since $\exists\ \hat{\sigma}$, s.t. $f(\hat{\sigma}) = p$. for any $p \in (f_{min}, f_{max})$.

we want to know $\hat{\sigma}$ is uniqueness.

$\because f$ is continuous and increasing. $\Rightarrow f(a) > f(b)$. for $a > b$. and $a, b \in [0, \infty)$.

Assume ~~A~~ H_0. $\therefore \exists$. $\sigma_a, \sigma_b \in [0, \infty)$. for $f(\sigma_a) = f(\sigma_b)$. and $\sigma_a \neq \sigma_b$.

contradiction.

then. we can know. for p. $\exists$ unique $\hat{\sigma}$ s.t. $f(\hat{\sigma}) = p$.

$\Rightarrow$ for any $p \in (f_{min}, f_{max})$. $\exists\ \hat{\sigma}$. s.t. $f(\hat{\sigma}) = p$

when $\sigma = \hat{\sigma}$, $f(\sigma) = f(\hat{\sigma}) = p$. $\Rightarrow f(\sigma) - p = 0$.

at point $\hat{\sigma}$. $\Rightarrow \hat{\sigma} = \arg\min_{\sigma \in [0, \infty)} |f(\sigma) - p|$