

~~1. martingale of each n.~~

1. Insert the soln into PDE,

$$\begin{aligned} \text{LHS} &= \frac{1}{2} \cdot 4 - (x_1 - \frac{1}{2})^2 - (x_2 - \frac{1}{2})^2 + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2} \\ &= 0 = \text{RHS.} \quad v. \end{aligned}$$

$$2. \partial_i v(x) = \frac{v(x+he_i) - v(x-he_i)}{2h.}$$

$$\partial_{ii} v(x) = \frac{v(x+he_i) + v(x-he_i) - 2v(x)}{2h^2.}$$

$$\text{Let } v_i^+ = v(x+he_i)$$

$$v_i^- = v(x-he_i).$$

$$\Rightarrow \frac{1}{2} \cdot \sum_{i=1}^2 \frac{v_i^+ + v_i^- - 2v(x)}{h^2} - v(x) + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2} = 0$$

$$\frac{2+h^2}{2} v(x) = \sum_{i=1}^2 v_i^+ \cdot \frac{1}{4} + \sum_{i=1}^2 v_i^- \cdot \frac{1}{4} + \frac{h^2}{2} (x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2})$$

$$v(x) = \frac{2}{h^2+2} \left[ \sum_{i=1}^2 v_i^+ \cdot \frac{1}{4} + \sum_{i=1}^2 v_i^- \cdot \frac{1}{4} + \frac{h^2}{2} (x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}) \right]$$

$$y = \frac{2}{h^2+2}$$

$$l^h(x) = \frac{h^2}{2} (x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2})$$

$$p^h(x \pm he_i | x) = \frac{1}{4}$$