Physics 216: Homework #1

Due on January 24, 2020 at 11:59pm (Pages 11)

Dr. Ostrovskaya Section 509

Amari West

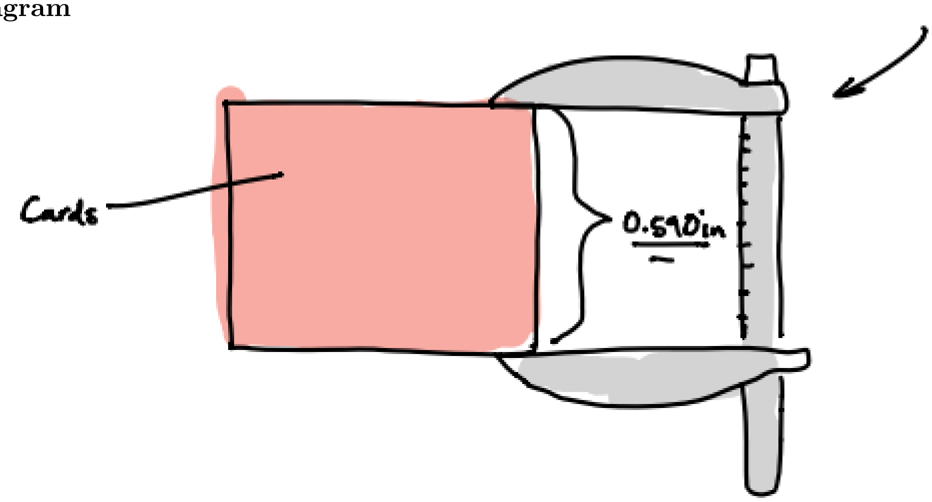
Problem 1

You have a set of calipers that can measure thickness of a few inches with an uncertainty of ±0.005 inches. You measure the thickness of a deck of 52 cards and get 0.590 in:

a) If you now calculate the thickness of 1 card, what is your answer, including its uncertainty?

b) You can improve this result by measuring several decks together. If you want to know the thickness of 1 card with an uncertainty of only 0.00002in, how many decks do you need to measure together?

Diagram



Given

- The measurement tool has an uncertainty of ±0.005.
- The thickness of 52 cards is 0.590 in.

Find

- The thickness of a single card along with its uncertainty.
- The amount of decks must be measured together to find a measurement of a single card with the uncertainty of 0.00002 in.

Theory

- If q is the quotient of two values (x and y) with uncertainties, then δq is

$$\delta q = |q|\sqrt{(\frac{\delta x}{x})^2 + (\frac{\delta y}{y})^2}$$

- The thickness, t, of a single card can be found with this equation, with T being the thickness of the deck and N being the number of cards

$$t = \frac{T}{N}$$

Problem 1 continued on next page. . .

- When calculating the uncertainty while multiplying by an exact number, simply multiply the uncertainty by the number as shown below

$$\delta q = |N|\delta x$$

Assumptions

- Each card has $\frac{1}{52}$ the thickness of a single deck.
- Every deck of cards has the same thickness.

Solution A

1. Divide the thickness of the cards by the amount.

$$\begin{aligned} q &= \frac{T}{N} \\ &= \frac{0.590}{52} \\ &= \underline{\underline{0.0113}} \end{aligned}$$

2. Calculate the new uncertainty.

$$\begin{aligned} \delta t &= |N|\delta x \\ &= \frac{0.005}{52} \\ &= \underline{\underline{0.0001}} \end{aligned}$$

Therefore, the thickness of a single card is 0.0113 ± 0.0001 in.

Solution B

1. Take the uncertainty equation that involves exact values and set it up to solve for the number of cards.

$$\begin{aligned} \delta q &= |N|\delta x \\ \frac{\delta q}{\delta x} &= |N| \end{aligned}$$

2. Plug in the uncertainty found earlier into the δq and plug in the target uncertainty value into δx.

$$\begin{aligned} \frac{0.0001}{0.00002} &= |N| \\ \underline{\underline{5}} &= |N| \end{aligned}$$

The amount of cards needed is a factor of 5; therefore, it is logical to assume that 5 decks of cards are needed to reach that uncertainty.

Conclusion

Based on the calculations above, the thickness of a single card is 0.0113 inches with an uncertainty of 0.0001 inches, and the amount of decks needed to decreased that uncertainty to 0.00002 is about 5.

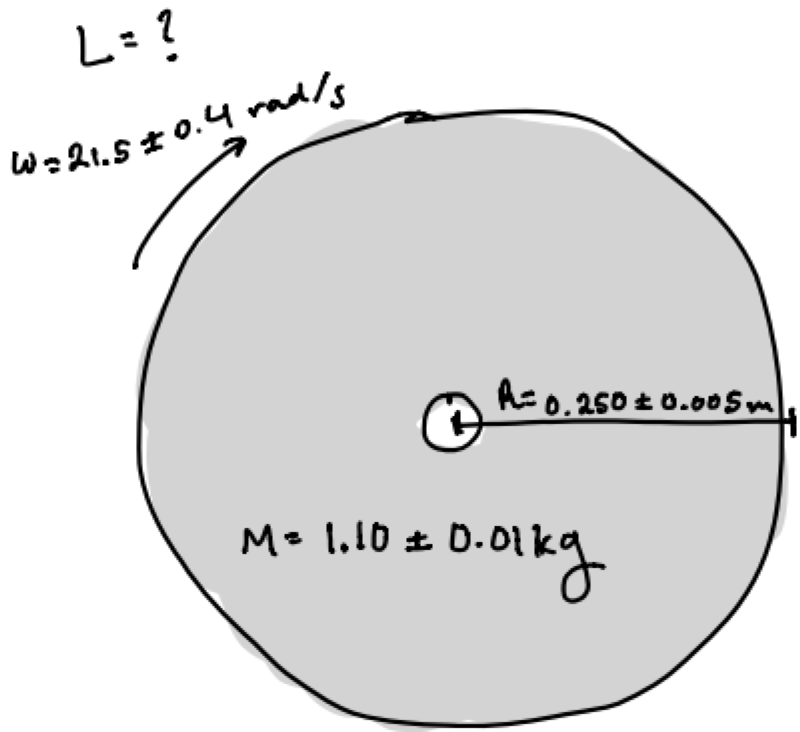
Problem 2

In an experiment on the conservation of angular momentum, a student needs to find the angular momentum L of a uniform disk of mass M and a radius R as it rotates with angular velocity ω. She makes the following measurements.

$$M = 1.10 \pm 0.01\,\text{kg}$$
$$R = 0.250 \pm 0.005\,\text{m}$$
$$\omega = 2.15 \pm 0.4\,\text{rad/s}$$

and calculates L using $L = \frac{1}{2}MR^2\omega$. What is her answer for L with its uncertainty?

Diagram



Given

- $M = 1.10 \pm 0.01\,\text{kg}$
- $R = 0.25 \pm 0.005\,\text{m}$
- $\omega = 21.5 \pm 0.4\,\text{rad/s}$

Find

- The angular momentum, L, along with its uncertainty.

Problem 2 continued on next page...

Theory

- The equation for angular momentum is described as:

$$L = \frac{1}{2}MR^2\omega$$

- The equations needed to find uncertainty in this case is:

$$\frac{\delta L}{|L|} = \sqrt{(\frac{0}{0.5})^2 + (\frac{\omega M}{M})^2 + (\frac{\omega R}{R})^2 + (\frac{\delta\omega}{\omega})^2}$$
$$\frac{\delta R^2}{|R^2|} = \sqrt{(\frac{\omega R}{R})^2 + (\frac{\omega R}{R}^2)}$$

Assumptions

- The angular velocity is constant.
- There is no friction.

Solution

1. To find angular momentum, L, plug in mass, radius, and angular velocity.

$$\begin{aligned} L &= \frac{1}{2}MR^2\omega \\ &= \frac{1}{2}(1.10\,\mathrm{kg})(0.250\,\mathrm{m})^2(21.5\,\mathrm{s}^{-1}) \\ &= \underline{\underline{0.739\,\frac{\mathrm{kg}*\mathrm{m}^2}{\mathrm{s}}}} \end{aligned}$$

2. To find the uncertainties, δL, plug in the uncertainties of the other variables.

$$\begin{aligned} \frac{\delta R^2}{|R|} &= \sqrt{(\frac{0.005\,\mathrm{m}}{0.250\,\mathrm{m}})^2 + (\frac{0.005\,\mathrm{m}}{0.250\,\mathrm{m}})^2} \\ &= \underline{\underline{0.03\,\mathrm{m}}} \\ \frac{\delta L}{|L|} &= \sqrt{(\frac{0}{0.5})^2 + (\frac{0.01\,\mathrm{kg}}{1.10\,\mathrm{kg}})^2 + (0.03\,\mathrm{m})^2 + (\frac{0.4\,\frac{\mathrm{rad}}{\mathrm{s}}}{21.5\,\frac{\mathrm{rad}}{\mathrm{s}}})^2} \\ &= \underline{\underline{0.03\,\frac{\mathrm{kg}*\mathrm{m}^2}{\mathrm{s}}}} \end{aligned}$$

Conclusion

The angular momentum is $0.739 \pm 0.03\,\frac{\mathrm{kg}*\mathrm{m}^2}{\mathrm{s}}$.

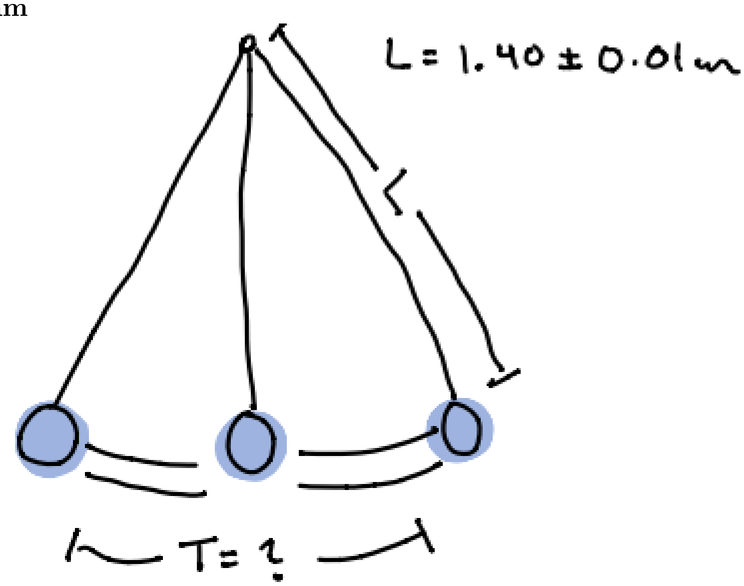
Problem 3

According to theory, the period T of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$.

a) If L is measured as $L = 1.40 \pm 0.01\,\text{m}$ what is the predicted value of T?

b) Would you say that a measured value of $T = 2.39 \pm 0.01$ is consistent with the theoretical prediction of part a.

Diagram



Given

- The length of the pendulum is $1.40 \pm 0.01\,\text{m}$
- The measured value of $T = 2.39 \pm 0.01\,\text{s}$ is consistent with the theoretical prediction of part(a).

Theory

- Period can be calculated with the following:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Assumptions

- All calculations are done with the assumption that the most ideal conditions are present (i.e. lack of air resistance)
- $g = 9.81\,\frac{\text{m}}{\text{s}^2}$

Problem 3 continued on next page...

Solution A

1. To solve for the period, T, plug in the values to the following equation.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g}} \\ &= 2\pi\sqrt{\frac{1.40\,\mathrm{m}}{9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}}} \\ &= 2\pi(0.378\,\mathrm{s}) \\ &= \underline{\underline{2.38\,\mathrm{s}}} \end{aligned}$$

2. To find the uncertainty of T, take the derivative of and plug the values into the following:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g}} \\ \ln T &= 2\ln(\pi\sqrt{\frac{L}{g}}) \\ \ln T &= \ln 2\pi + \frac{1}{2}\ln L - \frac{1}{2}\ln g \\ \frac{\delta T}{T}(T) &= \frac{\delta L}{2L}(T) \\ \delta T &= \frac{\delta L T}{2L} \\ &= \frac{(0.01\,\mathrm{m})(2.39\,\mathrm{s})}{2(1.20\,\mathrm{m})} \\ &= \underline{\underline{0.00854\,\mathrm{s}}} \end{aligned}$$

Solution B

- The measured value yielded nearly the same result but but the calculated value had a greater uncertainty.

Conclusion

- The calculation yielded $2.38 \pm 0.0832\,\mathrm{s}$ which has a smaller uncertainty then the measured value of $2.39 \pm 0.00854\,\mathrm{s}$.

Problem 4

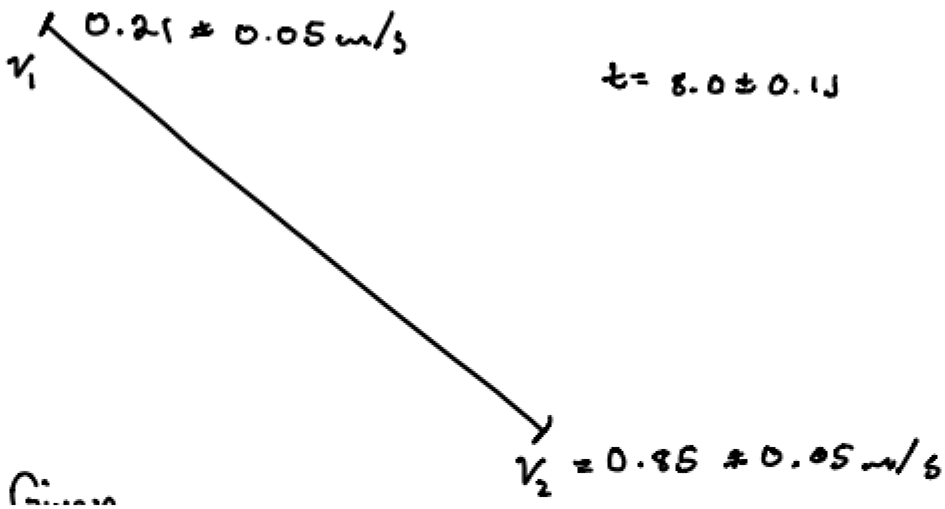
To find the acceleration of a glider moving down a sloping air track, you measure its velocity at two points (v_1 and v_2) and the time t it takes between them.

$$v_1 = 0.21 \pm 0.05\,\frac{\mathrm{m}}{\mathrm{s}}$$
$$v_2 = 0.85 \pm 0.05\,\frac{\mathrm{m}}{\mathrm{s}}$$
$$t = 8.0 \pm 0.1\,\mathrm{s}$$

a) Assuming all uncertainties are independent and random, and acceleration is calculated using $a = \frac{v_2 - v_1}{t}$, what should you report for a and its uncertainty?

b) You calculate using an air resistances model that the acceleration should be $0.13 \pm 0.01\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Does your measurement agree with this prediction?

Diagram



Given

- $v_1 = 0.21 \pm 0.05\,\frac{\mathrm{m}}{\mathrm{s}}$
- $v_2 = 0.85 \pm 0.05\,\frac{\mathrm{m}}{\mathrm{s}}$
- $t = 8.0 \pm 0.1\,\mathrm{s}$

Find

- a and its uncertainty.
- How does the measurement compare to a calculation of $0.13 \pm 0.01\,\mathrm{m}s^2$.

Theory

- The equation for acceleration is as follows:

$$a = \frac{v_2 - v_1}{t}$$

Problem 4 continued on next page...

Assumption

- There is no air resistance.

Solution

1. Plug all the variables into the acceleration equation to get acceleration:

$$
\begin{aligned}
a &= \frac{v_2 - v_1}{t} \\
&= \frac{0.85\,\frac{\mathrm{m}}{\mathrm{s}} - 0.21\,\frac{\mathrm{m}}{\mathrm{s}}}{8.0\,\mathrm{s}} \\
&= \underline{\underline{0.080\,\frac{\mathrm{m}}{\mathrm{s}^2}}}
\end{aligned}
$$

2. To find the uncertainty of a, the uncertainty of Δv must be found.

$$
\begin{aligned}
\delta\Delta v &= \sqrt{(0.05\,\frac{\mathrm{m}}{\mathrm{s}})^2 + (0.05\,\frac{\mathrm{m}}{\mathrm{s}})^2} \\
&= \underline{\underline{0.07071\,\frac{\mathrm{m}}{\mathrm{s}}}}
\end{aligned}
$$

3. Now, plug in $\delta\Delta V$, ΔV, and t into the equation to get δa.

$$
\begin{aligned}
\delta a &= 0.080\sqrt{(\frac{0.07\,\frac{\mathrm{m}}{\mathrm{s}}}{0.64\,\frac{\mathrm{m}}{\mathrm{s}}})^2(\frac{0.1\,\frac{\mathrm{m}}{\mathrm{s}}}{8.0\,\frac{\mathrm{m}}{\mathrm{s}}})^2} \\
&= \underline{\underline{0.01\,\frac{\mathrm{m}}{\mathrm{s}^2}}}
\end{aligned}
$$

Conclusion

1. The calculation yielded $0.080 \pm 0.01\,\frac{\mathrm{m}}{\mathrm{s}^2}$ therefore the measurement does not agree.

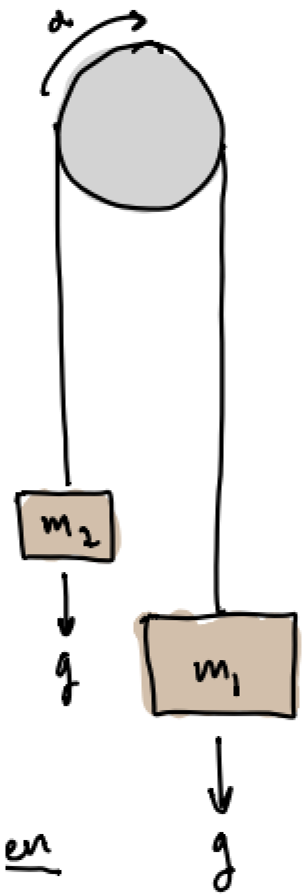
Problem 5

An Atwood machine consists of two masses m_1 and m_2 (with $m_1 > m_2$) attached to the ends ends of a light string that passes over a light, friction-less pulley. When the masses are released, the mass m_1 is easily shown to accelerate down with an acceleration

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Suppose that m_1 and m_2 are measured as $m_1 = 100 \pm 1$ gram and $m_2 = 50 \pm 1$ gram. Derive a formula of the uncertainty in the expected acceleration in terms of the masses and their uncertainties, and the calculate δa for the given numbers.

Diagram



Given

- $m_1 = 100 \pm 1$ gram
- $m_2 = 50 \pm 1$ gram

Find

- Derive a formula of uncertainty that gives an expected acceleration in terms of the masses and their uncertainties.
- Calculate δa.

Problem 5 continued on next page. . .

Theory

- This equation shows how the masses are related to find acceleration.

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Assumption

1. Plug in variables to calculate a.

$$\begin{aligned} a &= g\frac{m_1 - m_2}{m_1 + m_2} \\ &= (9.81\,\frac{\mathrm{m}}{\mathrm{s}^2})\frac{100\,\mathrm{g} - 50\,\mathrm{g}}{100\,\mathrm{g} + 50\,\mathrm{g}} \\ &= \underline{\underline{3.27\,\frac{\mathrm{m}}{\mathrm{s}^2}}} \end{aligned}$$

2. To calculate the uncertainty, the following equation needs to be derived.

$$\begin{aligned} \ln a &= \ln g\frac{m_1 - m_2}{m_1 + m_2} \\ &= \ln g + \ln(m_1 - m_2) - \ln(m_1 + m_2) \end{aligned}$$

3. Take the derivative.

$$\begin{aligned} \frac{\delta a}{a} &= \frac{\delta m_1 - \delta m_2}{m_1 - m_2} - \frac{\delta m_1 + \delta m_2}{m_1 + m_2} \\ \delta a &= a(\frac{\delta m_1 - \delta m_2}{m_1 - m_2} - \frac{\delta m_1 + \delta m_2}{m_1 + m_2}) \end{aligned}$$

4. Plug everything in.

$$\begin{aligned} \delta &= (\frac{0\,\mathrm{g}}{50\,\mathrm{g}} - \frac{2\,\mathrm{g}}{150\,\mathrm{g}})(3.27\,\mathrm{m}s^2) \\ &= \underline{\underline{0.044\,\frac{\mathrm{m}}{\mathrm{s}^2}}} \end{aligned}$$

Conclusion

1. The acceleration of the system is $3.27 \pm 0.044\,\frac{\mathrm{m}}{\mathrm{s}^2}$.

2. The following equation was derived to reach that uncertainty:

$$\frac{\delta a}{a} = \frac{\delta m_1 - \delta m_2}{m_1 - m_2} - \frac{\delta m_1 + \delta m_2}{m_1 + m_2}$$