Phys 216: Homework 3 CI

Due on Feburary 26, 2020 at 11:59pm
18 Pages

Professor Dr. O Section 509

Amari West

Problem 1

For a normal population with known variance σ^2, what is the confidence level for the CI

$$\bar{x} - \frac{2.14\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{2.14\sigma}{\sqrt{n}}$$

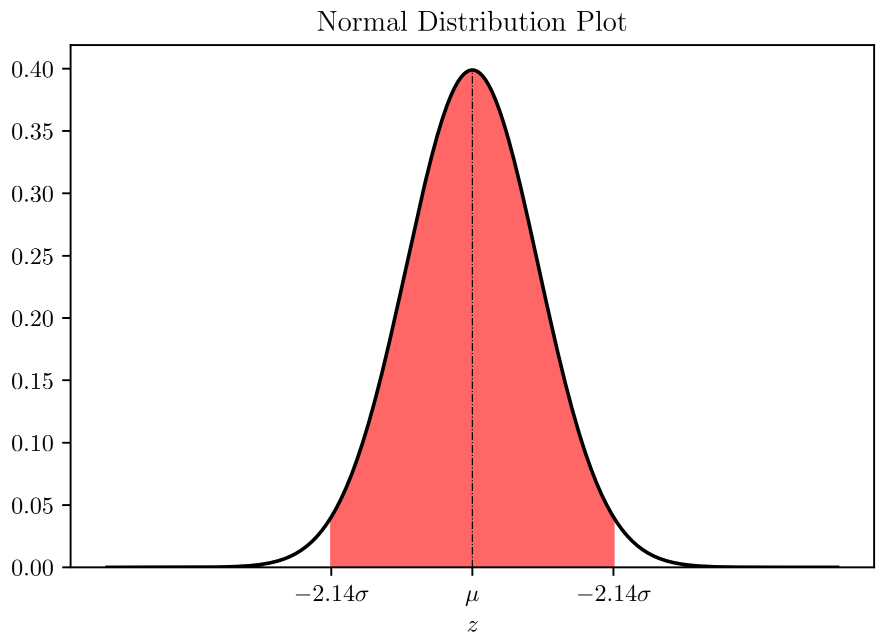
Given

- σ^2 is known.
- The given z-values are -2.14 and 2.14.

Find

- The confidence level for the interval

Diagram



Theory

The confidence level for the CI is found by using the value given as a coefficient of σ and using it to find the p-value. With that, the smaller p-value is subtracted from the larger p-value and the difference multiplied by 100% becomes the answer.

$$P(L \leq \mu \leq U) = \frac{P}{100\%}$$

Assumption

The data follows a normal distribution of $N(\mu, (\sigma)^2)$

Solution

To find the confidence level, first find the p-value and plug in given variables into the following equation.

$$\begin{aligned}\frac{P}{100\%} &= P(L \leq \mu \leq U) \\ &= P(-2.14 \leq \mu \leq 2.14) \\ &= 0.9838 - 0.0162 \\ &= 0.9676\end{aligned}$$

Now, multiply both sides by 100%.

$$P = \underline{\underline{96.76\%}}$$

Conclusion

The confidence level for the interval is $\boxed{96.76\%.}$

Problem 2

An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

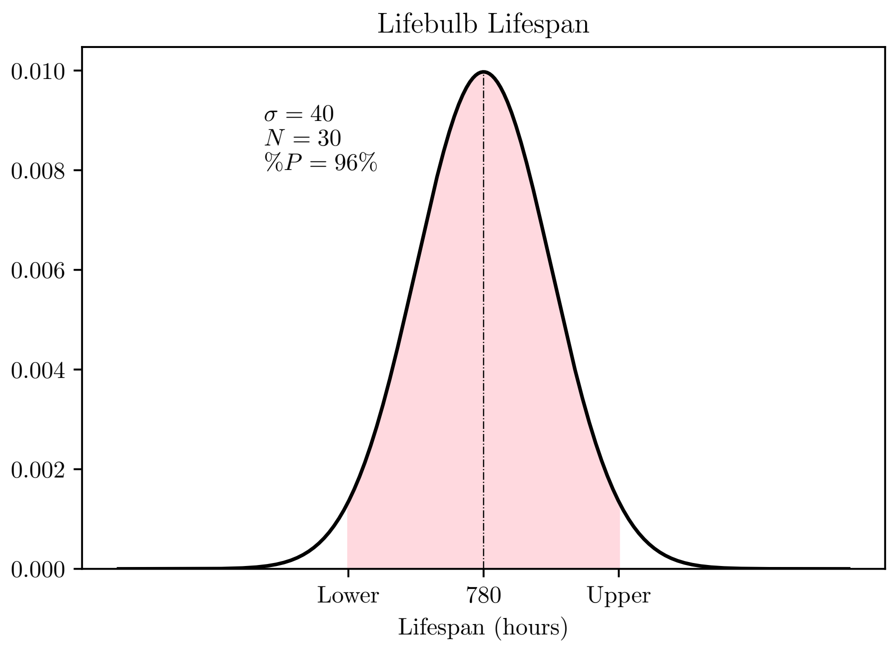
Given

- $\sigma = 40$ hours
- $N = 30$ bulbs
- $\mu = 780$ hours
- $P = 0.96$

Find

- The upper and lower bound of the interval.

Diagram



Theory

To find the upper and lower bound of the interval with the acceptable confidence level, all known values need to be plugged into the following equations.

$$P(L \leq \mu \leq U) = \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Assumption

The data follows the normal distribution curve $N(780, (40)^2)$

Solution

Note: Python and JupyterNotebooks were used to compute the answer. Among the modules used were `scipy.stats`, `numpy`, and `sympy`.

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # Unknown
x_upper = widgets.FloatText() # Unknown
mu = widgets.FloatText() # mu = 780
sigma = widgets.FloatText(value=1) # sigma = 40
num = widgets.FloatText(value=1) # num = 30
p = widgets.FloatText() # p = .96

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = abs(norm.ppf(half_alpha))

upper = mu.value + z_alpha * (sigma.value / m.sqrt(num.value))
lower = mu.value - z_alpha * (sigma.value / m.sqrt(num.value))

output('The upper estimate of $\mu$ is' + equ(upper))
output('The lower estimate of $\mu$ is' + equ(lower))

```

Output:

```
The upper estimate of mu is 794.9984614107295
The lower estimate of mu is 765.0015385892705

```

Conclusion

The interval with a confidence level of 96% has a lower bound of $\boxed{795}$ and an upper bound of $\boxed{765.}$

Problem 3

You are working part-time at a road construction firm. Your boss knows you are learning some statistics in your classes this semester and calls you into the office. He says, "We've taken sample densities from loads of asphalt we've gotten from our supplier. The information from their company says their process produces a consistent density with a standard deviation of 2.65 pounds per cubic foot. Here's the data we've collected (see file "CIData.txt"). Can you give me an estimate of the average density that our supplier's process is producing? I want to be 80% confident of the range you provide." Download the data file "CIData.txt". Use Python or Excel to calculate the sample average (please include one decimal place). You also need to know the number of measurements in the sample.

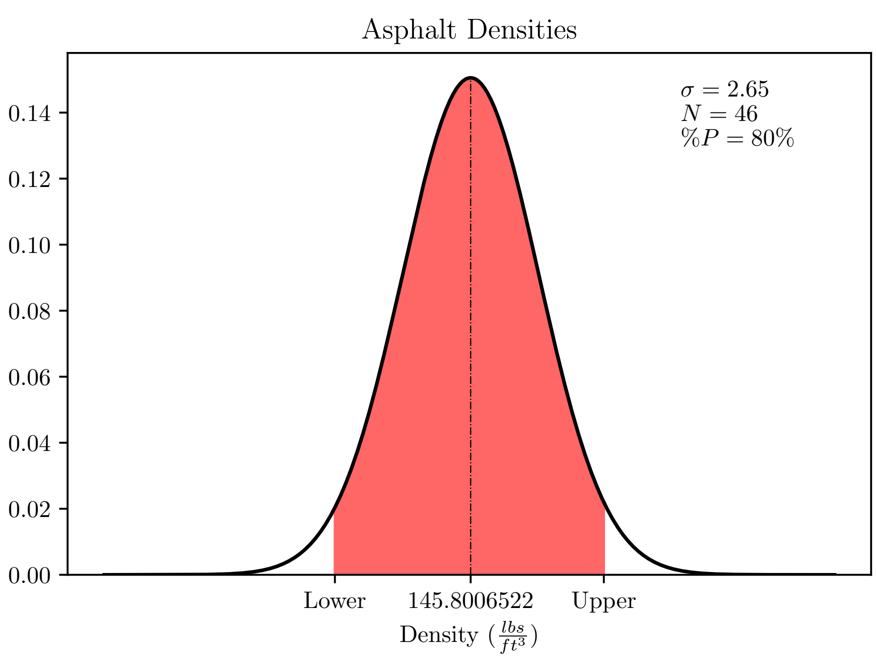
Given

- $\sigma = 2.65$
- $\bar{x} = 145.8$
- $N = 46$
- $P = 0.80$

Find

- A range of the average that has an 80% confidence level.

Diagram



Theory

To find the upper and lower bound of the interval with the acceptable confidence level, all known values need to be plugged into the following equations.

Problem 3 continued on next page...

$$P(L \leq \mu \leq U) = \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Assumption

The data follows the normal distribution curve $N(145.8, (2.65)^2)$

Solution

Note: Python and JupyterNotebooks were used to compute the answer. Among the modules used were `scipy.stats`, `numpy`, and `sympy`.

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # Unknown
x_upper = widgets.FloatText() # Unknown
mu = widgets.FloatText() # mu = 145.8
sigma = widgets.FloatText(value=1) # sigma = 2.65
num = widgets.FloatText(value=1) # num = 46
p = widgets.FloatText() # p = .80

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = abs(norm.ppf(half_alpha))

upper = mu.value + z_alpha * (sigma.value / m.sqrt(num.value))
lower = mu.value - z_alpha * (sigma.value / m.sqrt(num.value))

output('The upper estimate of $\mu$ is' + equ(upper))
output('The lower estimate of $\mu$ is' + equ(lower))
```

Output:

```
The upper estimate of mu is 146.30072934480376
The lower estimate of mu is 145.29927065519627
```

Conclusion

The interval with a confidence level of 80% has a lower bound of $\boxed{146.3}$ and an upper bound of $\boxed{145.3.}$

Problem 4

A civil engineer is analyzing the compressive strength of concrete. Compressive strength is approximately normally distributed with variance $\sigma^2 = 1000$ psi^2 A random sample of 12 specimens has a mean compressive strength of $\bar{x} = 3255.42$ psi. a) Construct a 95% two-sided CI on mean compressive strength. b) Construct a 99% two-sided CI on mean compressive strength. c) Compare the width of the 99% CI with the width of the 95% CI: evaluate the ratio $\frac{99\%Cl}{95\%Cl}$

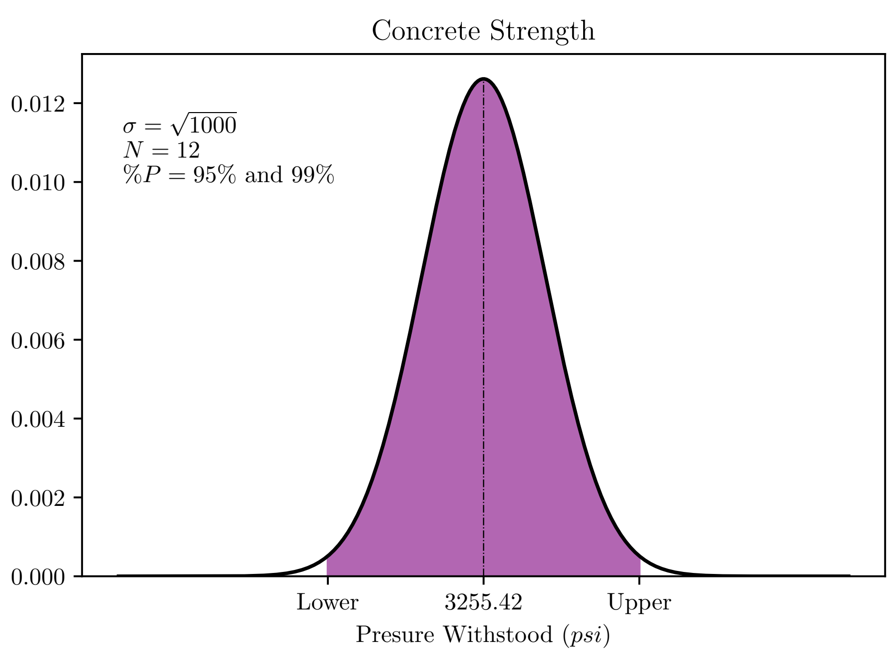
Given

- $\sigma^2 = 1000$ psi^2
- $N = 12$
- $\bar{x} = 3255.45$ psi

Find

- A 95% two-sided CI on mean compressive strength.
- A 99% two-sided CI on mean compressive strength.
- The ratio of the widths of part A and part B

Diagram



Theory

To find the upper and lower bound of the interval with the acceptable confidence level, all known values need to be plugged into the following equations.

Problem 4 continued on next page. . .

$$P(L \leq \mu \leq U) = \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Assumption

The data follows the normal distribution curve $N(3255.45, 1000)$

Solution

Note: Python and JupyterNotebooks were used to compute the answer. Among the modules used were `scipy.stats`, `numpy`, and `sympy`.

Part A

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # Unknown
x_upper = widgets.FloatText() # Unknown
mu = widgets.FloatText() # mu = 3255.42
sigma = widgets.FloatText(value=1) # sigma = sqrt(1000)
num = widgets.FloatText(value=1) # num = 12
p = widgets.FloatText() # p = .95

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = abs(norm.ppf(half_alpha))

upper = mu.value + z_alpha * (sigma.value / m.sqrt(num.value))
lower = mu.value - z_alpha * (sigma.value / m.sqrt(num.value))

output('The upper estimate of $\mu$ is' + equ(upper))
output('The lower estimate of $\mu$ is' + equ(lower))
```

Output:

```
The upper estimate of mu is 3273.311941437181
The lower estimate of mu is 3237.5280585628193
```

Problem 4 continued on next page...

Part B

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # Unknown
x_upper = widgets.FloatText() # Unknown
mu = widgets.FloatText() # mu = 3255.42
sigma = widgets.FloatText(value=1) # sigma = sqrt(1000)
num = widgets.FloatText(value=1) # num = 12
p = widgets.FloatText() # p = .99

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = abs(norm.ppf(half_alpha))

upper = mu.value + z_alpha * (sigma.value / m.sqrt(num.value))
lower = mu.value - z_alpha * (sigma.value / m.sqrt(num.value))

output('The upper estimate of $\mu$ is' + equ(upper))
output('The lower estimate of $\mu$ is' + equ(lower))
```

Output:

```
The upper estimate of mu is 3278.933996897288
The lower estimate of mu is 3231.906003102712
```

Part C

The ratio evaluation was done by hand. First, the difference between the upper and lower bounds were found for both ranges.

From part A...

$$\begin{aligned}(U - L) &= 3273.31 - 3237.53 \\ &= 35.78\end{aligned}$$

From part B...

$$\begin{aligned}(U - L) &= 3278.93 - 3231.91) \\ &= 47.02\end{aligned}$$

Now, the ratio is evaluated.

$$\frac{47.02}{35.78} = \underline{\underline{1.314}}$$

Problem 4 continued on next page...

<u>**Conclusion**</u>

For Part A, the interval with a confidence level of 95% has a lower bound of $\boxed{3237.53}$ and an upper bound of $\boxed{3273.31.}$

For Part B, the interval with a confidence level of 95% has a lower bound of $\boxed{3231.91}$ and an upper bound of $\boxed{3278.93.}$

The ratio between the two ranges is $\boxed{1.314.}$

Problem 5

In a random sample of 80 teenagers, the average number of texts handled in a day is 50. The 96% confidence interval for the mean number of texts handled by teens daily is given as 46 to 54. a) Find the value of standard deviation σ that was used to calculate this CI. b) If the number of participants per sample were doubled, by what factor would the confidence interval change (keeping the same confidence level)? Evaluate width of 96% CI from part b) over width of 96% CI from part a)

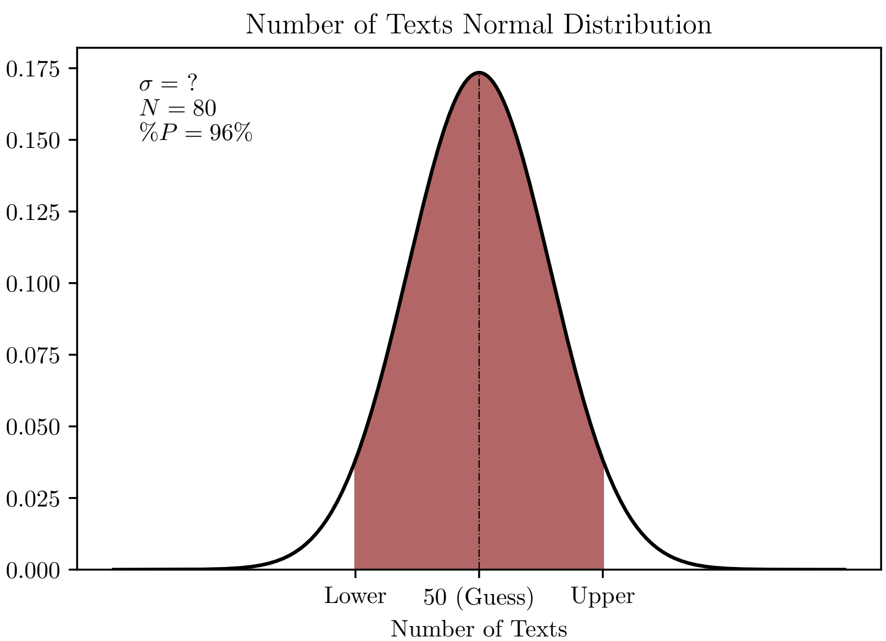
Given

- $N = 80$
- $\bar{x} = 50$
- $P = 0.96$
- Lower Bound = 46
- Upper Bound = 54

Find

- The standard deviation.
- The factor at which the confidence interval would change if the sample doubled.

Diagram



Theory

To find the upper and lower bound of the interval with the acceptable confidence level, all known values need to be plugged into the following equations.

$$P(L \leq \mu \leq U) = \bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

Assumption

The data follows a normal distribution.

Solution

Part A

Note: Python and JupyterNotebooks were used to compute the answer. Among the modules used were `scipy.stats`, `numpy`, and `sympy`.

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # L = 46
x_upper = widgets.FloatText() # U = 54
mu = widgets.FloatText() # mu = 50
sigma = widgets.FloatText(value=1) # sigma = unknown
num = widgets.FloatText(value=1) # num = 80
p = widgets.FloatText() # p = .96

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = norm.ppf(half_alpha)

sigma_x = (x_lower.value - mu.value) * m.sqrt(num.value) / z_alpha

output('The value of sigma is' + equ(sigma_x))

```

Output:

```
The value of sigma is 17.420380580501465

```

Part B

Note: Python and JupyterNotebooks were used to compute the answer. Among the modules used were `scipy.stats`, `numpy`, and `sympy`.

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # L = unknown
x_upper = widgets.FloatText() # U = unknown
mu = widgets.FloatText() # mu = 50
sigma = widgets.FloatText(value=1) # sigma = 17
num = widgets.FloatText(value=1) # num = 160
p = widgets.FloatText() # p = .96

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = abs(norm.ppf(half_alpha))

upper = mu.value + z_alpha * (sigma.value / m.sqrt(num.value))
lower = mu.value - z_alpha * (sigma.value / m.sqrt(num.value))

output('The upper estimate of $\mu$ is' + equ(upper))
output('The lower estimate of $\mu$ is' + equ(lower))

```

Output:

```
The upper estimate of mu is 52.82836533251332
The lower estimate of mu is 47.17163466748668

```

From part A...

$$\begin{aligned}(U - L) &= 54 - 46 \\ &= 8\end{aligned}$$

From part B...

$$\begin{aligned}(U - L) &= 53 - 47 \\ &= 6\end{aligned}$$

Now, the ratio is evaluated.

$$\frac{6}{8} = \underline{\underline{0.75}}$$

Problem 5 continued on next page...

Conclusion

Since only a whole number of texts can be sent, all answers will be expressed with whole numbers. The value of σ is $\boxed{17}$. When the sample size is doubled, the lower bound is $\boxed{47}$ and the upper bound is $\boxed{53.}$ The ratio between these ranges is $\boxed{0.75}$

Problem 6

Hotel managers wish to learn the average length of stay of all visitors that are senior citizens. A statistician determines that for a 90% confidence level estimate of the average length of stay to within ± 1.5 days, 65 senior visitors' check-in/check-out records will have to be examined. How many records should be looked at to obtain a 90% confidence level estimate to within ± 0.5 days?

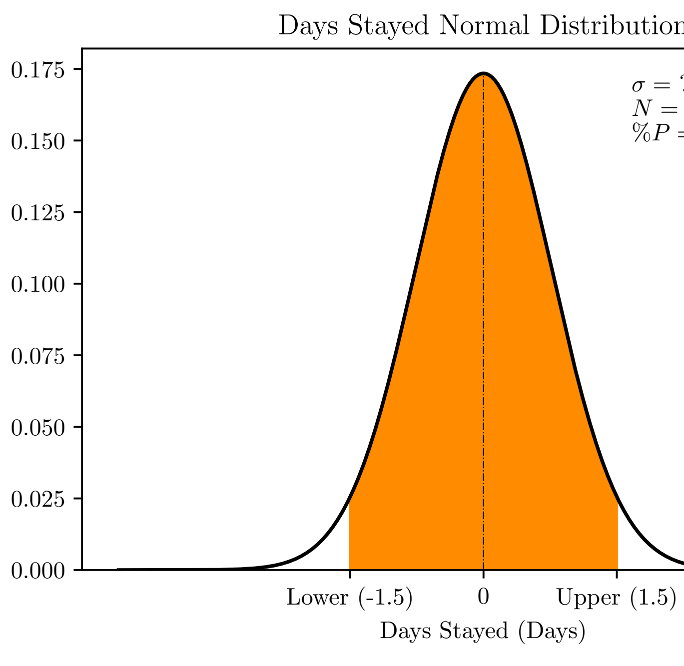
Given

- $P = 0.90$
- The intervals at which the confidence level is recorded
- The original sample size is 65

Find

- σ needs to be found to find the final answer.
- The number of data points to establish a 90% confidence level with 0.5 days

Diagram



Theory

To find the upper and lower bound of the interval with the acceptable confidence level, all known values need to be plugged into the following equations.

$$P(L \leq \mu \leq U) = \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Problem 6 continued on next page...

Assumption

The data follows the normal distribution curve $N(0, (\sigma))$

Solution

First, σ must be found.
Note: Python and JupyterNotebooks were used to compute the answer. Among the modules used were `scipy.stats`, `numpy`, and `sympy`.

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # L = -1.5
x_upper = widgets.FloatText() # U = 1.5
mu = widgets.FloatText() # mu = 0
sigma = widgets.FloatText(value=1) # sigma = unknown
num = widgets.FloatText(value=1) # num = 65
p = widgets.FloatText() # p = .90

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = norm.ppf(half_alpha)

sigma_x = (x_lower.value - mu.value) * m.sqrt(num.value) / z_alpha

output('The value of sigma is' + equ(sigma_x))

```

Output:

```
The value of sigma is 7.352257018067546

```

Now, find the number of samples required to have a 90% confidence estimate in a ± 0.5 days.

Input:

```
from scipy.stats import norm
import numpy as np
from sympy import *
from IPython.display import *
import ipywidgets as widgets
import math as m

def output(input):
return display(Latex(input))
```

```

def equ(input):
equation = ' $' + latex(input) + '$ '
return equation

x_lower = widgets.FloatText() # L = -0.5
x_upper = widgets.FloatText() # U = 0.5
mu = widgets.FloatText() # mu = 0
sigma = widgets.FloatText(value=1) # sigma = 7.352257018067546
num = widgets.FloatText(value=1) # num = unknown
p = widgets.FloatText() # p = .90

alpha = 1 - p.value
half_alpha = alpha / 2

z_alpha = norm.ppf(half_alpha)

num_x = ((sigma.value * z_alpha)/(x_upper.value - mu.value)) ** 2

output('The sample number is' + equ(num_x))

```

`Output:`

```
The sample number is 584.9999999999999

```

Conclusion

At least $\boxed{585}$ records should be looked at to obtain a 90% confidence level estimate with ±0.5 days.