ENGR 216: Homework 4

Due on March 6, 2020 at 11:55pm
14 Pages

Dr. O Section 509

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Problem 1

A utility pole supports a bundle of wires that apply the 400 N and 650 N forces shown and a guy wire apples the force **P**.

Part A

With $\alpha = 60°$, what value **P** will produce a resultant force that is vertical? Include a diagram.

Part B

If the resultant force is to be vertical and **P** is to be as small as possible, determine the value α should have and the corresponding value of **P**. Include a diagram.

Given

(i) A wire applies a force of 400N in the negative x-direction.

(ii) A wire applies a force of 650N at 30° to the horizontal.

(iii) For Part A, $\alpha = 60°$

Find

Part A

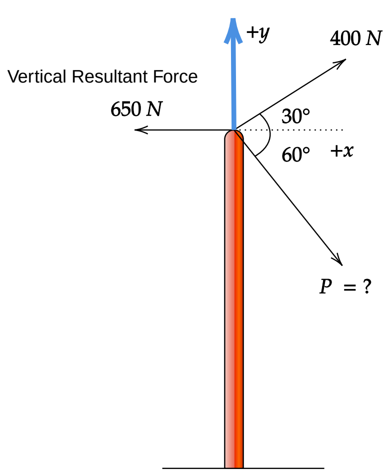
The value of **P** that will produce a vertical resultant force.

Part B

The smallest value of **P** and the value of α

Diagram

Part A



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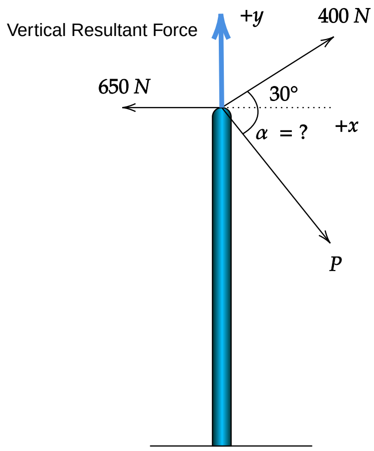
Part B



Theory

To solve this problem, forces will need to be broken up into their components, and unknown variables must be solved for via systems of equations.

Assumptions

a) The pillar or pole that the point sits on does not create a normal force.
b) The point is in static equilibrium.

Solution

Before solving either part, the forces must be broken up into their components and set equivalent to 0.

$$F_x = 400\cos 30^\circ + 650 + \boldsymbol{P}\cos\alpha = 0$$
$$F_y = 400\sin 30^\circ + \boldsymbol{P}\sin\alpha = 0$$

Part A

Since the resultant force is stated to be vertical, the x-component of the force is equal to zero; therefore, by setting $R_x = 0$ and $\alpha = 60^\circ$ the value of $\boldsymbol{P}$ can be found.

$$\begin{aligned}
0 &= 400\cos 30^\circ - 650 + \boldsymbol{P}\cos 60^\circ \\
0 &= 400\cos 30^\circ - 650 + \boldsymbol{P}\cos 60^\circ \\
-\boldsymbol{P}\cos 60^\circ &= 400\cos 30^\circ - 650 \\
\boldsymbol{P} &= \frac{-400\cos 30^\circ + 650}{\cos 60^\circ} \\
\boldsymbol{P} &= -100(4\sqrt{3} - 13) \\
\boldsymbol{P} &= \underline{\underline{607.18}}
\end{aligned}$$

Part B

To find the smallest value of **P**, set $R_x = 0$ and $\alpha = 0$ and use the equation for the net force in the x-direction to solve.

$$\begin{aligned} R_x &= 400\cos 30^\circ - 650 + \boldsymbol{P}\cos\alpha \\ 0 &= 400\cos 30^\circ - 650 + \boldsymbol{P}\cos\alpha \\ \boldsymbol{P} &= 650 - 400\cos 30^\circ \\ &= \underline{\underline{303.6}} \end{aligned}$$

With the new value for **P**, solve for α as an unknown.

$$\begin{aligned} 0 &= 400\cos 30^\circ - 650 + 303.6\cos\alpha \\ \cos\alpha &= \frac{650 - 400\cos 30^\circ}{303.6} \\ \alpha &= \arccos\left(\frac{650 - 400\cos 30^\circ}{303.6}\right) \\ &= \underline{\underline{0.4688}} \end{aligned}$$

<u>Conclusion</u>

Part A

The value of **P** that will produce a vertical resultant is $\boxed{607.2\,\text{N}}$.

Part B

The smallest **P** value is $\boxed{303.6\,\text{N}}$ and the smallest α is $\boxed{0.4688\,\text{degrees.}}$

Problem 2

The system shown below is in equilibrium. Determine the tensions in the cables AB and BC and weight W_2. Note that $W_1 = 85$ lbf. FBD is required.

Given

(i) The system is in equilibrium.

(ii) $W_1 = 85$ lbf.

Find

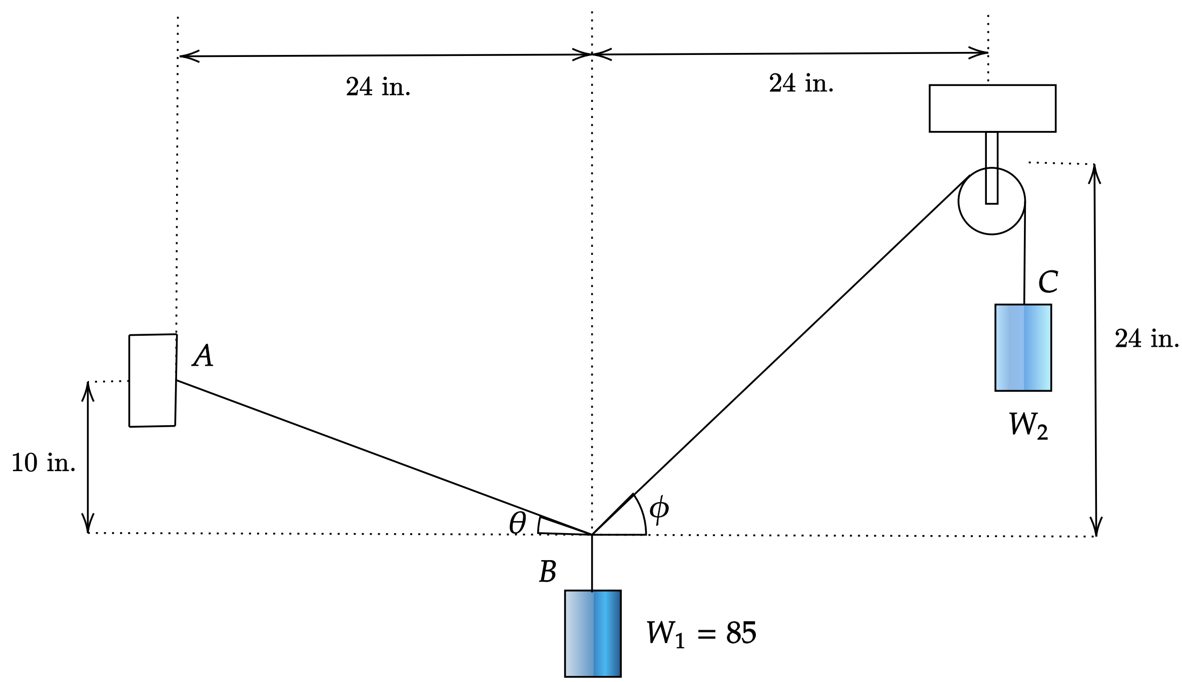
(i) Tension in AB.

(ii) Tension in BC.

(iii) Weight of W_2.

(iv) the value of ϕ and θ

Diagram



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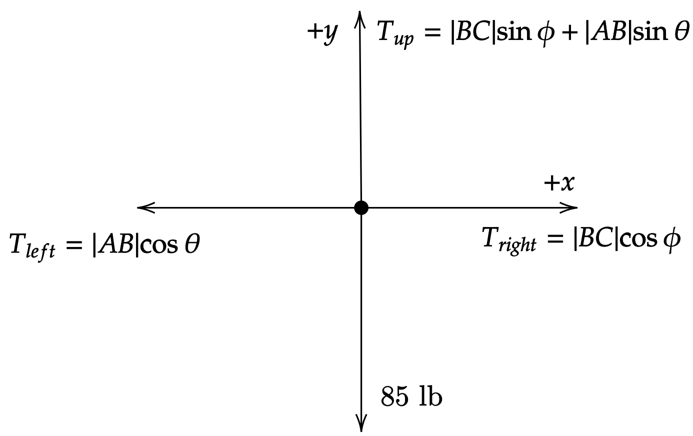
Free-Body



Theory

To solve this problem, all the forces along the y axis and the x axis must be summed up using Newton's Second Law, and the resulting system of equations must be solved for the unknowns.

Assumptions

(i) The system does not move at either point.

Solution

Set up the following system of equations based on the free-body diagrams:

$$\begin{aligned}\Sigma F_x &= |BC|\cos\phi - |AB|\cos\theta = 0\\ \Sigma F_y &= |BC|\sin\phi + |AB|\sin\theta - 85 = 0\\ \Sigma F_y &= |BC| - W_2 = 0\end{aligned}$$

Since the angles are unknown, is trigonometry to find them.
Solve for θ

$$\begin{aligned}\arctan\left(\frac{10}{24}\right) &= \theta\\ &= 22.62^\circ\end{aligned}$$

Solve for ϕ

$$\begin{aligned}\arctan\left(\frac{24}{24}\right) &= \phi\\ &= 45^\circ\end{aligned}$$

Use matrices on the calculator to solve for W_2, AB, and BC to get the solution.

Conclusion

Problem 2 continued on next page...

- $W_2 = \underline{\underline{78.86}}$
- $AB = \underline{\underline{60.41}}$
- $BC = \underline{\underline{78.86}}$

Problem 3

The cable-pulley systems shown are used to support a weight W. Determine the cable tension T in terms of W.
Hints:

a) Draw FBD for point A.

b) Draw FBDs for points B and C.

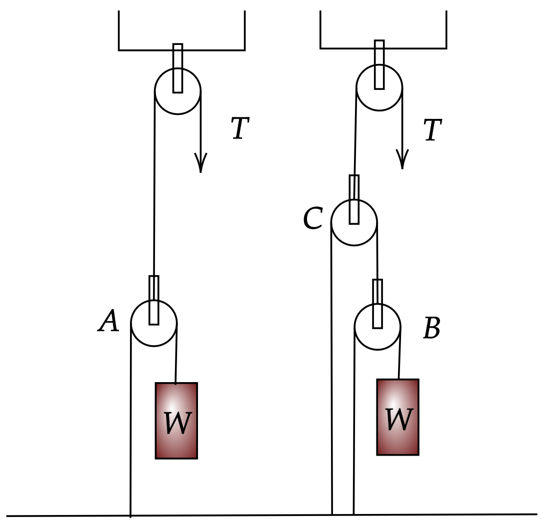
Given

(i) A diagram of the forces acting on the two weights.

Find

(i) Draw Free-Body diagrams for each point.

Diagram



Theory

In order to find all of the forces acting on each point, isolate all the points and draw the forces acting on all of them.

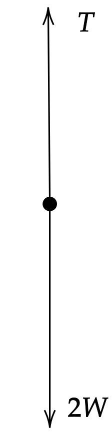
Assumptions

(i) The points are all in static equilibrium.

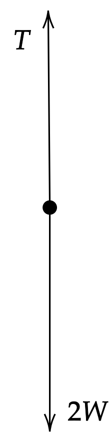
Problem 3 continued on next page. . .

Solution

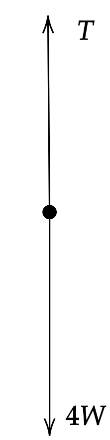
Point A



Point B



Point C



Conclusion

The forces above are depicted in the FBD that were drawn.

Problem 4

A 50-kg disk is supported by a cable and a wall as shown to the right.
Find the tension in the cable and the force exerted on the disk by the wall (note that for the latter you need to indicate the <u>direction</u> of the force, that is report the force using the polar vector representation). FBD is required.

<u>Given</u>

(i) The weight of the disk is 50kg.

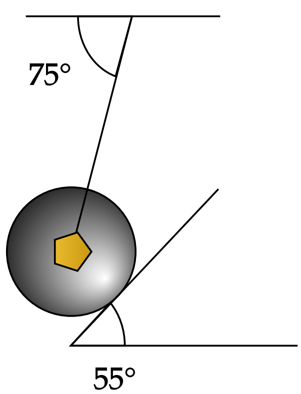
(ii) The angle at which the string and the normal force acts on the disk.

<u>Find</u>

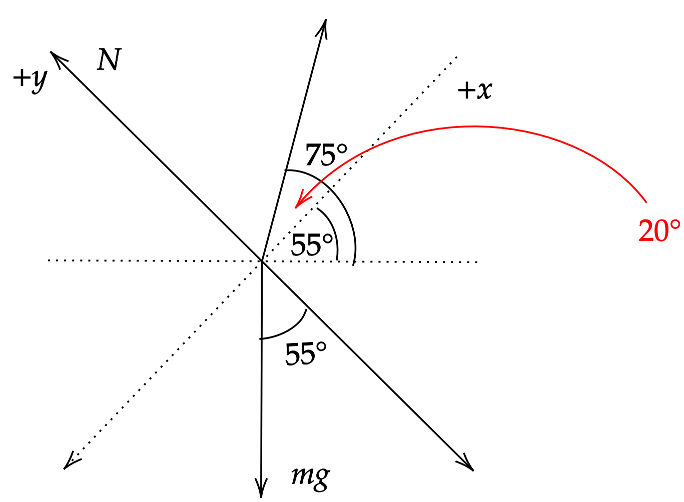
(i) Find the tension for the cable.

(ii) Find the normal force.

<u>Diagram</u>



Free-Body



Problem 4 continued on next page...

Theory

To solve this problem, all the forces along the y axis and the x axis must be summed up using Newton's Second Law, and the resulting system of equations must be solved for the unknowns.

Assumptions

(i) The system is in static equilibrium.

Solution

First, the direction of the coordinate plane was reestablished at the angle of the incline of where the disk rests. From there, the angle at which the tension was directed was reestablished to be 20°. After these adjustments were made all the forces were summed up into two equations.

$$\Sigma F_x = T\cos 20^\circ - mg\sin 55^\circ$$
$$\Sigma F_y = N + T\sin 20^\circ - mg\cos 55^\circ$$

Use matrices to solve for N and T_1.

Conclusion

The following is the solutions for the magnitudes and angles of the forces.

- $\underline{\underline{N = 135.1\angle 90^\circ}}$
- $\underline{\underline{T_1 = 427.6\angle 20^\circ}}$

Problem 5

Two cables are tied together at C and are loaded as shown.

Part A

If $W = 190$ lbf, determine the tension in cable AC and in cable BC. FBD is required.

Part B

Determine the range of values for W such that the tension will not exceed 240 lbf in either cable.

Given

Part A

(i) $W = 190$ lbf.

Part B

(i) AB nor BC cannot exceed 240 lbf.

Find

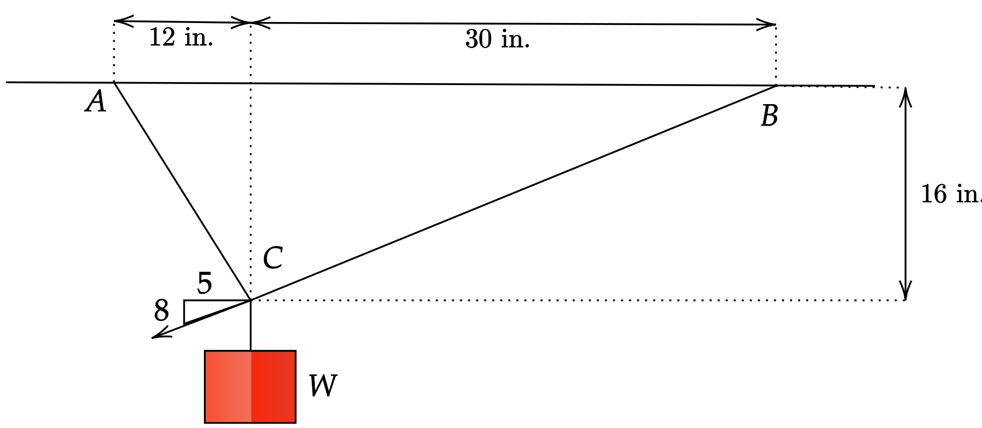
Part A

(i) Tension in AC and tension in BC.

Part B

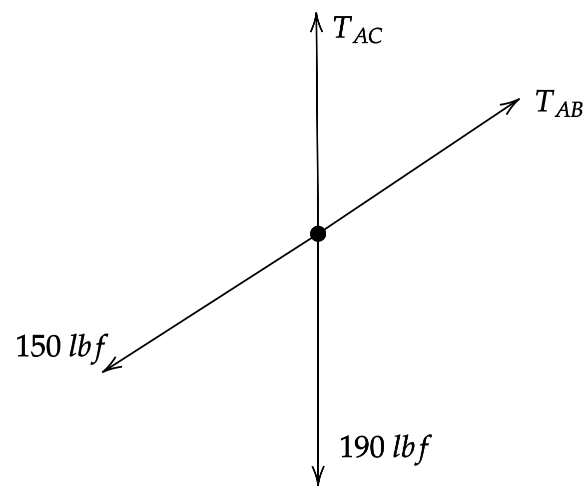
(i) The range of values for W where neither tension will surpass 240 lbf.

Diagram

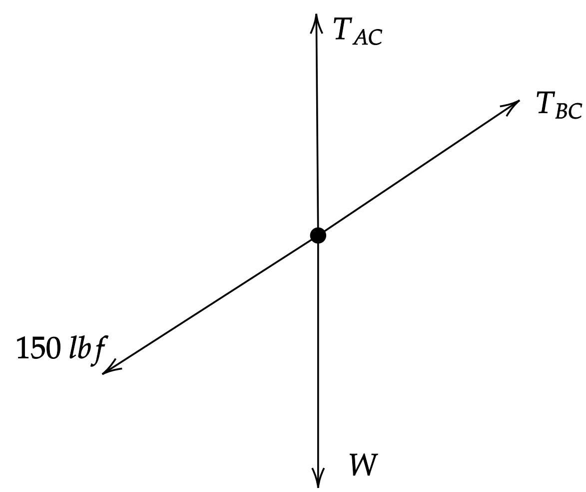


Problem 5 continued on next page...

Free-Body: Part A @ Point C



Free-Body: Part B @ Point C



Theory

To solve this problem, all the forces along the y axis and the x axis must be summed up using Newton's Second Law, and the resulting system of equations must be solved for the unknowns.

Assumptions

(i) The system is in static equilibrium.

Solution

Part A
Set up the following system of equations based on the free-body diagrams:

$$\Sigma F_x = |CB|\cos\phi - |AC|\cos\theta - 150\cos\phi = 0$$
$$\Sigma F_y = |CB|\sin\phi + |AC|\sin\theta - 150\sin\phi - 190 = 0$$

Since the angles are unknown, is trigonometry to find them.
Solve for θ

$$\begin{aligned}\arctan\left(\frac{16}{12}\right) &= \theta \\ &= 53.13^\circ\end{aligned}$$

Solve for ϕ

$$\begin{aligned}\arctan\left(\frac{16}{30}\right) &= \phi \\ &= 28.07^\circ\end{aligned}$$

Note: α and ϕ is the same angle.
Now, plug in the angles and use matrices on the calculator to solve for CB and AC to get the solution for Part A.
Part B
To solve for the range of values for W in which neither tension will surpass 240 lbf, set W as an unknown and use a guess-and-check method which includes plugging in 240 as the magnitudes for either tension until the smallest value is produced. This method of solving the problem yielded the value of 148.23 lbf for W

Conclusion

- $W = \underline{\underline{0 < W < 148.23}}$
- $AC = \underline{\underline{169.65}}$
- $CB = \underline{\underline{265.3}}$