1) $A = \{(4,2,0),(1,-1,1),(5,3,3)\}$
$B = \{(1,-2,1),(1,5,2),(1,0,1)\}$

Fórmulas genéricas para a base A

Gerando o sistema:

$$a_1(4,2,0) + a_2(1,-1,1) + a_3(5,3,3) = (x,y,z)$$

$$\begin{cases} 4a_1 + a_2 + 5a_3 = x \\ 2a_1 - a_2 + 3a_3 = y \\ a_2 + 3a_3 = z \end{cases}$$

Eq. 3:

$$\begin{aligned} a_2 + 3a_3 &= z \\ a_2 &= -3a_3 + z \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} 2a_1 - a_2 + 3a_3 &= y \\ 2a_1 + 3a_3 - z + 3a_3 &= y \\ 2a_1 &= -6a_3 + y + z \end{aligned}$$

Substituindo Eq. 1:

$$\begin{aligned} 4a_1 + a_2 + 5a_3 &= x \\ 2(-6a_3 + y + z) - 3a_3 + z + 5a_3 &= x \\ -10a_3 &= x - 2y - 3z \\ a_3 &= \tfrac{1}{10}(-x + 2y + 3z) \end{aligned}$$

Substituindo Eq. 3:

$$\begin{aligned} a_2 &= -3a_3 + z \\ a_2 &= -\tfrac{3}{10}(-x + 2y + 3z) + z \\ a_2 &= \tfrac{1}{10}(3x - 6y + z) \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} 2a_1 &= -6a_3 + y + z \\ 2a_1 &= -\tfrac{6}{10}(-x + 2y + 3z) + y + z \\ a_1 &= \tfrac{1}{10}(3x - y - 4z) \end{aligned}$$

Formulas da base A:

$$\begin{aligned} a_1(x,y,z) &= \tfrac{1}{10}(3x - y - 4z) \\ a_2(x,y,z) &= \tfrac{1}{10}(3x - 6y + z) \\ a_3(x,y,z) &= \tfrac{1}{10}(-x + 2y + 3z) \end{aligned}$$

Fórmulas genéricas para a base B

Gerando o sistema:

$$b_1(1,-2,1)+b_2(1,5,2)+b_3(1,0,1)=(x,y,z)$$

$$\left\{\begin{array}{rcrcrcl} b_1 & + & b_2 & + & b_3 & = & x \\ -2b_1 & + & 5b_2 & & & = & y \\ b_1 & + & 2b_2 & + & b_3 & = & z \end{array}\right.$$

Eq. 2:

$$\begin{aligned} -2b_1+5b_2 &= y \\ b_1 &= \tfrac{5}{2}b_2-\tfrac{y}{2} \end{aligned}$$

Substituindo Eq. 3:

$$\begin{aligned} b_1+2b_2+b_3 &= z \\ \tfrac{5}{2}b_2-\tfrac{y}{2}+2b_2+b_3 &= z \\ b_3 &= -\tfrac{9}{2}b_2+\tfrac{y}{2}+z \end{aligned}$$

Substituindo Eq. 1:

$$\begin{aligned} b_1+b_2+b_3 &= x \\ \tfrac{5}{2}b_2-\tfrac{y}{2}+b_2+-\tfrac{9}{2}b_2+\tfrac{y}{2}+z &= x \\ b_2 &= -x+z \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} b_1 &= \tfrac{5}{2}b_2-\tfrac{y}{2} \\ b_1 &= \tfrac{5}{2}(-x+z)-\tfrac{y}{2} \\ b_1 &= \tfrac{1}{2}(-5x-y+5z) \end{aligned}$$

Substituindo Eq. 3:

$$\begin{aligned} b_3 &= -\tfrac{9}{2}b_2+\tfrac{y}{2}+z \\ b_3 &= -\tfrac{9}{2}(-x+z)+\tfrac{y}{2}+z \\ b_3 &= \tfrac{1}{2}(9x+y-7z) \end{aligned}$$

Formulas da base B:

$$\begin{aligned} b_1(x,y,z) &= \tfrac{1}{2}(-5x-y+5z) \\ b_2(x,y,z) &= -x+z \\ b_3(x,y,z) &= \tfrac{1}{2}(9x+y-7z) \end{aligned}$$

$\boldsymbol{M_{B\to A}}$:

$$b_1 = x_1 \cdot a_1 + y_1 \cdot a_2 + z_1 \cdot a_3 = (1, -2, 1)$$

$$\begin{array}{rcrcl} x_1 &=& a_1(1,-2,1) &=& \frac{1}{10} \\ y_1 &=& a_2(1,-2,1) &=& \frac{8}{5} \\ z_1 &=& a_3(1,-2,1) &=& -\frac{1}{5} \end{array}$$

$$b_2 = x_2 \cdot a_1 + y_2 \cdot a_2 + z_2 \cdot a_3 = (1, 5, 2)$$

$$\begin{array}{rcrcl} x_2 &=& a_1(1,5,2) &=& -1 \\ y_2 &=& a_2(1,5,2) &=& -\frac{5}{2} \\ z_2 &=& a_3(1,5,2) &=& \frac{3}{2} \end{array}$$

$$b_3 = x_3 \cdot a_1 + y_3 \cdot a_2 + z_3 \cdot a_3 = (1, 0, 1)$$

$$\begin{array}{rcrcl} x_3 &=& a_1(1,0,1) &=& -\frac{1}{10} \\ y_3 &=& a_2(1,0,1) &=& \frac{2}{5} \\ z_3 &=& a_3(1,0,1) &=& \frac{1}{5} \end{array}$$

$$M_{B\to A} = \begin{bmatrix} \frac{1}{10} & -1 & -\frac{1}{10} \\ \frac{8}{5} & -\frac{5}{2} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix}$$

$\boldsymbol{M_{A\to B}}$:

$$b_1 = x_1 \cdot b_1 + y_1 \cdot b_2 + z_1 \cdot b_3 = (4, 2, 0)$$

$$\begin{array}{rcrcl} x_1 &=& b_1(4,2,0) &=& -11 \\ y_1 &=& b_2(4,2,0) &=& -4 \\ z_1 &=& b_3(4,2,0) &=& 19 \end{array}$$

$$b_2 = x_2 \cdot b_1 + y_2 \cdot b_2 + z_2 \cdot b_3 = (1, -1, 1)$$

$$\begin{array}{rcrcl} x_2 &=& b_1(1,-1,1) &=& \frac{1}{2} \\ y_2 &=& b_2(1,-1,1) &=& 0 \\ z_2 &=& b_3(1,-1,1) &=& \frac{1}{2} \end{array}$$

$$b_3 = x_3 \cdot b_1 + y_3 \cdot b_2 + z_3 \cdot b_3 = (5, 3, 3)$$

$$\begin{array}{rcrcl} x_3 &=& b_1(5,3,3) &=& -\frac{13}{2} \\ y_3 &=& b_2(5,3,3) &=& 2 \\ z_3 &=& b_3(5,3,3) &=& \frac{27}{2} \end{array}$$

$$M_{A\to B} = \begin{bmatrix} -11 & \frac{1}{2} & -\frac{13}{2} \\ -4 & 0 & 2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix}$$

a) $v = (0,1,2)_A$ em B:

$$M_{A\to B}: \begin{bmatrix} -11 & \frac{1}{2} & -\frac{13}{2} \\ -4 & 0 & 2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{25}{2} \\ -4 \\ \frac{55}{2} \end{bmatrix}$$

$$v = (-\frac{25}{2}, -4, \frac{55}{2})_B$$

b) $v = (1,3,-1)_B$ em A:

$$M_{b\to a}: \begin{bmatrix} \frac{1}{10} & -1 & -\frac{1}{10} \\ \frac{8}{5} & -\frac{5}{2} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{5} \\ -\frac{63}{10} \\ \frac{41}{10} \end{bmatrix}$$

$$v = (-\frac{14}{5}, -\frac{63}{10}, \frac{41}{10})_A$$

2) $S = \{(1,1,1,1,1), (2,0,-1,1,3), (3,1,0,2,4), (2,2,5,8,-1), (0,1,0,2,3)\}$

a) S é li ou ld?

Gerando o sistema:

$$x_1(1,1,1,1,1) + x_2(2,0,-1,1,3) + x_3(3,1,0,2,4) + x_4(2,2,5,8,-1) + x_5(0,1,0,2,3) = (0,0,0,0,0)$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 2x_4 = 0 \\ x_1 + x_3 + 2x_4 + x_5 = 0 \\ x_1 - x_2 + 5x_4 = 0 \\ x_1 + x_2 + 2x_3 + 8x_4 + 2x_5 = 0 \\ x_1 + 3x_2 + 4x_3 - x_4 + 3x_5 = 0 \end{cases}$$

Escalonamento:

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] \begin{array}{c} L_2 = L_2 - L_1 \to \\ L_3 = L_3 - L_1 \to \\ L_4 = L_4 - L_1 \to \\ L_5 = L_5 - L_1 \to \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]$$

$$\begin{array}{c} L_3 = L_3 - \frac{3}{2}L_2 \to \\ L_4 = L_4 - \frac{L_2}{2} \to \\ L_5 = L_5 + \frac{L_2}{2} \to \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 6 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right] \begin{array}{c} L_4 = L_4 - 2L_3 \to \\ L_5 = L_5 - L_3 \to \end{array} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \end{array}\right]$$

Substituição:

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 2x_4 = 0 \\ -2x_2 - 2x_3 + x_5 = 0 \\ 3x_4 - \frac{3}{2}x_5 = 0 \\ 2x_5 = 0 \\ 5x_5 = 0 \end{cases}$$

Eq. 4:

$$\begin{aligned} 2x_5 &= 0 \\ x_5 &= 0 \end{aligned}$$

Eq. 3:

$$\begin{aligned} 3x_4 &= 0 \\ x_4 &= 0 \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} -2x_2 - 2x_3 + x_5 &= 0 \\ -2x_2 - 2x_3 + 0 &= 0 \end{aligned}$$
$$x_2 = -x_3$$

Substituindo Eq. 1:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 2x_4 &= 0 \\ x_1 - 2x_3 + 3x_3 + 0 &= 0 \end{aligned}$$
$$x_1 = -x_3$$

Resultados:

$$\begin{aligned} x_1 &= -x_3 \\ x_2 &= -x_3 \\ x_3 &= ? \\ x_4 &= 0 \\ x_5 &= 0 \end{aligned}$$

Resposta: O conjunto S é LD, pois x_3 pode ter qualquer valor para encontrar $(0,0,0,0,0)$. Por exemplo:

$$-1(1,1,1,1,1) + (-1)(2,0,-1,1,3) + 1(3,1,0,2,4) + 0(2,2,5,8,-1) + 0(0,1,0,2,3) = (0,0,0,0,0)$$

b) S forma uma base do $\mathbb{R}$-espaço vetorial $\mathbb{R}^5$?
Resposta: Não, pois apenas conjuntos LI podem ser bases de espaços vetoriais.

3)
b)

$$\begin{cases} x - z = 0 \\ y + x = 0 \end{cases}$$

$$z = -x$$

$$y = -x$$

$$W = \{(x, -x, -x) | x \in \mathbb{R}\}$$

i) $x = 0$

$$\begin{aligned} & (x, -x, -x) \\ = & (0, -0, -0) \\ = & 0 \end{aligned}$$

Logo $0 \in W$.

ii) $u, v \in W \to u + v \in W$

$$\begin{aligned} u &= (u_1, -u_1, -u_1) \\ v &= (v_1, -v_1, -v_1) \\ u + v &= (u_1, -u_1, -u_1) + (v_1, -v_1, -v_1) \\ &= (u_1 + v_1, -u_1 - v_1, -u_1 - v_1) \\ &= (u_1 + v_1) \cdot (1, -1, -1) \end{aligned}$$

Logo $u+v \in W$.

iii) $a \in \mathbb{R}, v \in W \rightarrow a \cdot v \in W$

$$\begin{aligned} v &= (v_1, -v_1, -v_1) \\ a \cdot v &= (av_1, -av_1, -av_1) \\ &= (a \cdot v_1) \cdot (1, -1, -1) \end{aligned}$$

Logo $a \cdot v \in W$. Portanto W é um sub-espaço de $\mathbb{R}^3$.

c)

$$S = \{(x,y,z) | x,y,z \in \mathbb{R} \wedge x+2y=0 \wedge y+z=0\} \in \mathbb{R}^3$$

$$\begin{cases} x+2y &= 0 \\ y+z &= 0 \end{cases}$$

$$x = -2y$$

$$z = -y$$

$$S = \{(-2y, y, -y) | x \in \mathbb{R}\}$$

i) $y = 0$

$$\begin{aligned} & (-2y, y, -y) \\ =\ & (-0, 0, -0) \\ =\ & 0 \end{aligned}$$

Logo $0 \in S$.

ii) $u, v \in S \rightarrow u+v \in S$

$$\begin{aligned} u &= (2u_2, u_2, -u_2) \\ v &= (2v_2, v_2, -v_2) \\ u+v &= (2u_2, u_2, -u_2) + (2v_2, v_2, -v_2) \\ &= (-2u_2 - 2v_2, u_2 + v_2, -u_2 - v_2) \\ &= (u_2 + v_2) \cdot (-2, 1, -1) \end{aligned}$$

Logo $u+v \in S$.

iii) $a \in \mathbb{R}, v \in W \rightarrow a \cdot v \in S$

$$\begin{aligned} v &= (-2v_2, v_2, -v_2) \\ a \cdot v &= (-2av_2, av_2, -av_2) \\ &= (a \cdot v_2) \cdot (-2, 1, -1) \end{aligned}$$

Logo $a \cdot v \in S$. Portanto S é um sub-espaço de $\mathbb{R}^3$.

4)

Mostre que o conjunto é base:
$\{(-1,1,1,1,0,1,1)$,$(1,0,-1,1,1,-1,0)$,$(2,2,1,1,-1,1,1)$,$(1,0,0,1,2,1,1)$,
$(2,0,2,0,2,0,2)$,$(1,-1,-1,-1,-1,-1,1)$,$(3,0,2,0,2,-1,2)\}$

Gerando o sistema:

$$\begin{aligned} & a_1(-1,1,1,1,0,1,1) && + \quad a_2(1,0,-1,1,1,-1,0) && + \quad a_3(2,2,1,1,-1,1,1) \\ + \quad & a_4(1,0,0,1,2,1,1) && + \quad a_5(2,0,2,0,2,0,2) && + \quad a_6(1,-1,-1,-1,-1,-1,1) \\ + \quad & a_7(3,0,2,0,2,-1,2) && = \quad (x_1,x_2,x_3,x_4,x_5,x_6,x_7) \end{aligned}$$

$$
\left\{
\begin{array}{rcrcrcrcrcrcrcr}
-a_1 & + & a_2 & + & 2a_3 & + & a_4 & + & 2a_5 & + & a_6 & + & 3a_7 & = & x_1 \\
a_1 & & & + & 2a_3 & & & & & - & a_6 & & & = & x_2 \\
a_1 & - & a_2 & + & a_3 & & & + & 2a_5 & - & a_6 & + & 2a_7 & = & x_3 \\
a_1 & + & a_2 & + & a_3 & + & a_4 & & & - & a_6 & & & = & x_4 \\
 & & a_2 & - & a_3 & + & 2a_4 & + & 2a_5 & - & a_6 & + & 2a_7 & = & x_5 \\
a_1 & - & a_2 & + & a_3 & + & a_4 & & & - & a_6 & - & a_7 & = & x_6 \\
a_1 & & & + & a_3 & + & a_4 & + & 2a_5 & + & a_6 & + & 2a_7 & = & x_7
\end{array}
\right.
$$

Escalonamento:

$$\left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 1 & 0 & 2 & 0 & 0 & -1 & 0 & x_2 \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & x_3 \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & x_4 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & x_5 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & x_6 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & x_7 \end{array}\right] \begin{array}{r} L_2 = L_2 + L_1 \rightarrow \\ L_3 = L_3 + L_1 \rightarrow \\ L_4 = L_4 + L_1 \rightarrow \\ L_6 = L_6 + L_1 \rightarrow \\ L_7 = L_7 + L_1 \rightarrow \end{array} \left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & x_1 + x_4 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & x_5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & x_1 + x_6 \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & x_1 + x_7 \end{array}\right]$$

$$\begin{array}{r} L_4 = L_4 - 2L_2 \rightarrow \\ L_5 = L_5 - L_2 \rightarrow \\ L_7 = L_7 - L_2 \rightarrow \end{array} \left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & -5 & 0 & -2 & 0 & -3 & -x_1 - 2x_2 + x_4 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & -x_1 - x_2 + x_5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & x_1 + x_6 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & -x_2 + x_7 \end{array}\right]$$

$$\begin{array}{r} L_4 = L_4 + \frac{5}{3}L_3 \rightarrow \\ L_5 = L_5 + \frac{5}{3}L_3 \rightarrow \\ L_6 = 6 - L_3 \rightarrow \\ L_7 = L_7 + \frac{L_3}{3} \rightarrow \end{array} \left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & \frac{8}{3} & \frac{20}{3} & -1 & \frac{22}{3} & \frac{2}{3}x_1 - x_2 + \frac{5}{3}x_3 + x_5 \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & -x_3 + x_6 \\ 0 & 0 & 0 & \frac{4}{3} & \frac{10}{3} & 2 & \frac{11}{3} & \frac{1}{3}x_1 - x_2 + \frac{1}{3}x_3 + x_7 \end{array}\right]$$

$$\begin{array}{r} L_5 = L_5 - \frac{8}{5}L_4 \rightarrow \\ L_6 = L_6 - \frac{3}{5}L_4 \rightarrow \\ L_7 = L_7 - \frac{4}{5}L_4 \rightarrow \end{array} \left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & 0 & -\frac{4}{5} & -1 & -\frac{6}{5} & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 0 & 0 & 0 & 0 & -\frac{24}{5} & 0 & -\frac{31}{5} & -\frac{2}{5}x_1 + \frac{6}{5}x_2 - 2x_3 - \frac{3}{5}x_4 + x_6 \\ 0 & 0 & 0 & 0 & -\frac{2}{5} & 2 & -\frac{3}{5} & -\frac{1}{5}x_1 + \frac{3}{5}x_2 - x_3 - \frac{4}{5}x_4 + x_7 \end{array}\right]$$

$$\begin{array}{r} L_6 = L_6 - 6L_5 \rightarrow \\ L_7 = L_7 - \frac{L_5}{2} \rightarrow \end{array} \left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & 0 & -\frac{4}{5} & -1 & -\frac{6}{5} & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 & 2x_1 - 12x_2 + 4x_3 + 9x_4 - 6x_5 + x_6 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{2} & 0 & -\frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_5 + x_7 \end{array}\right]$$

$$L_7 = L_7 - \frac{5}{12}L_6 \rightarrow \left[\begin{array}{rrrrrrr|l} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & 0 & -\frac{4}{5} & -1 & -\frac{6}{5} & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 & 2x_1 - 12x_2 + 4x_3 + 9x_4 - 6x_5 + x_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{5}{12} & -\frac{5}{6}x_1 + \frac{9}{2}x_2 - \frac{13}{6}x_3 - \frac{15}{4}x_4 + 2x_5 - \frac{5}{12}x_6 + x_7 \end{array}\right]$$

Substituição:

$$\begin{cases} -a_1+a_2+2a_3+a_4+2a_5+a_6+3a_7 &= x_1 \\ a_2+4a_3+a_4+2a_5+3a_7 &= x_1+x_2 \\ 3a_3+a_4+4a_5+5a_7 &= x_1+x_3 \\ \frac{5}{3}a_4+\frac{14}{3}a_5+\frac{16}{3}a_7 &= \frac{2}{3}x_1-2x_2+\frac{5}{3}x_3+x_4 \\ -\frac{4}{5}a_5-a_6-\frac{6}{5}a_7 &= -\frac{2}{5}x_1+\frac{11}{5}x_2-x_3-\frac{8}{5}x_4+x_5 \\ 6a_6+a_7 &= 2x_1-12x_2+4x_3+9x_4-6x_5+x_6 \\ -\frac{5}{12}a_7 &= -\frac{5}{6}x_1+\frac{9}{2}x_2-\frac{13}{6}x_3-\frac{15}{4}x_4+2x_5-\frac{5}{12}x_6+x_7 \end{cases}$$

Eq 7:

$$\begin{aligned} -\frac{5}{12}a_7 &= -\frac{5}{6}x_1+\frac{9}{2}x_2-\frac{13}{6}x_3-\frac{15}{4}x_4+2x_5-\frac{5}{12}x_6+x_7 \\ a_7 &= \frac{1}{5}(10x_1-54x_2+26x_3+45x_4-24x_5+5x_6-12x_7) \end{aligned}$$

Substituindo Eq 6:

$$\begin{aligned} 6a_6+a_7 &= 2x_1-12x_2+4x_3+9x_4-6x_5+x_6 \\ a_6 &= \frac{1}{5}(-x_2-x_3-x_5+2x_7) \end{aligned}$$

Substituindo Eq 5:

$$\begin{aligned} -\frac{4}{5}a_5-a_6-\frac{6}{5}a_7 &= -\frac{2}{5}x_1+\frac{11}{5}x_2-x_3-\frac{8}{5}x_4+x_5 \\ a_5 &= \frac{1}{10}(-25x_1+137x_2-63x_3-115x_4+62x_5-15x_6+31x_7) \end{aligned}$$

Substituindo Eq 4:

$$\begin{aligned} \frac{5}{3}a_4+\frac{14}{3}a_5+\frac{16}{3}a_7 &= \frac{2}{3}x_1-2x_2+\frac{5}{3}x_3+x_4 \\ a_4 &= x_1-5x_2+2x_3+4x_4-2x_5+x_6-x_7 \end{aligned}$$

Substituindo Eq 3:

$$\begin{aligned} 3a_3+a_4+4a_5+5a_7 &= x_1+x_3 \\ a_3 &= \frac{1}{5}(7x_2-3x_3-5x_4+2x_5+x_7) \end{aligned}$$

Substituindo Eq 2:

$$\begin{aligned} a_2+4a_3+a_4+2a_5+3a_7 &= x_1+x_2 \\ a_2 &= \frac{1}{5}(15x_1-81x_2+39x_3+70x_4-36x_5+5x_6-18x_7) \end{aligned}$$

Substituindo Eq 1:

$$\begin{aligned} -a_1+a_2+2a_3+a_4+2a_5+a_6+3a_7 &= x_1 \\ a_1 &= \frac{1}{5}(20x_1-118x_2+57x_3+100x_4-53x_5+10x_6-24x_7) \end{aligned}$$

Substituindo $(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ *por* $(0, 0, 0, 0, 0, 0, 0)$*:*

$$\begin{aligned}
a_1 &= \frac{1}{5}(20(0) - 118(0) + 57(0) + 100(0) - 53(0) + 10(0) - 24(0)) &&= 0\\
a_2 &= \frac{1}{5}(15(0) - 81(0) + 39(0) + 70(0) - 36(0) + 5(0) - 18(0)) &&= 0\\
a_3 &= \frac{1}{5}(7(0) - 3(0) - 5(0) + 2(0) + (0)) &&= 0\\
a_4 &= (0) - 5(0) + 2(0) + 4(0) - 2(0) + (0) - (0) &&= 0\\
a_5 &= \frac{1}{10}(-25(0) + 137(0) - 63(0) - 115(0) + 62(0) - 15(0) + 31(0)) &&= 0\\
a_6 &= \frac{1}{5}(-(0) - (0) - (0) + 2(0)) &&= 0\\
a_7 &= \frac{1}{5}(10(0) - 54(0) + 26(0) + 45(0) - 24(0) + 5(0) - 12(0)) &&= 0
\end{aligned}$$

Conclusão 1: O conjunto é LI, pois é necessário que $\{a_1, a_2, \ldots, a_7\}$ sejam todos 0 para que $\{x_1, x_2, \ldots, x_7\}$ sejam todos 0.
Conclusão 2: Qualquer vetor $\mathbb{R}^7$ pode ser escrito de forma única nesta base, portanto ela forma uma base do $\mathbb{R}$-espaço vetorial $\mathbb{R}^7$

Coordenadas do vetor $(0, 1, 1, 1, 1, 0, 1)$*:*

$$\begin{aligned}
a_1 &= \frac{1}{5}(20(0) - 118(1) + 57(1) + 100(1) - 53(1) + 10(0) - 24(1)) &&= -\frac{38}{5}\\
a_2 &= \frac{1}{5}(15(0) - 81(1) + 39(1) + 70(1) - 36(1) + 5(0) - 18(1)) &&= -\frac{26}{5}\\
a_3 &= \frac{1}{5}(7(1) - 3(1) - 5(1) + 2(1) + (1)) &&= \frac{2}{5}\\
a_4 &= (0) - 5(1) + 2(1) + 4(1) - 2(1) + (0) - (1) &&= -\frac{2}{5}\\
a_5 &= \frac{1}{10}(-25(0) + 137(1) - 63(1) - 115(1) + 62(1) - 15(0) + 31(1)) &&= \frac{52}{5}\\
a_6 &= \frac{1}{5}(-(1) - (1) - (1) + 2(1)) &&= -\frac{1}{5}\\
a_7 &= \frac{1}{5}(10(0) - 54(1) + 26(1) + 45(1) - 24(1) + 5(0) - 12(1)) &&= -\frac{19}{5}
\end{aligned}$$

Conclusão 3: As coordenadas do vetor $(0, 1, 1, 1, 1, 0, 1)$ são:

$$\begin{aligned}
&- \frac{38}{5}(-1, 1, 1, 1, 0, 1, 1) &&- \frac{26}{5}(1, 0, -1, 1, 1, -1, 0) &&+ \frac{2}{5}(2, 2, 1, 1, -1, 1, 1)\\
&- \frac{2}{5}(1, 0, 0, 1, 2, 1, 1) &&+ \frac{52}{5}(2, 0, 2, 0, 2, 0, 2) &&- \frac{1}{5}(1, -1, -1, -1, -1, -1, 1)\\
&- \frac{(}{3}, 0, 2, 0, 2, -1, 2)
\end{aligned}$$