Physics 216: Homework 3

Due on Feburary 12, 2020 at 11:59pm

Dr. Ostrovskaya Section 509

Pages: 13

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Problem 1

A certain type of storage battery lasts on the average 3.0 years, with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.

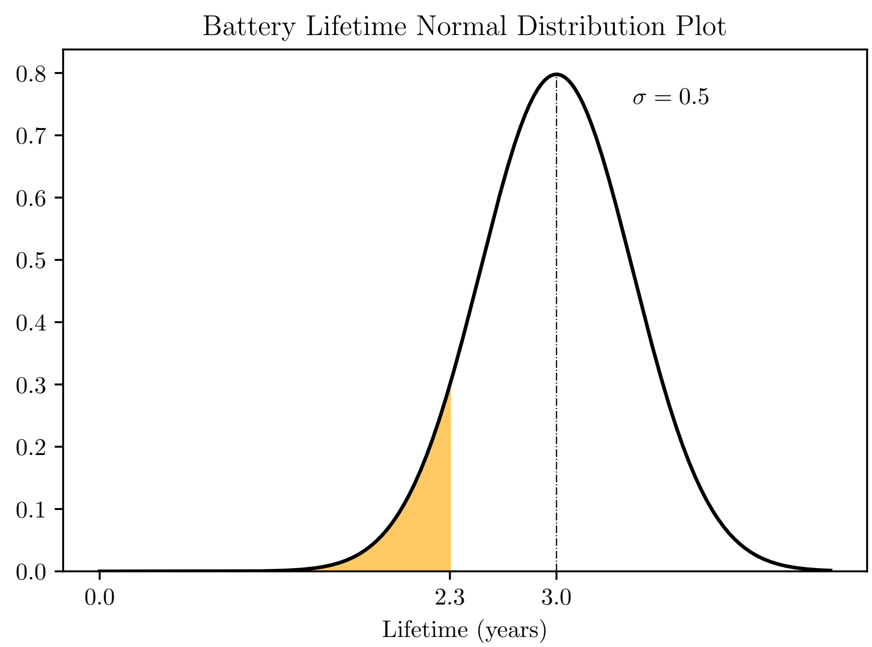
Given

- $\mu = 3.0\,\text{years}$
- $\sigma = 0.5\,\text{years}$
- $x = 2.3\,\text{years}$

Find

The probability that a given battery will last less than 2.3 years.

Diagram



Theory

To find the probability of a battery will die within the specified range of duration, a z-value must be found with the following formula and used with a z-table.

$$z = \frac{x - \mu}{\sigma}$$

Since the range in question does not contain the average, $P(2.3 < x < 3.0)$ will be subtracted from $P(0.0 < x < 3.0)$ which has a z-value equal to 0.5. Once this has been done, the probability can be calculated.

Problem 1 continued on next page. . .

Assumptions

The normal lifespan of a battery is $N(3.0, (0.5)^2)$.

Solution

First, calculate z

$$\begin{aligned} z &= \frac{2.3 - 3.0}{0.5} \\ &= 1.4 \end{aligned}$$

Find the z-value

$$\begin{aligned} P(0.0 < x < 2.3) &= 0.5 - P(0.0 < z < 1.4) \\ &= 0.5 - 0.4192 \\ &= \underline{\underline{0.0808}} \end{aligned}$$

Conclusion

The probability that a battery will last between 0.0 hours and 2.3 years is $\boxed{8.1\%.}$

Problem 2

An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

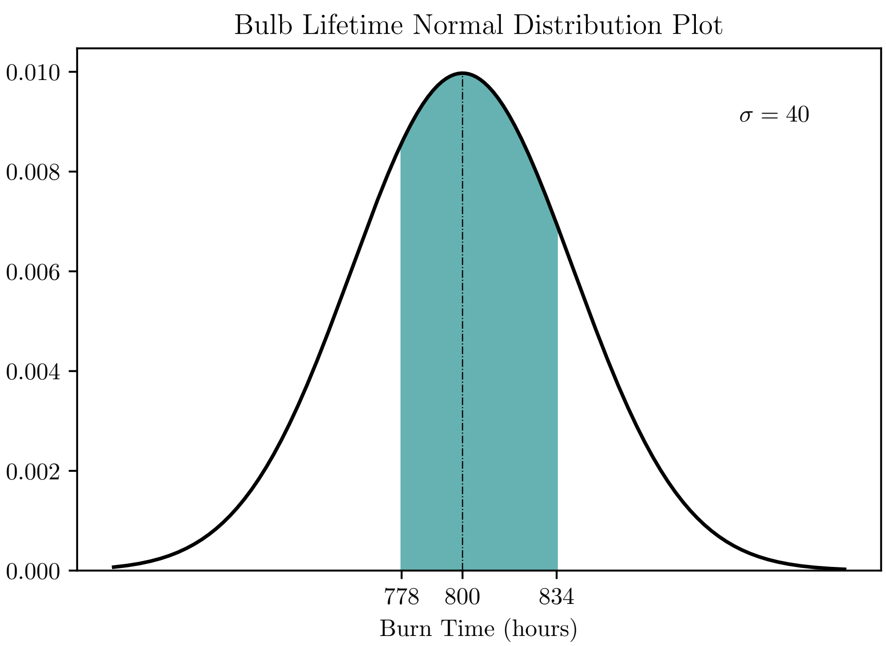
Given

- $\mu = 800\,\text{hours}$
- $\sigma = 40\,\text{hours}$
- $x_1 = 778\,\text{hours}$
- $x_2 = 834\,\text{hours}$

Find

The probability of a bulb that burns between 778 hours and 834 hours within a population using a z-table that does not include the left half of the area under a bell curve.

Diagram



Theory

To find the probability of a bulb will burn within the specified range of duration, a z-value must be found with the following formula and used with a z-table.

$$z = \frac{x - \mu}{\sigma}$$

However, since the z-table used only deals with half of the curve, two separate z calculations must be made, using

$$z_1 = \frac{x_1 - \mu}{\sigma}$$

and

$$z_2 = \frac{x_2 - \mu}{\sigma}$$

Once the z-values have been calculated, they can be added together to find the probability or $\int_{z_1}^{z_2} f(z)dz$ of the curve.

Assumptions

The normal burn time for these light bulbs is $N(800, (40)^2)$

Solution

First, calculate z_1

$$\begin{aligned} z_1 &= \frac{834 - 800}{40} \\ &= 0.85 \end{aligned}$$

Next, calculate z_2

$$\begin{aligned} z_2 &= \frac{778 - 800}{40} \\ &= 0.55 \end{aligned}$$

Use the z-table to find the area under the curve for both values. Note: since the z-table only deals with half of the curve each side is calculated separately.

$$\begin{aligned} P(778 < x < 800) &= P(0.55 < z < 0.85) \\ &= P(0.55 < z) + P(0.85 > z) \\ &= .2088 + .3023 \\ &= \underline{\underline{0.5111}} \end{aligned}$$

Conclusion

The probability that a light bulb will burn between 778 hours and 800 hours is $\boxed{51.1\%.}$

Problem 3

In an industrial process the diameter of a ball bearing is an important component part. They buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean 3.0 and standard deviation $\sigma = 0.005$. On the average, how many manufactured ball bearings will be scrapped?

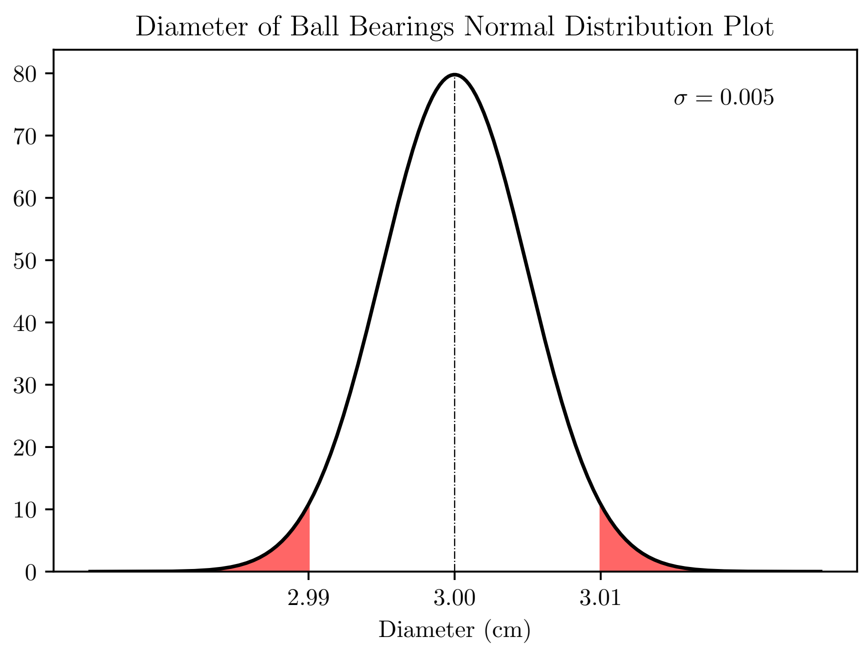
Given

- $\mu = 3.0\,\mathrm{cm}$
- $\sigma = 0.005\,\mathrm{cm}$
- $x_1 = 2.99\,\mathrm{cm}$
- $x_2 = 3.01\,\mathrm{cm}$
- Acceptable Value $= 3.0 \pm 0.01\,\mathrm{cm}$

Find

The probability that the manufactured ball bearings will be scrapped.

Diagram



Problem 3 continued on next page. . .

Theory

To find the probability of creating a ball bearing with a diameter outside of specified range, a z-value must be found with the following formula and used with a z-table.

$$z = \frac{x - \mu}{\sigma}$$

Since both ranges are at the same distance from the mean, it is possible to find a single probability using only one of the x-values and multiply that probability by 2 to get the final answer. This is possible due to the fact that the z-table in use only handles half of the curve.

Assumptions

The normal diameter of these ball bearings is $N(3.0, (0.005)^2)$

Solution

First, find the respective z-values using x_1 and x_2

$$\begin{aligned} z &= \frac{3.01 - 3.0}{0.005} \\ &= 2 \end{aligned}$$

$$\begin{aligned} z &= \frac{2.99 - 3.0}{0.005} \\ &= -2 \end{aligned}$$

Next, find the probability and subtract it from 0.5, but due to symmetry this is done twice. The negative 2 is turned into a positive since there are no negatives on the z-table that is used.

$$\begin{aligned} P(x < 2.99 \cup 3.01 < x) &= 2(0.5) - P(2 < z) \\ &= 2(0.5 - .4772) \\ &= 1 - 0.9544 \\ &= \underline{\underline{0.0456}} \end{aligned}$$

Conclusion

According to the calculations, $\boxed{4.56\%}$ of the ball bearings will be removed.

Problem 4

Gauges are used to reject all components in which a certain dimension is not within the specification $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 2.0. Determine the value d such that the specifications "cover" 95% of the measurements.

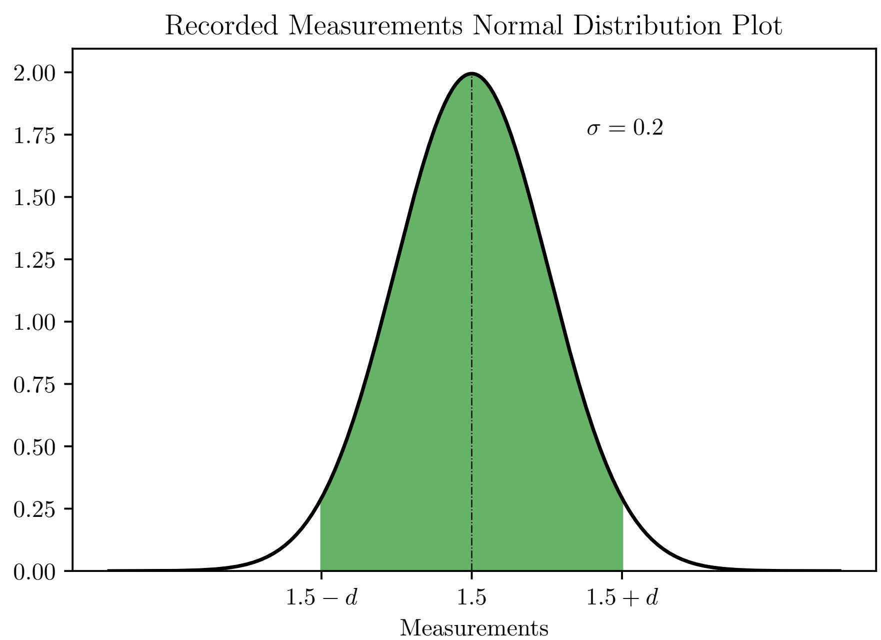
Given

- $\mu = 1.50$
- $\sigma = 2.0$
- $x_1 = 1.50 - d$
- $x_2 = 1.50 + d$
- Probability = 95%

Find

The value of d such that specifications include 95% of the measurements.

Diagram



Theory

To find the value of d, the following formula may be used.

$$z = \frac{x - \mu}{\sigma}$$

Since the probability is already given, the z-table can be used backwards to find the appropriate z-value, and the formula above can be reconfigured to find d.

Assumptions

The normal value of the measurements is $N(1.50, (2.0)^2)$

Solution

First, plug in x_2 into the equation used to find z and use half of the given percentage, 95%, to find the z-value to plug in as z.

$$\begin{aligned} 1.96 &= \frac{(1.50 + d) - 1.50}{2.0} \\ &= \frac{d}{2.0} \end{aligned}$$

Now, solve for d

$$\begin{aligned} d &= (1.96)(2.0) \\ &= \underline{\underline{0.392}} \end{aligned}$$

Conclusion

The value for d is $\boxed{0.392.}$

Problem 5

A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance that exceeds 43 ohms?

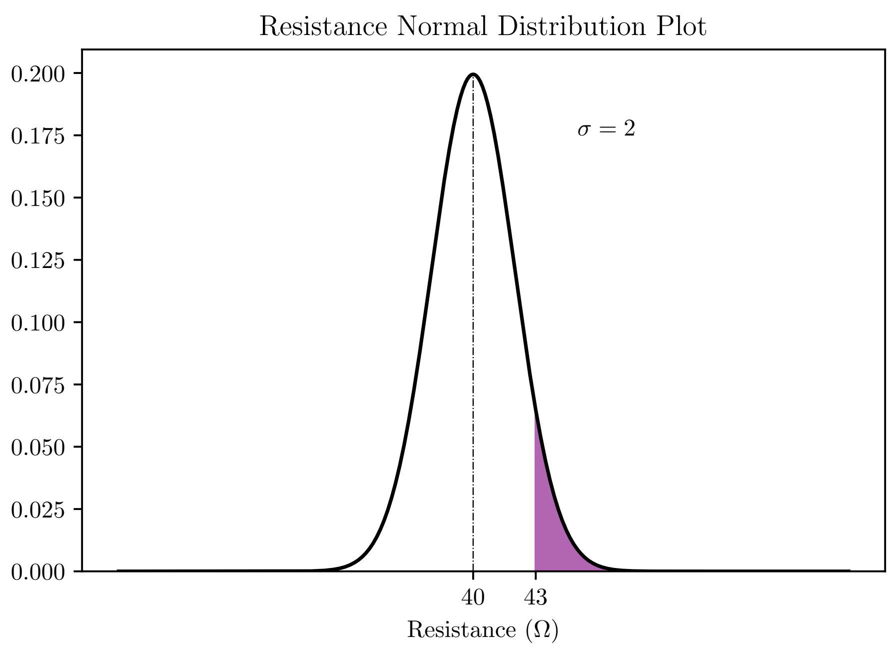
Given

- $\mu = 40\,\Omega$
- $\sigma = 2\,\Omega$
- $x = 43\,\Omega$

Find

The percentage of resistors that will have a resistance over 43 Ω.

Diagram



Theory

To find the probability of a resistor with a resistance over 45 Ω, a z-value must be found with the following formula and used with a z-table.

$$z = \frac{x - \mu}{\sigma}$$

Since the range in question does not contain the average, $P(43 < x < \infty)$ will be subtracted from $P(40 < x < \infty)$ which has a z-value equal to 0.5. Once this has been done, the probability can be calculated.

Problem 5 continued on next page...

Assumptions

The normal value of resistance is $N(40, (2)^2)$

Solution

First, calculate z

$$\begin{aligned} z &= \frac{43 - 40}{2} \\ &= 1.5 \end{aligned}$$

Find the z-value

$$\begin{aligned} P(43 < x < \infty) &= 0.5 - P(0.0 < z < 1.5) \\ &= 0.5 - 0.4332 \\ &= \underline{\underline{0.0668}} \end{aligned}$$

Conlcusion

The probability of the resistance being over 43 Ω is $\boxed{6.68\%.}$

Problem 6

On an examination the average grade was 74 and the standard deviation was 7. If 12% of the class are given A's, and the grades are curved to follow a normal distribution, what's the lowest possible A and the highest possible B? What is the sixth decile?

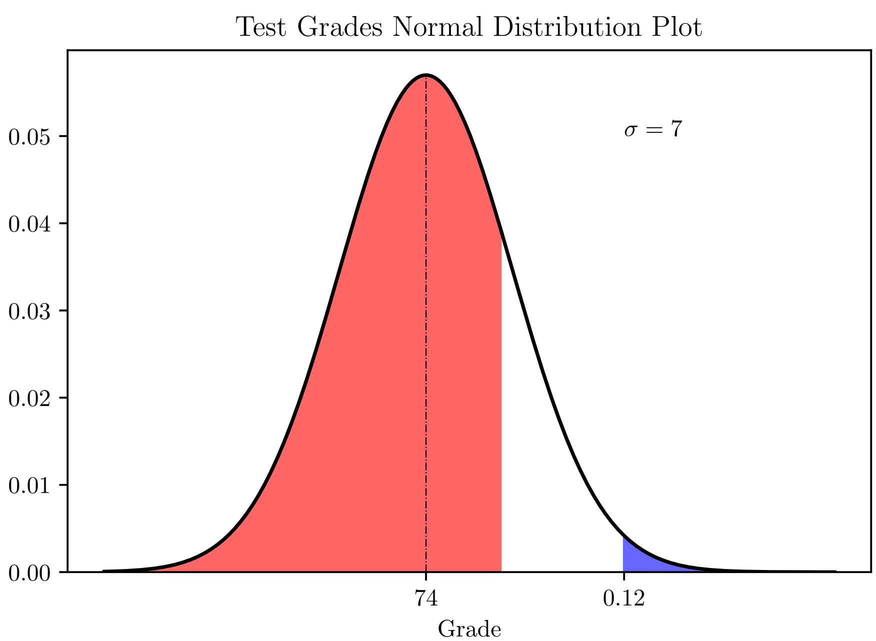
Given

- $\mu = 74$
- $\sigma = 7$
- 12% of students made an A

Find

- The value of x
- Lowest possible A
- Highest possible B
- The probability that a person scored within the sixth decile

Diagram



Theory

To find the value of the lowest A and highest B, the following formula may be used to x which would lead to the answers. Since area of 0.12 is already given, it can be subtracted from 0.5 to get a new value that should then assist in finding the z-value

$$x = \sigma z + \mu$$

Problem 6 continued on next page...

To find the sixth decile, the value of 0.1 can be used to find a desired z-value which could then be plugged into the formula above.

Assumptions

The normal value of resistance is $N(74, (7)^2)$

Solution

Part A

To find the lowest A and highest B, first subtract 0.12 from 0.50

$$0.50 - 0.12 = 0.38$$

Plug in the new value into the z-table to get 1.18, and use this value to find x

$$\begin{aligned} x &= (7)(1.18) + 74 \\ &= \underline{\underline{82.26}} \end{aligned}$$

This x value shows that the lowest A is 83 and the highest B is 82

Part B

To find the grade that marks the sixth decile, first use the z-table to plug in 0.10 to find 0.26. Now plug it into the above equation to find the solution.

$$\begin{aligned} x &= (7)(0.26) + 74 \\ &= \underline{\underline{75.82}} \end{aligned}$$

Therefore $D_6 = 75.82$.

Conclusion

The lowest A is $\boxed{83}$ and the highest B is $\boxed{82}$ and the sixth decile is $\boxed{75.82.}$