1. $\dfrac{1+i}{\sqrt{3}-i} = \dfrac{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}{2\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right)} = \dfrac{\sqrt{2}}{2}\left[\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right]$

$= \dfrac{\sqrt{2}}{2} e^{i\frac{5}{12}\pi}$ 选 A

2. C

3. $Ln(z_1^2 \cdot z_2^2) = Ln\, z_1^2 + Ln\, z_2^2 \neq 2Ln\, z_1 + 2Ln\, z_2$

$\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2} = \dfrac{\bar{z}_1 \cdot z_2}{\bar{z}_2 \cdot z_2} = \dfrac{\bar{z}_1 \cdot z_2}{|z_2|^2}$

$Arg\left(\dfrac{z_1}{\bar{z}_2}\right) = Arg\, z_1 - Arg\, \bar{z}_2 = Arg\, z_1 + Arg\, z_2$

$Re(z_1\bar{z}_2) = Re(\bar{z}_1 z_2)$ 选 (A)

4. $\cos i = \dfrac{e^{i\cdot i} + e^{-i\cdot i}}{2} = \dfrac{e^{-1}+e}{2}$

$\ln i = \ln|i| + i\cdot\frac{\pi}{2} = \frac{\pi}{2} i$

$i\sin i = i\,\dfrac{e^{i\cdot i} - e^{-i\cdot i}}{2i} = \dfrac{e^{-1}-e}{2}$ 选 (B)

5. $\lim\limits_{z\to 0} \dfrac{1}{2i}\left(\dfrac{z}{\bar{z}} - \dfrac{\bar{z}}{z}\right) = \lim\limits_{z\to 0} \dfrac{1}{2i} \dfrac{z^2-\bar{z}^2}{\bar{z}z} = \lim\limits_{z\to 0} \dfrac{2xy}{x^2+y^2} = \lim\limits_{\substack{x\to 0\\ y=kx}} \dfrac{2x\cdot kx}{x^2+k^2x^2} = \dfrac{2k}{1+k^2}$

不存在 选 (B)

6. $\sqrt[3]{1-i} = \sqrt[3]{\sqrt{2}\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right)} = \sqrt[6]{2}\left(\cos\dfrac{-\frac{\pi}{4}+2k\pi}{3} + i\sin\dfrac{-\frac{\pi}{4}+2k\pi}{3}\right)$, $k=0,1,2$

7. $z = (\sqrt{3}-i)^7 = \left[2\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right)\right]^7 = 2^7\left(\cos\frac{-7\pi}{6} + i\sin\frac{-7\pi}{6}\right)$

$\therefore Im\, z = 2^7 \sin\frac{-7\pi}{6} = 64$

8 $\oint_{|z|=2} (|z| - e^z\sin z)\,dz = \oint_{|z|=2} (\underbrace{2 - e^z\sin z}_{\text{解析}})\,dz = 0$

9. $z = \dfrac{(z+i)^{100}}{(z-i)^{100}}$，则 $|z| = 1$ （∵分子，分母共轭）

10. $g(\zeta) = \oint_{|z|=3} \dfrac{2z^2-z-2}{z-\zeta}\,dz = 2\pi i(2\zeta^2-\zeta-2)$

$\therefore g(2) = 2\pi i(8-2-2) = 8\pi i$

11. $u_x = v_y = 6x+2$ $\quad$ ∴ $u = \int (6x+2)\,dx = 3x^2+2x+c(y)$

$u_y = c'(y) = -v_x = -6y$ $\quad$ ∴ $\cancel{c'(y)=}$ $c(y) = -3y^2+c$

∴ $u(x,y) = 3x^2+2x-3y^2+c$

由 $f(0)=-1$ $\Rightarrow$ $\cancel{u(x,y)}$ $u(0,0)=-1$ $\quad$ ∴ $c=-1$

∴ $u(x,y) = 3x^2+2x-3y^2-1$ $\quad$ ∴ $u(1,1) = 3+2-3-1 = 1$

12. $f(z) = 3x^2+3y^2$ $\quad$ ∴ $u(x,y) = 3x^2+3y^2$, $v(x,y)=0$

$u_x = 6x$ $\quad u_y = 6y$ $\quad v_x = 0$ $\quad v_y = 0$

由 $\begin{cases} 6x=0 \\ 6y=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$ $\quad$ ∴ $f(z)$ 仅在 $(0,0)$ 处可导，处处不解析.

13. $i^i = e^{i\ln i} = e^{i[\ln|i| + i(\frac{\pi}{2}+2k_1\pi)]} = e^{-(\frac{\pi}{2}+2k_1\pi)}$

∴ $e^z = e^{-(\frac{\pi}{2}+2k_1\pi)}$ $\quad$ ∴ $z = -(\frac{\pi}{2}+2k_1\pi) + 2k\pi i$

∴ $\mathrm{Im}(z) = 2k\pi$

14. $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} = 2x + i\cdot 2y$

$f'(\frac{1}{2}) = 2\times\frac{1}{2} = 1$

15. 令 C: $z = t+it = (1+i)t$ $\quad 0\le t\le 1$

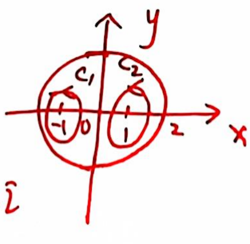
∴ $\int_C (\bar{z})^2 dz = \int_0^1 (1-i)^2 t^2 (1+i)\,dt = 2(1-i)\int_0^1 t^2\,dt = \frac{2}{3}(1-i)$

16. C: $z = e^{i\theta} = \cos\theta + i\sin\theta$

$\oint_{|z|=1} \mathrm{Im}(z)\,dz = \int_0^{2\pi} \sin\theta\; i e^{i\theta}\,d\theta = \int_0^{2\pi} i\sin\theta(\cos\theta + i\sin\theta)\,d\theta$

$= i\int_0^{2\pi} \sin\theta\cos\theta\,d\theta - \int_0^{2\pi} \sin^2\theta\,d\theta = -\pi$

17. $\oint_{|z|=2} \frac{\sin\frac{\pi}{4}z}{z^2-1}\,dz = \oint_{C_1} \frac{\sin\frac{\pi}{4}z}{z^2-1}\,dz + \oint_{C_2} \frac{\sin\frac{\pi}{4}z}{z^2-1}\,dz$



$= \oint_{C_1} \frac{\frac{\sin\frac{\pi}{4}z}{z-1}}{z+1}\,dz + \oint_{C_2} \frac{\frac{\sin\frac{\pi}{4}z}{z+1}}{z-1}\,dz = 2\pi i \left.\frac{\sin\frac{\pi}{4}z}{z-1}\right|_{z=-1} + 2\pi i \left.\frac{\sin\frac{\pi}{4}z}{z-1}\right|_{z=1} = \sqrt{2}\pi i$

18. $\oint_{|z|=3} \frac{\sin z}{(z-\frac{\pi}{2})^4} dz = \frac{2\pi i}{(4-1)!} (\sin z)'''\Big|_{z=\frac{\pi}{2}} = 0$

19. $\oint_{|z|=2} \frac{\bar{z}}{|z|} dz = \oint_{|z|=2} \frac{\bar{z}|z|}{|z|^2} dz = \oint_{|z|=2} \frac{\bar{z}\cdot 2}{\bar{z} z} dz = \oint_{|z|=2} \frac{2}{z} dz = 2\times 2\pi i = 4\pi i$

20 $\int_0^i z e^{z^2} dz = \frac{1}{2}\int_0^i e^{z^2} dz^2 = \frac{1}{2} e^{z^2}\Big|_0^i = \frac{1}{2}e^{-1} - \frac{1}{2}$

21. $u = x^2+2xy-y^2 \qquad v = y^2+axy-x^2$

$\frac{\partial u}{\partial x} = 2x+2y \qquad \frac{\partial u}{\partial y} = 2x-2y \qquad \frac{\partial v}{\partial x} = ay-2x \qquad \frac{\partial v}{\partial y} = 2y+ax$

由 $\begin{cases} 2x+2y = 2y+ax \\ 2x-2y = 2x-ay \end{cases} \Rightarrow a=2$

22. 令 $z = x+iy$, $|z|$

$\left|\frac{e^z}{z}\right| = \frac{|e^z|}{|z|} = \frac{|e^{x+iy}|}{1} = \frac{e^x}{1} \leq e^1$

$\therefore \left|\int_\gamma \frac{e^z}{z} dz\right| \leq e\cdot 2\pi\cdot\frac{1}{2} = e\pi$