

Munkres' Assignment Algorithm

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Procedure

	p	q	r
a	1	2	3
b	2	4	6
c	3	6	9

	p	q	r
a	0	1	2
b	0	2	4
c	0	3	6

	p	q	r
a	0*	1	2
b	0	2	4
c	0	3	6

	p	q	r
a	0*	1	2
b	0	2	4
c	0	3	6

	p	q	r
a	0*	1	2
b	0	2	4
c	0	3	6

- **Step 0:** Create a $n \times n$ matrix called the *cost matrix* in which each element represents the cost of assigning one of n workers to one of n jobs.
- **Step 1:** For each row of the matrix, find the smallest element and subtract it from every element in its row.
- **Step 2:** Find a zero (Z) in the cost matrix. If there isn't starred zero in its row or column, star it (Z). Repeat for each element in the matrix.
- **Step 3:** Cover each column containing a starred zero. If all columns are covered, the starred zeros describe a complete set of unique assignments. In this case, Go to DONE, otherwise, Go to Step 4.
- **Step 4:** If there is not noncovered zero, Go to Step 6. Find a noncovered zero(, which is produced by steps other than the third) and prime it. If the noncovered zero is in a row without any starred zero, Go to Step 5. Otherwise, cover this row (containing the starred and newly primed zeros) and uncover the column containing the starred zero. Go to the beginning of step 4.

Procedure

Result of step 4

	p	q	r
a	0*	1	2
b	0	2	4
c	0	3	6

Result of step 6

	p	q	r
a	0*	0	1
b	0	1	3
c	0	2	5

During step 4

	p	q	r
a	0*	0'	1
b	0	1	3
c	0	2	5

During step 4

	p	q	r
a	0*	0'	1
b	0	1	3
c	0	2	5

Result of step 4

	p	q	r
a	0*	0'	1
b	0'	1	3
c	0	2	5

- **Step 6:** Find the smallest noncovered value and add it to every element of each covered row, and subtract it from every element of each noncovered column. Without altering any stars, primes, or covered lines, return to Step 4.
- **Step 4:** If there is not noncovered zero, Go to Step 6. Find a noncovered zero(, which is produced by steps other than the third) and prime it. If the noncovered zero is in a row without any starred zero, Go to Step 5. Otherwise, cover this row (containing the starred and newly primed zeros) and uncover the column containing the starred zero. Go to the beginning of step 4.
- **Step 4:** If there is not noncovered zero, Go to Step 6. Find a noncovered zero(, which is produced by steps other than the third) and prime it. If the noncovered zero is in a row without any starred zero, Go to Step 5. Otherwise, cover this row (containing the starred and newly primed zeros) and uncover the column containing the starred zero. Go to the beginning of step 4.
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Procedure

During step 5

	p	q	r
a	0*	0'	1
b	0'	1	3
c	0	2	5

Result of step 5

	p	q	r
a	0	0*	1
b	0*	1	3
c	0	2	5

Result of step 3

	p	q	r
a	0	0*	1
b	0*	1	3
c	0	2	5

Result of step 4

	p	q	r
a	0	0*	1
b	0*	1	3
c	0	2	5

Result of step 6

	p	q	r
a	0	0*	0
b	0*	1	2
c	0	2	4

- **Step 5:** Construct a series of alternating primed and starred zeros as follows. Let Z0 (1,0) represent the noncovered primed zero found in Step 4. Let Z1 (0,0) denote the starred zero in the column of Z0 (if any). Let Z2 (0,1) denote the primed zero in the row of Z1 (there will always be one). Continue until the series terminates at a primed zero that has no starred zero in its column. Unstar each starred zero of the series, star each primed zero of the series, erase all primes and uncover every line in the matrix. Return to Step 3.
- **Step 5:** Construct a series of alternating primed and starred zeros as follows. Let Z0 (1,0) represent the noncovered primed zero found in Step 4. Let Z1 (0,0) denote the starred zero in the column of Z0 (if any). Let Z2 (0,1) denote the primed zero in the row of Z1 (there will always be one). Continue until the series terminates at a primed zero that has no starred zero in its column. Unstar each starred zero of the series, star each primed zero of the series, erase all primes and uncover every line in the matrix. Return to Step 3.
- **Step 3:** Cover each column containing a starred zero. If all columns are covered, the starred zeros describe a complete set of unique assignments. In this case, Go to DONE, otherwise, Go to Step 4.
- **Step 4:** If there is not noncovered zero, Go to Step 6. Find a noncovered zero(, which is produced by steps other than the third) and prime it. If the noncovered zero is in a row without any starred zero, Go to Step 5. Otherwise, cover this row (containing the starred and newly primed zeros) and uncover the column containing the starred zero. Go to the beginning of step 4.
- **Step 6:** Find the smallest noncovered value and add it to every element of each covered row, and subtract it from every element of each noncovered column. Without altering any stars, primes, or covered lines, return to Step 4.

Procedure

Result of step 6

	p	q	r
a	0	0*	0
b	0*	1	2
c	0	2	4

Result of step 4

	p	q	r
a	0	0*	0'
b	0*	1	2
c	0	2	4

Result of step 6

	p	q	r
a	0	0*	0'
b	0*	0	1
c	0	1	3

During step 4

	p	q	r
a	0	0*	0'
b	0*	0'	1
c	0	1	3

Result of step 4

	p	q	r
a	0	0*	0'
b	0*	0'	1
c	0'	1	3

- **Step 4:** If there is not noncovered zero, Go to Step 6. Find a noncovered zero(, which is produced by steps other than the third) and prime it. If the noncovered zero is in a row without any starred zero, Go to Step 5. Otherwise, cover this row (containing the starred and newly primed zeros) and uncover the column containing the starred zero. Go to the beginning of step 4.
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Procedure

Result of step 4

	p	q	r
a	0	0*	0'
b	0*	0'	1
c	0'	1	3

During step 5

	p	q	r
a	0	0*	0'
b	0*	0'	1
c	0'	1	3

Result of step 5

	p	q	r
a	0	0	0*
b	0	0*	1
c	0*	1	3

Result of step 3

	p	q	r
a	0	0	0*
b	0	0*	1
c	0*	1	3

Done

	p	q	r
a	0	0	0*
b	0	0*	1
c	0*	1	3

- **Step 4:** If there is not noncovered zero, Go to Step 6. **Find a noncovered zero**(, which is produced by steps other than the third) and prime it. **If the noncovered zero is in a row without any starred zero, Go to Step 5.** Otherwise, cover this row (containing the starred and newly primed zeros) and uncover the column containing the starred zero. Go to the beginning of step 4.
- **Step 5:** Construct a series of alternating primed and starred zeros as follows. Let Z0 (1,0) represent the noncovered primed zero found in Step 4. Let Z1 (0,0) denote the starred zero in the column of Z0 (if any). Let Z2 (0,1) denote the primed zero in the row of Z1 (there will always be one). Continue until the series terminates at a primed zero that has no starred zero in its column. Unstar each starred zero of the series, star each primed zero of the series, erase all primes and uncover every line in the matrix. Return to Step 3.
- **Step 3:** **Cover each column containing a starred zero.** If **all columns are covered**, the starred zeros describe a complete set of unique assignments. In this case, **Go to DONE**, otherwise, Go to
- **Step 4:** Jobs {p, q, r} to employees {c, b, a} respectively is the optimal assignment with the minimum cost.