Elements Of Data Science - F2020

Week 7: Model Evaluation and Hyperparameter Tuning

10/26/2020

TODOs

- Readings:
 - PDSH Chapter 5: Feature Engineering
 - (Recommended) PML Chapter 4
- HW2, Out this week?
- Answer and submit Quiz 7, Sunday Nov 9th, 11:59pm ET
- Midterm
 - Release Monday Oct 26th, 11:59pm ET
 - Due Saturday Oct 31st, 11:59pm ET
 - Have 24hrs after starting exam to finish
 - 30 questions (fill in the blank/multiple choice/short answer)
 - Online via Gradescope
 - Questions asked/answered privately via Piazza
 - Open-book, open-note, open-python

Generating Predictions for a Single Value

```
In [2]: from sklearn.linear_model import LogisticRegression
In [3]: X = np.random.rand(5,2)
        y = (np.random.rand(5) > .5).astype(int)
        lr = LogisticRegression().fit(X,y)
        # use reshape(1,-1) when generating a single prediction
        # recall that sklearn expects X to be two dimensional
        lr.predict(np.array([.2,.3]).reshape(1,-1))
Out[3]: array([0])
In [4]: # Note: this is different from when training on a single feature: .reshape(-1,1)
        lr.fit(X[:,0].reshape(-1,1),y)
        lr.predict(np.array(.2).reshape(1,-1))
Out[4]: array([0])
```

Today

- Model Evaluation and Selection
- Hyperparameter Tuning
- Regularization

Questions?

Model Evaluation and Hyperparameter Tuning

- How well are any of our models working?
- How can we compare different models?
- How do we decide on hyperparameter settings?
- How can we keep our models from "overfitting"?
- How do we do all this both for Regression and Classification?

How well are our models performing?

Regression

- Mean Squared Error (MSE) and Root Mean Squared Error (RMSE)
- R^2
- Adjusted R^2

Classification

- Accuracy
- Precision/Recall/F1
- ROC Area Under the Curve (AUC)

Data Setup for Regression

In [6]: df_wine

Out[6]:

	alcohol	ash	alcalinity_of_ash	magnesium	hue	proline	class
0	1.514341	0.231400	-1.166303	1.908522	0.361158	1.010159	0
1	0.245597	-0.825667	-2.483841	0.018094	0.404908	0.962526	0
2	0.196325	1.106214	-0.267982	0.088110	0.317409	1.391224	0
3	1.686791	0.486554	-0.806975	0.928300	-0.426341	2.328007	0
4	0.294868	1.835226	0.450674	1.278379	0.361158	-0.037767	0
•••							•••
173	0.873810	0.304301	0.300954	-0.331985	-1.388840	-0.021890	2
174	0.491955	0.413653	1.049555	0.158126	-1.126341	0.009866	2
175	0.331822	-0.388260	0.151234	1.418411	-1.607590	0.279786	2
176	0.208643	0.012696	0.151234	1.418411	-1.563840	0.295664	2
177	1.391162	1.361368	1.498716	-0.261969	-1.520090	-0.593486	2

178 rows × 7 columns

Regression with Simple Linear Model

```
In [7]: from sklearn.linear_model import LinearRegression
        lr = LinearRegression().fit(X_1d,y_r)
        argmax_y_r = np.argmax(y_r)
        y_pred = lr.predict(X_1d)
        fig, ax = plt.subplots(1, 1, figsize=(8, 6))
        sns.scatterplot(x=X_r.proline, y=y_r, color='grey');
        ax.plot(X_1d,y_pred,color='k');
        ax.vlines(X_1d[argmax_y_r],y_r.iloc[argmax_y_r],y_pred[argmax_y_r],color='r');
```

How Good is This Fit? MSE and RMSE

• Mean Squared Error: $\sum_{i} (y_i - \hat{y}_i)^2$

- But this is the squared error! (alcohol^2)
- Root Mean Squared Error: $\sqrt{\sum_i y_i \hat{y}_i)^2}$

Is this good? Need a Baseline Comparison

- What's a baseline to compare against?
- Simple one for Regression: always predict the mean of the targets

```
In [10]: from sklearn.dummy import DummyRegressor
    dummyr = DummyRegressor(strategy='mean') # default strategy
    dummyr.fit(X_1d,y_r)
    dummy_rmse = root_mean_squared_error(y_r,dummyr.predict(X_1d))
    f'{dummy_rmse = :0.2f}'
Out[10]: 'dummy_rmse = 1.00'
```

Comparing against the mean: ${\it R}^2$

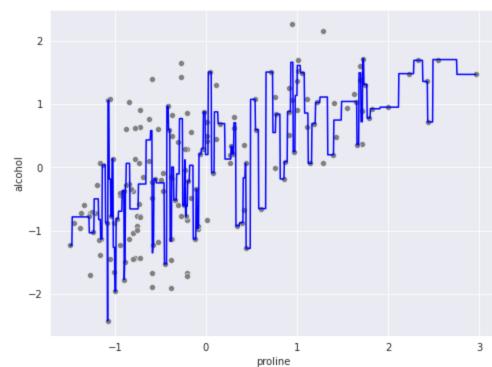
• the proportion of variance explained by the model

$$R^{2} = 1 - \frac{\sum (y_{i} - \hat{y}_{i})^{2}}{\sum (y_{i} - \bar{y})^{2}}$$

- maximum value of 1
- a value below 0 means the model is predicting worse than just predicting the mean
- ullet sklearn uses ${m R}^2$ as the default for regression scoring

Can we do better?

```
In [12]: from sklearn.tree import DecisionTreeRegressor
         dtr = DecisionTreeRegressor(max_depth=10)
         dtr.fit(X_1d,y_r)
         r2_dtr = dtr.score(X_1d, y_r)
         print(f'\{r2_1r = :0.2f\} \setminus n\{r2_dtr = :0.2f\}')
         r2 lr = 0.41
          r2_dtr = 0.76
In [13]: X_1d_sorted = np.sort(X_1d.flatten()).reshape(-1,1)
         X_{query} = np.linspace(X_1d.min(), X_1d.max(), 1000).reshape(-1, 1)
         y_pred = dtr.predict(X_query)
         fig, ax = plt.subplots(1, 1, figsize=(8, 6))
         sns.scatterplot(x=X_r.proline, y=y_r,color='gray');
         ax.plot(X_query,y_pred,color='b');
```



But is this what we want? Interpretation vs Prediction

Always good to ask:

- do we want our model to very closely fit our data for interpretation?
- do we want our model to predict well on new, unseen data?

Generalization:

how well will model predict on data that it hasn't seen yet?

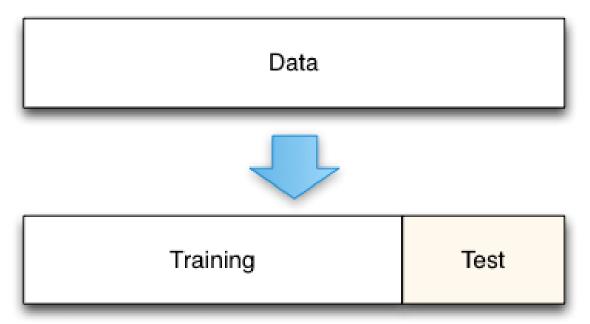
But we used all of our data to train?

Need to do a Train/Test Split

Train/Test Split

- Training Set: portion of dataset used for training
- Test/Held-Aside/Out of sample: portion of dataset used for evaluation

• Want the test set to reflect the same distribution as training



Train/Test split with Sklearn

- How big should test be?
 - Large enough to capture variance of dataset.
 - Depends on the dataset and the models being trained

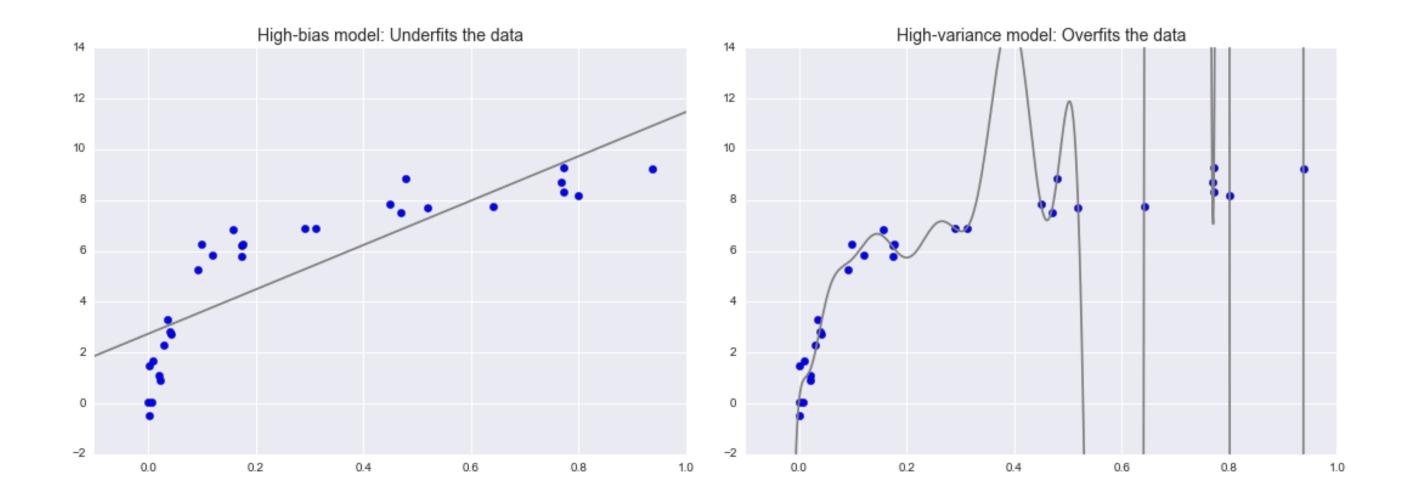
Training and Evaluate on Different Data

• DecisionTree model is doing worse than the Dummy model on the test set!

Overfitting and Underfitting

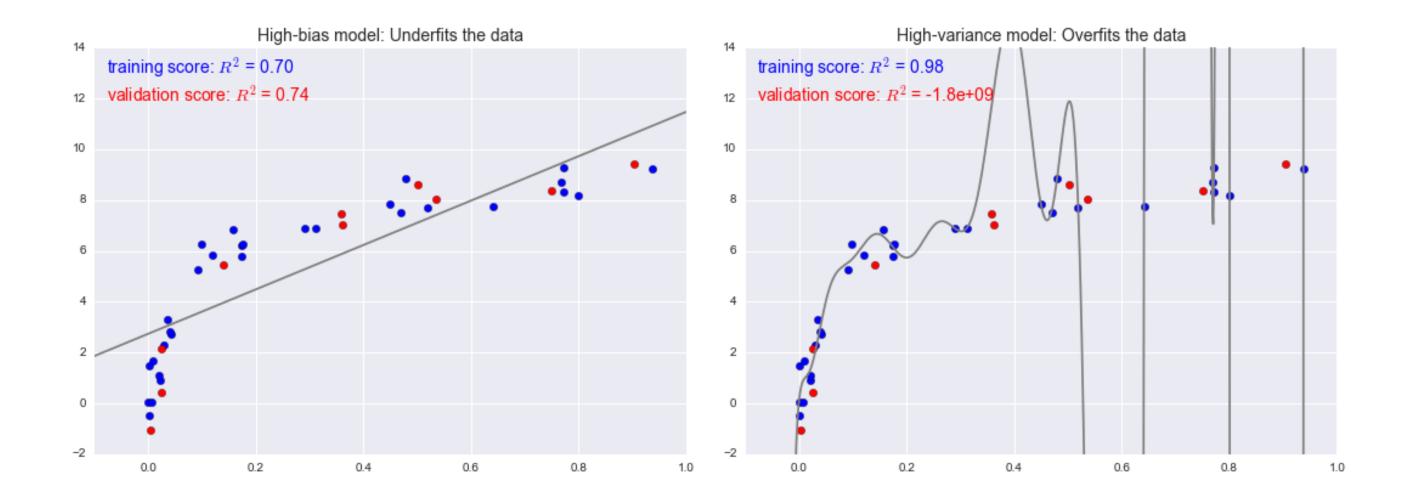
- Overfitting: poor generalization due to complexity
 - learning noise in training data
- Underfitting: poor generalization due to simplicity
 - not flexibile enough to learn concept
- Need to find a balance between simplicity and complexity
- Need to find a balance between **bias** and **variance**

Bias-Variance Tradeoff



From PDSH

Bias-Variance Tradeoff



From PDSH

Bias-Variance Tradeoff Continued

- How close is the model to the underlying concept?
- How sensitive is the model to the training set?

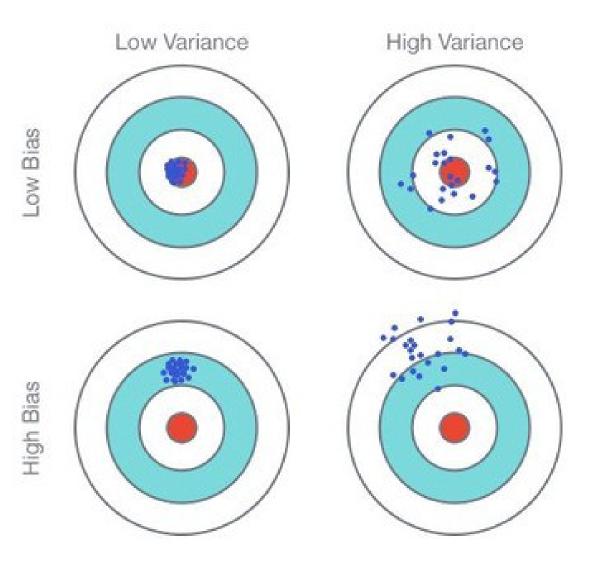
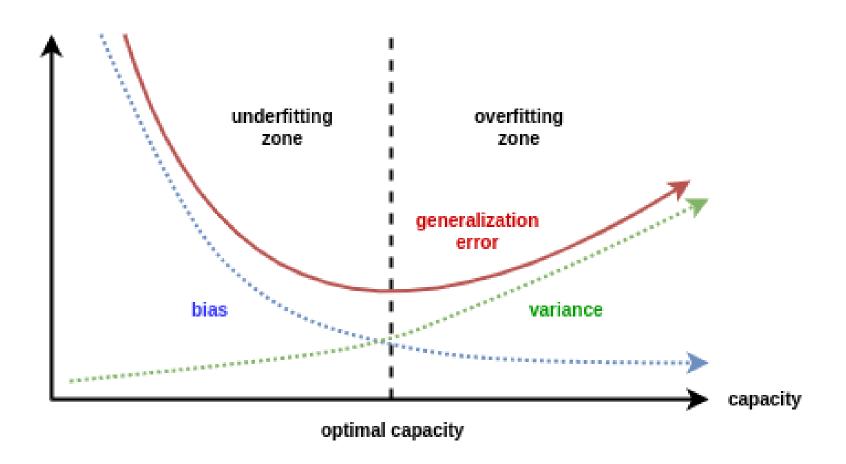


Fig. 1: Graphical Illustration of bias-<u>variance trade</u>-off , Source: Scott Fortmann-Roe., Understanding Bias-<u>Variance Trade</u>-off

Bias-Variance Tradeoff Continued



- We'd like to:
 - reduce the Bias (use a model complex enough to capture the concept)
 - without introducing too much Variance (overfit the data)
 - all in order to minimize Generalization Error

Overfitting/Underfitting Revisited

- Overfitting: poor generalization due to complexity
 - learning noise in training data
 - model has high variance and low bias
- Underfitting: poor generalization due to simplicity
 - not flexibile enough to learn concept
 - model has high bias and low variance

Avoiding Overfitting/Underfitting

- Never train and evaluate on the same set of data!
 - train test split
 - cross-validation
- Keep the model as simple as possible (Occom's Razor)

Avoiding Overfitting/Underfitting

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 - train test split
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- Keep the model as simple as possible (Occom's Razor)

```
In [16]: fig, ax = plt.subplots(1, 3, figsize=(16, 4))
    for i in range(3):
        sns.scatterplot(x=X_train_r.flatten(), y=y_train_r, color="gray", ax=ax[i])
        sns.scatterplot(x=X_test_r.flatten(), y=y_test_r, color="black", s=100, ax=ax[i]);
        ax[0].plot(X_query, dummyr.predict(X_query), color='r');
        ax[1].plot(X_query,lr.predict(X_query), color='g');
        ax[2].plot(X_query,dtr.predict(X_query), color='b');
        rediction of the prediction of the prediction
```

Overfitting? Simplify the model

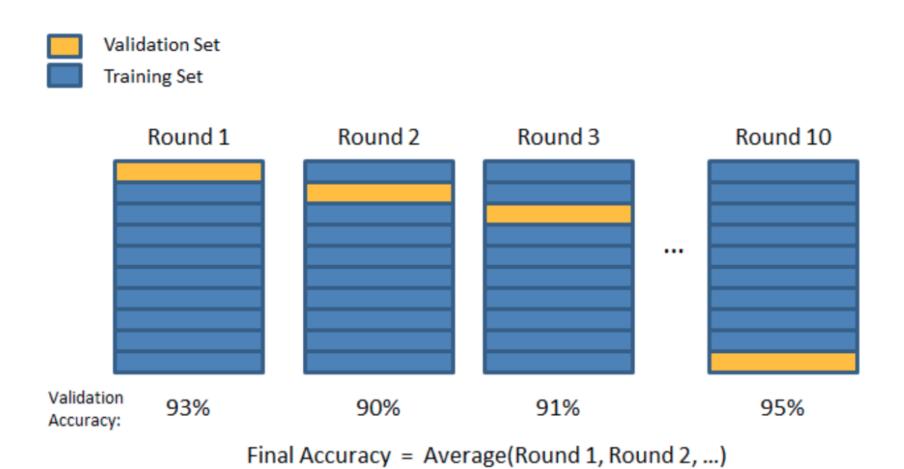
```
In [17]: max_depths = [2,3,4]
         fig, ax = plt.subplots(1,3,figsize=(16,4))
         for i in range(3):
             dtr = DecisionTreeRegressor(max_depth=max_depths[i]).fit(X_train_r,y_train_r)
             sns.scatterplot(x=X_train_r.flatten(), y=y_train_r, color="gray", ax=ax[i])
             sns.scatterplot(x=X_test_r.flatten(), y=y_test_r, color="black", s=100, ax=ax[i]);
             ax[i].plot(X_query, dtr.predict(X_query), color='b');
             ax[i].set_title(f'depth {max_depths[i]}, r2: {dtr.score(X_test_r,y_test_r):0.2f}')
```

- But now we might be overfitting on the test set!
- How to choose hyperparameters: **Cross-Validation**

k-Fold Cross-Validation

- 1. split dataset into k equal sized subsets (folds)
- 2. for each subset (fold)
 - train on the other k-1 subsets combined
 - test on this subset to get a score
- 3. average across all scores
- Result is a set of samples of model performance
- Can use to set hyperparameters without overfitting on train or test
- Can also use to estimate range of generalization performance

Example: 10-Fold Cross-Validation



k-Fold Cross-Validation Continued

- Can be used for:
 - tuning hyperparameters
 - model selection
 - any time we need estimate of model performance
- Issue: each fold requires training the model
 - Training time can be an issue for large k
- What values can k take?
 - min: 2
 - max: *n*, the size of the dataset (aka Leave-One-Out CV)

k-Fold Cross-Validation in sklearn

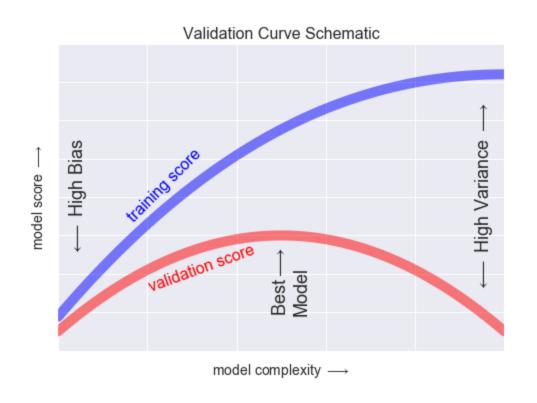
Tuning Hyperparameters with CV

```
In [20]: mean_scores = []
         for depth in [1, 2, 3, 5, 10]:
             dtr = DecisionTreeRegressor(max_depth=depth)
             scores = cross_val_score(dtr,X_train_r,y_train_r,cv=5)
             mean_scores.append( (depth, scores.mean()) )
         for depth, mean_score in mean_scores:
             print(f'\{depth = :2d\} : \{mean\_score: 0.3f\}')
         depth = 1 : 0.289
         depth = 2 : 0.346
         depth = 3 : 0.341
         depth = 5 : 0.050
         depth = 10 : -0.142
In [21]: # find the depth that gives best score (highest R^2)
         sorted(mean_scores, key=lambda x:x[1], reverse=True)[0] # sorted is ascending by default
Out[21]: (2, 0.3460316005269502)
```

Visualize Tuning: Validation Curve

Validation Curve

- Show model complexity vs model performance on both train and test/validation
- Want to find point where performance on validation set begins to decline (overfitting)



From PDSH

Validation Curve in sklearn

```
In [22]: from sklearn.model_selection import validation_curve
         depth = [1, 2, 3, 5, 8, 10]
         train_scores, test_scores = validation_curve(DecisionTreeRegressor(),
                                                       X_train_r, y_train_r,
                                                       param_name='max_depth',
                                                       param_range=depth,
                                                       cv=5)
         mean_train_scores = np.mean(train_scores,axis=1) # take the mean across rows
         mean_test_scores = np.mean(test_scores,axis=1)
In [23]: plt.plot(depth, mean_train_scores, 'o-', color='b', label='training score');
         plt.plot(depth, mean_test_scores, 'o-', color='r', label='validation score');
         plt.xlabel('max_depth'), plt.ylabel('R^2'); plt.title('Validation Curve for DecisionTree');
                    Validation Curve for DecisionTree
            0.8
            0.6
```

More Than One HyperParameter? Grid Search

Grid Search: Search over a 'grid' of hyperparameter settings

Example: KNN "number of neighbors" and "distance metric"

Grid Search in sklearn

```
In [25]: from sklearn.model_selection import GridSearchCV
         from sklearn.neighbors import KNeighborsRegressor
         params = {'n_neighbors':[1,2,3,5,10],
                   'metric':['euclidean','manhattan']}
         gscv = GridSearchCV(KNeighborsRegressor(),
                             params, # grid of size 10
                             cv=3, # do 3-fold CV at every grid point
                             refit=True) # refit True trains one more time on the entire training set
         gscv.fit(X_train_r,y_train_r)
                                       # How many times are we training a model here? (2*5*3 + 1 = 31)
         print(gscv.best_params_)
         {'metric': 'euclidean', 'n_neighbors': 5}
In [26]: scores = cross_val_score(gscv.best_estimator_, X_train_r, y_train_r, cv=5)
         print(f'\{np.mean(scores):0.2f\} +- \{2*np.std(scores):0.2f\}')
         0.35 + - 0.21
```

Review So Far

- Regression Metrics
 - MSE and RMSE
 - R^2
- Model Selection
 - Comparison to Baseline Model
 - Underfitting/Overfitting, Bias/Variance
 - Train/Test Split
- Hyperparameter Tuning
 - Cross-Validation
 - Grid Search
 - Validation Curve

Data Setup for Classification

Out[28]:

train test

12

15

```
In [27]: idx_binary = df_wine['class'].isin([0,1])
                                                     # reduce to binary classification
        X_bc = df_wine.loc[idx_binary,['alcalinity_of_ash','magnesium']] # only 2 features for ease of plotting
         y_bc = df_wine.loc[idx_binary,'class']
                                                 # pull out classification target [0,1]
         X_{train_bc}, X_{test_bc}, y_{train_bc}, y_{test_bc} = train_{test_split}(X_{bc}, y_{test_bc})
                                                                      y_bc,
                                                                      stratify=y_bc, # maintain label proportions
                                                                      random_state=0
         pd.DataFrame({'train':y_train_bc.value_counts(), 'test':y_test_bc.value_counts()}).sort_index()
Out[27]:
            train test
         1 53 18
In [28]: X_mc = df_wine.loc[:,['alcalinity_of_ash', 'magnesium']]
                                                                      # multiple features for multiclass classification task
         y_mc = df_wine.loc[:,'class']
                                                                       # pull out classification target [0,1,2]
        X_train_mc, X_test_mc, y_train_mc, y_test_mc = train_test_split(X_mc,
                                                                      stratify=y_mc, # maintain label proportions
                                                                      random_state=123
         pd.DataFrame({'train':y_train_mc.value_counts(),'test':y_test_mc.value_counts()}).sort_values(by="train")
```

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Default Metric in Classification: Accuracy

• Accuracy: out of all the observations, how many did I get right?

```
In [29]: from sklearn.dummy import DummyClassifier
    from sklearn.tree import DecisionTreeClassifier
    dummyc = DummyClassifier(strategy='prior').fit(X_train_bc,y_train_bc) # works like 'most-frequent'
    dtc = DecisionTreeClassifier(max_depth=2).fit(X_train_bc,y_train_bc)

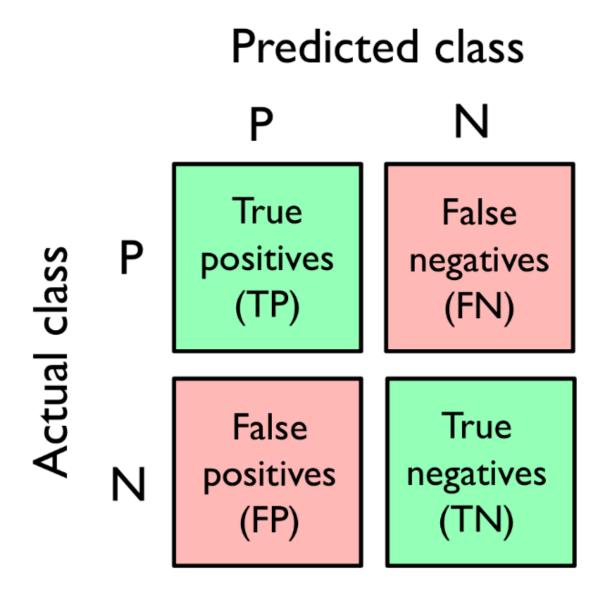
    print(f'{dummyc.score(X_test_bc,y_test_bc) = :0.2f}') # default classification score is accuracy
    print(f'{dtc.score(X_test_bc,y_test_bc) = :0.2f}')

    dummyc.score(X_test_bc,y_test_bc) = 0.55
    dtc.score(X_test_bc,y_test_bc) = 0.79
```

- But what if the cost of calling a negative a positive is different from calling a positive a negative?
- Examples:
 - disease testing
 - medical product failures
 - incarceration

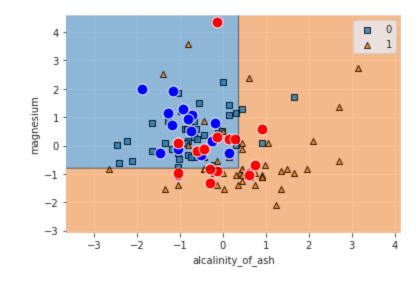
Errors in Classification

• There are different kinds of error in classification



Visualizing Errors with a Confusion Matrix

```
In [30]: fig,ax = plt.subplots(1,1,figsize=(6,4));
    plot_decision_regions(X_train_bc.values,y_train_bc.values,dtc,ax=ax);
    sns.scatterplot(x=X_bc.columns[0],y=X_bc.columns[1],data=X_test_bc[y_test_bc == 0],color="blue",s=120);
    sns.scatterplot(x=X_bc.columns[0],y=X_bc.columns[1],data=X_test_bc[y_test_bc == 1],color="red",s=120);
```



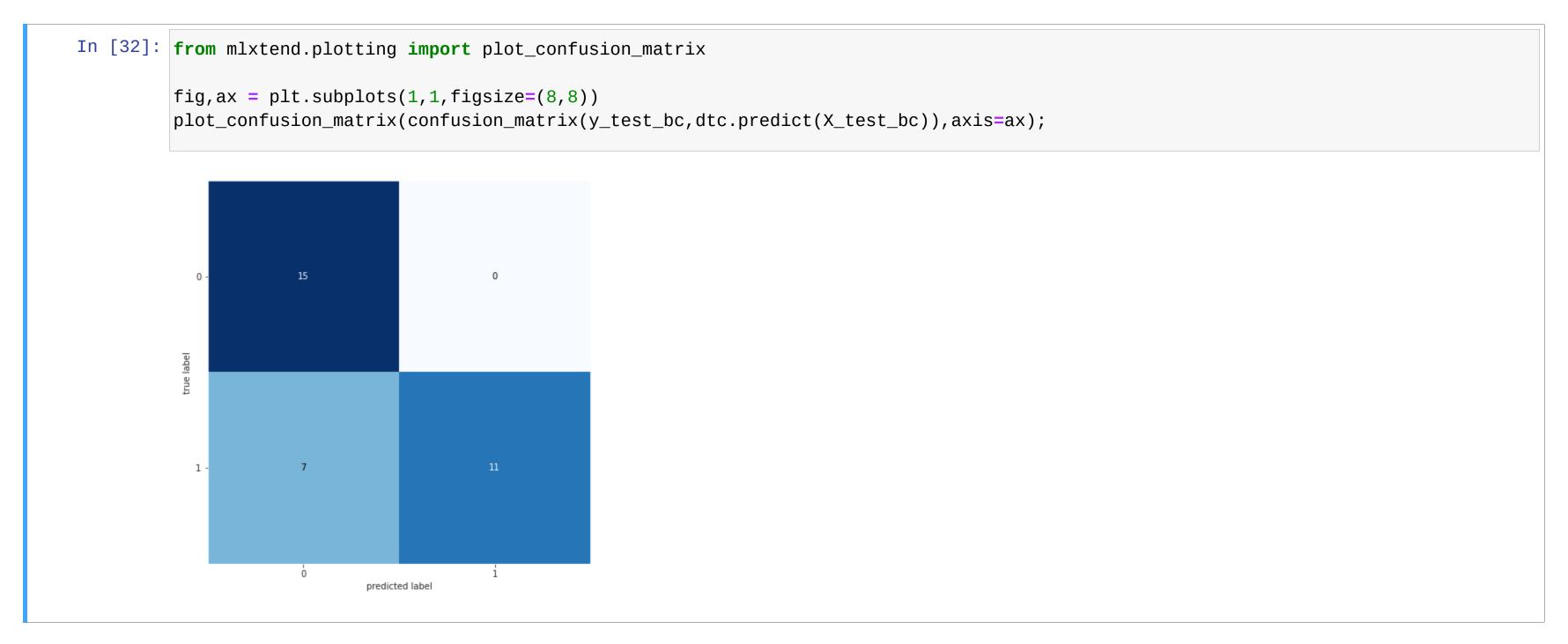
```
In [31]: from sklearn.metrics import confusion_matrix

print('training set error\n', confusion_matrix(y_train_bc,dtc.predict(X_train_bc)))
print()
print('test set error\n', confusion_matrix(y_test_bc,dtc.predict(X_test_bc)))

training set error
[[41 3]
       [14 39]]

test set error
[[15 0]
[ 7 11]]
```

Plot Confusion Matrix with mlxtend



Weighing Errors: Precision vs. Recall

Precision

• Out of the observations I predicted positive (TP+FP), how many are truly positive (TP)?

$$precision = \frac{TP}{TP+FP}$$

Recall

• Out of the truly positive (TP+FN), how many obsersevations did I predict positive (TP)?

$$recall = \frac{TP}{TP+FN}$$

Using Other Measures in sklearn

```
In [33]: dummyc_precision_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='precision')
    dummyc_recall_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='recall')

print(f'dtc precision: {np.mean(dummyc_precision_scores):0.2f} +- {2*np.std(dummyc_precision_scores):0.2f}')
    print(f'dtc recall : {np.mean(dummyc_recall_scores):0.2f} +- {2*np.std(dummyc_recall_scores):0.2f}')
    print()

dtc_precision_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='precision')
    dtc_recall_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='precision')
    print(f'dtc precision: {np.mean(dtc_precision_scores):0.2f} +- {2*np.std(dtc_precision_scores):0.2f}')

print(f'dtc recall : {np.mean(dtc_recall_scores):0.2f} +- {2*np.std(dtc_recall_scores):0.2f}')

dtc precision: 0.55 +- 0.04
    dtc recall : 1.00 +- 0.00

dtc precision: 0.85 +- 0.36
    dtc recall : 0.66 +- 0.38
```

How do we decide if something is positive or negative?

Usually set a threshold:

$$\hat{y}_i = \begin{cases} 1 & \text{if } P(y_i = 1 | x_i) > \text{threshold,} \\ 0 & \text{o.w.} \end{cases}$$

Usually, threshold = .5, but it doesn't have to be.

What happens if we change it?

- High threshold → High Precision, Low Recall
- Low threshold → High Recall, Low Precision

Combining Precision and Recall: F_1 -score

Usually, we just want one number to optimize

 F_1 -score: harmonic mean of precision and recall

eg. weighted average of the precision and recall

$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

Note that F_1 ignores False Negatives!

Combining Precision and Recall: F_1 -score

Usually, we just want one number to optimize

 F_1 -score: harmonic mean of precision and recall

eg. weighted average of the precision and recall

$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

Note that F_1 ignores False Negatives!

```
In [34]:
dummyc_f1_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='f1')
dtc_f1_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='f1')
print(f'dummyc f1 = {np.mean(dummyc_f1_scores):0.2f} +- {2*np.std(dummyc_f1_scores):0.2f}')
print(f'dtc f1 = {np.mean(dtc_f1_scores):0.2f} +- {2*np.std(dtc_f1_scores):0.2f}')
dummyc f1 = 0.71 +- 0.03
dtc f1 = 0.72 +- 0.31
```

Paying attention to False Negatives: ROC

Receiver Operating Characteristic

displays FPR vs TPR

False Positive Rate (FPR) =
$$\frac{FP}{FP+TN}$$
 = $\frac{\text{negatives we got wrong}}{\text{all negatives}}$

True Positive Rate (TPR) = Recall =
$$\frac{TP}{TP+FN}$$
 = $\frac{positives \text{ we got right}}{all \text{ positives}}$

How do these change as we move our threshold?

Paying attention to False Negatives: ROC

Receiver Operating Characteristic

displays FPR vs TPR

False Positive Rate (FPR) =
$$\frac{FP}{FP+TN}$$
 = $\frac{\text{negatives we got wrong}}{\text{all negatives}}$

True Positive Rate (TPR) = Recall =
$$\frac{TP}{TP+FN}$$
 = $\frac{positives we got right}{all positives}$

How do these change as we move our threshold?

Plotting ROC Curves

```
In [36]: def plot_roc(curves):
    fig,ax = plt.subplots(1,1,figsize=(6,6))
    lw = 2
    for fpr,tpr,model_name in curves:
        l1, = ax.plot(fpr, tpr, lw=lw, label=model_name)
    ax.plot([0, 1], [0, 1], color='k', lw=lw, linestyle='--')
    ax.set_xlim([0.0, 1.0])
    ax.set_ylim([0.0, 1.05])
    ax.set_ylabel('False Positive Rate')
    ax.set_ylabel('True Positive Rate')
    ax.set_aspect('equal', 'box')
    ax.set_title('Receiver operating characteristic example')
    ax.legend()
```

Plotting ROC Curves

```
In [37]: curves = [(fpr_dtc, tpr_dtc, 'dtc2')]
          fpr_dummyc, tpr_dummyc, _ = roc_curve(y_train_bc, dummyc.predict_proba(X_train_bc)[:,1]) # Compare dummy
          curves.append((fpr_dummyc,tpr_dummyc,'dummy'));
          plot_roc(curves);
                  Receiver operating characteristic example
             0.2
                         False Positive Rate
```

Plotting ROC Curves

```
In [38]: for depth in [3,4]:
              fpr, tpr, _ = roc_curve(y_train_bc,DecisionTreeClassifier(max_depth=depth).fit(X_train_bc,y_train_bc).predict_proba(X_train_
              curves.append((fpr,tpr,'dtc'+str(depth)))
          plot_roc(curves);
                  Receiver operating characteristic example
             1.0
             0.2
                         False Positive Rate
```

ROC AUC

- But again, we'd like one number to optimize
- ROC Area Under the Curve
 - How much area falls under the ROC curve?

```
In [39]: dummyc_rocauc_scores = cross_val_score(dummyc, X_train_bc, y_train_bc, cv=5, scoring='roc_auc')
    dtc_rocauc_scores = cross_val_score(dtc, X_train_bc, y_train_bc, cv=5, scoring='roc_auc')

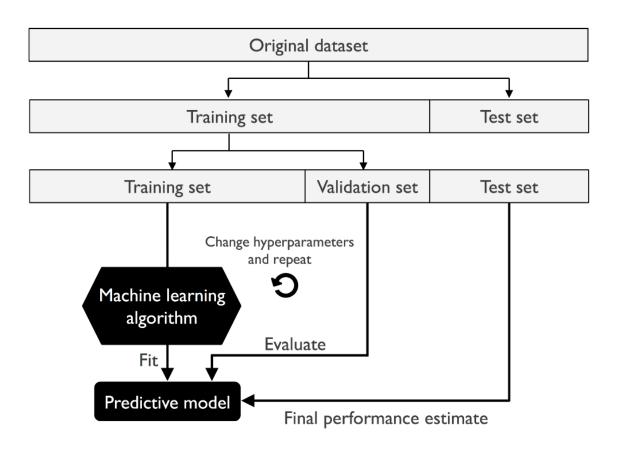
print(f'dummyc rocauc = {np.mean(dummyc_rocauc_scores):0.2f} +- {2*np.std(dummyc_rocauc_scores):0.2f}')

print(f'dtc rocauc = {np.mean(dtc_rocauc_scores):0.2f} +- {2*np.std(dtc_rocauc_scores):0.2f}')

dummyc rocauc = 0.50 +- 0.00
    dtc rocauc = 0.78 +- 0.22
```

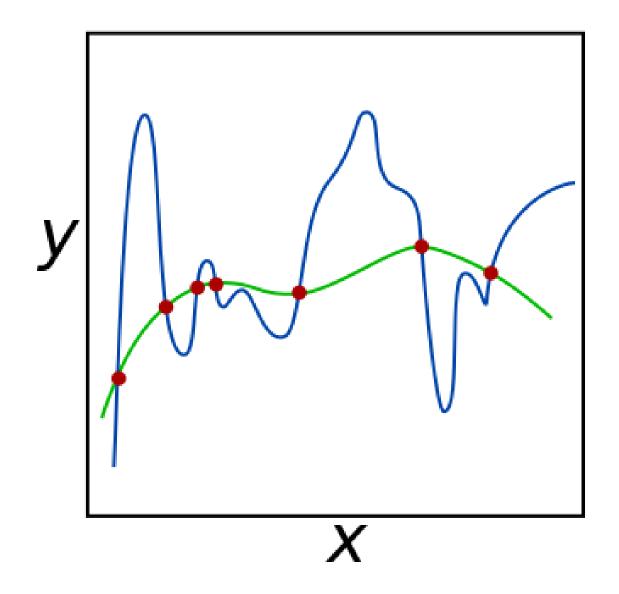
Review: Steps to Choosing a Model

- 1. Create Held-Aside Set (Train/Test Split)
- 2. Determine Metric (or combination of metrics)
- 3. Get a Baseline for comparison
- 4. Use Cross-Validation to fit Hyperparameters and Choose Model
- 5. Evaluate Chosen Model on Held-Aside Set



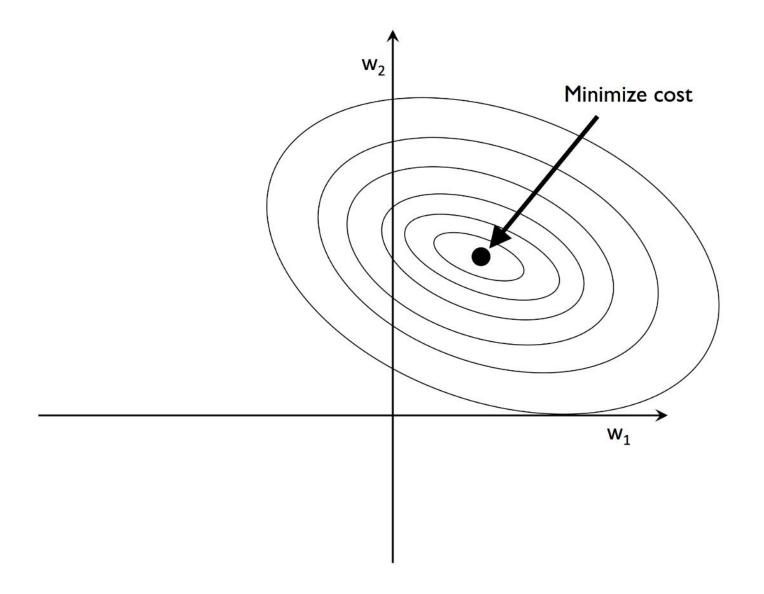
Avoiding Overfitting in Linear Models: Regularization

- Use to avoid overfitting in linear models
- Idea: can we reduce complexity of our linear model by minimizing weights?



Regression: Finding the Weights

- Linear models learn by finding weights that minimize a cost.
- Can we get close to the solution while still keeping weights small (simpler model)?



Regularization: Add a cost for large weights

Penalizing extreme weights (w)

If the original cost function looks like:

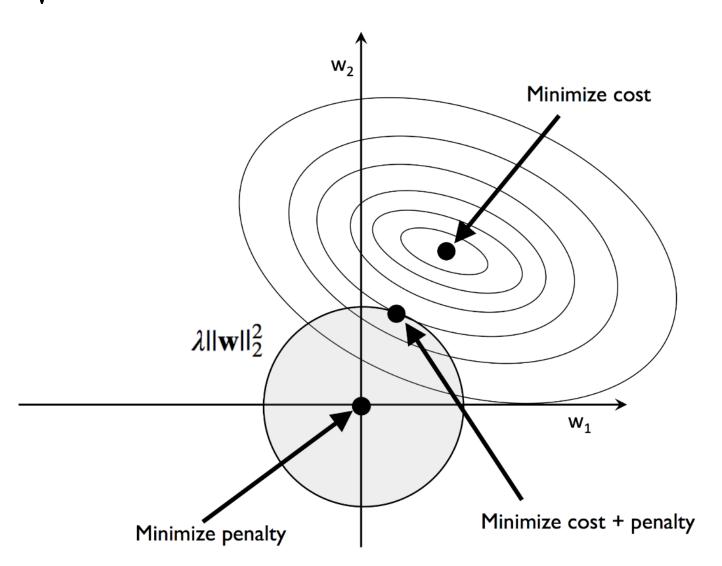
$$arg min_w C(f(w, x), y)$$

we can add a **regularization term**:

$$\arg\min_{w} C(f(w, x), y) + \lambda g(w)$$

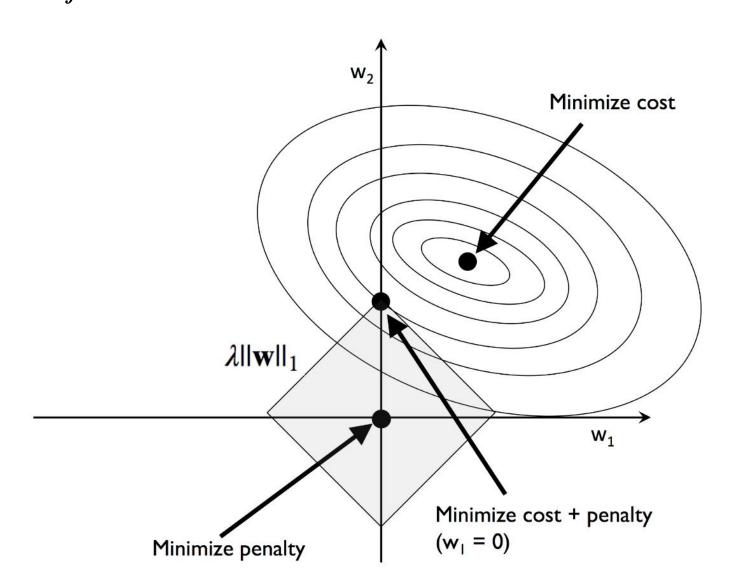
Regularization: Ridge

- Coefficients are encouraged to be "small"
- Uses L_2 norm: $\|w\|_2 = \sqrt{\sum_{j=1}^m w_j^2}$



Regularization: LASSO

- Coefficients are (likely) driven to zero
- Uses L_1 norm: $\|w\|_1 = \sum_{j=1}^m |w_j|$



From PML

Regularization: ElasticNet

- ullet Mixture of L_1 and L_2
- $\alpha L_1 + (1 \alpha)L_2$

- introduces a new hyperparameter α or l1_ratio
- 11_ratio = 1 is LASSO (L_1)
- 11_ratio = 0 is Ridge (L_2)

Regularization: ElasticNet

- ullet Mixture of L_1 and L_2
- $\alpha L_1 + (1 \alpha)L_2$

- introduces a new hyperparameter α or l1_ratio
- 11_ratio = 1 is LASSO (L_1)
- 11_ratio = 0 is Ridge (L_2)

```
In [40]: from sklearn.linear_model import LogisticRegression

lr = LogisticRegression(penalty='l2', # default

C=1.0, # weight on regularization, 1/lambda above

ll_ratio=None # only used when penalty is 'elasticnet'

)
```