1. SA $(D=\{x_i: i\in N\}$ $\alpha=0.01)$.

$n = \text{len}(D)$.

$0 \to b$.

For $i=1,2,\dots, n$.

$b+\alpha \to b$

return b.

4 $b_{n+1}=(1-\alpha)b_n+\alpha x_{n+1}$.

$$\begin{cases} b_1=(1-\alpha)b_0+\alpha x_1 \\ b_2=(1-\alpha)^2 b_0+\alpha(1-\alpha)x_1+\alpha x_2 \\ b_3=(1-\alpha)^3 b_0+\alpha(1-\alpha)^2 x_1+\alpha(1-\alpha)x_2+\alpha x_3 \end{cases}$$

$\therefore b_n = \sum_{i=1}^{n} (1-\alpha)^{n-i} \alpha x_i + (1-\alpha)^n b_0.$

when $b_0=0 \Rightarrow \sum_{i=1}^{n} (1-\alpha)^{n-1} \alpha x_i$

$\Rightarrow b = \mathbb{E}[x_i]$ $\Rightarrow \mathbb{E}[b_n] = \alpha b \frac{1-(1-\alpha)^n}{\alpha}$

$= (1-(1-\alpha)^n) b.$

$\lim_{n\to\infty} \mathbb{E}[b_n] = \lim_{n\to\infty} b(1-(1-\alpha)^n)$

when $\alpha=0 \Rightarrow b=(1-0)b$.

5. $\lim_{n\to\infty} \mathbb{E}[(b_n-b)^2] = \lim_{n\to\infty} \mathbb{E}[b_n^2]-2b^2+b^2 = \lim_{n\to\infty} (\text{var}(b_n)+\mathbb{E}(b_n)^2]-b^2$

$= \lim_{n\to\infty} \text{var}(b_n)$

$x_i \sim N(b, \sigma^2)$

$\text{var}(x_i) = \sigma^2$.

$\text{Var}(b_n) = \sigma^2 \alpha^2 \cdot \frac{1-(1-\alpha)^{2n}}{1-(1-\alpha)^2}$

$\lim_{n\to\infty} \mathbb{E}[(b_n-b)^2] = \lim_{n\to\infty} \text{Var}(b_n) = \frac{\alpha^2\sigma^2}{2\alpha-\alpha^2} \neq 0.$

updated:

$\Rightarrow$ SA $x_i \sim N(b, \alpha^2)$ $\text{Var}(x_i)=\sigma^2$.

$\text{Var}(b_n) = \text{Var}(\frac{1}{n}\sum_{i=1}^{n} x_i) = \frac{1}{n^2}$

$n\sigma^2 = \frac{\sigma^2}{n}$.

$\lim_{n\to\infty} \text{Var}(b_n) = \lim_{n\to\infty} \frac{\sigma^2}{n} = 0$ $\Rightarrow b_n \to b$ in L^2.

2. $r = \frac{2}{2+h^2}$ $\quad P^h(x \pm h e_i \mid x) = \frac{1}{4}$

$l^h(x) = \frac{h^2}{2}\left(x_1^2 + x_2^2 - x_1 \cdot x_2 - \frac{3}{2}\right)$

$V(s) = R(s) + r \sum_{s' \in S} P(s, s')\, V(s')$.

If $s \notin \partial S$,

$\Rightarrow$ ① $P(s, s') = \begin{cases} \frac{1}{4}, & \text{if } \|s' - s\| = h \\ 0 & \text{others} \end{cases}$

② $R(s) = \gamma\, l^h(s)$

$= \frac{h^2}{2+h^2}\left(s_1^2 + s_2^2 - s_1 \cdot s_2 - \frac{3}{2}\right)$

③ $\gamma = \frac{2}{2+h^2}$