Homework 4

DUE: November 3 2022, 9PM

Instructions

Upload a PDF file, named with your UC Davis email ID and homework number (e.g., `xtai_hw1.pdf`), to Gradescope (accessible through Canvas). You may handwrite your answers and scan them, or type them up.

Students may choose to collaborate with each other on the homework, but must clearly indicate with whom they collaborated.

Problem 1 (20 points)

A student has trouble waking up for class. They have two different old alarm clocks. The first goes off 80% of the time, and the second goes off 50% of the time. Calculate the following probabilities:

a. Neither alarm clock goes off
b. One alarm clock goes off
c. Both alarm clocks go off
d. The first alarm clock goes off, given that the second goes off.

Problem 2 (20 points)

The following table represents the joint distribution of X and Y:

X \ Y	1	2
1	0.2	0.1
2	0.0	0.2
3	0.3	0.2

a. Find the marginal distribution of X and the marginal distribution of Y.
b. Find the conditional distribution of $Y|X = 1$, $Y|X = 2$ and $Y|X = 3$.
c. Are X and Y independent? Why or why not?

Problem 3 (15 points)

On any given day, there is a 10% chance of rain. A person works in a casino with no windows. When it rains, customers wear rain boots 80% of the time. When it doesn't rain, customers wear rain boots 5% of the time. If the casino worker sees a customer in rain boots, what is the chance of rain?

Problem 4 (25 points)

People with the disease D have a 90% probability of testing positive on the D-test. If they do not have disease D, they have a 99% probability that they will test negative. We know that 5% of all people test positive.

a. What is the probability of having disease D?
b. A person just tested positive on the D-test. What is the probability that they are actually healthy?

Problem 5 (20 points)

A random experiment involves rolling a four-sided dice twice. Let X represent the sum of the numbers on the dice's face.

a. Write the probability mass function of the random variable X.
b. Find $P(X = 2)$.
c. Find $P(X \leq 4)$.
d. Find $P(X = 2|X \leq 4)$.
e. Find $E(X)$.
f. Find $Var(X)$.
g. If $Y = 2X + 1$, what is $E(Y)$? What is $Var(Y)$?