# ECE1254 Assignment 2

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## 1 Problem 1

Some notes: numerical artifact

In last week's homework, the final voltage and current vector we obtained was:

$$\begin{bmatrix} 5 \\ 0 \\ -17.14 \\ -11.43 \\ -11.43 \\ 0.0005 \\ -0.0005 \\ -0.0003 \end{bmatrix}$$

The result obtained using my own LU Decomposition code, with optimized features such as switching small value pivots, is similar to last week:

$$\begin{bmatrix} 5 \\ 1.14e-5 \\ -17.14 \\ -11.43 \\ -11.43 \\ 0.0005 \\ -0.0005 \\ -0.00029 \end{bmatrix}$$

## 2 Problem 2

### 2.1 2 a

The resistor grid circuit was implemented with three random current sources. In the particular instance generated for Figure 1, one current source is generated near the voltage source at one tip of the grid, another is generated near the diagonally opposite end of the grid, and yet another is generated somewhere in the middle.

We observe that the sources only affect node voltages nearby. The vast majority of nodes do not see drastic voltage drops, and the voltage never dipped below 0.94V.

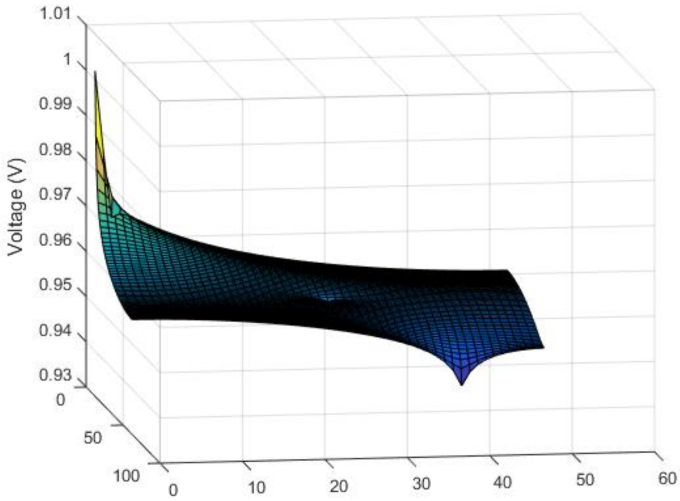


Figure 1: The voltage values of the resistor grid visualized as a surface

## 2.2 2 b, c, d

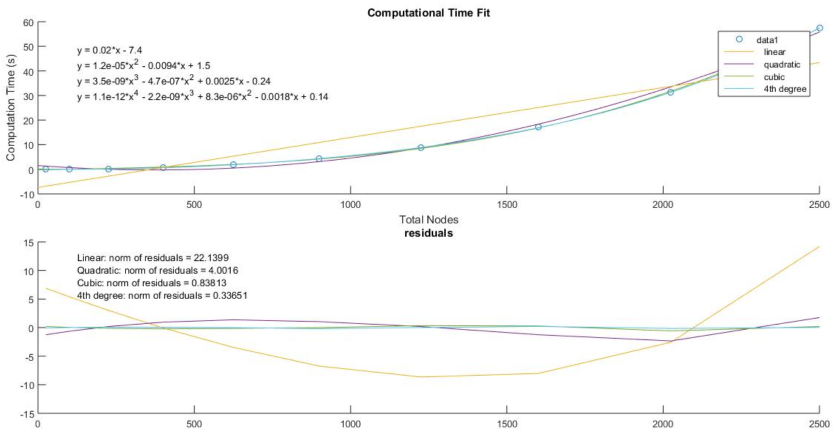


Figure 2: The voltage values of the resistor grid visualized as a surface

The computational time for a range of grid sizes is experimentally measured and plotted in Figure 2. Graphically, it seems the lowest order polynomial with a close fit to the data (residue ¡ 1) is the third order polynomial, which is confirmed when the residues are checked. Therefore, the computation time relation is:

$$t = 3.5 \times 10^{-9} n^3 - 4.6 \times 10^{-7} n^2 + 2.5 \times 10^{-3} n - 0.24$$

The fitting was done using the Fitting functionality in MATLAB's figure GUI. The randomization of current sources are removed in the code to reduce variability between tests.

We have seen in class that the most expensive computation in LU decomposition is traversing the matrix and modifying each row, which, in the circuit model, is of order $O(n^3)$. This would agree with the polynomial fit obtained in this exercise.

# 3 Problem 3

## 3.1 3 a

It is possible to show that if we let temperature $T$ be the analogy to voltage, heat flow $Q = k\frac{dT}{dx}$ be the analogy to current, and discretizing any derivatives, a heat equation problem with convection and source terms could be mapped to a resistor circuit:

$$Q_{out}^{(m)}(x) - Q_{in}^{(m)}(x) = Q^{(conv)}(x) - H(x)$$

where the (m) superscript refers to heat flow through conduction in the metal $Q^{(m)}(x) = K_m \frac{T(x+\Delta x)-T(x)}{\Delta x^2}$, and (conv) superscript refers to heat flow through convection with the environment $Q^{(conv)}(x) = k_\alpha(T(x) - T_0)$. This is a KCL equation with $H(x)$ being current sources, and the other terms are currents flowing through a resistor grid.

The geometry of the circuit looks like Figure 3. Temperature at the two sides are held constant by two 250V voltage sources. The convection to the environment is represented by $R_{conv}$ and the ambient temperature is held constant by a set of 400V voltages. The input heat flow $H(x)$ is represented by incoming current sources. There are also a row or resistors connecting each voltage source, for the reason that this makes the circuit easier to generate for the function developed in problem 2.

## 3.2 3 b

There are two types of resistors, corresponding to conduction through metal and convection through air. They are simply the inverse of the thermal conductivities:

$$R_m = \frac{\Delta x^2}{K_m}$$

$$R_{conv} = \frac{1}{K_\alpha}$$

## 3.3 3 c, d

The difference between the thermal conductivity of metal and air,according to Wikipedia, is approximately ten thousand times. This allows calculation of suitable $\Delta x$:

$$10^4 = \frac{R_{conv}}{R_m} = \frac{K_m}{K_\alpha}\frac{1}{\Delta x^2}$$

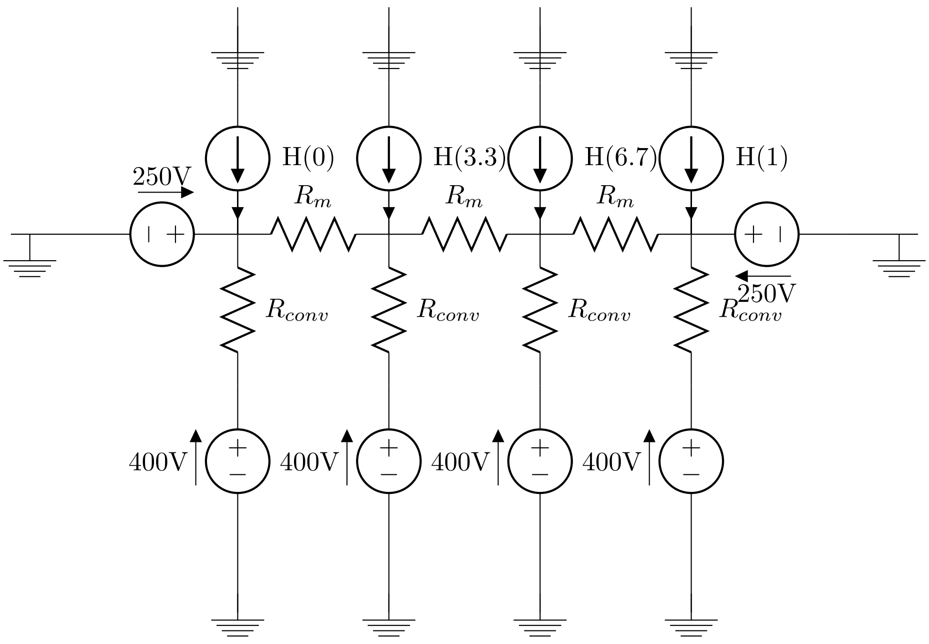


Figure 3: Equivalent circuit of the conducting rod problem with heat sinks

which yields $\Delta x = 0.1$, or $N = 10$.

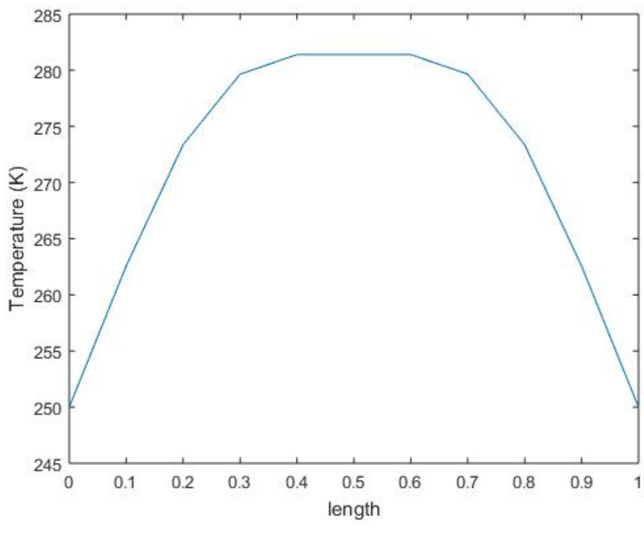


Figure 4: The result of the conducting rod problem with heat sink boundary conditions and $\Delta x = 0.1$

The node voltage solution of the thermal circuit is shown in Figure 4. I tried a variety of $\Delta x$ close to 0.1 and the behaviour of all solutions is similar. The only difference is the profile is smoother for smaller increments.

### 3.4 3 e, f

The condition

$$\frac{\Delta T(0)}{\Delta x} = \frac{\Delta T(1)}{\Delta x} = 0$$

implies that

$$\frac{T(\Delta x) - T(0)}{\Delta x} = \frac{T(1) - T(1 - \Delta x)}{\Delta x} = 0$$

So the heat flow at the very end elements of the rod need to be controlled by current sources. The modified thermal circuit model is shown in Figure 5

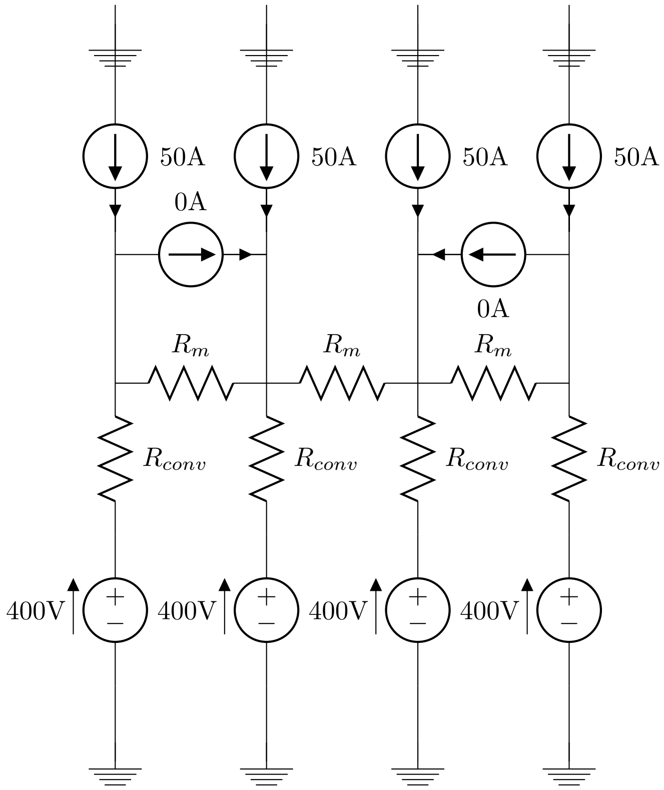


Figure 5: Equivalent circuit of the conducting rod problem with insulation at the ends.

The results are shown in Figure 6.

### 3.5 3 g

Both results make sense.

In the first case,the incoming heat source is sinusoidal, while ambient convection is constant all along the rod. Therefore, the rod is hottest where the heat

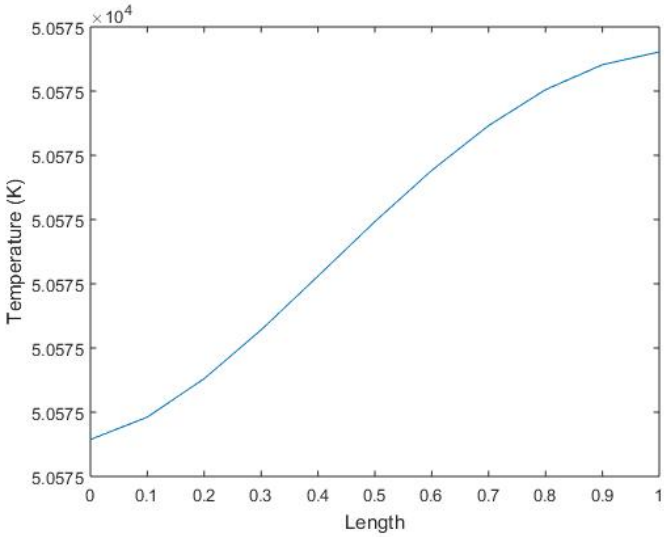


Figure 6: The result of the conducting rod problem with insulating boundaries and $\Delta x = 0.1$. Ignoring numerical artifacts, the profile is constant at 50575K

source is the highest, and decays sinusoidally to the edges held constant at 250K.

In the second case, no heat can escape from the sides, and the heat source is also a constant; therefore, a constant solution is expected. The heat builds up until the temperature difference is large enough for convection to become significant. I verified this analytically by verifying that at a temperature of 50575K, the convection to the environment exactly cancels the incoming heat flow.