1) by definition, $m(X) = \frac{1}{N} \sum_{i=1}^{N} x_i$

$$\text{so, } m(a+bX) = \frac{1}{N} \sum_{i=1}^{N} (a+bx_i)$$
$$= \frac{1}{N} \Big(\underbrace{\sum_{i=1}^{N} a}_{=Na} + \sum_{i=1}^{N} bx_i \Big)$$
$$= \frac{1}{N} \cdot \Big(Na + b \sum_{i=1}^{N} x_i \Big)$$
$$= a + b \cdot \underbrace{\frac{1}{N} \sum_{i=1}^{N} x_i}_{\text{same as definition}}$$
$$\therefore m(a+bX) = a + b \times m(X) \checkmark$$

2) by definition, $cov(X,Y) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))(y_i - m(Y))$

$$\text{so, } cov(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))((a+by_i) - m(a+bY))$$

from 1), we have $m(a+bY) = a + bm(Y)$ so substituting this:

$$cov(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))((\cancel{a}+by_i) - (\cancel{a} + bm(Y))$$
$$= \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))\, b(y_i - m(Y))$$
$$= b \cdot \underbrace{\frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))(y_i - m(Y))}_{\text{definition of } cov(X,Y)}$$
$$\therefore cov(X, a+bY) = b \times cov(X,Y) \checkmark$$

3) ① by definition of the covariance,

$$cov(X,X) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))(x_i - m(X)) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))^2 = s^2 \checkmark$$

② $Cov(a+bX, a+bX) = \frac{1}{N} \sum_{i=1}^{N} ((a+bx_i) - m(a+bX))((a+bx_i) - m(a+bX))$

using $m(a+bX) = a + bm(X)$ from 1), we have:

$$Cov(a+bX, a+bX) = \frac{1}{N} \sum_{i=1}^{N} ((\cancel{a}+bx_i) - \cancel{a} - bm(X))((\cancel{a}+bx_i) - \cancel{a} - bm(X))$$
$$= \frac{1}{N} \sum_{i=1}^{N} (bx_i - bm(X))(bx_i - bm(X))$$
$$= \frac{1}{N} \sum_{i=1}^{N} b(x_i - m(X)) \cdot b(x_i - m(X))$$
$$= b^2 \cdot \underbrace{\frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))^2}_{\text{definition of } cov(X,X)}$$
$$\therefore Cov(a+bX, a+bX) = b^2 \times Cov(X,X) \checkmark$$

4) No, not necessarily.
This is because for non-linear transformations, the transformed median can shift. For example, the median of X might not be the same as arcsinh (median(X))
↳ this applies to other quantiles, including the IQR (for the same reasons)
↳ also applies to the range, because extreme values in non-linear graphs can be distorted.

5) No, $m(g(X)) = g(m(X))$ is not always true
we know that $E[g(X)] \neq g(E[X])$ unless the function is linear
↳ Jensen's inequality