CSC4140 Assignment I

Computer Graphics

February 4, 2022

Learn to use VirtualBox and Mathematic Review

This assignment is 5% of the total mark.

Strict Due Date: 11:59PM, Feb 14^{th}, 2022

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This assignment represents my own work in accordance with University regulations.

Signature:

1 Readme

1.1 File firectory

Please organize files in the following structure.

```
/
├── bin
│   ├── compile_run.sh //linux executable file
│   └── lenna.png //sample image
├── doc
│   └── 120090453_HW1_1.pdf
├── src
│   ├── CMakeLists.txt
│   ├── q2_1.cpp
│   ├── q2_2.cpp
│   ├── q2_3.cpp
│   └── q2_4.cpp
├── CMakeLists.txt
└── main.cpp //main program
```

1.2 How to run the program

1. Make sure your work directory under *"./bin/"* when you run the executable file.

2. Run the executable file *"./compile_run.sh 2-x"* when you want to check the outcome in Section 2-x. E.g. if you want to check Section 2-1, input the following command in the terminal:

./compile_run.sh 2-1

2 Basic operations of vector and matrix

2.1 Basic vector operations

2.1.1 Define v and w

The vectors are defined as tow float vectors $\boldsymbol{v} = \begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}$ and $\boldsymbol{m} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 4 \end{bmatrix}$, which is homogeneously normalized as $\boldsymbol{v} = \begin{bmatrix} 0.333333 \\ 0.5 \\ 0.666667 \\ 1 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 0 \\ 0.25 \\ 0.5 \\ 1 \end{bmatrix}$.

2.1.2 vector add

$$\boldsymbol{v}+\boldsymbol{w}=\begin{bmatrix}0.333333\\0.75\\1.16667\\1\end{bmatrix}$$

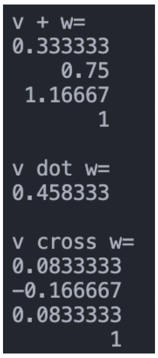
2.1.3 vevtor inner product

$$\boldsymbol{v}\cdot\boldsymbol{w}=0.458333$$

2.1.4 vector cross product

$$\boldsymbol{v}\times\boldsymbol{w}=\begin{bmatrix}0.0833333\\-0.166667\\0.0833333\\1\end{bmatrix}$$

2.1.5 Result crops



```
v + w=
0.333333
    0.75
 1.16667
       1

v dot w=
0.458333

v cross w=
0.0833333
-0.166667
0.0833333
        1
```

Figure 1. Basic vector operations

2.2 Basic matrix operations

2.2.1 Define i and j

The matrixs $\boldsymbol{i}$ and $\boldsymbol{j}$ are defined as following:

$$\boldsymbol{i}=\begin{bmatrix}1&2&3&4\\5&6&7&8\\9&10&11&12\\13&14&15&16\end{bmatrix}\quad\boldsymbol{j}=\begin{bmatrix}4&3&2&1\\8&7&6&5\\12&11&10&9\\16&15&14&13\end{bmatrix}$$

2.2.2 matrix add i+j

$$\boldsymbol{i}+\boldsymbol{j}=\begin{bmatrix} 5 & 5 & 5 & 5 \\ 13 & 13 & 12 & 14 \\ 21 & 21 & 21 & 21 \\ 29 & 29 & 29 & 29 \end{bmatrix}$$

2.2.3 matrix add i*j

$$\boldsymbol{i}\times\boldsymbol{j}=\begin{bmatrix} 120 & 110 & 98 & 92 \\ 280 & 254 & 222 & 208 \\ 440 & 398 & 346 & 324 \\ 600 & 542 & 470 & 440 \end{bmatrix}$$

2.2.4 matrix mutiply vector add i*v

The vector is defined as $\boldsymbol{v}=\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}$ and the matrix is defined as $\boldsymbol{i}=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$. Therefore,

$$\boldsymbol{i}\times\boldsymbol{v}=\begin{bmatrix} 22 \\ 52 \\ 82 \\ 112 \end{bmatrix}$$

2.2.5 Result crops

```
matrix add i+j:
  5  5  5  5
 13 13 12 14
 21 21 21 21
 29 29 29 29
matrix multiply i*j:
120 110  98  92
280 254 222 208
440 398 346 324
600 542 470 440
matrix multiply vector i*v:
 22
 52
 82
112
```

Figure 2. Basic matrix operations

2.3 SVD decomposition of "lenna"

2.3.1 convert and normalize

The image is converted to gray scale by *cv::convert()*.

Figure 3. "lenna" and it converted to gray scale

2.3.2 Depositie image to U, S, V

The image is depositied to U, S, V with the shape of $(512, 512)$ by *Eigen::JacobiSVD*.

2.3.3 Save the feature map as an image

The restored images of the feature map with first 1, 10, 50 singular value are saved in the following ways:

$$\text{Restoration with first 1 singular values: } \boldsymbol{U} \times \boldsymbol{S}[:, 1] \times \boldsymbol{V}[:, 1]^T$$

$$\text{Restoration with first 10 singular values: } \boldsymbol{U} \times \boldsymbol{S}[:, 10] \times \boldsymbol{V}[:, 10]^T$$

$$\text{Restoration with first 50 singular values: } \boldsymbol{U} \times \boldsymbol{S}[:, 1:50] \times \boldsymbol{V}[:, 1:50]^T$$

Figure 4. Feature images with first 1, 10, 50 singular values

2.3.4 Explanation

Actually, SVD deposition devides the matrix into the sum of matrixs that $Rank = 1$.

In other words, the matrix can be written in the following way:

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + ... + \sigma_r u_r v_r^T = \sum_{i=1}^{r} \sigma_i u_i v_i^T (\sigma > 0)$$

where r is the number of the matrix's eigenvalues.

Suppose that $B = \sum_{i=1}^{k} \sigma_i u_i v_i^T (0 < k < r)$. When k increases, B tends to approach A. Therefore, B is the estimation of A when k is large enough. It indicates the outcome in Section 2.3.4.

2.4 Basic transformation operations

The initial point is $[1, 2, 3]$ and the rotation center is $[4, 5, 6]$. Then the point is rotated with angle $(45°, 30°, 60°)$ in turn. Therefore, the new point is $[1.6403, 0.912883, 3.82577]$.

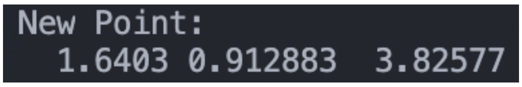


Figure 5. The new point