

# 量子力学与统计物理

Quantum mechanics and statistical physics

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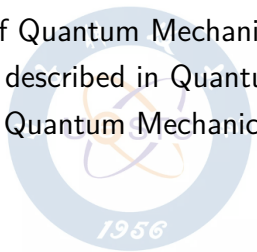
# 目录

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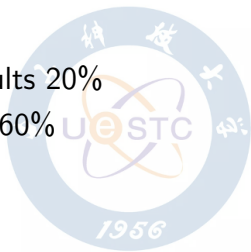
## 课程目标

- ① Learn the formal theory of Quantum Mechanics
- ② How physical systems are described in Quantum Mechanics.
- ③ How to solve problems in Quantum Mechanics.



## 分数构成

- ① Normal results 20%
- ② Midterm examination results 20%
- ③ Final examination results 60%

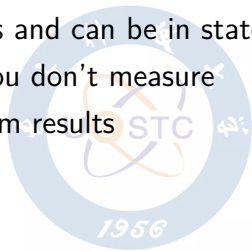


## 参考书目

- 《量子力学》卷 I, II, 曾谨言, 科学出版社, 2008
- Principles of quantum mechanics, shankar
- Modern quantum mechanics, shankar
- Lectures on quantum mechanics, weinberg
- Principles of quantum mechanics, Dirac

## 三条军规

- ① Objects are wave-particles and can be in states of superposition
- ② Rule 1 holds as long as you don't measure
- ③ Measurement gives random results



# 第一讲：普朗克能量量子假说



## 经典物理伟大成就

- ① Newtonian mechanics
- ② Maxwell's electromagnetism
- ③ Thermodynamic laws

### Great successes in Classical Physics

*"There is nothing new to be discovered in physics now. All that remains is more and more precise measurements"*

*... Lord Kelvin (1900)*



*"But the beauty and clearness ... is obscured by two small puzzling clouds "*

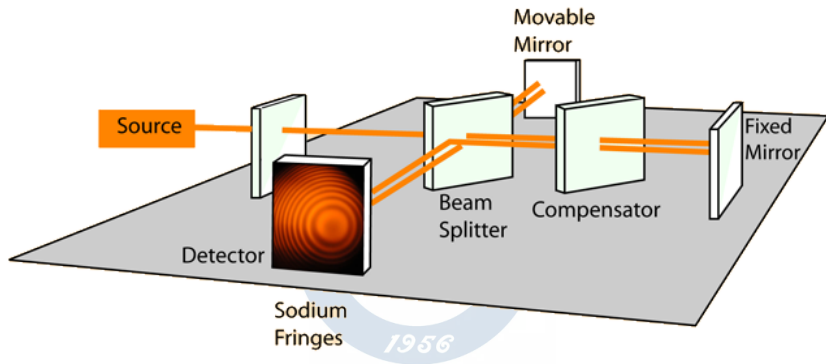
*... Lord Kelvin (1900.4)*



- ① Michelson-Morley experiment
- ② Black body radiation

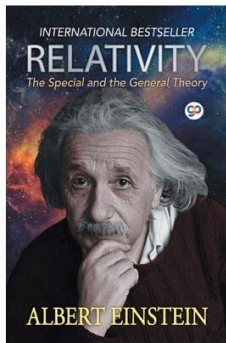
两朵乌云

## 迈克尔逊-莫雷实验



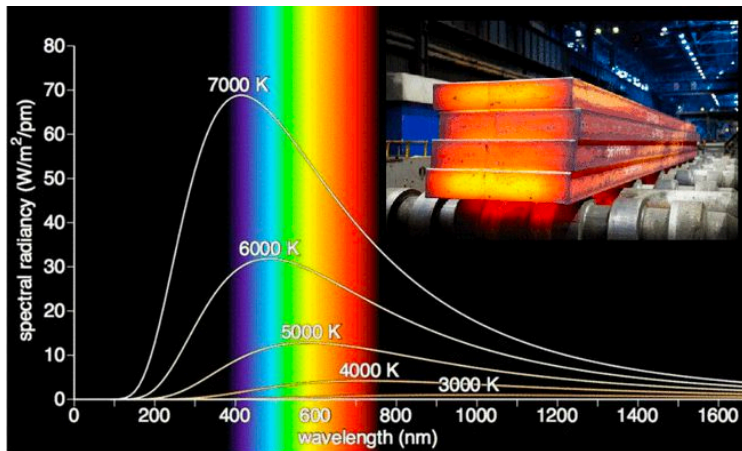
There is no displacement of the interference bands. ...the Stationary Ether is thus shown to be incorrect

## The theory of relativity is established



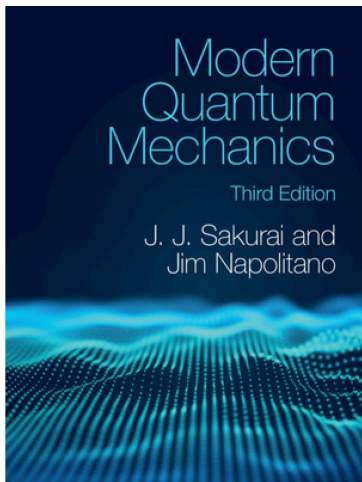
Greatly changed our view of time and space. Mainly useful in two aspects: high-speed motion, and strong gravitational field.

# 黑体辐射实验



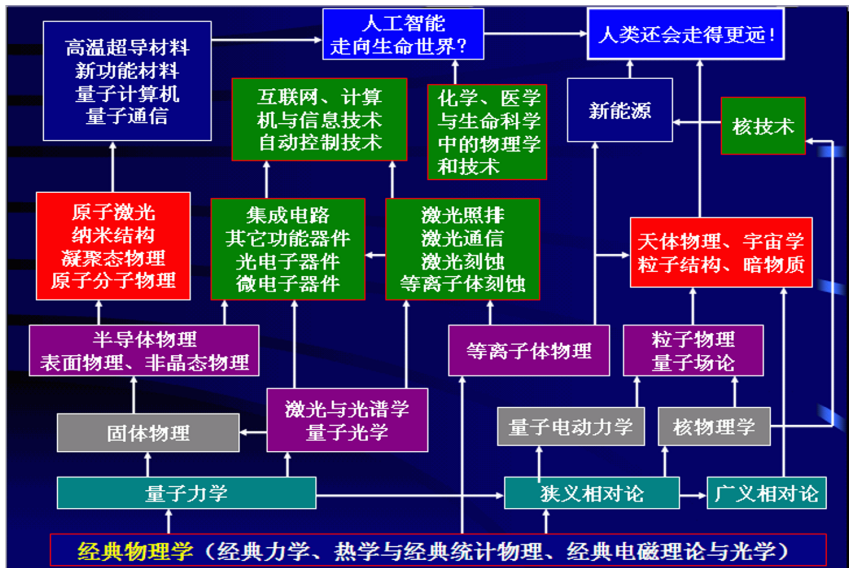
No mathematical function to describe the curves exactly

## Quantum mechanics is established



It is a theory about matter.

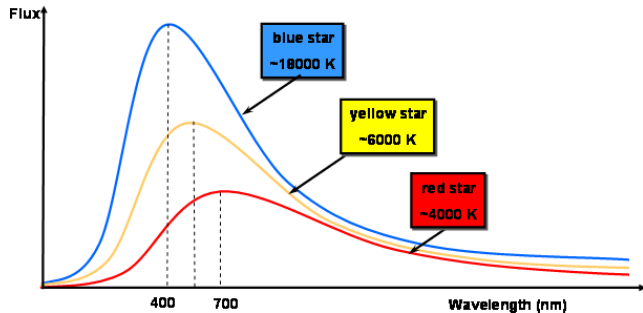
# 现代科学基石



# Black body radiation

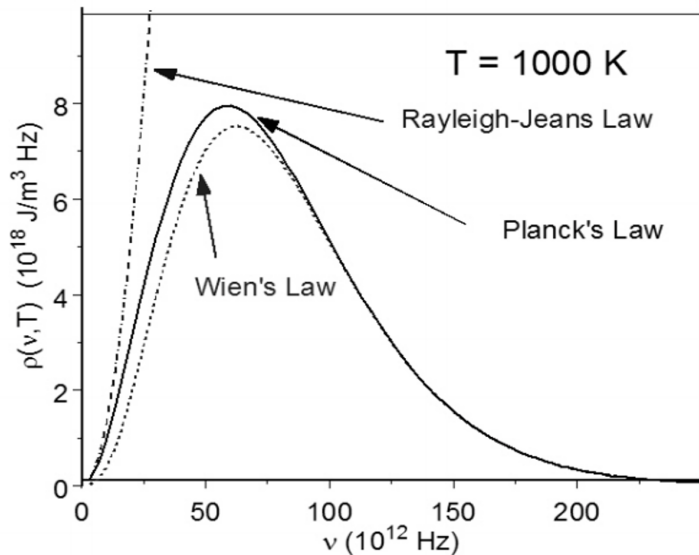
## Definition of Black body

Absorb all electromagnetic waves in any temperature



**Most interestingly**, what is the mathematical function that describes all of these curves?

## 三个经验公式



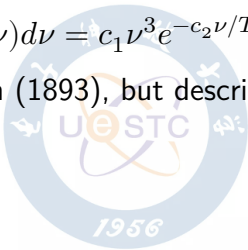


## 维恩公式

$$\rho(\nu)d\nu = c_1 \nu^3 e^{-c_2 \nu/T} d\nu$$

Derived from electromagnetism (1893), but described well only in high frequency region.

Nobel Prize in physics (1911)



## 瑞-金公式

$$\rho(\nu, T) d\nu = \frac{8\pi}{c^3} \nu^2 kT d\nu$$

Derived from thermodynamics (1900), but described well only in low frequency region.

Nobel Prize in physics (1904)

Ultraviolet Catastrophe:

$$\int_0^{\infty} \frac{8\pi}{c^3} \nu^2 kT \rightarrow \infty$$

## 普朗克公式

On 1900-10-19, at the German Physical Society, Max Planck presented a resolution to **Ultraviolet Catastrophe**

$$\rho(\nu, T)d\nu = \frac{8\pi}{c^3} \frac{h\nu^3}{e^{h\nu/KT} - 1} d\nu \quad (1)$$

Obtained from experimental data via interpolation technique (1900), described well in whole region

**Nobel Prize in physics (1918)**

## Problem

How to derive the formula from existing theory.

## Solution

On 1900-12-14, Planck gives out his solution based on the Energy Quantum Hypothesis

1956

## Energy quantum hypothesis

Assuming the oscillators of the cavity could only radiate at a discrete amounts of energy

$$E = n\varepsilon \quad (2)$$

where, the  $\varepsilon$  is the unit of the energy (quanta) determined by the oscillator's frequency

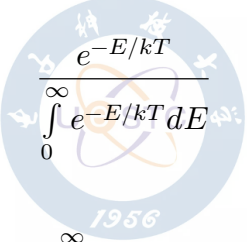
$$\varepsilon = h\nu \quad (3)$$

and the  $h = 6.6260693(11) \times 10^{-34} J \cdot s$  is the Planck constant.

Based on Boltzmann distribution law,

$$\frac{N_i}{N} = \frac{\exp\left(-\frac{E_i}{kT}\right)}{\sum_i \exp\left(-\frac{E_i}{kT}\right)}$$

- when the energy is continuous, the distribution between  $E - dE$  should be


$$\frac{e^{-E/kT}}{\int_0^{\infty} e^{-E/kT} dE}$$

the average energy

$$\langle E \rangle = \int_0^{\infty} E \frac{e^{-E/kT}}{\int_0^{\infty} e^{-E/kT} dE} dE$$

$$\begin{aligned}\langle E \rangle &= -kT \frac{Ee^{-E/kT}|_0^\infty - \int_0^\infty e^{-E/kT} dE}{\int_0^\infty e^{-E/kT} dE} \\ &= kT\end{aligned}$$

- when the energy is discrete, the distribution should be

$$\frac{e^{-E/kT}}{\int_0^\infty e^{-E/kT} dE} \rightarrow \frac{e^{-E/kT}}{\sum_0^\infty e^{-E/kT}} \rightarrow \frac{e^{-nh\nu/kT}}{\sum_0^\infty e^{-nh\nu/kT}}$$

the average energy

$$\begin{aligned}\langle E \rangle &= \sum_0^{\infty} nh\nu \frac{e^{-nh\nu/kT}}{\sum_0^{\infty} e^{-nh\nu/kT}} \\ &= -h\nu \frac{d}{dx} \frac{ne^{-nx}}{\sum_0^{\infty} e^{-nx}} \\ &= \frac{h\nu}{e^{h\nu/kT} - 1}\end{aligned}$$

We get

$$(\text{continuous}) \quad kT \rightarrow \frac{h\nu}{e^{h\nu/kT} - 1} \quad (\text{discrete})$$



In Rayleigh-Jeans formula

$$\rho(\nu, T)d\nu = \frac{8\pi}{c^3}\nu^2 kT d\nu$$

the item  $kT$  should be replaced by

$$\rho(\nu, T)d\nu = \frac{8\pi}{c^3} \frac{h\nu^3}{e^{h\nu/kT} - 1} d\nu$$

It is the Planck's formula exactly

Planck's Energy Quantum Hypothesis broke through the constraints of classical physics and opened the door of quantum mechanics

**Revolutionary Significance**



THE END

In 1927, Dirac got the Planck's formula from Quantum Mechanism.



## 学术讨论

能量量子化只是一种数学处理技术？



# Thanks for your attention!

