Feedback on three topics related to GRASP

MIC 2024: 15th Metaheuristics International Conference
June 4-7, 2024. Lorient (France)

Xavier GANDIBLEUX
Nantes Université – France

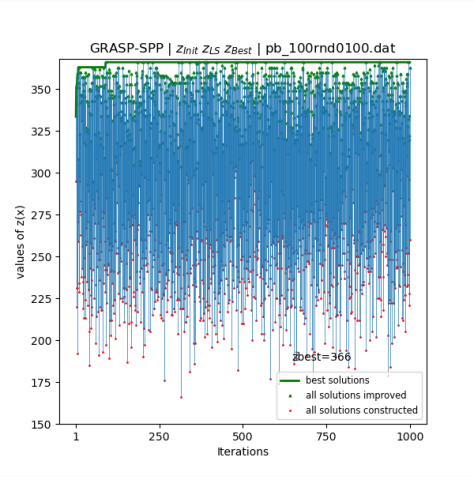
GRASP metaheuristic

Thomas A. Féo and Mauricio G.C. Resende. A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters*, 8:67-71, 1989.

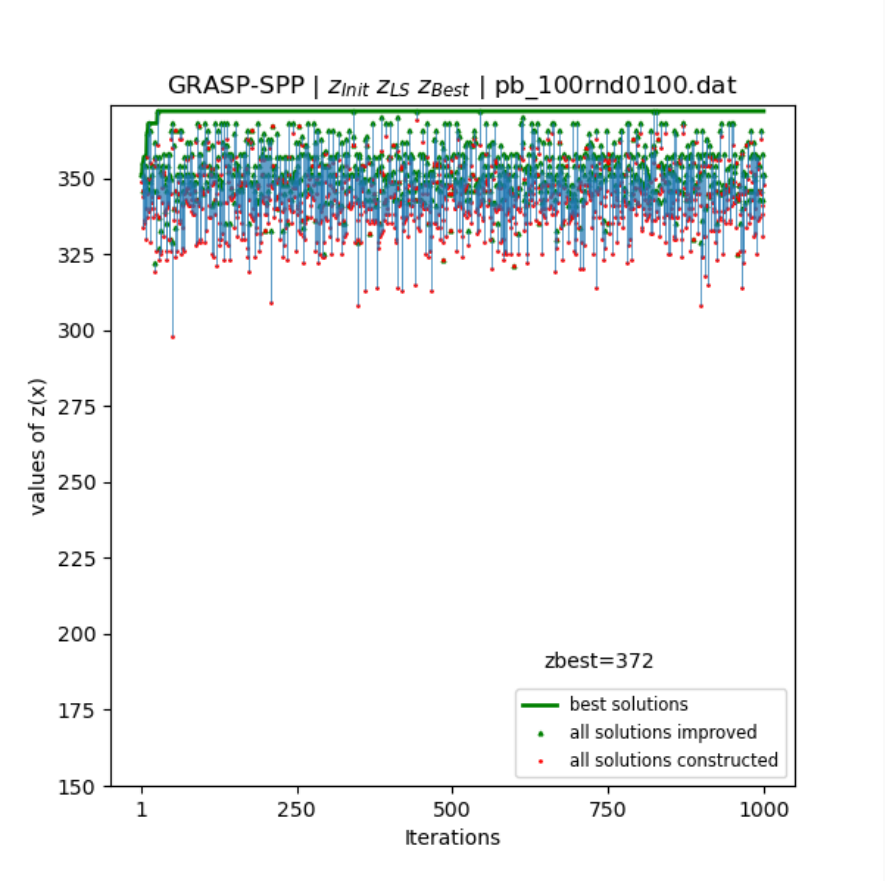
$S^* \leftarrow \emptyset$, the best solution found
repeat
  $S \leftarrow$ `greedyRandomizedConstruction`(problem, α)
  $S' \leftarrow$ `localSearchImprovement`(S)
  `updateSolution`(S', S^*)
until `isFinished?`(StoppingRule)
return S^*

- $\alpha \in [0, 1]$, the compromize between random and greedy.
- stoppingRule (ex : nIter, elapsedTime, etc.).

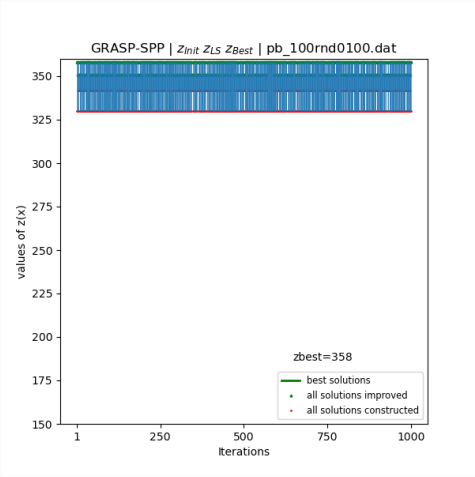
GRASP metaheuristic



$\alpha = 0.00$

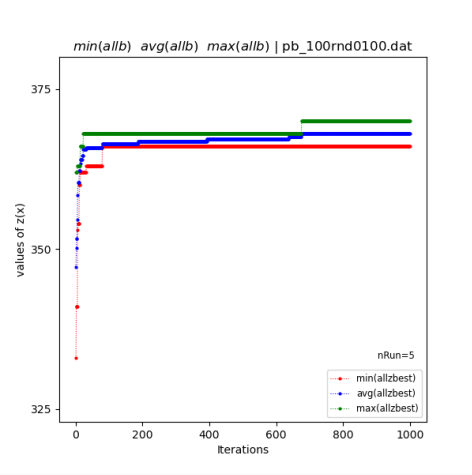


$\alpha = 0.70$

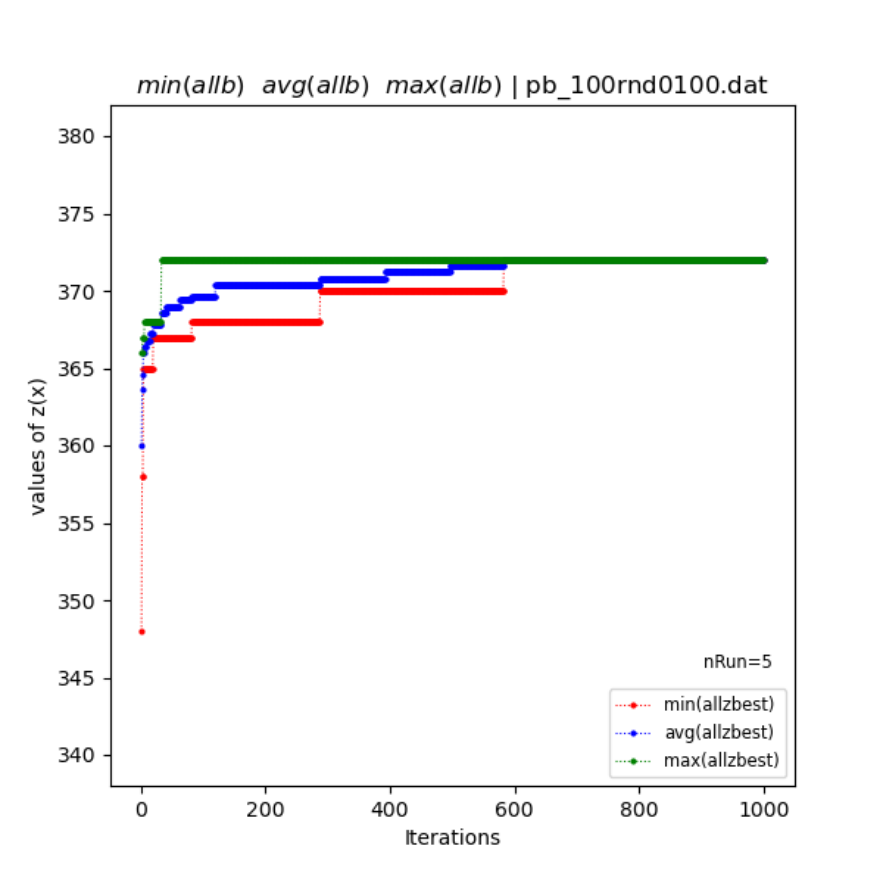


$\alpha = 1.00$

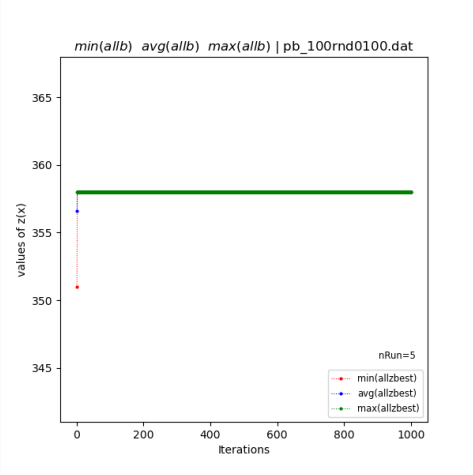
GRASP metaheuristic



$\alpha = 0.00$



$\alpha = 0.70$



$\alpha = 1.00$

Topic 1

Topic 1

During MIC'1997,
Sophia-Antipolis (France):

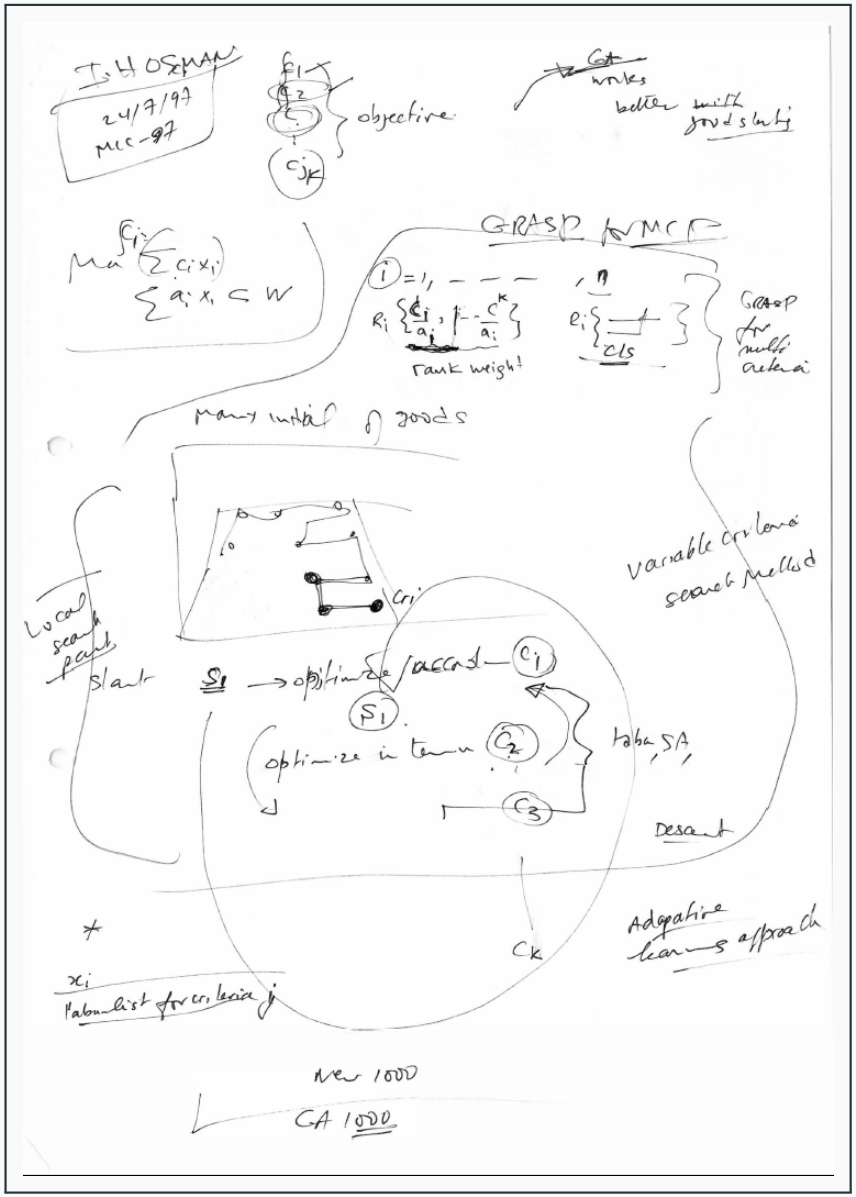
Private communication with
Ibrahim Osman (American
University of Beirut, Lebanon):

adapting GRASP to
MultiObjective Optimization

$$\begin{array}{ll} \min & F(x) \\ s.t. & x \in X \end{array}$$

where $X \subseteq \mathbb{R}^n$, $Y = F(X) \subseteq \mathbb{R}^p$

to compute $X_{PE} \subset X$, $Y_{PN} \subset Y$

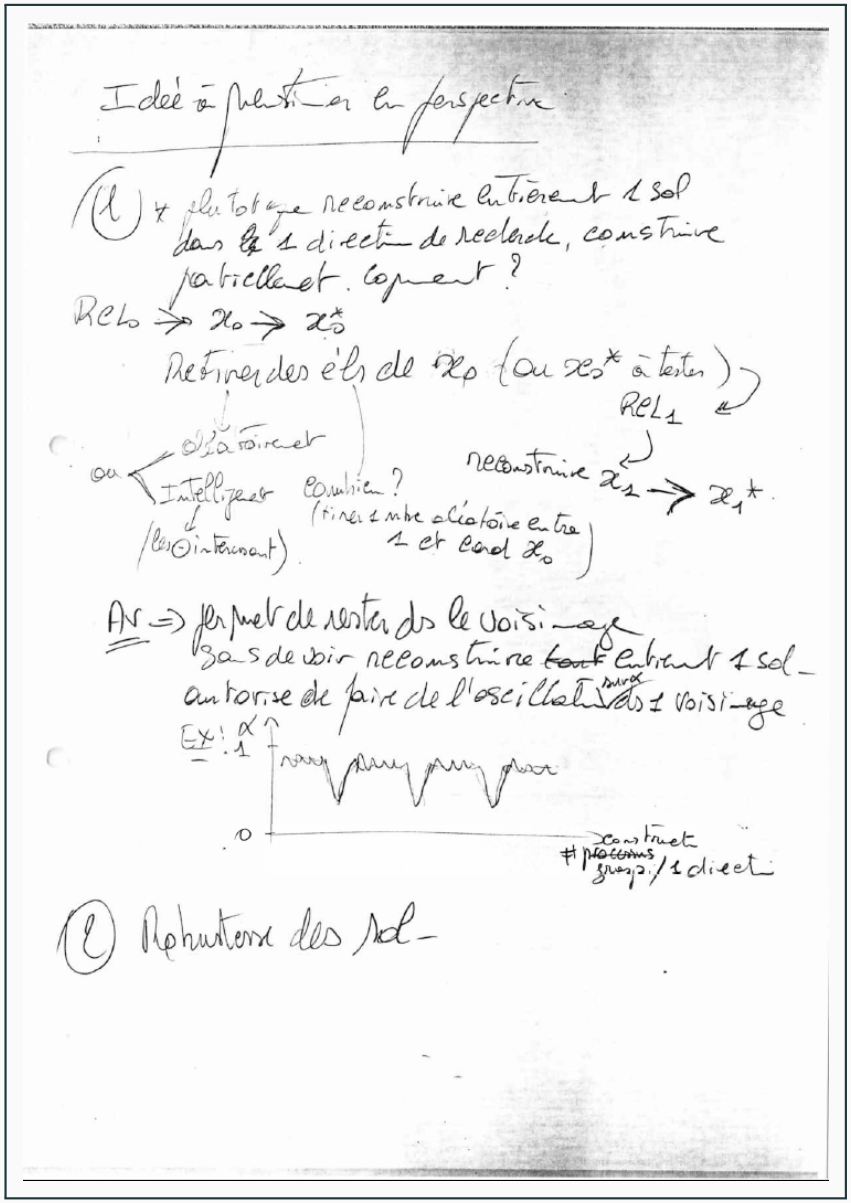


Topic 1

- Master thesis of D. Vancoppenolle
- MCDM'1998 Charlotteville (USA):

Xavier Gandibleux, David Vancoppenolle, Daniel Tuyttens, A first making use of GRASP for solving MOCO problems. MCDM'1998: 14th International Conference on Multiple Criteria Decision Making, June 8-12 1998, Charlottesville (VA), USA.

$$
\circlearrowleft d \begin{cases} x_0 \leftarrow construction(\lambda^d) \\ x'_0 \leftarrow localSearch(x_0) \\ \circlearrowleft i \begin{cases} x \leftarrow deconstruction(x'_{i-1}) \\ x_i \leftarrow reconstruction(x, \lambda^d) \\ x'_i \leftarrow localSearch(x_i) \end{cases} \end{cases}
$$



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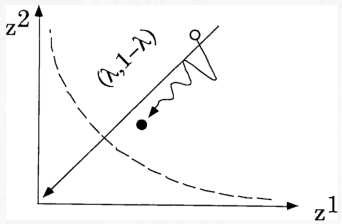
Matthias Ehrgott, Xavier Gandibleux. A survey and annotated bibliography of multiobjective combinatorial optimization. *OR Spectrum*. 22(4): 425-460 (2000)

***MO*GRASP**

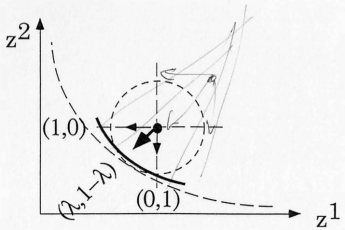
▷ GRASP construction

$\lambda = (\lambda_1, \ldots, \lambda_p) \geq 0$

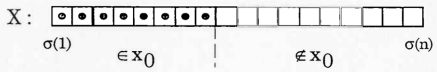
$$s_1(z,\lambda) = \sum_{j=1}^{p} \lambda_j z_j(x), \quad \sum_{j=1}^{p} \lambda_j = 1$$



▷ GRASP local search



ex :* *1,0-exchange
1,1-exchange
1,2-exchange...



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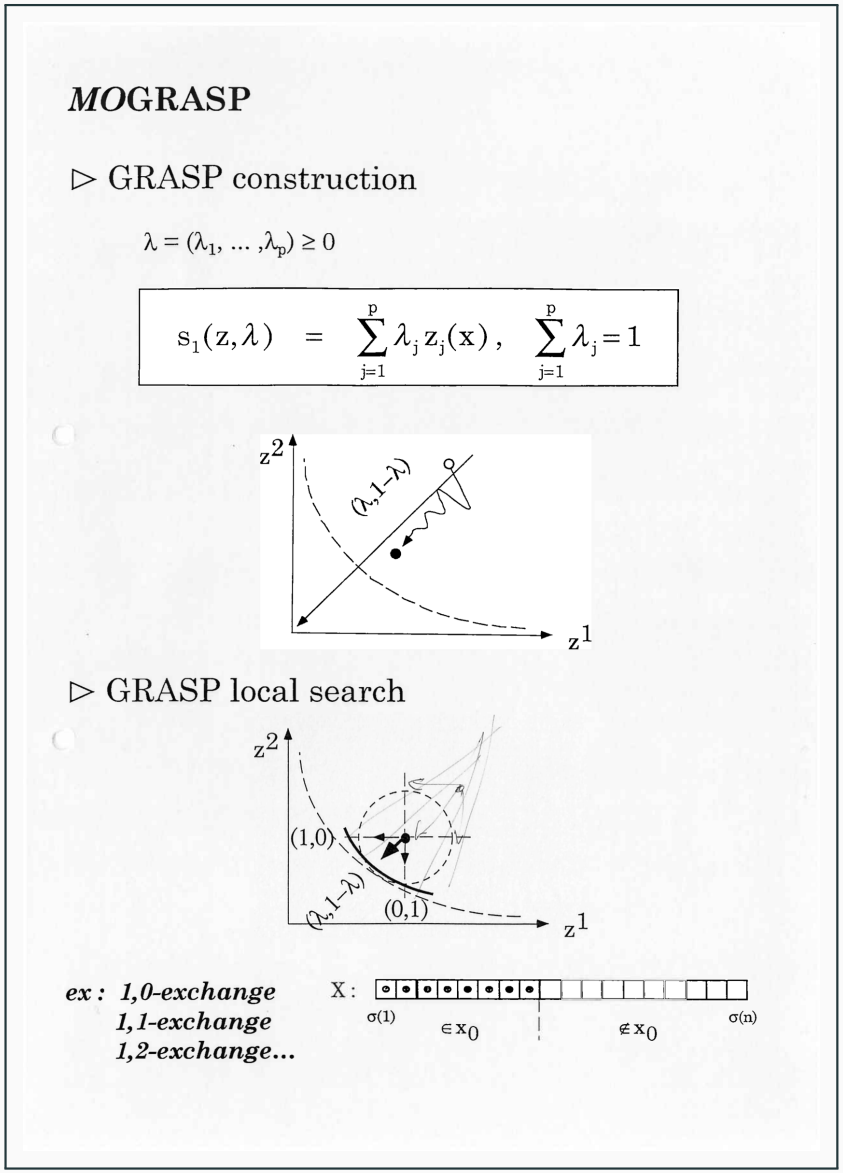
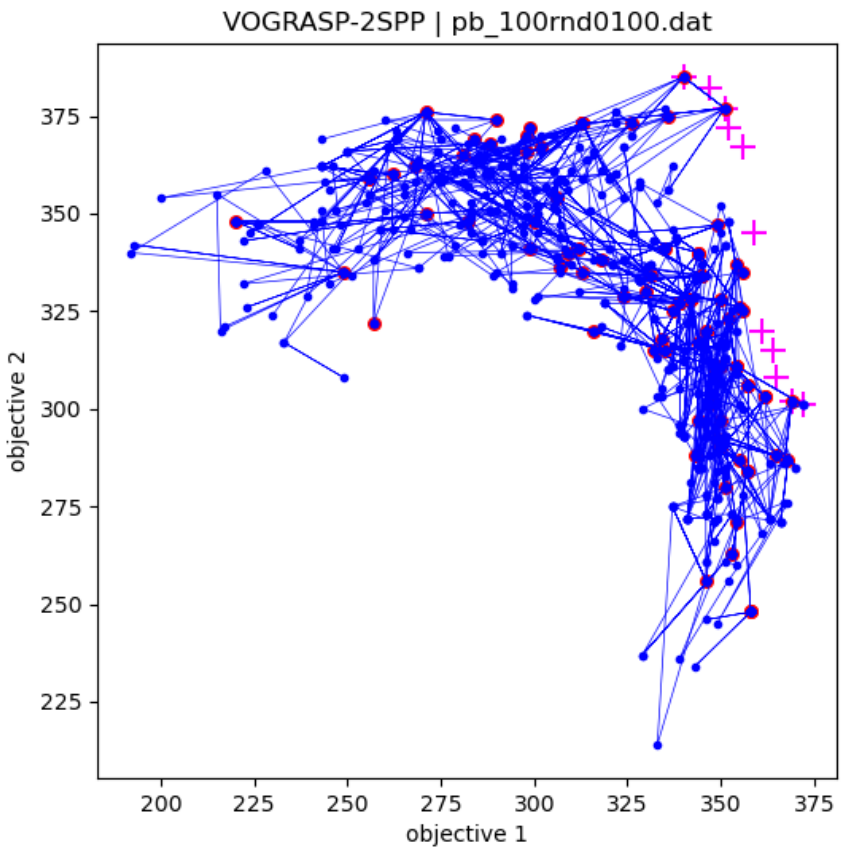
$\rightsquigarrow$ LNS (Shaw, 1998):

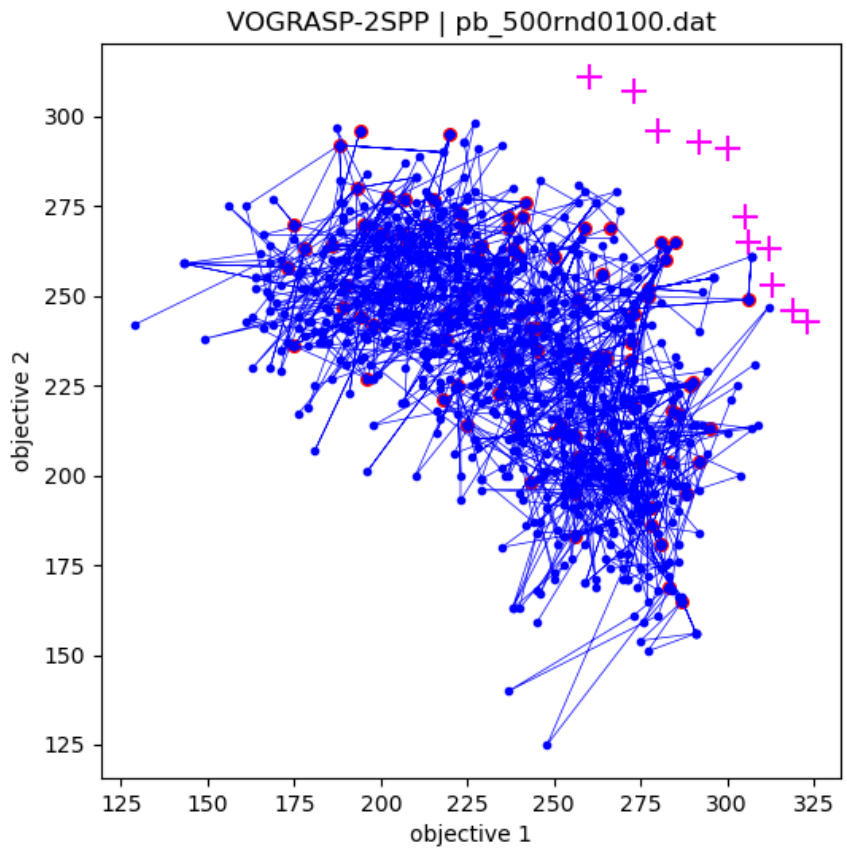
In LNS, an initial solution is gradually improved by alternately destroying and repairing the solution (Pisinger and Røpke 2010)



Topic 1

Illustration (1-SPP):





- $\alpha = 0.7$ value for computing U_{limit} into RCL
- nb$\lambda = 100$ # search directions from 0.0 to 1.0, step 0.01
- nbDR $= 15$ # deconstruction/reconstruction

GRASP and multiObjective Optimization

Examples of contributions :

2004: D. S. Vianna and J. E. C. Arroyo. A GRASP algorithm for the multi-objective knapsack problem. *XXIV International Conference of the Chilean Computer Science Society*, Arica, Chile, pp. 69-75.

2009: Li, H., Landa-Silva, D. (2009). An Elitist GRASP Metaheuristic for the Multi-objective Quadratic Assignment Problem. In: Ehrgott, M., Fonseca, C.M., Gandibleux, X., Hao, JK., Sevaux, M. (eds) Evolutionary Multi-Criterion Optimization. EMO 2009. *Lecture Notes in Computer Science*, vol 5467. Springer, Berlin, Heidelberg.

2011: Arroyo, J.C., de Souza Pereira, A.A. A GRASP heuristic for the multi-objective permutation flowshop scheduling problem. *The International Journal of Advanced Manufacturing Technology*. 55, 741–753.

2015: Rafael Martí, Vicente Campos, Mauricio G.C. Resende, Abraham Duarte, Multiobjective GRASP with Path Relinking, *European Journal of Operational Research*, Volume 240, Issue 1, Pages 54-71.

2018: López-Sánchez, A.D., Hernández-Díaz, A.G., Gortázar, F. and Hinojosa, M.A., A multiobjective GRASP–VND algorithm to solve the waste collection problem. *International Transactions in Operational Research*, 25(2): 545-567. 2018.

2021: Pedro Casas-Martínez, Alejandra Casado-Ceballos, Jesús Sánchez-Oro, and Eduardo G. Pardo. Multi-Objective GRASP for Maximizing Diversity. *Electronics* 10(11):1232. 2021.

2022: Xing Wan, Zuo Xingquan, Li Xiaodong, and Zhao Xinchao. A Hybrid Multiobjective GRASP for a Multi-Row Facility Layout Problem with Extra Clearances. *International Journal of Production Research* 60(3), 957–976. 2022.

GRASP and deconstruction/reconstruction

GRASP $\approx$ LNS with 100% of deconstruction.

Investigation on 1-objective Weighted Set Packing Problem (1-WSPP):

- Valentin Antuori (2014):
  GRASP_DR v.s. LNS
- Elizabeth Gandibleux and Awen Jacq-Bodet (2023):
  GRASP_DR := GRASP + DR[rnd/VND]

Deconstruction (randomly):

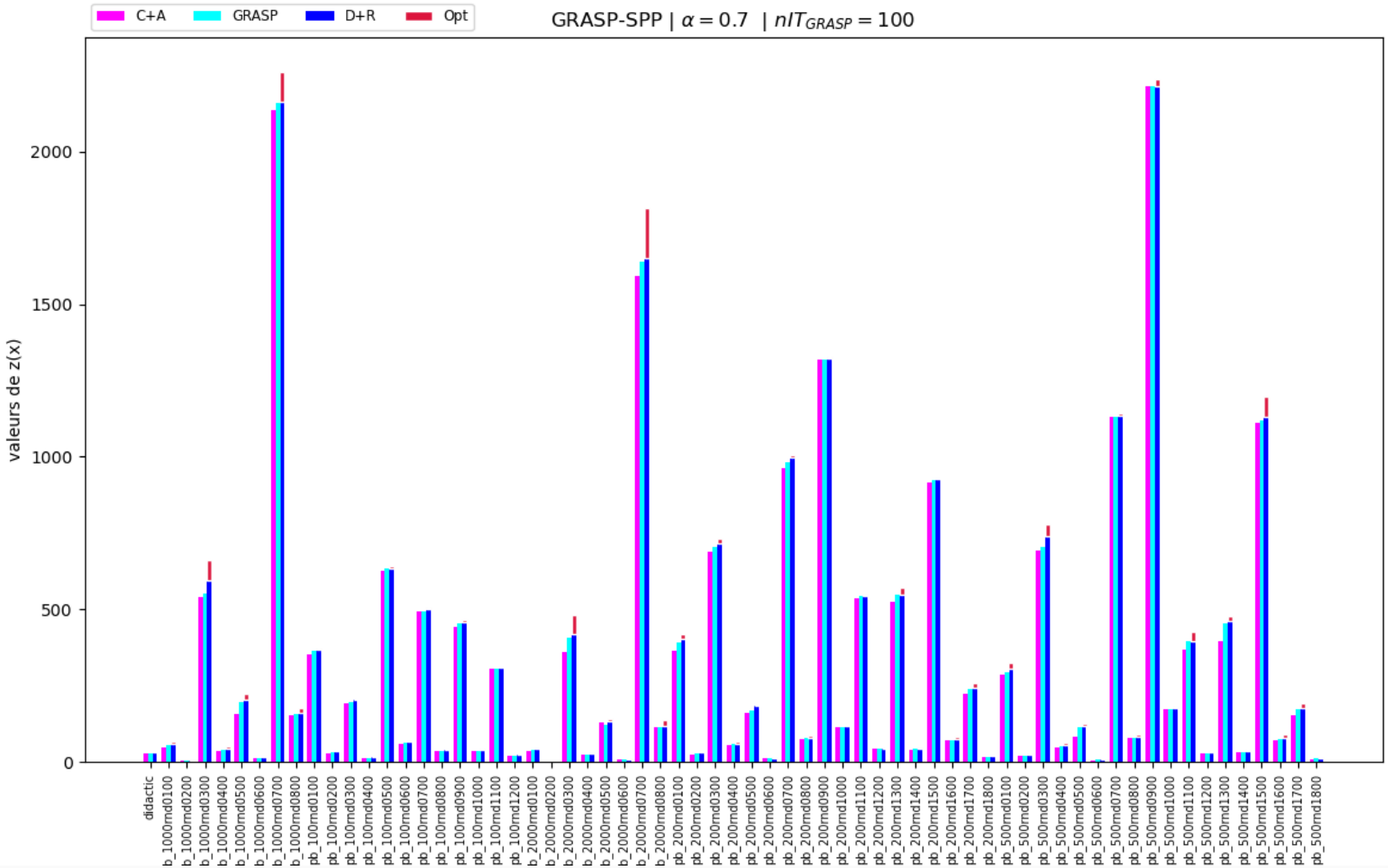
- identify the `nList1` variables at 1 into x'_0
- remove 33% $\leqslant$ rand(1:nList1) $\leqslant$ 50% of variables at 1 into x'_0

Reconstruction (following VND principle):

- 0-1 exchange $\longrightarrow$ 1-1 exchange $\longrightarrow$ 2-1 exchange

GRASP and deconstruction/reconstruction

1-WSPP | Nb instances : 65 | example of 1 run :



GRASP and deconstruction/reconstruction

1 run with $\alpha = 0.70$ | nbIterGrasp $= 100$ | nbIterDR $= 10$ | bests :

instance	C+A	GRASP	GRASP_DR	bestKnown
100rnd0100	351	368	**370**	372
100rnd0200	29	32	**33**	34
100rnd0300	194	195	**203**	203
100rnd0400	13	**15**	**15**	16
200rnd0100	366	400	**415**	416
200rnd0200	25	29	**30**	32
200rnd0300	689	**721**	**721**	731
200rnd0400	55	58	**60**	64
500rnd0100	285	309	**312**	323
500rnd0200	19	21	**22**	25
500rnd0300	694	724	**749**	776
500rnd0400	47	54	**55**	62
1000rnd0100	49	**67**	**67**	67
1000rnd0200	**3**	**3**	**3**	4
1000rnd0300	541	582	**592**	661
1000rnd0400	37	**43**	**43**	48
2000rnd0100	37	**40**	**40**	40
2000rnd0200	**2**	**2**	**2**	2
2000rnd0300	361	428	**434**	478
2000rnd0400	25	26	**28**	32

Nb instances : 65; #best: C+A : 4 | GRASP : 26 | GRASP_DR : 62

Topic 2

Topic 2

During EURO'1997, Barcelona (Spain):

- Private communication with Naoki Katoh (Kyoto University, Japan) and Hiroyuki Morita (Osaka Prefecture University, Japan): strenghten EMO with LS for MOCO problems.

↳ concept of seeding solutions in a EMO

Hiroyuki Morita, Xavier Gandibleux, and Naoki Katoh. Experimental feedback on biobjective permutation scheduling problems solved with a population heuristic. *Foundations of Computing and Decision Sciences Journal*. 26(1):23–50, 2001.

↳ seeding solutions generated with λ-GRASP

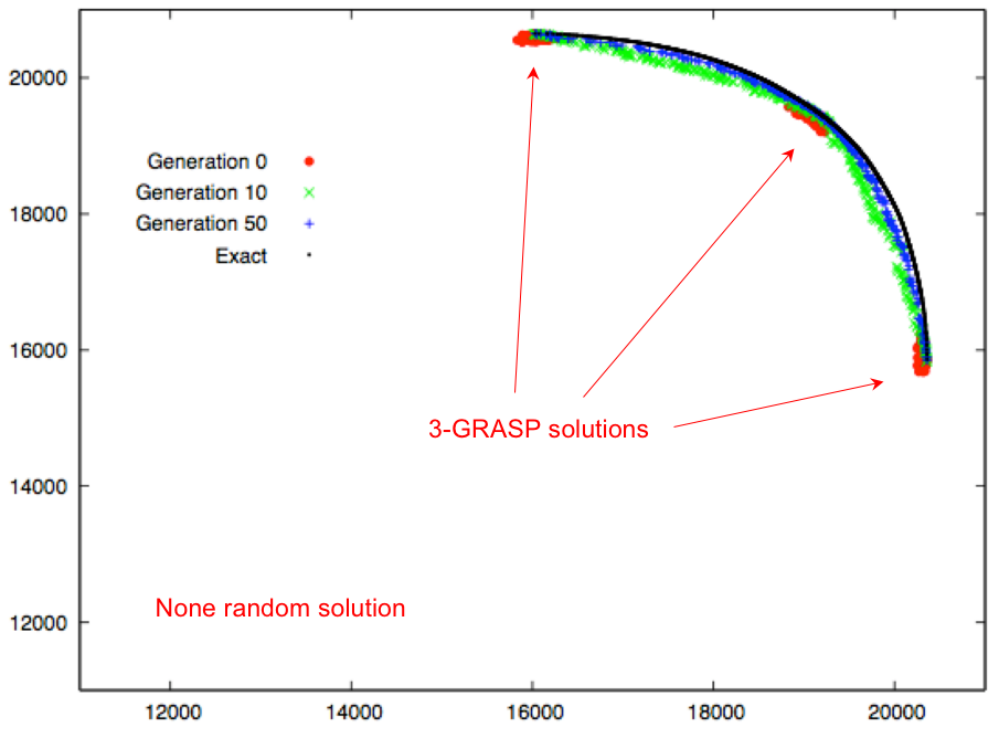
Xavier Delorme, Xavier Gandibleux, and Fabien Degoutin. Evolutionary, constructive and hybrid procedures for the bi-objective set packing problem. *European Journal of Operational Research*. 204(2):206–217, July 2010.

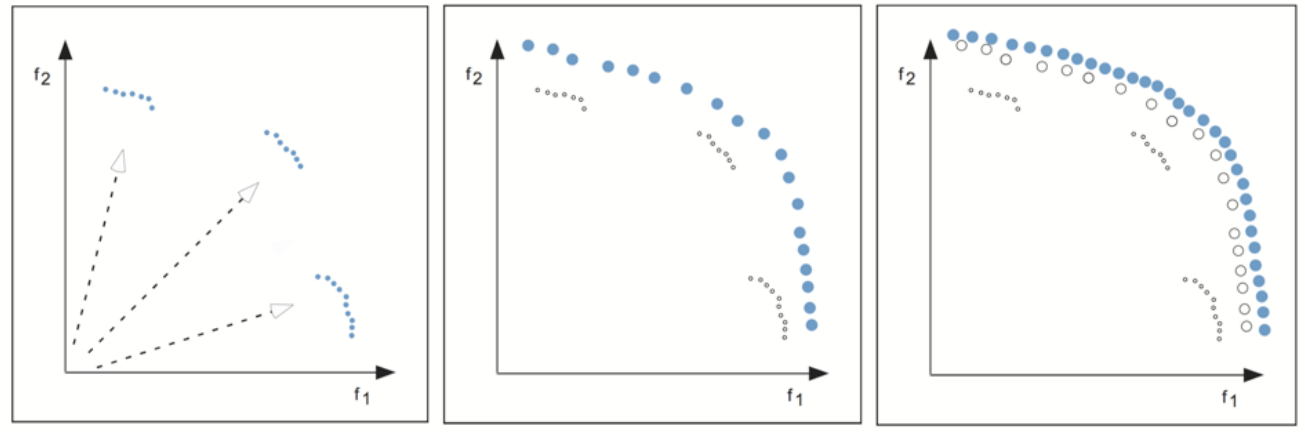
↳ methodology for MOMH in 3 stages

Xavier Gandibleux: Peek - Shape - Grab: A Methodology in Three Stages for Approximating the Non-dominated Points of Multiobjective Discrete/Combinatorial Optimization Problems with a Multiobjective Metaheuristic. In: Trautmann, H., et al. Evolutionary Multi-Criterion Optimization. EMO 2017. Lecture Notes in Computer Science, vol 10173, 221-235. Springer. 2017.

...visual demonstration...

Topic 2


20000
18000
16000
14000
12000
12000
14000
16000
18000
20000
Generation 0
Generation 10
Generation 50
Exact
3-GRASP solutions
None random solution


f2
f1
f2
f1
f2
f1

MOMH using seeding solutions

Examples of contributions :

2005: Christian Haubelt, Jürgen Gamenik, and Jürgen Teich . Initial population construction for convergence improvement of MOEAs. In Carlos Coello Coello, Arturo Hernandez Aguirre, and Eckart Zitzler, editors, Evolutionary Multi-Criterion Optimization. *Lecture Notes in Computer Sciences*, volume 3410, pages 191–205. Springer Verlag, Berlin, Germany, 2005.

2009: Sophie N. Parragh, Karl F. Doerner, Richard F. Hartl, Xavier Gandibleux. A heuristic two-phase solution approach for the multi-objective dial-a-ride problem. *Networks*. 54: 227-242. 2009.

2018: Ana D. López-Sánchez, Alfredo G. Hernández-Díaz, Francisco Gortázar, and Miguel Hinojosa, A multiobjective GRASP–VND algorithm to solve the waste collection problem. *International Transactions in Operational Research*. 25(2): 545-567. 2018.

2020: Takfarinas Saber, Xavier Gandibleux, Michael O'Neill, Liam Murphy, Anthony Ventresque. A comparative study of multi-objective machine reassignment algorithms for data centres. *Journal of Heuristics*. 26(1): 119-150, 2020.

2021: Ana. D. López-Sánchez, Jesus. Sánchez-Oro, Manuel Laguna. A New Scatter Search Design for Multiobjective Combinatorial Optimization with an Application to Facility Location. *INFORMS Journal on Computing* 33(2):629-642. 2021.

2022: Yu Xue, Xu Cai, Ferrante Neri, A multi-objective evolutionary algorithm with interval based initialization and self-adaptive crossover operator for large-scale feature selection in classification, *Applied Soft Computing*, Volume 127, 2022.

Topic 3

Topic 3

ReactiveGRASP: component to automatize the tuning of α

Marcelo Prais, Celso C. Ribeiro. Reactive GRASP:An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS Journal on Computing*. 12, 164-176. 2000.

- n classes of probabilities, each class is associated to one value α
- N_α iterations before to update the probabilities

Xavier Delorme, Xavier Gandibleux, Joaquin Rodriguez. GRASP for set packing problems. *European Journal of Operational Research*. Volume 153, Issue 3, 2004.

Refactoring my material for my lecture about GRASP...

...elaborated in 2019 a script drawing the evolution of probabilities...

ReactiveGRASP: component to automatize the tuning of α

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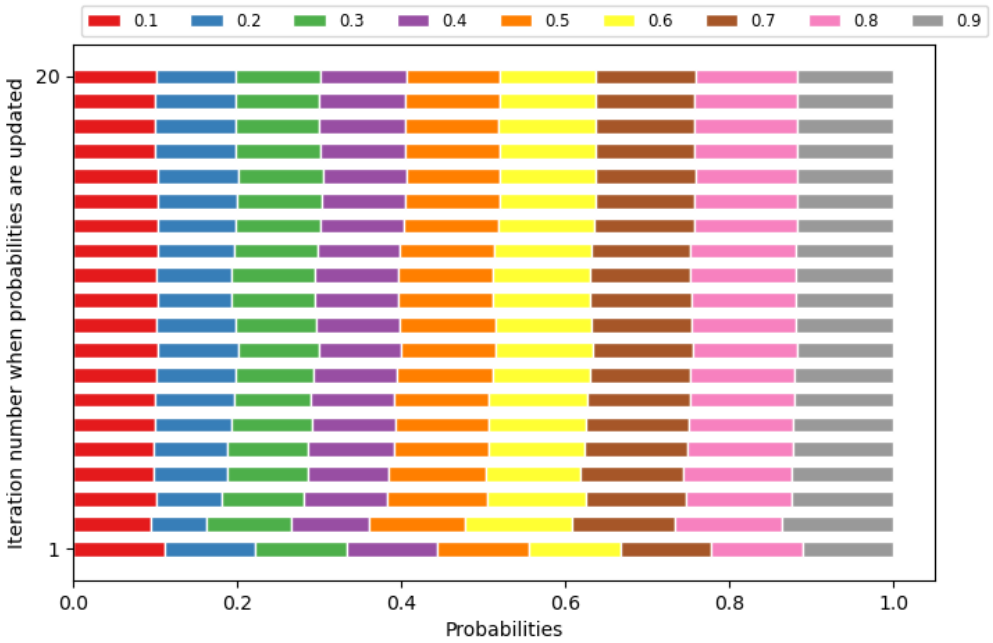
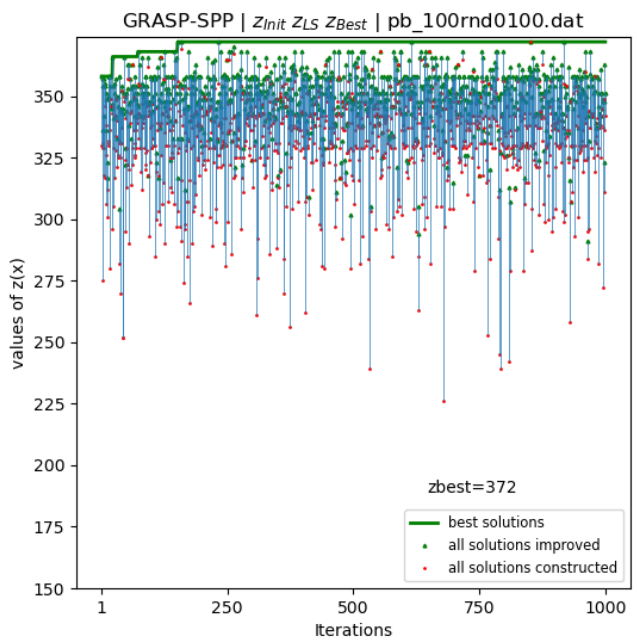


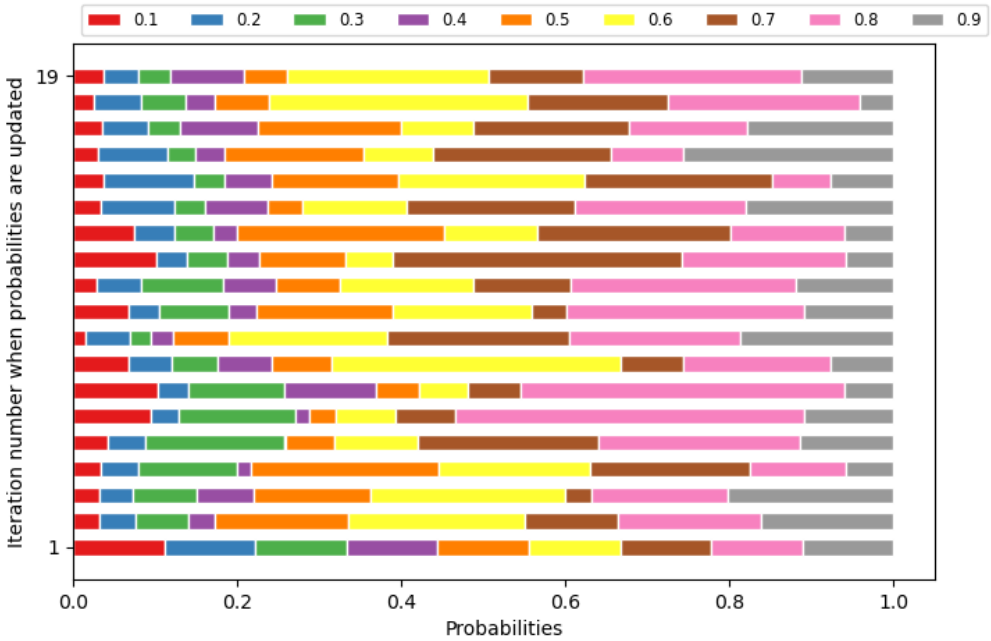


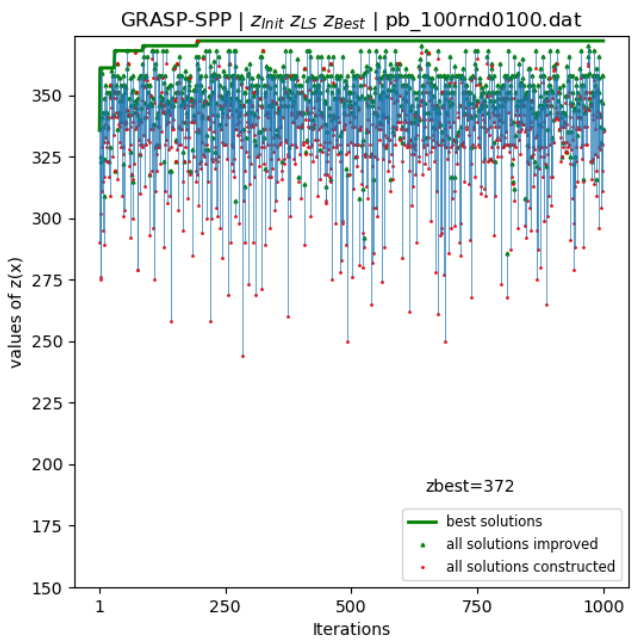
- no literature examining in detail the "Reactive" component (private communication with Celso Ribeiro)
- same on 1-StPP, 1-TSP (Athanael Jousselin, Axel Verneuil, 2024)

Topic 3

Tentative 1:

- 2019: Amplify the changes on probabilities: tested on 1-WSPP





Topic 3

Tentative 2:

- 2024: tentative 1 + freezing iteratively one class (i.e. $p_i \geqslant p_i^{min}$):
  ↳ ReactiveGRASP strengthened, tested on 1-WSPP

Sketch:

◦ parameters:
- #classes + value α associated to each class
- #iterations GRASP to perform (init, search)

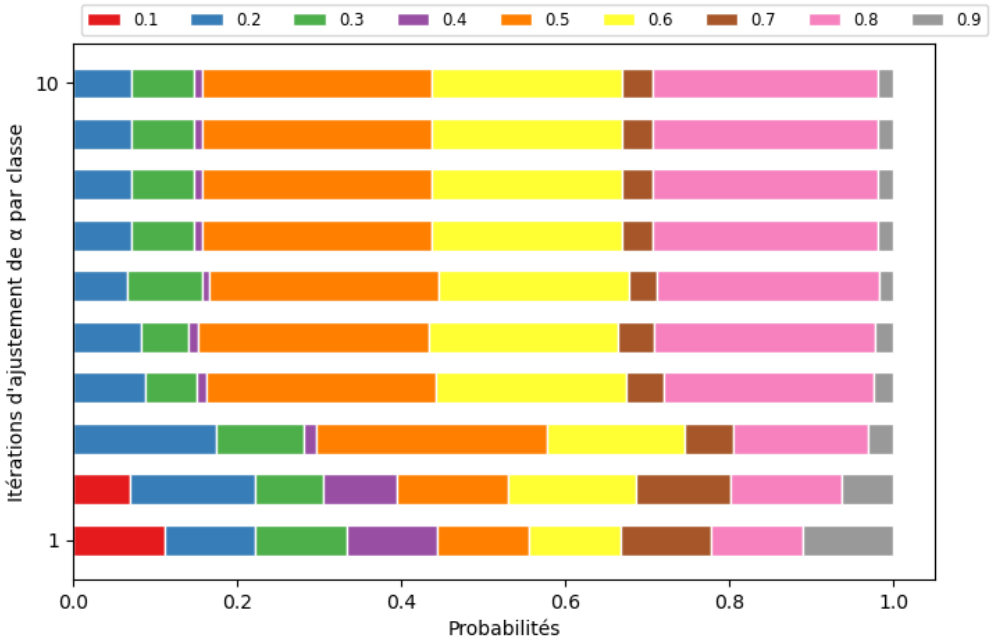
◦ steps:
  ReactiveGRASP (Prais, Ribeiro 2000) +
1. initialize each class, set N_α according #classes and #iterations
2. acceptance threshold for updating the classes; automatically maintained
3. amplification mechanism; distort probabilities for valuable classes visited
4. freezing mechanism; prevent the probability decrease of a good class

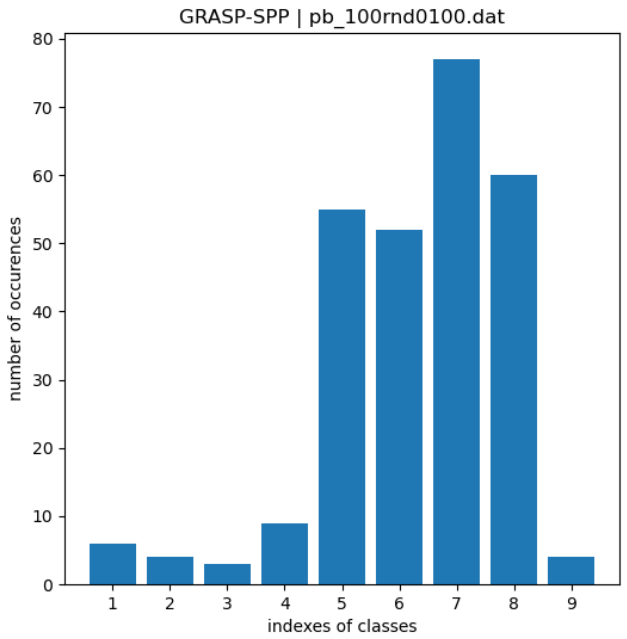
$\Rightarrow$ after N_α iterations, at most 1 class is frozen

Topic 3

pb_100rnd_0100 (100 variables; 500 constraints; density: 2%; max1: 2)
z=372 (optimal)

1000 iterations:





1 run

30 runs

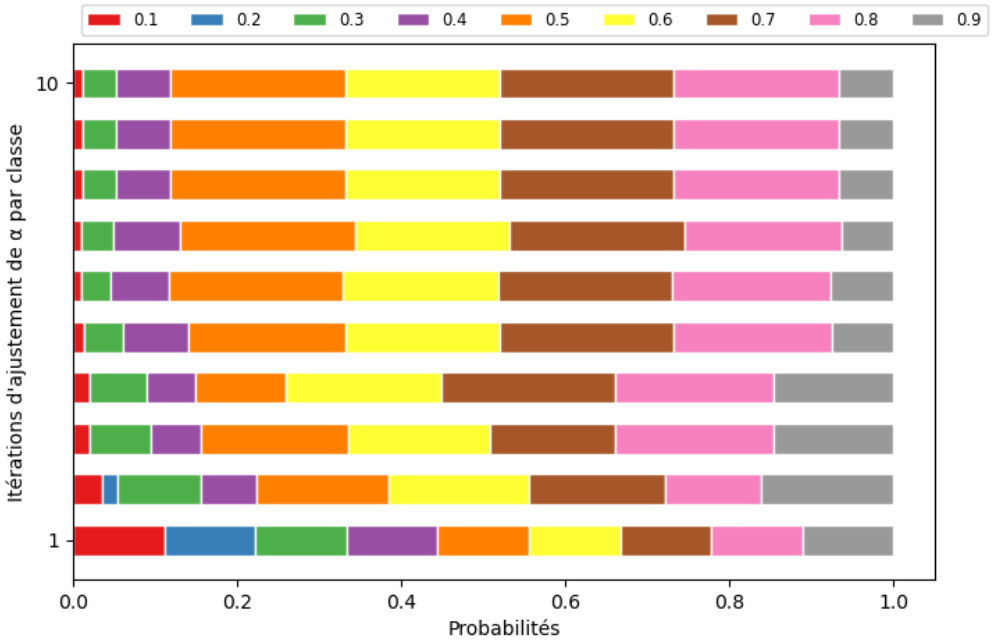
z: [372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372, 372]

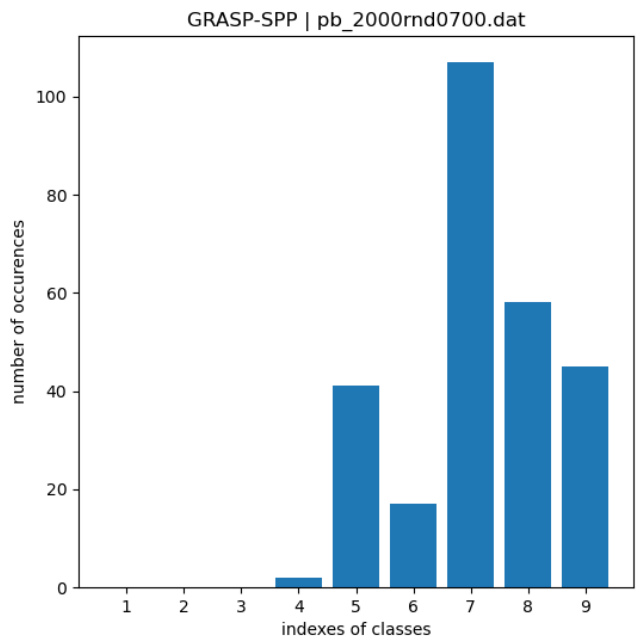
Δ(%): [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]

Topic 3

pb_2000rnd_0700 (2000 vars; 2000 consts; density: 0.56%; max1: 20)
z=1811 (best)

1000 iterations:





1 run

30 runs

z: [1703, 1693, 1708, 1692, 1687, 1691, 1713, 1697, 1698, 1689, 1726, 1703, 1693, 1700, 1702, 1686, 1677, 1683, 1689, 1704, 1688, 1689, 1694, 1698, 1689, 1691, 1697, 1700, 1695, 1691]

Δ(%): [6.342, 6.97, 6.03, 7.033, 7.35, 7.096, 5.721, 6.718, 6.655, 7.223, 4.925, 6.342, 6.97, 6.529, 6.404, 7.414, 7.99, 7.605, 7.223, 6.279, 7.287, 7.223, 6.907, 6.655, 7.223, 7.096, 6.718, 6.529, 6.844, 7.096]

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Nantes Université – France

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Xavier Gandibleux. Peek - Shape - Grab: A Methodology in Three Stages for Approximating the Non-dominated Points of Multiobjective Discrete/Combinatorial Optimization Problems with a Multiobjective Metaheuristic. *Lecture Notes in Computer Science*, vol 10173, 221-235. 2017.