Time Domain

1 Definition

$$\begin{aligned} f_s &= \text{sampling rate} \\ \lambda &= \text{wavelength} \\ T &= \text{period} \\ \omega &= \text{angular frequency} \end{aligned}$$

2 Wavelength λ

$$\lambda = \frac{c}{f}$$

3 Period T

$$T = 1\,ms = 1000\,Hz$$

4 Angular Frequency ω

$$\begin{aligned} \omega &= 2\pi f \\ &= \frac{2\pi}{T} \end{aligned}$$

$$\begin{aligned} \omega_0 &= 2\pi T \\ &= \pi \frac{f_0}{fs} \end{aligned}$$

5 Unit Pulse & Unit Step

unit pulse:

$$\delta(n) = \begin{cases} 1, & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

unit step:

$$u(n) = \begin{cases} 1, & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

6 Harmonic Signals

$$n = [1 : t \cdot f_s]$$

$$\begin{aligned} x[n] &= sin(\omega_0 n + \varphi) \\ &= sin(2\pi f_0 T + \varphi) \\ &= sin(2\pi \frac{f_0}{fs} n + \varphi) \end{aligned}$$

$$\begin{aligned} x[n] &= e^{-\mathrm{i}\omega_0 n} \\ &= cos(\omega_0 n) + \mathrm{i}\, sin(\omega_0 n) \end{aligned}$$

Frequency Domain

1 Definition

$$\begin{aligned} f_s &= \text{sampling rate} \\ \lambda &= \text{wavelength} \\ T &= \text{period} \\ \omega &= \text{angular frequency} \end{aligned}$$

LTI Systems

$$y[n] = T\{\, x[n] \,\}$$

Linear time-invariant systems (LTI systems) are a class of systems used in signals and systems that are both linear and time-invariant. Linear systems are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs. Time-invariant systems are systems where the output does not depend on when an input was applied. These properties make LTI systems easier to represent and understand.

1. memory freedom
2. causality
3. stability (BIBO)

Filter Design

1 In General

The lowpass general, standard form is given by

$$\left|H(j\omega)\right| = \left| \frac{A_0}{1 + \epsilon^2 A_N^2(\frac{\omega}{\omega_g})} \right|$$

with the design criteria defined by

2 Butterworth

The Butterworth filter is a type of signal processing filter designed to have a frequency response as flat as possible in the passband. It is also referred to as a maximally flat magnitude filter, given by

$$A_N(\frac{\omega}{\omega_g}) = (\frac{\omega}{\omega_g})^N$$

so that

$$\left|H(j\omega)\right| = \left| \frac{A_0}{1 + \epsilon^2 (\frac{\omega}{\omega_g})^{2N}} \right|$$

The order N is therefore given by

$$\begin{aligned} \epsilon(\frac{\omega}{\omega_g})^N &= \lambda \\ \Rightarrow (\frac{\omega}{\omega_g})^N &= \frac{\lambda}{\epsilon} \\ \Rightarrow N \cdot log_{10}(\frac{\omega_s}{\omega_g}) &= log_{10}(\frac{\lambda}{\epsilon}) \\ \Rightarrow N &\geq \frac{log_{10}(\eta_s)}{log_{10}(\frac{\omega_s}{\omega_g})} \end{aligned}$$

for $\eta_s = \frac{\omega_s}{\omega_g}$ and $\eta = \frac{\omega}{\omega_g}$. To find the poles we use

$$s_{\infty n} = \omega_g \epsilon^{-\frac{1}{N}} e^{j\frac{\pi}{2}(\frac{(2n-1)}{N}+1)}$$

and find $2N$ equidistant poles in a circle with radius $r = \omega_g \epsilon^{\frac{1}{N}}$.

3 Tchebychev I & II

The tchebychev filter is given by

$$\left|H(j\omega)\right| = \left| \frac{A_0}{1 + \epsilon^2 T_N^2(\frac{\omega}{\omega_g})} \right|$$

and uses the tchebychev polynomial T_N defined by

$$T_N(\eta) = \left\{ \begin{array}{ll} cos[N \cdot arccos(\eta)], & \text{for } \left|\eta\right| \leq 1 \\ cosh[N \cdot arccosh(\eta)], & \text{for } \left|\eta\right| > 1 \end{array} \right\}$$