

解析求解：混合SfM框架

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个人主页：<https://xgao-3dv.github.io/>



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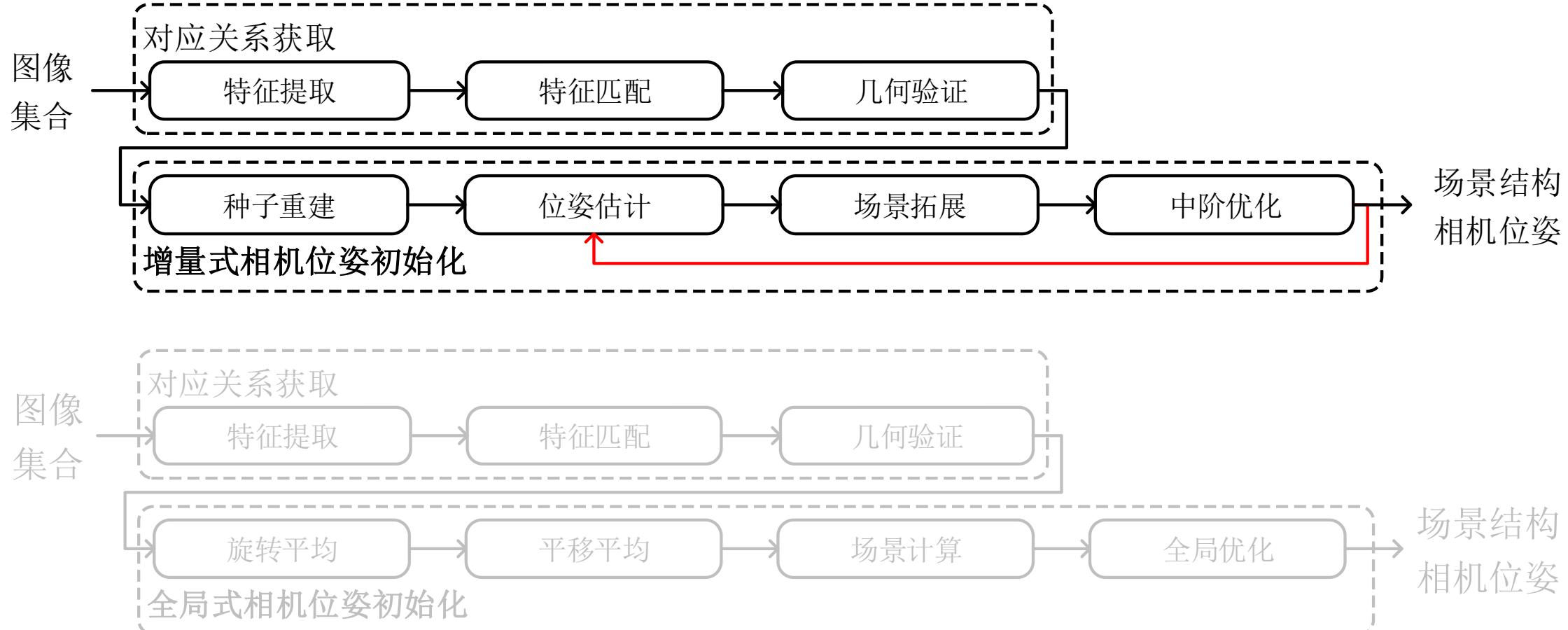
- 增量全局结合的**混合式方法**
 - 混合式SfM方法
 - 增量式运动平均方法
- 自顶向下分组的**分布式方法**
 - 分布式SfM方法
 - 分布式旋转平均方法
- 自底向上聚合的**层级式方法**

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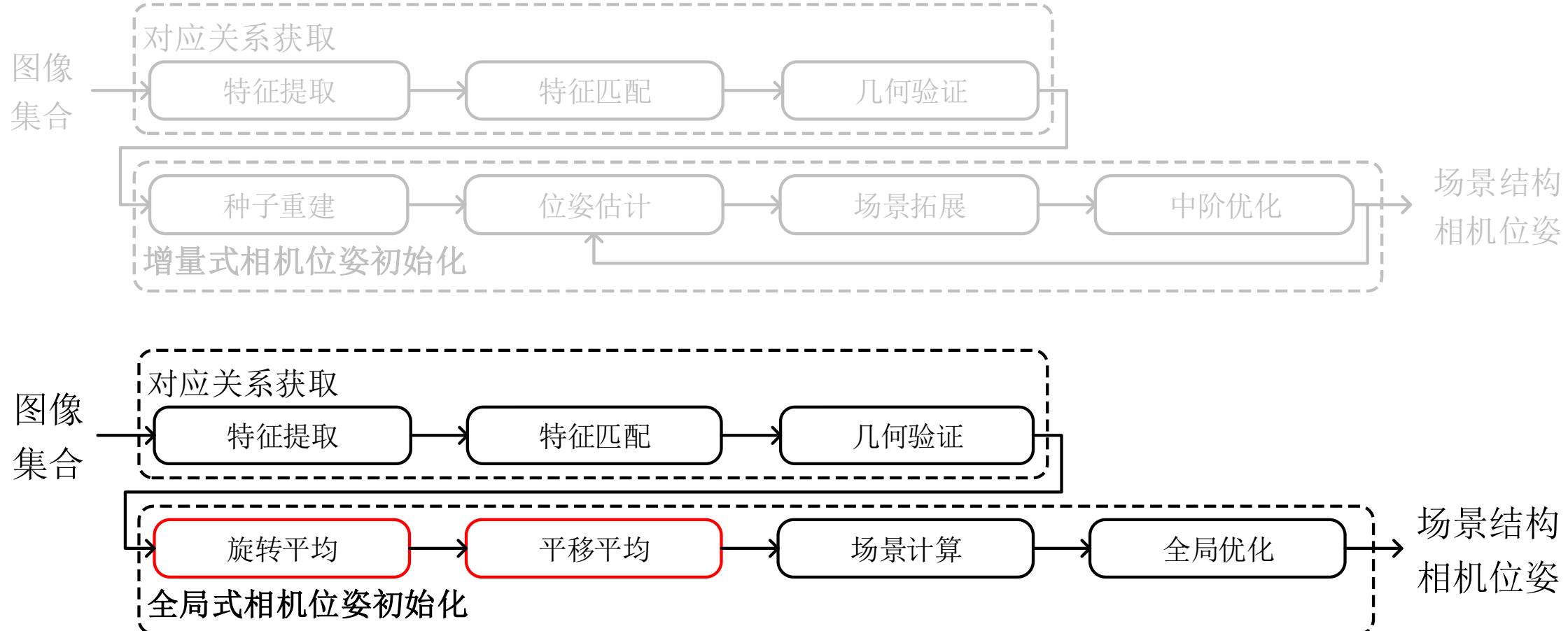
混合式方法

- 增量式SfM vs. 全局式SfM



混合式方法

- 增量式SfM vs. 全局式SfM



混合式方法

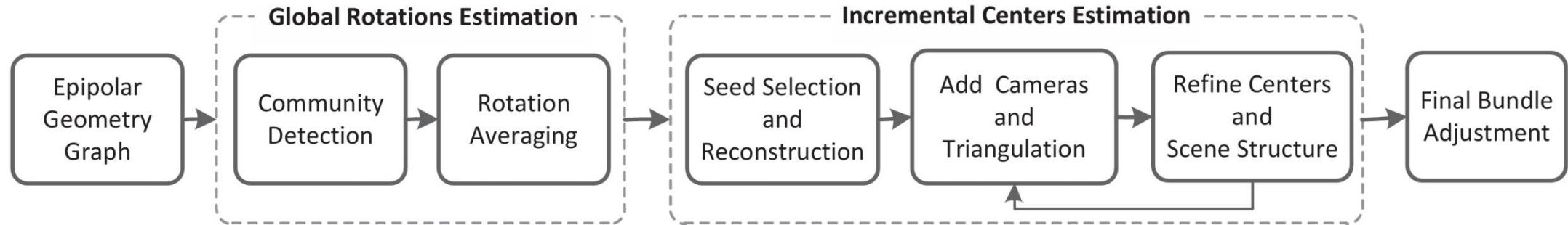
- 增量式SfM vs. 全局式SfM

- 增量式SfM
 - ☺ 精度更高、鲁棒性更强、
 - ☹ 效率较低、场景漂移、
- 全局式SfM
 - ☺ 效率更高、一致性更好、
 - ☹ 精度较差、场景缺失、

混合式方法

- 代表性方法

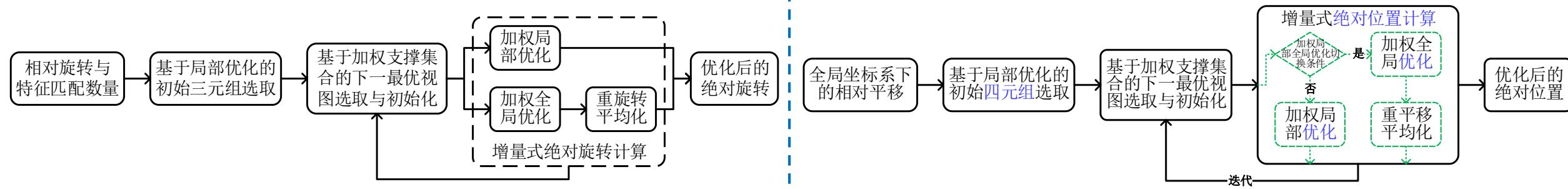
- 混合式SfM方法: HSfM^[1]



- 增量式运动平均方法:

IRA^[2]

ITA^[3]



[1] Cui H, Gao X, Shen S, et al. HSfM: Hybrid structure-from-motion. In Proc. CVPR 2017.

[2] Gao X, Zhu L, Xie Z, et al. Incremental rotation averaging. IJCV 2021.

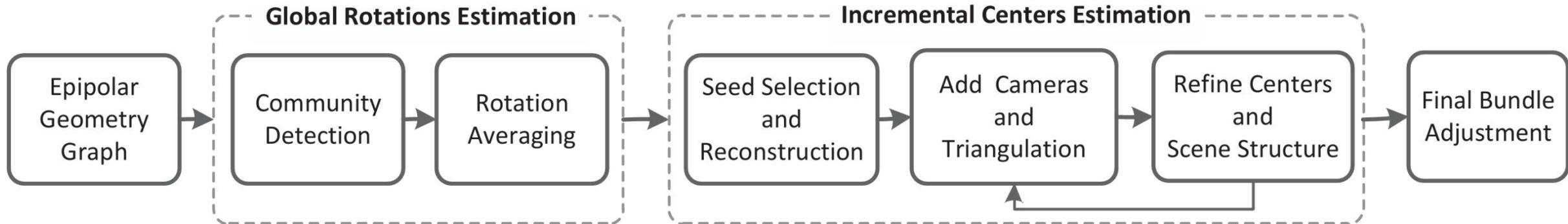
[3] Gao X, Zhu L, Fan B, et al. Incremental translation averaging. IEEE TCSVT 2022.

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混合式SfM方法

- HSfM: Hybrid Structure from Motion^[1]



- **关键思想:**

- 全局式相机绝对旋转估计 + 增量式相机绝对位置估计
- 全局式方法可有效避免增量式方法中因累积误差所导致的**场景漂移**现象
- 相对于平移平均问题，旋转平均问题**可解性**更好，求解方法更为成熟、可靠
- 在已知相机旋转的前提下增量式求解相机位置，问题**复杂度**更低，求解效果更好

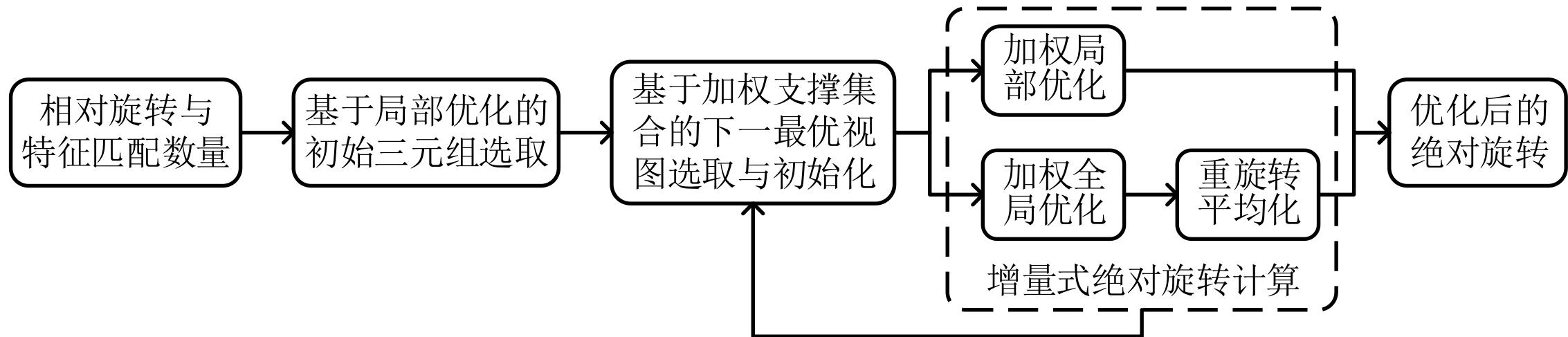
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增量式运动平均方法

- IRA: Incremental Rotation Averaging^[2]



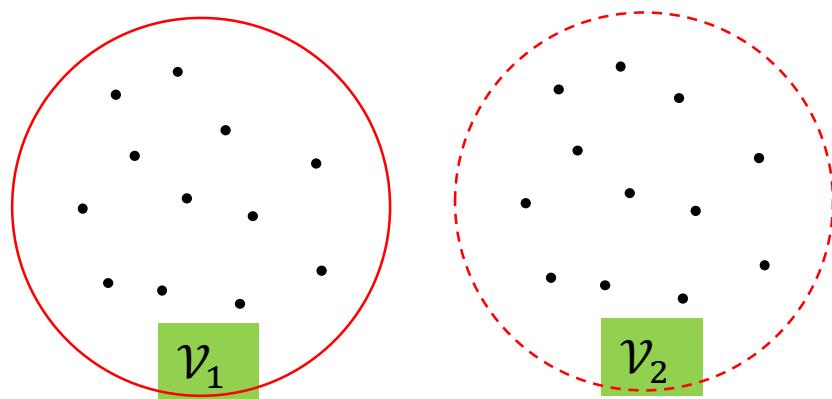
- **关键思想:**

- IRA采用与增量式SfM类似的**增量式**参数估计流程：更加精确、鲁棒
- 旋转平均问题相对于SfM问题待估计的**参数量**更少：更加简单、高效

[2] Gao X, Zhu L, Xie Z, et al. Incremental rotation averaging. IJCV 2021.

增量式运动平均方法

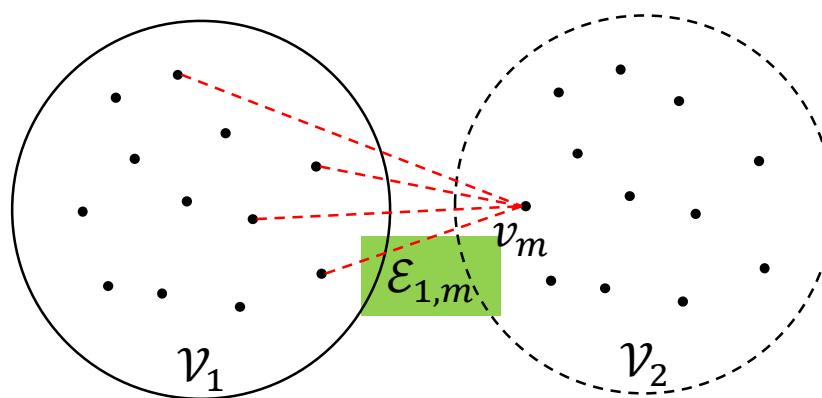
- IRA下一最优视图 (Next Best View, NBV) 选取与初始化
 - \mathcal{V}_1 与 \mathcal{V}_2 : 当前已估计与未估计绝对旋转的顶点集合



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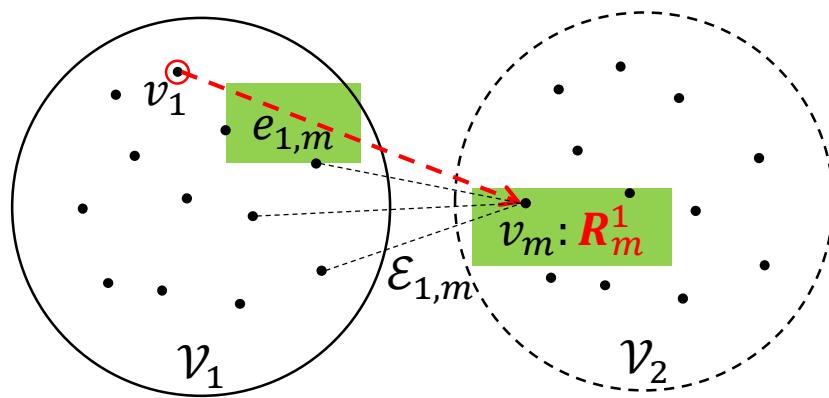
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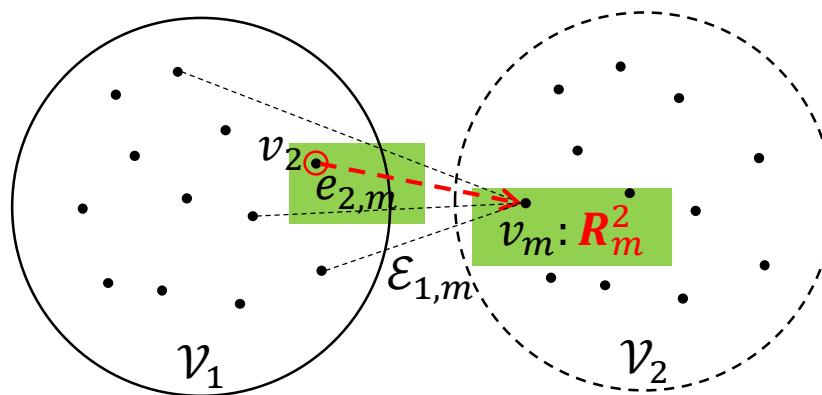
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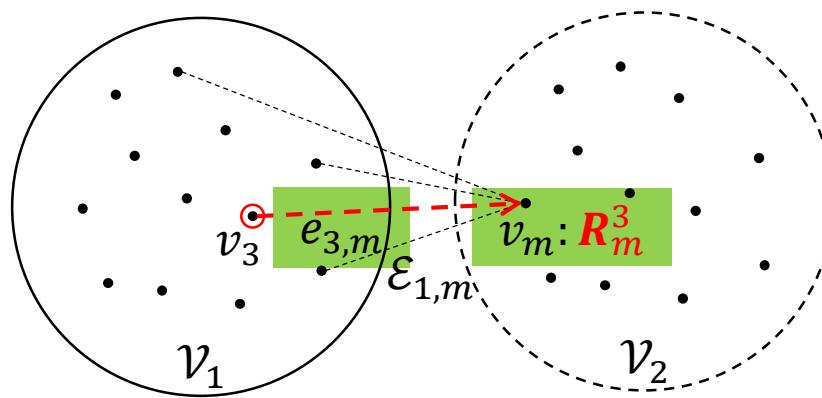
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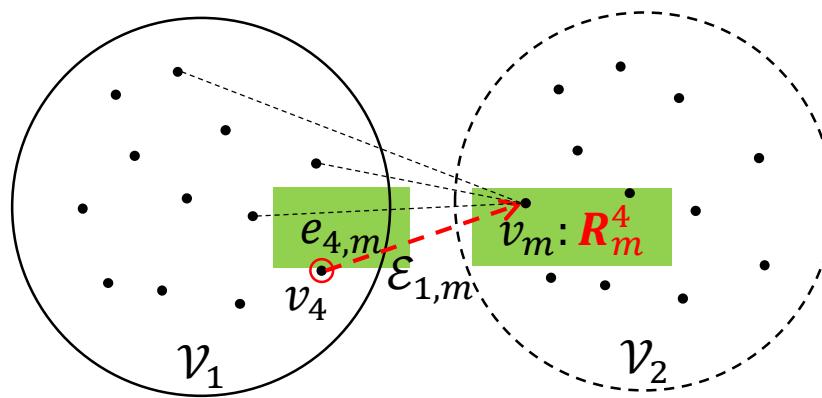
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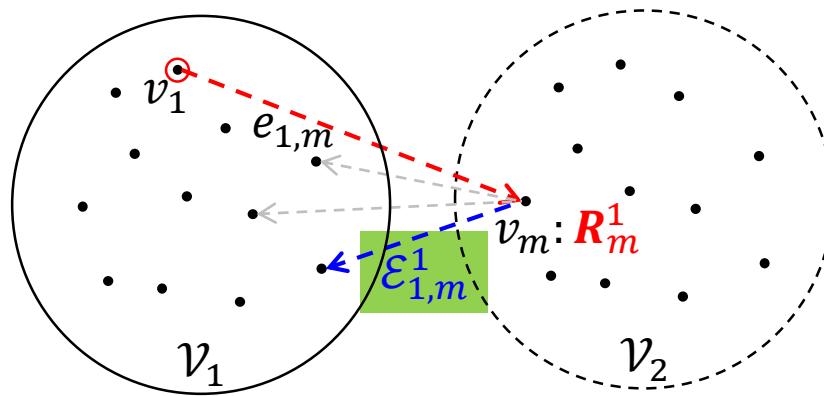
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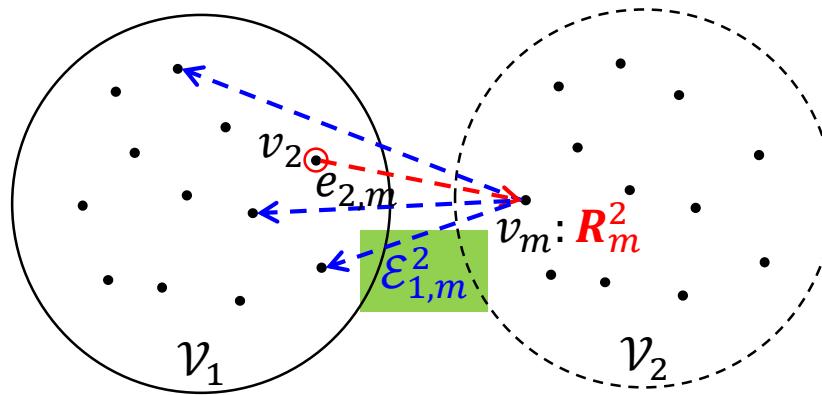
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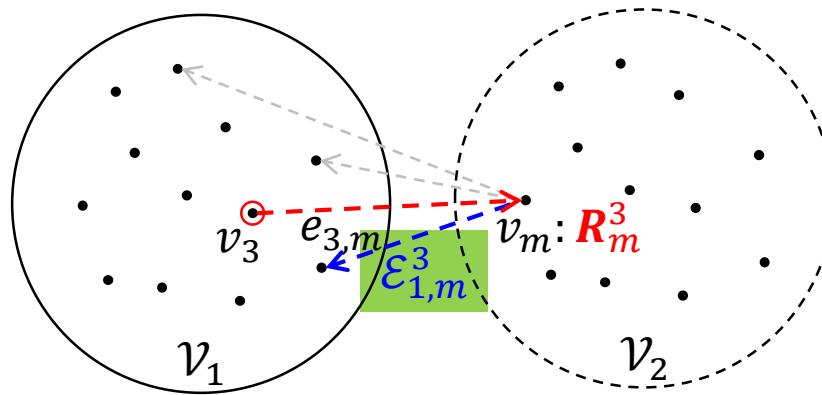
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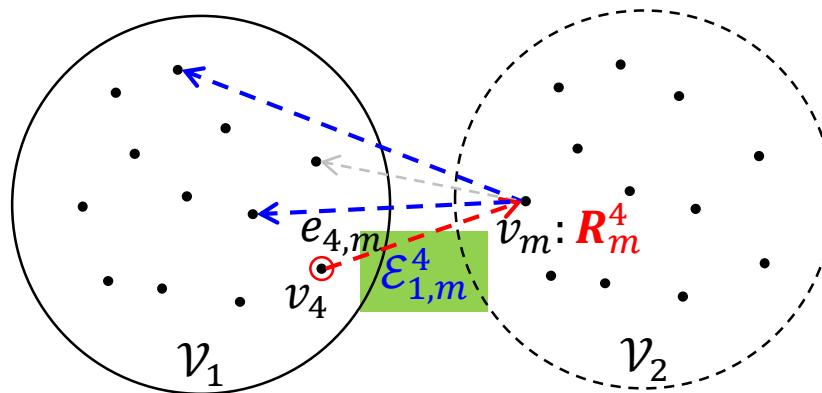
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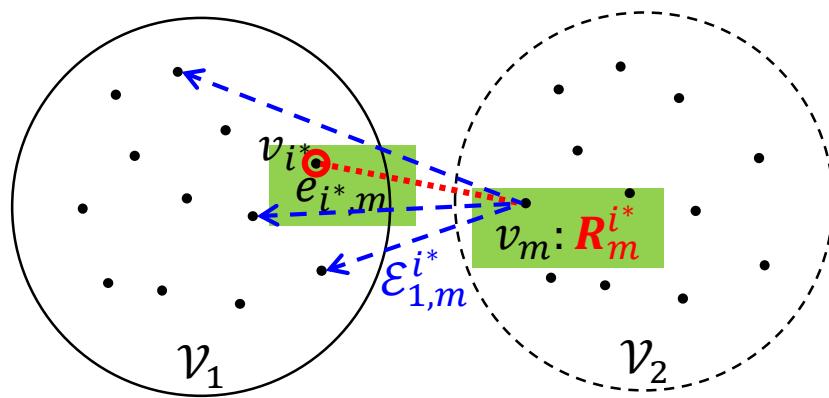
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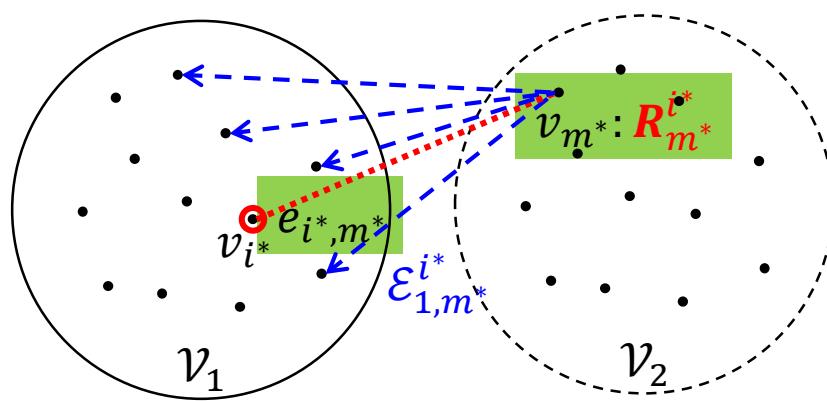
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- 最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$ 对应的 $R_m^{i^*}$ 即为 v_m 的初始化结果



增量式运动平均方法

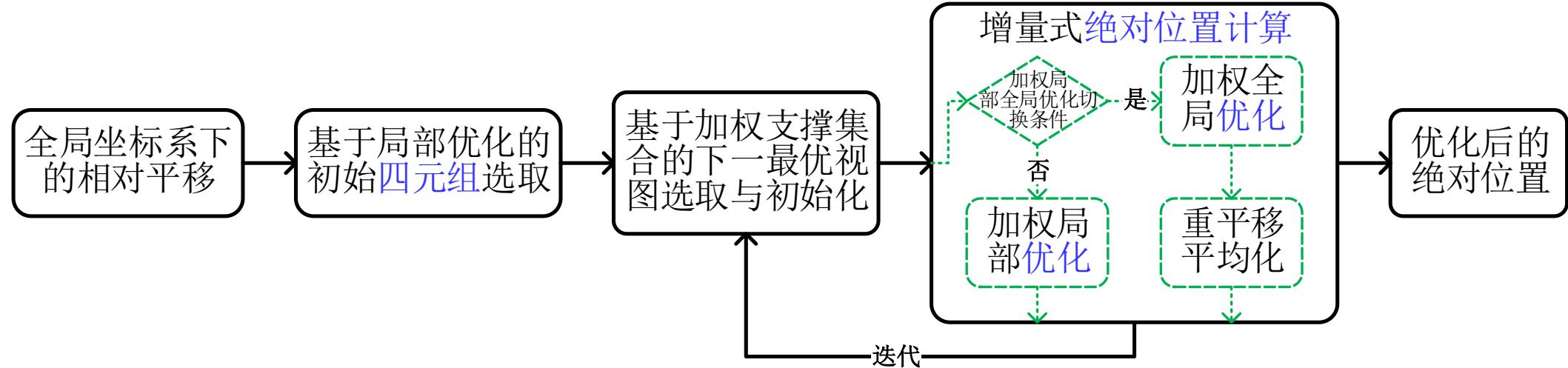
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- 最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$ 对应的 $R_m^{i^*}$ 即为 v_m 的初始化结果
- 对 \mathcal{V}_2 中各顶点重复上述过程并获取全局最大支撑边集 $\mathcal{E}_{1,m}^{i^*}$, 则 v_{m^*} 与 $R_{m^*}^{i^*}$ 即为 **NBV** 及其 **初始化结果**



增量式运动平均方法

- ITA: Incremental Translation Averaging^[3]



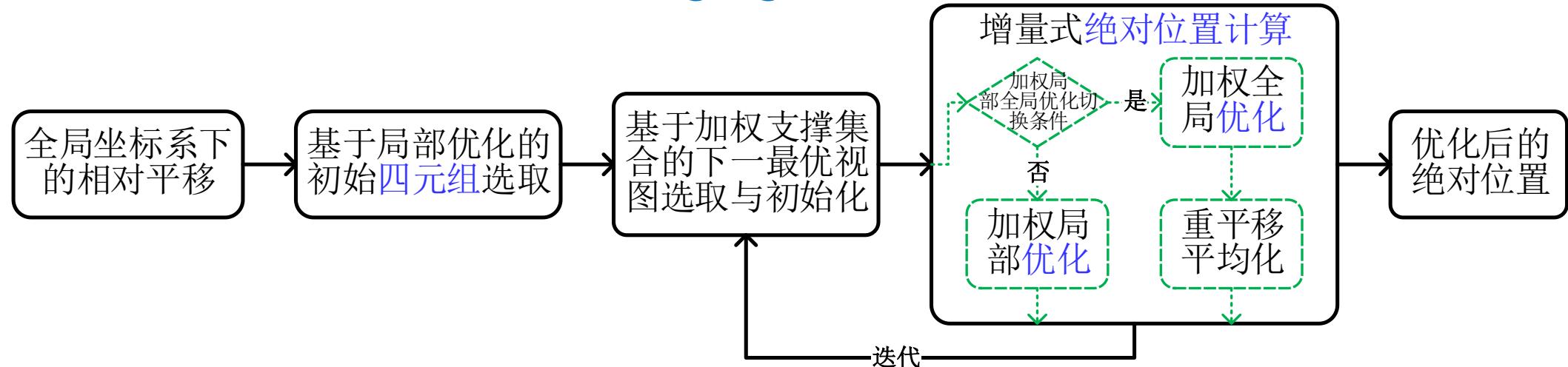
- IRA \Rightarrow ITA:

- 参数求解算子
- 流程关键技术
- 优化目标函数

[3] Gao X, Zhu L, Fan B, et al. Incremental translation averaging. *IEEE TCSVT* 2022.

增量式运动平均方法

• ITA: Incremental Translation Averaging



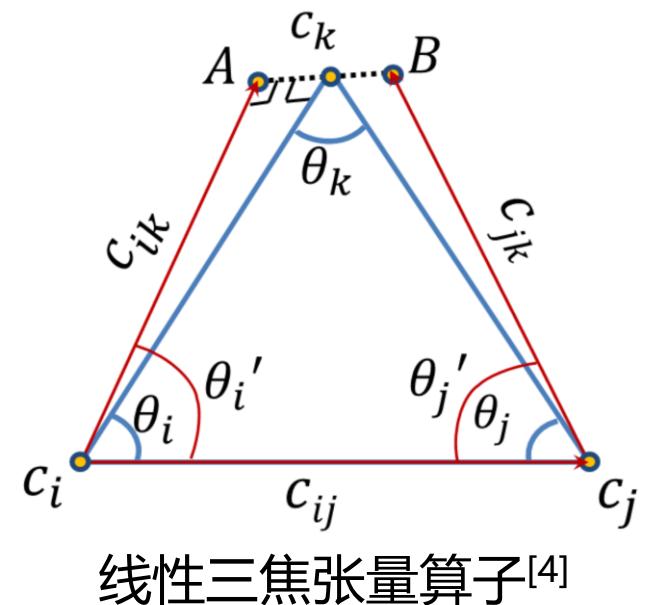
• IRA \Rightarrow ITA:

- 参数求解算子

IRA, 已知 R_i 、 $R_{i,j}$, 求 R_j : $R_j = R_{i,j}R_i$

ITA, 已知 c_i 、 c_j 、 $c_{i,j}$ 、 $c_{i,k}$ 、 $c_{j,k}$, 求 c_k :

$$c_k = \frac{1}{2} \left(R_i(\theta'_i) s_{i,j}^{i,k} (c_j - c_i) + R_j(-\theta'_j) s_{i,j}^{j,k} (c_i - c_j) + c_i + c_j \right)$$



[4] Jiang N, Cui Z, Tan P. A global linear method for camera pose registration. In Proc. ICCV 2013.

线性三焦张量算子^[4]

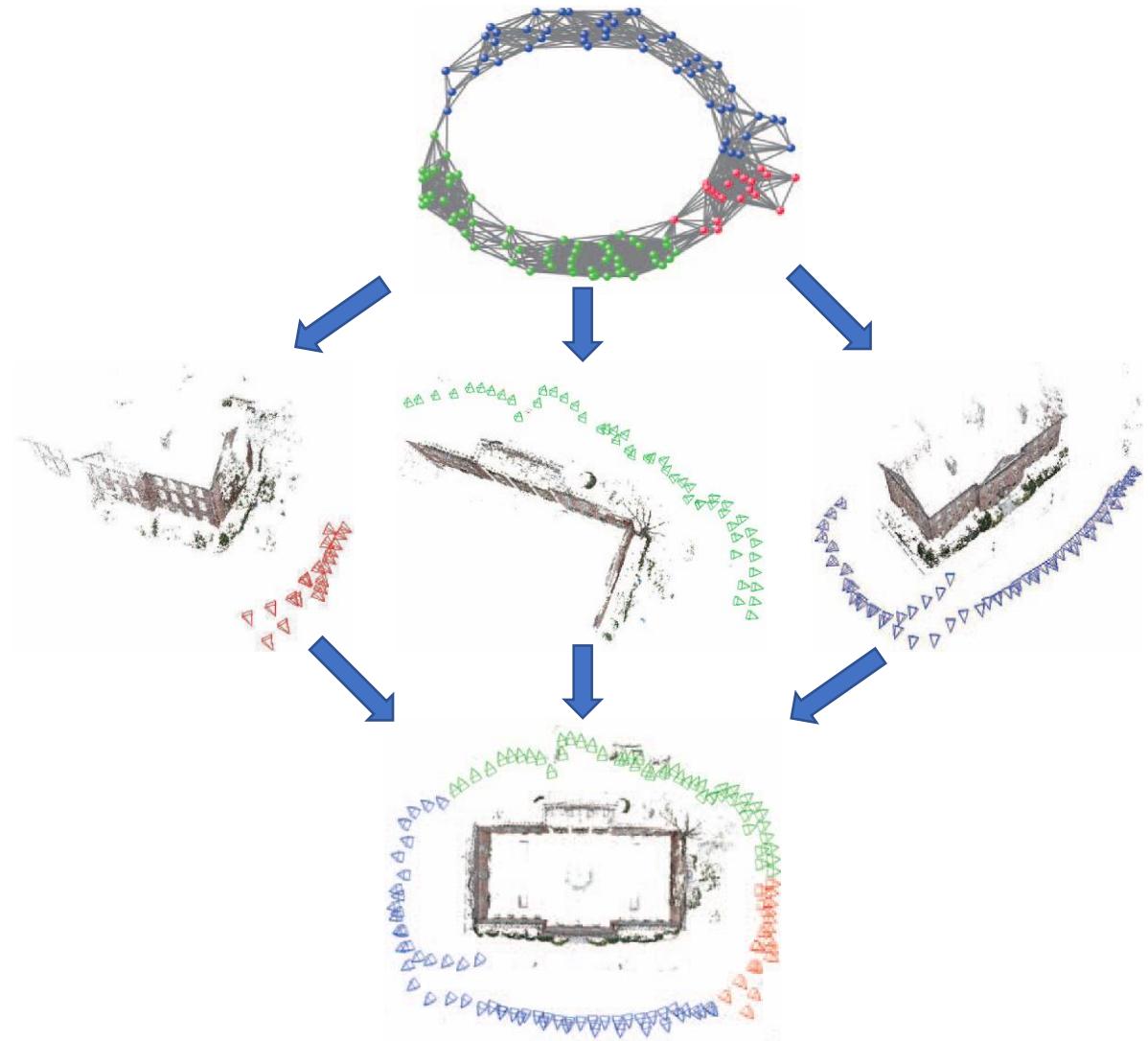
By X. Gao

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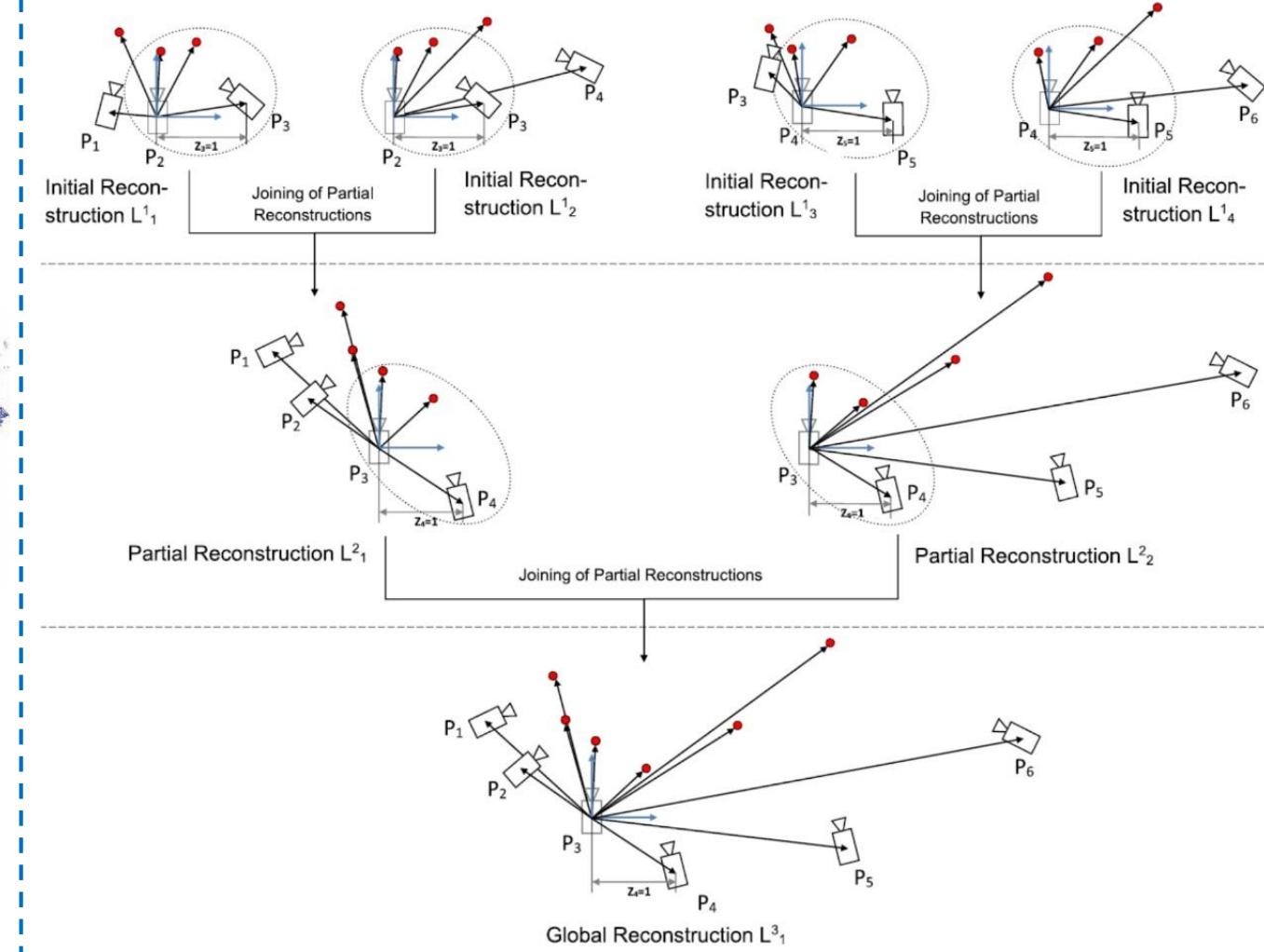
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分布式方法 vs. 层级式方法

自顶向下分组的分布式方法典型流程

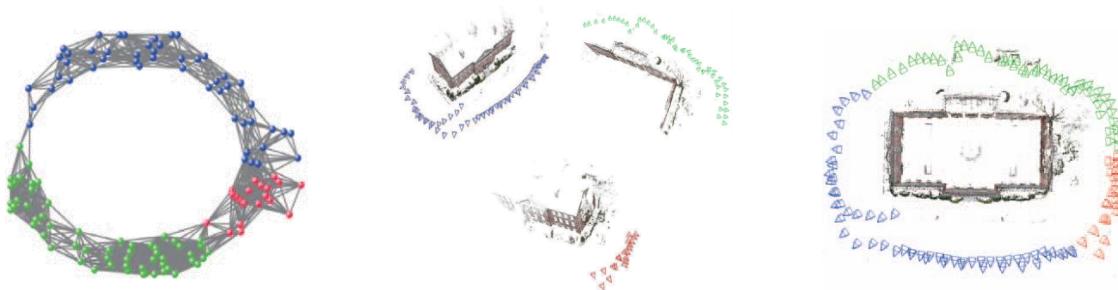
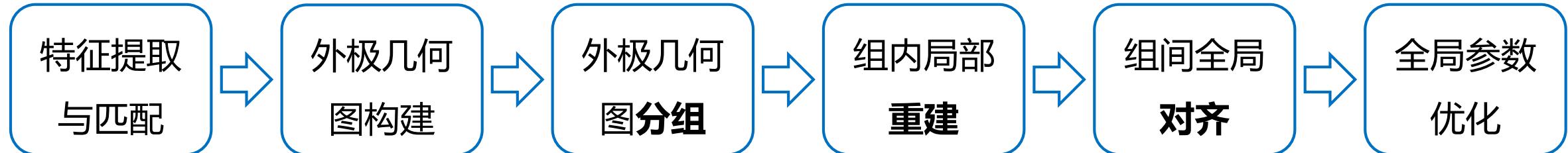


自底向上聚合的层级式方法典型流程



分布式方法

- 基本流程



- 核心步骤

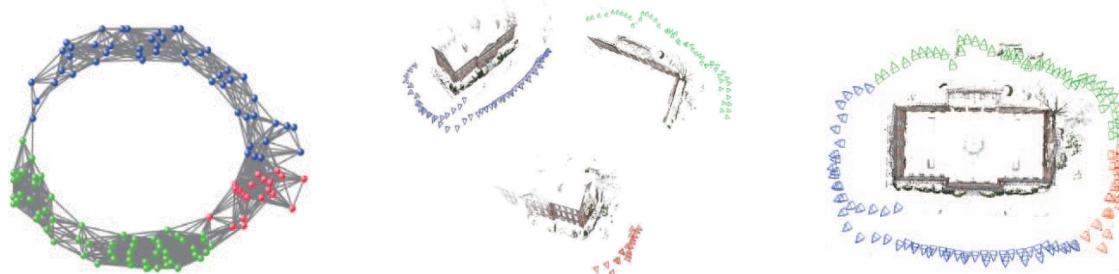
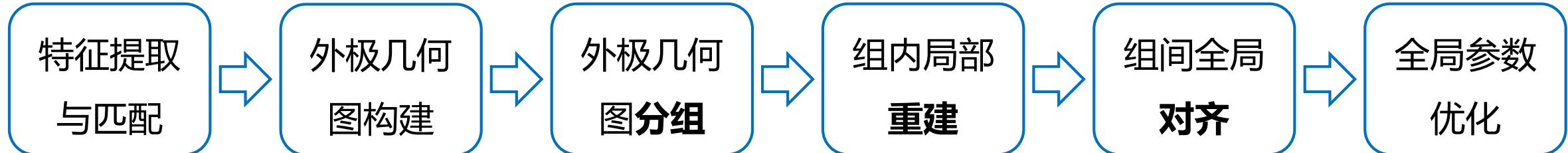
- 外极几何图分组：① 加权方式；② 分组方式
- 组内局部重建：① 增量式求解；② 全局式求解
- 组间全局对齐：① 相对变换估计方式；② 绝对变换估计方式

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分布式SfM方法

- 基本流程



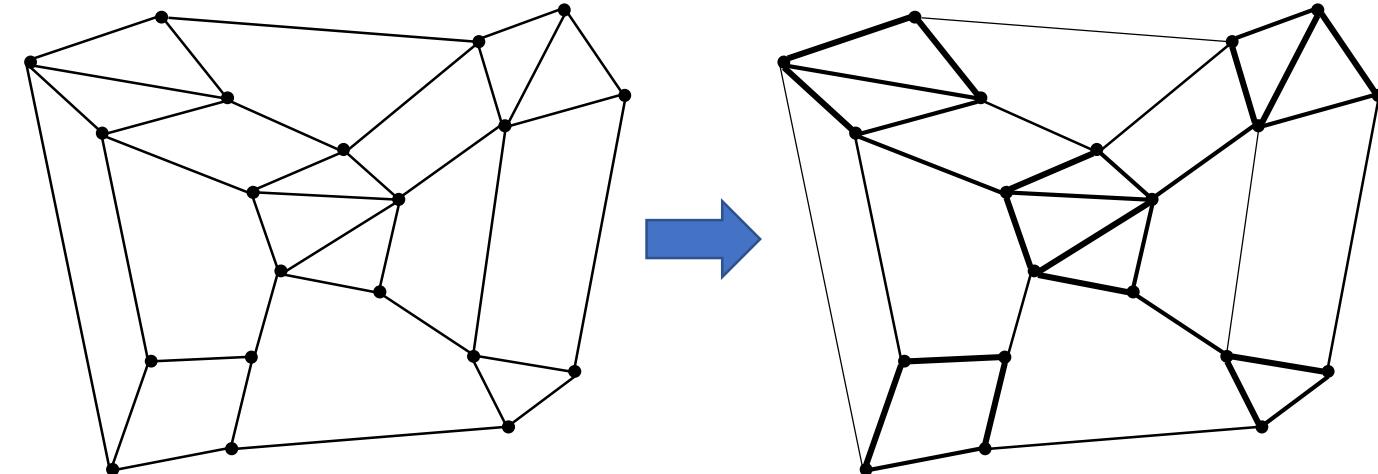
- SfM方法

- [5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. [In Proc. ACCV 2014](#).
- [6] Cui H, Shen S, Gao X, et al. CSfM: Community-based structure from motion. [In Proc. ICIP 2017](#).
- [7] Zhu S, Zhang R, Zhou L, et al. Very large-scale global SfM by distributed motion averaging. [In Proc. CVPR 2018](#).
- [8] Locher A, Havlena M, Van Gool L. Progressive structure from motion. [In Proc. ECCV 2018](#).
- [9] Fang M, Pollok T, Qu C. Merge-SfM: Merging partial reconstructions. [In Proc. BMVC 2019](#).
- [10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. [Pattern Recognition, 2020](#).
- [11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. [ISPRS P&RS, 2021](#).
- [12] Chen Y, Yu Z, Song S, et al. AdaSfM: From coarse global to fine incremental adaptive structure from motion. [In Proc. ICRA 2023](#).
- [13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. [IEEE TGRS, 2022](#).

分布式SfM方法

• 加权方式

- 基于词汇树视觉相似度: [5]
- 局部特征匹配内点对数: [7], [10], [12]
- 自定义图像对加权方式: [8], [11], [13]

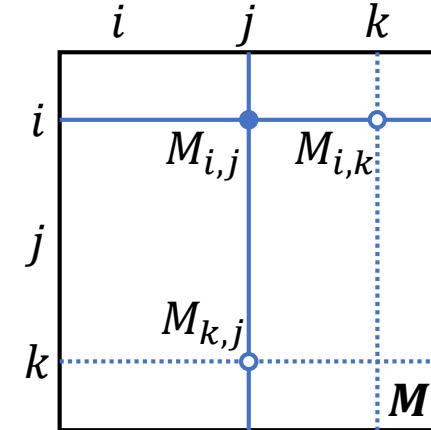


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- [11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. ISPRS P&RS, 2021.
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- [13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. IEEE TGRS, 2022.

分布式SfM方法

- 加权方式：自定义图像对加权方式
 - 衡量图像对与其它图像匹配情况的一致性：

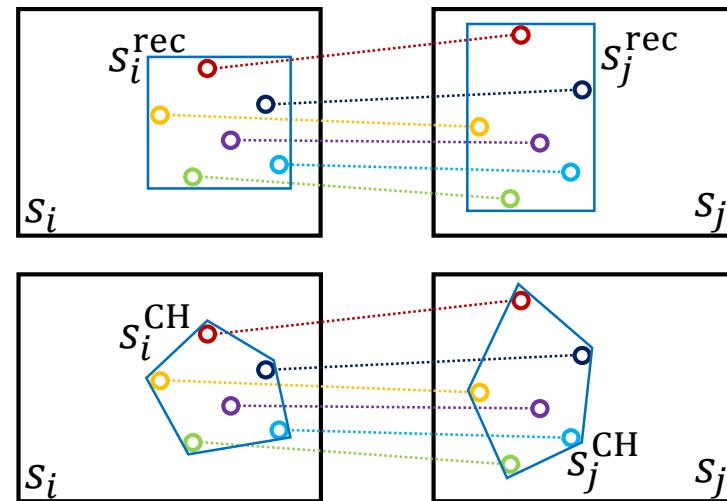
$$d_{i,j} = 1 - \frac{\sum_{n \in \mathcal{N}} M_{i,n} + M_{n,j}}{\sum_{n \in \mathcal{V}} M_{i,n} + M_{n,j}}, \mathcal{N} = \{k \mid M_{i,k} \cdot M_{k,j} > 0\}^{[8]}$$



- 衡量图像对之间匹配点数量以及分布情况：

$$\omega_{i,j} = \omega_1 \frac{M_{i,j}}{\min(\max_k M_{i,k}, \max_k M_{k,j})} + \omega_2 \frac{S_i^{\text{rec}} + S_j^{\text{rec}}}{S_i + S_j}^{[11]}$$

$$\omega_{i,j} = \omega \frac{M_{i,j}}{\max_{m,n} M_{m,n}} + (1 - \omega) \frac{S_i^{\text{CH}} + S_j^{\text{CH}}}{S_i + S_j}^{[13]}$$



[8] Locher A, Havlena M, Van Gool L. Progressive structure from motion. In Proc. ECCV 2018.

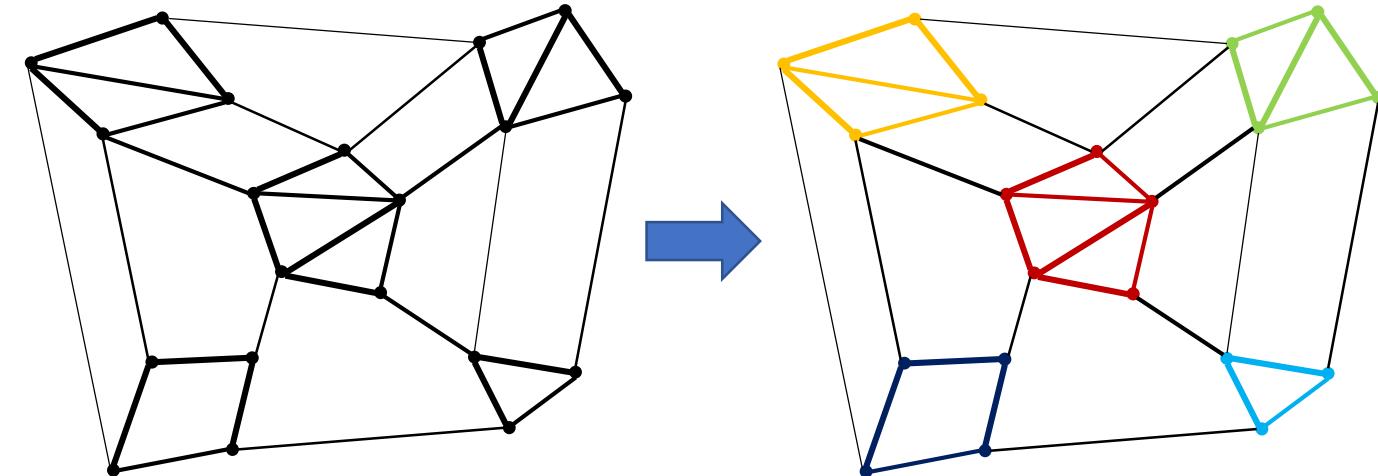
[11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. ISPRS P&RS, 2021.

[13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. IEEE TGRS, 2022.

分布式SfM方法

• 分组方式

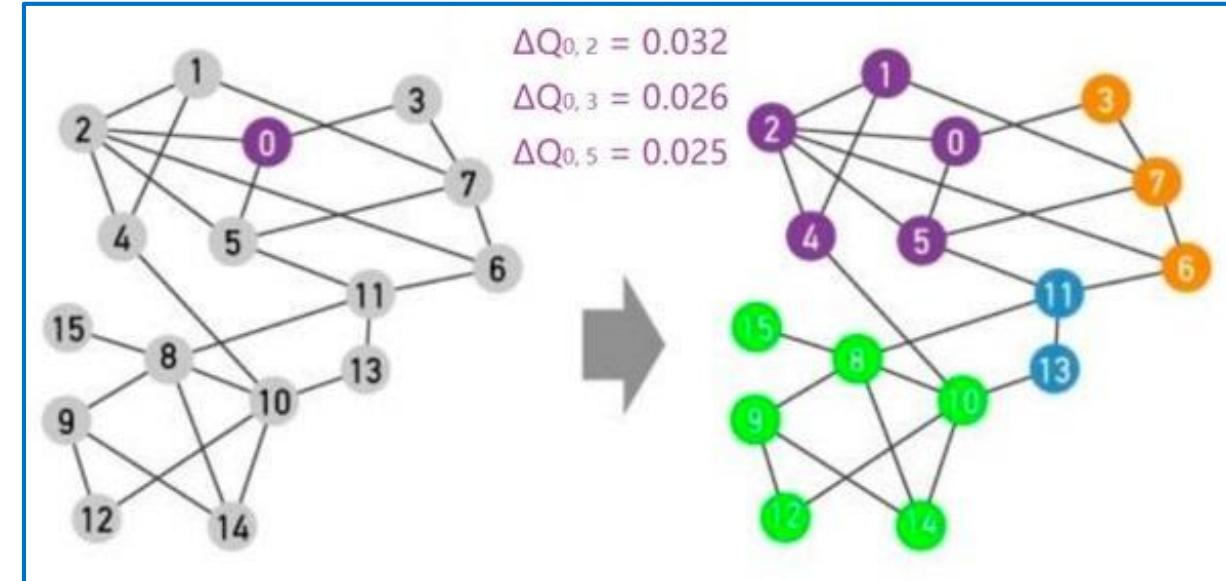
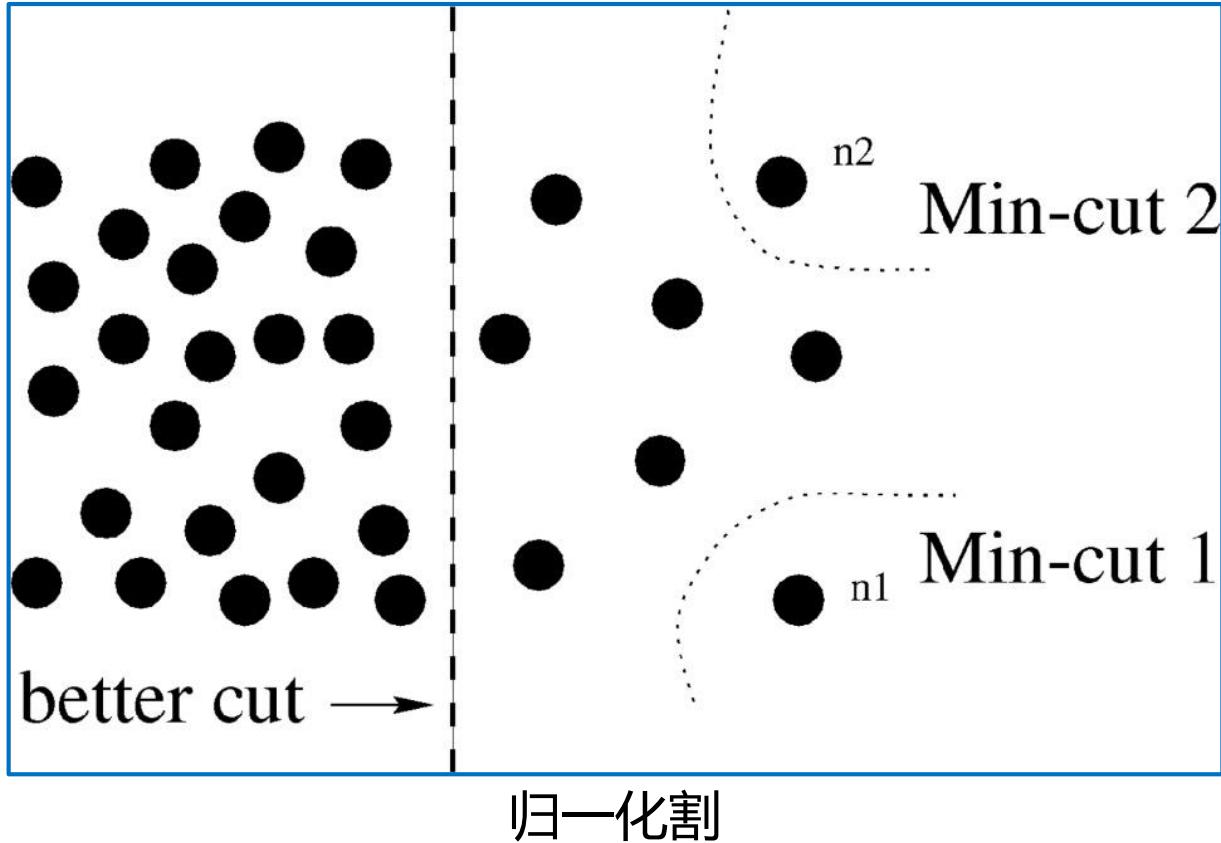
- 归一化割: [5], [7], [10], [12], [13]
- 社区发现: [6], [9]
- 单链聚类: [8], [11]



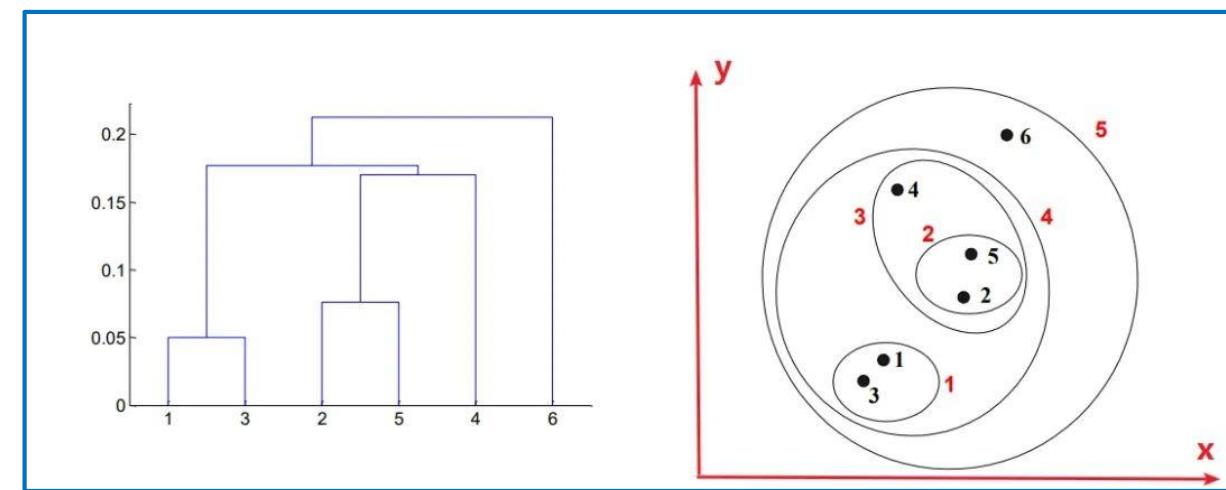
- [5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. In Proc. ACCV 2014.
- [6] Cui H, Shen S, Gao X, et al. CSfM: Community-based structure from motion. In Proc. ICIP 2017.
- [7] Zhu S, Zhang R, Zhou L, et al. Very large-scale global SfM by distributed motion averaging. In Proc. CVPR 2018.
- [8] Locher A, Havlena M, Van Gool L. Progressive structure from motion. In Proc. ECCV 2018.
- [9] Fang M, Pollok T, Qu C. Merge-SfM: Merging partial reconstructions. In Proc. BMVC 2019.
- [10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. Pattern Recognition, 2020.
- [11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. ISPRS P&RS, 2021.
- [12] Chen Y, Yu Z, Song S, et al. AdaSfM: From coarse global to fine incremental adaptive structure from motion. In Proc. ICRA 2023.
- [13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. IEEE TGRS, 2022.

分布式SfM方法

- 分组方式



社区发现

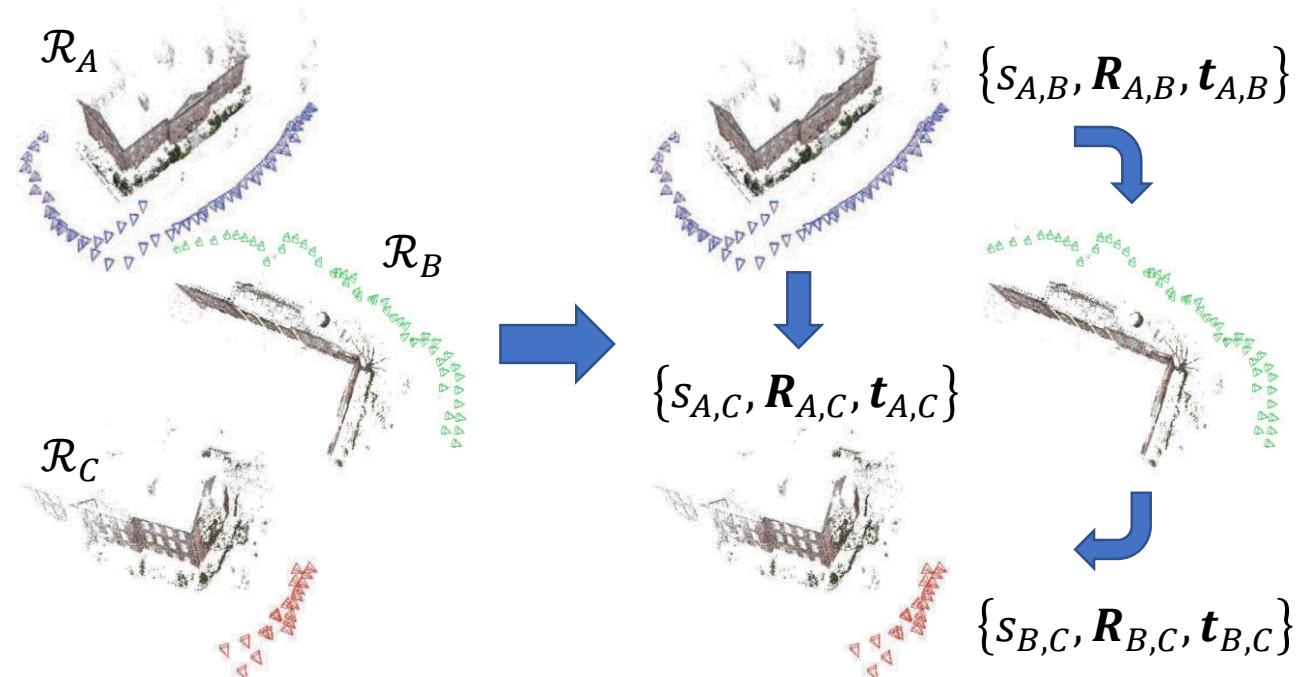


单链聚类

分布式SfM方法

• 相对变换估计方式

- 组间公共可见三维点: [6], [8], [11], [13]
- 组间连接边相对运动: [5], [7], [9]
- 组间公共相机: [10], [12]



- [5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. *In Proc. ACCV 2014*.
- [6] Cui H, Shen S, Gao X, et al. CSfM: Community-based structure from motion. *In Proc. ICIP 2017*.
- [7] Zhu S, Zhang R, Zhou L, et al. Very large-scale global SfM by distributed motion averaging. *In Proc. CVPR 2018*.
- [8] Locher A, Havlena M, Van Gool L. Progressive structure from motion. *In Proc. ECCV 2018*.
- [9] Fang M, Pollok T, Qu C. Merge-SfM: Merging partial reconstructions. *In Proc. BMVC 2019*.
- [10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. *Pattern Recognition*, 2020.
- [11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. *ISPRS P&RS*, 2021.
- [12] Chen Y, Yu Z, Song S, et al. AdaSfM: From coarse global to fine incremental adaptive structure from motion. *In Proc. ICRA 2023*.
- [13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. *IEEE TGRS*, 2022.

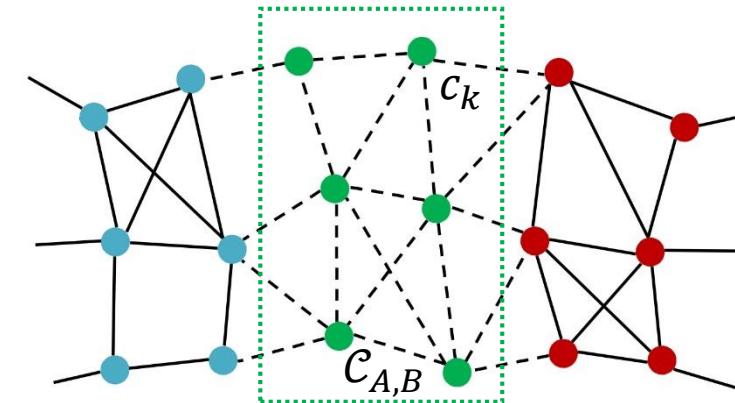
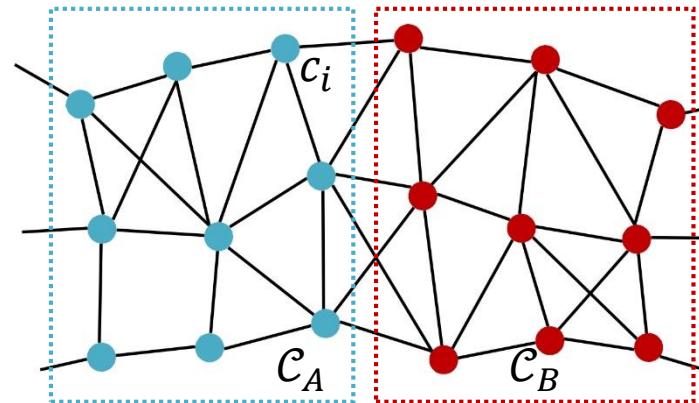
分布式SfM方法

- 相对变换估计方式：组间边上相对运动

- 相对尺度→相对旋转→相对平移^[5]

- 相对尺度估计

$$\hat{\mathbf{R}}_k^A = \operatorname{mean}_{c_i \in \mathcal{C}_A} (\mathbf{R}_{i,k} \mathbf{R}_i^A)$$



$$\hat{\mathbf{c}}_k^A = \arg \min_{\mathbf{c}_k^A} \sum_{c_i \in \mathcal{C}_A} \frac{\|[\mathbf{t}_{i,k}]_\times \hat{\mathbf{R}}_k^A (\mathbf{c}_i^A - \mathbf{c}_k^A)\|^2}{\|\hat{\mathbf{R}}_k^A (\mathbf{c}_i^A - \mathbf{c}_k^A)\|^2}$$

$$\hat{s}_{A,B} = \operatorname{median}_{c_{k_1}, c_{k_2} \in \mathcal{C}_{A,B}} \frac{\|\mathbf{c}_{k_2}^B - \mathbf{c}_{k_1}^B\|}{\|\mathbf{c}_{k_2}^A - \mathbf{c}_{k_1}^A\|}$$

[5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. In Proc. ACCV 2014.

分布式SfM方法

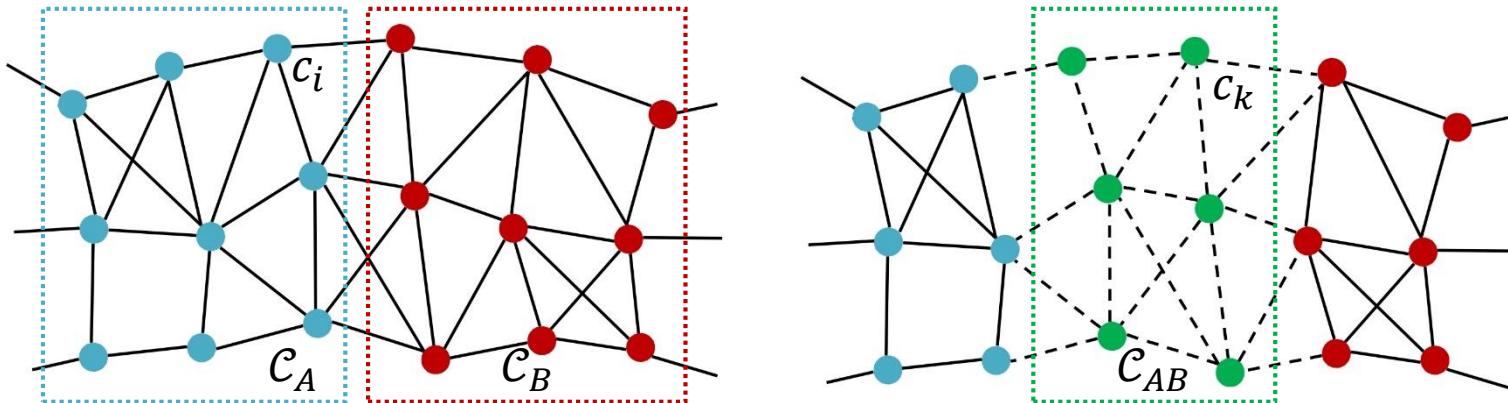
- 相对变换估计方式：组间边上相对运动

- 相对尺度→相对旋转→相对平移^[5]

- 相对旋转估计

$$\hat{\mathbf{R}}_{A,B} = \underset{c_k \in \mathcal{C}_{A,B}}{\text{mean}} \left(\hat{\mathbf{R}}_k^B {}^\top \hat{\mathbf{R}}_k^A \right)$$

- 相对平移估计



$$\hat{\mathbf{t}}_{A,B} = \arg \min_{\mathbf{t}_{A,B}} \sum_{c_k \in \mathcal{C}_{A,B}} \left\| \mathbf{t}_{A,B} - \hat{\mathbf{c}}_k^B + \hat{s}_{A,B} \hat{\mathbf{R}}_{A,B} \hat{\mathbf{c}}_k^A \right\|_1$$

[5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. In Proc. ACCV 2014.

分布式SfM方法

- 相对变换估计方式：组间边上相对运动

- 相对旋转→相对尺度、平移^[9]

- 相对旋转估计

$$\mathbf{R}_{A,B} = \mathbf{R}_j^B \mathbf{R}_{i,j} \mathbf{R}_i^A \Rightarrow \mathbf{R}_j^B \mathbf{R}_{A,B} = \mathbf{R}_{i,j} \mathbf{R}_i^A$$

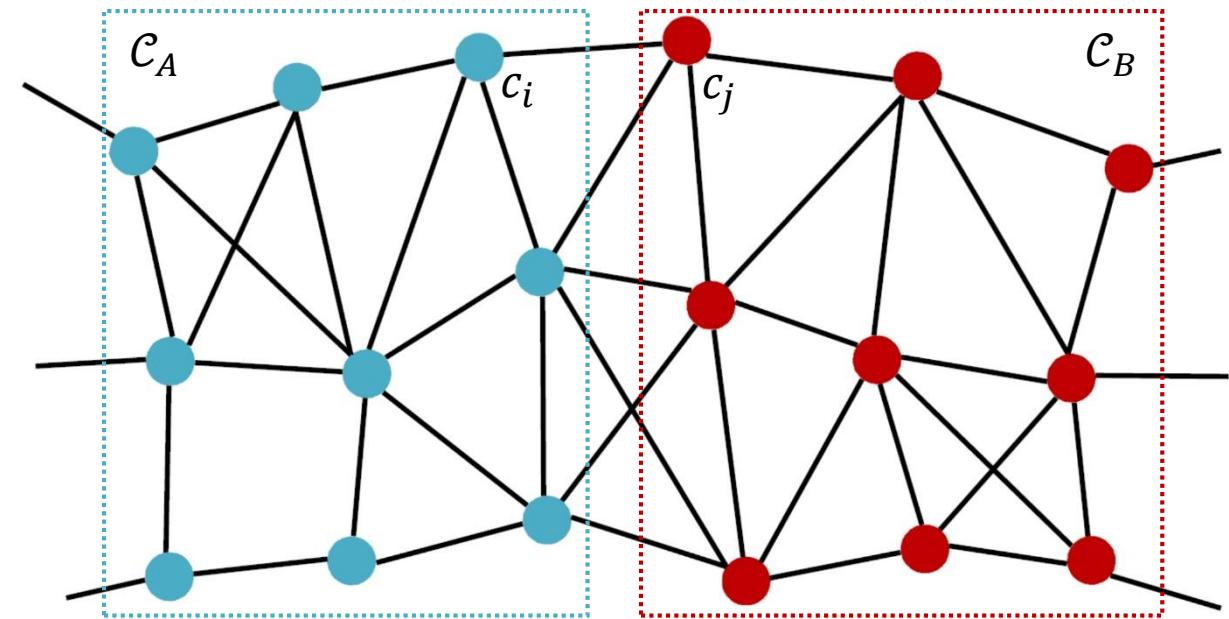
$$\Rightarrow \widehat{\mathbf{R}}_{A,B} = \arg \min_{\mathbf{R}_{A,B}} \|\mathbf{A}\mathbf{R}_{A,B} - \mathbf{b}\|_1$$

- 相对尺度、平移估计

$$\lambda_{i,j} \mathbf{t}_{i,j} = \mathbf{R}_j^B (\mathbf{c}_i^B - \mathbf{c}_j^B) = \mathbf{R}_j^B (s_{A,B} \widehat{\mathbf{R}}_{A,B} \mathbf{c}_i^A + \mathbf{t}_{A,B} - \mathbf{c}_j^B)$$

$$\Rightarrow \lambda_{i,j} \mathbf{t}_{i,j} - \mathbf{R}_j^B \mathbf{t}_{A,B} - s_{A,B} \mathbf{R}_j^B \widehat{\mathbf{R}}_{A,B} \mathbf{c}_i^A = -\mathbf{R}_j^B \mathbf{c}_j^B$$

$$\Rightarrow \{\widehat{\lambda}_{i,j}\}, \widehat{s}_{A,B}, \widehat{\mathbf{t}}_{A,B} = \arg \min_{\{\lambda_{i,j}\}, s_{A,B}, \mathbf{t}_{A,B}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_1$$



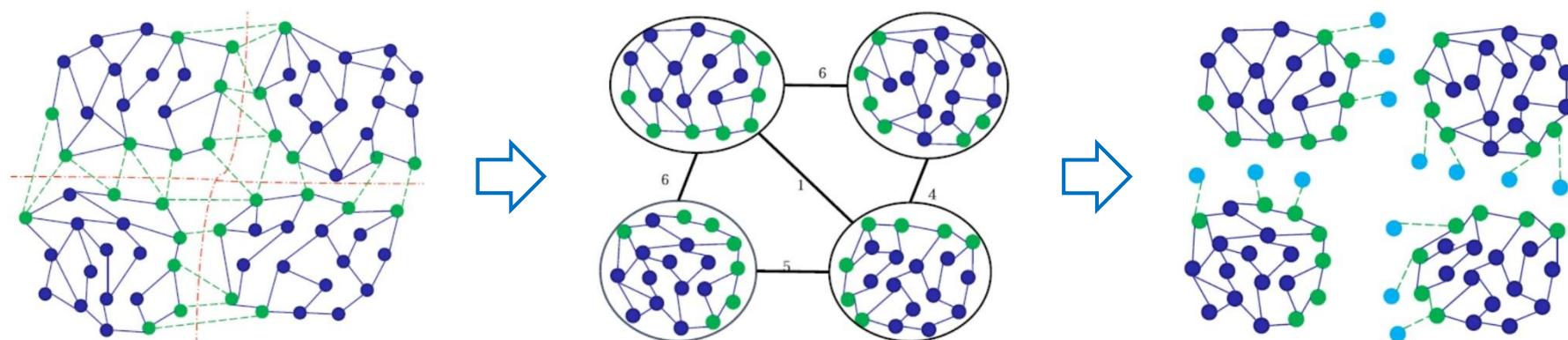
[9] Fang M, Pollok T, Qu C. Merge-SfM: Merging partial reconstructions. In Proc. BMVC 2019.

分布式SfM方法

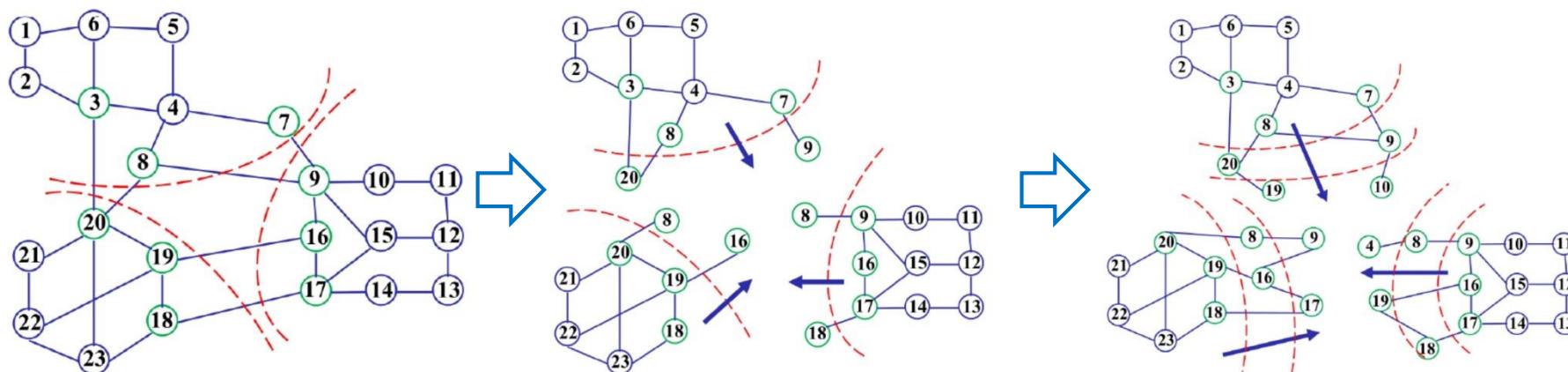
- 相对变换估计方式：组间公共相机

- 子集拓展方式

- 一次性拓展^[10]



- 层级式拓展^[12]



[10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. *Pattern Recognition*, 2020.

[12] Chen Y, Yu Z, Song S, et al. AdaSfM: From coarse global to fine incremental adaptive structure from motion. In Proc. ICRA 2023.

分布式SfM方法

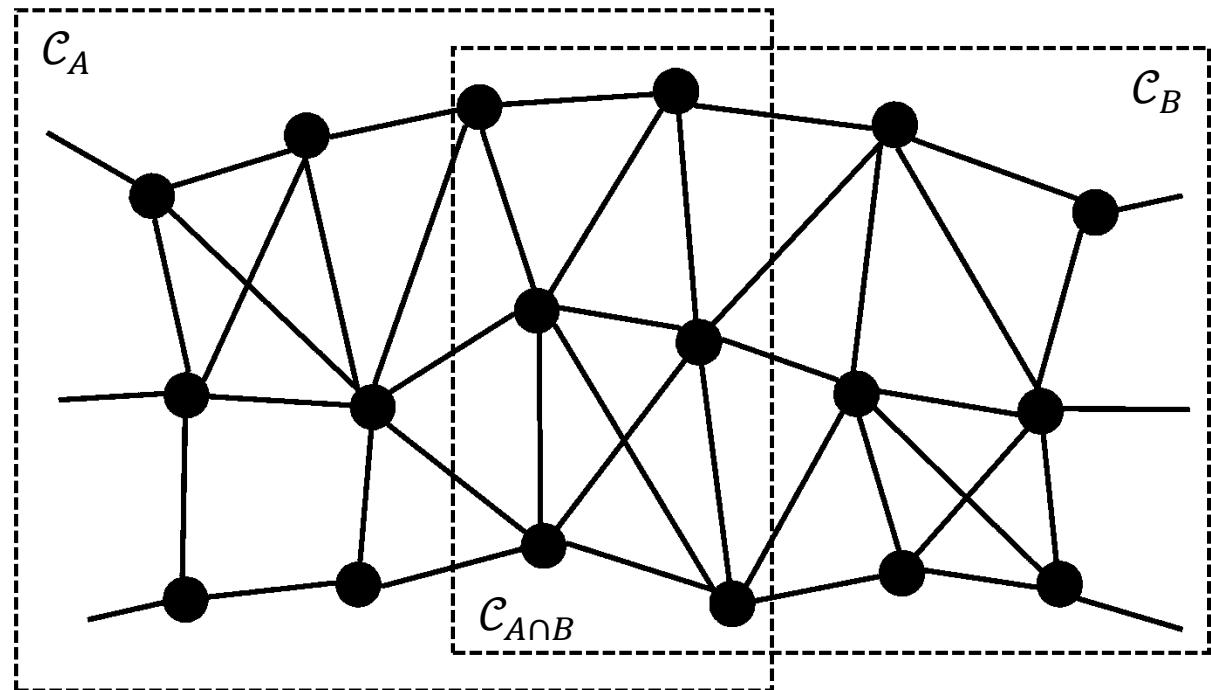
- 相对变换估计方式：组间公共相机^[10]

- 相对尺度估计

$$\hat{s}_{A,B} = \underset{c_{k_1}, c_{k_2} \in \mathcal{C}_{A \cap B}}{\text{median}} \frac{\|c_{k_2}^B - c_{k_1}^B\|}{\|c_{k_2}^A - c_{k_1}^A\|}$$

- 欧式变换估计

$$\hat{\mathbf{R}}_{A,B}, \hat{\mathbf{t}}_{A,B} = \underset{c_k \in \mathcal{C}_{A \cap B}}{\text{RANSAC}}(c_k^B = \hat{s}_{A,B} \mathbf{R}_{A,B} c_k^A + \mathbf{t}_{A,B})$$

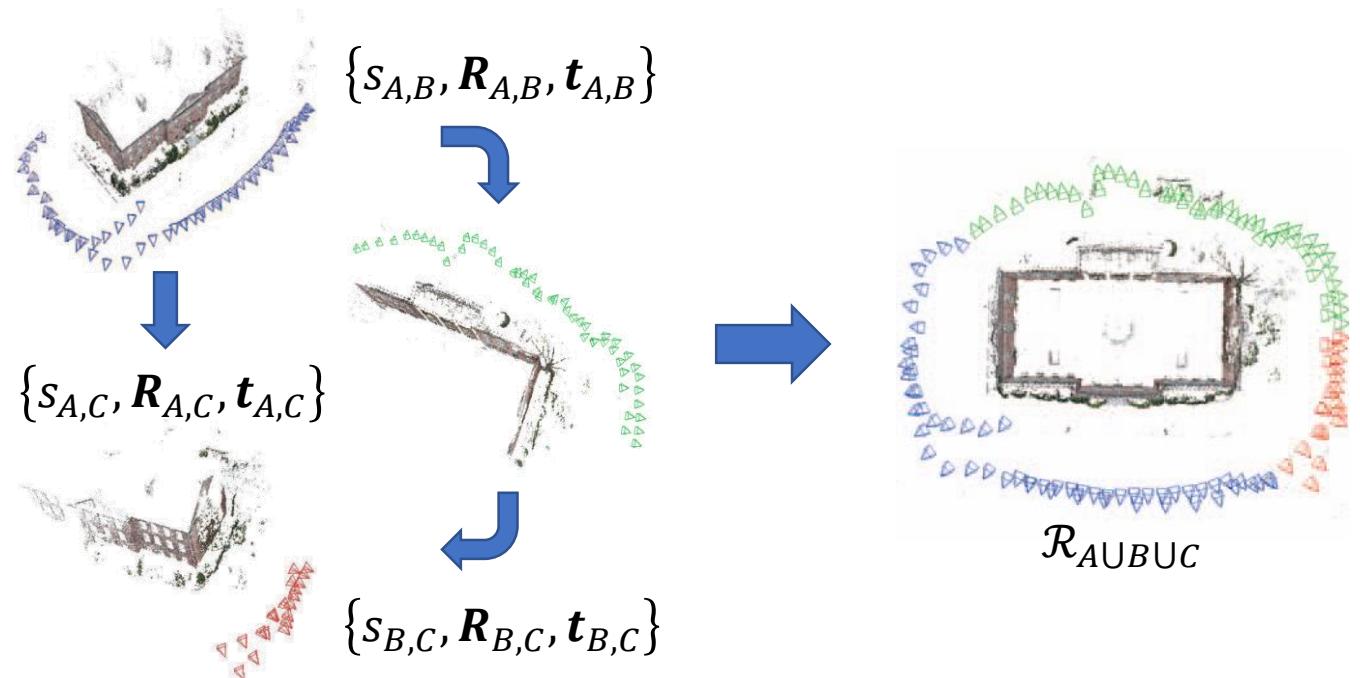


[10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. *Pattern Recognition*, 2020.

分布式SfM方法

• 绝对变换估计方式

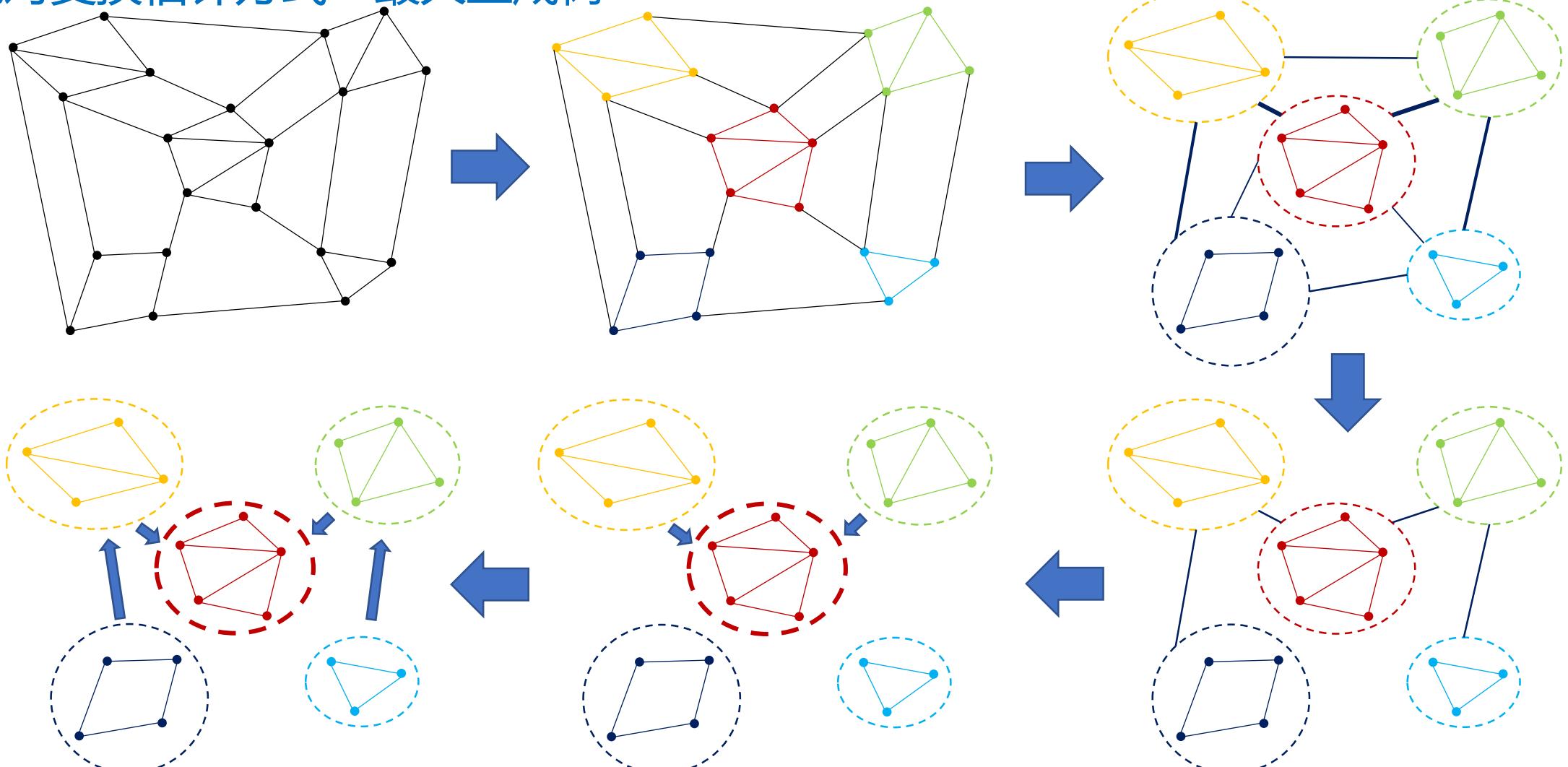
- 最大生成树: [9]
- 增量式扩展: [10], [11]
- 全局对齐基准: [5], [7], [12], [13]
- 运动平均方法: [6], [8]



- [5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. In Proc. ACCV 2014.
- [6] Cui H, Shen S, Gao X, et al. CSfM: Community-based structure from motion. In Proc. ICIP 2017.
- [7] Zhu S, Zhang R, Zhou L, et al. Very large-scale global SfM by distributed motion averaging. In Proc. CVPR 2018.
- [8] Locher A, Havlena M, Van Gool L. Progressive structure from motion. In Proc. ECCV 2018.
- [9] Fang M, Pollok T, Qu C. Merge-SfM: Merging partial reconstructions. In Proc. BMVC 2019.
- [10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. Pattern Recognition, 2020.
- [11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. ISPRS P&RS, 2021.
- [12] Chen Y, Yu Z, Song S, et al. AdaSfM: From coarse global to fine incremental adaptive structure from motion. In Proc. ICRA 2023.
- [13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. IEEE TGRS, 2022.

分布式SfM方法

- 绝对变换估计方式：最大生成树^[9]

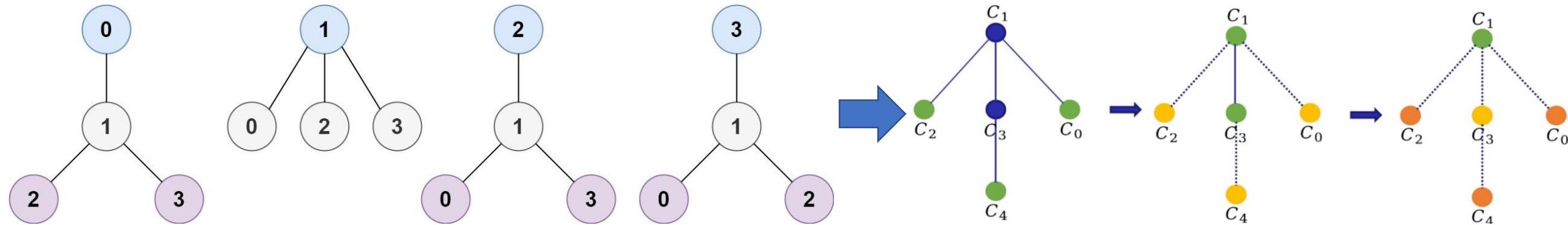


[9] Fang M, Pollok T, Qu C. Merge-SfM: Merging partial reconstructions. In Proc. BMVC 2019.

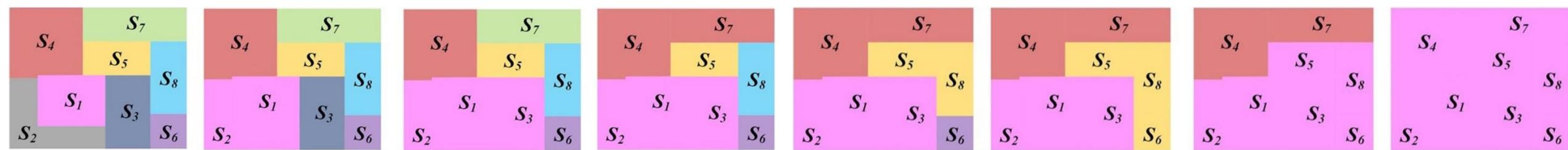
分布式SfM方法

- 绝对变换估计方式：增量式扩展

- 最小高度树^[10]



- 对齐序列表^[11]



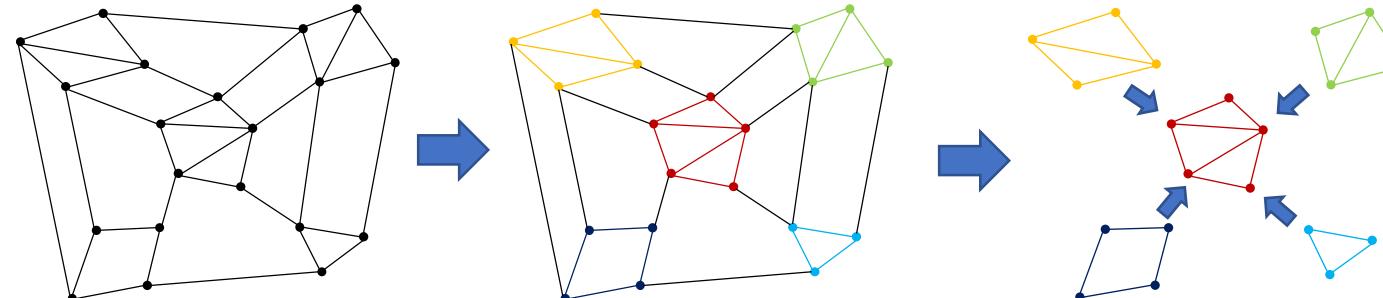
[10] Chen Y, Shen S, Chen Y, et al. Graph-based parallel large scale structure from motion. *Pattern Recognition*, 2020.

[11] Xu B, Zhang L, Liu Y, et al. Robust hierarchical structure from motion for large-scale unstructured image sets. *ISPRS P&RS*, 2021.

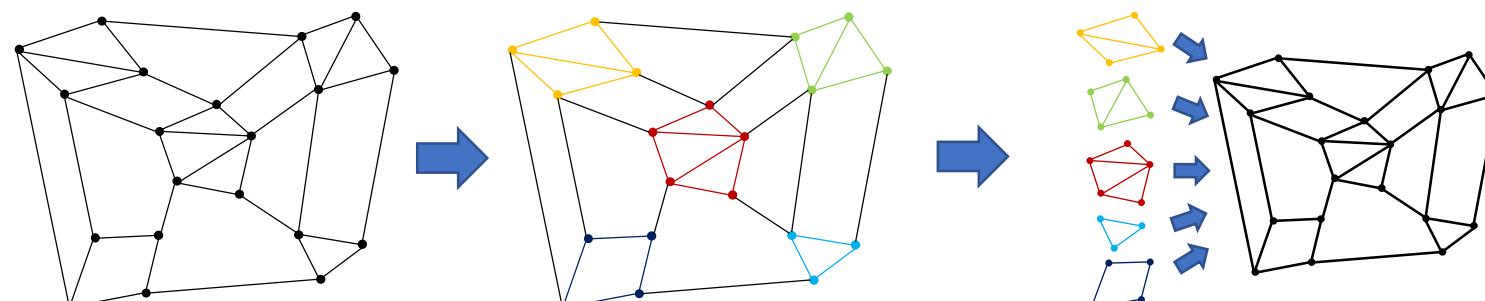
分布式SfM方法

- 绝对变换估计方式：全局对齐基准

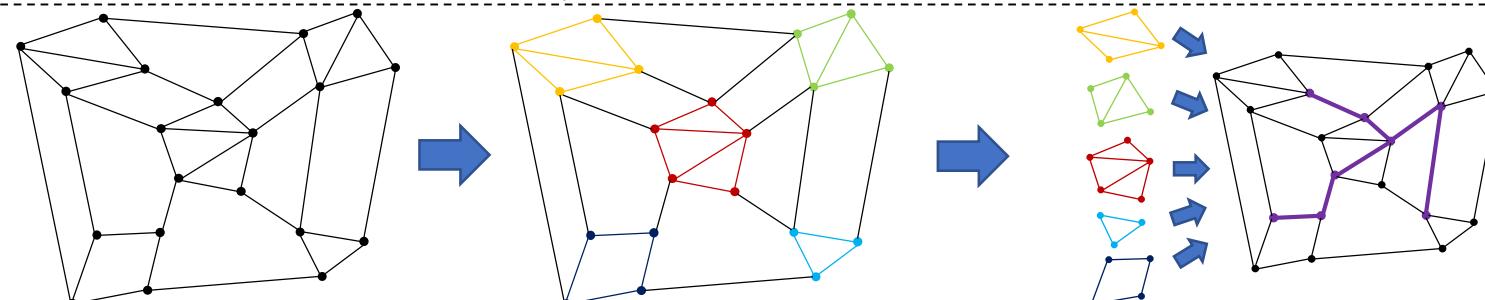
- 最大分组子集^[5]



- 全体相机集合^[12]



- 连通支配集合^[13]



[5] Bhowmick B, Patra S, Chatterjee A, et al. Divide and conquer: Efficient large-scale structure from motion using graph partitioning. In Proc. ACCV 2014.

[12] Chen Y, Yu Z, Song S, et al. AdaSfM: From coarse global to fine incremental adaptive structure from motion. In Proc. ICRA 2023.

[13] Jiang S, Li Q, Jiang W, et al. Parallel structure from motion for UAV images via weighted connected dominating set. IEEE TGRS, 2022.

分布式SfM方法

- 绝对变换估计方式：运动平均方法

- 多阶段运动平均^[6]: 尺度平均→旋转平均→平移平均
 - 尺度平均: $s_i/s_j = s_{i,j} \Rightarrow \log(s_i) - \log(s_j) = \log(s_{i,j}) \Rightarrow \{s_i^*\} = \arg \min_{\{s_i\}} \|A_s x_s - b_s\|_1$
 - 旋转平均: $R_j^\top R_i = R_{i,j} \Rightarrow \omega_i - \omega_j = \omega_{i,j} \Rightarrow \{R_i^*\} = \arg \min_{\{R_i\}} \|A_\omega x_\omega - b_\omega\|_1$
 - 平移平均: $s_j R_j^\top (t_i - t_j) = t_{i,j} \Rightarrow t_i - t_j = \frac{1}{s_j} R_j t_{i,j} \Rightarrow \{t_i^*\} = \arg \min_{\{t_i\}} \|A_t x_t - b_t\|_1$
- 一次性运动平均^[8]: $\{T_i^*\} = \arg \min_{\{T_i\}} \|\delta(T_j^{-1} T_i T_{i,j}^{-1})\|_2$

[6] Cui H, Shen S, Gao X, et al. CSfM: Community-based structure from motion. In Proc. ICIP 2017.

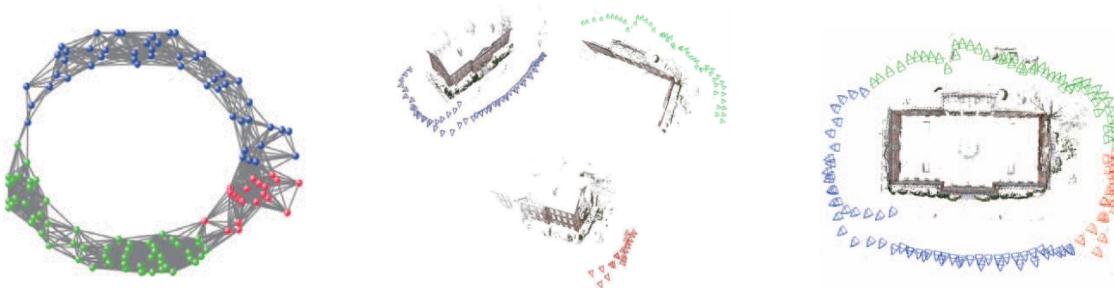
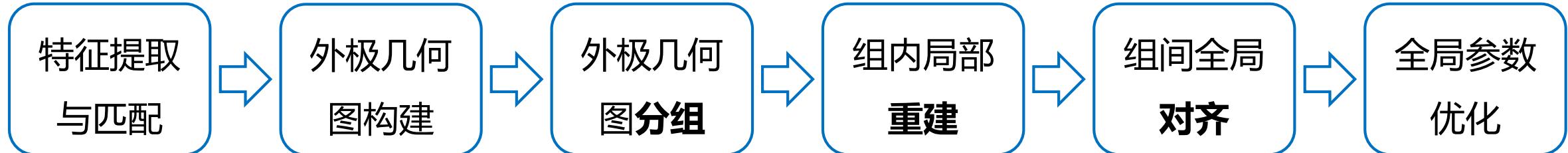
[8] Locher A, Havlena M, Van Gool L. Progressive structure from motion. In Proc. ECCV 2018.

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 - 分布式SfM方法
 - 分布式旋转平均方法
- 自底向上聚合的层级式方法

分布式旋转平均方法

- 基本流程



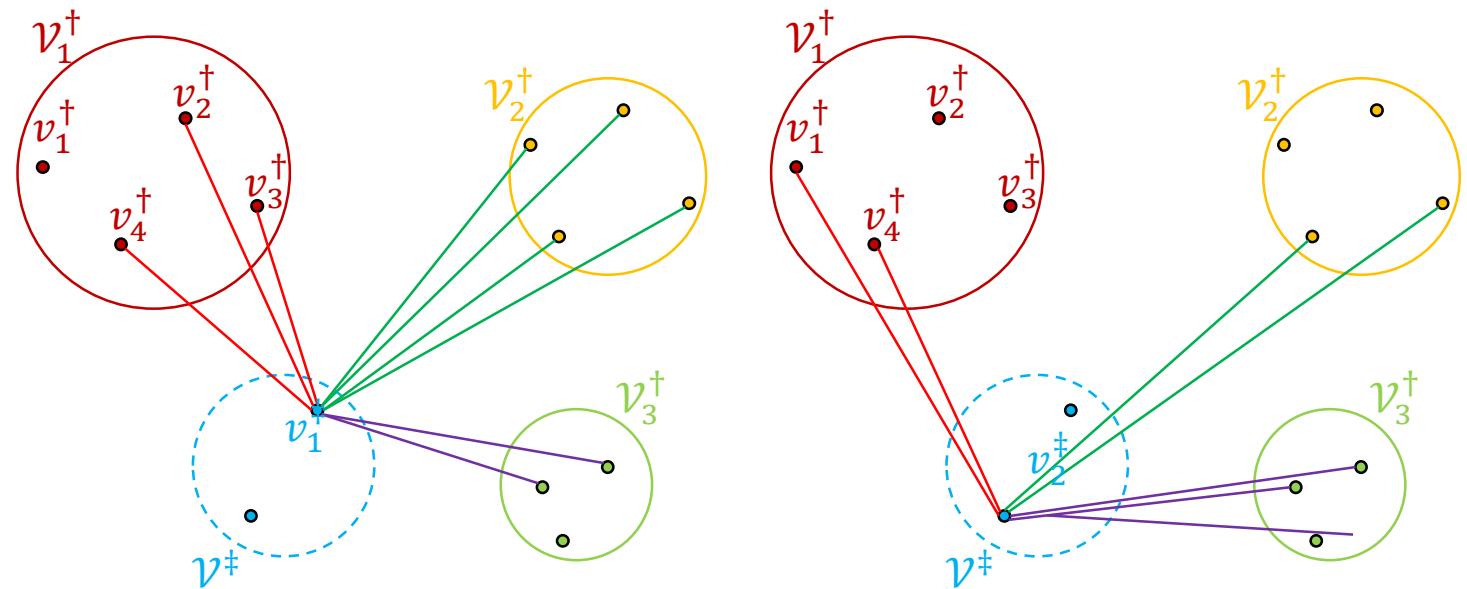
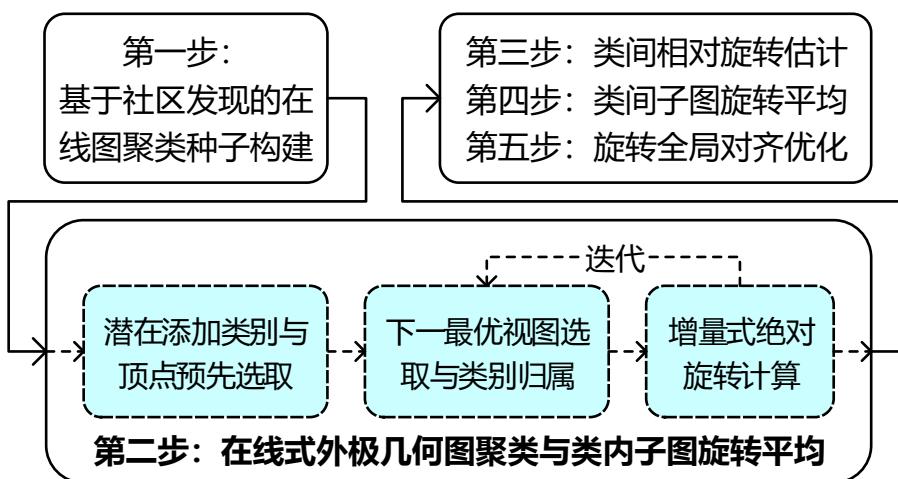
- 旋转平均方法

- [1] Cui H, Gao X, Shen S, et al. HSfM: Hybrid structure-from-motion. In Proc. CVPR 2017.
- [14] Gao X, Luo J, Li K, et al. Hierarchical RANSAC-based rotation averaging. IEEE SPL, 2020.
- [15] Gao X, Zhu L, Cui H, et al. IRA++: Distributed incremental rotation averaging. IEEE TCSVT, 2022.
- [16] Gao X, Cui H, Li M, et al. IRAv3: Hierarchical incremental rotation averaging on the fly. IEEE TCSVT, 2023.
- [17] Gao X, Cui H, Huang W, et al. IRAv3+: Hierarchical incremental rotation averaging via multiple connected dominating sets. IEEE TCSVT, 2024.

分布式旋转平均方法

• 分组方式

- 社区发现: [1], [14], [15]
- 任务驱动: [16], [17]



[1] Cui H, Gao X, Shen S, et al. HSfM: Hybrid structure-from-motion. In Proc. CVPR 2017.

[14] Gao X, Luo J, Li K, et al. Hierarchical RANSAC-based rotation averaging. IEEE SPL, 2020.

[15] Gao X, Zhu L, Cui H, et al. IRA++: Distributed incremental rotation averaging. IEEE TCSVT, 2022.

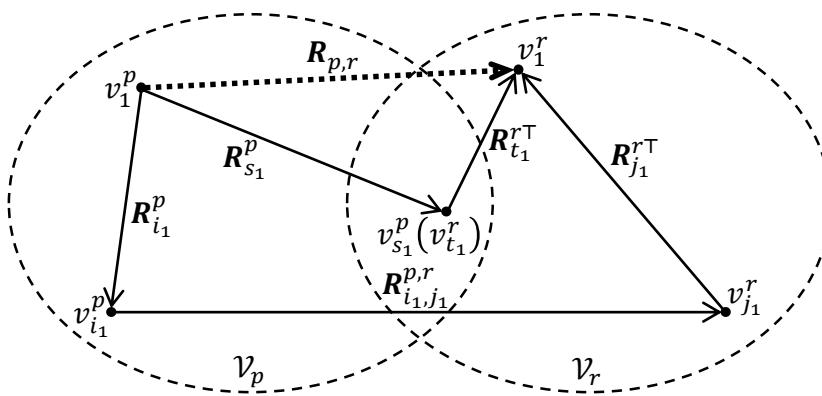
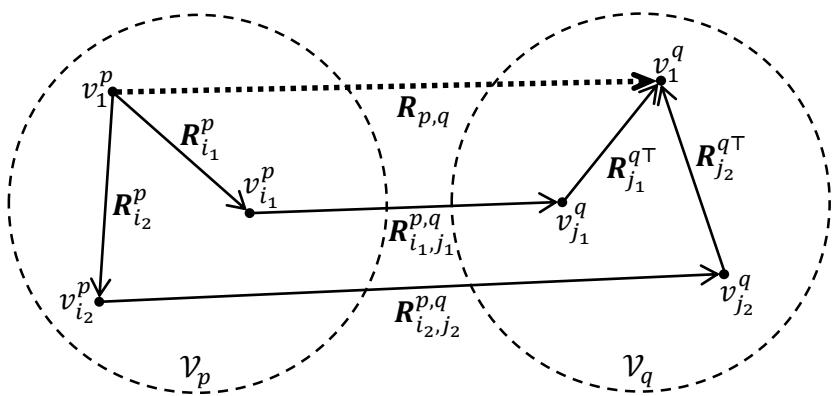
[16] Gao X, Cui H, Li M, et al. IRAv3: Hierarchical incremental rotation averaging on the fly. IEEE TCSVT, 2023.

[17] Gao X, Cui H, Huang W, et al. IRAv3+: Hierarchical incremental rotation averaging via multiple connected dominating sets. IEEE TCSVT, 2024.

分布式旋转平均方法

- 相对变换估计方式

- 组间连接边相对运动: [1], [14], [15], [16]
- 公共相机引导连接边: [17]

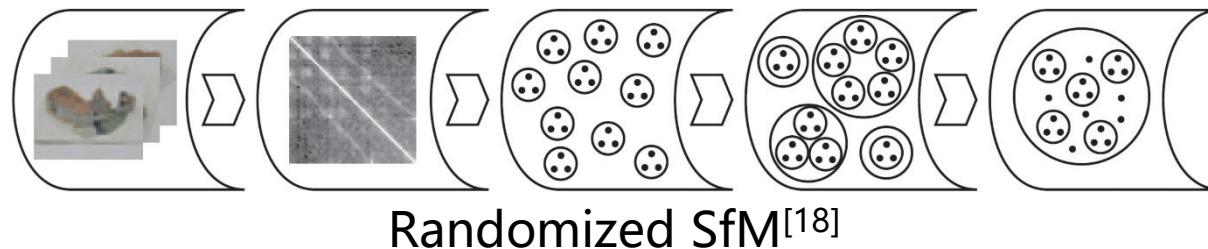


- [1] Cui H, Gao X, Shen S, et al. HSfM: Hybrid structure-from-motion. In Proc. CVPR 2017.
- [14] Gao X, Luo J, Li K, et al. Hierarchical RANSAC-based rotation averaging. IEEE SPL, 2020.
- [15] Gao X, Zhu L, Cui H, et al. IRA++: Distributed incremental rotation averaging. IEEE TCSVT, 2022.
- [16] Gao X, Cui H, Li M, et al. IRAv3: Hierarchical incremental rotation averaging on the fly. IEEE TCSVT, 2023.
- [17] Gao X, Cui H, Huang W, et al. IRAv3+: Hierarchical incremental rotation averaging via multiple connected dominating sets. IEEE TCSVT, 2024.

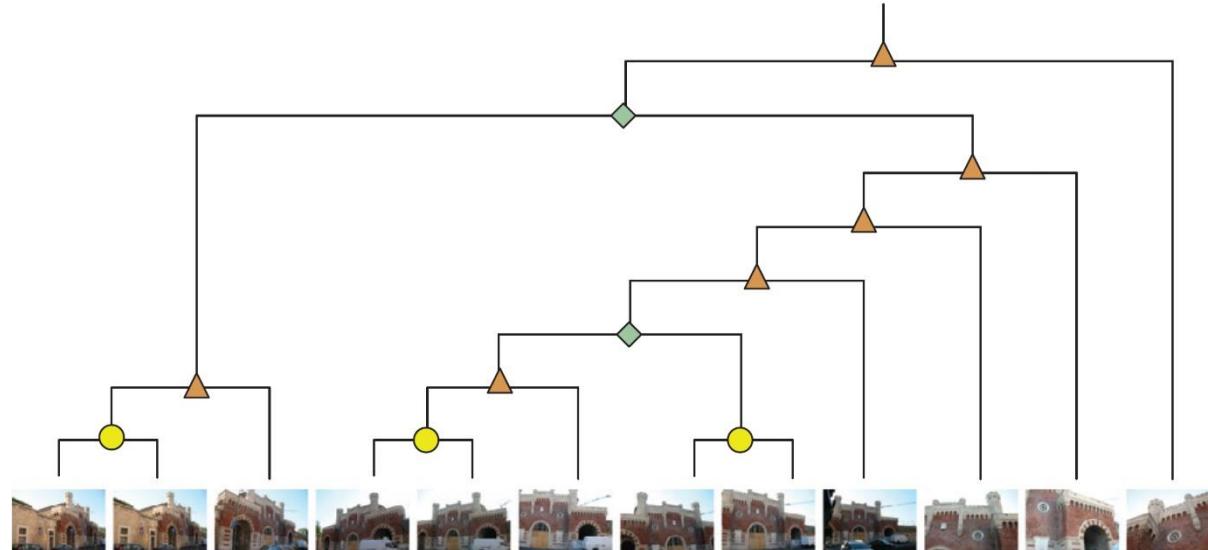
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 - 分布式SfM方法
 - 分布式旋转平均方法
- 自底向上聚合的层级式方法

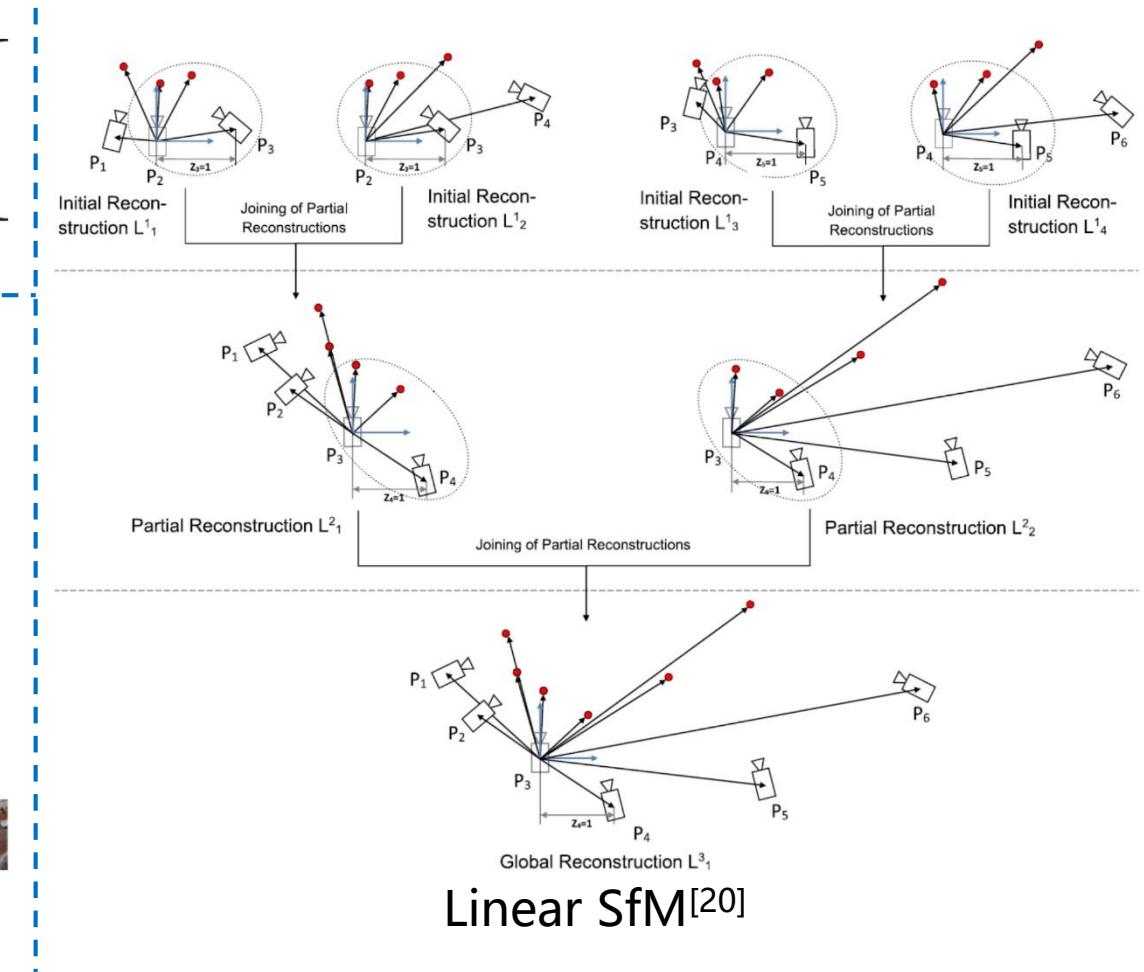
层级式方法



Randomized SfM^[18]



Hierarchical SfM^[19]



Linear SfM^[20]

[18] Havlena M, Torii A, Knopp J, et al. Randomized structure from motion based on atomic 3D models from camera triplets. In Proc. CVPR 2009.

[19] Toldo R, Gherardi R, Farenzena M, et al. Hierarchical structure-and-motion recovery from uncalibrated images. CVIU, 2015.

[20] Zhao L, Huang S, Dissanayake G. Linear SfM: A hierarchical approach to solving structure-from-motion problems by decoupling the linear and nonlinear components. ISPRS P&RS, 2018.

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- [1] Cui H, Gao X, Shen S, et al. HSfM: Hybrid structure-from-motion. In Proc. CVPR 2017.
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Thanks for your attention!

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