

量子力学与统计物理

Quantum mechanics and statistical physics

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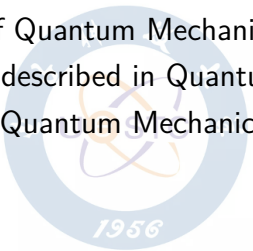
目录

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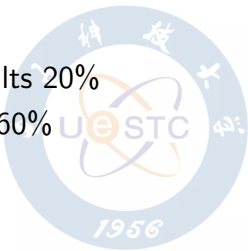
课程目标

- ① Learn the formal theory of Quantum Mechanics
- ② How physical systems are described in Quantum Mechanics.
- ③ How to solve problems in Quantum Mechanics.



分数构成

- ① Normal results 20%
- ② Midterm examination results 20%
- ③ Final examination results 60%

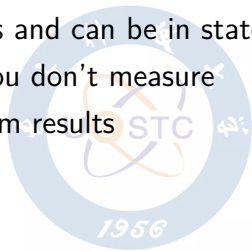


参考书目

- 《量子力学》卷 I, II, 曾谨言, 科学出版社, 2008
- Principles of quantum mechanics, shankar
- Modern quantum mechanics, shankar
- Lectures on quantum mechanics, weinberg
- Principles of quantum mechanics, Dirac

三条军规

- Objects are wave-particles and can be in states of superposition
- Rule 1 holds as long as you don't measure
- Measurement gives random results



第一讲：普朗克能量子假说



伟大成就

Great successes in Classical Physics

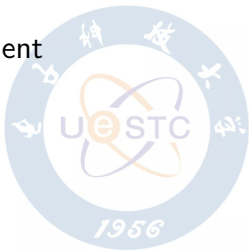
- ① Newtonian mechanics
- ② Maxwell's electromagnetism
- ③ Thermodynamic laws

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurements" ...Lord Kelvin 1900

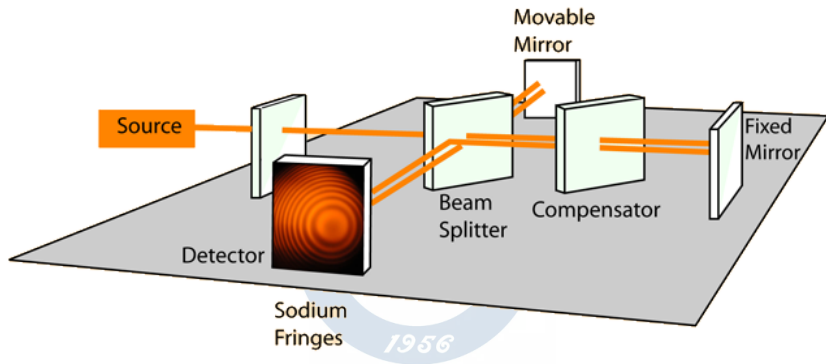
"The beauty and clearness ... is obscured by two small puzzling clouds " ...Lord Kelvin 1900.4

两朵乌云

- Michelson-Morley experiment
- Black body radiation

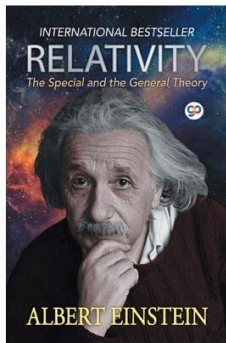


迈克尔逊-莫雷实验



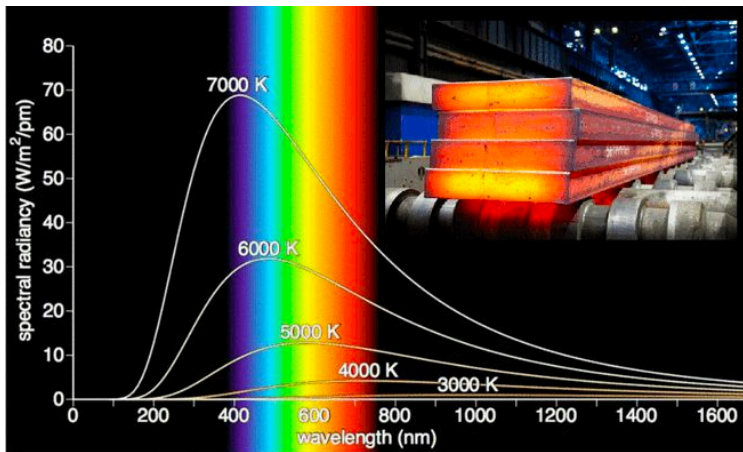
There is no displacement of the interference bands. ...the Stationary Ether is thus shown to be incorrect

The theory of relativity is established



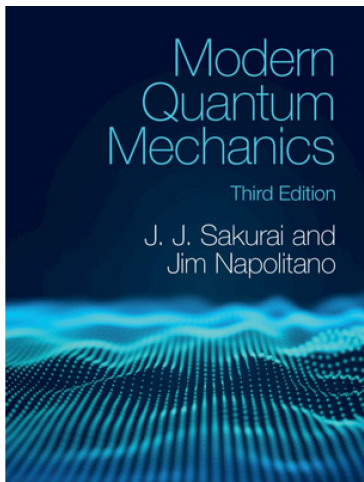
Greatly changed our view of time and space. Mainly useful in two aspects: high-speed motion, and strong gravitational field.

黑体辐射实验



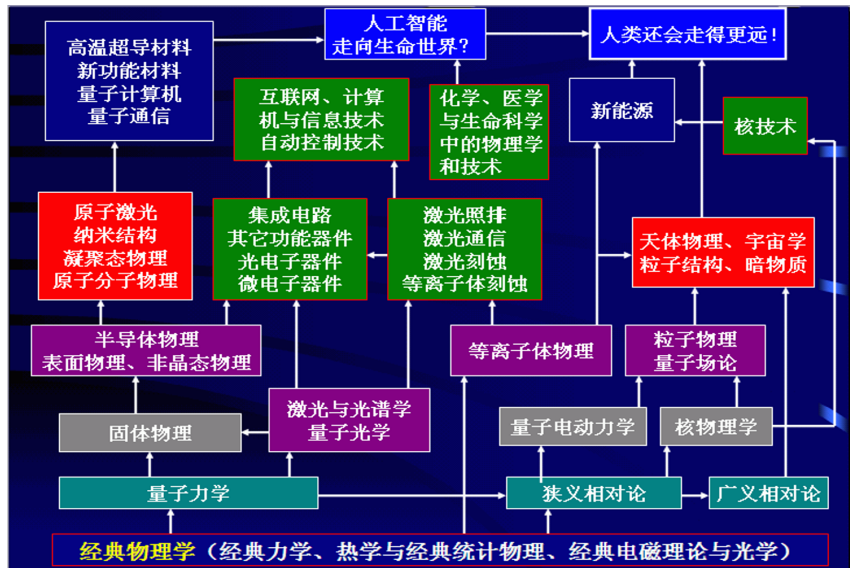
No mathematical function to describe the curves exactly

Quantum mechanics is established



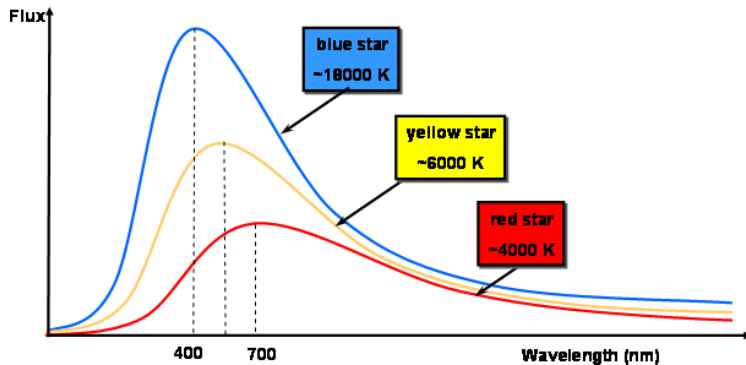
It is a theory about matter.

现代科学基石



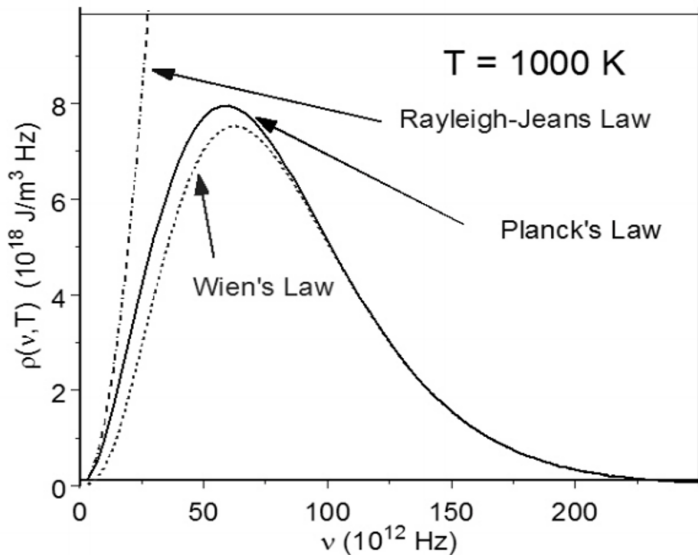
Definition

Black body: absorb all electromagnetic waves in any temperature



Most interestingly, what is the mathematical function that describes all of these curves?

三个经验公式

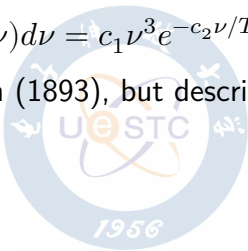


维恩公式

$$\rho(\nu)d\nu = c_1 \nu^3 e^{-c_2 \nu/T} d\nu$$

Derived from electromagnetism (1893), but described well only in high frequency region.

Nobel Prize in physics (1911)



瑞-金公式

$$\rho(\nu, T)d\nu = \frac{8\pi}{c^3}\nu^2 kT d\nu$$

Derived from thermodynamics (1900), but described well only in low frequency region.

Nobel Prize in physics (1904)

Ultraviolet Catastrophe:

$$\int_0^\infty \frac{8\pi}{c^3}\nu^2 kT \rightarrow \infty$$

普朗克公式

On 1900-10-19, at the German Physical Society, Max Planck presented a resolution to **Ultraviolet Catastrophe**

$$\rho(\nu, T)d\nu = \frac{8\pi}{c^3} \frac{h\nu^3}{e^{h\nu/KT} - 1} d\nu \quad (1)$$

Obtained from experimental data via interpolation technique (1900), described well in whole region

Nobel Prize in physics (1918)

The problem

How to derive the formula from existing theory.

Solution

On 1900-12-14, Planck gives out his solution based on the Energy Quantum Hypothesis

Energy quantum hypothesis

Assuming the oscillators of the cavity could only radiate at a discrete amounts of energy

$$E = n\varepsilon \quad (2)$$

where, the ε is the unit of the energy (quanta) determined by the oscillator's frequency

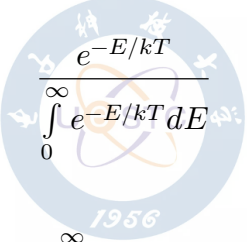
$$\varepsilon = h\nu \quad (3)$$

and the $h = 6.6260693(11) \times 10^{-34} J \cdot s$ is the Planck constant.

Based on Boltzmann distribution law,

$$\frac{N_i}{N} = \frac{\exp\left(-\frac{E_i}{kT}\right)}{\sum_i \exp\left(-\frac{E_i}{kT}\right)}$$

- when the energy is continuous, the distribution between $E - dE$ should be


$$\frac{e^{-E/kT}}{\int_0^{\infty} e^{-E/kT} dE}$$

the average energy

$$\langle E \rangle = \int_0^{\infty} E \frac{e^{-E/kT}}{\int_0^{\infty} e^{-E/kT} dE} dE$$

$$\begin{aligned}\langle E \rangle &= -kT \frac{Ee^{-E/kT}|_0^\infty - \int_0^\infty e^{-E/kT} dE}{\int_0^\infty e^{-E/kT} dE} \\ &= kT\end{aligned}$$

- when the energy is discrete, the distribution should be

$$\frac{e^{-E/kT}}{\int_0^\infty e^{-E/kT} dE} \rightarrow \frac{e^{-E/kT}}{\sum_0^\infty e^{-E/kT}} \rightarrow \frac{e^{-nh\nu/kT}}{\sum_0^\infty e^{-nh\nu/kT}}$$

the average energy

$$\begin{aligned}\langle E \rangle &= \sum_0^{\infty} nh\nu \frac{e^{-nh\nu/kT}}{\sum_0^{\infty} e^{-nh\nu/kT}} \\ &= -h\nu \frac{d}{dx} \frac{ne^{-nx}}{\sum_0^{\infty} e^{-nx}} \\ &= \frac{h\nu}{e^{h\nu/kT} - 1}\end{aligned}$$

We get

$$(\text{continuous}) \quad kT \rightarrow \frac{h\nu}{e^{h\nu/kT} - 1} \quad (\text{discrete})$$

In Rayleigh-Jeans formula

$$\rho(\nu, T) d\nu = \frac{8\pi}{c^3} \nu^2 kT d\nu$$

the item kT should be replaced by $\frac{h\nu}{e^{h\nu/kT}-1}$

$$\rho(\nu, T) d\nu = \frac{8\pi}{c^3} \frac{h\nu^3}{e^{h\nu/KT} - 1} d\nu$$

It is the Planck's formula exactly

讨论：

能量量子化只是一种数学处理技术？



Revolutionary Significance

Planck's Energy Quantum Hypothesis broke through the constraints of classical physics and opened the door of quantum mechanics



THE END

In 1927, **Dirac** got the Planck's formula from Quantum Mechanism.



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