

## SEMTM0010 Transport and Mobility Modelling

### 2024 / 2025 coursework

#### General points.

- This coursework is open-book, and you are permitted to consult any material (e.g., lecture slides, exercises) that has been posted on the unit's Blackboard page, or indeed your own notes. You are also strongly encouraged to research on the internet, however, significant sources should be cited in an appropriate manner.
- Note this is an individual assessment, and your answers must be your own, and may not be obtained with any outside help. Collaboration is not permitted. If cheating or collaboration are detected, as with any assessment, then this will be subject to the University's rules on academic integrity:  
<http://www.bristol.ac.uk/students/study/teaching/integrity/>.
- Please consult the Blackboard announcement of 28/10/2024 for details of how you will be supported with this coursework and the associated 'rules of engagement'.
- Please take note of the University's rules on the use of AI. The level permitted for this assignment is **category 2: minimal** — which means for example, it is ok to use spelling and grammar checkers to help identify mistakes, but not to rewrite chunks of text. Furthermore, note the use of AI to generate computer code is not permitted.
- The coursework consists of six compulsory questions, Q1–6, one for each section of the course. Each question is worth 20 marks. Each question is annotated with its mark breakdown. All six questions should be attempted.
- The computing elements should use Python/Matlab to solve problems as the lecturer for each section of the course has directed.
- Your submission should constitute a single pdf and might comprise a blend of (for example), pages of (neat) hand-written calculation or argument, figures — hand-drawn or powerpoint/similar, typed text, commented computer code, and computer output (please label and annotate all graphs). There are many ways of addressing this. For example, the use of Matlab livescript or Jupyter in Python is a really neat way of combining (latex) documentation and code. However, livescript/Jupyter/latex are not compulsory and any method which gives a clear and easy-to-follow narrative is fine. However: must emphasise: your submission should be bound together as a single pdf.
- Your coursework should be submitted via Blackboard, with a deadline of 13:00 GMT on Tuesday 3rd December 2024.

### Q1. Demand Modelling.

This question concerns the modelling of morning commuter flows aggregated at MSOA level in England and Wales, as described in the provided Matlab data file

TMMcoursework2024Q1.mat. In this file you will find three data structures:

- **MSOAdata** which provides a code and full name for each of the relevant 7,264 MSOAs, together with centroid coordinates in the form of a northing and easting in the OS grid system.
- **UTLAdata** which provides a code and full name for each of the 174 upper tier local authorities (UTLAs) to which the MSOAs belong. Note the **MSOAdata** structure provides a lookup index into the UTLA structure, so that you can determine the parent UTLA for each MSOA.
- **demandData** which provides counts for pairs of origin-destination pairs of MSOAs, indexed according to the **MSOAdata** structure.

(The field names should make their purpose clear — but if you are uncertain, post on the discussion forum.)

(a) Describe, develop, and demonstrate code to:

- (i) Build a **sparse** origin-destination demand matrix and compute its row sums (total origin flows, called  $O_i$  in lectures) and column sums (total destination flows, called  $D_j$  in lectures).  
(2 marks)
- (ii) Compute a matrix of the distances between the centroids of origins and destinations, being careful to state its units.  
(2 marks)
- (iii) Investigate best fits to the origin-destination demand data in the form  $cO_iD_j/d_{ij}^\alpha$ , where  $d_{ij}$  is the distance data computed in (a)(ii), and  $c$  and  $\alpha$  are constants to be fitted. You may use a combination of graphical and analytical techniques and full marks will be reserved for scripts which explain methods, discuss results and provide other insightful comment.  
(6 marks)

(b) Describe, develop, and demonstrate code to:

- (i) Compute the centroid of each UTLA in OS grid coordinates. (You may use a simple average of the MSOA constituent members.) Compute a matrix of distances between UTLA centroids.  
(2 marks)
- (ii) Compute the  $174 \times 174$  origin-destination matrix that describes flows at UTLA-UTLA level, and the corresponding row sums (origin flows) and column sums (destination flows).  
(2 marks)

(c) Using your results from part (b), compute, compare, and comment on the performance of the origin-constrained and destination-constrained gravity models on UTLA-UTLA demand data.

(6 marks)

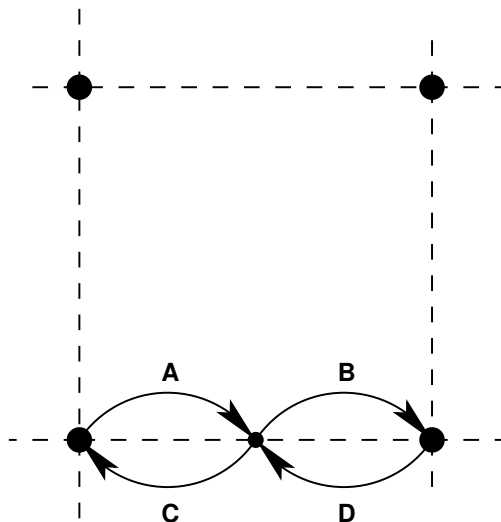
## Q2. Network modelling.

In this question we are going to consider simple models of dense residential street networks and modifications to them that represent the implementation of low traffic neighbourhoods (LTNs).

- (a) See lab sheet 02 Q1. Describe, develop, and demonstrate code which builds a Manhattan-style grid street network. The network's junctions should constitute an  $n \times n$  grid, where  $n$  is a parameter. The 'horizontal'/'vertical' distance between neighbouring junctions is the same, and initially all streets should be two-way. Your implementation should use Matlab's `graph` data structure.

(2 marks)

- (b) We shall assume that the network's streets are residential, and each street is (in the morning) the origin for a mixture of trips which terminate at the four corners of the network (perhaps before continuing subsequently to further destinations). Describe, develop, and demonstrate code which builds a new network in which the midpoint of each edge is a new node which will model the origin of the demand on that street, as indicated roughly below, for just one street.



(4 marks)

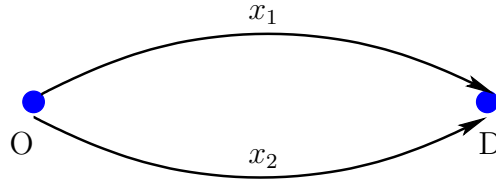
- (c) Investigate designs for LTNs. You might want to try and make some streets one-way (for example, by closing links A and B in the above figure) or blocking off the ends of streets at some junctions (for example, by closing links B and D in the above figure). Good designs will try to maximise the number of blocked-off streets (which are thus free of through traffic) and make large numbers of the remaining through streets one-way, in order to simplify the traffic flow. However, each street's residents must retain a viable route to each corner of the network (and be able to return to their homes in the evening). Describe, develop, and demonstrate Matlab code which implements and displays your modified networks and which verifies the required connectivity. Note: highest marks will be reserved for solutions that work for any value of  $n$ , but if you find that too hard, setting  $n$  to a fixed value larger or equal to 10 is fine.

(10 marks)

- (d) Let us suppose that all travellers take the shortest path from their residence to their respective destination (although noting that depending how many links you have deleted — that shortest path may not be unique). Also, for simplicity, you may assume that each street generates exactly the same demand to each of the four corners of the network. Describe, develop, and demonstrate Matlab code which computes the traffic flow on each street for your proposed LTN design. How do flows compare with the setting where no LTN interventions are made? Discuss briefly possible trade-offs in your design: e.g., close off fewer streets, and maybe improve the capacity of others?

*(4 marks)*

**Q3. Assignment modelling.**



- (a) We analyse the parallel network with a single origin-destination pair and two links, shown above. Denote the link flows by  $x_1, x_2 \geq 0$  and prescribe total demand  $d$ , so that  $x_1 + x_2 = d$ . Per-user link costs  $c_1(x_1), c_2(x_2)$  are given by

$$c_1(x_1) = 1 + \frac{1}{x_1},$$

$$c_2(x_2) = 3 + x_2.$$

Link 2 represents a standard congestible choice (e.g., car traffic) whose cost increases as it becomes more popular. However, link 1 represents a choice whose cost *decreases* as it becomes more popular — representing a public transport option where there is an economy of scale and more users enable a more frequent timetable and cheaper tickets.

- (i) By considering link costs and a statement of Wardrop's equilibrium, demonstrate that  $x_1 = 0$  and  $x_2 = d$  is a user equilibrium (UE) assignment, and derive a condition on  $d$  for which  $x_1 = d$  and  $x_2 = 0$  is also a UE assignment. (4 marks)
- (ii) Find an additional UE assignment where both links are used and their costs are equal, by solving an appropriate quadratic for  $x_1$ , and find the minimum demand  $d$  for which this assignment is valid (i.e.,  $x_1, x_2 \geq 0$ ). (4 marks)
- (iii) Find the total system cost for the assignments you found in parts (i) and (ii) and discuss and interpret your results in terms of the suggested application setting. (4 marks)

[Note part (a) should be solved with by-hand calculation and the use of a computer is not required.]

- (b) We are now going to compute UE for large networks. Study and adapt the provided livescript `TMMcourseworkQ3.mlx` which loads in the network provided in `BigNetworkExperimentsBetaSkeleton.mat` and formulates it so that `quadprog` can be used to solve the UE problem.

Show graphically how the network 'fills up' as demand  $d$  is increased from 0 over an appropriate range that you should choose. You should certainly ensure that the peak Price of Anarchy (PoA) is captured — and this may require you to also compute the System Optimal (SO) assignment. Finally, for the UE assignment with a fixed value of  $d$  that you should choose, demonstrate that there are multiple routes with the same total cost.

(8 marks)

#### Q4. Microscopic modelling.

- (a) The *Optimal Velocity Model*, is specified in eqs. 1 and 2 – referred to as *Model specification 1* from now on:

$$\frac{dv_\alpha}{dt} = \frac{v'_e(d_\alpha) - v_\alpha}{\tau} \quad (1)$$

$$v'_\alpha(d_\alpha) = \frac{v_0}{2} \left[ \tanh(d_\alpha - d_c) + \tanh(d_c) \right] \quad (2)$$

Consider an alternative specification – referred to as *Model specification 2*, specified by eqs. 3 – 5:

$$\frac{dv_\alpha}{dt} = \frac{v'_e(d_\alpha, v_\alpha) - v_\alpha}{\tau} \quad (3)$$

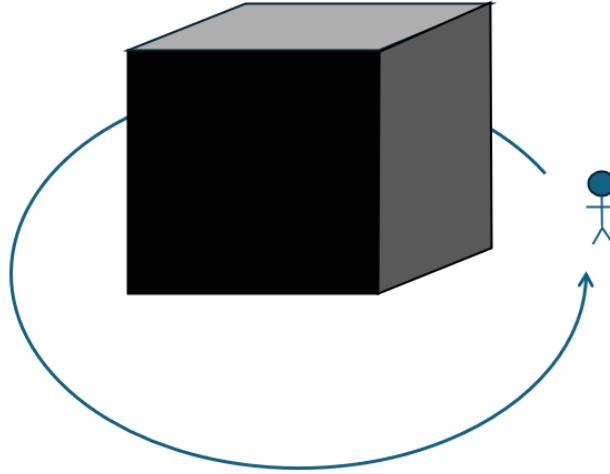
$$v'_\alpha(d_\alpha, v_\alpha) = \frac{v_0}{2} f(d_\alpha, v_\alpha) \quad (4)$$

$$f(d_\alpha, v_\alpha) = \begin{cases} \tanh\left(\frac{d_\alpha}{v_\alpha + \epsilon} - 1\right) + \tanh(1), & \text{if } d_\alpha > 10\text{m} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where  $\epsilon$  is an infinitesimally small positive number.

- (i) Analyse this new model specification and explain what the behavioural interpretation of the model is. (2 marks)
- (ii) Come up with two specific examples (either specific modelling scenarios or specific properties of the models), where one of the two model specifications is more realistic than the other one. (2 marks)
- (iii) Compare model specifications 1 and 2; explain what the differences and similarities in model dynamics is for the following three cases:
1. Steady state traffic (i.e., free flowing traffic with no particular perturbations or obstructions).
  2. A queue of cars at a traffic light that has just turned green (assume that cars are spaced at 5 metres initially (net gap, bumper to bumper).
  3. A sudden crash of one of the cars. The car has crashed into a stationary object and has reduced its speed to zero in approximately zero seconds.
- (6 marks)

- (b) Consider the pilgrimage to Mecca where each pilgrim will walk 7 times around a central cubic building called Kaaba.



To simulate this scenario, write down a modified set of Social Force Model equations and add the following behaviours:

- The walking direction of each person should be circular, around a central point, in a counter-clockwise direction.
- If the local density around a pedestrian exceeds  $6\text{m}^{-2}$  (measured within a 5-metre radius around that person), an additional social force will act upon that pedestrian away from the central point, in order to seek out a lower density environment.
- When a pedestrian has completed the seven laps, the person will change their walking direction to be tangentially away from the centre.

*(10 marks)*

### Q5. Data-driven modelling

For all parts of the question, provide your discussion in bullet points.

- (a) Consider the following time-series model for the count of vehicles,  $Y(t)$ , on a road in Bristol. One unit of time is 12 hours, so at  $t = 0$ , the total count for 12 hours (7am-7pm) is given, and at  $t = 1$  the count for the following 12 hours is given. Counts are given in 1000s of vehicles, so  $Y(t) = 5$  means 5,000 vehicles were counted.

$$Y(t) = a(t) + b(t) + c(t) + d(t),$$

$$a(t) = \alpha a(t-1),$$

$$b(t) = \beta Y(t-1),$$

$$c(t) = \sum_{j=1}^{s/2} \cos\left(\frac{2\pi j}{s} c(t-1)\right),$$

$$d(t) \sim \text{Normal}(0, \sigma).$$

The parameters of the model have been estimated to take the following values, based on observational data:  $\alpha = 0.99$ ,  $\beta = 0.5$ ,  $s = 4$ , and  $\sigma = 0.1$

- (i) Describe the dynamics produced by this model for the initial condition  $Y(0) = 0$ ,  $a(0) = 0.75$ ,  $c(0) = 0.5$ . Include a sketch/plot in your answer.

(5 marks)

- (ii) Discuss the usefulness of this model with the given parameter estimates for forecasting and explaining the dynamics of vehicle traffic counts on the road in Bristol, clearly justifying the arguments you make.

(7 marks)

- (b) This questions extends the worked direct demand modelling example on pedestrian traffic data from Melbourne covered in the lab sheet for data-driven modelling.

- (i) Develop and implement in code (Python or Matlab) a regression-based direct demand model for central Melbourne pedestrian traffic to forecast traffic on all links of part of the network 2 hours in advance. Provide a clear explanation and rationale for the model you choose, and for any additional data you obtain.

(5 marks)

- (ii) Evaluate the performance of your model using data (note that marking places a higher emphasis on the quality of the evaluation than on the prediction accuracy).

(3 marks)

Guideline to allocation of marks for part (b):

- Simple answers build on the code and data provided in the lab sheet.
- Good answers consider additional data. Excellent answers start from scratch, obtaining all data from primary sources.
- Use a similar spatial and temporal coverage as shown in the lab sheet example, there are no extra marks for considering larger areas or longer time-frames.



### Q6. Practitioner studies.

For all parts of the question, please provide your answers as bullet points.

Suppose you are a transport consultant. You are being tasked to help design a pedestrian bridge from a public railway station to a large stadium that is used for sports matches, concerts, and political rallies. This bridge will have to deal with high volumes of pedestrian traffic. There are no level changes and the length of the bridge is fixed. Figure 1 below gives an indication of passenger traffic at the railway station for a typical event starting at 19:00 finishing at 20:30. Your customer is concerned with safety, construction costs, and user experience in this order of importance. The authorities have provided a simple web tool based on a model for pedestrian traffic that you must use in your work. You can find it here:

<https://seis.bristol.ac.uk/~nb14397/TMMcalculation.html>

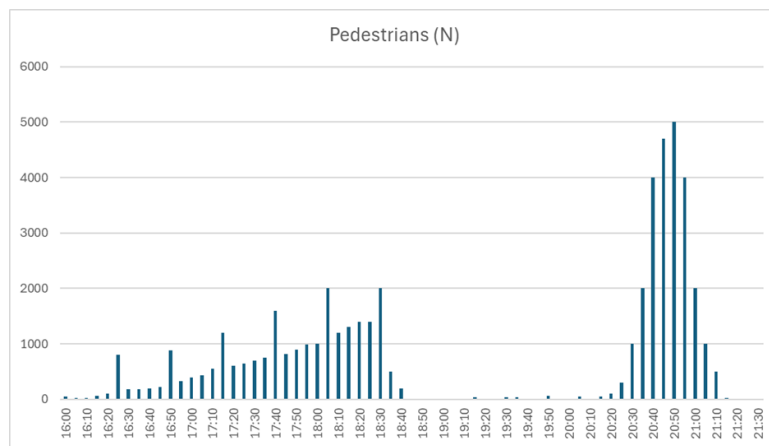


Figure 1: Passengers numbers at the railway station (sum of arriving and departing).

- (a) For the benefit of your customer, and for writing your report, you should understand the model in the web tool. Investigate and then describe this model, its properties, and what it predicts about pedestrian traffic by using the output the website provides, calling the website no more than 100 times (to save energetic costs and because we can imagine there may be licensing fees). In your answer, you should focus on how you use the online model output and your justification for this rather than simply stating an inferred model formulation.

(4 marks)

- (b) Use the model provided in the web tool and the information provided in the question to make recommendations about the design of the bridge, focusing on elements relevant to foot traffic and considering the relevant stakeholders. You may suggest adapting the model, if necessary. Explain and justify your approach.

(9 marks)

- (c) If you were to be given access to one other model covered in the lecture slides on microscopic modelling to make your design recommendation, which model would it be, and why? Stick to the same brief as in part (b) and state clearly where in the lecture notes the model was covered. We expect a sufficiently detailed explanation as to why you choose this model including an explanation on how it would be used.

(7 marks)