<u>**CS311: Homework 4**</u>

Spring 2017

Problem 1

(20 points). Determine if the following functions are **onto**, **one-to-one**, or **bijective**. The domain and codomain of the functions is R. Justify your answer.

a.) $x^2 + 4$

b.) $3x + 1$

c.) $4x^2 + x + 1$

d.) $\frac{x^2+2}{x^2+3}$

Problem 2

(15 points). Find the composite functions $g \circ f$ and $f \circ g$. The domain and codomain of the functions is R.

a.) $f(x) = x^2 + 2x + 1$, $g(x) = \sqrt{2x} + 1$.

b.) $f(x) = 6x + 3$, $g(x) = 4x + 1$.

c.) $f(x) = x^2 + 2$, $g(x) = x + 3$.

Problem 3

Part A (5 points): Given the function f where the domain $A = \{a, b, c, d\}$. Determine if f is invertible given the following: $f(a) = b$, $f(b) = a$, $f(d) = b$, $f(c) = c$.

Part B (5 points): Given $f(x) = x^2 + 3$, $g(x) = \frac{1}{x^2+3}$, determine if g is the inverse function of f. Give a counterexample if it is NOT, or show that they are inverses through the composite function $g \circ f$.

Problem 4

(20 points). Determine if f is strictly increasing, strictly decreasing, or none, given the domain of x.

a.) $f(x) = x + 4$, $x \in Z$.
b.) $f(x) = x^2 - 1$, $x \in N$.
c.) $f(x) = x^2 + 2$, $x \in Z$
d.) $f(x) = \frac{1}{x+1}$, $x \in N$.

Problem 5

Suppose S, R, T are sets. Show that:

$$(S - R) \cup (T - R) = (S \cup T) - R.$$

Part A (20 points): Prove this by showing that the left-hand side of the equation is a subset of the right-hand side, and then by showing that the right-hand side is a subset of the left-hand side. *Hint:* assume that $x \in ((S - R) \cup (T - R))$, then show through equivalences, inference rules, and set definitions that $x \in ((S \cup T) - R)$.

Part B (15 points): Prove the same statement again by using set identities. For this problem, you may use an alternative definition of set difference: $A - B = A \cap \overline{B}$.