University of California San Diego

MATH 181E
Mathematical Statistics - Time Series

# Summary Report: A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series

March 25, 2023



| *Author* | *Student ID* |
|---|---|
| Hanbei Xiong | A16131305 |

# Introduction

This paper examines the effectiveness of iterated forecasts versus direct forecasts in time series analysis, focusing on the relative performance of these methods for macroeconomic time series, such as price, wage, and money series versus non-price series. The study finds that the optimal forecasting method depends on the type of series being forecasted. For non-price series, iterated forecasts with short lags tend to perform best, while for price, wage, and money series, direct forecasts and iterated forecasts with long lags are more successful. However, the study also finds that the iterated AIC forecast, which can choose between short and long lags depending on the data, performs well on average for all series. The results suggest that forecasters should consider the type of series being analyzed and the length of the forecast horizon when choosing a forecasting method.[2]

# Methods

The authors explain the forecasting models and methods of comparison used in the study. They note that many macroeconomic time series are non-stationary and require transformation to approximate stationarity. The transformed series are used to estimate the forecasting models, which are then used to compute h-step-ahead forecasts of the original series. The authors also explain that all forecasts are recursive, meaning they are based only on values of the series up to the date of the forecast. The parameters of the models are re-estimated in each period using data from the beginning of the sample through the current forecasting date. The order of the model is also selected recursively, which means it can change over the sample as new information is added to the forecast data set.

For the Univariate models, the authors used four different methods to determine the lag order p for their forecasting models:

1. $p = 4$ (fixed)
2. $p = 12$ (fixed)
3. $p$ chosen by the Akaike Information Criterion (AIC) with $0 \leq p \leq 12$
4. $p$ chosen by the Bayesian Information Criterion (BIC) with $0 \leq p \leq 12$

For iterated forecasts, the AIC and BIC were computed using the sum of squared residuals from the one-step-ahead regression, while for direct forecasts, they were computed using the SSR from the estimated h-step-ahead regression. The AIC and BIC were recomputed at each date, so the order of the selected forecasting model could change from one period to the next. The authors explain that these four choices cover leading cases of theoretical interest, and provide benchmarks against which to compare the BIC and AIC forecasts. They also note that while the direct estimator with AIC model selection achieves an efficiency bound for direct estimators if the true lag order is infinite, the BIC with iterated forecasts is asymptotically efficient if the true lag order is finite and if the maximum lag considered exceeds the true lag order.

For the Multivariate models, the paper also considers bivariate vector autoregressions (VARs) for iterated and direct forecasts. The VARs are specified in terms of the stationary transforms of the two series of interest. The iterated forecast is obtained by iterating forward the VAR and then applying the transformation, while the h-step direct forecast for one series is obtained from the OLS regression of the series against a constant and p lags of both series. The same four methods of lag determination used in the univariate models are also used for the bivariate VARs.

The mean squared forecast error (MSFE) is computed for each of the 170 series, for each forecasting method (iterated and direct) with four different lag choices, and for each horizon (3, 6, 12, and 24 months). The MSFEs are used to assess the empirical efficiency of comparable direct and indirect forecasts by comparing the MSFEs for a given series and horizon.

The study compares the efficiency of direct and iterated forecasting methods for 170 macroeconomic series, using 4 different lag selection methods and 4 different forecast horizons. The mean relative MSFE of the direct estimator compared to the iterated estimator is of interest. A parametric bootstrap is used to test the null hypothesis that the iterated forecasting model is correctly specified and to estimate the distribution of relative MSFEs under this hypothesis. The bootstrap p-value is used to test the hypothesis that the direct estimator is more efficient than the iterated estimator. The study also fits a static factor model with four factors to the residuals of the autoregressive models of order p estimated for each series and uses this model to generate pseudo-random data for the bootstrap.

# Results

For the Univariate models, the study finds that the preference between the two methods depends on the method of lag selection and the type of series being forecasted. For short-lag selection methods, such as BIC, the direct estimator is generally preferred, while for long-lag models, the iterated estimator is preferable. However, for series on nominal prices, wages, and money, the direct estimator provides statistically significant improvements over the indirect estimator at all horizons and at all points in the distribution, for both short-lag models. On average, direct forecasts produce higher MSFEs than the iterated forecasts, but the relative performance of the iterated forecasts improves as the horizon lengthens. The study suggests that iterated forecasts with a data-dependent lag choice that can select long-lagged models should be best in some average sense.

For the Multivariate models, the results showed that long-lag direct forecasts did not offer significant improvements over long-lag iterated forecasts, while short-lag direct forecasts sometimes outperformed iterated forecasts for certain pairs of series. The iterated method with AIC lag selection tended to produce the lowest MSFE on average across all horizons. Results for nominal price, wage, and money series were different from other series, where long-lag iterated methods and, at short horizons, long-lag direct methods outperformed short-lag iterated methods.

# Conclusion

The main finding of the study is that iterated forecasts tend to have smaller mean squared forecast errors (MSFEs) than direct forecasts for monthly US macroeconomic time series. This is particularly true when using AIC lag-length selection for the iterated forecasts, and the performance of direct forecasts deteriorates as the forecast horizon increases. The heterogeneity in the lag order of the one-period model is handled adequately by using AIC lag-length selection for the iterated forecasts.

# Discussion

## 1 Topic covered in MATH181E

### 1.1 Bias-variance trade off

The paper argues that the reduction in estimation variance arising from estimating the one-period ahead model outweighs the reduction in bias obtained from the direct multiperiod model. This conclusion reflects the bias-variance tradeoff, as it suggests that there is a tradeoff between bias and variance in forecasting models.

### 1.2 Lagged regression model

The paper discusses the use of lagged regression models for forecasting in the context of vector autoregression (VAR) models. Specifically, the paper compares iterated and direct forecasts using different lag length selection methods in the context of VAR models.

### 1.3 Differencing

The paper uses differencing to eliminate the trend of series and transform the macroeconomic time series to be stable.

### 1.4 Recursive Prediction

The authors compare direct forecasts (which use a single model to directly forecast a multi-step ahead forecast) with iterated forecasts (which involve using a one-step ahead model to forecast each subsequent period recursively).

### 1.5 One-step-ahead AR model / h-step-ahead AR model

One-step-ahead AR model was used as a baseline model which helps the author to develop the h-step-ahead iterative models. The parameters of iterated AR forcast are estimated recursively by OLS.

### 1.6 Akaike's Information Criterion (AIC) / Bayesian Information Criterion (BIC)

Both AIC and BIC are used as model selection criteria for choosing the optimal lag order for the iterated forecasts. Here are the formulas:

$$AIC = 2k - 2\ln(L)$$

$$BIC = k\ln(n) - 2\ln(L)$$

where k is the number of parameters in the model, L is the likelihood of the data given the model, and n is the sample size.

### 1.7 Trivial Concepts: lag, autoregression

## 2 New Concepts

### 2.1 Bootstrap

The parametric bootstrap method involves generating many resamples from the original dataset, where each resample is generated by randomly selecting observations from the original dataset with replacement. For each resample, the model is fit using the iterated or direct method, and the resulting mean squared forecast error (MSFE) is recorded.

This process is repeated many times to create a distribution of MSFEs for each method. The parametric bootstrap method assumes that the MSFEs follow a specific distribution, such as the normal distribution, and uses this assumption to estimate the actual null distribution of the test statistic of interest, such as the difference in MSFEs between the iterated and direct methods.

By comparing the observed test statistic to the estimated null distribution, the method can determine whether the difference in MSFEs between the two methods is statistically significant. This approach allows for a more rigorous comparison of the two methods and helps to address potential biases or confounding factors that may be present in the data.

### 2.2 Mean Squared Forcast Error

The mean squared forecast error (MSFE) is a commonly used measure of the accuracy of a forecasting model. It measures the average squared difference between the forecasted values and the actual values over a given time period.

To calculate the MSFE, you first need to obtain a set of forecasts and the corresponding actual values. Let's denote the forecasted values as F and the actual values as A, and assume that there are n observations. Then, the MSFE is calculated as follows [1]:

$$MSFE = \frac{1}{n}\sum_{i=1}^{n}(F_i - A_i)^2$$

where $\sum$ represents the sum over all n observations.

This formula calculates the squared difference between the forecasted and actual values for each observation, sums these squared differences, and then divides the sum by the total number of observations. The resulting value gives an indication of the average amount of error between the forecasted values and the actual values, with higher values indicating larger errors.

### 2.3 Vector Autoregression (VAR) model

A Vector Autoregression (VAR) model is a statistical model used to analyze the relationship between multiple time series variables. This is used as one of the methods for forecasting in the context of multivariate time series. In a VAR model, each variable is expressed as a function of its past values as well as the past values of all other variables in the system [1].

### 2.4 Lag Length Selection Method

Selecting the appropriate lag length for a time series model is an important step in building an accurate and effective model. The lag length determines the number of past time periods that are included in the model, and it can have a significant impact on the accuracy of the model's forecasts.

The paper used Information criteria like AIC and BIC to select the lag length. These criteria balance the tradeoff between model fit and complexity, and select the lag length that provides the best balance.

## References

[1] Michail Grammatikopoulos and Gary Koop. Forecasting the yield curve with large bayesian vars - an ssvs approach. 08 2018.

[2] Massimiliano Marcellino, James H. Stock, and Mark W. Watson. A comparison of direct and iterated multistep ar methods for forecasting macroeconomic time series. *Journal of Econometrics*, 135(1):499–526, 2006.