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Final project

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1 Introduction

Though sub-replacement fertility has become the main population problem in Taiwan, this is not always the case in Zhubei. There are more and more people moving into Zhubei. With move-in population growing, the Hsinchu government has proposed many schools building programs to deal with it. To continue assisting students in the eastern district of Zhubei, County magistrate Yang Wen-ke and the county government team held a meeting to understand the progress of the establishment of Wen Xing Elementary School. In 2022, Jia Feng Elementary School enrolled students in their first year.

Zhubei are now trying hard to accommodate more students. What makes this need to be conducted urgently is that the population in urban areas is positively growing.[1] Therefore, there will be more students in Zhubei in the future. Moreover, children born in dragon year are going to be the age of entering elementary schools. By that time, things would get harder. Therefore, this problem has to be solved as soon as possible.

Although the Hsinchu County Education Bureau has conceived of allowing schools to over-enrollment, this is not a proper long-term plan. Yet, if we build schools to meet the needs, there

[1]Source: `https://www.parenting.com.tw/article/5097404`.

are many things that need to be considered. For instance, the school capacity has to be large enough to contain all needy students in Zhubei. At the same time, there are limited teachers that can be assigned. In addition, building schools is time-consuming and expensive. The plan has to be designed well, otherwise it would be a waste of government expense. As the result, deciding where to build a new school and its size is a tough issue.

In this project, we try to help the Hsinchu government to figure out the best school building locations. With new schools construction, needy students can eliminate the hassle of commuting and have more time to be accompanied by their parents. This arrangement can increase study time, reduce living stress, and alleviate financial burdens. We have three scenarios. In the first one, we assume that the capacity and the cost of school have been made and fixed. There are two kinds of school, big and small. They have corresponding building costs, the numbers of maximum students enrollment and the numbers of teachers. What's our primary goal is to minimize the total building cost and total distances of all students in need. We are allowed to give weight to each cost for further evaluation. In the second one, we make the school sizes continuous variables. That is, the building cost, the maximum students enrollment and the teachers are continuous variables. Each of the values will be determined by the linear regression model. What's our primary goal is also to minimize the total building cost and total distances of all students in need. In the third one, we will relax the limit of student capacity, a school can accommodate 120% of its capacity, but it will get punishment relatively.

2 Problem description

Based on the current situation and data, the Zhubei government has to build new schools at appropriate locations to meet schooling needs. In order to get closer to reality, we keep existing schools and add new ones, instead of demolishing and rebuilding everything. Also, we refer to government's open data as the statistics we used. [2] Our task is to find out area in which added schools should be built and the proportion of students in a specific district who go to schools in different districts. At the same time, we try to minimize total costs. To estimate in the same

[2] Source: `https://doe.hcc.edu.tw/doe_front/index.php?action=basic_table_school&page_uuid=3e330d3a-105f-418a-9bb9-48d9c8e43e6f&city=1&type=3&search_option=2`.

criteria, we all use money as the unit. In addition, we set the weight of commuting cost to 50 as a long-term consideration.

In situation 1, to simplify the problem, there will be only two sizes, which are mentioned as big school and small school in this document. The capacity of students, number of teachers needed and school's building cost are fixed in both sizes of schools. We set the maximum capacity of big school as 1250 students, and that of small school as 300 students. There will always be exactly 110 teachers in big school and 40 teachers in small one. As for the maximum number of teachers provided in Zhubei, it is set to be 2400. Finally, building a big and small school cost $1200 million and $500 million dollars, respectively.

In situation 2, we will make the capacity of students, number of teachers needed and school's building cost in both sizes of schools become continuous variables. Then, values of each variables would be determined by the linear regression model, made by government's open data. The capacity of students is the upper limit, which cannot be exceed. However, an added small school need at least 300 students and an added big school can at most accommodate 1800 students.

In situation 3, we modify the situation two, relax the capacity limit so that the number of students can be greater than the capacity of the school, but will receive punishment. Since most of the schools in Zhubei have over-enrollment around 120%, we set the upper bound at 120% of the capacity.

The numbers of students in each district in Zhubei are given, which are calculated based on the total citizens and percentage of age 6 to 12 from government's data. Students in each district can go to schools in different districts and every district has their own distribution proportion. For example, in district A, 30% students will go to school in district B, 40% students will go to school in district C, and the remain 30% will stay in school in district A. The percentage differs in each district. The number of students in a school can not exceed the maximum capacity mentioned before. We don't consider how students go to school, whether it's by school bus, public transportation or parents' cars. The only factor we care about is the distances among districts.

To sum up, we are going to find some locations for building new schools such that students in Zhubei can shorten their commuting time and the government can have another way to solve over-enrollment problem. We would like to minimize the cost of building new schools, total commuting distance. At the same time, limitation mentioned above have to be satisfied, too.

3 Mathematical model

In this section, we use mathematical models to precisely describe three different situations presented in Section 2.

Situation 1:

Table 1: Notation Table of Situation 1

Variable	Description
L	The set of districts in Zhubei, which also serves as candidate locations for a new school, $L = \{1, ..., 30\}$
K	The fixed number of teachers in Zhubei city
S_i^B	The number of existing big schools in district i , $\forall\ i \in L$
S_i^S	The number of existing small schools in district i , $\forall\ i \in L$
P_i	The number of students in districts i , $\forall\ i \in L$
T_B	The number of teachers for big school
T_S	The number of teachers for small school
N_B	The number of students in big school
N_S	The number of students in small school
d_{ij}	The distance between district i where the students are and district j where the school is located , $\forall\ i, j \in L$
x_j^B	The number of big schools built in district j, $\forall\ j \in L$ (decision variable)
x_j^S	The number of small schools built in district j, $\forall\ j \in L$ (decision variable)
C_B	The building cost of a big school
C_S	The building cost of a small school
z_{ij}	The percentage of students in district i who study at school built in district j, $z_{ij} \in [0, 1]\ \ \forall i \in L,\ j \in L$ (decision variable)

$$\min \quad \sum_{i\in L} \left[(x_i^B - S_i^B)C_B + (x_i^S - S_i^S)C_S\right] + 50\sum_{i\in L}\sum_{j\in L} d_{ij} z_{ij} P_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in L} z_{ij} P_i \le x_j^B N_B + x_j^S N_S,\ \forall j \in L, \tag{2}$$

(The total schools in district j has a capacity limitation.)

$$\sum_{j\in L} z_{ij} = 1,\ \forall i \in L, \tag{3}$$

(All students in district i must attend school.)

$$\sum_{i\in L} \left(x_i^B T_B + x_i^S T_S\right) \le K, \tag{4}$$

(The demand for teachers will not exceed.)

$$\sum_{i\in L} P_i \le \sum_{i\in L} (x_i^B N_B + x_i^S N_S), \tag{5}$$

(The number of students will not exceed the total capacity of schools.)

$$x_i^S \ge S_i^S,\ \forall i \in L, \tag{6}$$

(The number of small schools in district i is not less than the original number.)

$$x_i^B \ge S_i^B,\ \forall i \in L, \tag{7}$$

(The number of big schools in district i is not less than the original number.)

$$x_i^B \ge 0,\ \forall i \in L, \tag{8}$$

$$x_i^S \ge 0,\ \forall i \in L, \tag{9}$$

$$z_{ij} \in [0, 1],\ \forall i, j \in L. \tag{10}$$

Situation 2:

Table 2: Notation Table of Situation 2

Variable	Description
L	The set of districts in Zhubei, which also serves as candidate district for a new school, $L = \{1, ..., 30\}$
Q	The set of position of newly built school in a district, $Q = \{1, 2\}$
K	The fixed number of teachers in Zhubei city
P_i	The number of students in district i, $\forall i \in L$,
y_{ij}	The number of students in school at position j within district i, $\forall i \in L$, $\forall j \in Q$
T_{ij}	The number of teachers in school at position j within district i, $\forall i \in L$, $\forall j \in Q$
S_{ij}	The original number of students in school at position j within district i, $\forall i \in L$, $\forall j \in Q$
d_{ij}	The distance between district i where the students are and district j where the school is located, $\forall i, j \in L$
C_{ij}	The building cost of the school at position j within district i, $\forall i \in L$, $\forall j \in Q$
F_{ij}	If there is a school at position j within district i, $F_{ij} = 1$; otherwise, $F_{ij} = 0$, $\forall i \in L$, $\forall j \in Q$ (decision variable)
B_{ij}	If there is a school newly built at position j within district i, $B_{ij} = 1$; otherwise, $B_{ij} = 0$, $\forall i \in L$, $\forall j \in Q$ (decision variable)
z_{ij}	The percentage of students in districts i who study at school built in district j, $z_{ij} \in [0, 1]$, $\forall i \in L$, $j \in L$ (decision variable)

$$\min \quad \sum_{i \in L} \sum_{j \in Q} C_{ij} B_{ij} + 50 \sum_{i \in L} \sum_{j \in L} d_{ij} z_{ij} P_i \tag{11}$$

$$\text{s.t.} \quad \sum_{i \in L} \sum_{j \in Q} y_{ij} = 17410, \tag{12}$$

$$C_{ij} = 10^8 (0.0075 y_{ij} + 2.8257 F_{ij}),\ \forall i \in L, \tag{13}$$

(the building cost of school at location j in district i)

$$T_{ij} = 0.0762 y_{ij} + 15.351 F_{ij},\ \forall i \in L, \tag{14}$$

(the number of teachers for school j in district i)

$$\sum_{j \in L} z_{ij} = 1,\ \forall i \in L, \tag{15}$$

(All of the students in district i must attend school.)

$$\sum_{j \in L} z_{ji} P_j \leq \sum_{j \in Q} y_{ij},\ \forall i \in L, \tag{16}$$

(The number of students attend school cannot exceed the total capacity of schools in district i.)

$$\sum_{i \in L} T_{ij} \leq K, \tag{17}$$

(The demand for teachers will not exceed.)

$$\sum_{i \in L} P_i \leq \sum_{i \in L} \sum_{j \in Q} y_{ij}, \tag{18}$$

(The number of students will not exceed the total capacity of schools.)

$$C_{ij} \leq 10^{11} y_{ij},\ \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, C_{ij} = 0), \tag{19}$$

$$T_{ij} \leq 10^4 y_{ij},\ \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, T_{ij} = 0), \tag{20}$$

$$y_{ij} \geq S_{ij},\ \forall i \in L, j \in Q, \tag{21}$$

$$y_{ij} \leq 10^5 F_{ij},\ \forall i \in L, j \in Q, \tag{22}$$

$$y_{ij} - S_{ij} \leq 1800 B_{ij},\ \forall i \in L, j \in Q, \tag{23}$$

(The maximum capacity is 1800.)

$$y_{ij} \geq 300 B_{ij},\ \forall i \in L, j \in Q. \tag{24}$$

(The minimum capacity is 300.)

Situation 3:

We add a decision variable x_{ij}, the number of students in school at position j within district i, $\forall i \in L, \forall j \in Q$.

$$\min \quad \sum_{i \in L} \sum_{j \in Q} C_{ij} B_{ij} + 50 \sum_{i \in L} \sum_{j \in L} d_{ij} z_{ij} P_i + 10^5 \sum_{i \in L} \sum_{j \in Q} (x_{ij} - y_{ij}) \tag{25}$$

$$\text{s.t.} \quad \sum_{i \in L} \sum_{j \in Q} x_{ij} = 17410, \tag{26}$$

$$C_{ij} = 10^8 (0.0075 y_{ij} + 2.8257 F_{ij}), \forall i \in L, \tag{27}$$

$$T_{ij} = 0.0762 y_{ij} + 15.351 F_{ij}, \forall i \in L, \tag{28}$$

$$\sum_{j \in L} z_{ij} = 1, \forall i \in L, \tag{29}$$

$$\sum_{j \in L} z_{ji} P_j \leq \sum_{j \in Q} x_{ij}, \forall i \in L, \tag{30}$$

(The number of students will not exceed the total capacity of schools.)

$$\sum_{i \in L} T_{ij} \leq K, \tag{31}$$

$$\sum_{i \in L} P_i \leq \sum_{i \in L} \sum_{j \in Q} x_{ij}, \tag{32}$$

$$C_{ij} \leq 10^{11} y_{ij}, \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, c_{ij} = 0), \tag{33}$$

$$T_{ij} \leq 10^4 y_{ij}, \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, c_{ij} = 0), \tag{34}$$

$$y_{ij} \geq S_{ij}, \forall i \in L, j \in Q, \tag{35}$$

$$y_{ij} \leq 10^5 F_{ij}, \forall i \in L, j \in Q, \tag{36}$$

$$y_{ij} - S_{ij} \leq 1800 B_{ij}, \forall i \in L, j \in Q, \tag{37}$$

$$y_{ij} \geq 300 B_{ij}, \forall i \in L, j \in Q, \tag{38}$$

$$x_{ij} \leq 1.2 y_{ij}, \forall i \in L, j \in Q. \tag{39}$$

(The number of students of a school can not over enroll 120% of the capacity.)

4 Evaluation

We use Gurobi program to analyze and predict the best solution that minimize the cost of building new schools and the commuting cost for the students, which is measured by distance. Zhubei City had built two elementary schools in these two years, we will demonstrate the effectiveness by comparing our results with government's policy.

Table 3: Situation 1 solution

	Situation 1	**Reality**
Total Cost	3.6 billion	2.58 billion

In Situation 1, we get the result as shown in Table 3. Based on the result, our model does not perform well, as it not only costs more than reality but results in many vacancies despite being feasible. This is because of the lack of flexibility in school size, and in turn lead to imprecise predictions. To improve the model's precision, we adopt linear methods for school size, building cost, and the number of teacher in Situation 2.

Table 4: Situation 2 solution

	Situation 2 without weight	**Situation 2 with weight**	**Reality**
Total Cost	2.444 billion	2.920 billion	2.580 billion
Districts of New School	15, 17	15, 17	8, 29

As shown in Table 4 above, our result of Situation 2 is presented. We can find that our model solves the problem since it results in a lower cost than the actual situation.

We initially believe that increasing the weight of commuting costs in the original model would lead to more precise results. However, our findings indicate that there's no significant difference between the two models when the weight is small (i.e., 50). (It is a deviation when the weight is large enough to affect the result, so we don't consider.)

Since we had seen the issue of over-enrollment in Zhubei in many reports, we suppose it also plays a crucial role in cost. Hence, we consider the issue of over-enrollment in the model in Situation 3 by permitting the fixed proportion (0.2 of the capacity) of over-enrollment of each school and adding the cost of over-enrollment to the objective function.

Table 5: Comparison of Different Situation and Reality

	Situation 3 without weight	**Situation 3 with weight**	**Situation 2**	**Reality**
Total Cost	2.442 billion	2.842 billion	2.444 billion	2.580 billion
Districts of New School	24, 29	24, 29	15, 17	8, 29

We demonstrate our solution and compare it within the two situations in Table 5. In Situation 3, we believe that the model addresses the over-enrollment issue more accurately, making it more realistic than Situation 2. In the result, the cost is lower than in both Situation 2 and the actual situation, indicating that this model is better. Additionally, in Situation 2, we found that the weight did not affect the locations of new schools. The results in this situation are consistent with the previous findings, leading us to conclude that the weight has a minimal impact on the outcome. Hence, we can ignore the weight in the model.

5 Conclusion

We designed three situations to address our research problems. For the model in Situation 1, due to the worse result compared with the actual situation and the plethora vacancies in several schools, we conclude that it is not an appropriate model. We make a presumption that the lack of flexibility in school size is the reason of the imprecision.

In Situation 2, the linearization of the school size, building cost, and the number of teacher effectively solve the problem incurred in Situation 1, proving our presumption in Situation 1. Thus, it results in a lower cost than the actual situation.

In Situation 3, we consider the issue of over-enrollment, and obtain the least cost among all the situations mentioned in our report. It hence demonstrates that moderate degree of over-enrollment is an effective way to reduce the total cost. From the three situation, we conclude that the model in situation 3 is the best one and also the closest to the reality.

In conclusion, our final model predicts the optimal location more effectively than the government's current method. Compared to the decision made by the government, we also build two schools, but in different districts and at a lower cost.

To enhance our study, we hope to gain access to more detailed or non-public datasets (e.g., the real distance from student's home to school, the actual building cost of schools) to conduct more precise analyses. This would help the government and residents reduce costs while improving decision-making.