PHYS 206: Homework 4 Statics

Due on April 8th, 2020 at 11:55pm
7 Pages

Dr. O Section 509

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Problem 1

A uniform in shape and mass pipe strut BDC is loaded and supported as shown. The mass of the strut is m kg; the acceleration due to gravity is $g\,\mathrm{m/s^2}$. The strut is supported by a smooth pin at point C and a cable at point B. There is an external force **P** (downward) at point D. The cable is light and does not stretch.

a) Find the unknown reaction forces (reaction at the pin and tension in the cable) in terms of given parameters like L, H, m, g, and **P**.

b) Using the following numerical parameter values: the mass of the strut is 50 kg, the acceleration due to gravity is 9.81 $\mathrm{m/s^2}$, L is 0.8 m, H is 1 m, and **P** is 750 N, calculate the magnitude and direction of the force exerted by the pin on the strut at point C and the tension force in cable AB.

Given

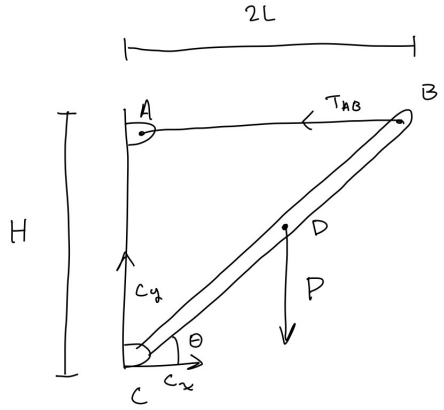
- The mass of the strut, m.
- The acceleration due to gravity, g.
- Smooth pin @ point C
- Cable @ point B
- Downward force **P**

Find

a) The reaction forces with respect to point C.

b) Using the given values to find the force at point C and the tension force in AB.

Diagram



Theory

To solve a problem involving statics, first assume that the net force and net moment acting on the structure is equal to 0. Use this assumption to find all of the unknown variables by solving the system of equations.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_a = 0$$

Problem 1 continued on next page...

Assumptions

a) There is no external net force acting upon the structure.

b) **P** and the force of gravity are their own separate forces.

Solution

First, sum up all of the forces in both x and y directions along with the net moment.

$$\begin{aligned}\Sigma F_x &= C_x - T_{AB} = 0\\ \Sigma F_y &= C_y - P - mg = 0\\ \Sigma M_a &= \frac{HT_{AB}}{2} + L(P+mg) = 0\end{aligned}$$

To answer the first part of the question, solve the system for the unknowns in terms of the given parameters which yields.

$$\begin{aligned}T_{AB} &= \frac{2(P+mg)L}{H}\\ C_y &= P + mg\\ C_x &= \frac{2(P+mg)L}{H}\end{aligned}$$

To find the values for the second part, simply plug in the values.

$$\begin{aligned}T_{AB} &= \frac{2\left(750\,\mathrm{N} + 50\,\mathrm{kg}\cdot 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.8\,\mathrm{m})}{1\,\mathrm{m}} = 1984.8\,\mathrm{N}\\ C_y &= 750\,\mathrm{N} + 50\,\mathrm{kg}\cdot 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2} = 1240.5\\ C_x &= T_{AB} = 1984.8\,\mathrm{N}\end{aligned}$$

Now, find the value of $\vec{C}$ and θ.

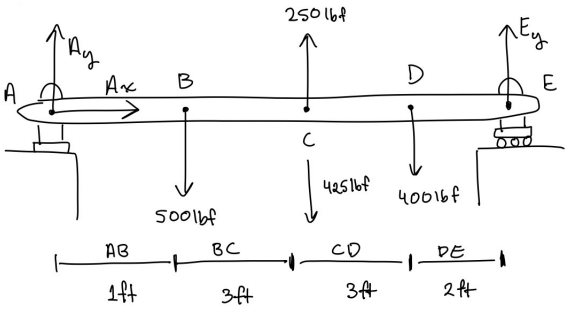
$$\begin{aligned}\vec{C} &= \sqrt{(1984.8\,\mathrm{N})^2 + (1240.5\,\mathrm{N})^2} = 2340.57\,\mathrm{N}\\ \theta &= \arctan\left(\frac{1240.5}{1984.8}\right) = 32^\circ\end{aligned}$$

Conclusion

$$\begin{aligned}T_{AB} &= 1984.8\,\mathrm{N}\\ \vec{C} &= 2340.57\,\mathrm{N}\\ \theta &= 32^\circ\end{aligned}$$

Problem 2

A rectangular beam ABCDE is loaded and supported as shown. The beam has a uniform cross section and weighs 425 lbf (pound-force). There is a smooth pin at point A, a 500 lbf downward force at point B, a 250 lbf upward force at point C, a 400 lbf downward force at point D, and a smooth roller at point E. The distances between the points A, B, C, D, and E are as follows: $AB = 1$ ft, $BC = 3$ ft, $CD = 3$ ft, $DE = 2$ ft. Determine the reactions at supports A and E.

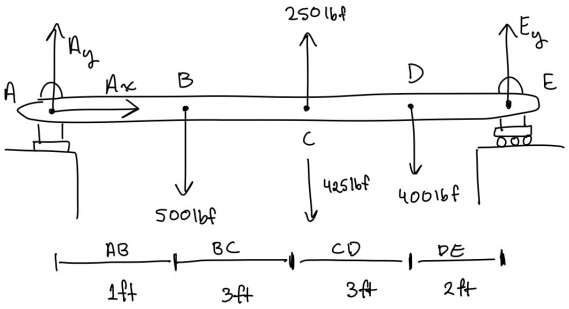


Solution:

$$A_y = 607\,\text{lbf}$$
$$E_y = 468\,\text{lbf}$$

Problem 3

A light bar ABC is supported by a smooth pin at A and a cable at B. An external 850 N downward force is applied at point C. Determine the force exerted by the cable at point B and the reaction at support A. You may disregard the weight of the bar for the analysis. The cable is light and does not stretch.



Solution:

$$T_B = -1413.58\,\text{Nmm}$$
$$\vec{A} = 1349.2\,\text{Nmm}$$
$$\theta = \arctan\left(\frac{1310.6}{320.46}\right) = 13.74^\circ$$

Problem 4

A weightless bar is supported by a smooth pin at A and a smooth roller at B. A 20 kN external force and a 15 kN external force act on the bar as shown. The bar is in static equilibrium. What is the external reaction at A?

Given

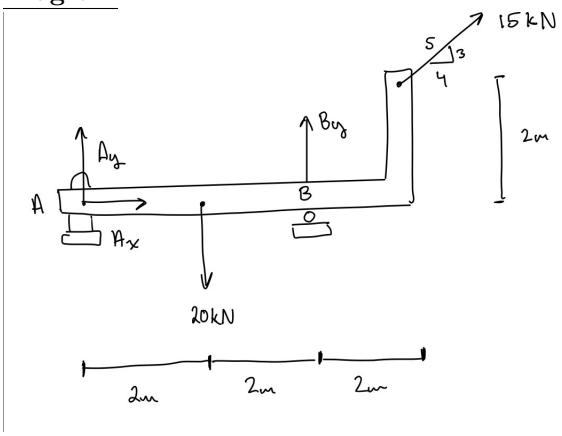
- Smooth pin @ point A
- Smooth roller @ point B
- The system is at static equilibrium
- External force of 15 kN Present
- External force of 20 kN Present

Find

a) External Reaction at A

b) Find B_y

Diagram



Theory

To solve a problem involving statics, first assume that the net force and net moment acting on the structure is equal to 0. Use this assumption to find all of the unknown variables by solving the system of equations.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_a = 0$$

Assumptions

a) The net force is equal to 0

Solution

First, to ease the burden of further calculations, find the components of the 15kN force.

$$x : 15\,\text{kN} \cdot \cos\theta = 15\,\text{kN} \cdot \frac{4}{5} = 12\,\text{kN}$$
$$y : 15\,\text{kN} \cdot \sin\theta = 15\,\text{kN} \cdot \frac{3}{5} = 9\,\text{kN}$$

Now, sum up all of the forces in both x and y directions along with the net moment.

$$\Sigma F_x = 12\,\text{kN} + A_x = 0$$
$$\Sigma F_y = A_y - 20\,\text{kN} + B_y + 9\,\text{kN} = 0$$
$$\Sigma M_A = -20\,\text{kN} \cdot 2\,\text{m} + B_y \cdot 4\,\text{m} + 9\,\text{kN} \cdot 6\,\text{m} + 12\,\text{kN} \cdot 2\,\text{m} = 0$$

Solve for the unknowns.

$$B_y = -9.5\,\text{kN}$$
$$A_x = -12\,\text{kN}$$
$$A_y = 20.5\,\text{kN}$$

Now find the magnitude of $\vec{A}$ and θ.

$$\vec{A} = \sqrt{(-12)^2 + (20.5)^2} = 23.75\,\text{kN}$$
$$\theta = \arctan\left(\frac{20.5}{-12}\right) = -59.65^\circ$$

Conclusion

$\vec{A} = 23.75$ kN @ 59.65° CW