12.1.2 Suppose that the purity of a chemical solution y is related to the amount of a catalyst x through a linear regression model with $\beta_0 = 123.0$, $\beta_1 = -2.16$, and with an error standard deviation $\sigma = 4.1$.

(a) What is the expected value of the purity when the catalyst level is 20?

(b) How much does the expected value of the purity change when the catalyst level increases by 10?

(c) What is the probability that the purity is less than 60.0 when the catalyst level is 25?

(d) What is the probability that the purity is between 30 and 40 when the catalyst level is 40?

(e) What is the probability that the purity of a solution with a catalyst level of 30 is smaller than the purity of a solution with a catalyst level of 27.5?

12.1.5 Consider the linear regression model

$$y = 675.30 - 5.87x$$

with $\sigma = 7.32$, which relates the viscosity y of a substance to the temperature x. If a sample of the substance is prepared at temperature 80, what is the probability that its viscosity is less than 220?

12.2.3 A data set has $n = 30$, $\sum_{i=1}^{30} x_i = -67.11$, $\sum_{i=1}^{30} y_i = 1322.7$, $\sum_{i=1}^{30} x_i^2 = 582.0$, $\sum_{i=1}^{30} y_i^2 = 60{,}600$, and $\sum_{i=1}^{30} x_i y_i = -3840$. Calculate $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\sigma}^2$. What is the fitted value when $x = -2.0$?

12.3.2 In a simple linear regression analysis with $n = 22$ data points, an estimate $\hat{\beta}_1 = 56.33$ is obtained with $\text{s.e.}(\hat{\beta}_1) = 3.78$.

(a) Calculate a two-sided 95% confidence interval for the slope parameter β_1.

(b) Test the null hypothesis $H_0 : \beta_1 = 50.0$ against a two-sided alternative hypothesis.

12.5.9 A linear regression model is fitted to the data

x	y
17.1	45.9
18.4	48.2
19.8	50.3
20.3	50.9
21.6	52.8
22.1	55.5
23.5	57.9

with x as the input variable and y as the output variable. Find $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}^2$. Construct a 99% prediction interval for an observation of the output variable when the input variable is equal to 20.

12.6.4 A data set has $n = 25$, $\sum_{i=1}^{25} x_i = 1356.25$, $\sum_{i=1}^{25} y_i = -6225$, $\sum_{i=1}^{25} x_i^2 = 97{,}025$, $\sum_{i=1}^{25} y_i^2 = 10{,}414{,}600$, and $\sum_{i=1}^{25} x_i y_i = -738{,}100$. Compute the analysis of variance table and calculate the coefficient of determination R^2.