

Table 3.1 Balls and boxes counting problems.

Number of Ways to Put Balls into Boxes				
$S = S_1 = \{1, 2, \dots, s\}$, or $S = S_2 = \{s \cdot 1\}$, a multiset of balls $T = T_1 = \{U_1, U_2, \dots, U_t\}$, or $T = T_2 = \{t \cdot U_1\}$, a multiset of boxes Box U_i contains u_i balls				
Conditions on S and $T \rightarrow$ on $u_i \downarrow$	$T = T_1$ distinct $S = S_1$ distinct	$T = T_1$ distinct $S = S_2$ identical	$T = T_2$ identical $S = S_1$ distinct	$T = T_2$ identical $S = S_2$ identical
$0 \leq u_i \leq 1$ Assume $t \geq s$	1	2	3	4
$u_i \geq 0$	5	6	7	8
$u_i \geq 1$	9	10	11	12
for $i = 1, \dots, t$, $0 \leq u_i \leq n_i$, $n_i \in \mathbb{Z}^{>0}$	13	14		
$u_i \in N_i \subset \mathbb{Z}^{\geq 0}$ for $i = 1, \dots, t$	15	16		