Example 1 *Let uniform mesh be denoted by*

$$\Pi_{T,N} = \{iT/N : i = 0, ..., N\}.$$

- *Write pseudocode.*

Algorithm 1 Use (1), generate $\hat{W}$ to simulate a discrete path $\langle W, \Pi_{T,N} \rangle$.

```
1: procedure EXACTBM1D(T, N)                                    ▷ T, N is ...
2:     ...
3:     ...
```

- *Prove that $\hat{W}$ is an exact sampling.*
- *Draw 10 path simulations of $t \mapsto \frac{W(t)}{\sqrt{2t \log \log t}}$ on interval $t = [100, 110]$ with mesh size $h = 0.1$.*

Algorithm 1

1. procedure : exactBM1D(T, N)
2. when $\hat{w}_0 \to 0$, $h = T/N$.
3. for $i = 0 \dots N-1$.

   $W_{t_{i+1}} = W_{t_i} + \sqrt{t_{i+1} - t_i}\, Z_{i+1} = w_i + \sqrt{y} \cdot z$.

   Return $\langle w, w_1, w_2 \dots w_n \rangle$

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2. Prove $\hat{w}$ is an exact sampling.

1). $\hat{w}$ is an exact sampling when $w_0 = 0$

2). $\hat{W}_i = \sum_{j=0}^{i-1} \sqrt{y} \cdot z_j = \sqrt{y} \cdot \sum_{j=1}^{i-1} z_j$. $z_i \sim N(0,1)$. $\forall i = 0, 1 \dots N-1$

Since z_{i+1} & z_i are iid.

$\hat{w}_{i+1} - \hat{w}_i = \sqrt{y}\, z_i$

$\hat{w}_{i+2} - \hat{w}_{i+1} = \sqrt{y} \cdot z_{i+1}$.

$\Rightarrow$. $\hat{w}_i$ has indep. increments

3) $\hat{w}_i - \hat{w}_j = \sqrt{y} \sum_{j=n}^{i-1} z_n$.

Ex 3

Example 3 *Consider Arithmetic asian option price on BSM by exact sampling.*

- *Write a pseudocode for Arithmetic asian option price on BSM*
- *To the Gbm class, add a method*

  arasian(otype, strike, maturity, nstep, npath)

  for the price by exact sampling.
- *Use your code to compute Arithmetic asian option of*

$$S_0 = 100.0, \sigma = 0.20, r = 0.0475, K = 110.0, T = 1.0, otype = 1, nstep = 5.$$

Algorithm 2.

1. Produre: EXACT Arasian. (otype. strike, maturity. nstep. npath)
2. $\hat{W}_0 \to 0$. $h = T/N$.
3. For $i = 0 \ldots n-1$,
   For $j = 1, 2, \ldots N$;

   $N(0,1) \to Z$,

   $\hat{W}_{i+1} = \hat{W}_i + \sqrt{Y} \cdot Z$.

   $\frac{S_0 + S_1 + S_2 + \ldots S_n}{n+1} \to S_j$.

   $Sum + S_j \to Sum$

   return Sum/N.

Ex 1 : $dS_t = \mu S_t dt + \sigma S_t dW_t$,

$S_0 = S \Rightarrow \frac{dS_t}{S_t} = \mu dt + \sigma dW_t$.

$d \ln S_t = \frac{1}{S_t} dS_t - \frac{1}{2S_t^2} (dS_t)^2$

$= (\mu - \frac{\sigma^2}{2}) dt + \sigma dW_t$

$\Rightarrow \ln S_t - \ln S_0 = (\mu - \frac{\sigma^2}{2}) t + \sigma W_t = (\mu - \frac{\sigma^2}{2}) t + \sigma W_t$

$\Rightarrow \ln S_t = \ln S + (\mu - \frac{\sigma^2}{2}) t + \sigma W_t$.