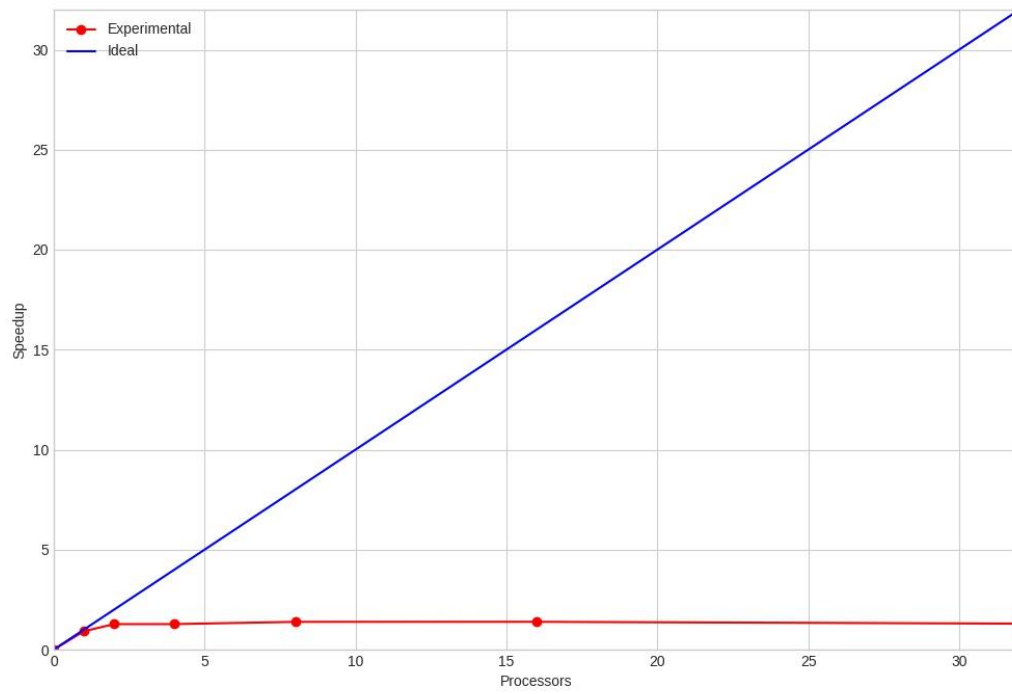
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|----------------------|----------------------|----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.008896715447154473 | 0.029058073529411765 | 0.025598360655737715 | 0.014379032258064519 | 0.039511904761904762 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.008868425675675678 | 0.032401844827586207 | 0.028294964028776976 | 0.014864406779661018 | 0.042999999999999990 | 0.9188815060908087 | 0.91888150609080865 |
| Parallel | 2 | 0.009191398305084746 | 0.050077076190476195 | 0.048024000000000011 | 0.019301587301587302 | 0.031022388059701485 | 1.2736577431068654 | 0.63682887155343271 |
| Parallel | 4 | 0.011204182432432435 | 0.096145564516129017 | 0.090946153846153829 | 0.019913385826771657 | 0.031005847953216384 | 1.2743371773454757 | 0.31858429433636892 |
| Parallel | 8 | 0.012515636363636364 | 0.043328792592592591 | 0.036481751824817520 | 0.021401515151515147 | 0.028496402877697842 | 1.3865576273427827 | 0.17331970341784783 |
| Parallel | 16 | 0.013201676691729322 | 0.044000635135135129 | 0.037318518518518522 | 0.021340425531914894 | 0.028440559440559440 | 1.3892801526806937 | 0.08683000954254336 |
| Parallel | 32 | 0.014492514084507042 | 0.045230027777777779 | 0.038357664233576637 | 0.023820143884892082 | 0.030452380952380956 | 1.2974980453479279 | 0.04054681391712275 |

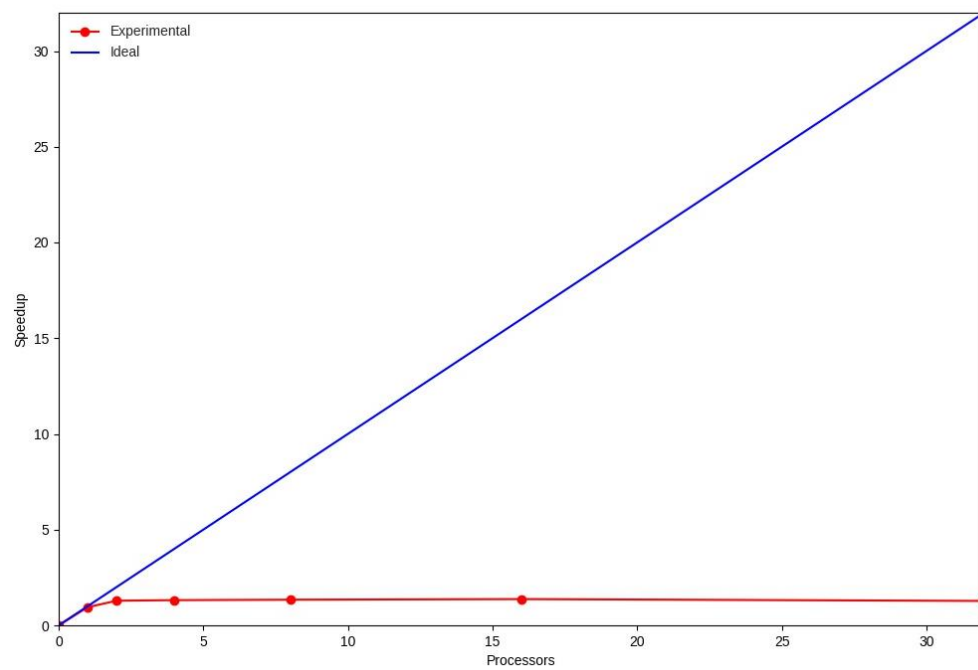
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.010087121019108 | 0.036213965034965 | 0.030090909090909 | 0.020731543624161 | 0.051184397163121 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.010867993670886 | 0.037110419540230 | 0.031809523809524 | 0.023020408163265 | 0.053950920245399 | 0.948721484829276 | 0.948721484829276 |
| Parallel | 2 | 0.011345147435898 | 0.046236284023669 | 0.038641379310345 | 0.024546762589928 | 0.039789772727273 | 1.286370684093850 | 0.643185342046924 |
| Parallel | 4 | 0.016560068493151 | 0.077273337837838 | 0.065822695035461 | 0.033049295774648 | 0.038874251497006 | 1.316665792705040 | 0.329166448176261 |
| Parallel | 8 | 0.016287631901841 | 0.055709273972603 | 0.045048611111111 | 0.032242647058824 | 0.038247126436782 | 1.338254711702930 | 0.167281838962866 |
| Parallel | 16 | 0.018123030303030 | 0.055445168674699 | 0.047655172413793 | 0.034033333333333 | 0.037318435754190 | 1.371557947933920 | 0.085722371745870 |
| Parallel | 32 | 0.021019171597633 | 0.059039172222222 | 0.050306122448980 | 0.037516778523490 | 0.040153846153846 | 1.274707209043230 | 0.039834600282601 |

Setup 1



Setup 2



SIZE-10000000-O1

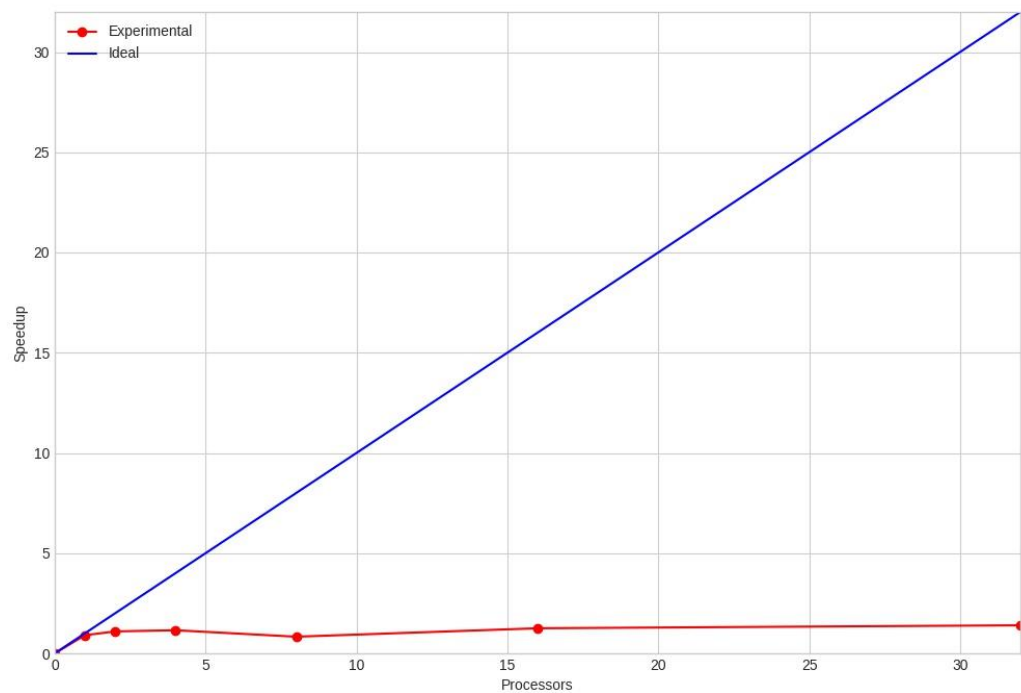
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|---------------------|----------------------|----------------------|--------------------|----------------------|
| Serial | 1 | 0.019658585034013607 | 0.07008259459459461 | 0.06697600000000002 | 0.024878787878787879 | 0.092118881118881099 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.019643524691358025 | 0.05949182530120482 | 0.05634558823529411 | 0.025575757575757571 | 0.08199999999999990 | 1.1234009892546477 | 1.123400989254647708 |
| Parallel | 2 | 0.022915609999999996 | 0.07979133009708737 | 0.08218333333333332 | 0.030262773722627735 | 0.054733944954128456 | 1.6830301780016823 | 0.841515089000841154 |
| Parallel | 4 | 0.031055724832214762 | 0.13908114084507042 | 0.13910144927536233 | 0.033698529411764711 | 0.052311377245508987 | 1.7609722008760464 | 0.440243050219011611 |
| Parallel | 8 | 0.029744593984962410 | 0.08437416129032257 | 0.08090579710144927 | 0.036284671532846716 | 0.048869230769230769 | 1.8850077987493379 | 0.235625974843667235 |
| Parallel | 16 | 0.030803588235294114 | 0.08530852238805971 | 0.08159259259259259 | 0.038059259259259258 | 0.048882812500000011 | 1.8844840631639408 | 0.117780253947746300 |
| Parallel | 32 | 0.032141650684931508 | 0.08905139007092200 | 0.08477083333333332 | 0.040014084507042247 | 0.050208053691275165 | 1.8347431208011342 | 0.057335722525035444 |

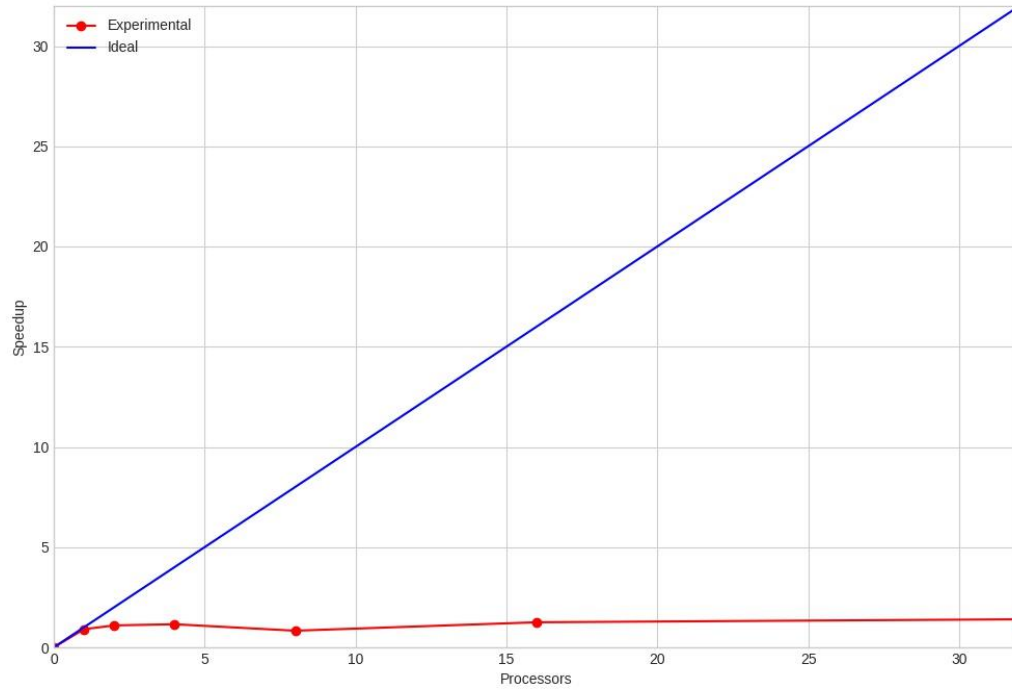
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.023749299465241 | 0.066009826589595 | 0.059243421052632 | 0.037229299363057 | 0.095538011695906 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.025776119718310 | 0.072986490445860 | 0.063859154929578 | 0.041855172413793 | 0.105574324324324 | 0.904936046783626 | 0.904936046783626 |
| Parallel | 2 | 0.032130418750000 | 0.101925450549451 | 0.087700000000000 | 0.054406060606061 | 0.087197802197802 | 1.095647015038120 | 0.547823507519060 |
| Parallel | 4 | 0.038839555555556 | 0.157831178010471 | 0.134448979591837 | 0.068270833333333 | 0.082786885245902 | 1.154023507613920 | 0.288505876903480 |
| Parallel | 8 | 0.049537601226994 | 0.181939437500000 | 0.148290540540541 | 0.097366013071896 | 0.114924528301887 | 0.831310888176497 | 0.103913861022062 |
| Parallel | 16 | 0.038868171641791 | 0.133718668965517 | 0.109801418439716 | 0.072515384615385 | 0.076310559006211 | 1.251963200638200 | 0.078247700039888 |
| Parallel | 32 | 0.039340127272727 | 0.118987705882353 | 0.099127659574468 | 0.069021582733813 | 0.068248366013072 | 1.399857861470380 | 0.043745558170949 |

Setup 1



Setup 2



SIZE-10000000-O2

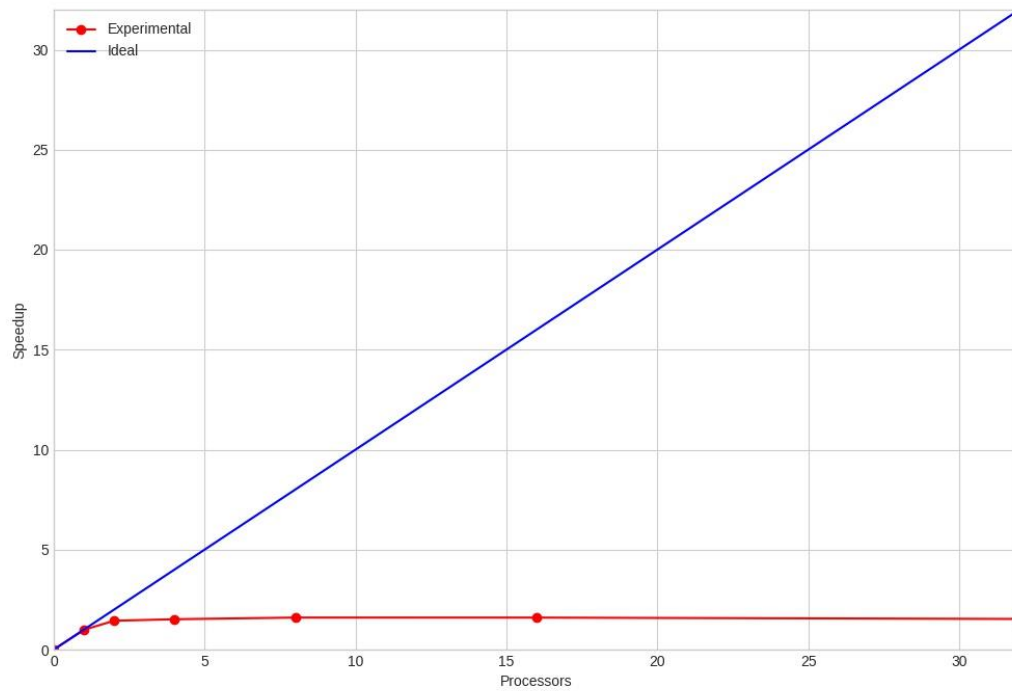
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|----------------------|---------------------|----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.019674050632911392 | 0.058347114942528734 | 0.05460000000000001 | 0.025838926174496641 | 0.080563636363636365 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.019624315789473684 | 0.058951970414201194 | 0.05582692307692307 | 0.024668918918918918 | 0.081421052631578950 | 0.9894693541752365 | 0.98946935417523652 |
| Parallel | 2 | 0.023685285714285714 | 0.082923150943396223 | 0.08062204724409450 | 0.030413533834586465 | 0.055826923076923086 | 1.4430964832618292 | 0.72154824163091458 |
| Parallel | 4 | 0.031254773333333333 | 0.137185698529411759 | 0.13757142857142859 | 0.034937007874015741 | 0.053079136690647483 | 1.5178023115404520 | 0.37945057788511299 |
| Parallel | 8 | 0.029855333333333334 | 0.083174135714285724 | 0.08077099236641221 | 0.035551470588235295 | 0.050315789473684210 | 1.6011601369341957 | 0.20014501711677446 |
| Parallel | 16 | 0.030963713235294114 | 0.084687566176470594 | 0.08144285714285715 | 0.037425531914893617 | 0.050352941176470579 | 1.5999787595581991 | 0.09999867247238745 |
| Parallel | 32 | 0.031973932432432427 | 0.088384294520547940 | 0.08372857142857142 | 0.040046153846153842 | 0.052600000000000001 | 1.5316280677497407 | 0.04786337711717940 |

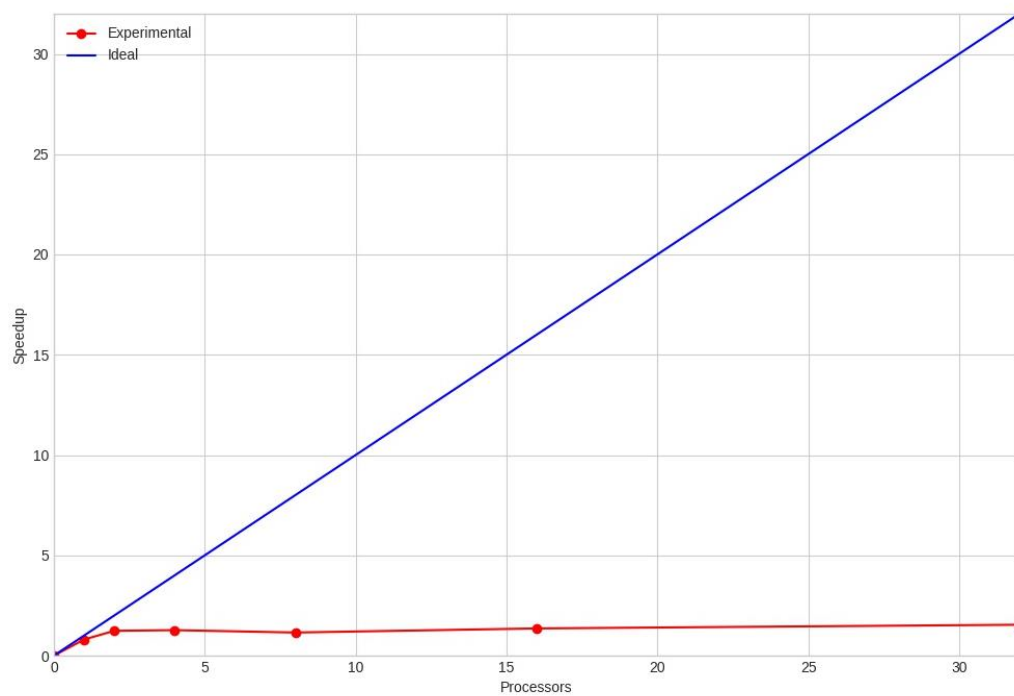
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.025699723684211 | 0.073814032258065 | 0.061570370370370 | 0.044160839160839 | 0.105519480519481 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.030251937853107 | 0.079392960451977 | 0.070323170731707 | 0.047928571428572 | 0.131447236180905 | 0.802751610343934 | 0.802751610343934 |
| Parallel | 2 | 0.033469870503597 | 0.098074905405405 | 0.086721854304636 | 0.052185714285714 | 0.085414201183432 | 1.235385674249550 | 0.617692837124773 |
| Parallel | 4 | 0.044579741007194 | 0.164559744966443 | 0.140451388888889 | 0.077746666666667 | 0.083104046242775 | 1.269727351316000 | 0.317431837829000 |
| Parallel | 8 | 0.045358840764331 | 0.148841680000000 | 0.122109589041096 | 0.083265306122449 | 0.091664516129032 | 1.151148612086110 | 0.143893576510764 |
| Parallel | 16 | 0.041041486301370 | 0.134117513333333 | 0.112735294117647 | 0.071700000000000 | 0.077923076923077 | 1.354149305778130 | 0.084634331611133 |
| Parallel | 32 | 0.040991197674419 | 0.113452395604396 | 0.097962732919255 | 0.066516556291391 | 0.068577142857143 | 1.538697532781360 | 0.048084297899418 |

Setup 1



Setup 2



SIZE-10000000-O3

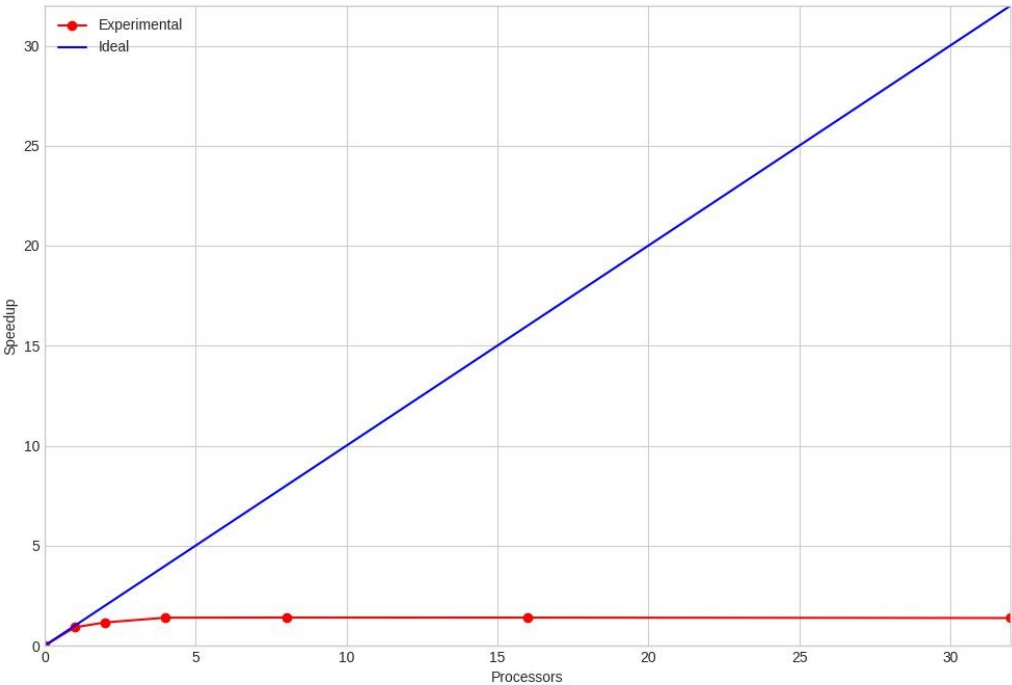
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|----------------------|----------------------|----------------------|--------------------|---------------------|
| Serial | 1 | 0.017476393617021274 | 0.05700595833333333 | 0.048692307692307688 | 0.028228571428571426 | 0.076999999999999985 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.017520804054054057 | 0.06377591515151515 | 0.056413043478260864 | 0.027717391304347826 | 0.083999999999999977 | 0.9166666666666667 | 0.9166666666666667 |
| Parallel | 2 | 0.020920747747747749 | 0.09426002173913044 | 0.082096774193548383 | 0.037511111111111113 | 0.066548387096774200 | 1.1570528356761993 | 0.57852641783809966 |
| Parallel | 4 | 0.024963326086956523 | 0.13851411333333336 | 0.127653543307086642 | 0.037274809160305337 | 0.055181250000000001 | 1.3954015177256764 | 0.34885037943141911 |
| Parallel | 8 | 0.025622139534883719 | 0.08502094244604316 | 0.073753731343283571 | 0.039546099290780144 | 0.054897810218978105 | 1.4026060364313253 | 0.17532575455391566 |
| Parallel | 16 | 0.026313617391304348 | 0.08642998026315790 | 0.073177304964539003 | 0.042210884353741492 | 0.055015748031496073 | 1.3995992557606980 | 0.08747495348504362 |
| Parallel | 32 | 0.027812402777777776 | 0.08710851034482758 | 0.074930069930069926 | 0.043323741007194237 | 0.055843537414965984 | 1.3788524789864780 | 0.04308913996832744 |

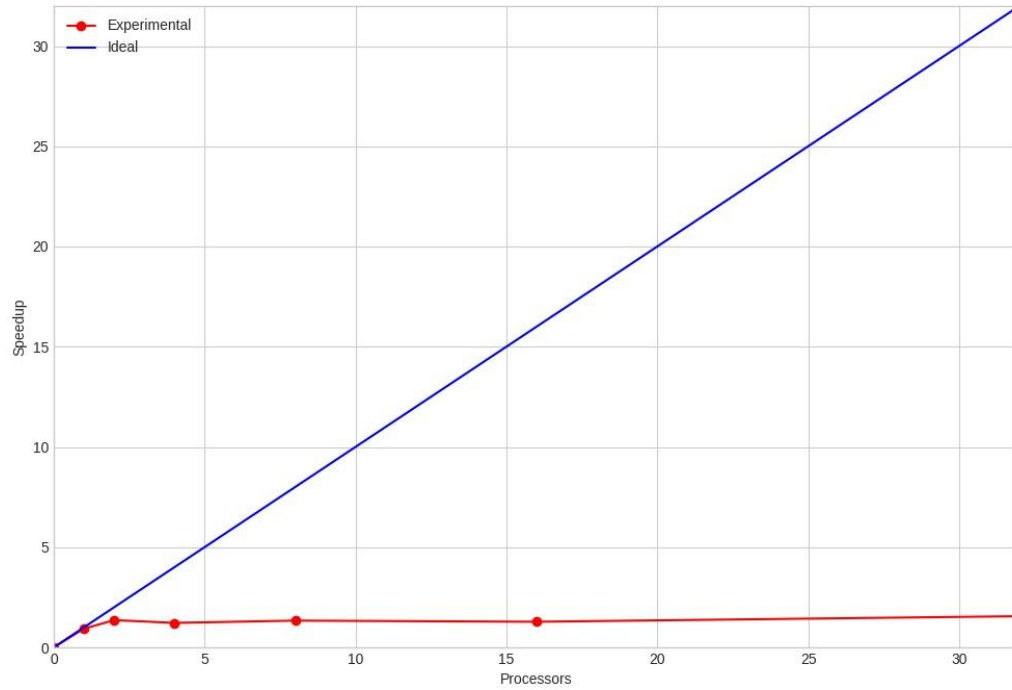
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Serial | 1 | 0.023772425000000 | 0.076552297752809 | 0.061456250000000 | 0.045311111111111 | 0.106619318181818 | 1.000000000000000 | 1.000000000000000 |
| Parallel | 1 | 0.026220480446927 | 0.079383034285714 | 0.064469879518072 | 0.050971098265896 | 0.114443181818182 | 0.931635388739946 | 0.931635388739946 |
| Parallel | 2 | 0.025154666666667 | 0.091462148648649 | 0.073232394366197 | 0.051394366197183 | 0.078473118279570 | 1.358673142081270 | 0.679336571040634 |
| Parallel | 4 | 0.039984112582782 | 0.173315025000000 | 0.139860759493671 | 0.081710344827586 | 0.087343023255814 | 1.220696447265710 | 0.305174111816427 |
| Parallel | 8 | 0.034955660493827 | 0.135582041176471 | 0.104111842105263 | 0.076503448275862 | 0.079901098901099 | 1.334391136644950 | 0.166798892080619 |
| Parallel | 16 | 0.039657792592593 | 0.137353694915254 | 0.107623376623377 | 0.079640211640212 | 0.083497175141243 | 1.276921261126050 | 0.079807578820378 |
| Parallel | 32 | 0.036242883977901 | 0.113269318181818 | 0.092020134228188 | 0.064516778523490 | 0.068553333333333 | 1.555275476735650 | 0.048602358647989 |

Setup 1



Setup 2



SIZE-15000000-O1

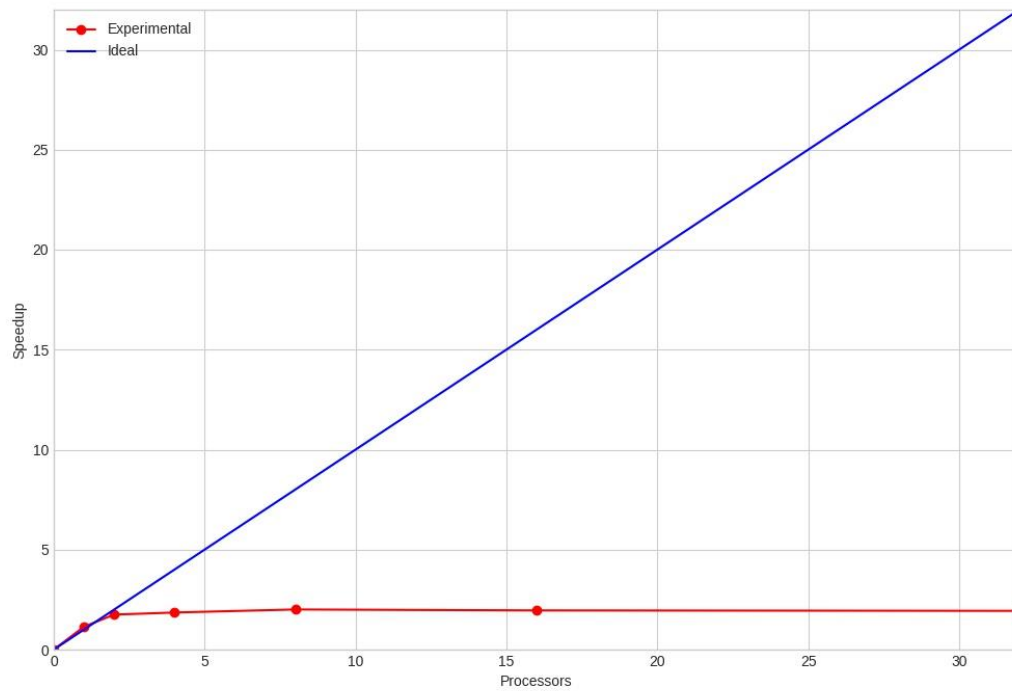
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|---------------------|----------------------|---------------------|--------------------|----------------------|
| Serial | 1 | 0.029844231788079476 | 0.10323951111111111 | 0.09911940298507461 | 0.037732824427480910 | 0.13650322580645163 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.029847993464052286 | 0.08584162416107381 | 0.08210273972602739 | 0.037788732394366198 | 0.11944360902255638 | 1.1428256976116120 | 1.1428256976116120 |
| Parallel | 2 | 0.031061466019417467 | 0.10686448120300751 | 0.10655284552845529 | 0.045386861313868612 | 0.07803597122302157 | 1.7492346627728201 | 0.874617331386410068 |
| Parallel | 4 | 0.044713659574468080 | 0.18288996732026142 | 0.18118840579710146 | 0.050233082706766921 | 0.07360714285714284 | 1.8544834170696975 | 0.463620854267424387 |
| Parallel | 8 | 0.043888920289855068 | 0.12284303614457831 | 0.11873381294964028 | 0.052492537313432834 | 0.06803125000000000 | 2.0064782847066844 | 0.250809785588335554 |
| Parallel | 16 | 0.045231340740740748 | 0.12486989115646259 | 0.12158823529411766 | 0.052926470588235290 | 0.06966473988439308 | 1.9594306392728285 | 0.122464414954551779 |
| Parallel | 32 | 0.046928263888888891 | 0.12563974100719424 | 0.11957600000000000 | 0.057634920634920657 | 0.07047368421052631 | 1.9369389770893064 | 0.060529343034040825 |

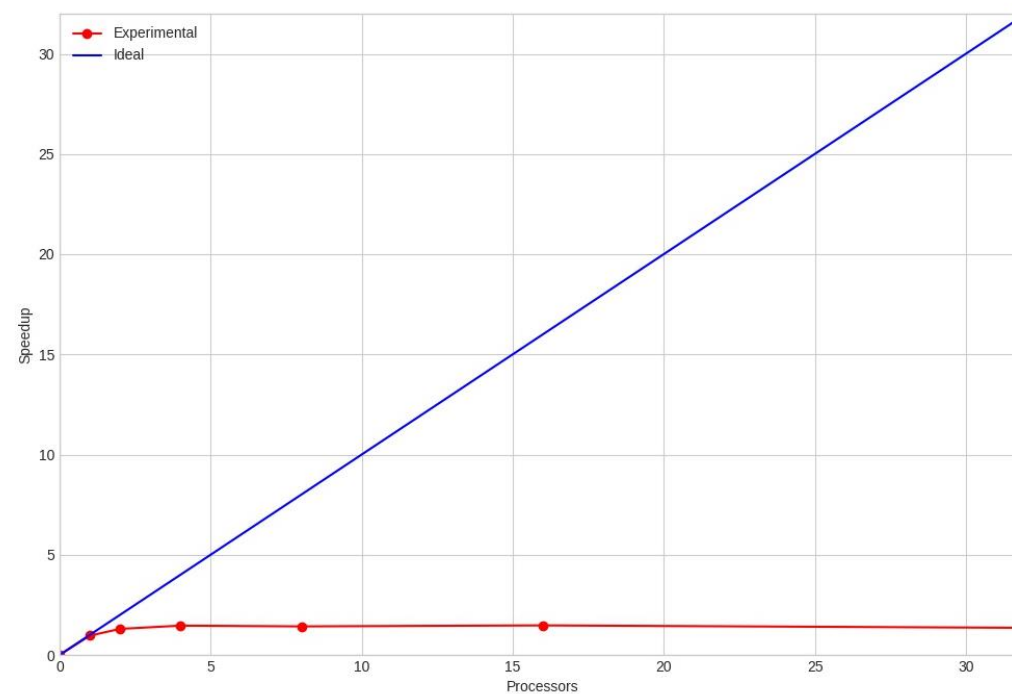
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.030851587500000 | 0.081404937853107 | 0.071432624113475 | 0.047701388888889 | 0.118773255813953 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.031745514084507 | 0.084954261904762 | 0.073316901408451 | 0.050813432835821 | 0.123337500000000 | 0.962993864914998 | 0.962993864914998 |
| Parallel | 2 | 0.035385237804878 | 0.108894285714286 | 0.093721518987342 | 0.058026490066225 | 0.091485875706215 | 1.298268775339330 | 0.649134387669665 |
| Parallel | 4 | 0.049846189024390 | 0.166949686390533 | 0.150208588957055 | 0.075973684210526 | 0.081261627906977 | 1.461615511196970 | 0.365403877799242 |
| Parallel | 8 | 0.049135879518072 | 0.147312337349398 | 0.127480263157895 | 0.079013698630137 | 0.083457627118644 | 1.423156395821130 | 0.177894549477642 |
| Parallel | 16 | 0.050159206896552 | 0.142320857954545 | 0.124000000000000 | 0.079735099337748 | 0.080604519774011 | 1.473530965099160 | 0.092095685318698 |
| Parallel | 32 | 0.055761556756757 | 0.151069045977011 | 0.132820359281437 | 0.085440476190476 | 0.087870056497175 | 1.351692038775140 | 0.042240376211723 |

Setup 1



Setup 2



SIZE-15000000-O2

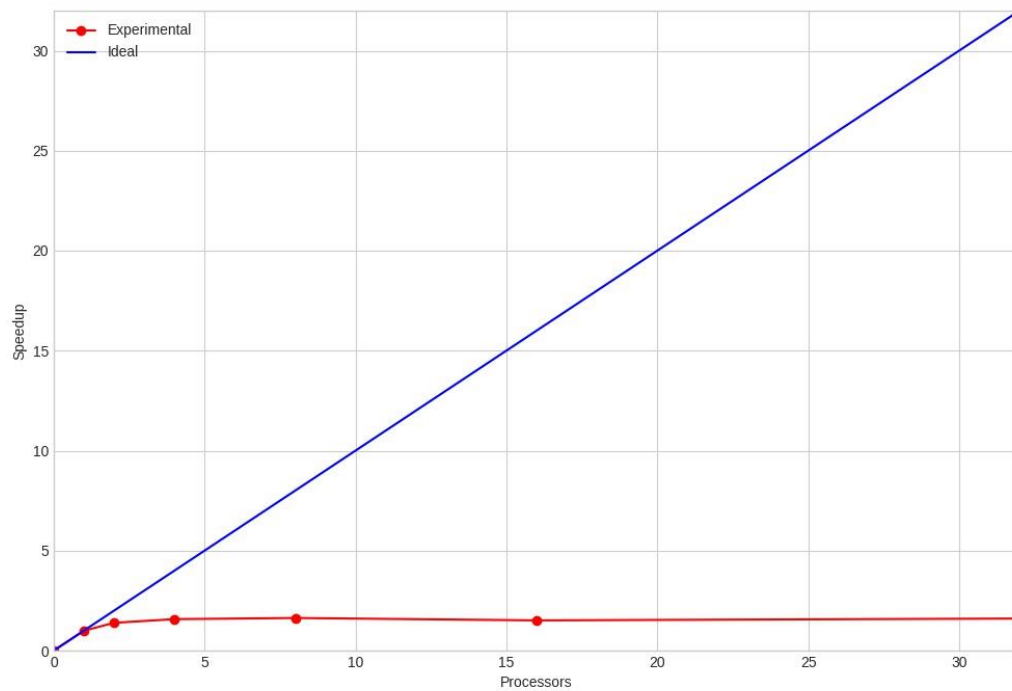
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|---------------------|----------------------|---------------------|--------------------|---------------------|
| Serial | 1 | 0.029912354166666669 | 0.08472194160583943 | 0.07967153284671533 | 0.038300699300699306 | 0.11800625000000001 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.029832862068965522 | 0.08502527586206897 | 0.08006382978723403 | 0.037999999999999999 | 0.11861061946902654 | 0.9949045922554653 | 0.99490459225546535 |
| Parallel | 2 | 0.035152242424242419 | 0.11359202803738321 | 0.11383333333333336 | 0.044204379562043788 | 0.08493333333333333 | 1.3893985478806907 | 0.69469927394034536 |
| Parallel | 4 | 0.043912820105820108 | 0.18065211038961040 | 0.17846478873239435 | 0.051398692810457516 | 0.07505347593582885 | 1.5722956002850024 | 0.39307390007125059 |
| Parallel | 8 | 0.044170434782608695 | 0.12343172727272726 | 0.11907913669064749 | 0.052893939393939389 | 0.07236686390532543 | 1.6306669051512679 | 0.20383336314390849 |
| Parallel | 16 | 0.045464676258992802 | 0.12728349999999999 | 0.12151908396946563 | 0.055323741007194248 | 0.07812408759124087 | 1.5104976408483604 | 0.09440610255302252 |
| Parallel | 32 | 0.046848581560283688 | 0.12476709090909094 | 0.12118115942028984 | 0.055521428571428569 | 0.07363865546218484 | 1.6025041367111728 | 0.05007825427222415 |

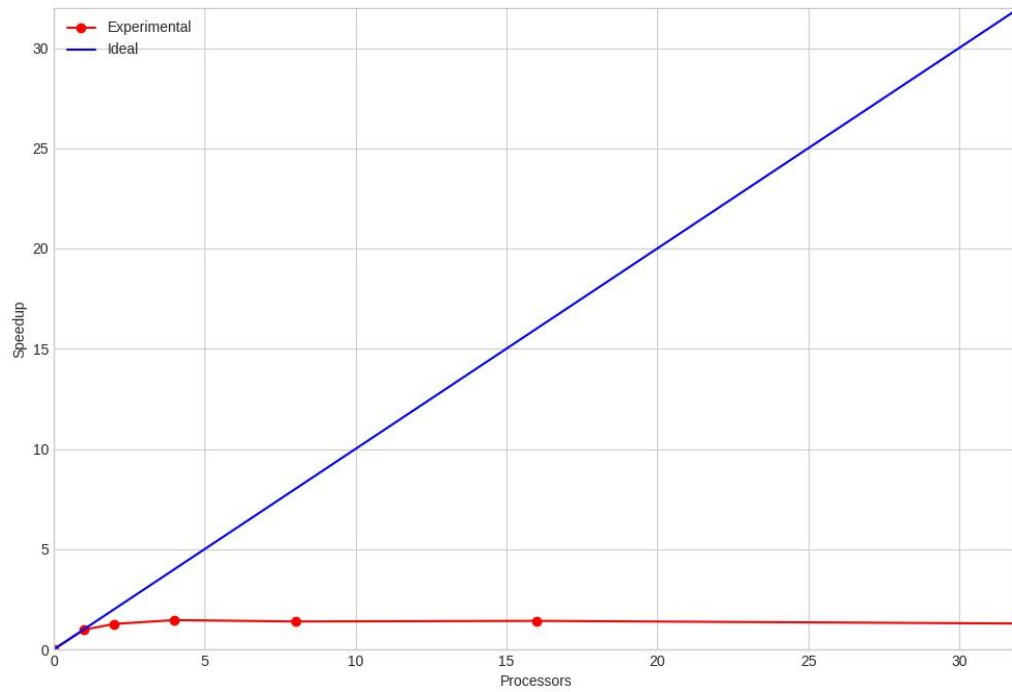
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|--------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.031709038461539 | 0.085590620689655 | 0.072120000000000 | 0.053867647058824 | 0.123779874213836 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.034762275449102 | 0.085366256684492 | 0.076152317880795 | 0.051750000000000 | 0.126340659340659 | 0.979731108416032 | 0.979731108416032 |
| Parallel | 2 | 0.040616957219251 | 0.1111131680851064 | 0.097779569892473 | 0.061768211920530 | 0.097610810810811 | 1.268095953569600 | 0.634047976784798 |
| Parallel | 4 | 0.053412619047619 | 0.165121975155280 | 0.148958333333333 | 0.076785185185185 | 0.084772727272727 | 1.460137926383060 | 0.365034481595764 |
| Parallel | 8 | 0.052130270588235 | 0.148751457627119 | 0.131654545454545 | 0.079147239263804 | 0.088982954545455 | 1.391051520441560 | 0.173881440055195 |
| Parallel | 16 | 0.054440450549451 | 0.145034597826087 | 0.130188571428571 | 0.078859756097561 | 0.087098360655738 | 1.421150447401470 | 0.088821902962592 |
| Parallel | 32 | 0.059270878205128 | 0.153714445161290 | 0.137891304347826 | 0.084710144927536 | 0.096006666666667 | 1.289284156105510 | 0.040290129878297 |

Setup 1



Setup 2



SIZE-15000000-O3

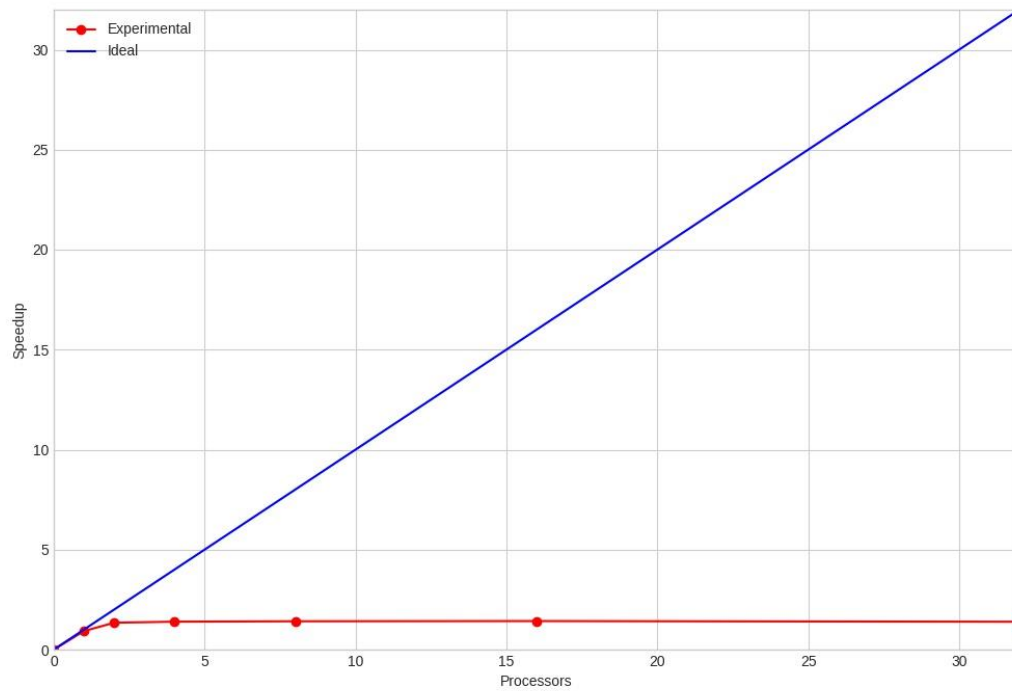
Setup 1

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|----------------------|---------------------|---------------------|----------------------|---------------------|--------------------|----------------------|
| Serial | 1 | 0.026320587412587415 | 0.08330458125000000 | 0.07155223880597017 | 0.041756944444444430 | 0.11296666666666669 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.026518329032258064 | 0.09224133834586466 | 0.08050375939849624 | 0.042056338028169007 | 0.12203703703703700 | 0.9256752655538699 | 0.9256752655538699 |
| Parallel | 2 | 0.029626548076923075 | 0.10952739200000000 | 0.10480180180180182 | 0.049100719424460426 | 0.08409999999999999 | 1.3432421720174399 | 0.671621086008719947 |
| Parallel | 4 | 0.039208448087431694 | 0.18119369945355196 | 0.16678306878306878 | 0.059235294117647067 | 0.08095161290322581 | 1.3954838281198116 | 0.348870957029952911 |
| Parallel | 8 | 0.038679937062937063 | 0.12625039999999998 | 0.10732575757575759 | 0.062060150375939843 | 0.07977600000000000 | 1.4160482684850919 | 0.177006033560636489 |
| Parallel | 16 | 0.039615115942028989 | 0.12708212413793107 | 0.10805555555555554 | 0.062035460992907798 | 0.07919879518072288 | 1.4263684997844885 | 0.089148031236530534 |
| Parallel | 32 | 0.041008386861313868 | 0.12909945255474453 | 0.11038931297709922 | 0.06404999999999996 | 0.08114035087719298 | 1.3922378378378379 | 0.043507432432432436 |

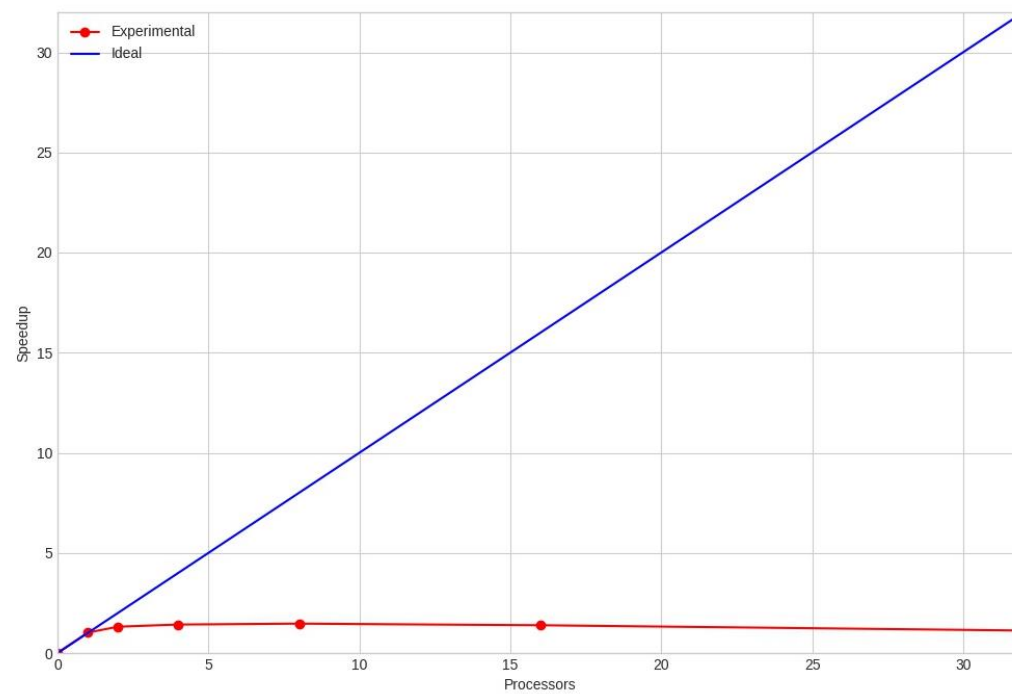
Setup 2

| Version | Threads | Init | Counting | User | Sys | Elapsed | Speedup | Efficiency |
|----------|---------|-------------------|-------------------|-------------------|--------------------|-------------------|--------------------|--------------------|
| Serial | 1 | 0.027534050314465 | 0.087121941558442 | 0.070690647482014 | 0.050808219178082 | 0.121328767123288 | 1.0000000000000000 | 1.0000000000000000 |
| Parallel | 1 | 0.028145027624309 | 0.083589573033708 | 0.067870967741936 | 0.0525555555555556 | 0.118950276243094 | 1.01999567344705 | 1.019995673447050 |
| Parallel | 2 | 0.031490460937500 | 0.109456836601307 | 0.086895104895105 | 0.061905109489051 | 0.092325000000000 | 1.31414857431127 | 0.657074287155633 |
| Parallel | 4 | 0.045977406451613 | 0.166669339622642 | 0.138979591836735 | 0.083917808219178 | 0.085276073619632 | 1.42277619000690 | 0.355694047501725 |
| Parallel | 8 | 0.043897550000000 | 0.142094638709677 | 0.113503703703704 | 0.078140740740741 | 0.082650000000000 | 1.46798266331867 | 0.183497832914833 |
| Parallel | 16 | 0.048487370588235 | 0.147832578651685 | 0.120932515337423 | 0.083526315789474 | 0.087470930232558 | 1.38707530376906 | 0.086692206485566 |
| Parallel | 32 | 0.054499648809524 | 0.169527575539568 | 0.135848275862069 | 0.098617021276596 | 0.107950354609929 | 1.12393115855618 | 0.035122848704881 |

Setup 1



Setup 2



Considerations

Speedup

The maximum computed speedup is 2 in the 15000000-O1 8 threads test case with the Setup 1. This means that the parallel version is 2 times faster than the serial one. The reason why it doesn't go over this value probably being the lack of parallelization in the sorting nested loop, which is the last for in the counting_sort function and the most expensive one, with a complexity of $O(k)$ with k equals to $(\max(\text{array}) - \min(\text{array}) + 1)$. The lowest speedup is 0.78 in the 2500000-O3 8 threads test case with Setup 2. It is even lower than the 1 thread OpenMP version. The reason probably being the saturation of hardware resources.

We can observe that with 32 threads, in most cases, the speedup decreases due to the lack of hardware resources, on both the setups.

Efficiency

In most cases the relative efficiency of each experimental setup is comparable. The difference between the various comparisons is around 5% to 10% with the efficiency of the Setup 2 being slightly lower than the Setup 1. This is due to the overhead introduced by the virtualization of the OS in the Setup 2, and the different processor. However, it is noted that the Setup 2 runs slightly more efficiently with the GCC O-3 optimization.

In general, the efficiency is constant and not depending on size of the array, so we can assume that this parallel solution is partially scalable.

Initialization Time

We noted that the initialization time increases with the number of threads used compared to the sequential one. In general, the parallelization does not seem to be the best solution, probably due to the overhead introduced by the synchronizations and communication time of the various threads.

Elapsed Time

In the parallel solution the elapsed time increases linearly proportional to the size of the array on both experimental setups between size 2500000 and 10000000. However, it is noted that with bigger array sizes the elapsed time difference slightly decreases. Considering the difference between the elapsed time of the serial version and that of the parallel version with only one thread, the overhead introduced by OpenMP can be seen. This overhead is greater in

the Setup 2 due to virtualization. The overhead introduced by OpenMP on average does not exceed 10%.

Despite the hardware used for the project, which is a dual core for Setup 1 and a quad core for Setup 2, we can see that in the most cases the Elapsed Time with 8 or 16 threads is less than the others. This could be caused by the synchronization required by the threads. When a thread waits the end of the others causes the idle state of a core. Incrementing the number of threads, this idle time is reduced, causing better performances.

Code Considerations

Due to the dependence between the data of the various parallelized for loops, it was not possible to insert the “nowait” clause in the pragmas.

The scope of each variable in the pragmas was evaluated in accordance with the correctness of the execution and trying to maintain the minimum possible overhead of managing the variables.

In maxmin function we could not remove the critical region to calculate the global maximum and minimum but this region of code is very small (only four lines) so it is not a relevant problem to performance.

Since all the data on which the program works are of type int, the FP pipe is free and the compiler, instead of implementing all operations in the backend as on integers and saturating only one pipe, can transform some operations on integers into operations on floats by exploiting in this way the FP pipe. This compiler optimization (symmetric multithreading) also helps improve performance.

Test case

In the test folder there is a *test.c* file which contains the test cases.

The tests are performed to verify that the sorting is executed correctly.

We tested the Counting Sort algorithm several times, sorting different arrays and asserting that the expected results and the ones expected were the same.

API

Functions

- void `maxmin` (`ELEMENT_TYPE *a`, `int len`, `ELEMENT_TYPE *max`, `ELEMENT_TYPE *min`)
This function calculates the maximum and minimum of the array passed as an argument.
- void `counting_sort` (`ELEMENT_TYPE *a`, `int len`)
This function sorts the array 'a' by implementing the counting sort algorithm.
- void `generate` (`ELEMENT_TYPE **v`, `int len`)
This function generates an array of the given size.

2.1.1.3 maxmin()

```
void maxmin (  
    ELEMENT_TYPE * a,  
    int len,  
    ELEMENT_TYPE * max,  
    ELEMENT_TYPE * min )
```

This function calculates the maximum and minimum of the array passed as an argument.

Parameters

| | |
|------------|---|
| <i>a</i> | pointer to the array used in the counting sort on which to calculate the minimum and maximum. |
| <i>len</i> | array size. |
| <i>max</i> | pointer to the variable used to store the maximum. |
| <i>min</i> | pointer to the variable used to store the minimum. |

2.1.1.1 counting_sort()

```
void counting_sort (
    ELEMENT_TYPE * a,
    int len )
```

This function sorts the array 'a' by implementing the counting sort algorithm.

Parameters

| | |
|------------|------------------------------------|
| <i>a</i> | pointer to the array to be sorted. |
| <i>len</i> | array size. |

See also

https://it.wikipedia.org/wiki/Counting_sort

2.1.1.2 generate()

```
void generate (
    ELEMENT_TYPE ** v,
    int len )
```

This function generates an array of the given size.

Parameters

| | |
|------------|---|
| <i>v</i> | pointer to the array used to store the generated array. |
| <i>len</i> | array size. |

2.1.1.3 maxmin()

```
void maxmin (
    ELEMENT_TYPE * a,
    int len,
    ELEMENT_TYPE * max,
    ELEMENT_TYPE * min )
```

This function calculates the maximum and minimum of the array passed as an argument.

HOW TO RUN

1. Create a build directory and launch cmake:

```
mkdir build
```

```
cd build
```

```
cmake ..
```

2. Generate executables with **make**
3. To generate measures run **make generate_measures**
4. To extract mean times and speedup curves from them run **make extract_measures**

Results can be found in the measures/measure directory, divided by problem size and the gcc optimization option used.

Scripts to generate and extract measures were given by Capitani Giuseppe, Falanga Armando e Terrone Luigi under GNU free license.

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