Types and Programming Language Project Report*

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Introduction

In current Internet environment, users tend to share more personal data online. This phenomenon makes it increasingly important for applications to protect confidentiality. For existing approaches to achieve privacy control, programmers are forced to ensure compliance by their own efforts, even when both the application and the policies may be evolving rapidly. This can cause considerable burdens to application developers.

This academic paper, called A Language for Automatically Enforcing Privacy Policies.¹, proposes a new programming model that makes the system responsible for automatically producing outputs consistent with programmer-specified policies. This automation makes it easier for programmers to enforce policies specifying how each sensitive value should be displayed in a given context, therefore solves the problem mentioned above. Furthermore, they have implemented this programming model in a new functional constraint language named **Jeeves**.

We carried out our course project based on this paper. More specifically, we first read and comprehended this paper, including the language design, the semantics, the evaluation and typing rules, the partial property proof and the implementation details as we did in our class. Then we proved the progress and preservation parts which is left out in the paper by ourselves. Finally, we successfully run the implementation codes provided by the authors and wrote our own use cases to understand the implementation details.

1 Language Design

1.1 Jeeves

Jeeves allows the programmer to specify policies explicitly and upon data creation rather than implicitly across the code base. The Jeeves system trusts the programmer to correctly specify policies describing high- and low-confidentiality views of sensitive values and to correctly provide context values characterizing output channels. Figure 1 shows the jeeves syntax, which looks like a high-level language since it's based on and translated from $\lambda_{\rm J}$ described in next subsection.

Several key words in this syntax tell what Jeeves wants to do and what it is capable of doing.

• Level provides variables the means of abstraction to specify policies incrementally and independently of the sensitive value declaration. Level variables can be constrained directly (by explicitly passing around a level variable) or indirectly (by constraining another level variable when there is a dependency).

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 $^{^{1}}$ http://www.cs.cmu.edu/ jyang2/papers/popl088-yang.pdf

- Policies, introduced through policy expressions, provide declarative rules describing when to set a level variable to top or bottom.
- Context construct relieves the programmer of the burden of structuring code to propagate values from the output context to the policies. Statements such as print that release information to the viewer require a context parameter.

```
 \begin{array}{lll} Level & ::= & \perp \mid \top \\ Exp & ::= & v \mid Exp_1 \; (op) \; Exp_2 \\ & \mid \; \mathbf{if} \; Exp_1 \; \mathbf{then} \; Exp_t \; \mathbf{else} \; Exp_f \\ & \mid \; Exp_1 \; Exp_2 \\ & \mid \; \langle Exp_{\perp} \mid Exp_{\top} \rangle \left( \ell \right) \\ & \mid \; \mathbf{level} \; \ell \; \mathbf{in} \; Exp \\ & \mid \; \mathbf{policy} \; \ell : \; Exp_p \; \mathbf{then} \; Level \; \mathbf{in} \; Exp \\ Stmt \; ::= & \quad \mathbf{let} \; x : \tau \; = \; Exp \\ & \mid \; \mathbf{print} \; \{Exp_c\} \; Exp \end{array}
```

Figure 1: Jeeves syntax

1.2 Lambda J

One interesting thing is the authors don't formally implement Jeeves from the scratch. Instead, they introduce λ_J , a simple constraint functional language based on the λ -calculus, and then they show how to translate Jeeves from λ_J . Here we describe the λ_J language show in Figure 2.

Basically, The λ_J language extends the λ -calculus with logical variables. Expressions (e) include the standard λ expressions extended with the **defer** construct for introducing logic variables, the **assert** construct for introducing constraints, and the **concretize** construct for producing concrete values consistent with the constraints. λ_J evaluation produces irreducible values (v), which are either concrete (c) or symbolic (σ) . Concrete values are what one would expect from λ -calculus, while symbolic values are values that cannot be reduced further due to the presence of logic variables. Symbolic values also include the **context** construct which allows constraints to refer to a value supplied at concretization time. The context variable is an implicit parameter provided in the **concretize** expression. In the semantics we model the behavior of the context variable as a symbolic value that is constrained during evaluation of concretize. λ_J contains a **let** rec construct that handles recursive functions in the standard way using **fix**.

A novel feature of λ_J is that logic variables are also associated with a default value that serves as a default assumption: this is the assigned value for the logic variable unless it is inconsistent with the constraints. The purpose of default values is to provide some determinism when logic variables are underconstrained.

2 Semantics

Here is the semantics...

```
n \mid b \mid \lambda x : \tau . e \mid record \ x \overline{:} v
           | error | ()
              x \mid \mathbf{contex} \ \tau
\sigma ::=
           \mid c_1 (op) \sigma_2 \mid \sigma_1 (op) c_2 \mid
           \mid \sigma_1 (op) \sigma_2 \mid
           | if \sigma then v_t else v_f
v ::= c \mid \sigma
             v \mid e_1 \ (op) \ e_2
e ::=
           | if e_1 then e_t else e_f | e_1 e_2
           | let x : \tau = e_1 in e_2
           | let rec f: \tau = e_1 in e_2
             defer x : \tau\{e\} defaut v_d
              assert e
              concretize e with v_c
```

Figure 2: $\lambda_{\rm J}$ syntax

3 Properties

Lemma 1. (ConcreteFunction). if v is a value of type $\tau_1 \to \tau_2$, then $v = \lambda x : \tau_1.e$, where e has type τ_2 .

Proof. According to the $\lambda_{\rm J}$ syntax, we can get Lemma 1 immediately.

Theorem 1. (Progress). Suppose e is a closed, well-typed expression. Then e is either a value v or there is some e' such that $\vdash \langle \Sigma, \Delta, e \rangle \to \langle \Sigma', \Delta', e' \rangle$.

Proof. According to the dynamic demantics 3 and static semantics 5 of $\lambda_{\rm J}$, we will proof that any $\lambda_{\rm J}$ program holds the *progress* property by structral induction over the syntax of $\lambda_{\rm J}$. According to Figure 2, there are nine kinds of expressions including the value expression v. Next we discuss each of them respectively.

- For the value expression v, according to the definition we can conclude that it holds the property immediately.
- For the expression e_1 (op) e_2 , according to the hyposthesis, we know that subexpression e_1 and e_2 are both closed and well-typed expressions. Thus if e_1 is not a value, according to E-OP1, there must be a expression e'_1 such that $e_1 \to e'_1$. Therefore the expression e_1 (op) e_2 can be reduced to e'_1 (op) e_2 , and the same as if the e_1 is already a value but e_2 is not according to E-OP2. While if both e_1 and e_2 are values, we know that e_1 (op) e_2 is a value as well. Let $c = e_1$ (op) e_2 , according to E-OP, $\vdash \langle \Sigma, \Delta, e_1 \ (op) \ e_2 \rangle \to \langle \Sigma', \Delta', c \rangle$. Hence, we can conclude that expression e_1 (op) e_2 holds the progress property.
- For the expression if e_1 then e_t else e_f , the conditional expression e_1 is a either a **concrete** expression or a **symbolic** expression. If expression e_1 is **concrete** and not a value, according to E-COND, there must be an expression e'_1 such

```
\mathscr{G} \vdash \langle \Sigma, \Delta, e \rangle \rightarrow \left\langle \Sigma', \Delta', e' \right\rangle
                                                                  \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e_1 \rangle \rightarrow \left\langle \Sigma', \Delta', e_1' \right\rangle}{\mathscr{G} \vdash \langle \Sigma, \Delta, e_1 \ e_2 \rangle \rightarrow \left\langle \Sigma', \Delta', e_1' \ e_2 \right\rangle} \quad \text{E-APP1} \quad \frac{\mathscr{G} \vdash \left\langle \Sigma, \Delta, e_2 \right\rangle \rightarrow \left\langle \Sigma', \Delta', e_2' \right\rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, v \ e_2 \right\rangle \rightarrow \left\langle \Sigma', \Delta', v \ e_2 \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                                                        E-APP2
                                                                                                                                                                        E-APPLAMBDA \frac{c' = c_1 \ (op) \ c_2}{\mathscr{G} \vdash \langle \Sigma, \Delta, c_1 \ (op) \ c_2 \rangle \rightarrow \langle \Sigma', \Delta', c' \rangle}
                                                                                                                                                                                                                                                                                                                                                                                                                E-OP
                    \overline{\mathscr{G} \vdash \langle \Sigma, \Delta, \lambda x. e \ \upsilon \rangle \rightarrow \langle \Sigma', \Delta', e[x \mapsto \upsilon] \rangle}
\frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e_1 \rangle \rightarrow \left\langle \Sigma', \Delta', e_1' \right\rangle}{\mathscr{G} \vdash \langle \Sigma, \Delta, e_1 \ (op) \ e_2 \rangle \rightarrow \left\langle \Sigma', \Delta', e_1' \ (op) \ e_2 \right\rangle}
                                                                                                                                                                                  E-OP1 \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e_2 \rangle \rightarrow \left\langle \Sigma', \Delta', e_2' \right\rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, \upsilon \right. \left(op\right) \left. e_2 \right\rangle \rightarrow \left\langle \Sigma', \Delta', \upsilon \right. \left(op\right) \left. e_2' \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                                                                E-OP2
                                                                                                       \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e_c \rangle \rightarrow \langle \Sigma', \Delta', e_c' \rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, \text{if } e_c \text{ then } e_t \text{ else } e_f \right\rangle \rightarrow \left\langle \Sigma', \Delta', \text{if } e_c' \text{ then } e_t \text{ else } e_f \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                                                      E-COND
                                                                                                                                                        \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e_t \rangle \rightarrow \langle \Sigma', \Delta', e_t' \rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, \text{if true then } e_t \text{ else } e_f \right\rangle \rightarrow \left\langle \Sigma', \Delta', e_t' \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                          E-CONDTRUE
                                                                                                                                               \frac{\mathscr{G} \vdash \left\langle \Sigma, \Delta, e_f \right\rangle \rightarrow \left\langle \Sigma', \Delta', e_f' \right\rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, \text{if false then } e_t \text{ else } e_f \right\rangle \rightarrow \left\langle \Sigma', \Delta', e_f' \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                         E-CONDFALSE
                                                                                  \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e_t \rangle \rightarrow \langle \Sigma', \Delta', e_t' \rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, \mathbf{if} \ \sigma \ \mathbf{then} \ e_t \ \mathbf{else} \ e_f \right\rangle \rightarrow \left\langle \Sigma', \Delta', \mathbf{if} \ \sigma \ \mathbf{then} \ e_t' \ \mathbf{else} \ e_f \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                          E-CONDSYMT
                                                                                  \frac{\mathscr{G} \vdash \left\langle \Sigma, \Delta, e_f \right\rangle \rightarrow \left\langle \Sigma', \Delta', e_f' \right\rangle}{\mathscr{G} \vdash \left\langle \Sigma, \Delta, \text{if } \sigma \text{ then } v_t \text{ else } e_f' \right\rangle \rightarrow \left\langle \Sigma', \Delta', \text{if } \sigma \text{ then } v_t \text{ else } e_f' \right\rangle}
                                                                                                                                                                                                                                                                                                                                                                           E-CONDSYMF
         \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e \rangle \rightarrow \langle \Sigma', \Delta', e' \rangle}{\mathscr{G} \vdash \langle \Sigma, \Delta, \mathbf{defer} \ x : \tau\{e\} \ \mathbf{default} \ v_d \rangle \rightarrow \langle \Sigma', \Delta', \mathbf{defer} \ x : \tau\{e'\} \ \mathbf{default} \ v_d \rangle}
                                                                                                                                                                                                                                                                                                                             E-DEFERCONSTRAINT
                                                   \frac{fresh\ x'}{\mathscr{G}\vdash \langle \Sigma, \Delta, \mathbf{defer}\ x:\tau\{e\}\ \mathbf{default}\ v_d\rangle \rightarrow \langle \Sigma\cup \{\mathscr{G}\Rightarrow v_c[x\mapsto x']\}, \Delta\cup \{\mathscr{G}\Rightarrow x'=v_d\}, x'\rangle}
                                                                                                                                                                                                                                                                                                                                                                                                 E-DEFER
                                                                                                                                               \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e \rangle {\rightarrow} \langle \Sigma', \Delta', e' \rangle}{\mathscr{G} \vdash \langle \Sigma, \Delta, \mathbf{assert} \ e \rangle {\rightarrow} \langle \Sigma', \Delta', \mathbf{assert} \ e' \rangle}
                                                                                                                                                                                                                                                                                                                            E-ASSERTCONSTRAINT
                                                                                                                                                                                                                                                                                                                                                                                             E-ASSERT
                                                                                                                                                                                                               \mathscr{G} \vdash \langle \Sigma, \Delta, \mathbf{assert} \ v \rangle \rightarrow \langle \Sigma \cup \{\mathscr{G} \Rightarrow v\}, \Delta, () \rangle
                                            \frac{\mathscr{G} \vdash \langle \Sigma, \Delta, e \rangle \rightarrow \langle \Sigma', \Delta', e' \rangle}{\mathscr{G} \vdash \langle \Sigma, \Delta, \mathbf{concretize} \ e \ \mathbf{with} \ \upsilon_c \rangle \rightarrow \langle \Sigma', \Delta', \mathbf{concretize} \ e' \ \mathbf{with} \ \upsilon_c \rangle}
                                                                                                                                                                                                                                                                                                                                               E-CONCRETIZEEXP
                                                                                                          \frac{\text{MODLE}(\Delta, \Sigma \cup \{\mathscr{G} \cap \mathbf{context} = v_c\}) = \mathscr{M} \quad c = \mathscr{M}[[v_\nu]]}{\mathscr{G} \vdash \langle \Sigma, \Delta, \mathbf{concretize} \ v_\nu \ \mathbf{with} \ v_c \rangle \rightarrow \langle \Sigma, \Delta, c \rangle}
                                                                                                                                                                                                                                                                                                                                                 E-CONCRETIZESAT
                                                                                                                   MODLE(\Delta,\Sigma \cup \{\mathscr{G} \cap \mathbf{context} = v_c\}) = UNSAT
                                                                                                                                                                                                                                                                                                                                   E-CONCRETIZEUNSAT
                                                                                                          \mathscr{G} \vdash \langle \Sigma, \Delta, \mathbf{concretize} \ \upsilon_{\nu} \ \mathbf{with} \ \upsilon_{c} \rangle \rightarrow \langle \Sigma, \Delta, \mathbf{error} \rangle
```

Figure 3: Dynamic semantics for $\lambda_{\rm J}$.

```
\delta ::= \mathbf{concretize} \mid \mathbf{sym} determinism tag
\beta ::= \mathbf{int}_c \mid \mathbf{bool}_c \mid \mathbf{unit} \mid \mathbf{int} \mid \mathbf{bool} base type
\tau ::= \beta \mid \tau_1 \xrightarrow{nr} \tau_2 \mid \tau_1 \to \tau_2 type
```

Figure 4: $\lambda_{\rm J}$ types

```
\tau_1 <: \tau_2
                                                                                                                                                                                                                                                                                                                                        S-BOOL
                                                                                                                                         S-REFLEXIVE
                                                                                                                                                                                                                                                 S-INT
                                                                                                                                                                                                                                                                                   \overline{\mathrm{bool}_c <: \mathrm{bool}}
                                                                                                                                                                                                         \overline{\mathbf{int}_c <: \mathbf{int}}
                                                                                                             \overline{\tau < : \tau}
                                                                                                                                                                                                                                                                                  \frac{\tau_1' <: \tau_1 \quad \tau_2 <: \tau_2'}{\tau_1 \rightarrow \tau_2 <: \tau_1' \rightarrow \tau_2'}
                                                                                                                                                                                                                               S-RECFUN
                                                                                                                                                                                                                                                                                                                                              S-FUN
                                                                                                                                                            \overline{\tau_1} \xrightarrow{nr} \tau_2 < : \tau_1 \rightarrow \tau_2
                                                                                                                                                                        rep \tau
                                                                                                                                                                                                                   OK-SUBTYPE
                                                                                                                                                                                                                                                                                                               OK-BASETYPE
                                                                                                                                                                                                                     \frac{\mathbf{rep}\ (\tau_1 \overset{nr}{\rightarrow} \tau')\ \mathbf{rep}\ \tau_2}{\mathbf{rep}\ (\tau_1 \overset{nr}{\rightarrow} \tau') \overset{nr}{\rightarrow} \tau_2}
                                                                             \frac{\mathbf{rep} \ \tau_2}{\mathbf{rep} \ \beta_1 \to \tau_2}
                                                                                                                                OK-BASEFUNCTION
                                                                                                                                                                                                                                                                                                    OK-HOFUNCTION
                                                                                                                                                                                                         \frac{\mathbf{rep}\ \tau_1{\rightarrow}\tau_2\ \mathbf{rep}\ \tau_1'{\rightarrow}\tau_2'}{\mathbf{rep}\ (\tau_1{\rightarrow}\tau_2){\rightarrow}(\tau_1'{\rightarrow}\tau_2')}
                                        rep \tau_1 \rightarrow \tau_2
                                                                                                                                                                                                                                                                                               OK-RECFUNCTION
                                                                                                  OK-RECFUNCTIONBASE
                                rep (\tau_1 \rightarrow \tau_2) \rightarrow \beta
                                                                                                                                                         \Gamma; \gamma \vdash e : \langle \tau, \delta \rangle
                       \tfrac{x \in \Gamma}{\Gamma; \gamma \vdash x : \Gamma(x)}
                                                                         T-VAR
                                                                                                                                                           T-INT
                                                                                                                                                                                                                                                                                                                                          T-UNIT
                                                                                                                                                                                                                                               T-BOOL
                                                                                                                                                                                                                                                                                        \overline{\Gamma;\gamma\vdash():\mathbf{unit}}
                                                                                                             \overline{\Gamma; \gamma \vdash n : \mathbf{int}_c}
                                                                                                                                                                                              \Gamma; \gamma \vdash b : \mathbf{bool}_c
                                                                                                                                                                                                            \frac{\Gamma; \gamma \vdash e_1 : \tau_1 \quad \Gamma; \gamma \vdash e_2 : \tau_2 \quad \tau_1, \tau_2 < : \tau \quad \mathbf{rep} \ \tau}{\Gamma; \gamma \vdash e_1 \ (op) \ e_2 : \tau}
                                                                             \frac{\mathbf{rep} \ \tau}{\Gamma; \gamma \vdash \mathbf{context} \ \tau : \tau}
                                                                                                                                                  T-CONTEXT
                                                                                                                                       \frac{\Gamma; \gamma \vdash e : \mathbf{bool}_c \quad \Gamma; \gamma \vdash e_t : \tau_1 \quad \Gamma; \gamma \vdash e_f : \tau_2 \quad \tau_1, \tau_2 <: \tau \quad \mathbf{rep} \ \tau}{\Gamma; \gamma \vdash \mathbf{if} \ e \ \mathbf{then} \ e_t \ \mathbf{else} \ e_f : \tau}
                                                                                                                                                                                                                                                                                                                                 T-CONDC
                                                                                                \underline{\Gamma;} \gamma \vdash e{:}\mathbf{bool} \quad \Gamma; \mathbf{sym} \vdash e_t{:}\beta_1 \quad \Gamma; \mathbf{sym} \vdash e_f{:}\beta_2 \quad \beta_1, \beta_2 < :\beta_c \quad \mathbf{rep} \ \beta_c
                                                                                                                                                                                                                                                                                                                       T-CONDSYM
                                                                                                                                                       \Gamma; \gamma \vdash \mathbf{if} \ e \ \mathbf{then} \ e_t \ \mathbf{else} \ e_f : \beta_c
                                                                                                                                                             \frac{\Gamma; \gamma \vdash e_1 : \tau_1 \xrightarrow{nr} \tau_2 \quad \Gamma, x : \tau_d; \gamma \vdash e_2 : \tau_1' \quad \tau_1' < : \tau_1 \quad \mathbf{rep} \ \tau_1 \quad \mathbf{rep} \ \tau_2}{\Gamma; \gamma \vdash (e_1 \ e_2) : \tau_2} \quad \text{T-APP}
\frac{\Gamma,\!x\!:\!\tau_d;\!\gamma\vdash\!e\!:\!\tau'}{\Gamma;\!\gamma\vdash\!(\lambda x\!:\!\tau_d.e)\!:\!\tau_d\!\to\!\tau'}
                                                                                                      T-LAMBDA
                                                                                               \frac{\Gamma, f: \tau_1 \rightarrow \tau_2; \gamma \vdash e_1: \tau_1 \rightarrow \tau_2 \quad \Gamma, f: \tau_1 \overset{nr}{\rightarrow} \tau_2; \gamma \vdash e_2: \tau_2 \quad \mathbf{rep} \ \tau \quad \mathbf{rep} \ \tau_2}{\Gamma; \gamma \vdash \mathbf{let} \ \mathbf{rec} \ f: \tau_1 \overset{nr}{\rightarrow} \tau_2 = e_1 \ \mathbf{in} \ e_2: \tau_2}
                                                                                                                                                                                                                                                                                                               T-APPLETREC
                                                                          \frac{\gamma{=}\mathbf{concrete} \quad \Gamma;\!\gamma{\vdash}e_1{:}\tau_1{\to}\tau_2 \quad \Gamma;\!\gamma{\vdash}e_2{:}\tau_1' \quad \tau_1'{<:}\tau_1 \quad \mathbf{rep} \ \tau_1 \quad \mathbf{rep} \ \tau_2}{\Gamma;\!\gamma{\vdash}(e_1\ e_2){:}\tau_2}
                                                                                                                                                                                                                                                                                                              T-APPCURREC
                                                                          \frac{\Gamma{,}x{:}\beta{;}\gamma{\vdash}e_c{:}\mathbf{bool}\quad \Gamma{;}\gamma{\vdash}v{:}\beta}{\Gamma{;}\gamma{\vdash}(\mathbf{defer}x{:}\beta\{e_c\}\ \mathbf{default}\ v){:}\beta}
                                                                                                                                                                                                                                                 \frac{\Gamma; \gamma \vdash e_c \text{:bool}}{\Gamma; \gamma \vdash (\mathbf{assert} \ e_c) \text{:unit}}
                                                                                                                                                                                                  T-DEFER
                                                                                                                                                                                                                                                                                                                                T-ASSERT
                                                                                                                                                                                       \Gamma; \gamma \vdash e_1 : \beta \quad \Gamma; \gamma \vdash^c e_1 : \beta' \quad \Gamma; \gamma \vdash v : \beta'
                                                                                                                                                                                                                                                                                                            T-CONCRETIZE
                                                                                                                                                                                       \Gamma; \gamma \vdash (\mathbf{concretize} \ e_1 \ \mathbf{with} \ v) : \beta_c
```

Figure 5: Static semantics for $\lambda_{\rm J}$ describing simple type-checking and enforce restriction on scope of nondeterminism and recursion. Recall that β refers to base (non-function) types.

that $\vdash \langle \Sigma, \Delta, \mathbf{if} \ e_1 \ \mathbf{then} \ e_t \ \mathbf{else} \ e_f \rangle \rightarrow \langle \Sigma', \Delta', \mathbf{if} \ e_1' \ \mathbf{then} \ e_t \ \mathbf{else} \ e_f \rangle$. Similarly, if expression e_1 is a **concrete** value, it must be **true** or **false**. According to E-CONDTRUE and E-CONDFALSE, **if** e_1 **then** e_t **else** e_f can be reduced as well. On the other hand, if the expression e_1 is **symbolic**, according to E-CONDSYMT and E-CONDSYMF, it holds the *progress* property.

- For the expression e_1 e_2 , if the sub-expression e_1 or e_2 is not a value, similar as the expression e_1 (op) e_2 , according to E-APP1 and E-APP2, it can be at least reduce one step and evetually both e_1 and e_2 are values. Then according to the previous **Concrete Function Lemma**, e_1 e_2 must be the form as $\lambda x : e.v$, according to E-APPLAMBDA, it holds the property immediately.
- For the expression defer $x : \tau\{e\}$ default v_d , if sub-expression e is not a value, by the induction hypothesis, there must be an expression e' such that $\vdash \langle \Sigma, \Delta, e \rangle \rightarrow \langle \Sigma', \Delta', e' \rangle$, then by E-DEFERCONSTRAINT, the expression can perform a reduction $\vdash \langle \Sigma, \Delta, \text{defer } x : \tau\{e\} \text{ default } v_d \rangle \rightarrow \langle \Sigma', \Delta', \text{defer } x : \tau\{e'\} \text{ default } v_d \rangle$. In addition, if sub-expression e is a value, according to static semantics, it must be some concrete value v_c , then according to E-DEFER, a fresh variable named x' will be generated and a new defaut condition will be added to the Δ environment, which indicates that the expression defer is progressive.
- For the expression assert e, similar as expression of defer $x : \tau\{e\}$ default v_d , according to E-ASSERTCONSTRAINT and E-ASSERT, it holds the progress property as well.
- For the expression **concretize** e **with** v_c , if sub-expression e is not a value, according to the hypothesis, there exists an expression e' such that $\vdash \langle \Sigma, \Delta, e \rangle \rightarrow \langle \Sigma', \Delta', e' \rangle$. According to E-CONCRETIZEEXP, it holds the **progress** property. On the contrary, if sub-expression e is a value, then a MODEL will be built to model the constraints, and it can be reduced to a concrete value c that satisfies the model or an **error** will be generated while the model cannot be satisfied according to E-CONCRETIZESAT and E-CONCRETIZEUNSAT. Therefore, we can conclude the expression **concretize** e **with** v_c holds the property as well.
- For the expression let $x : \tau = e_1$ in e_2 and let $\operatorname{rec} f : \tau = e_1$ in e_2 , expressions e_1 and e_2 are composed by one ore more above expressions. Thus, it holds the property accordingly.

Theorem 2. (Preservision). If $\Gamma \vdash e : \tau \delta$ and $e \rightarrow e'$, then $\Gamma \vdash e' : \tau \delta$.

Proof. According to the static semantics 5, we can proof that $\lambda_{\rm J}$ program is type-preserved by structral induction over the syntax of $\lambda_{\rm J}$. According to T-VAR, T-INT, T-BOOL, T-UNIT, T-CONTEXT, we know the simple expressions are all type-preserved. Next we will discuss the operation, condition, λ abstraction, application, let, assert, defer and concretize expressions respectively. At the beginning, by the typing rules from Figure 5, we conclude that all the δ value is the same for both sides except for the T-CONCRETIZE in which, however, the type is also preserved for it only cast the symbolic type to the corresponding concrete type. Therefore, we will not discuss the δ type in the following proof. In addition, by the hypothesis, all the sub-expressions are well-typed while proof the specific expression.

• For the operation expression e_1 (op) e_2 , assume that expression e_1 is with the type τ_1 and expression e_2 is with the type τ_2 , according to T-OP there must be

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a type τ that is the common super type of e_1 and e_2 , such that the expression e_1 (op) e_2 is with the type τ . According to E-OP, E-OP1 and E-OP2, after one step evaluation, the expression e_1 will be some e'_1 with the type τ'_1 , such that $\tau'_1 <: \tau_1$, or the expression e_2 will be reduced to some e'_2 with the type τ'_2 and $\tau'_2 <: \tau_2$, assume that the evaluated expression is with the type τ' , according to subtyping, we know $\tau' <: \tau$, which indicates that the type is preserved.

- For the conditional expression if e_1 then e_t else e_f , assume expressions e_t and e_f are with the type τ_1 and τ_2 respectively. According to T-CONDC and T-CONDSYM, if the expression e_1 is with the symbolic type, if e_1 then e_t else e_f must be with some type τ which is the common super type of τ_1 and τ_2 . Then according to E-CONDSYMT, after one step evaluation e_t will be reduced to some e_t' with the type τ_1' . According to the hypothesis, $\tau_1' <: \tau_1$. Therefore the type τ is the common super type of τ_1' and τ_2 as well. And the E-CONDSYMF is the same. In addition, if the expression e_1 is with the concrete type, there will be only one branch being taken, either τ_1 or τ_2 , either of which holds the subtyping relation naturally. Consequently, the conditional expression is type-preserved.
- For the λ abstraction expression $\lambda x : \tau.e$, according to E-APPLAMBDA, the type of the expression will not change and it is type-preserved naturally.
- For the application expression e_1 e_2 , assume the type of e_1 and e_2 are $\tau_1 \to \tau_2$ and τ'_1 respectively. In addition, $\tau'_1 <: \tau_1$, and according to T-APP, the type of application expression is τ_2 . Then according to E-APP1 and E-APP2, the evaluated expression will preserve the expression type due to the hypothesis that the sub-expressions e_1 and e_2 preserve the corresponding type respectively.
- For the let expression let $x : \tau = e_1$ in e_2 ,
- For the recursive let expression let rec $f : \tau = e_1$ in e_2 ,
- For the expression assert e, defer $x : \tau\{e\}$ default v_d and concretize e with v_c are the same.

4 Evaluation

Here is the example area.

7

5 Conclusion

Here is the conclusion section...

References

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