

$$1) A = \{(4, 2, 0), (1, -1, 1), (5, 3, 3)\}$$

$$B = \{(1, -2, 1), (1, 5, 2), (1, 0, 1)\}$$

Fórmulas genéricas para a base A

Gerando o sistema:

$$a_1(4, 2, 0) + a_2(1, -1, 1) + a_3(5, 3, 3) = (x, y, z)$$

$$\begin{cases} 4a_1 + a_2 + 5a_3 = x \\ 2a_1 - a_2 + 3a_3 = y \\ a_2 + 3a_3 = z \end{cases}$$

Eq. 3:

$$\begin{aligned} a_2 + 3a_3 &= z \\ a_2 &= -3a_3 + z \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} 2a_1 - a_2 + 3a_3 &= y \\ 2a_1 + 3a_3 - z + 3a_3 &= y \\ 2a_1 &= -6a_3 + y + z \end{aligned}$$

Substituindo Eq. 1:

$$\begin{aligned} 4a_1 + a_2 + 5a_3 &= x \\ 2(-6a_3 + y + z) - 3a_3 + z + 5a_3 &= x \\ -10a_3 &= x - 2y - 3z \\ a_3 &= \frac{1}{10}(-x + 2y + 3z) \end{aligned}$$

Substituindo Eq. 3:

$$\begin{aligned} a_2 &= -3a_3 + z \\ a_2 &= -\frac{3}{10}(-x + 2y + 3z) + z \\ a_2 &= \frac{1}{10}(3x - 6y + z) \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} 2a_1 &= -6a_3 + y + z \\ 2a_1 &= -\frac{6}{10}(-x + 2y + 3z) + y + z \\ a_1 &= \frac{1}{10}(3x - y - 4z) \end{aligned}$$

Formulas da base A:

$$\begin{aligned} a_1(x, y, z) &= \frac{1}{10}(3x - y - 4z) \\ a_2(x, y, z) &= \frac{1}{10}(3x - 6y + z) \\ a_3(x, y, z) &= \frac{1}{10}(-x + 2y + 3z) \end{aligned}$$

Fórmulas genéricas para a base B

Gerando o sistema:

$$b_1(1, -2, 1) + b_2(1, 5, 2) + b_3(1, 0, 1) = (x, y, z)$$

$$\begin{cases} b_1 + b_2 + b_3 = x \\ -2b_1 + 5b_2 = y \\ b_1 + 2b_2 + b_3 = z \end{cases}$$

Eq. 2:

$$\begin{aligned} -2b_1 + 5b_2 &= y \\ b_1 &= \frac{5}{2}b_2 - \frac{y}{2} \end{aligned}$$

Substituindo Eq. 3:

$$\begin{aligned} b_1 + 2b_2 + b_3 &= z \\ \frac{5}{2}b_2 - \frac{y}{2} + 2b_2 + b_3 &= z \\ b_3 &= -\frac{9}{2}b_2 + \frac{y}{2} + z \end{aligned}$$

Substituindo Eq. 1:

$$\begin{aligned} b_1 + b_2 + b_3 &= x \\ \frac{5}{2}b_2 - \frac{y}{2} + b_2 + -\frac{9}{2}b_2 + \frac{y}{2} + z &= x \\ b_2 &= -x + z \end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned} b_1 &= \frac{5}{2}b_2 - \frac{y}{2} \\ b_1 &= \frac{5}{2}(-x + z) - \frac{y}{2} \\ b_1 &= \frac{1}{2}(-5x - y + 5z) \end{aligned}$$

Substituindo Eq. 3:

$$\begin{aligned} b_3 &= -\frac{9}{2}b_2 + \frac{y}{2} + z \\ b_3 &= -\frac{9}{2}(-x + z) + \frac{y}{2} + z \\ b_3 &= \frac{1}{2}(9x + y - 7z) \end{aligned}$$

Formulas da base B:

$$\begin{aligned} b_1(x, y, z) &= \frac{1}{2}(-5x - y + 5z) \\ b_2(x, y, z) &= -x + z \\ b_3(x, y, z) &= \frac{1}{2}(9x + y - 7z) \end{aligned}$$

$M_{B \rightarrow A}$:

$$b_1 = x_1 \cdot a_1 + y_1 \cdot a_2 + z_1 \cdot a_3 = (1, -2, 1)$$

$$\begin{aligned} x_1 &= a_1(1, -2, 1) = \frac{1}{10} \\ y_1 &= a_2(1, -2, 1) = \frac{8}{5} \\ z_1 &= a_3(1, -2, 1) = -\frac{1}{5} \end{aligned}$$

$$b_2 = x_2 \cdot a_1 + y_2 \cdot a_2 + z_2 \cdot a_3 = (1, 5, 2)$$

$$\begin{aligned} x_2 &= a_1(1, 5, 2) = -1 \\ y_2 &= a_2(1, 5, 2) = -\frac{5}{2} \\ z_2 &= a_3(1, 5, 2) = \frac{3}{2} \end{aligned}$$

$$b_3 = x_3 \cdot a_1 + y_3 \cdot a_2 + z_3 \cdot a_3 = (1, 0, 1)$$

$$\begin{aligned} x_3 &= a_1(1, 0, 1) = -\frac{1}{10} \\ y_3 &= a_2(1, 0, 1) = \frac{2}{5} \\ z_3 &= a_3(1, 0, 1) = \frac{1}{5} \end{aligned}$$

$$M_{B \rightarrow A} = \begin{bmatrix} \frac{1}{10} & -1 & -\frac{1}{10} \\ \frac{8}{5} & -\frac{5}{2} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix}$$

$M_{A \rightarrow B}$:

$$b_1 = x_1 \cdot b_1 + y_1 \cdot b_2 + z_1 \cdot b_3 = (4, 2, 0)$$

$$\begin{aligned} x_1 &= b_1(4, 2, 0) = -11 \\ y_1 &= b_2(4, 2, 0) = -4 \\ z_1 &= b_3(4, 2, 0) = 19 \end{aligned}$$

$$b_2 = x_2 \cdot b_1 + y_2 \cdot b_2 + z_2 \cdot b_3 = (1, -1, 1)$$

$$\begin{aligned} x_2 &= b_1(1, -1, 1) = \frac{1}{2} \\ y_2 &= b_2(1, -1, 1) = 0 \\ z_2 &= b_3(1, -1, 1) = \frac{1}{2} \end{aligned}$$

$$b_3 = x_3 \cdot b_1 + y_3 \cdot b_2 + z_3 \cdot b_3 = (5, 3, 3)$$

$$\begin{aligned} x_3 &= b_1(5, 3, 3) = -\frac{13}{2} \\ y_3 &= b_2(5, 3, 3) = 2 \\ z_3 &= b_3(5, 3, 3) = \frac{27}{2} \end{aligned}$$

$$M_{A \rightarrow B} = \begin{bmatrix} -11 & \frac{1}{2} & -\frac{13}{2} \\ -4 & 0 & 2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix}$$

a) $v = (0, 1, 2)_A$ em B :

$$M_{A \rightarrow B} : \begin{bmatrix} -11 & \frac{1}{2} & -\frac{13}{2} \\ -4 & 0 & 2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{25}{2} \\ -4 \\ \frac{55}{2} \end{bmatrix}$$

$$v = \left(-\frac{25}{2}, -4, \frac{55}{2}\right)_B$$

b) $v = (1, 3, -1)_B$ em A :

$$M_{b \rightarrow a} : \begin{bmatrix} \frac{1}{10} & -1 & -\frac{1}{10} \\ \frac{8}{5} & -\frac{5}{2} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{5} \\ -\frac{63}{10} \\ \frac{41}{10} \end{bmatrix}$$

$$v = \left(-\frac{14}{5}, -\frac{63}{10}, \frac{41}{10}\right)_A$$

2) $S = \{(1, 1, 1, 1, 1), (2, 0, -1, 1, 3), (3, 1, 0, 2, 4), (2, 2, 5, 8, -1), (0, 1, 0, 2, 3)\}$

a) S é li ou ld?

Gerando o sistema:

$$x_1(1, 1, 1, 1, 1) + x_2(2, 0, -1, 1, 3) + x_3(3, 1, 0, 2, 4) + x_4(2, 2, 5, 8, -1) + x_5(0, 1, 0, 2, 3) = (0, 0, 0, 0, 0)$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 2x_4 = 0 \\ x_1 + x_3 + 2x_4 + x_5 = 0 \\ x_1 - x_2 + 5x_4 = 0 \\ x_1 + x_2 + 2x_3 + 8x_4 + 2x_5 = 0 \\ x_1 + 3x_2 + 4x_3 - x_4 + 3x_5 = 0 \end{cases}$$

Escalonamento:

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{bmatrix} \begin{matrix} L_2 = L_2 - L_1 \rightarrow \\ L_3 = L_3 - L_1 \rightarrow \\ L_4 = L_4 - L_1 \rightarrow \\ L_5 = L_5 - L_1 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{bmatrix}$$

$$\begin{matrix} L_3 = L_3 - \frac{3}{2}L_2 \rightarrow \\ L_4 = L_4 - \frac{L_2}{2} \rightarrow \\ L_5 = L_5 + \frac{L_2}{2} \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 6 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{bmatrix} \begin{matrix} L_4 = L_4 - 2L_3 \rightarrow \\ L_5 = L_5 - L_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \end{bmatrix}$$

Substituição:

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 2x_4 = 0 \\ -2x_2 - 2x_3 + x_5 = 0 \\ 3x_4 - \frac{3}{2}x_5 = 0 \\ 2x_5 = 0 \\ 5x_5 = 0 \end{cases}$$

Eq. 4:

$$\begin{aligned}2x_5 &= 0 \\ x_5 &= 0\end{aligned}$$

Eq. 3:

$$\begin{aligned}3x_4 &= 0 \\ x_4 &= 0\end{aligned}$$

Substituindo Eq. 2:

$$\begin{aligned}-2x_2 - 2x_3 + x_5 &= 0 \\ -2x_2 - 2x_3 + 0 &= 0 \\ x_2 &= -x_3\end{aligned}$$

Substituindo Eq. 1:

$$\begin{aligned}x_1 + 2x_2 + 3x_3 + 2x_4 &= 0 \\ x_1 - 2x_3 + 3x_3 + 0 &= 0 \\ x_1 &= -x_3\end{aligned}$$

Resultados:

$$\begin{aligned}x_1 &= -x_3 \\ x_2 &= -x_3 \\ x_3 &= ? \\ x_4 &= 0 \\ x_5 &= 0\end{aligned}$$

Resposta: O conjunto S é LD, pois x_3 pode ter qualquer valor para encontrar $(0, 0, 0, 0, 0)$. Por exemplo:

$$-1(1, 1, 1, 1, 1) + (-1)(2, 0, -1, 1, 3) + 1(3, 1, 0, 2, 4) + 0(2, 2, 5, 8, -1) + 0(0, 1, 0, 2, 3) = (0, 0, 0, 0, 0)$$

b) S forma uma base do \mathbb{R} -espaço vetorial \mathbb{R}^5 ?

Resposta: Não, pois apenas conjuntos LI podem ser bases de espaços vetoriais.

3)

b)

$$\begin{cases} x - z &= 0 \\ y + x &= 0 \end{cases}$$

$$z = -x$$

$$y = -x$$

$$W = \{(x, -x, -x) | x \in \mathbb{R}\}$$

i) $x = 0$

$$\begin{aligned}(x, -x, -x) \\ &= (0, -0, -0) \\ &= 0\end{aligned}$$

Logo $0 \in W$.

ii) $u, v \in W \rightarrow u + v \in W$

$$\begin{aligned}u &= (u_1, -u_1, -u_1) \\ v &= (v_1, -v_1, -v_1) \\ u + v &= (u_1, -u_1, -u_1) + (v_1, -v_1, -v_1) \\ &= ((u_1 + v_1, -u_1 - v_1, -u_1 - v_1)) \\ &= (u_1 + v_1) \cdot (1, -1, -1)\end{aligned}$$

Logo $u + v \in W$.

iii) $a \in \mathbb{R}, v \in W \rightarrow a \cdot v \in W$

$$\begin{aligned} v &= (v_1, -v_1, -v_1) \\ a \cdot v &= (av_1, -av_1, -av_1) \\ &= (a \cdot v_1) \cdot (1, -1, -1) \end{aligned}$$

Logo $a \cdot v \in W$. Portanto W é um sub-espço de \mathbb{R}^3 .

4)

Mostre que o conjunto é base:

$$\{(-1, 1, 1, 0, 1, 1), (1, 0, -1, 1, 1, -1, 0), (2, 2, 1, 1, -1, 1, 1), (1, 0, 0, 1, 2, 1, 1), (2, 0, 2, 0, 2, 0, 2), (1, -1, -1, -1, -1, -1, 1), (3, 0, 2, 0, 2, -1, 2)\}$$

Gerando o sistema:

$$\begin{aligned} &a_1(-1, 1, 1, 0, 1, 1) + a_2(1, 0, -1, 1, 1, -1, 0) + a_3(2, 2, 1, 1, -1, 1, 1) \\ + &a_4(1, 0, 0, 1, 2, 1, 1) + a_5(2, 0, 2, 0, 2, 0, 2) + a_6(1, -1, -1, -1, -1, -1, 1) \\ + &a_7(3, 0, 2, 0, 2, -1, 2) = (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \end{aligned}$$

$$\left\{ \begin{array}{l} -a_1 + a_2 + 2a_3 + a_4 + 2a_5 + a_6 + 3a_7 = x_1 \\ a_1 + 2a_3 - a_6 = x_2 \\ a_1 - a_2 + a_3 + 2a_5 - a_6 + 2a_7 = x_3 \\ a_1 + a_2 + a_3 + a_4 - a_6 = x_4 \\ a_2 - a_3 + 2a_4 + 2a_5 - a_6 + 2a_7 = x_5 \\ a_1 - a_2 + a_3 + a_4 - a_6 - a_7 = x_6 \\ a_1 + a_3 + a_4 + 2a_5 + a_6 + 2a_7 = x_7 \end{array} \right.$$

Escalonamento:

$$\left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 1 & 0 & 2 & 0 & 0 & -1 & 0 & x_2 \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & x_3 \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & x_4 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & x_5 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & x_6 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & x_7 \end{array} \right] \begin{array}{l} L_2 = L_2 + L_1 \rightarrow \\ L_3 = L_3 + L_1 \rightarrow \\ L_4 = L_4 + L_1 \rightarrow \\ L_6 = L_6 + L_1 \rightarrow \\ L_7 = L_7 + L_1 \rightarrow \end{array} \left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & x_1 + x_4 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & x_5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & x_1 + x_6 \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & x_1 + x_7 \end{array} \right]$$

$$\begin{array}{l} L_4 = L_4 - 2L_2 \rightarrow \\ L_5 = L_5 - L_2 \rightarrow \\ L_7 = L_7 - L_2 \rightarrow \end{array} \left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & -5 & 0 & -2 & 0 & -3 & -x_1 - 2x_2 + x_4 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & -x_1 - x_2 + x_5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & x_1 + x_6 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & -x_2 + x_7 \end{array} \right]$$

$$\begin{array}{l} L_4 = L_4 + \frac{5}{3}L_3 \rightarrow \\ L_5 = L_5 + \frac{5}{3}L_3 \rightarrow \\ L_6 = 6 - L_3 \rightarrow \\ L_7 = L_7 + \frac{L_3}{3} \rightarrow \end{array} \left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & \frac{8}{3} & \frac{20}{3} & -1 & \frac{22}{3} & \frac{2}{3}x_1 - x_2 + \frac{5}{3}x_3 + x_5 \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & -x_3 + x_6 \\ 0 & 0 & 0 & \frac{4}{3} & \frac{10}{3} & 2 & \frac{11}{3} & \frac{1}{3}x_1 - x_2 + \frac{1}{3}x_3 + x_7 \end{array} \right]$$

$$\begin{array}{l} L_5 = L_5 - \frac{8}{5}L_4 \rightarrow \\ L_6 = L_6 - \frac{3}{5}L_4 \rightarrow \\ L_7 = L_7 - \frac{4}{5}L_4 \rightarrow \end{array} \left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & 0 & -\frac{4}{5} & -1 & -\frac{6}{5} & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 0 & 0 & 0 & 0 & -\frac{24}{5} & 0 & -\frac{31}{5} & -\frac{2}{5}x_1 + \frac{6}{5}x_2 - 2x_3 - \frac{3}{5}x_4 + x_6 \\ 0 & 0 & 0 & 0 & -\frac{2}{5} & 2 & -\frac{3}{5} & -\frac{1}{5}x_1 + \frac{3}{5}x_2 - x_3 - \frac{4}{5}x_4 + x_7 \end{array} \right]$$

$$\begin{array}{l} L_6 = L_6 - 6L_5 \rightarrow \\ L_7 = L_7 - \frac{L_5}{2} \rightarrow \end{array} \left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & 0 & -\frac{4}{5} & -1 & -\frac{6}{5} & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 & 2x_1 - 12x_2 + 4x_3 + 9x_4 - 6x_5 + x_6 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{2} & 0 & -\frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_5 + x_7 \end{array} \right]$$

$$L_7 = L_7 - \frac{5}{12}L_6 \rightarrow \left[\begin{array}{cccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & x_1 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & x_1 + x_2 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & x_1 + x_3 \\ 0 & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & \frac{16}{3} & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ 0 & 0 & 0 & 0 & -\frac{4}{5} & -1 & -\frac{6}{5} & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 0 & 0 & 0 & 0 & 0 & 6 & 1 & 2x_1 - 12x_2 + 4x_3 + 9x_4 - 6x_5 + x_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{5}{12} & -\frac{5}{6}x_1 + \frac{9}{2}x_2 - \frac{13}{6}x_3 - \frac{15}{4}x_4 + 2x_5 - \frac{5}{12}x_6 + x_7 \end{array} \right]$$

Substituição:

$$\left\{ \begin{array}{rcl} -a_1 + a_2 + 2a_3 + a_4 + 2a_5 + a_6 + 3a_7 & = & x_1 \\ a_2 + 4a_3 + a_4 + 2a_5 + 3a_7 & = & x_1 + x_2 \\ 3a_3 + a_4 + 4a_5 + 5a_7 & = & x_1 + x_3 \\ \frac{5}{3}a_4 + \frac{14}{3}a_5 + \frac{16}{3}a_7 & = & \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ -\frac{4}{5}a_5 - a_6 - \frac{6}{5}a_7 & = & -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ 6a_6 + a_7 & = & 2x_1 - 12x_2 + 4x_3 + 9x_4 - 6x_5 + x_6 \\ -\frac{5}{12}a_7 & = & -\frac{5}{6}x_1 + \frac{9}{2}x_2 - \frac{13}{6}x_3 - \frac{15}{4}x_4 + 2x_5 - \frac{5}{12}x_6 + x_7 \end{array} \right.$$

Eq 7:

$$\begin{aligned} -\frac{5}{12}a_7 &= -\frac{5}{6}x_1 + \frac{9}{2}x_2 - \frac{13}{6}x_3 - \frac{15}{4}x_4 + 2x_5 - \frac{5}{12}x_6 + x_7 \\ a_7 &= \frac{1}{5}(10x_1 - 54x_2 + 26x_3 + 45x_4 - 24x_5 + 5x_6 - 12x_7) \end{aligned}$$

Substituindo Eq 6:

$$\begin{aligned} 6a_6 + a_7 &= 2x_1 - 12x_2 + 4x_3 + 9x_4 - 6x_5 + x_6 \\ a_6 &= \frac{1}{5}(-x_2 - x_3 - x_5 + 2x_7) \end{aligned}$$

Substituindo Eq 5:

$$\begin{aligned} -\frac{4}{5}a_5 - a_6 - \frac{6}{5}a_7 &= -\frac{2}{5}x_1 + \frac{11}{5}x_2 - x_3 - \frac{8}{5}x_4 + x_5 \\ a_5 &= \frac{1}{10}(-25x_1 + 137x_2 - 63x_3 - 115x_4 + 62x_5 - 15x_6 + 31x_7) \end{aligned}$$

Substituindo Eq 4:

$$\begin{aligned} \frac{5}{3}a_4 + \frac{14}{3}a_5 + \frac{16}{3}a_7 &= \frac{2}{3}x_1 - 2x_2 + \frac{5}{3}x_3 + x_4 \\ a_4 &= x_1 - 5x_2 + 2x_3 + 4x_4 - 2x_5 + x_6 - x_7 \end{aligned}$$

Substituindo Eq 3:

$$\begin{aligned} 3a_3 + a_4 + 4a_5 + 5a_7 &= x_1 + x_3 \\ a_3 &= \frac{1}{5}(7x_2 - 3x_3 - 5x_4 + 2x_5 + x_7) \end{aligned}$$

Substituindo Eq 2:

$$\begin{aligned} a_2 + 4a_3 + a_4 + 2a_5 + 3a_7 &= x_1 + x_2 \\ a_2 &= \frac{1}{5}(15x_1 - 81x_2 + 39x_3 + 70x_4 - 36x_5 + 5x_6 - 18x_7) \end{aligned}$$

Substituindo Eq 1:

$$\begin{aligned} -a_1 + a_2 + 2a_3 + a_4 + 2a_5 + a_6 + 3a_7 &= x_1 \\ a_1 &= \frac{1}{5}(20x_1 - 118x_2 + 57x_3 + 100x_4 - 53x_5 + 10x_6 - 24x_7) \end{aligned}$$

Substituindo $(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ por $(0, 0, 0, 0, 0, 0, 0)$:

$$\begin{aligned}
a_1 &= \frac{1}{5}(20(0) - 118(0) + 57(0) + 100(0) - 53(0) + 10(0) - 24(0)) &= 0 \\
a_2 &= \frac{1}{5}(15(0) - 81(0) + 39(0) + 70(0) - 36(0) + 5(0) - 18(0)) &= 0 \\
a_3 &= \frac{1}{5}(7(0) - 3(0) - 5(0) + 2(0) + (0)) &= 0 \\
a_4 &= (0) - 5(0) + 2(0) + 4(0) - 2(0) + (0) - (0) &= 0 \\
a_5 &= \frac{1}{10}(-25(0) + 137(0) - 63(0) - 115(0) + 62(0) - 15(0) + 31(0)) &= 0 \\
a_6 &= \frac{1}{5}(-(0) - (0) - (0) + 2(0)) &= 0 \\
a_7 &= \frac{1}{5}(10(0) - 54(0) + 26(0) + 45(0) - 24(0) + 5(0) - 12(0)) &= 0
\end{aligned}$$

Conclusão 1: O conjunto é LI, pois é necessário que $\{a_1, a_2, \dots, a_7\}$ sejam todos 0 para que $\{x_1, x_2, \dots, x_7\}$ sejam todos 0.

Conclusão 2: Qualquer vetor \mathbb{R}^7 pode ser escrito de forma única nesta base, portanto ela forma uma base do \mathbb{R} -espaço vetorial \mathbb{R}^7

Coordenadas do vetor $(0, 1, 1, 1, 1, 0, 1)$:

$$\begin{aligned}
a_1 &= \frac{1}{5}(20(0) - 118(1) + 57(1) + 100(1) - 53(1) + 10(0) - 24(1)) &= -\frac{38}{5} \\
a_2 &= \frac{1}{5}(15(0) - 81(1) + 39(1) + 70(1) - 36(1) + 5(0) - 18(1)) &= -\frac{26}{5} \\
a_3 &= \frac{1}{5}(7(1) - 3(1) - 5(1) + 2(1) + (1)) &= \frac{2}{5} \\
a_4 &= (0) - 5(1) + 2(1) + 4(1) - 2(1) + (0) - (1) &= -\frac{2}{5} \\
a_5 &= \frac{1}{10}(-25(0) + 137(1) - 63(1) - 115(1) + 62(1) - 15(0) + 31(1)) &= \frac{52}{5} \\
a_6 &= \frac{1}{5}(-(1) - (1) - (1) + 2(1)) &= -\frac{1}{5} \\
a_7 &= \frac{1}{5}(10(0) - 54(1) + 26(1) + 45(1) - 24(1) + 5(0) - 12(1)) &= -\frac{19}{5}
\end{aligned}$$

Conclusão 3: As coordenadas do vetor $(0, 1, 1, 1, 1, 0, 1)$ são:

$$\begin{aligned}
& -\frac{38}{5}(-1, 1, 1, 1, 0, 1, 1) - \frac{26}{5}(1, 0, -1, 1, 1, -1, 0) + \frac{2}{5}(2, 2, 1, 1, -1, 1, 1) \\
& -\frac{2}{5}(1, 0, 0, 1, 2, 1, 1) + \frac{52}{5}(2, 0, 2, 0, 2, 0, 2) - \frac{1}{5}(1, -1, -1, -1, -1, -1, 1) \\
& -\left(\frac{1}{3}, 0, 2, 0, 2, -1, 2\right)
\end{aligned}$$