Programming

optimisation and operations research algorithms with Julia

for Business Tasks

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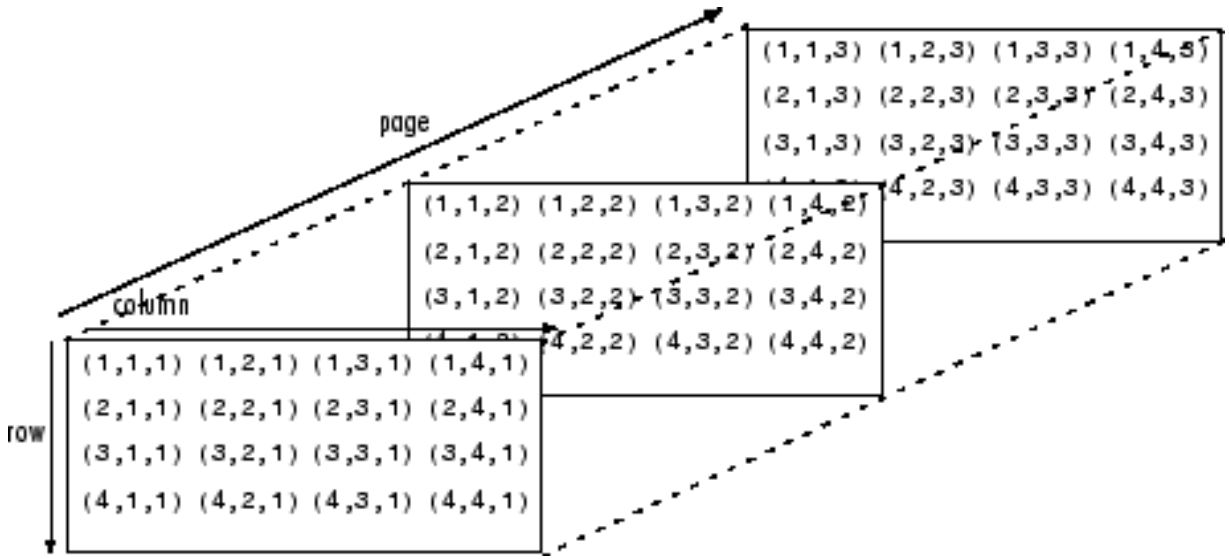
Data structure

Arrays (vectors, matrices, lists)



Constructing a vector (1/3)

A vector is an array with 1 dimension

Arrays contain ordered collections

Definition:

Let a collection of n items: $item_1$, $item_2$, ..., $item_n$

$$[item_1, item_2, \ldots, item_n]$$

Julia is 1-based indexing

Examples:

```
julia> ["Austria", "France", "Belgium"]
```

```
julia> primeNumbers = [2, 3, 5, 7, 11]
```

```
julia> primeNumbers = [2, 3, 5, "seven", 11]
```

Constructing a vector (1/3)

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julia> primeNumbers = [2, 3, 5, "seven", 11]
```

Constructing a vector (2/3)

Typed but non initialized vector:

```
Array{type}(undef,size)
```

```
Vector{type}(undef,size)
```

Examples:

```
julia> v = Array{Int64}(undef,5)
```

```
julia> v = Vector{Int64}(undef,5)
```

Constructing a vector (2/3)

Typed but non initialized vector:

```
Array{type}(undef,size)
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Vector{type}(undef,size)
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julia> v = Array{Int64}(undef,5)
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julia> v = Vector{Int64}(undef,5)
```

Constructing a vector (3/3)

Typed and initialized vector:

```
zeros(type,size)
```

```
ones(type,size)
```

See also `collect` and `fill` functions

Examples:

```
julia> v0 = zeros(Int64,5)
```

```
julia> v1 = ones(Int64,5)
```

```
julia> v3to6 = collect(3:6)
```

```
julia> v10 = fill(10,5)
```

Constructing a vector (3/3)

Typed and initialized vector:

```
zeros(type,size)
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```
ones(type,size)
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See also `collect` and `fill` functions

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```
julia> v10 = fill(10,5)
```

Accessing and editing a vector (1/3)

Access to individual data inside an array by indexing into the array:

```
nameArray[index]
```

```
julia> primeNumbers[3]
```

Access to the last data inside an array:

```
nameArray[end]
```

```
julia> primeNumbers[end]
```

Access to multiple consecutive data inside an array:

```
nameArray[indexStart:indexEnd]
```

```
julia> primeNumbers[3:4]
```

Accessing and editing a vector (1/3)

Access to individual data inside an array by indexing into the array:

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nameArray[index]
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Access to the last data inside an array:

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Access to multiple consecutive data inside an array:

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julia> primeNumbers[3:4]
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Accessing and editing a vector (1/3)

Access to individual data inside an array by indexing into the array:

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nameArray[index]
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Access to the last data inside an array:

```
nameArray[end]
```

```
julia> primeNumbers[end]
```

Access to multiple consecutive data inside an array:

```
nameArray[indexStart:indexEnd]
```

```
julia> primeNumbers[3:4]
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Accessing and editing a vector (1/3)

Access to individual data inside an array by indexing into the array:

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```

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julia> primeNumbers[3]
```

Access to the last data inside an array:

```
nameArray[end]
```

```
julia> primeNumbers[end]
```

Access to multiple consecutive data inside an array:

```
nameArray[indexStart:indexEnd]
```

```
julia> primeNumbers[3:4]
```

Accessing and editing a vector (2/3)

Edit an existing element of an array (arrays are mutable):

```
nameArray[index] = item
```

```
julia> primeNumbers[2] = "three"
```

Edit multiple consecutive element of an array:

```
nameArray[indexStart:indexEnd] = [item_1,...,item_n]
```

```
julia> v[2:3] = [3, 7]
```

Accessing and editing a vector (2/3)

Edit an existing element of an array (arrays are mutable):

$$nameArray[index] = item$$

```
julia> primeNumbers[2] = "three"
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Edit multiple consecutive element of an array:

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Edit multiple consecutive element of an array:

$$nameArray[indexStart:indexEnd] = [item_1, ..., item_n]$$

```
julia> v[2:3] = [3, 7]
```

Accessing and editing a vector (3/3)

`push!` function adds an element to the end of an array

```
push!(nameArray,item)
```

```
julia> push!(primeNumbers,13)
```

`pop!` function removes the last element of an array

```
pop!(nameArray)
```

```
julia> pop!(primeNumbers)
```

Accessing and editing a vector (3/3)

`push!` function adds an element to the end of an array

```
push!(nameArray,item)
```

```
julia> push!(primeNumbers,13)
```

`pop!` function removes the last element of an array

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pop!(nameArray)
```

```
julia> pop!(primeNumbers)
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Accessing and editing a vector (3/3)

`push!` function adds an element to the end of an array

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`pop!` function removes the last element of an array

```
pop!(nameArray)
```

```
julia> pop!(primeNumbers)
```

Constructing a matrix (1/4)

A matrix as multi-dimensional array

Definition:

Let a collection of items stored in a (e.g.) $n \times m$ grid (2D array),

$$[t_{1,1}\ t_{1,2} \ldots t_{1,m}\ ;\ t_{2,1}\ t_{2,2} \ldots t_{2,m}\ ;\ \ldots\ ;\ t_{n,1}\ t_{n,2}\ \ldots t_{n,m}]$$

Examples:

```
julia> [1 2 ; 3 4]
```

```
julia> T = ['a' 'b' 'c' ; 'x' 'y' 'z']
```

Constructing a matrix (1/4)

A matrix as multi-dimensional array

Definition:

Let a collection of items stored in a (e.g.) $n \times m$ grid (2D array),

$$[t_{1,1}\ \ t_{1,2} \ldots t_{1,m}\ ;\ t_{2,1}\ \ t_{2,2} \ldots t_{2,m}\ ;\ \ldots\ ;\ t_{n,1}\ \ t_{n,2}\ \ \ldots t_{n,m}]$$

Examples:

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Examples:

```
julia> [1 2 ; 3 4]
```

```
julia> T = ['a' 'b' 'c' ; 'x' 'y' 'z']
```

Constructing a matrix (2/4)

A matrix as an array of arrays

Definition:

Let a collection of items stored in (e.g.) a $n \times m$ grid (2D array),

```
[ [t1_1 t1_2...t1_m], [t2_1 t2_2...t2_m], [...], [t3_1 t3_2 ...t3_m] ]
```

Examples:

```
julia> [ [1 2], [3 4] ]
```

```
julia> A = [ [1 2], [3 4 6], [1] ]
```

Constructing a matrix (2/4)

A matrix as an array of arrays

Definition:

Let a collection of items stored in (e.g.) a $n \times m$ grid (2D array),

```
[ [t1_1  t1_2...t1_m], [t2_1  t2_2...t2_m], [...], [t3_1  t3_2  ...t3_m] ]
```

Examples:

```
julia> [ [1 2], [3 4] ]
```

```
julia> A = [ [1 2], [3 4 6], [1] ]
```

Constructing a matrix (2/4)

A matrix as an array of arrays

Definition:

Let a collection of items stored in (e.g.) a $n \times m$ grid (2D array),

```
[ [t1_1  t1_2...t1_m], [t2_1  t2_2...t2_m], [...], [t3_1  t3_2  ...t3_m] ]
```

Examples:

```
julia> [ [1 2], [3 4] ]
```

```
julia> A = [ [1 2], [3 4 6], [1] ]
```

Constructing a matrix (3/4)

Typed but non initialized matrix:

$$\texttt{Array\{}type\texttt{\}(undef,}dim_1\texttt{,}dim_2\texttt{,...,}dim_n\texttt{)}$$

$$\texttt{Matrix\{}type\texttt{\}(undef,}dim_1\texttt{,}dim_2\texttt{,...,}dim_n\texttt{)}$$

Examples:

```
julia> m = Array{Int64}(undef,2,3)
```

```
julia> m = Matrix{Int64}(undef,2,3)
```

Constructing a matrix (3/4)

Typed but non initialized matrix:

$$\texttt{Array\{}type\texttt{\}(undef,}dim_1\texttt{,}dim_2\texttt{,...,}dim_n\texttt{)}$$

$$\texttt{Matrix\{}type\texttt{\}(undef,}dim_1\texttt{,}dim_2\texttt{,...,}dim_n\texttt{)}$$

Examples:

```
julia> m = Array{Int64}(undef,2,3)
```

```
julia> m = Matrix{Int64}(undef,2,3)
```

Constructing a matrix (4/4)

Typed and initialized matrix:

`zeros(`$type, dim_1, dim_2, \ldots, dim_n$`)`

`ones(`$type, dim_1, dim_2, \ldots, dim_n$`)`

See also `fill` function

Examples:

```
julia> m0 = zeros(Float64,2,3,4)
```

```
julia> m1 = ones(Float64,2,3,4)
```

```
julia> fill(π,2,3)
```

Constructing a matrix (4/4)

Typed and initialized matrix:

`zeros(`$type, dim_1, dim_2, \ldots, dim_n$`)`

`ones(`$type, dim_1, dim_2, \ldots, dim_n$`)`

See also `fill` function

Examples:

```
julia> m0 = zeros(Float64,2,3,4)
```

```
julia> m1 = ones(Float64,2,3,4)
```

```
julia> fill(π,2,3)
```

Accessing a matrix

General syntax for indexing into an n-dimensional array:

$$nameArray[index_1, index_2, \ldots, index_n]$$

```
julia> m[2,1]
```

General syntax for indexing an array of arrays:

$$nameArray[index_1][index_2] \ldots [index_n]$$

```
julia> A[2][3]
```

All indices within an entire dimension or across the entire array:

```
:
```

```
julia> T[:,2]
```

Accessing a matrix

General syntax for indexing into an n-dimensional array:

$$nameArray[index_1, index_2, \ldots, index_n]$$

```
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General syntax for indexing an array of arrays:

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julia> A[2][3]
```

All indices within an entire dimension or across the entire array:

```
:
```

```
julia> T[:,2]
```

Accessing a matrix

General syntax for indexing into an n-dimensional array:

$nameArray$`[`$index_1$`, `$index_2$`, ... ,`$index_n$`]`

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julia> m[2,1]
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General syntax for indexing an array of arrays:

$nameArray$`[`$index_1$`][`$index_2$`] ... [`$index_n$`]`

```
julia> A[2][3]
```

All indices within an entire dimension or across the entire array:

```
:
```

```
julia> T[:,2]
```

Dimensions of an array

```
size(array, [dim])
```

```
julia> size(T)
julia> size(T,2)
```

```
length(array)
```

```
julia> length(T)
```

Dimensions of an array

```
size(array, [dim])
```

```
julia> size(T)
julia> size(T,2)
```

```
length(array)
```

```
julia> length(T)
```

Warning 1:

Observe this:

```
julia> [2, 3, 5, 7]
julia> [2 3 5 7]
```

Return an array with the same data as `A`, but with different dimension sizes or number of dimensions

```
reshape(array,dims)
```

```
julia> reshape([2 3 5 7], 4)
```

```
julia> reshape([2 3 5 7], (2,2))
```

Warning 1:

Observe this:

```
julia> [2, 3, 5, 7]
julia> [2 3 5 7]
```

Return an array with the same data as `A`, but with different dimension sizes or number of dimensions

```
reshape(array, dims)
```

```
julia> reshape([2 3 5 7], 4)
```

```
julia> reshape([2 3 5 7], (2,2))
```

Warning 2 (1/2)

Observe this:

```
julia> g = [2 3 4 5]
julia> h = g

julia> println(g)
julia> println(h)

julia> g[2] = 0

julia> println(h)
```

Warning 2 (2/2)

```
copy(array)
```

```
julia> copy(T)
```

```
deepcopy(array)
```

```
julia> deepcopy(T)
```

```
similar(array)
```

```
julia> similar(T)
```

Comprehension to construct arrays

[fct(var_1,var_2,...) for var_1=val_1, var_2=val_1,...]

```
julia> [i for i in 1:5]
```

```
julia> [i+j for i in 1:2 for j in 1:4]
```

```
julia> [i for i in 1:2, j in 1:4]
```

```
julia> [i for i in 1:10 if i % 2 == 1]
```

```
julia> [exp(i) for i in 1:3]
```

Array and operations on a list (1/2)

```
julia> L=[1,2,3,4,5]
```

Adding elements

- at the end

```
julia> push!(L,10)
```

- at the front

```
pushfirst!(array,item)
```

```
julia> pushfirst!(L,0)
```

- (replacing) at the given index

```
splice!(array,position(s),item(s))
```

```
julia> splice!(L,3,[L[3] 7 4])
```

Array and operations on a list (1/2)

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julia> splice!(L,3,[L[3] 7 4])
```

Array and operations on a list (2/2)

```
julia> L=[1,2,3,4,5]
```

Deleting elements

- at the end

```
julia> pop!(L)
```

- at the front

```
popfirst!(array,item)
```

```
julia> popfirst!(L)
```

- at the given index

```
splice!(array,position(s))
```

```
julia> splice!(L,2)
```

Array and operations on a list (2/2)

```
julia> L=[1,2,3,4,5]
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Deleting elements

- at the end

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julia> pop!(L)
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- at the given index

```
splice!(array,position(s))
```

```
julia> splice!(L,2)
```

Data structure

Tuple, dictionary, set

Constructing and accessing to a tuple

A tuple is an immutable array with 1 dimension

Definition:

Let a collection of n items: $item_1$, $item_2$, ..., $item_n$

```
(item1, item2, ..., itemn)
```

Examples:

```
julia> ("Austria", "France", "Belgium")
```

```
julia> primeNumbers = (2, 3, 5, "seven", 11)
```

Accessing to a tuple:

Same of a vector.

Constructing and accessing to a tuple

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julia> ("Austria", "France", "Belgium")
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```
julia> primeNumbers = (2, 3, 5, "seven", 11)
```

Accessing to a tuple:

Same of a vector.

Constructing and accessing to a tuple

A tuple is immutable

```
julia> vct = [2,7]                                    # array
```

```
julia> push!(vct,3)
```

Right!

```
julia> tpl = (2,7)                                    # tuple
```

```
julia> push!(tpl,3)
```

Wrong!

Constructing and accessing to a dictionary (1/3)

A dictionary is an 1D array where the sequence of elements is not ordered

Definition:

Let a collection of *n* couple *key-value*

```
Dict(key_1 => value_1, key_2 => value_2, ...)
```

Example:

```
julia> Dict(43 => "Austria", 32 => "Belgium", 33 =>
"France")
```

Constructing and accessing to a dictionary (1/3)

A dictionary is an 1D array where the sequence of elements is not ordered

Definition:

Let a collection of *n* couple *key-value*

```
Dict(key_1 => value_1, key_2 => value_2, ...)
```

Example:

```
julia> Dict(43 => "Austria", 32 => "Belgium", 33 => "France")
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Constructing and accessing to a dictionary (1/3)

A dictionary is an 1D array where the sequence of elements is not ordered

Definition:

Let a collection of *n* couple *key-value*

```
Dict(key1 => value1, key2 => value2, ...)
```

Example:

```
julia> Dict(43 => "Austria", 32 => "Belgium", 33 =>
"France")
```

Constructing and accessing to a dictionary (2/3)

Empty dictionary:

```
Dict()
```

```
julia> code = Dict()
```

Add an entry:

```
nameDictionary[key] = value
```

```
julia> code[49] = "Germany"
```

Delete an entry:

```
pop!(nameDictionary,key)
```

```
julia> pop!(code,33)
```

Constructing and accessing to a dictionary (3/3)

Return an iterator over all keys in a dictionary:

```
keys(dict)
```

```
julia> keys(code)
```

Return an iterator over all values in a dictionary:

```
values(dict)
```

```
julia> values(code)
```

Constructing and accessing to a set (1/2)

A set is a collection of unordered, unique values

Empty (zero-elements) set:

```
Set()
```

```
julia> a = Set()
```

Initialise a set with values:

```
nameSet([value_1, value_2, ..., value_n])
```

```
julia> a = Set([1,2,2,3,4])
```

Constructing and accessing to a set (1/2)

A set is a collection of unordered, unique values

Empty (zero-elements) set:

```
Set()
```

```
julia> a = Set()
```

Initialise a set with values:

```
nameSet([value_1, value_2, ..., value_n])
```

```
julia> a = Set([1,2,2,3,4])
```

Constructing and accessing to a set (2/2)

Some operations:

```
union(set1,set2)
```

```
julia> union([1, 2], [3, 4])
```

```
intersect(set1,set2)
```

```
julia> intersect([1, 2, 3], [3, 4, 5])
```

```
setdiff(set1,set2)
```

```
julia> setdiff([1,2,3], [3,4,5])
```

Summary

	mutable	ordered collections
array	yes	yes
tuple	no	yes
dictionary	yes	no
set	no	no

Data structure

Strings

Constructing and accessing to a string (1/4)

A string is an immutable finite sequence of characters

```
julia> s1 = "A string."                # ASCII characters
```

```
julia> s2 = """Also a string."""
```

```
julia> """A "special" string."""
```

```
julia> "\u2200 x \u2203 y"      # unicode characters
```

Constructing and accessing to a string (1/4)

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```
julia> """A "special" string."""
```

```
julia> "\u2200 x \u2203 y"       # unicode characters
```

Constructing and accessing to a string (2/4)

String interpolation

Use the $ sign

- to insert existing variables into a string and
- to evaluate expressions within a string.

```
julia> city = "Linz"
```

```
julia> "Hello $city"
```

```
julia> zc = 4020
```

```
julia> "zc of $city:  $zc, $(zc+10), $(zc+20)"
```

Constructing and accessing to a string (2/4)

String interpolation

Use the $ sign

- to insert existing variables into a string and
- to evaluate expressions within a string.

```
julia> city = "Linz"
```

```
julia> "Hello $city"
```

```
julia> zc = 4020
```

```
julia> "zc of $city:  $zc, $(zc+10), $(zc+20)"
```

Constructing and accessing to a string (3/4)

String concatenation

Two manners:

- use the `string()` function:

```
julia> string("Hello ", city)
```

`string()` converts non-string inputs to strings

- use the `*` operator:

```
julia> "Hello " * city
```

Constructing and accessing to a string (4/4)

```
julia> s1 = "A string."
```

```
julia> s1[3]
```

```
julia> s1[3:5]
```

```
julia> s1[begin:3]
```

```
julia> s1[3:end]
```

```
julia> s1[3] = 'b'                                    # error!
```

```
julia> s1 = s1[begin:3] * "w" * s1[5:end]
```

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Review and exercises

(notebook)