Modelling Guitar Effects: Transient, Non-linear Circuit Solver

Johnathon Caguiat, Philip Christian, Steven Dong, Ciaran Geaney

ECE1254 Final Project

April 20^{th}, 2016

Overview

- Objective and Background: Transience with Non-linearity
- Variable Time-stepping
- Diodes and Transistors: Equations, Stamps and Testing
- Modelling of Guitar effects and Insights Gained

Objective

- Create a circuit solver which can solve a system transience and nonlinearity.

- System of choice: guitar sound modulation.
  - Sound is a transient input voltage input
  - Additional transience and non-linearity can be added by choosing relevant circuit elements: capacitors, diodes and transistors

Adding Non-Linear Elements to Transient Systems

$$\begin{cases} Gx(t) + C\dot{x}(t) + \boldsymbol{F'(x)} = Bu(t) \\ y(t) = L^T x(t) \end{cases} \qquad \begin{cases} Gx(t) + C\dot{x}(t) + \mathbf{Hg(x)} = Bu(t) \\ y(t) = L^T x(t) \end{cases}$$

Example of how these might look for a two connection, non-linear resistor:

$$\mathbf{F'(x)} \qquad\qquad\qquad\qquad \mathbf{H} \qquad\qquad \mathbf{g(x)}$$

$$\begin{array}{cc} & 1 \\ & \\ \begin{array}{c} \\ n1 \\ \\ n2 \\ \\ \end{array} & \begin{bmatrix} \vdots \\ g \\ \vdots \\ -g \\ \vdots \end{bmatrix} \end{array} \quad \Longleftrightarrow \quad \begin{array}{cc} & n^* \\ & \\ \begin{array}{c} \\ n1 \\ \\ n2 \\ \\ \end{array} & \begin{bmatrix} & \vdots & \\ & 1 & \\ \cdots & \vdots & \cdots \\ & -1 & \\ & \vdots & \end{bmatrix} \end{array} \quad \begin{array}{cc} & 1 \\ n^* & \begin{bmatrix} \vdots \\ g \\ \vdots \end{bmatrix} \end{array}$$

Implementing Trapezoidal Rule

Add two x_{n+1} and x_n such that we can use an approximation for the first derivative:

$$\begin{array}{rl} & Gx_{n+1} + C\dot{x}_{n+1} + F'(x_{n+1}) = Bu_{n+1} \\ + & Gx_n \;+\; C\dot{x}_n \;+\; F'(x_n) = Bu_n \\ \hline \end{array}$$

$$G(x_{n+1} + x_n) + C\boxed{(x_{n+1} + x_n)} + F'(x_{n+1}) + F'(x_n) = B(u_{n+1} + u_n)$$

Substitute the approximation of the : $\dfrac{x_{n+1} - x_n}{\Delta t} = \dfrac{\boxed{\dot{x}_{n+1} + \dot{x}_n}}{2}$

$$G(x_{n+1} + x_n) + 2C\left(\frac{x_{n+1} - x_n}{\Delta t}\right) + F'(x_{n+1}) + F'(x_n) = B(u_{n+1} + u_n)$$

Implementing Trapezoidal Rule

Collect x terms and divide by 2:

$$\left(\frac{G}{2}+\frac{C}{\Delta t}\right)\boxed{x_{n+1}}+\left(\frac{G}{2}-\frac{C}{\Delta t}\right)x_n+\frac{F'(\boxed{x_{n+1}})+F'(x_n)}{2}=\frac{B(u_{n+1}+u_n)}{2}$$

X_{n+1} is now non-linear and requires Newton's Method to solve for each time step

Implementing Newton’s Method

Recall Newton’s Method:

$$x_{k+1} = x_k - \left[J\big(F(x_k)\big)\right]^{-1} F(x_k)$$

$F(x_k)$ form of the Trapezoidal Rule:

$$F(x_k) = \left(\frac{G}{2} + \frac{C}{\Delta t}\right)\boxed{x_k} + \left(\frac{G}{2} - \frac{C}{\Delta t}\right)x_n + \frac{F'(\boxed{x_k}) + F'(x_n)}{2} - \frac{B(u_{n+1} + u_n)}{2} = 0$$

Implementing Newton’s Method

The Jacobian of $F(x_k)$

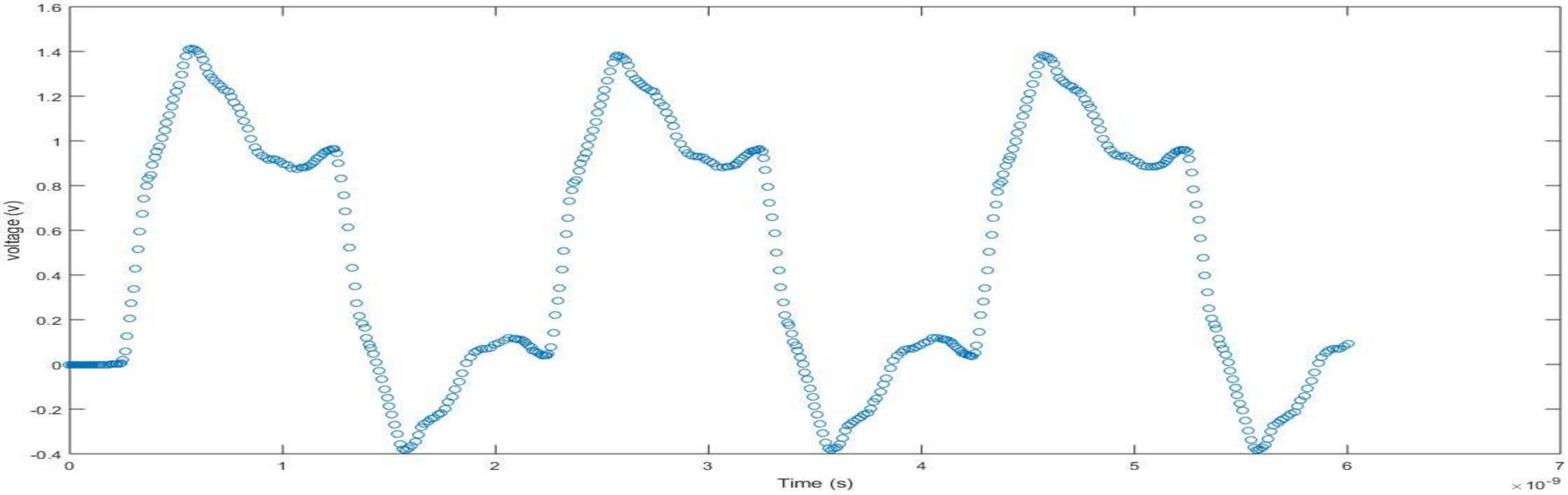
$$\mathrm{J}(F(x_\mathrm{k})) = \mathrm{J}\left(\left(\frac{G}{2}+\frac{C}{\Delta t}\right)x_k\right) + \mathrm{J}\left(\cancelto{0}{\left(\frac{G}{2}-\frac{C}{\Delta t}\right)x_n}\right) + J\left(\frac{F'(x_k)+\cancelto{0}{F'(x_n)}}{2}\right) - \mathrm{J}\left(\cancelto{0}{\frac{B(u_{n+1}+u_n)}{2}}\right)$$

$$\mathrm{J}(F(x_\mathrm{k})) = \left(\frac{G}{2}+\frac{C}{\Delta t}\right) + \frac{J(F'(x_k))}{2}$$

Note that we need $F'(x)$ and $\left[J\left(F(x_k)\right)\right]^{-1}$ for every guess of x_k.

This means that $F'(x)$, $J(F'(x)$ need to be calculated and $J(F'(x))$ inverted at every guess.

Variable Time Stepping



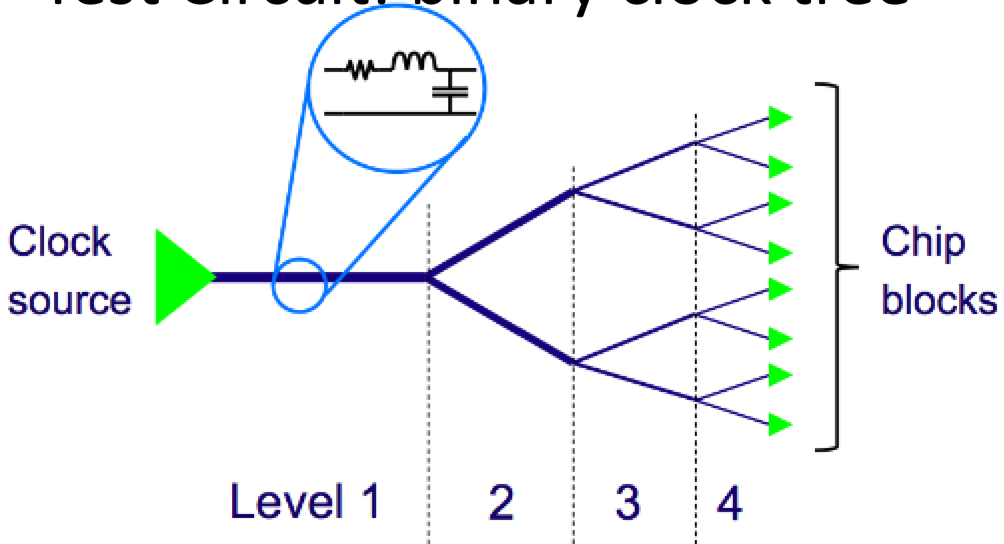
- Large Time Steps when function is smooth
- Small time steps when change is large

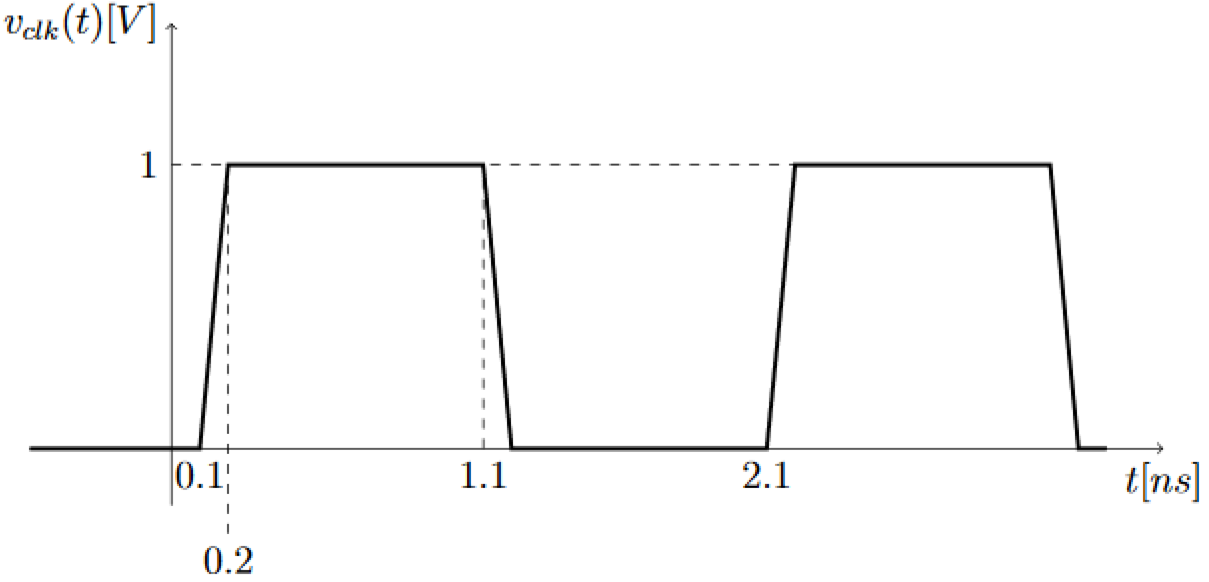
Time size depends on Error

- Key Equation:

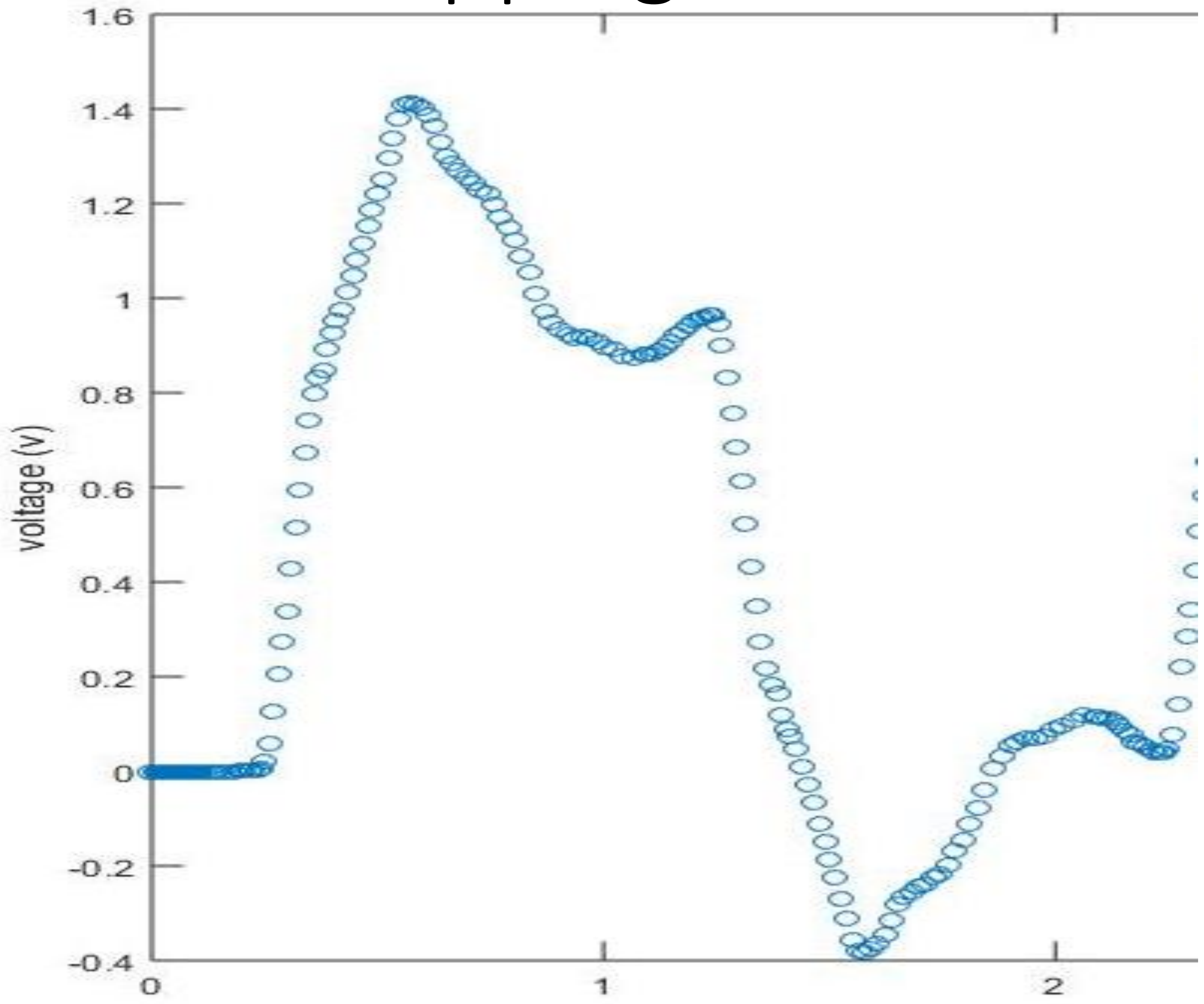
$$\Delta t_{new} = \min(0.9\sqrt{({\epsilon_{max}}/{\epsilon})}, \Delta t\_\mathrm{max})$$

- Error calculates difference in the result of two methods
  - In this case, 1-step trapezoidal and 2-step trapezoidal
- Test Circuit: binary clock tree

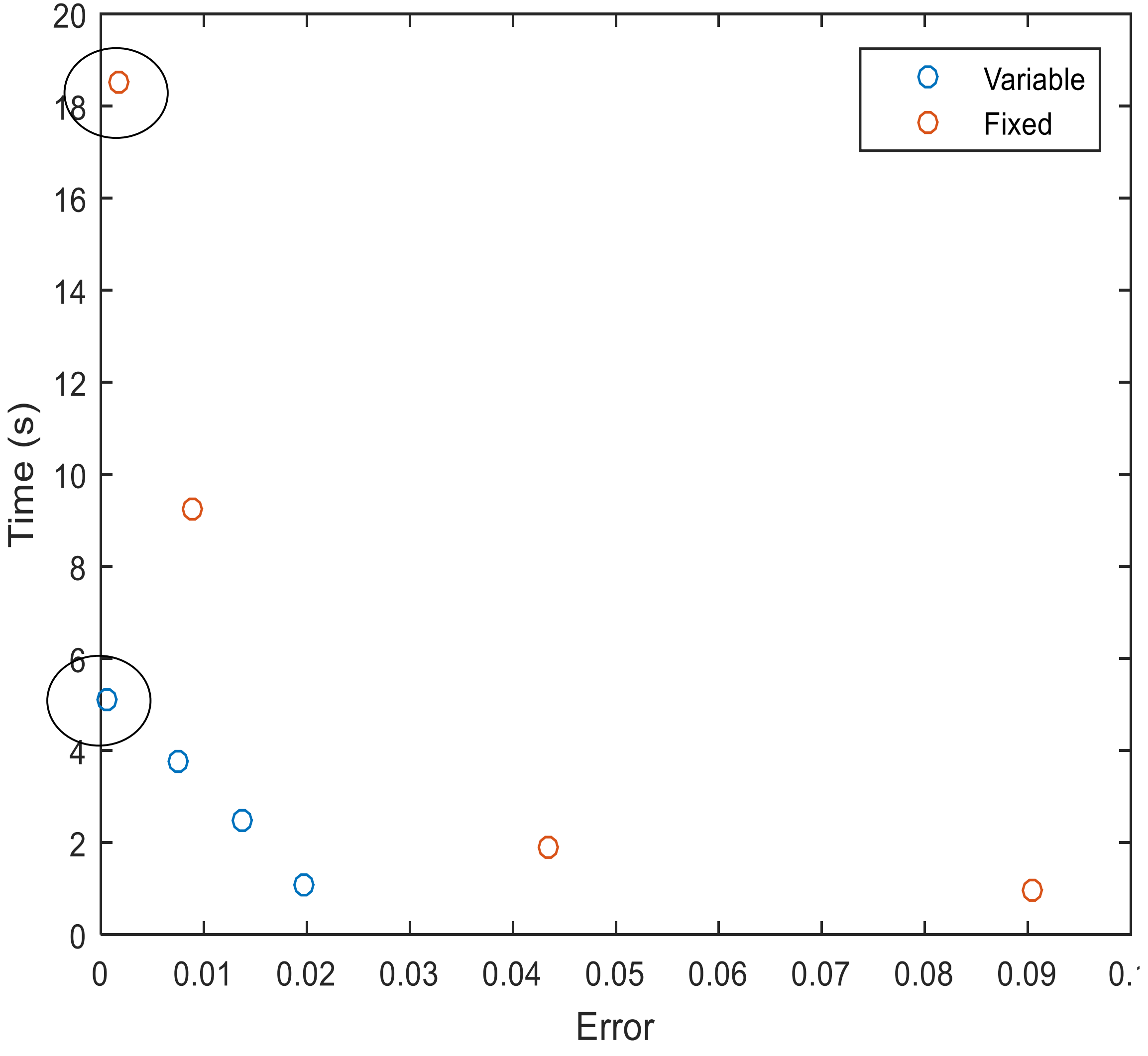


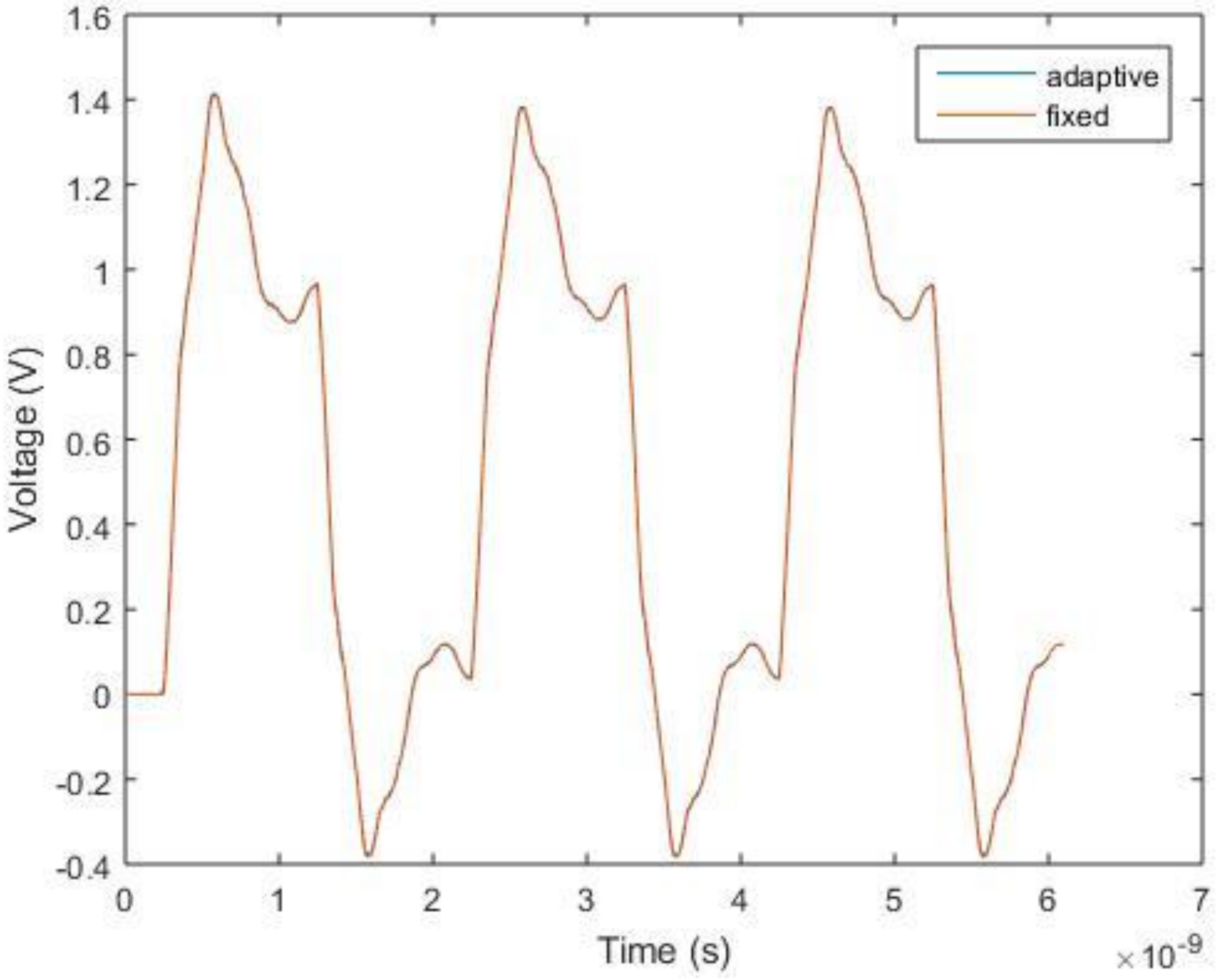


Variable Time Stepping



Error of Variable Time Stepping



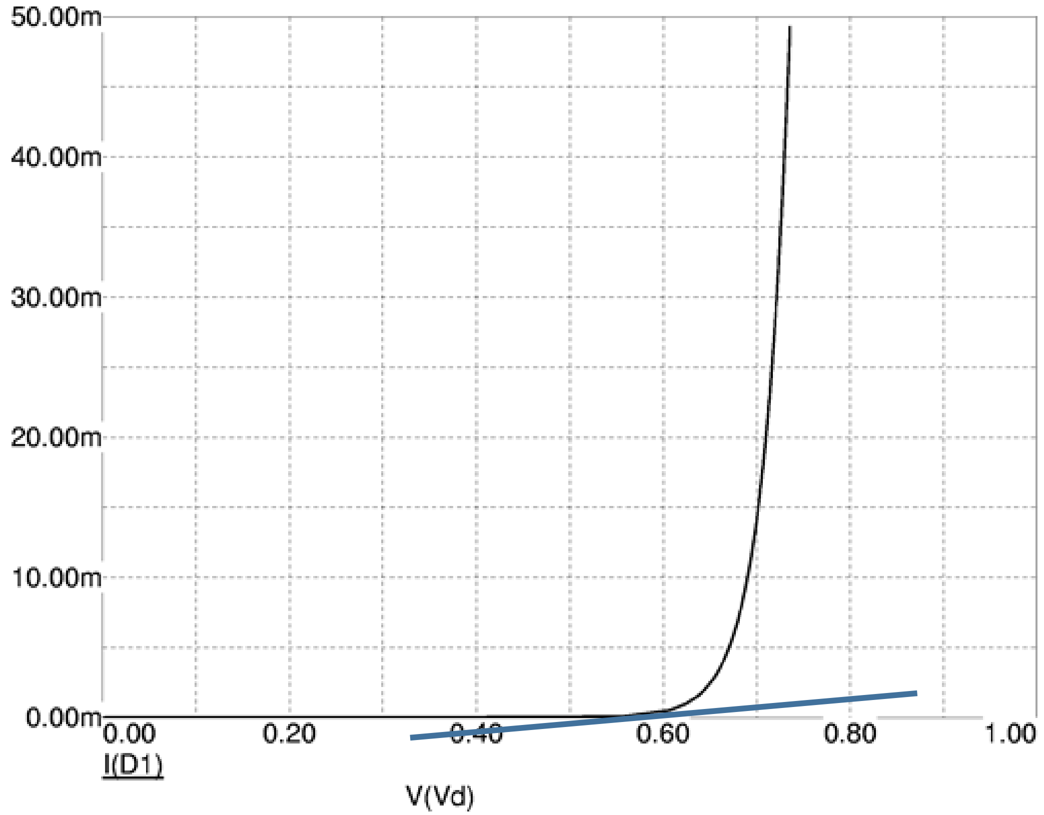


Newton's Method Convergence Strategies

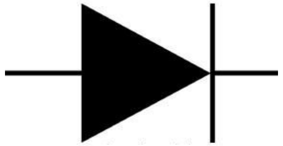
- Large shunt resistors to ground
- Final result of previous time step is used as Newton’s Method initial guess of the current time step
- Continuation Method
- Adding time delay to source signal

Some More Improvements

- Relaxed Newton’s Method convergence minimum error
- Restricted the maximum allowable range for Newton's Method
- dx=dx_max/norm(dx)*dx
- Scale down the amplitudes of incoming signal



Diode Model Equation and Stamps



The current for a diode is described by the following equation:

$$i = I_0\left(e^{\frac{V_+ - V_-}{\eta V_T}} - 1\right)$$

The stamp follows the format of a non-linear resistor, seen earlier:

$$\underbrace{\begin{array}{c c} & 1 \\ & \vdots \\ n1 & \\ n2 & \\ \end{array}\!\!\left[\begin{array}{c} \vdots \\ I_0\left(e^{\frac{V_+ - V_-}{\eta V_T}} - 1\right) \\ \vdots \\ -I_0\left(e^{\frac{V_+ - V_-}{\eta V_T}} - 1\right) \\ \vdots \end{array}\right]}_{F'(x)} \Longleftrightarrow \begin{array}{c} \\ n1 \\ \\ n2 \\ \\ \end{array}\underbrace{\overset{n^*}{\left[\begin{array}{ccc} & \vdots & \\ & 1 & \\ \cdots & \vdots & \cdots \\ & -1 & \\ & \vdots & \end{array}\right]}}_{H} \; n^* \underbrace{\overset{1}{\left[\begin{array}{c} \vdots \\ I_0\left(e^{\frac{V_+ - V_-}{\eta V_T}} - 1\right) \\ \vdots \end{array}\right]}}_{g(x)}$$

Jacobian of a Diode Stamp

For **F'(x)**:

$$\begin{bmatrix} & & \vdots & & \\ & \frac{I_0}{\eta V_T} e^{\frac{V_1 - V_2}{\eta V_T}} & & -\frac{I_0}{\eta V_T} e^{\frac{V_1 - V_2}{\eta V_T}} & \\ \cdots & \vdots & \cdots & \vdots & \cdots \\ & -\frac{I_0}{\eta V_T} e^{\frac{V_1 - V_2}{\eta V_T}} & & \frac{I_0}{\eta V_T} e^{\frac{V_1 - V_2}{\eta V_T}} & \\ & & \vdots & & \end{bmatrix}$$

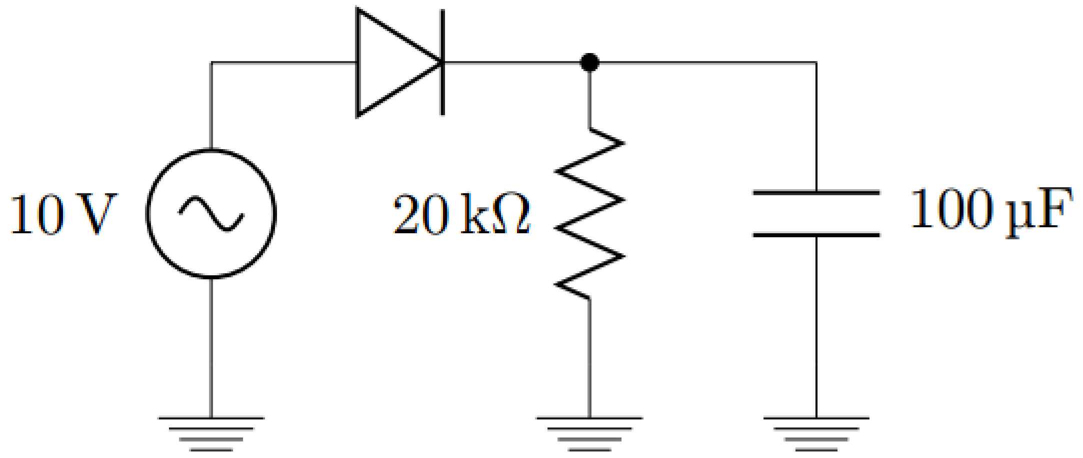
For **g(x)**:

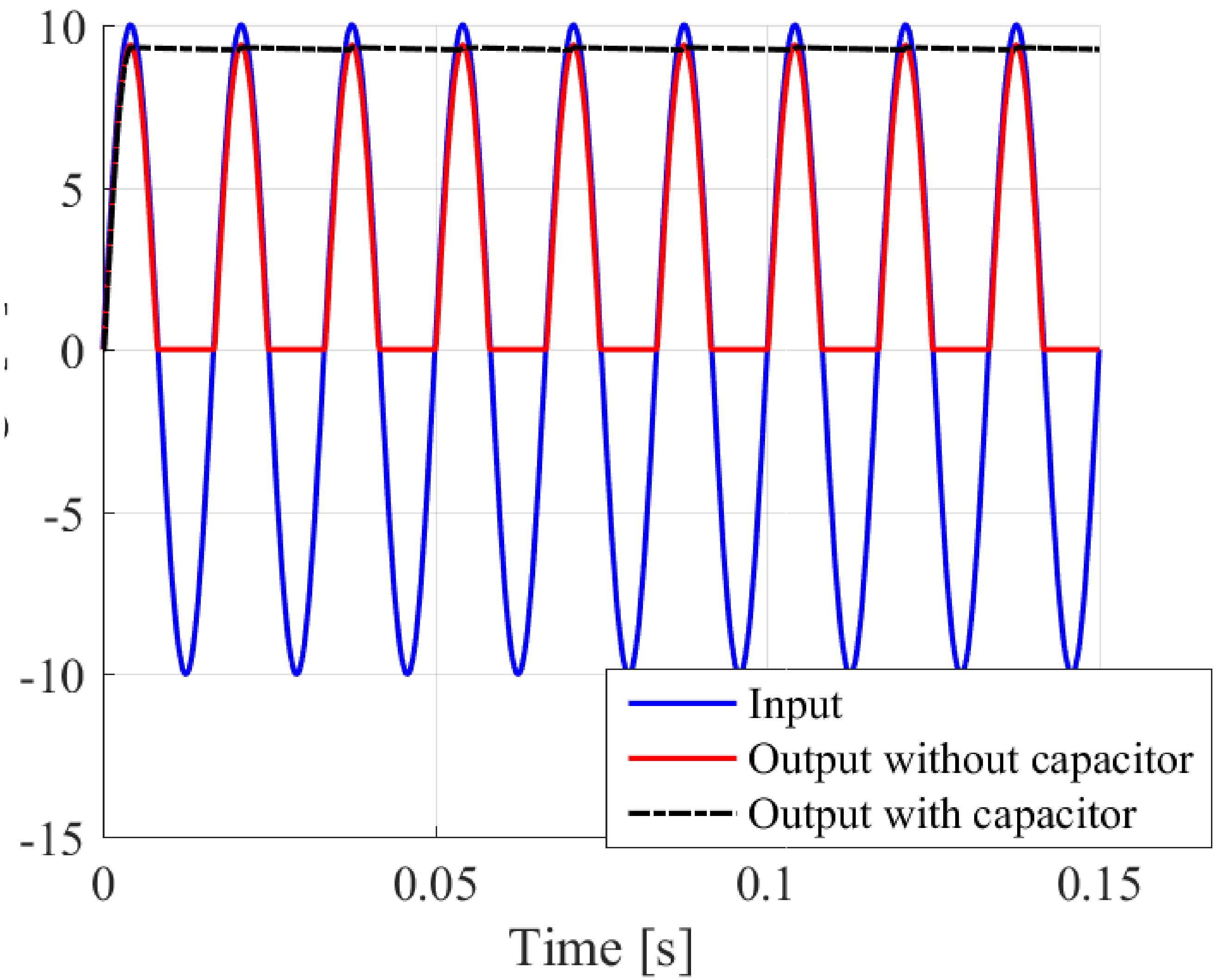
$$\begin{array}{cc} n_1 & n_2 \\ \end{array}$$

$$\underbrace{\begin{bmatrix} \frac{\partial i_D}{\partial v} & -\frac{\partial i_D}{\partial v} \end{bmatrix}}_{J_g}$$

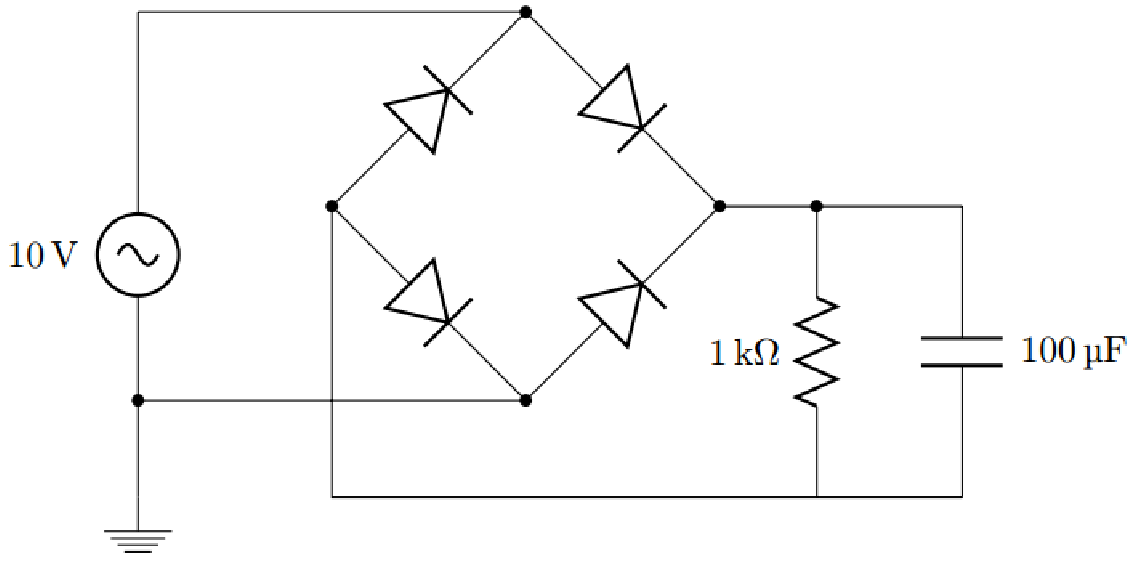
$$\begin{array}{c} n_1 \\ \\ n_2 \end{array} \underbrace{\begin{bmatrix} +1 \\ \\ -1 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} i_D \end{bmatrix}}_{g}$$

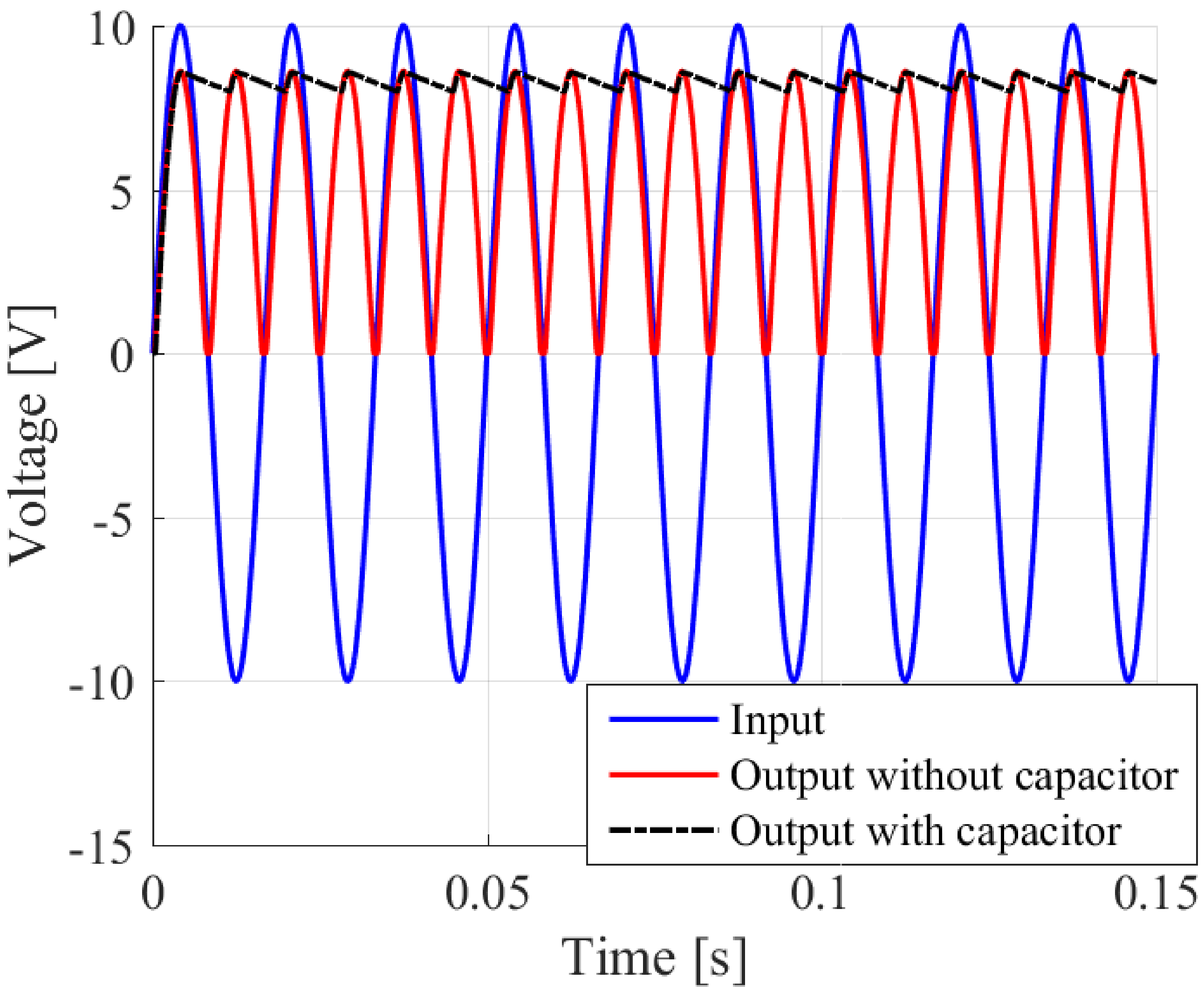
Diode Test 1: Half Wave Rectifier



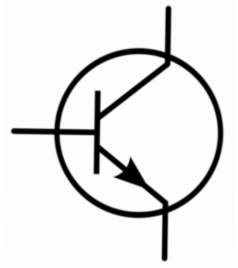


Diode Test 2– Bridge Rectifier





Transistor Model Equations

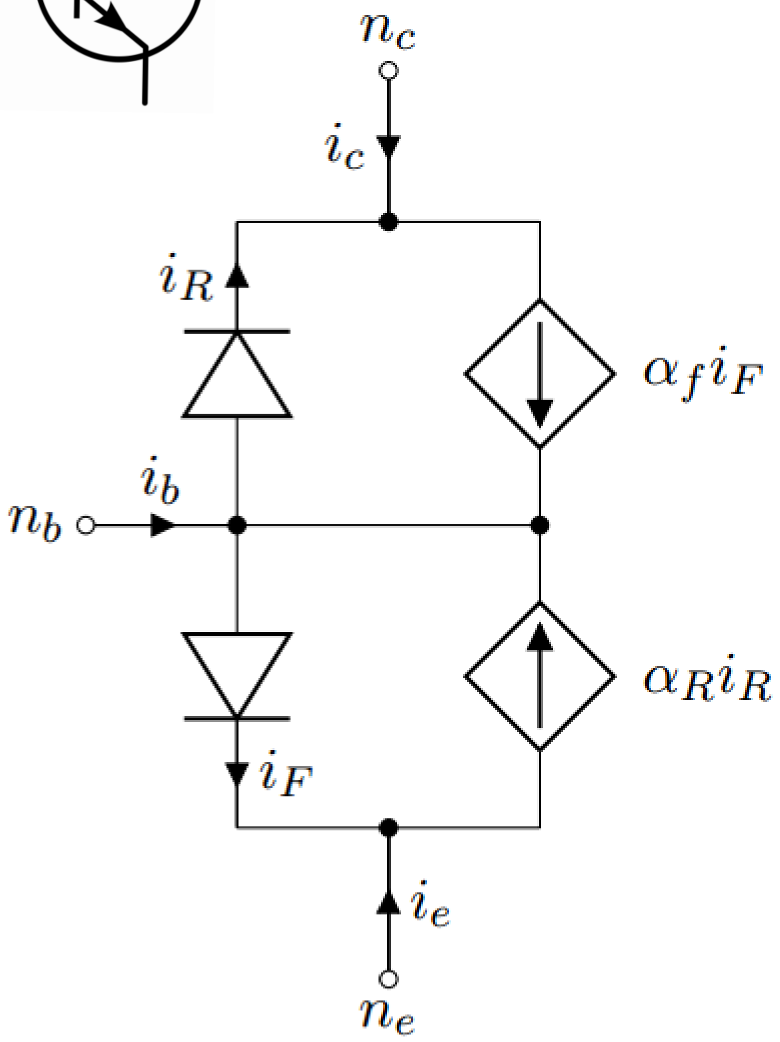


- Ebers-Moll BJT model
  - Simpler to implement than others (e.g. Gummel-Poon)

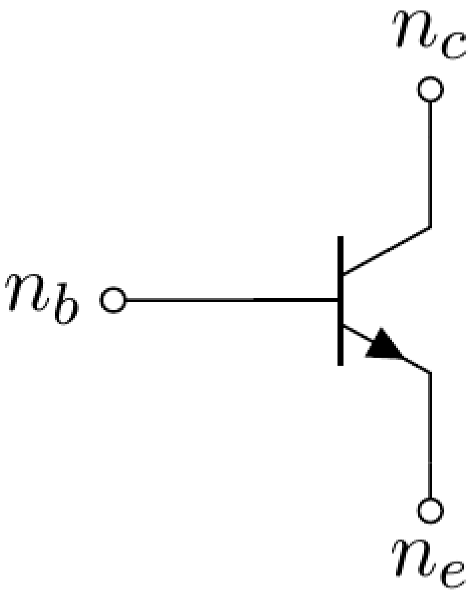
$$i_F = I_{Se}\left(e^{v_{be}/V_{Te}} - 1\right)$$
$$i_R = I_{Sc}\left(e^{v_{bc}/V_{Tc}} - 1\right)$$

- From KCL:

$$i_b = (1-\alpha_F)i_F + (1-\alpha_R)i_R$$
$$i_c = \alpha_F i_F - i_R$$
$$i_e = -i_F + \alpha_R i_R$$



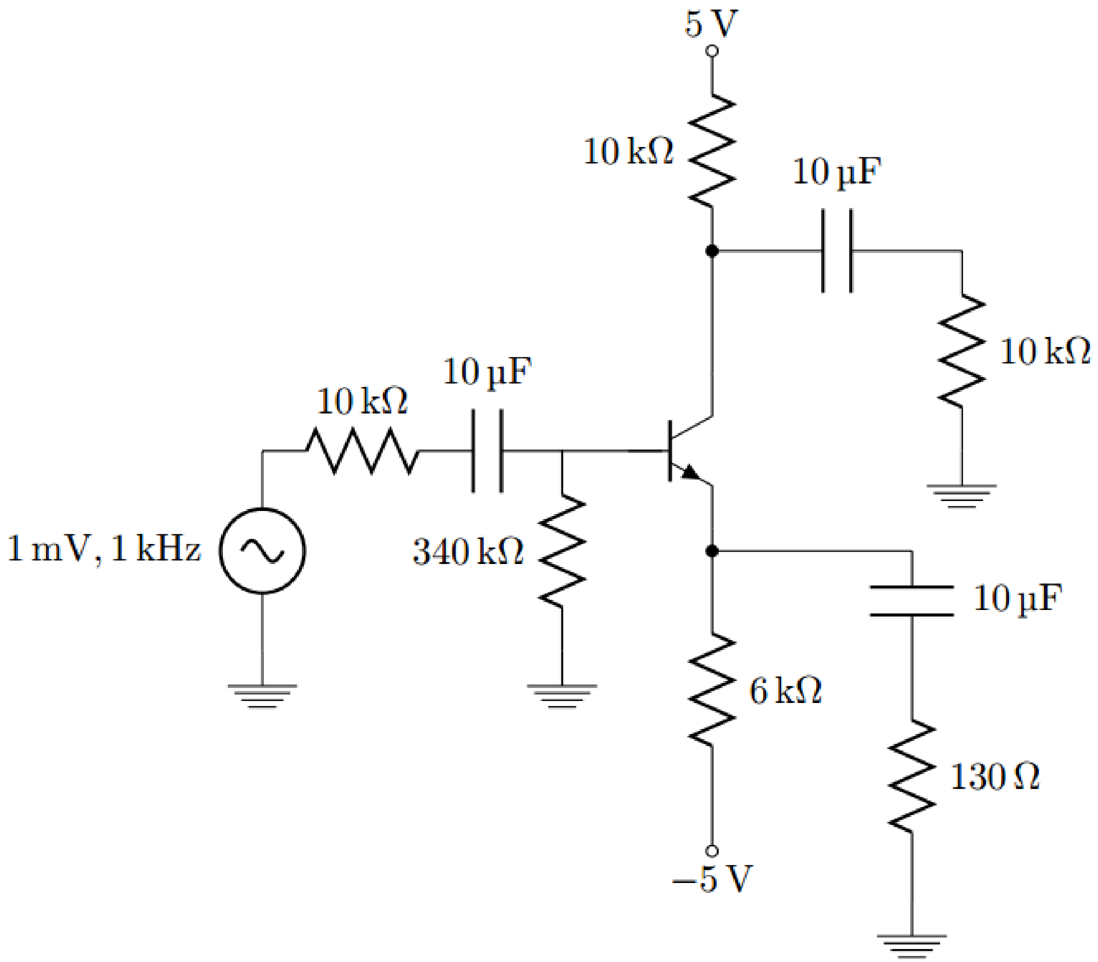
Transistor Stamp

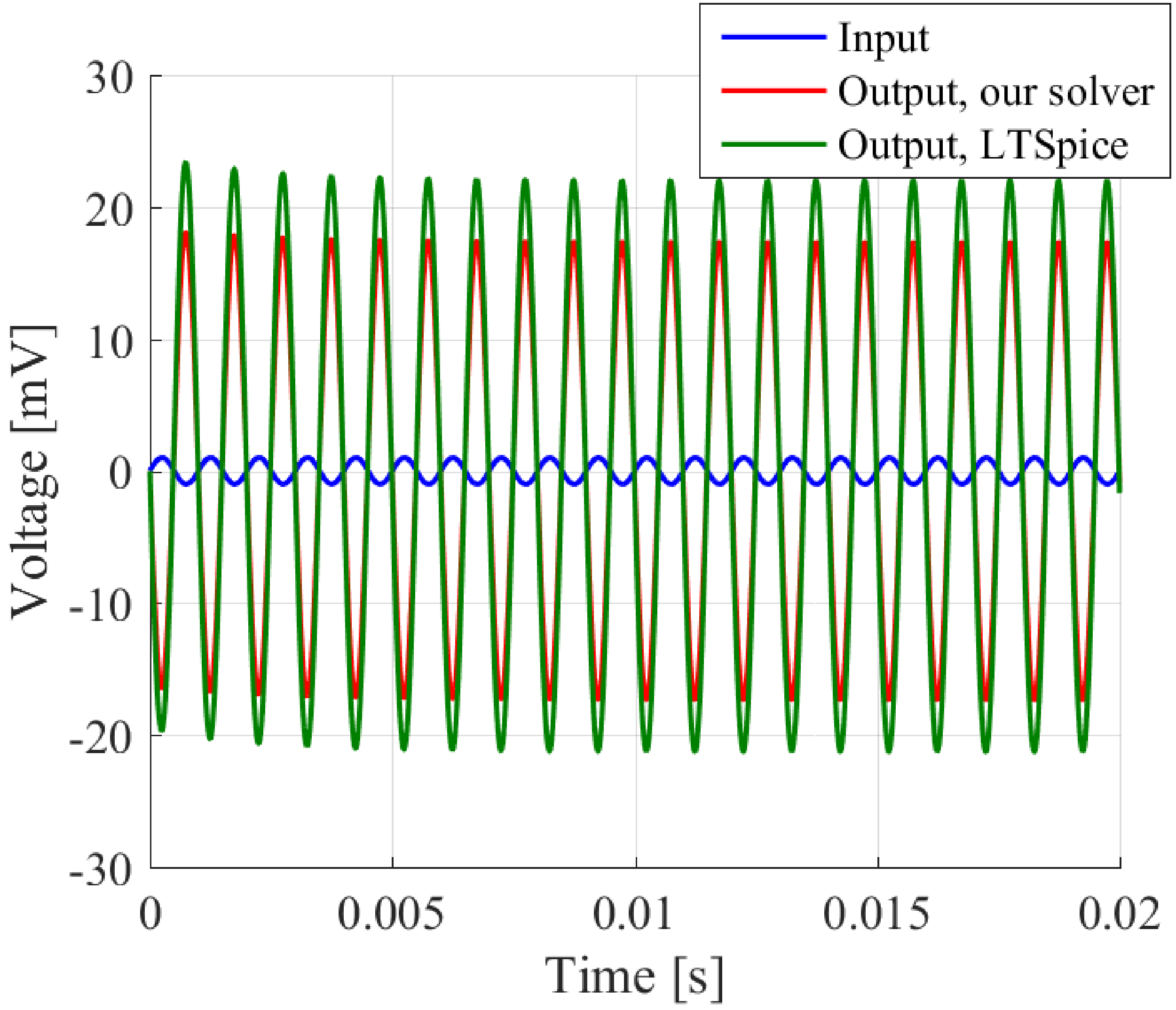


$$
\underbrace{\begin{array}{c} n_b \\ n_c \\ n_e \end{array}
\begin{bmatrix} (1-\alpha_F) & (1-\alpha_R) \\ \alpha_F & -1 \\ -1 & \alpha_R \end{bmatrix}}_{H}
\underbrace{\begin{bmatrix} i_F \\ \\ i_R \end{bmatrix}}_{g}
$$

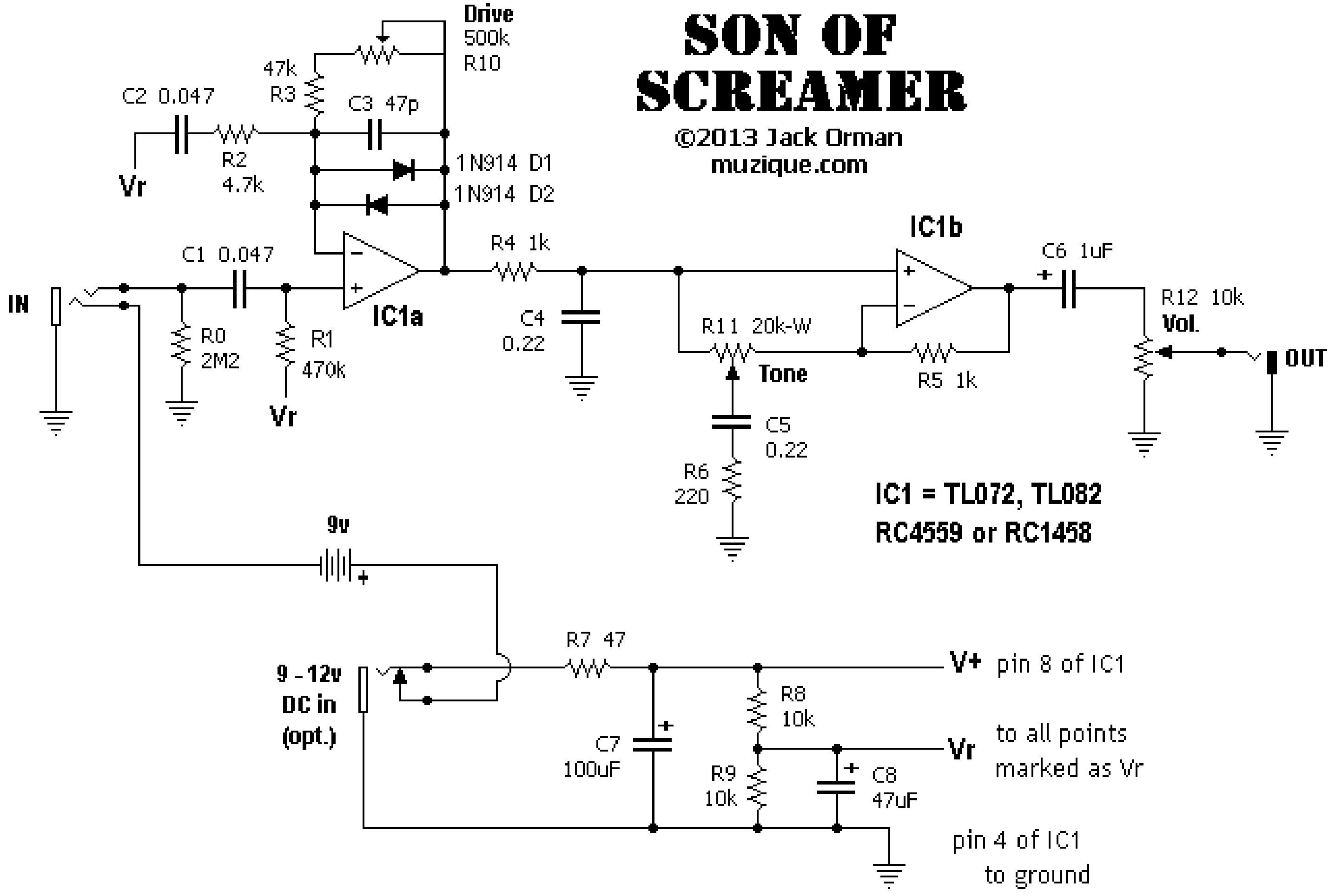
$$
\underbrace{\begin{array}{c} \begin{array}{ccc} n_c & n_b & n_e \end{array} \\
\begin{bmatrix} 0 & \dfrac{\partial i_F}{\partial v} & -\dfrac{\partial i_F}{\partial v} \\[2ex] -\dfrac{\partial i_R}{\partial v} & \dfrac{\partial i_R}{\partial v} & 0 \end{bmatrix} \end{array}}_{J_g}
$$

Transistor Testing – CE Amplifier

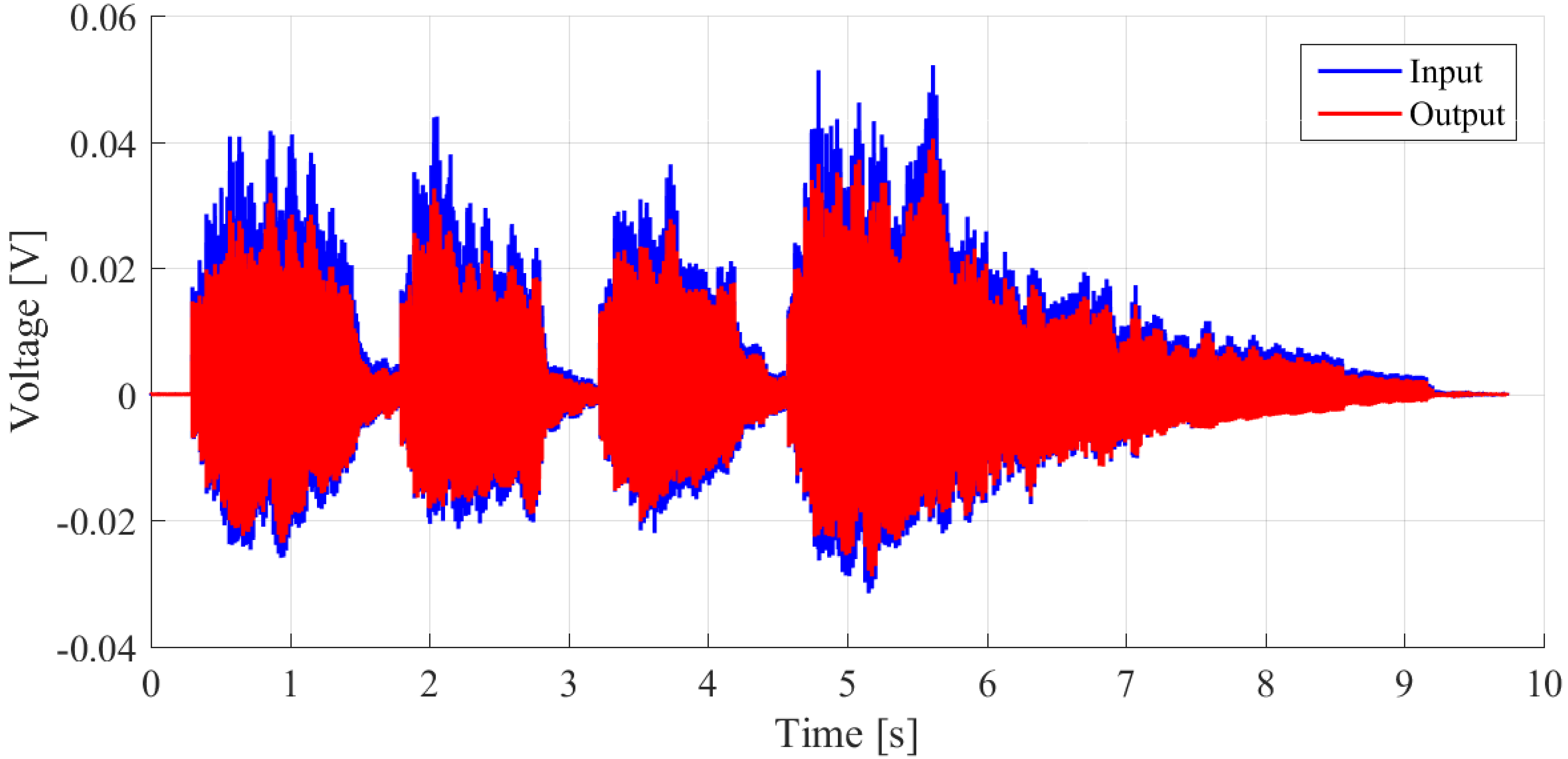




Guitar Effect 1: Tube Screamer Circuit

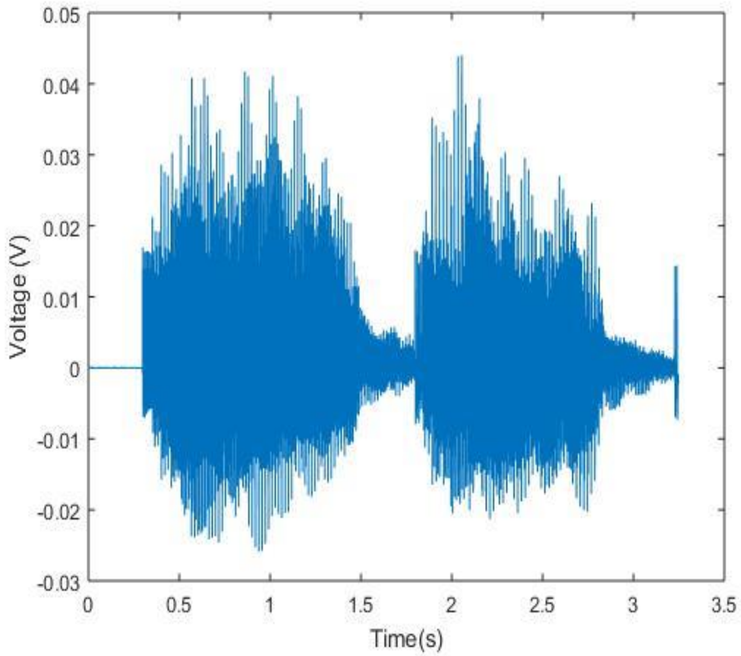


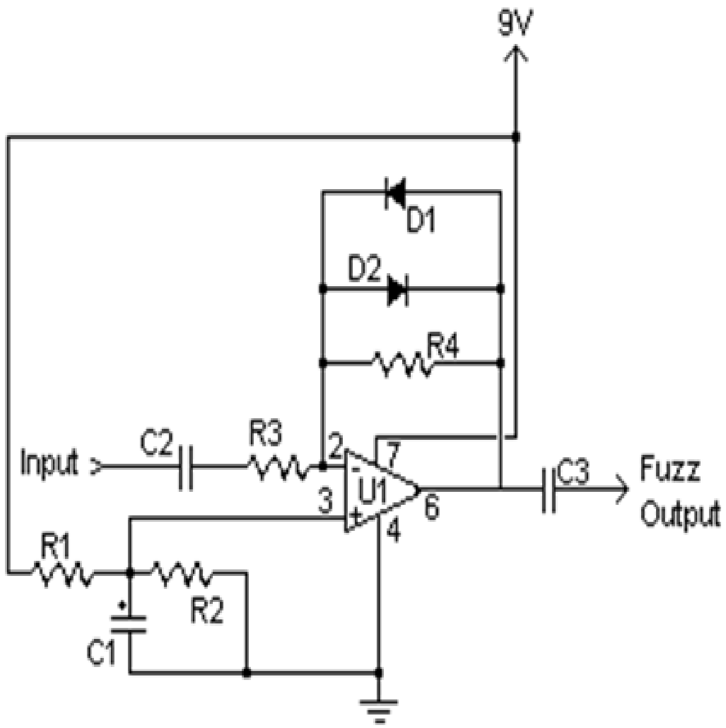
Tube Screamer Input vs. Output

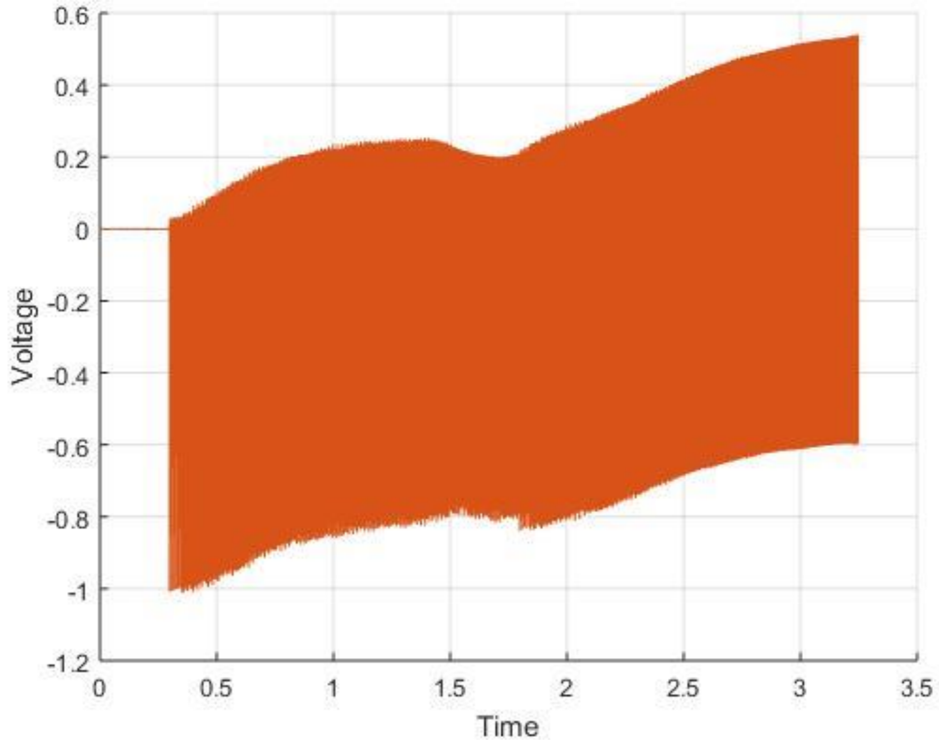


Guitar Effect 2: Fuzz Effect

Iteration	1	2	3	4	5	6
Node 3 Voltage	4.2908	4.2908	4.2907	4.2901	4.2567	NaN
Node 4 Voltage	4.8088	4.7736	4.7083	4.1638	-29.2446	NaN







Insights

- Diodes
  - Either the current through the diode becomes unmanageably large, which produces a singular (exceeding precision) inverse-Jacobian, or becomes a poorly scaled non-invertible Jacobian
- Transistors
  - Ebers-Moll model consists of diodes: the problems that are applicable to diodes are also applicable to transistors.
  - A circuit that contains more than one non-linear element (Fuzz Box) becomes very sensitive to voltage, the exponential characteristics cascades, which is harder to control.
- Modelling
  - Although the integration of both methods was depicted as simply integrating two while loops, it still required to revaluate, by hand, the form of the equations within
  - The error obtained when comparing the variable time step to fixed is smaller, and also computationally faster

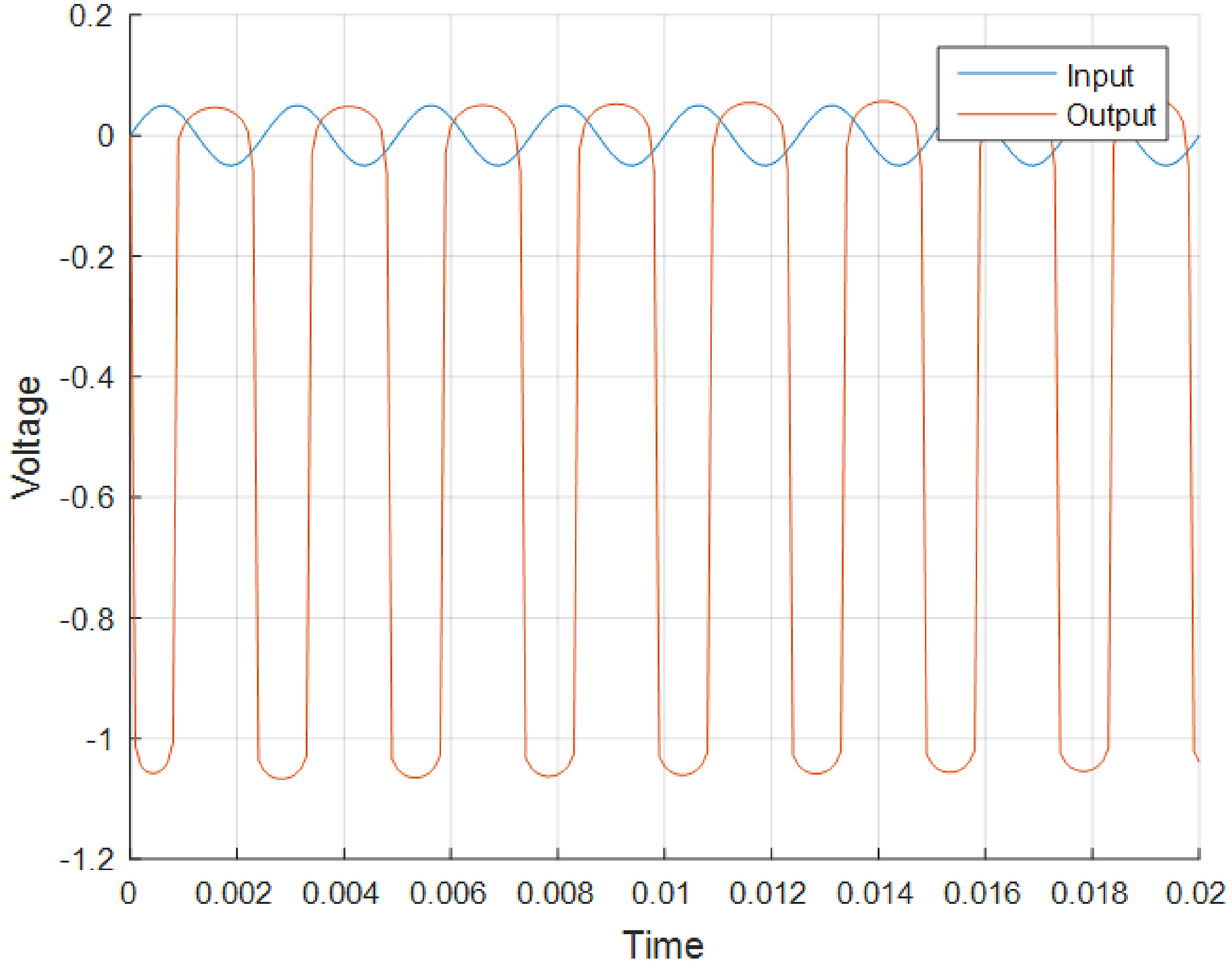
Improvements

- Try MOSFET instead of BJT since its I-V curve is quadratic
- Can use algorithm to improve shunt resistor method (Gmin stepping)
- The sensitivity of the diodes may have been manageable if we had time to integrate the variable time stepping
- Scaling units of the voltages, currents, and circuit elements

Questions?

Thank you for your time!

Result



Input
Output
Voltage
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
Time