ECE1254 Assignment 5

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1 1a

The constitutive equation for inductors is

$$V = V_{n1} - V_{n2} = L\frac{dI}{dt}$$

If KCL is applied at a node connected to an inductor as well as the usual set of capacitors and resistors, the current of the inductor, I_L, and its derivative $\frac{dI}{dt}$ would be left as an additional unknown.

Therefore, for each inductor, another row would be added to the G, C, and B matrices (where the additional row to the B matrix is all empty) to include the constitutive equation, and one column added to G and C matrices (where the additional column added to C matrix is all empty) to incorporate the unknown currents and its time derivative into the KCL equations at their respective nodes.

The stamping is similar to the voltage sources. The two nodes connected to the inductor get stamped 1 and -1 in the new row of G, while they get stamped -1 and 1 respectively in the new column of G. The difference for inductor is that another term is stamped with value of L in the new row added to the C matrix at the node corresponding to the $\frac{dI}{dt}$ term.

2 1b

The values used in the cable circuit model is shown in table 1. These values are found by multiplying the transmission line values by the differential length 0.05mm.

The binary transmission line is generated iteratively using a for loop. For this report, it is assumed that each RLC block takes up 0.05mm of the line,

Level	Resistance	Capacitance	Inductance
1	0.125Ω	0.01pF	25pH
2	0.1785Ω	0.007pF	35.7pH
3	0.255Ω	4.9fF	51pH
4	0.255Ω	4.9fF	51pH

Table 1: Circuit element values in transmission line model of cable.

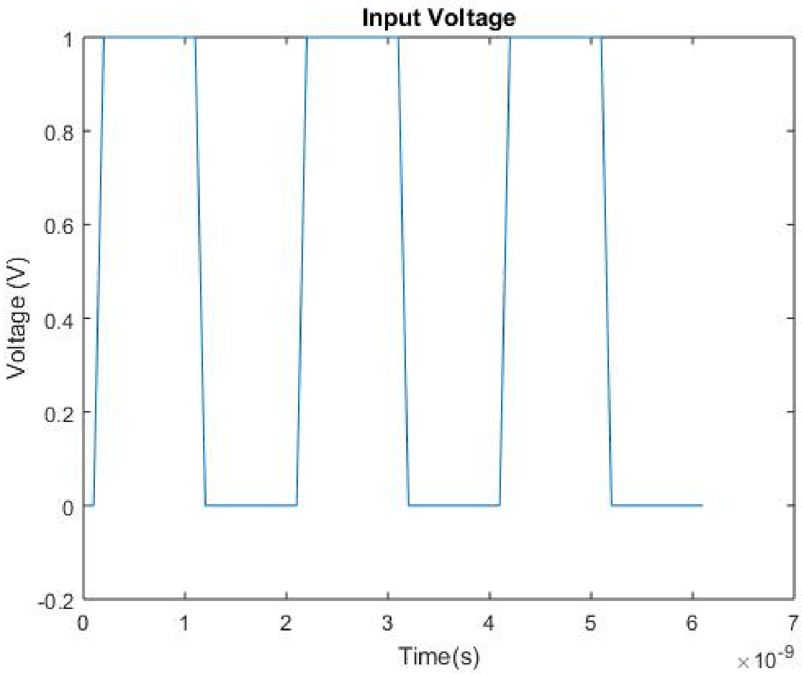

Figure 1: The input signal into the clock cable.

although analysis for the case where individual components takes up 0.05mm was also done.

3 1c

The input and output voltage values over time is seen in figures 1, 2, and 3. Both B.E. and T.R. preserved the overall behaviour of the signal. However, Trapezoidal Rule seems to be able to capture more of the finer oscillations in the output. As a result, in a later section, Trapezoidal Rule is found to converge to a lower error as a function of number of time steps faster than Backward Euler.

4 1d

The time step chosen is 0.01 ns. It needs to be small enough to capture the smallest changes in the clock generator wave form, which in this case is 0.1 ns. I also attempted 0.001 ns time steps. The results are almost the same but because the matrices are now 10 times as large, the wait time is noticeably longer. The calculations for 0.01 ns time steps were fast enough that the wait time is almost unnoticeable. Therefore, 0.01 ns is a better time step.

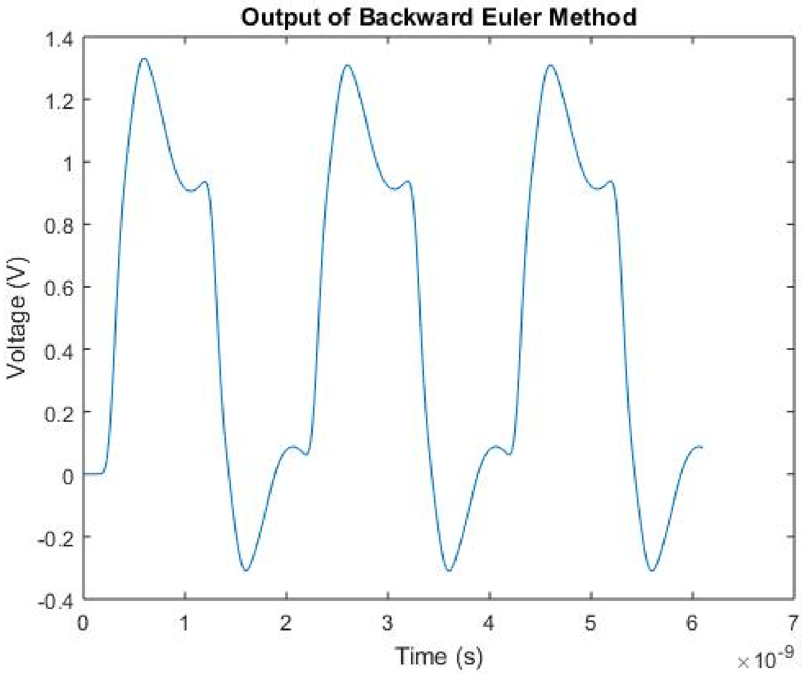

Figure 2: The output node signal from Backward Euler Method

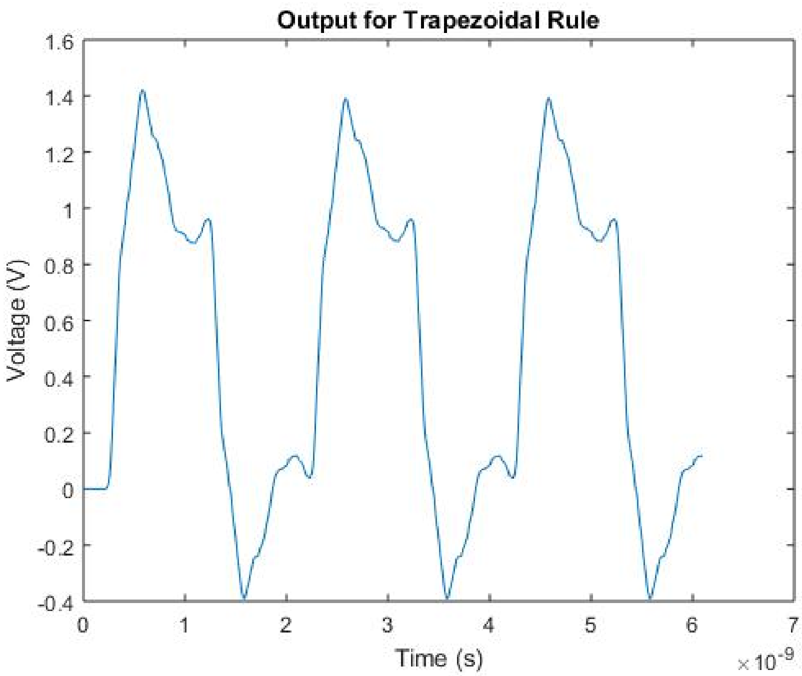

Figure 3: The output node signal from Trapezoidal Ruler Method.

Method	Time Steps	Computation Time	Error
Backward Euler	600	0.91	0.1438
	1200	1.81	0.0876
	6000	8.81	0.0319
	12000	17.83	0.0210
	60000	87.31	0.0075
Trapezoidal	600	0.94	0.0905
	1200	1.87	0.0434
	6000	9.24	0.0088
	12000	18.53	0.0048
	60000	92.52	0

Table 2: Comparison of run time and error of Backward Euler and Trapezoidal methods.

5 1e

0 initial condition is assumed for the entire length of the cable. The error at the nth time step at the very last output node is computed as $||x_n - x*||$, where x* is the solution of trapezoidal rule using the smallest time step in this study. Since the question specified that we want the maximum error, the infinity norm is used. Because the size of the solution matrix is different depending on the time step chosen, only time points that appear both in x* and x_n are considered.

The results can be seen in table 2. The time steps used are 10 ps, 5 ps, 1 ps, 0.5 ps, and 0.1 ps. Note that in the tables and plots, the corresponding total number of time steps for calculating the time interval 0 to 6 ns is used for convenience.

6 1f

Trapezoidal error convergence order is 0.98 +/- 0.09, which is approximately twice that of Backward Euler, which appears to be 0.63 +/-0.02. These values are obtained by fitting a linear function to a log-log plot and extracting the slope value of the fit.

Overall the computational costs are similar for both methods, but the error of trapezoidal rule drops off more quickly as a function of time step. Trapezoidal rule seems to be able to find a more accurate solution without taking significantly more time in simulation.

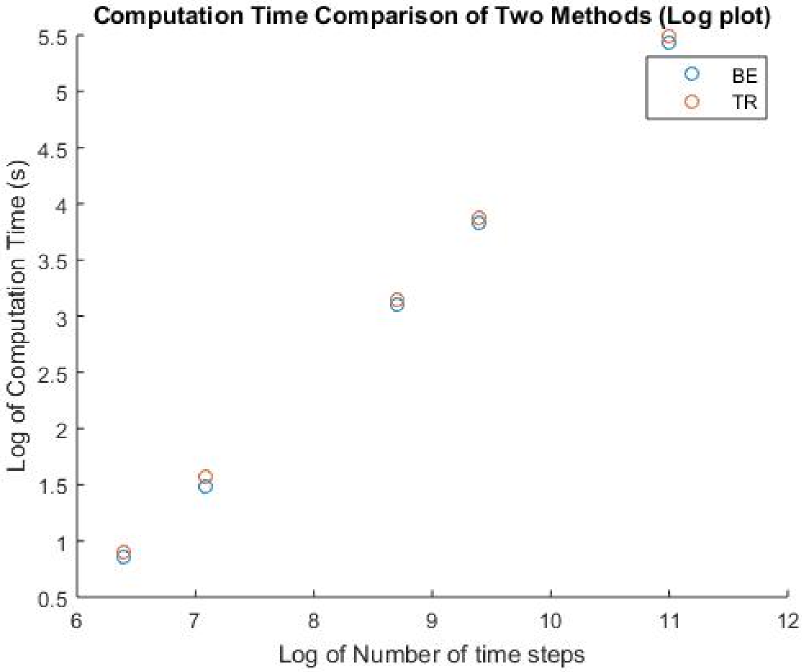

Figure 4: The run time of each method as a function of time steps

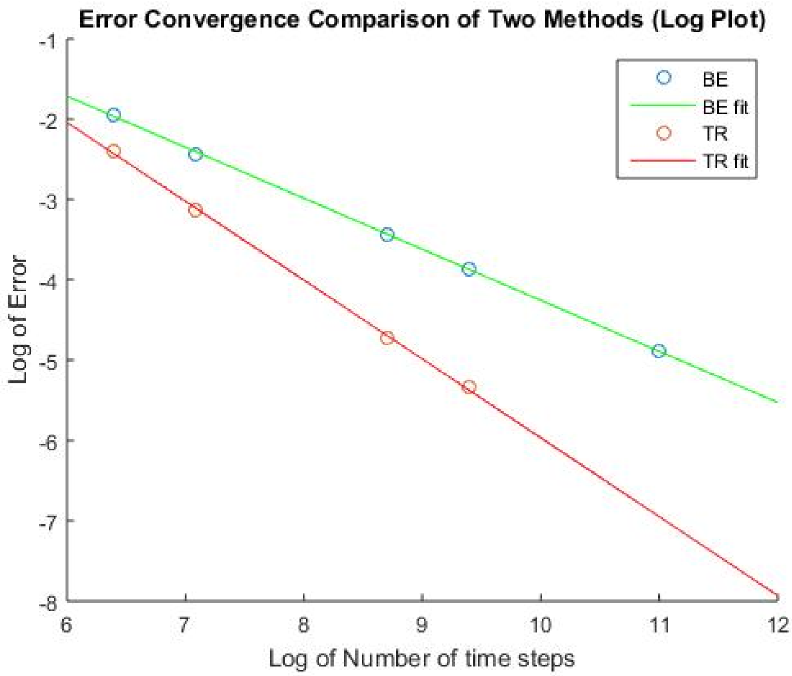

Figure 5: The maximum error is plotted and fitted as a function of time steps. The convergence order of Backward Euler is 0.63 and the convergence order of trapezoidal is 0.98