



中国地理信息产业协会

CHINA ASSOCIATION FOR GEOSPATIAL INDUSTRY AND SCIENCES

大规模三维重建中的增量式运动平均问题

Incremental Motion Averaging in Large-Scale 3D Reconstruction

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研究背景

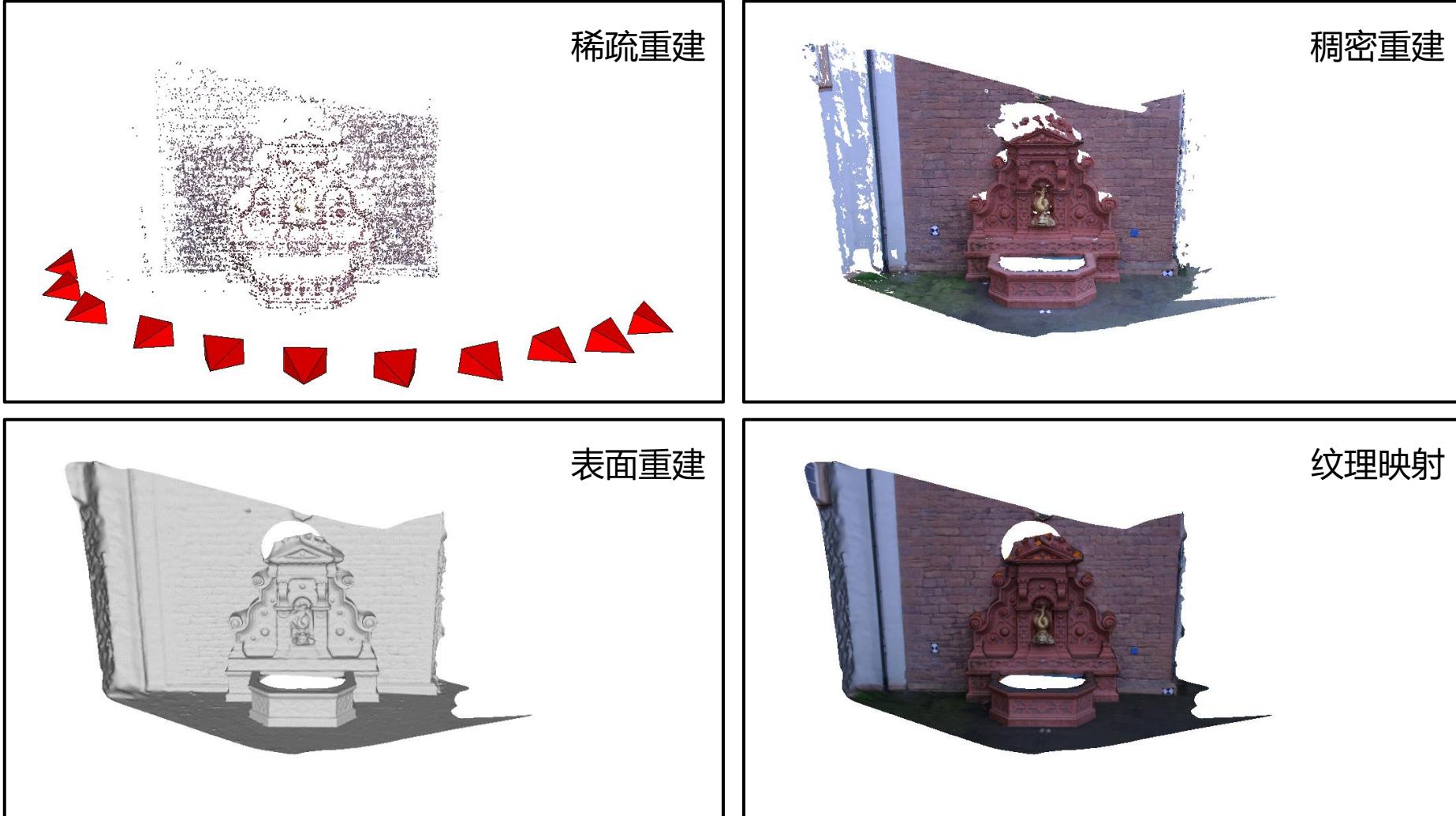
- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM: 输入图像集合, 输出**相机绝对位姿与场景稀疏表达**



Snavely et al. PIEEE 2010

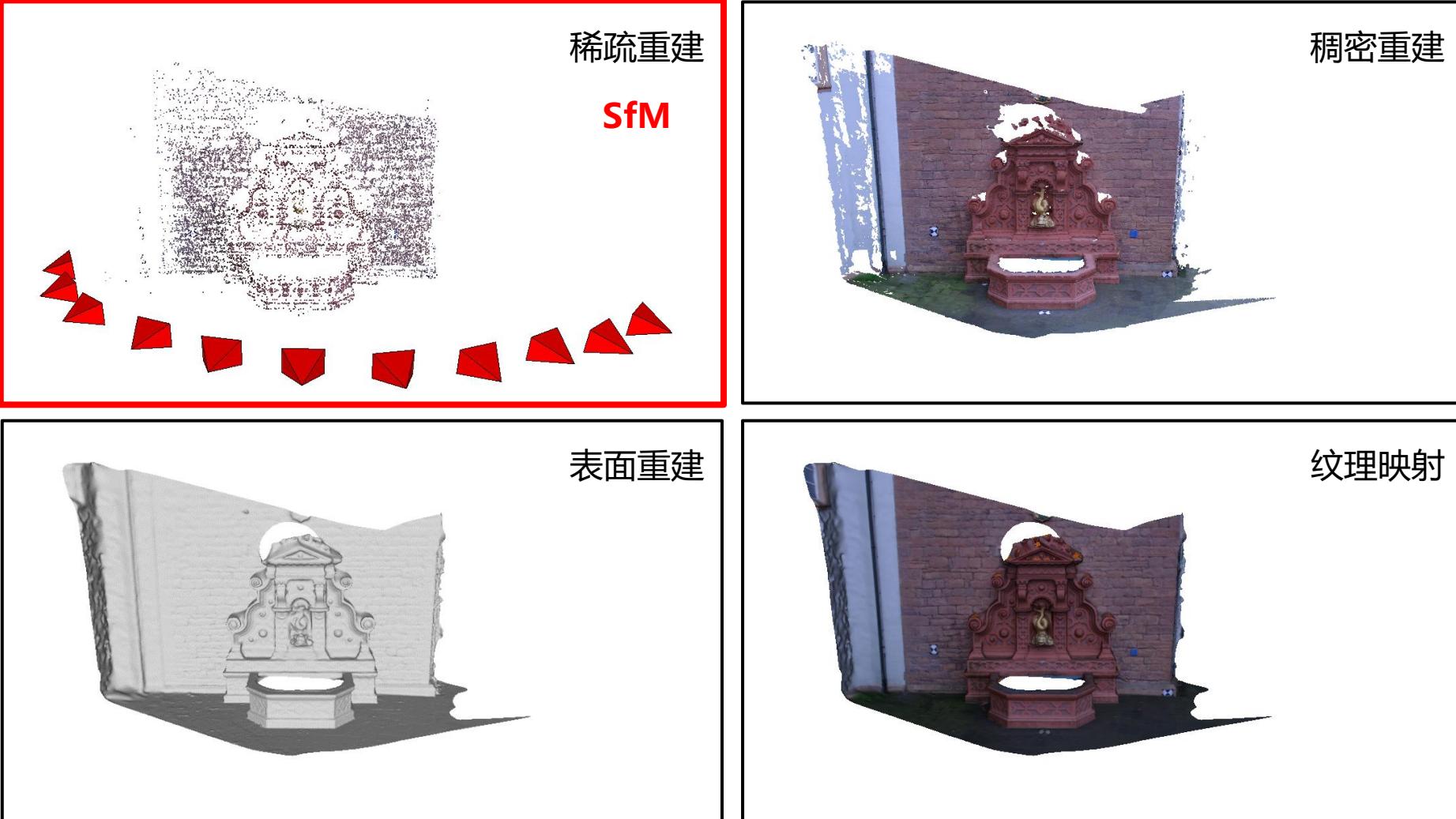
研究背景

- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM是基于图像的建模 (Image-Based Modeling) 流程中的核心步骤



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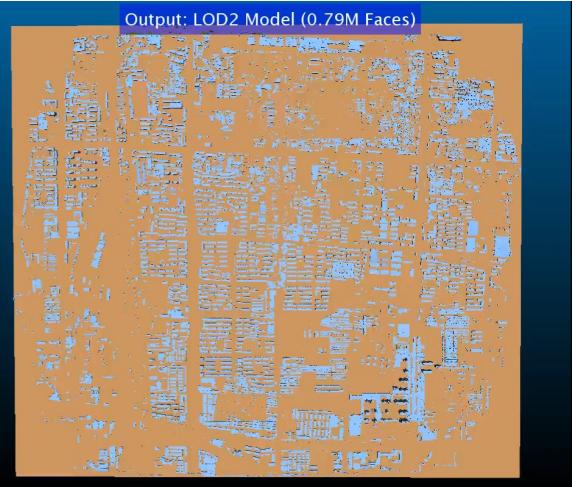
- 从运动恢复结构 (Structure from Motion, SfM)
 - SfM在室内建模、遥感测绘、古建保护、增强现实等领域有着重要的应用



Han et al. ISPRS P&RS 2021



Zhu et al. In Proc. ECCV 2018



Gao et al. ISPRS P&RS 2018



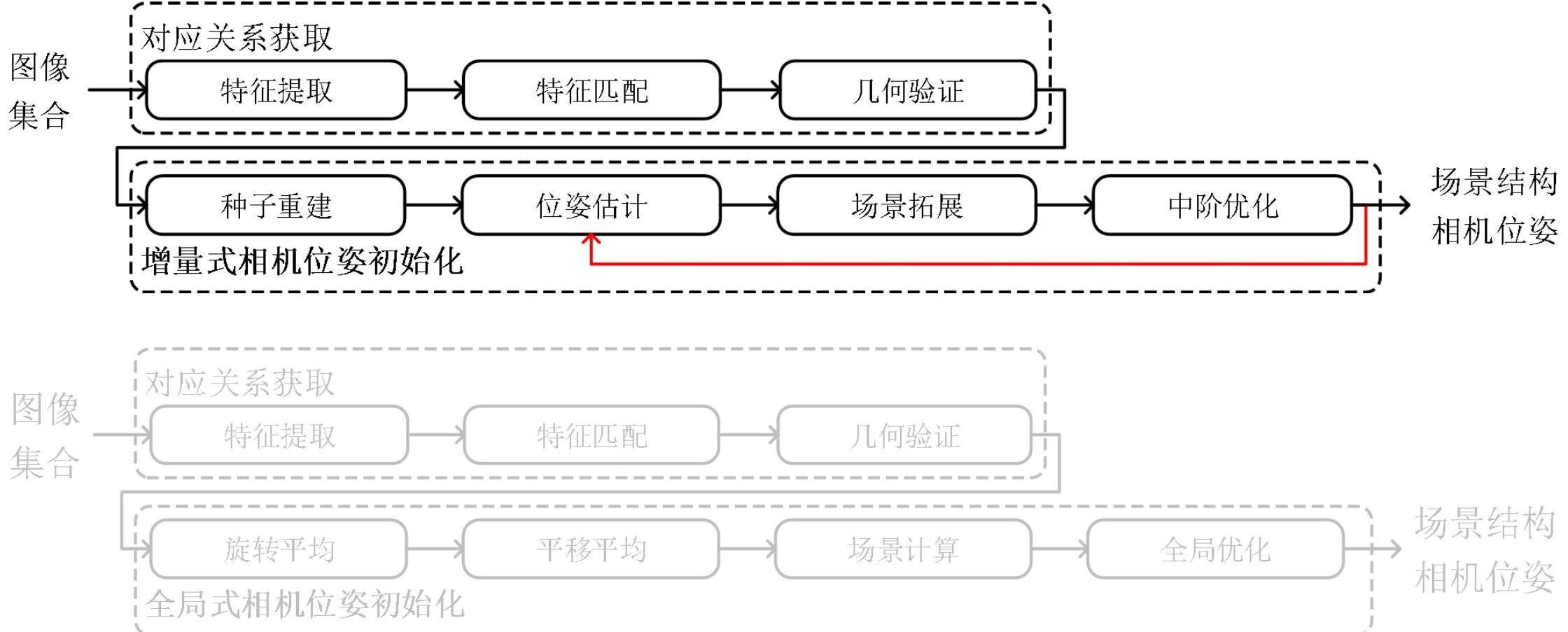
Sun et al. In Proc. CVPR 2021



研究背景

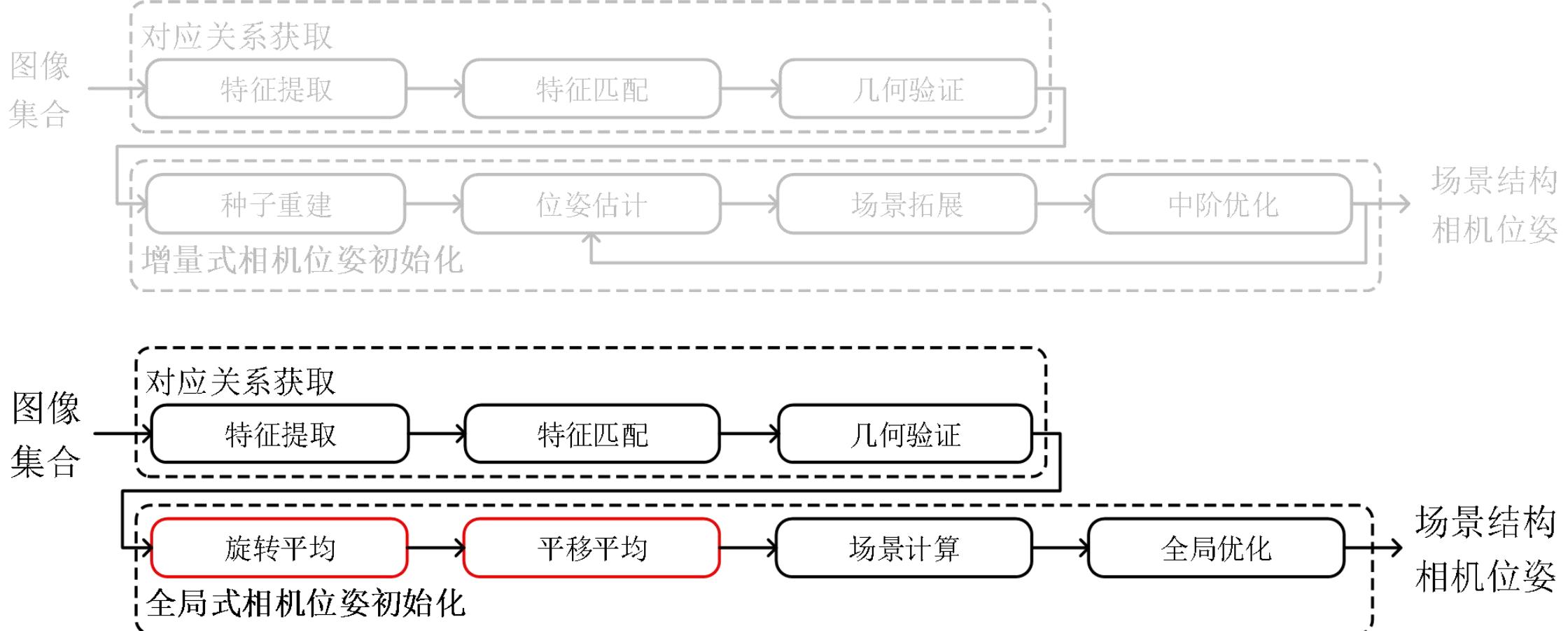
- 从运动恢复结构 (Structure from Motion, SfM)

- 根据相机位姿初始化方式，SfM可分为**增量式SfM**与**全局式SfM**两类



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研究背景

- 增量式SfM vs. 全局式SfM
 - 增量式SfM
 - ◆ 精度更高、鲁棒性更强、
 - ◆ 效率较低、场景漂移、
 - 全局式SfM
 - ◆ 效率更高、一致性更好、
 - ◆ 精度较差、场景缺失、

- **运动平均 (Motion Averaging)**

- 运动平均包括旋转平均 (Rotation Averaging, RA) 与平移平均 (Translation Averaging, TA) ,
是全局式SfM中的核心步骤
- RA: 给定相机之间的相对旋转 $\{\mathbf{R}_{i,j} | e_{i,j} \in \mathcal{E}\}$, 求解在同一基准下所有相机的绝对旋转 $\{\mathbf{R}_i | v_i \in \mathcal{V}\}$

$$\{\mathbf{R}_i^*\} = \arg \min \sum_{\substack{e_{i,j} \in \mathcal{E} \\ v_i, v_j \in \mathcal{V}}} \rho(d_{\mathbf{R}}(\mathbf{R}_{i,j}, \mathbf{R}_j \mathbf{R}_i^\top))$$

研究背景

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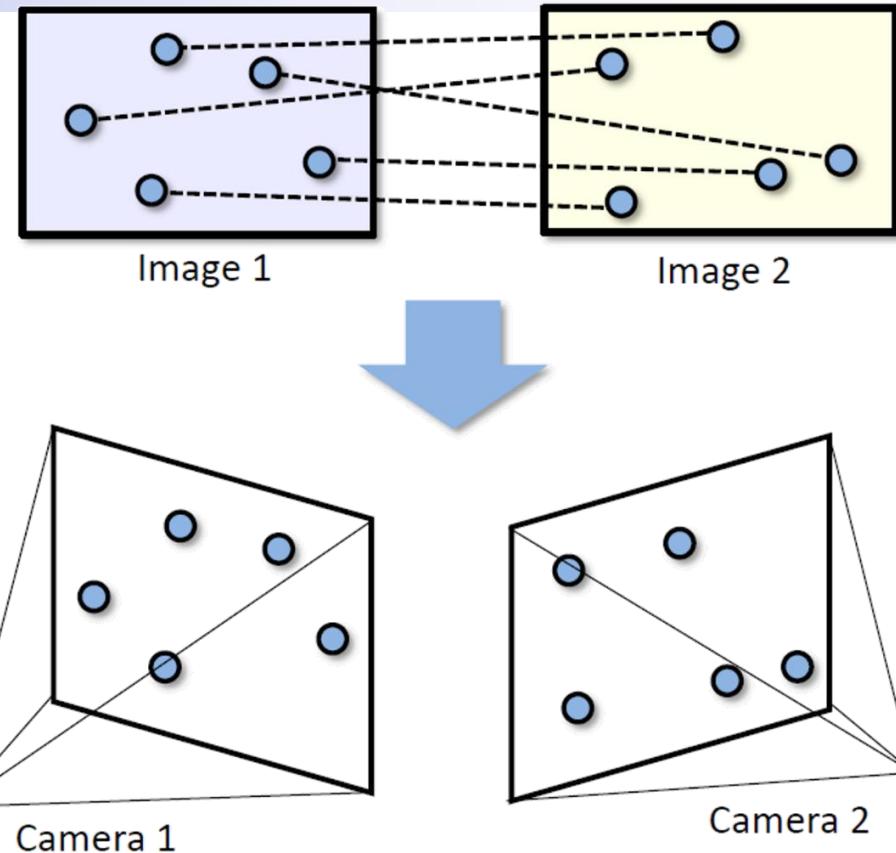
$$\{R_i^*\} = \arg \min \sum_{\substack{e_{i,j} \in \mathcal{E} \\ v_i, v_j \in \mathcal{V}}} \rho(d_R(R_{i,j}, R_j R_i^\top))$$

- TA: 给定相机之间的相对平移 $\{t_{i,j} | e_{i,j} \in \mathcal{E}\}$, 求解在同一基准下所有相机的绝对位置 $\{c_i | v_i \in \mathcal{V}\}$

$$\{c_i^*\} = \arg \min \sum_{\substack{e_{i,j} \in \mathcal{E} \\ v_i, v_j \in \mathcal{V}}} \rho \left(d_t \left(t_{i,j}, R_j \frac{c_i - c_j}{\|c_i - c_j\|_2} \right) \right)$$

研究背景

- 运动平均 (Motion Averaging, MA)
 - 相对运动 (旋转、平移) $\{R_{i,j}, t_{i,j} | e_{i,j} \in \mathcal{E}\}$ 的获取方式:
 - 局部特征的提取与匹配
 - 本质矩阵的估计与分解



研究背景

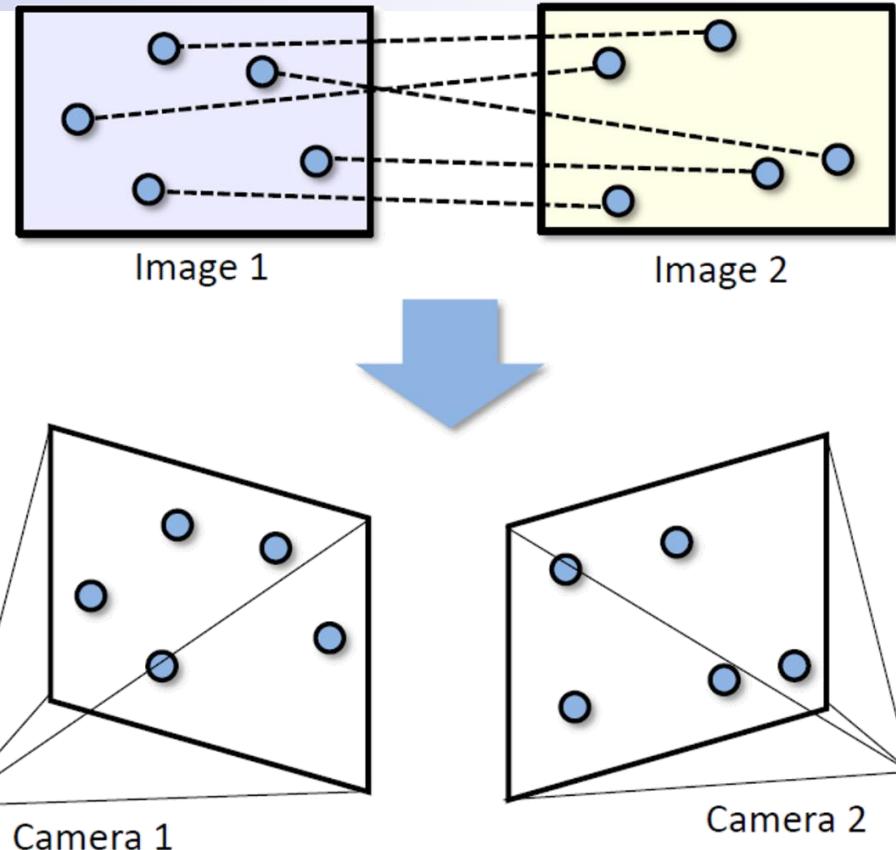
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- 局部特征的提取与匹配
 - 本质矩阵的估计与分解

- 相对于RA, TA问题更加困难, 主要体现在

- 通过本质矩阵分解得到的相对平移具有**尺度不确定性**^[1]
 - 相对平移估计精度比旋转更容易受到**特征误匹配**影响^[1]
 - 平移平均的问题**可解性**对外极几何图有着更高的要求^[2]



[1] D. Nister. An Efficient Solution to the Five-Point Relative Pose Problem[J]. *IEEE T-PAMI*, 2004.

[2] O. Ozyesil and A. Singer. Robust Camera Location Estimation by Convex[C]. *In Proc. CVPR*, 2015.

研究背景

• 运动平均 (Motion Averaging)

- 运动平均问题的主要难点在于如何利用存在测量外值的相对运动精确估计相机的绝对位姿
- 现有的运动平均方法主要包括两类：
 - 鲁棒优化方法：RA^[1-2]、TA^[3-4]，形式复杂、效率欠佳
 - 外值滤除方法：RA^[5-6]、TA^[7-8]，chicken-and-egg problem

- [1] A. Chatterjee and V. M. Govindu. Robust Relative Rotation Averaging[J]. *IEEE T-PAMI*, 2018.
- [2] Y. Shi and G. Lerman. Message Passing Least Squares Framework and its Application to Rotation Synchronization[C]. *In Proc. ICML*, 2020.
- [3] T. Goldstein, P. Hand, C. Lee, V. Voroninski, and S. Soatto. ShapeFit and ShapeKick for Robust, Scalable Structure from Motion[C]. *In Proc. ECCV*, 2016.
- [4] B. Zhuang, L. Cheong, and G. H. Lee. Baseline Desensitizing in Translation Averaging[C]. *In Proc. CVPR*, 2018.
- [5] X. Gao, J. Luo, K. Li, and Z. Xie. Hierarchical RANSAC-Based Rotation Averaging[J]. *IEEE SPL*, 2020.
- [6] S. H. Lee and J. Civera. HARA: A Hierarchical Approach for Robust Rotation Averaging[C]. *In Proc. CVPR*, 2022.
- [7] K. Wilson and N. Snavely. Robust Global Translations with 1DSfM[C]. *In Proc. ECCV*, 2014.
- [8] C. Sweeney, T. Sattler, T. Höllerer, T. M. Turk, and M. Pollefeys. Optimizing the Viewing Graph for Structure-from-Motion[C]. *In Proc. ICCV*, 2015.

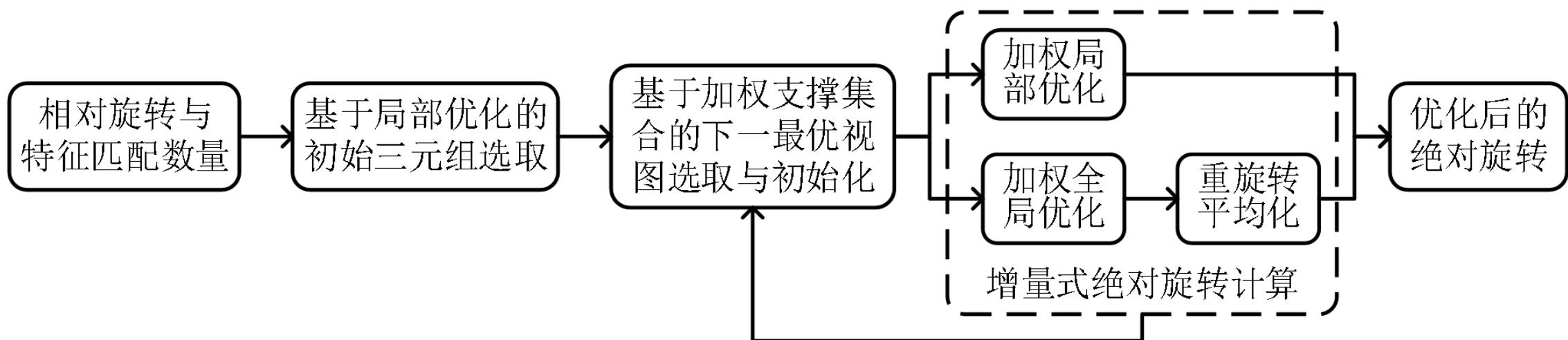
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- 针对上述问题，我们在大规模运动平均的鲁棒性问题研究方面开展了一些初步探索：
 - 可在滤除相对运动测量外值同时估计相机的绝对位姿
 - 以期实现简单、高效、精确、鲁棒的**大规模**运动平均

旋转平均 (Rotation Averaging, RA)

- 增量式旋转平均 (Incremental Rotation Averaging, IRA)

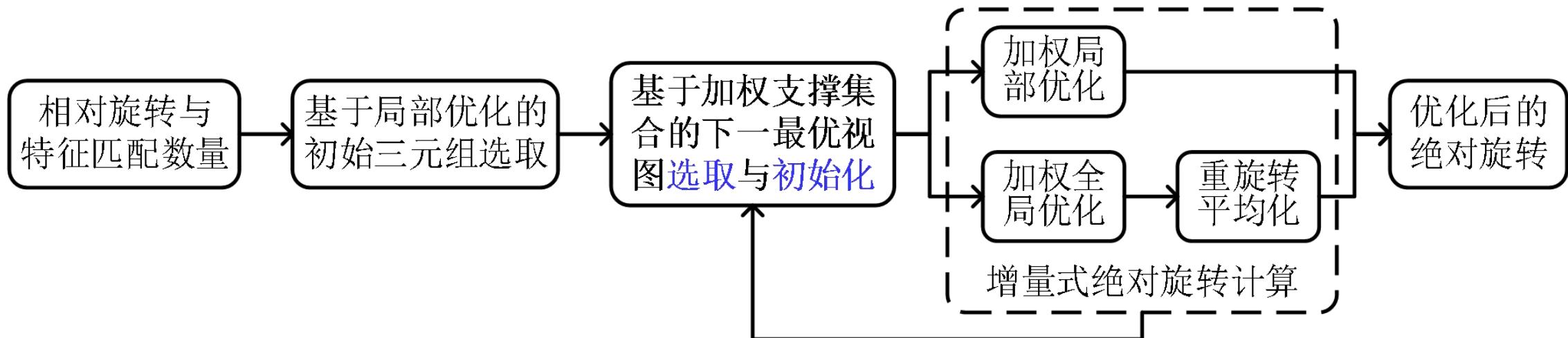
- IRA采用与增量式SfM类似的增量式参数估计流程
 - 更加精确、鲁棒
 - 旋转平均问题相对于SfM问题待估计的参数量更少
 - 更加简单、高效



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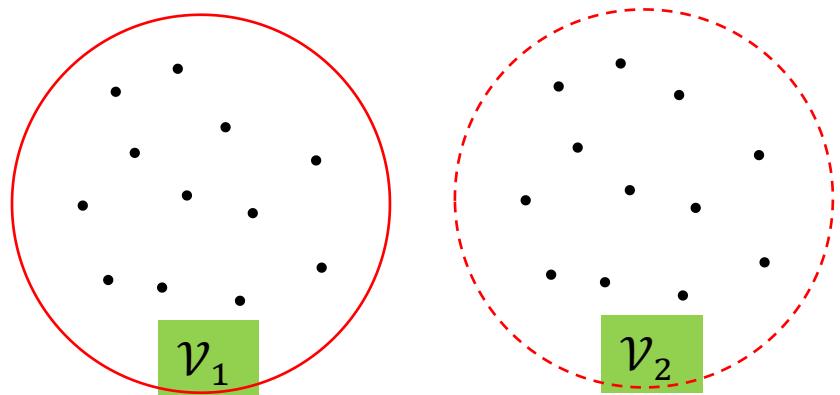
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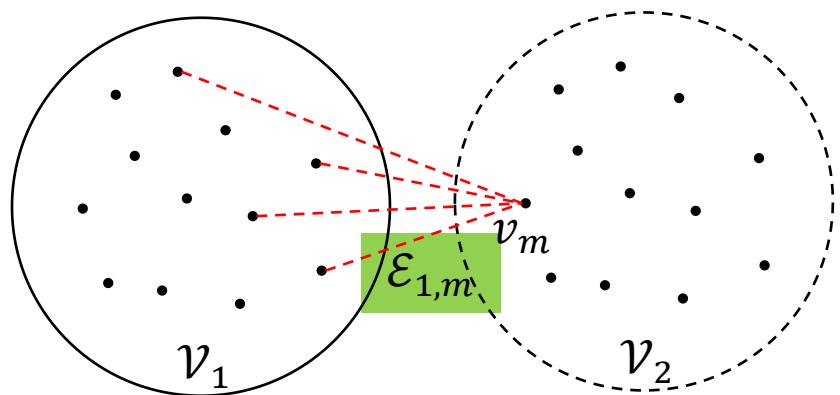
- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合



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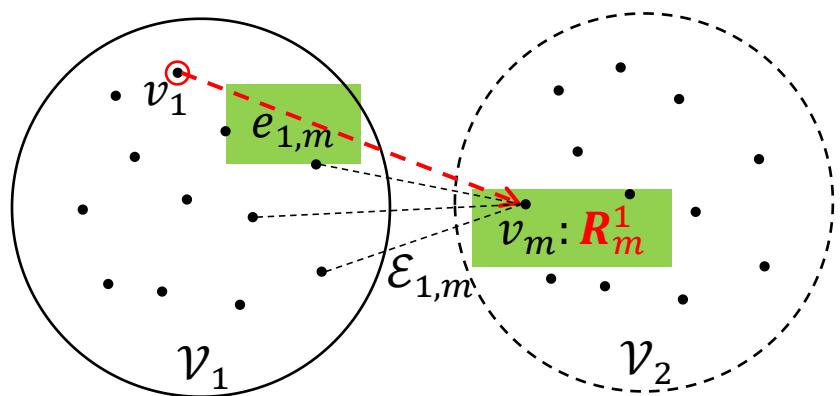
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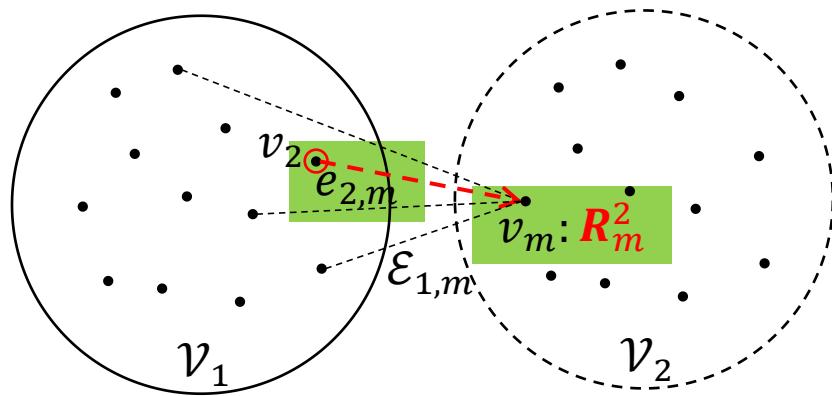
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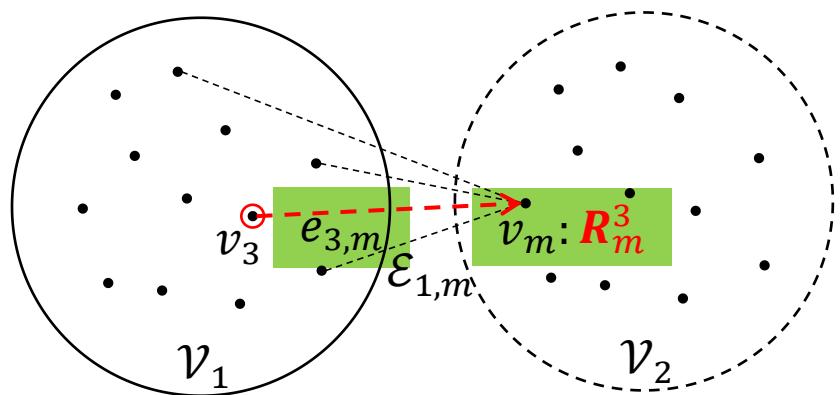
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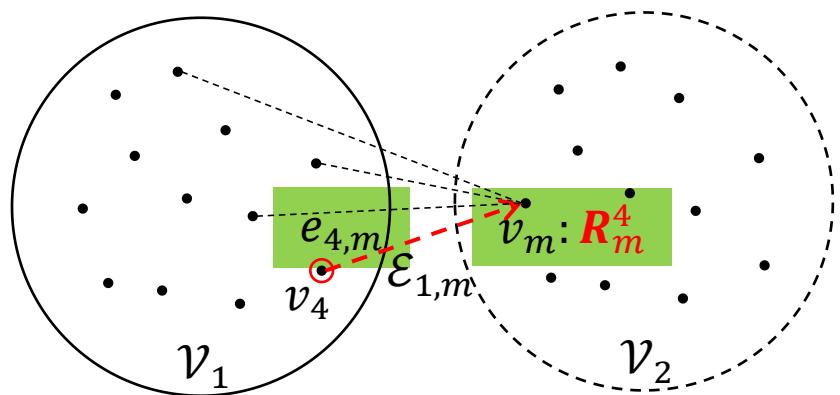
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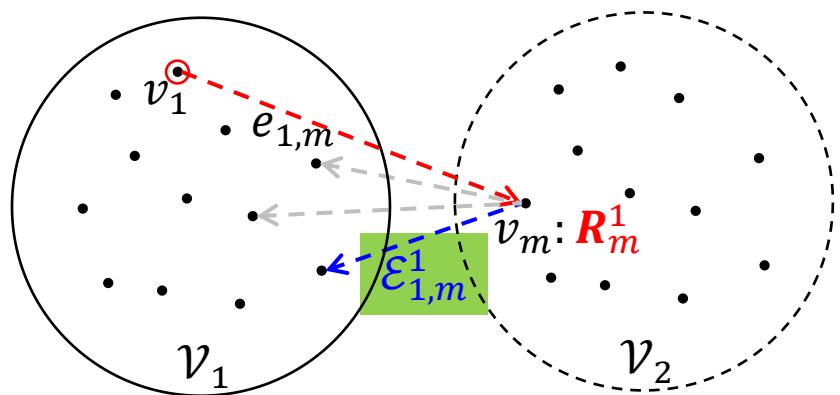
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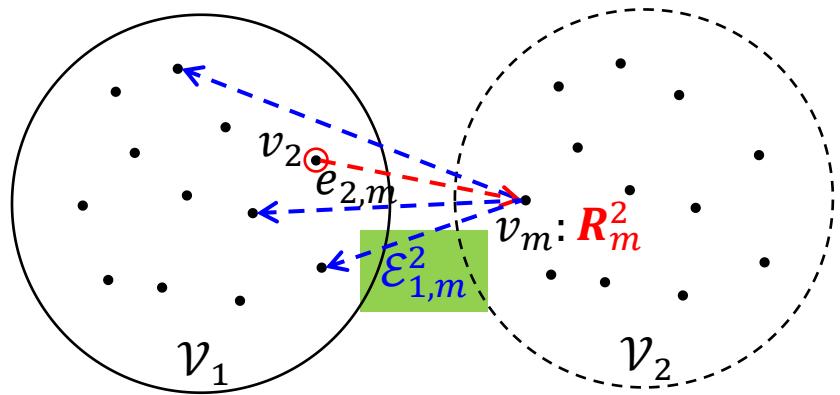
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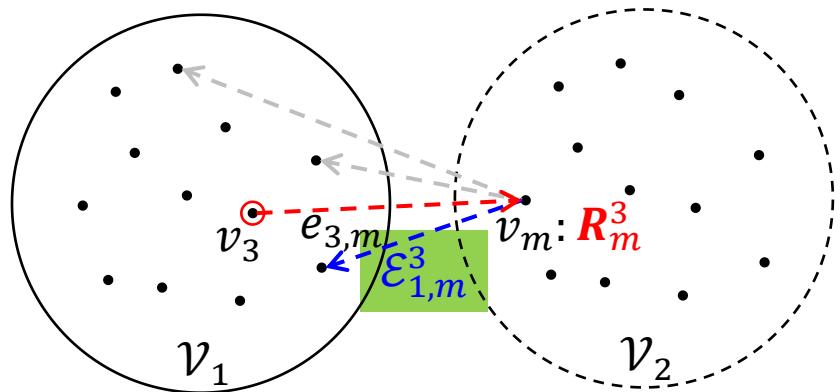
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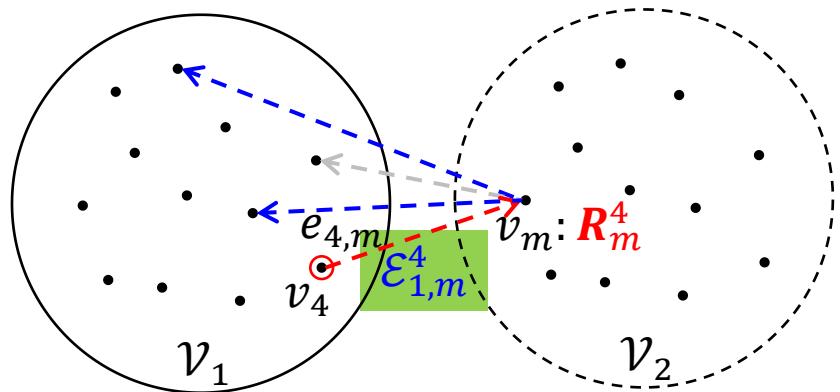
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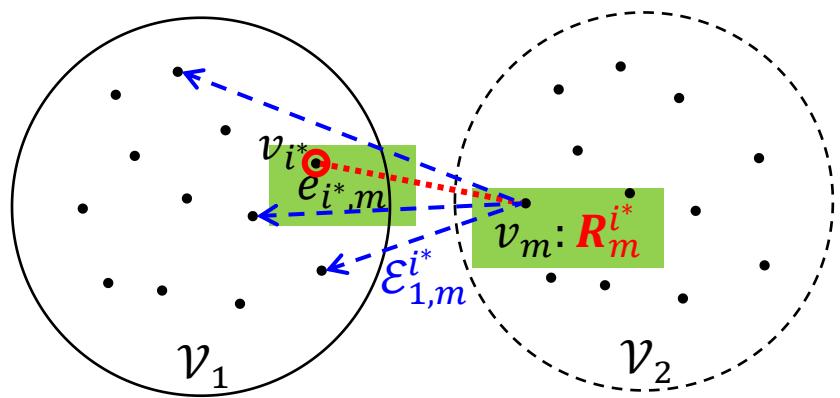
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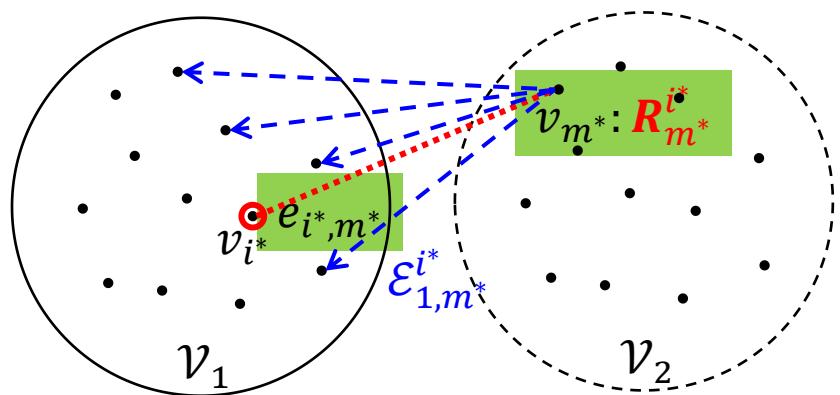
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- 最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$ 对应的 $R_m^{i^*}$ 即为 v_m 的初始化结果



旋转平均 (Rotation Averaging, RA)

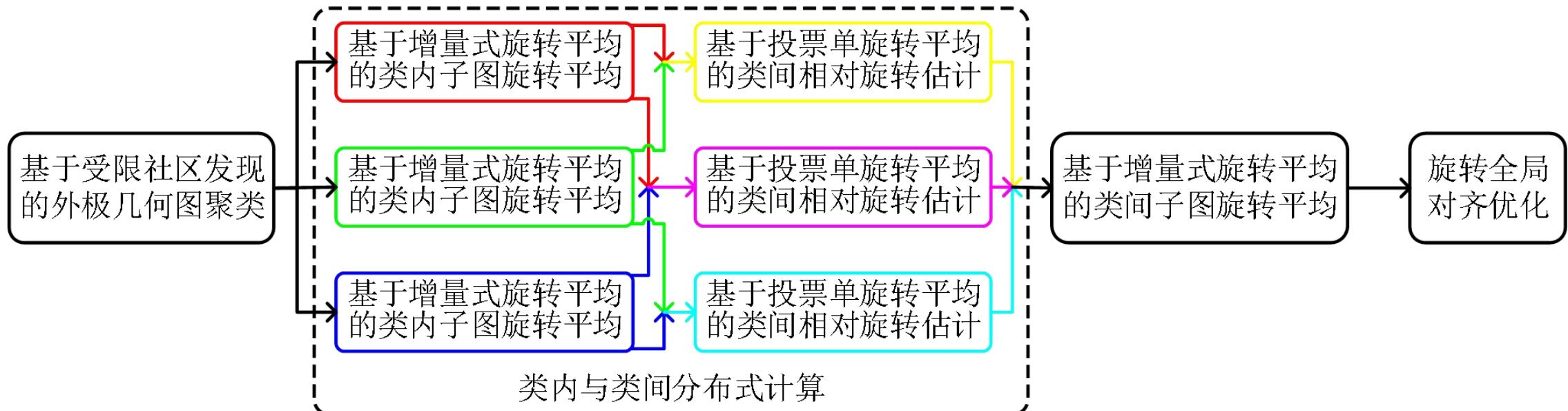
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- 最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$ 对应的 $R_m^{i^*}$ 即为 v_m 的初始化结果
- 对 \mathcal{V}_2 中各顶点重复上述过程并获取全局最大支撑边集 $\mathcal{E}_{1,m^*}^{i^*}$, 则 v_{m^*} 与 $R_{m^*}^{i^*}$ 即为 **NBV** 及其初始化结果



旋转平均 (Rotation Averaging, RA)

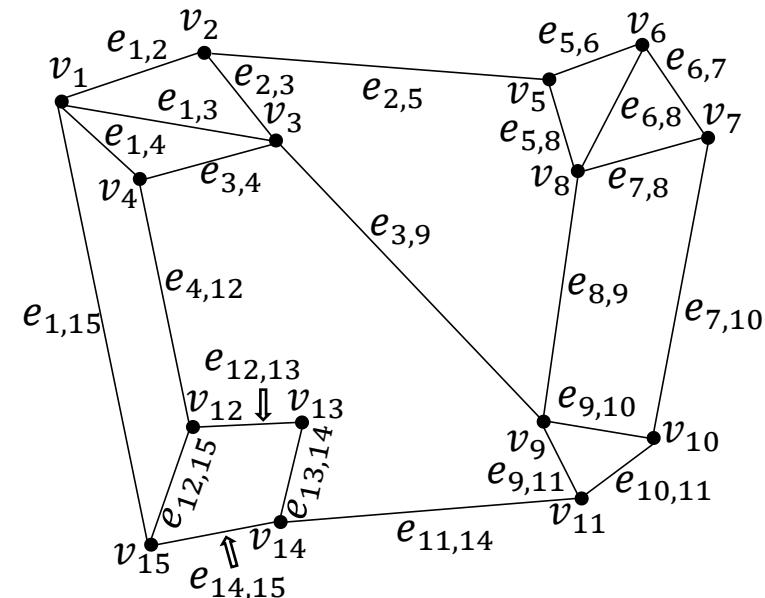
- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 解决IRA因其固有的增量式参数估计流程在面向大规模旋转平均时的累积误差与计算效率问题



旋转平均 (Rotation Averaging, RA)

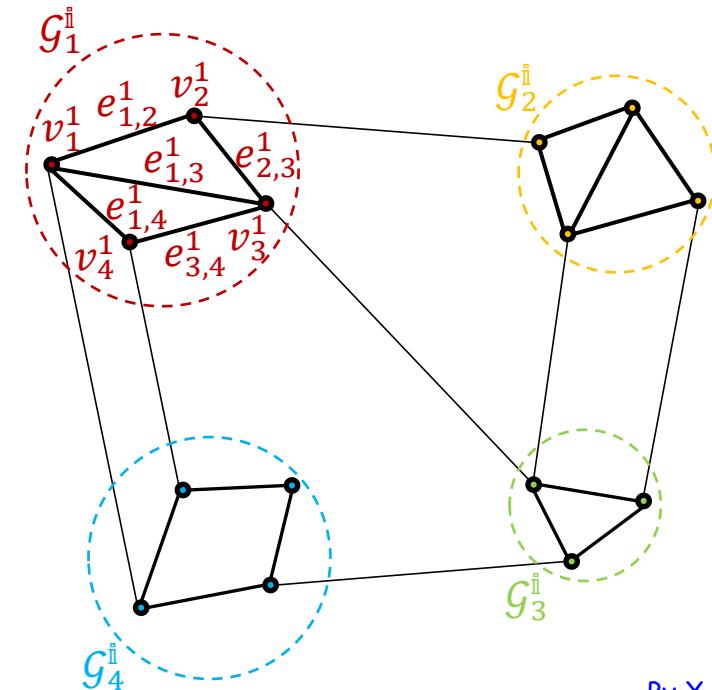
- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图:

$$\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}, \mathcal{V} = \{v_i\}, \mathcal{E} = \{e_{i,j}\}$$



旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)
 - 原始外极几何图
 - 基于社区发现的外极几何图聚类:
 - 类内子图: $\{G_p^{\text{II}} = \{\mathcal{V}_p^{\text{II}}, \mathcal{E}_p^{\text{II}}\}\}, \mathcal{V}_p^{\text{II}} = \{v_m^p\}, \mathcal{E}_p^{\text{II}} = \{e_{m,n}^p\}$

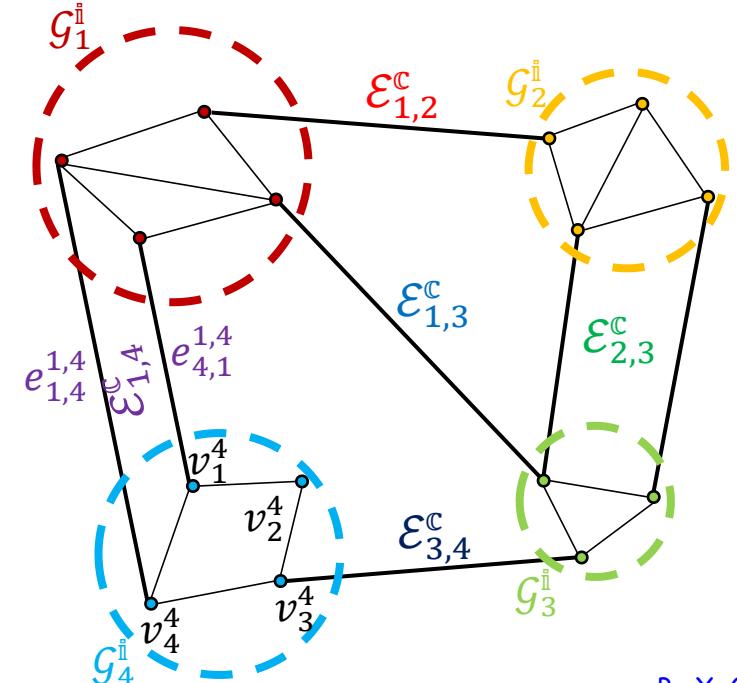


旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

- 原始外极几何图
- 基于社区发现的外极几何图聚类:

- 类内子图: $\{G_p^{\text{I}} = \{\mathcal{V}_p^{\text{I}}, \mathcal{E}_p^{\text{I}}\}\}, \mathcal{V}_p^{\text{I}} = \{v_m^p\}, \mathcal{E}_p^{\text{I}} = \{e_{m,n}^p\}$
- 类间子图: $G^{\text{C}} = \{\mathcal{V}^{\text{C}}, \mathcal{E}^{\text{C}}\}, \mathcal{V}^{\text{C}} = \{G_p^{\text{I}}\}, \mathcal{E}^{\text{C}} = \{\mathcal{E}_{p,q}^{\text{C}}\} = \{\{e_{m,n}^{p,q}\}\}$

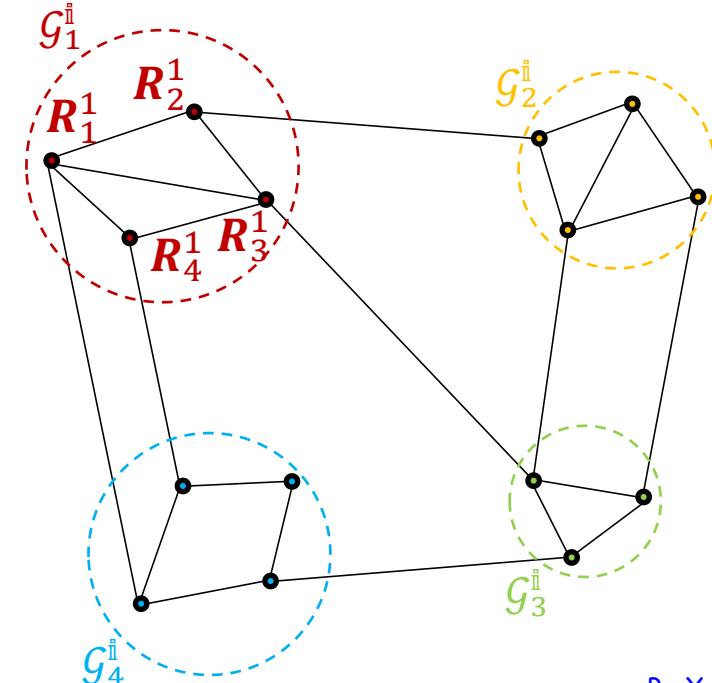


旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均：

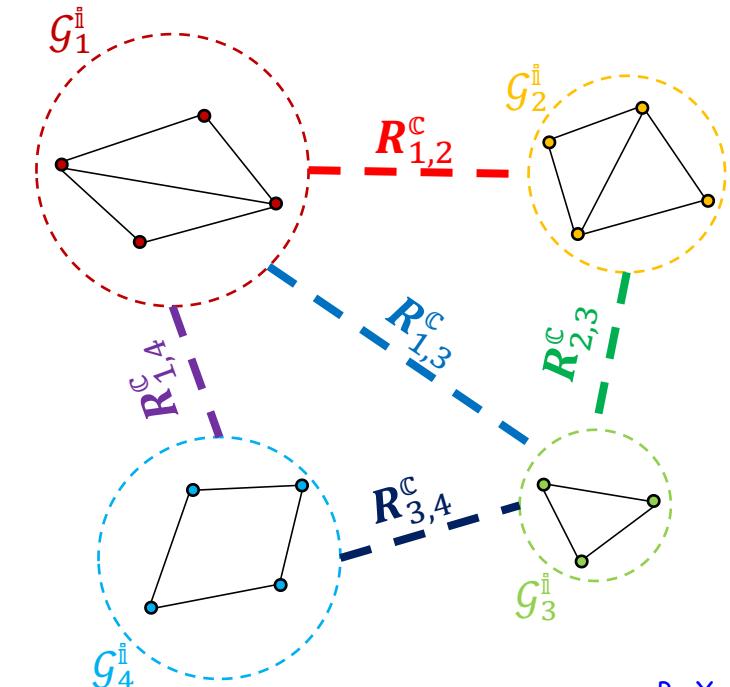
- $\{R_{m,n}^p | e_{m,n}^p \in \mathcal{E}_p^i\} \Rightarrow \{R_m^p | v_m^p \in \mathcal{V}_p^i\}$



旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

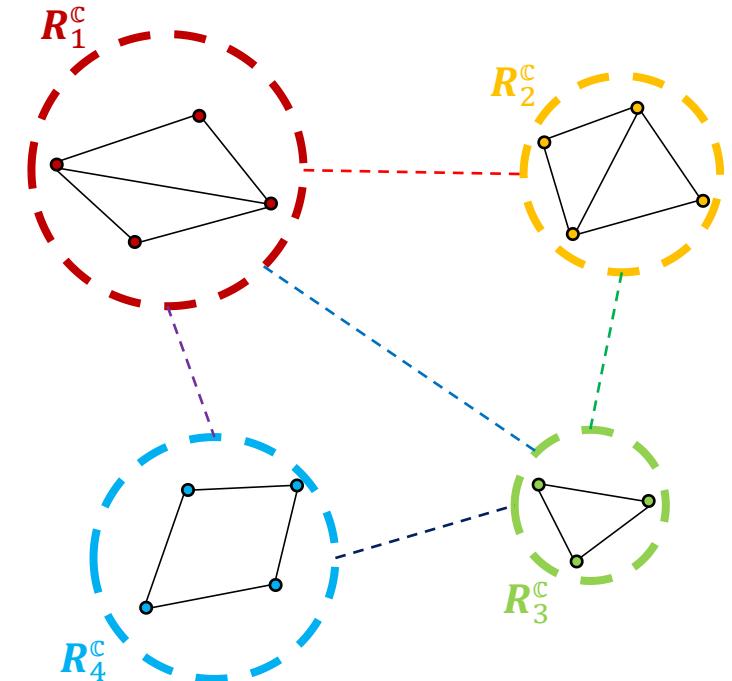
- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均
- 基于单旋转平均的类间相对旋转估计:
 - $\{R_{m,n}^{p,q} | e_{m,n}^{p,q} \in \mathcal{E}_{p,q}\} \Rightarrow \{R_p^c | \mathcal{E}_p^c \in \mathcal{E}^c\}$



旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

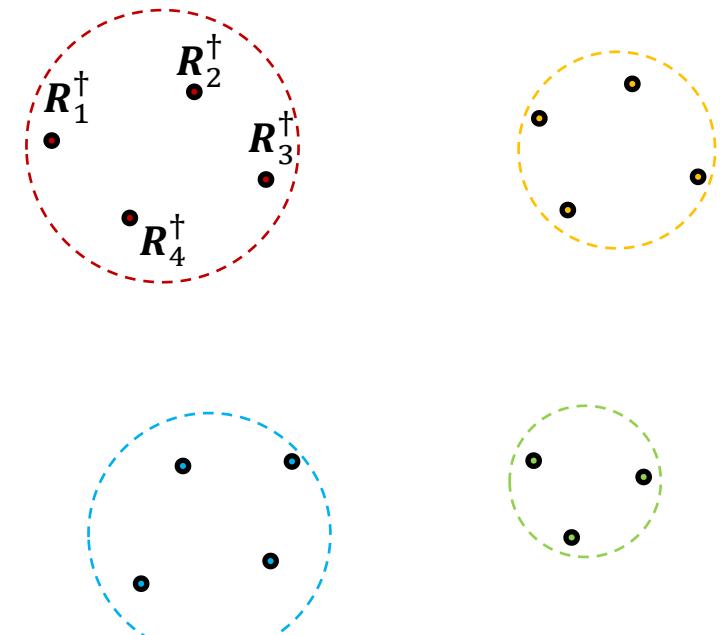
- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均
- 基于单旋转平均的类间相对旋转估计
- 基于IRA的类间旋转平均：
 - $\{R_{p,q}^c | \mathcal{E}_{p,q}^c \in \mathcal{E}^c\} \Rightarrow \{R_p^c | \mathcal{G}_p^i \in \mathcal{V}^c\}$



旋转平均 (Rotation Averaging, RA)

• 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

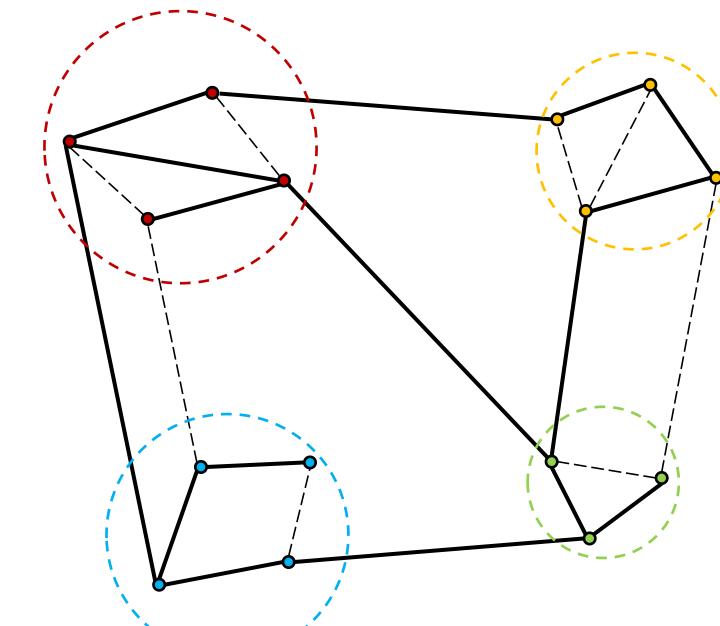
- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均
- 基于单旋转平均的类间相对旋转估计
- 基于IRA的类间旋转平均
- 绝对旋转全局对齐与优化：
 - 全局对齐： $R_i^\dagger = R_m^p R_p^c$



旋转平均 (Rotation Averaging, RA)

- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

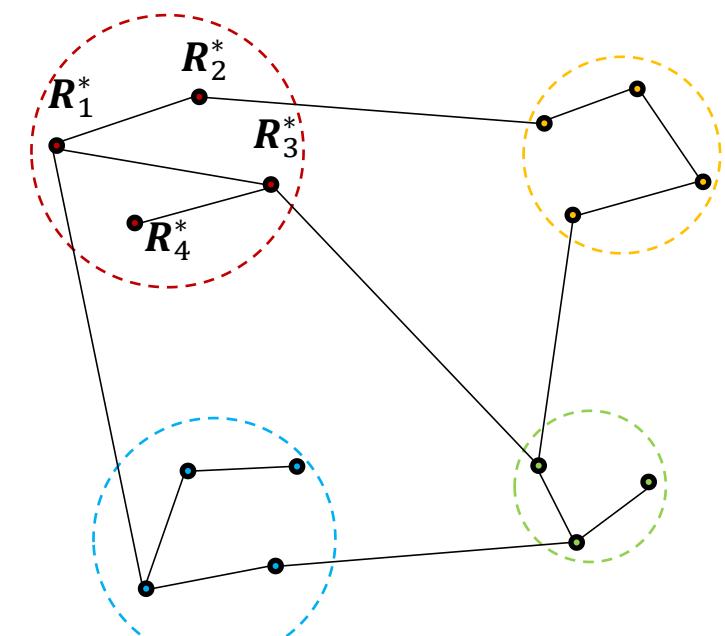
- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均
- 基于单旋转平均的类间相对旋转估计
- 基于IRA的类间旋转平均
- 绝对旋转全局对齐与优化：
 - 全局对齐： $\mathbf{R}_i^\dagger = \mathbf{R}_m^p \mathbf{R}_p^c$
 - 内值计算： $\mathcal{E}^I = \left\{ d_\theta^R \left(\mathbf{R}_{i,j}, \mathbf{R}_j^\dagger \mathbf{R}_i^{\dagger T} \right) < \theta_{th}^R \right\}$ for $v_i, v_j \in \mathcal{V}, e_{i,j} \in \mathcal{E}$



旋转平均 (Rotation Averaging, RA)

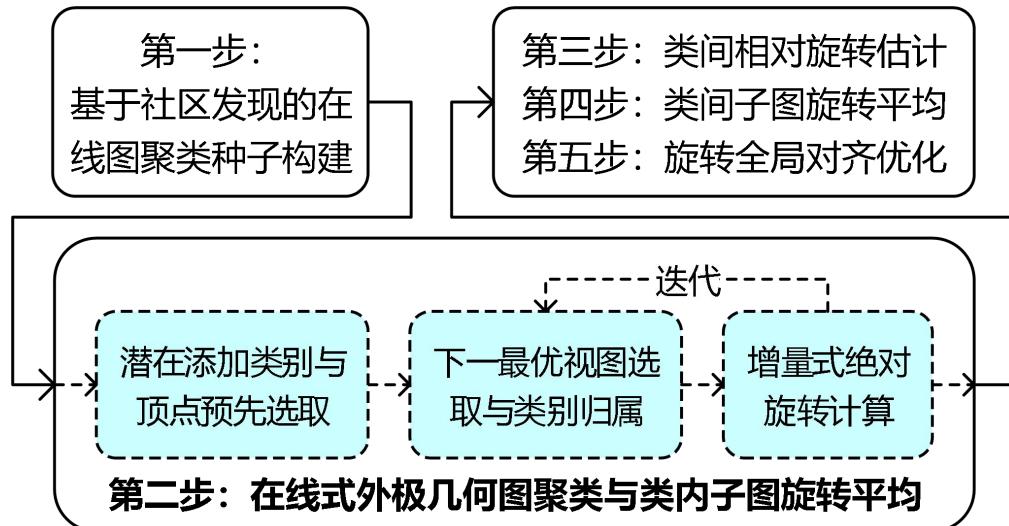
- 层级增量式旋转平均 (Hierarchical Incremental Rotation Averaging, IRA++)

- 原始外极几何图
- 基于社区发现的外极几何图聚类
- 基于IRA的类内旋转平均
- 基于单旋转平均的类间相对旋转估计
- 基于IRA的类间旋转平均
- 绝对旋转全局对齐与优化：
 - 全局对齐： $\mathbf{R}_i^\dagger = \mathbf{R}_m^p \mathbf{R}_p^c$
 - 内值计算： $\mathcal{E}^I = \left\{ d_\theta^R \left(\mathbf{R}_{i,j}, \mathbf{R}_j^\dagger \mathbf{R}_i^{\dagger T} \right) < \theta_{th}^R \right\}$ for $v_i, v_j \in \mathcal{V}, e_{i,j} \in \mathcal{E}$
 - 全局优化： $\{\mathbf{R}_i^*\} = \arg \min \sum_{v_i, v_j \in \mathcal{V}} \sum_{e_{i,j}^I \in \mathcal{E}^I} d_\theta^R(\mathbf{R}_{i,j}^I, \mathbf{R}_j \mathbf{R}_i^\top)^2$



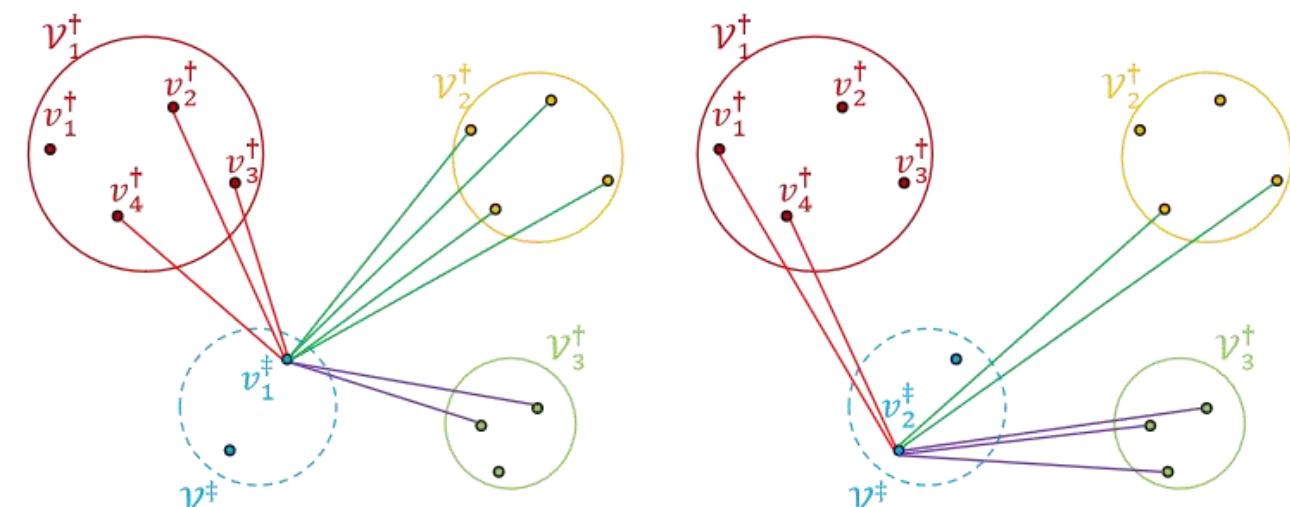
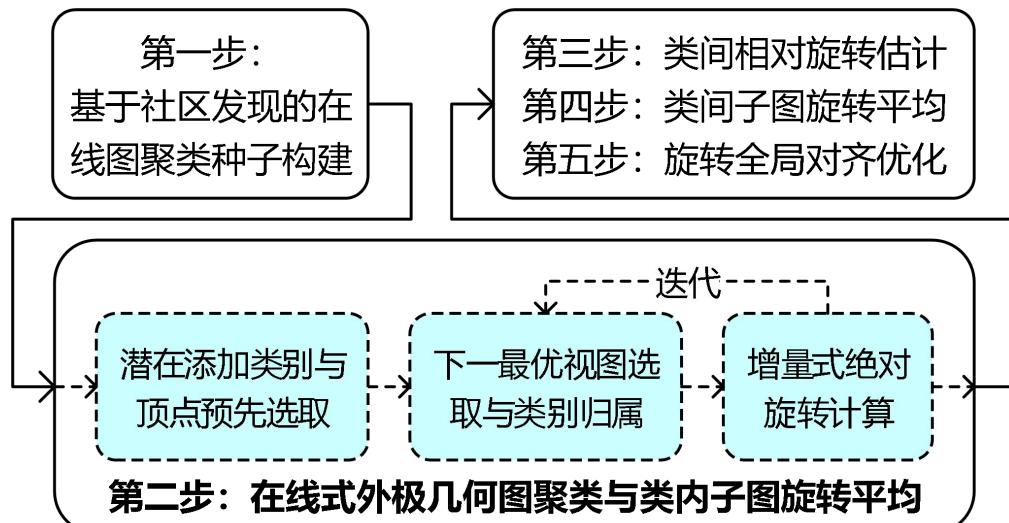
旋转平均 (Rotation Averaging, RA)

- 在线层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging on the Fly, IRAv3
 - 解决IRA++中基于社区发现的聚类方式所存在的**预先确定与任务无关**问题



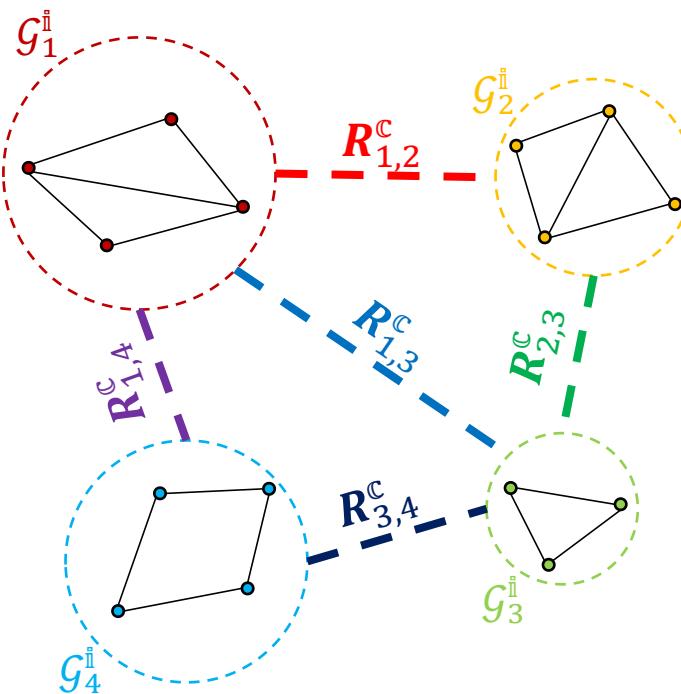
旋转平均 (Rotation Averaging, RA)

- 在线层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging on the Fly, IRAv3
 - 解决IRA++中基于社区发现的聚类方式所存在的**预先确定与任务无关**问题
 - \mathcal{V}_i^\dagger : 当前已估计绝对旋转的各类别顶点集合
 - \mathcal{V}^\ddagger : 当前未估计绝对旋转的顶点集合
 - 图中实线: \mathcal{V}^\ddagger 中顶点 v_j^\ddagger 在 \mathcal{V}_i^\dagger 中的支撑边集



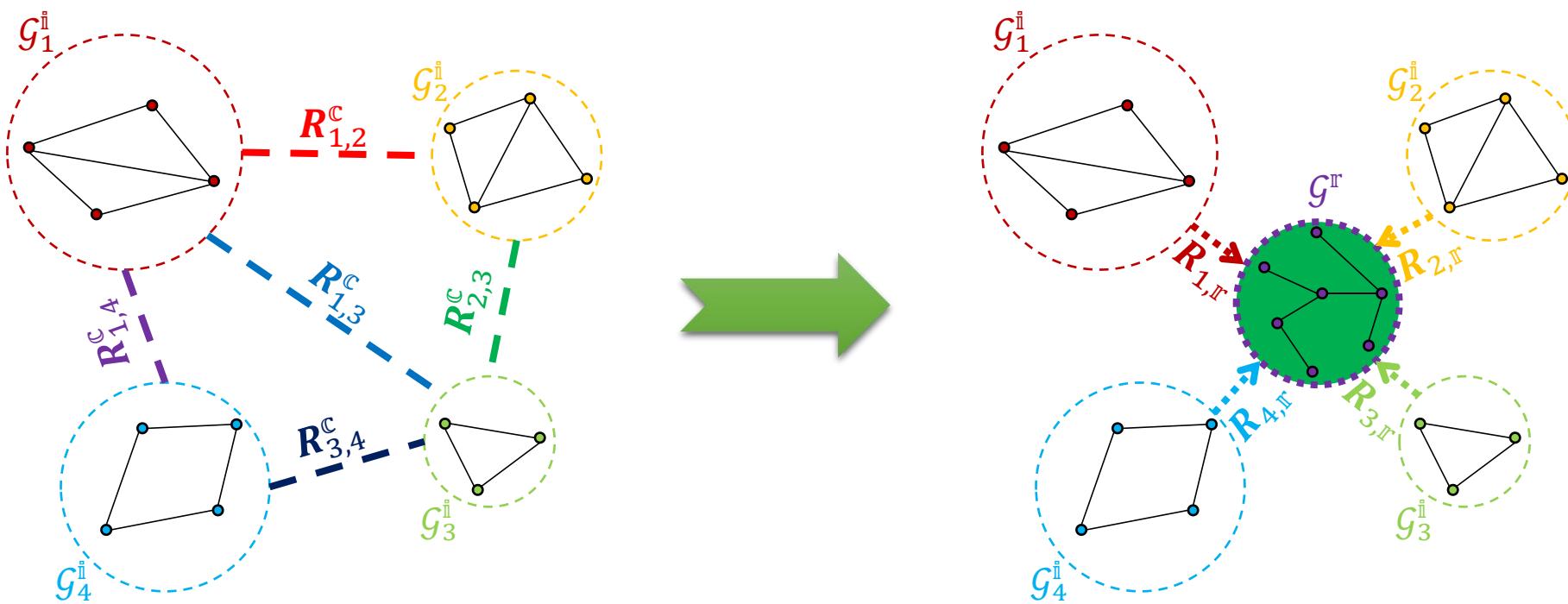
旋转平均 (Rotation Averaging, RA)

- 基准层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging via **Multiple Connected Dominating Sets**, IRAv3+
 - 解决IRA++/IRAv3中基于类间旋转平均的全局对齐存在的类间连接弱与对齐精度低的问题



旋转平均 (Rotation Averaging, RA)

- 基准层级增量式旋转平均
- Hierarchical Incremental Rotation Averaging via **Multiple Connected Dominating Sets**, IRAv3+
 - 解决**IRA++/IRAv3**中基于类间旋转平均的全局对齐存在的**类间连接弱与对齐精度低**的问题
 - 连通支配集：原始图的连通子集，且原始图中的全部顶点或属于该子集或与该子集某顶点相邻



旋转平均 (Rotation Averaging, RA)

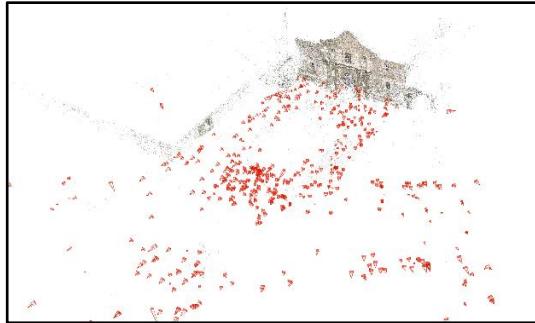
- 实验数据: 1DSfM

	$ \mathcal{V} $	$ \mathcal{V}_{\text{GT}} $	$ \mathcal{E} $	$\tilde{n}_{i,j}$	$\bar{n}_{i,j}$	$\tilde{r}_{i,j}^R$	$\bar{r}_{i,j}^R$	$\tilde{r}_{i,j}^t$	$\bar{r}_{i,j}^t$
ALM	627	577	97206	105	192	2.78°	9.09°	4.65°	18.80°
ELS	247	227	20297	106	160	2.89°	12.50°	8.75°	36.66°
GDM	742	677	48144	73	144	12.30°	33.33°	26.87°	52.65°
MDR	394	341	23784	61	128	9.34°	29.30°	13.46°	36.30°
MND	474	450	52424	180	310	1.67°	7.51°	3.33°	18.61°
NYC	376	332	20680	80	167	4.22°	14.14°	7.18°	28.86°
PDP	354	338	24710	87	128	1.81°	8.38°	3.07°	21.75°
PIC	2508	2152	319257	56	97	4.93°	19.09°	2.92°	7.71°
ROF	1134	1084	70187	65	188	2.97°	13.83°	4.01°	30.19°
TOL	508	472	23863	81	220	2.60°	11.58°	2.63°	19.92°
TFG	5433	5058	680012	71	109	3.01°	8.62°	6.56°	23.67°
USQ	930	789	25561	87	150	3.61°	9.02°	20.32°	43.22°
VNC	918	836	103550	229	408	2.59°	11.26°	4.24°	24.47°
YKM	458	437	27729	112	245	2.68°	11.16°	3.40°	21.20°

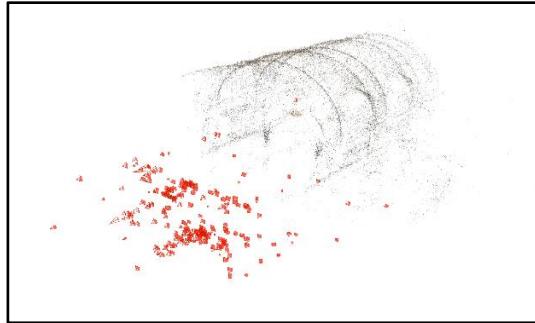
- $|\mathcal{V}|, |\mathcal{E}|, |\mathcal{V}_{\text{GT}}|$: 外极几何图顶点数与边数以及带有真值的顶点数
- $\tilde{n}_{i,j}, \bar{n}_{i,j}$: 匹配图像对之间的图像局部特征匹配对数的中值与均值
- $\tilde{r}_{i,j}^R, \bar{r}_{i,j}^R$: 匹配图像对之间的相对旋转测量值的角度误差中值与均值
- $\tilde{r}_{i,j}^t, \bar{r}_{i,j}^t$: 匹配图像对之间的相对平移测量值的角度误差中值与均值

旋转平均 (Rotation Averaging, RA)

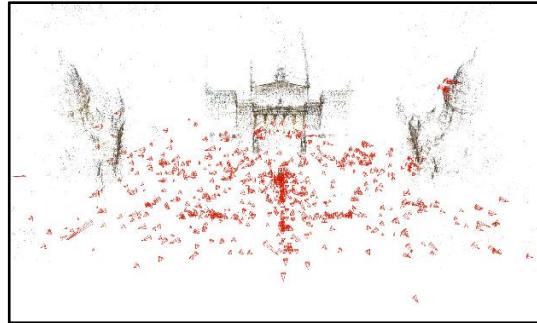
- 实验数据: 1DSfM



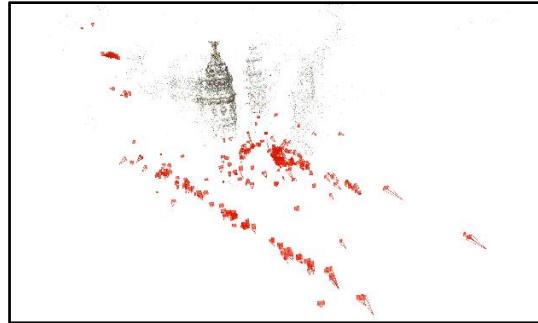
ALM



ELS



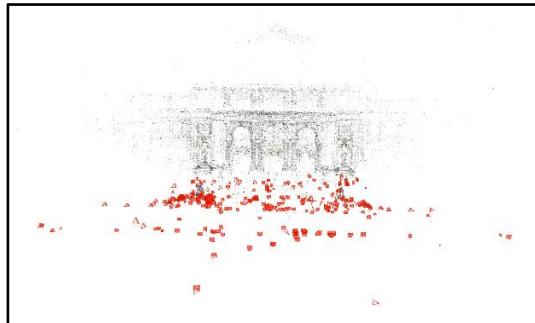
GDM



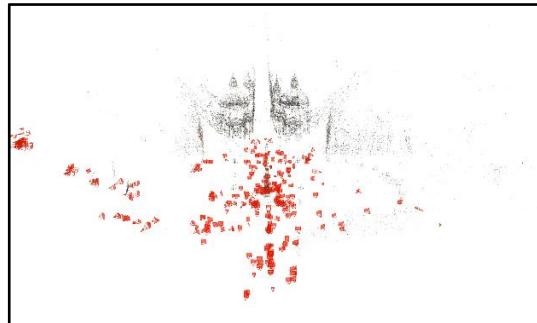
MDR



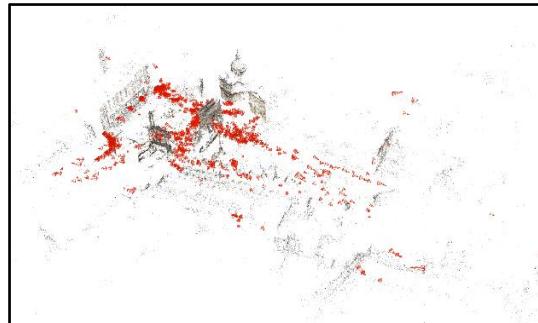
MND



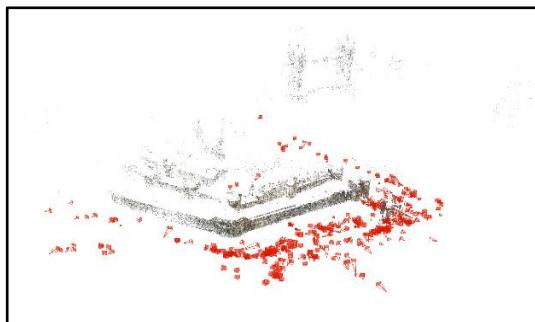
NYC



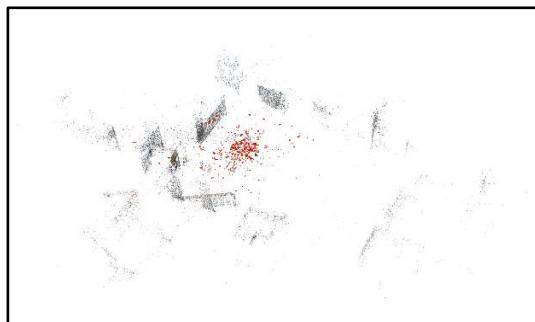
PDP



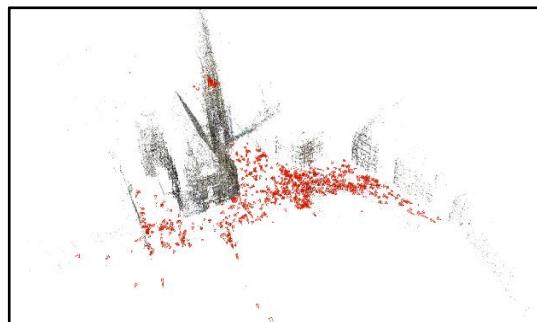
ROF



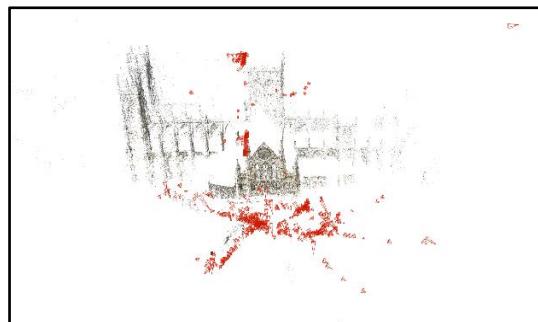
TOL



USQ



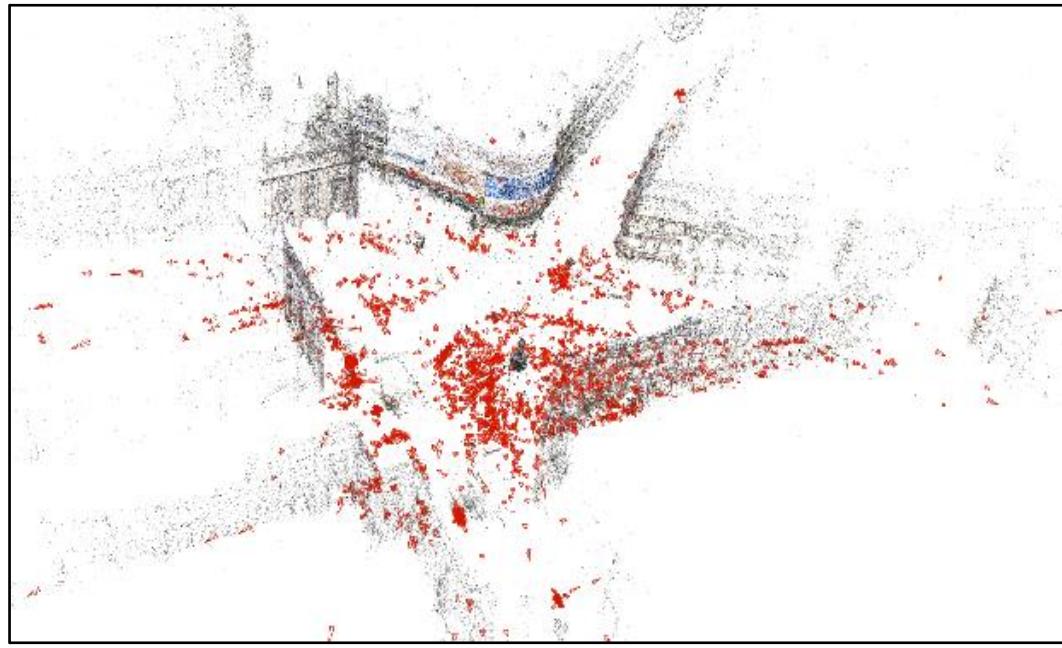
PDP



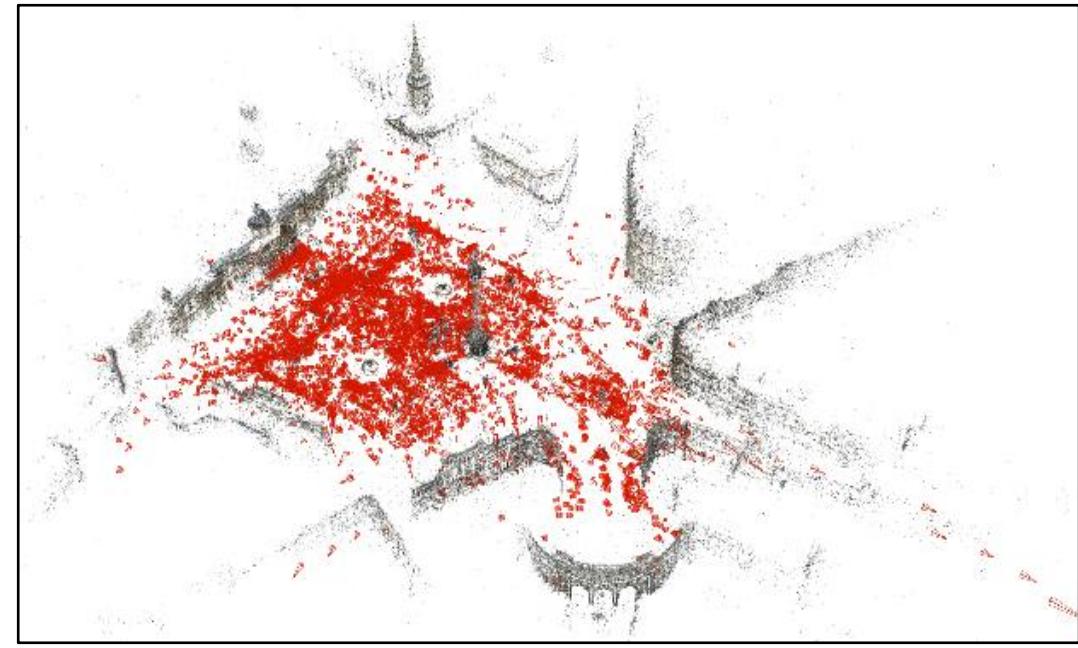
ROF

旋转平均 (Rotation Averaging, RA)

- 实验数据: 1DSfM



PIC



TFG

旋转平均 (Rotation Averaging, RA)

- 实验结果

Data	Robust loss-based			Outlier filtering-based			Deep learning-based			IRA series			
	IRLS- $\ell_{\frac{1}{2}}$ [1]	MPLS [2]	DESC [3]	OMSTS [4]	HRRA [5]	HARA [6]	NeuRoRA [7]	MSP [8]	RAGO [9]	IRA	IRA++	IRAv3	IRA3v+
ALM	2.14°	1.16°	1.63°	1.26°	1.03°	1.15°	1.16°	1.07°	0.88°	0.83°	0.80°	0.73°	0.72°
ELS	1.15°	0.88°	1.01°	0.75°	0.59°	0.62°	0.64°	0.83°	0.46°	0.51°	0.46°	0.44°	0.39°
GDM	28.20°	9.87°	9.43°	45.15°	4.04°	63.74°	2.94°	3.69°	2.68°	5.32°	2.88°	1.99°	2.20°
MDR	3.08°	1.26°	2.35°	1.12°	2.54°	1.50°	1.13°	1.09°	1.03°	0.85°	0.83°	0.75°	0.75°
MND	0.71°	0.51°	0.59°	0.68°	0.62°	0.51°	0.60°	0.50°	0.46°	0.51°	0.50°	0.44°	0.40°
NYC	1.40°	1.24°	1.48°	1.30°	1.24°	1.37°	1.18°	1.12°	0.71°	1.00°	0.95°	0.82°	0.81°
PDP	2.62°	1.93°	1.95°	1.73°	0.92°	0.92°	0.79°	0.76°	0.63°	0.90°	0.75°	0.72°	0.63°
PIC	3.12°	1.81°	2.43°	1.41°	4.87°	3.22°	1.91°	1.80°	0.58°	1.67°	1.70°	1.50°	1.55°
ROF	1.70°	1.37°	1.42°	1.85°	2.48°	2.42°	1.31°	1.19°	1.10°	1.51°	1.24°	1.09°	1.14°
TOL	2.45°	2.20°	2.49°	2.10°	2.05°	2.36°	1.46°	1.25°	1.20°	2.45°	1.33°	1.44°	1.09°
TFG	2.03°	—	1.84°	2.63°	4.88°	2.06°	2.25°	—	1.53°	3.30°	1.74°	1.49°	1.80°
USQ	4.97°	3.48°	4.31°	3.83°	3.77°	4.78°	2.01°	1.85°	1.92°	4.40°	3.70°	3.27°	2.77°
VNC	4.64°	2.83°	1.99°	3.30°	1.84°	1.49°	1.50°	1.10°	0.89°	1.02°	0.94°	0.86°	0.76°
YKM	1.62°	1.45°	1.60°	1.55°	1.57°	1.65°	0.99°	0.91°	0.92°	1.57°	1.38°	1.36°	0.98°
Rank	11.71	8.62	10.36	9.29	9.14	9.71	6.93	5.08	2.50	7.07	4.29	2.57	2.07

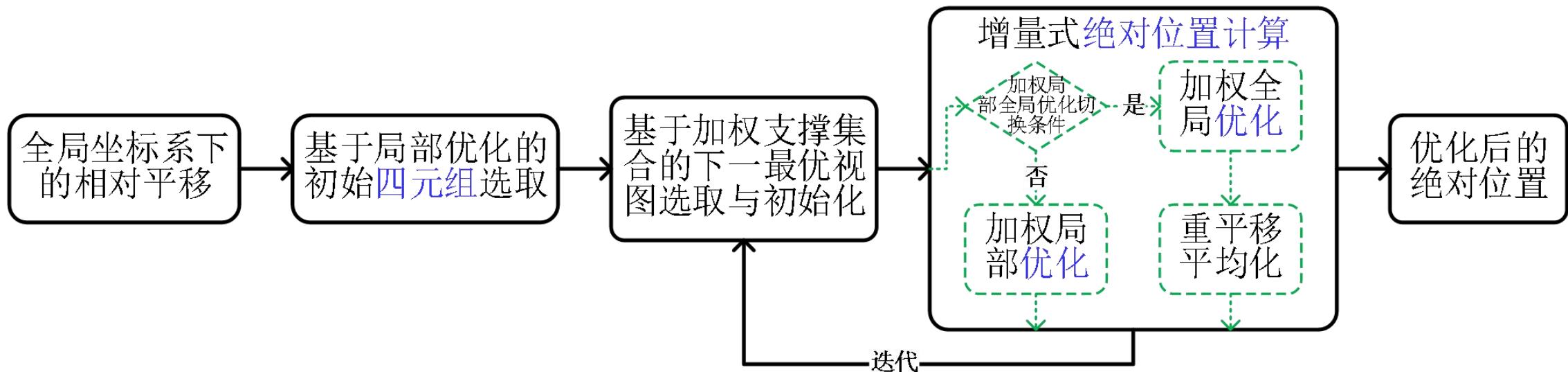
- [1] A. Chatterjee and V. M. Govindu. Robust Relative Rotation Averaging[J]. *IEEE T-PAMI*, 2018.
- [2] Y. Shi and G. Lerman. Message Passing Least Squares Framework and its Application to Rotation Synchronization[C]. *In Proc. ICML*, 2020.
- [3] Y. Shi, C. M. Wyeth, and G. Lerman. Robust Group Synchronization via Quadratic Programming[C]. *In Proc. ICML*, 2022.
- [4] H. Cui, S. Shen, W. Gao, L. Hong, and Z. Wang. Efficient and Robust Large-Scale Structure-from-Motion via Track Selection and Camera Prioritization[J]. *ISPRS P&RS*, 2019.
- [5] X. Gao, J. Luo, K. Li, and Z. Xie. Hierarchical RANSAC-Based Rotation Averaging[J]. *IEEE SPL*, 2020.
- [6] S. H. Lee and J. Civera. HARA: A Hierarchical Approach for Robust Rotation Averaging[C] *In Proc. CVPR*, 2022.
- [7] P. Purkait, T. J. Chin, and I. Reid. NeuRoRA: Neural Robust Rotation Averaging[C]. *In Proc. ECCV*, 2020.
- [8] L. Yang, H. Li, J. A. Rahim, Z. Cui, and P. Tan. End-to-End Rotation Averaging with Multi-Source Propagation[C]. *In Proc. CVPR*, 2021.
- [9] H. Li, Z. Cui, S. Liu, and P. Tan. RAGO: Recurrent Graph Optimizer for Multiple Rotation Averaging[C]. *In Proc. CVPR*, 2022.

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- IRA \Rightarrow ITA

- 相机位置算子
 - 流程关键技术
 - 优化目标函数



平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- IRA \Rightarrow ITA

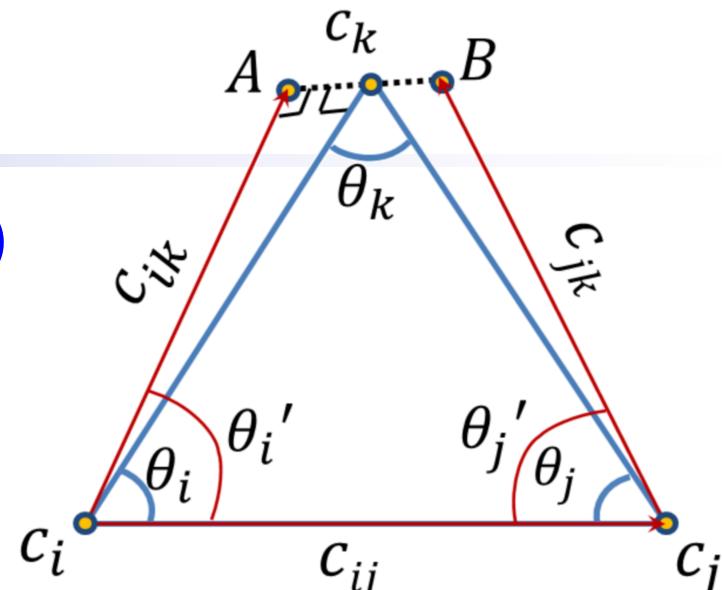
- 相机位置算子

IRA, 已知 R_i 、 $R_{i,j}$, 求 R_j : $R_j = R_{i,j}R_i$

ITA, 已知 c_i 、 c_j 、 c_{ij} 、 c_{ik} 、 c_{jk} , 求 c_k :

$$c_k = \frac{1}{2} \left(R_i(\theta'_i) s_{i,j}^{i,k} (c_j - c_i) + R_j(-\theta'_j) s_{i,j}^{j,k} (c_i - c_j) + c_i + c_j \right)$$

- 流程关键技术
- 优化目标函数



Jiang et al. In Proc. ICCV 2013

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- IRA \Rightarrow ITA

- 相机位置算子
- 流程关键技术

IRA, 基于局部优化的初始三元组选取:

$$i^*, j^*, k^* = \arg \max \left\{ \sum_{\substack{\nu_i, \nu_j \in \mathcal{V}_{t_{i,j,k}} \\ e_{i,j} \in \mathcal{E}_{t_{i,j,k}}}} n_{i,j} \cos \left(d_\theta^R(\mathbf{R}_{i,j}, \mathbf{R}_j^* \mathbf{R}_i^{*\top}) \right) \mid t_{i,j,k} \in \mathcal{T}^* \right\}$$

ITA, 基于局部优化的初始四元组选取:

$$i^*, j^*, k^*, l^* = \arg \max \left\{ \sum_{\substack{\nu_i, \nu_j \in \mathcal{V}_{q_{i,j,k,l}} \\ e_{i,j} \in \mathcal{E}_{q_{i,j,k,l}}}} \mathbf{c}_{i,j} \cdot \frac{\mathbf{c}_j^* - \mathbf{c}_i^*}{\|\mathbf{c}_j^* - \mathbf{c}_i^*\|_2} \mid q_{i,j,k,l} \in \mathcal{Q}^* \right\}$$

- 优化目标函数

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)
 - IRA \Rightarrow ITA

- 相机位置算子
- 流程关键技术
- 优化目标函数

IRA, 旋转矩阵角距离最小化: $\{\mathbf{R}_i^*\} = \arg \min \sum_{\substack{\nu_i, \nu_j \in \mathcal{V} \\ e_{i,j} \in \mathcal{E}}} d_\theta^R(\mathbf{R}_{i,j}, \mathbf{R}_j \mathbf{R}_i^\top)^2$

ITA, 位置向量弦距离最小化: $\{\mathbf{c}_i^*\} = \arg \min \sum_{\substack{\nu_i, \nu_j \in \mathcal{V} \\ e_{i,j} \in \mathcal{E}}} d_{ch}^t\left(\mathbf{c}_{i,j}, \frac{\mathbf{c}_j - \mathbf{c}_i}{\|\mathbf{c}_j - \mathbf{c}_i\|_2}\right)^2$

平移平均 (Translation Averaging, TA)

- 增量式平移平均 (Incremental Translation Averaging, ITA)

- 实验结果

Data	# reconstructed cameras estimation error/m elapsed time/s on different translation averaging methods							
	SATA[1]	VGO[2]	SFSK[3]	BATA[4]	ACEM[5]	Our ITA	ITA w/ IRA	ITA w/ IRA*
ALM	574 0.5 78	533 1.4 69	- 0.9 27	- 0.6 24	482 1.2 172	575 0.5 17	577 0.5 17	523 0.4 10
ELS	223 2.5 37	203 3.7 19	- 1.9 4	- 1.5 2	211 6.1 87	230 1.3 2	227 1.1 2	224 1.0 2
MDR	317 2.7 31	272 8.7 67	- 6.0 9	- 1.8 4	168 6.9 23	339 7.0 3	336 6.5 4	294 1.5 2
MND	452 0.4 62	416 2.0 133	- 0.8 19	- 0.3 10	416 1.0 224	446 0.4 10	452 0.4 10	429 0.4 9
NYC	338 0.8 38	294 2.8 71	- 1.4 11	- 0.6 4	277 2.2 67	327 0.6 3	328 0.7 4	304 0.4 3
PDP	340 2.0 43	302 2.9 23	- 3.6 7	- 4.2 4	275 3.5 60	326 0.6 5	322 0.6 4	299 0.5 5
PIC	2276 1.3 328	1928 5.2 544	- 1.2 464	- 1.0 114	- - -	2199 1.8 381	2231 1.5 402	1929 0.7 268
ROF	1077 2.9 131	966 6.8 385	- 4.3 62	- 1.6 29	- - -	1062 2.3 28	1053 2.6 27	997 2.2 22
TOL	465 1.9 52	409 9.3 145	- 2.3 18	- 2.2 7	414 5.0 121	453 1.8 7	452 2.0 7	422 1.9 5
USQ	570 5.5 41	701 4.5 141	- 8.9 28	- 4.3 10	- - -	703 5.6 8	697 5.3 9	588 3.0 6
VNC	842 2.7 117	771 6.7 185	- 1.9 74	- 1.9 30	674 4.2 273	783 1.0 27	778 1.2 26	720 0.8 17
YKM	417 2.3 46	409 3.9 31	- - -	- 0.9 9	341 2.7 91	414 0.8 5	411 0.8 6	385 0.7 6

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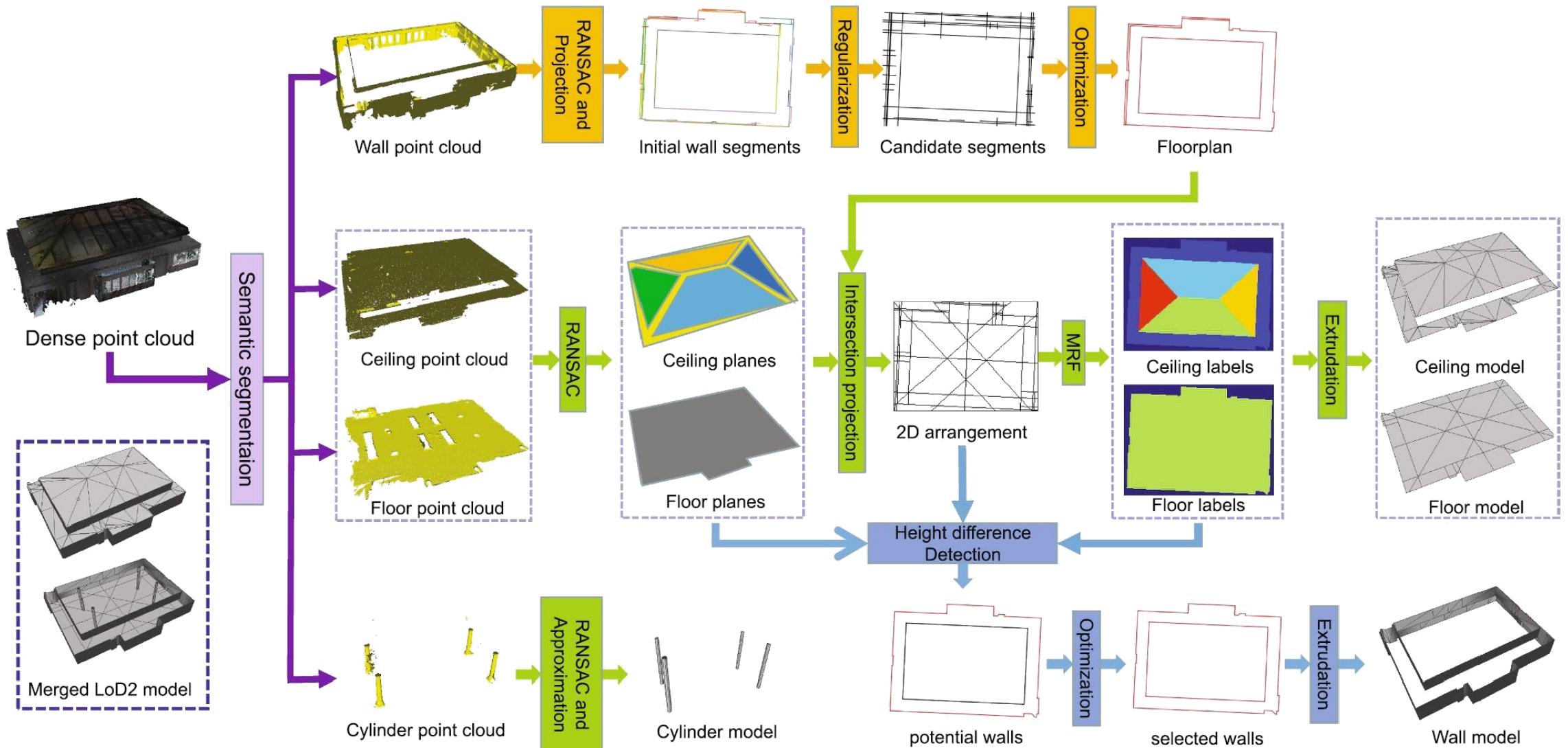
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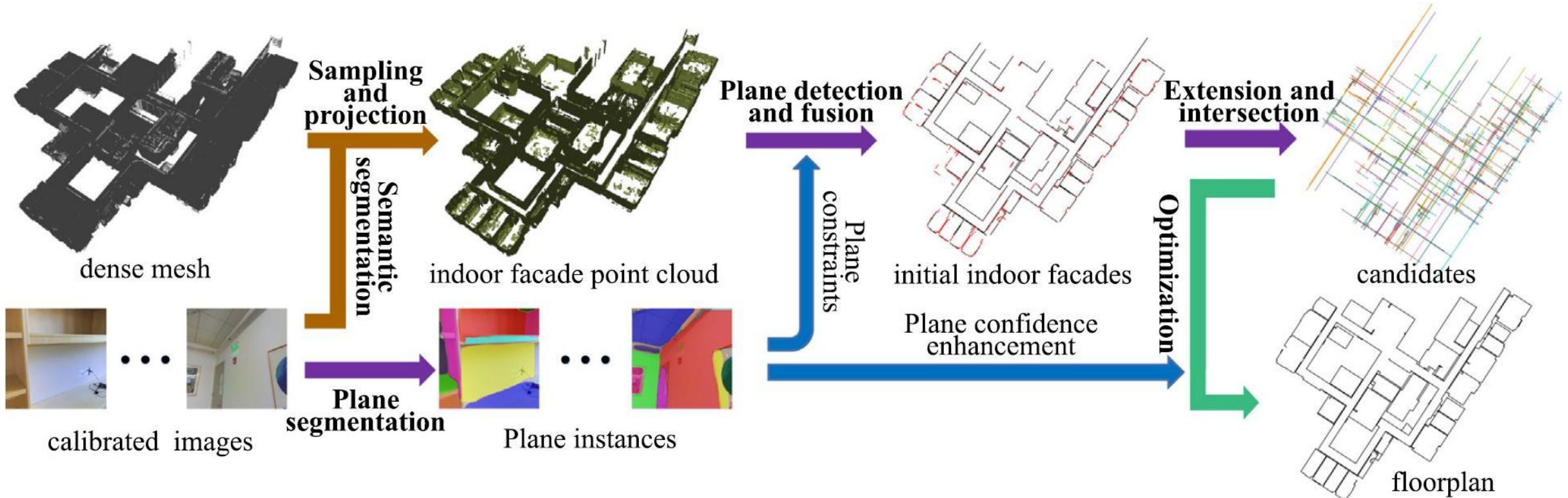
One More Thing

- **VecIM:** Han et al. ISPRS P&RS 2021



One More Thing

- **FloorUSG**: Han et al. ISPRS P&RS 2023



总结展望

- **总结**

- 增量式旋转平均: IRA, IJCV 2021
- 层级增量式旋转平均: IRA++, IEEE T-CSVT 2022
- 在线层级增量式旋转平均: IRAv3, IEEE T-CSVT 2023
- 基准层级增量式旋转平均: IRAv3+, IEEE T-CSVT Major Revision
- 增量式平移平均: ITA, IEEE T-CSVT 2022

- **展望**

- 在线/基准层级增量式旋转平均 \Rightarrow **在线**基准层级增量式旋转平均
- 增量式平移平均 \Rightarrow 增量式**尺度**平均 + **尺度已知的**增量式平移平均
- 增量式**尺度/旋转/平移**平均 \Rightarrow 增量式**运动**平均

相关成果

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