Supplementary Material

Resiliency assessment of urban rail transit networks: London as an example

Due to the word limit, a lot of the research that has been done has to be put in the appendix. I hope these additions will help you better understand the work that I have done.

1.unweighted network

For the topology analysis of the UW London Underground network, we check whether the network exhibits a small world effect. Watts and Strogaz developed the concept of the small-world effect in the network, which is characterized by small characteristic path lengths and high clustering coefficients. World attributes, networks with small-world effects often have higher hierarchical network efficiency and local network efficiency.

Local Efficiency[1]

Efficiency is the inverse of the geodesic path length between all pairs of nodes of a set group of nodes. Local efficiency evaluates the average efficiency of the subgraph of nodes neighboring a node, and averages it across the network.

$$E_{Local} = \frac{1}{N}\sum\left(E(G_i)\right)$$

where

$$E(G) = \frac{2}{n(n-1)}\sum_{i\neq j\in G}\frac{1}{d_{ij}}$$

d_{ij} = the geodesic path length between node pairs i and j
N = the number of nodes within the subgraph of node i
n = the number of nodes in the network

Global Efficiency[1]

Global efficiency evaluates the average efficiency of all nodes within the network.

$$E_{Global} = \frac{E(G)}{E(G_{ideal})}$$

where

$$E(G) = \frac{2}{n(n-1)}\sum_{i\neq j\in G}\frac{1}{d_{ij}}$$

G_{ideal}= network with all n(n-1)/2 possible edges
d_{ij} = the geodesic path length between all pairs of nodes in the network
N = the number of nodes in the network

The results of the small-world characteristics of the UW network show that it is not a small-world network. The data shows that its Eglobal=0.094, Elocal=0.006, which cannot meet the conditions for the establishment of a small world network (Eglobal>=0, Elocal<=1). This means that the subway cannot obtain a high level of fault tolerance, especially in the network Periphery, where the interruption will lead to the isolation of the subway station.

2.Weighted network

For the topology analysis of the Wd London Underground network, we analyze the passenger flow distribution of all stations in the London Underground network by constructing the probability distribution p(s) of the passenger intensity for each station. If the intensity distribution follows the power law $p(s) \propto s^{-a}$, then it is proved that a few stations bear most of the passenger load, and these weighted stations may be the key to network resilience.

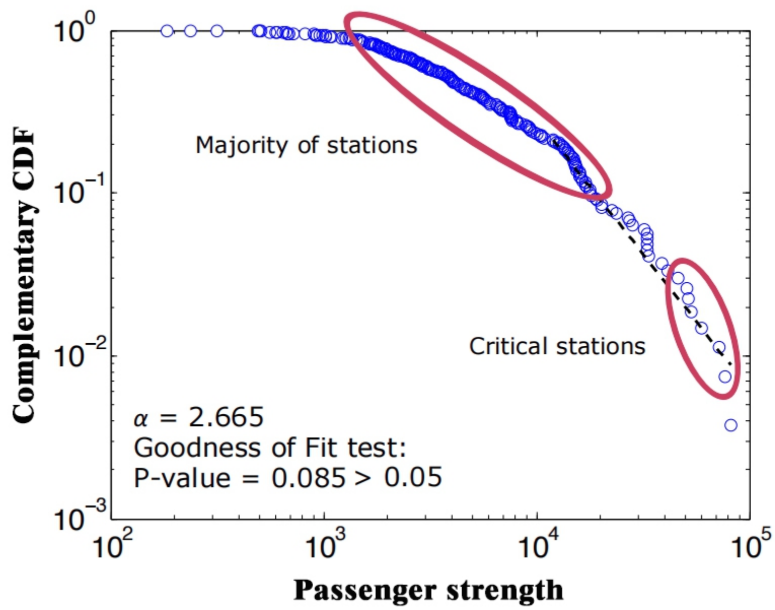
Analysis of Power-law distributions

The following steps were used to identify whether the passenger strength in the London metro network follows a power-law distribution[2]:
First, we estimate the parameters s_{min} and the scaling factor ε for the degree distribution. Estimation of the ε accurately requires accurate estimation of the s_{min} (empirical data tends to follow a power-law distribution for values of s greater than a lower bound s_{min}; therefore, all values below the s_{min} are discarded). If a very low or a very high value of s_{min} is chosen then the estimate for the scaling factor ε will be biased. For this reason, we choose the Kolmogorov-Smirnov (KS) statistic for accurate estimation of s_{min}[3]. KS test is used to estimate the value of s_{min} that makes the passenger strength distribution of the London metro fit best to the power law model. The KS statistic is used to quantify the maximum distance D between cumulative distribution functions of two non-normal distributions as follows:

$$D = \max_{s \geq s_{\min}} \left| t(s) - p(s) \right|$$

Here t(s) is the cumulative distribution function (CDF) of the station passenger strengths for the London metro network with the smallest s_{min}, and p(s) is the CDF for the power law model that best fits the data in the region $s \geq s_{min}$. A value of s_{min} is picked that minimizes the value of D, the distance, between the empirical and the synthetic distributions. The method of maximum likelihood estimation (MLE), technique for finding model parameters that are most consistent with the observed data, is used to estimate the scaling factor ε for the estimated s_{min}. The following maximum likelihood estimator of the scaling factor is derived for continuous distributions.

The topological analysis of the Wd London Underground network is shown in the figure below. The power law of passenger intensity distribution shows that although relatively few passengers depend on most subway stations, a very small number of platforms are responsible for most of the passenger traffic. These key stations are the main source of vulnerabilities, and damage to them may prevent the London Underground system from working properly.



3.Robustness analysis of the London metro network

We also performed robustness analysis of the London metro network. Robustness analysis is useful to understand the network topology on the basis of node removal. It also allows us to observe the inherent vulnerability of the system due to connectivity pattern of the nodes. The robustness of London metro Wd network is investigated by determining the impact of systematic targeted and random disruptions of stations on overall connectivity and passenger flow of the network.

Reference

[1] Latora, V. & Marchiori, M. 2002 Is the Boston subway a small-world network? Physica A: Statistical Mechanics and its Applications 314, 109-113.
[2] Clauset, A., Shalizi, C.R. & Newman, M.E.J. 2009 Power-law distributions in empirical data. SIAM Rev. 51, 661-703.
[3] Massey Jr, F.J. 1951 The Kolmogorov-Smirnov test for goodness of fit. Journal of the American statistical Association 46, 68-78