# Statistics and Prediction

# Assignment 2

**Due date:** April 8, 2020 (Before class)

**Submission method:** Please submit your assignment to me via E-mail: sherryecon@qq.com.

**Other requirements:**

1. Please use English to answer the questions.

2. Your assignment should be in **pdf** format.

3. The title of your submission email should be "Assignment 2 - your student ID - your name", for example, "Assignment 2 - 201901010101 - Zhang, San"

Please answer the following questions in Chapter 7 and Chapter 8 of *Bounded Rationality and Industrial Organization* (Spiegler, 2011).

### Exercise 7.1 (p.99) Basic Model of Competitive Obfuscation

Let $n = 2$ and $c = 0$. Modify the model by allowing prices to get values in $\{0, 1\}$ only. In addition, assume that for every firm $k = 1, 2$, the consumer observes $m_k = k$ independent sample points from the firm's price distribution and chooses the firm with the lowest average price in the sample. In case of a tie, he chooses firm 2. If the average price of both firms in the consumer's sample is 1, he opts out and chooses none of the firms. Formulate the interaction between the firms as a strategic game and find its Nash equilibrium.

### Exercise 7.2 (p.99) Basic Model of Competitive Obfuscation

Let $n = 2$ and $c = 0$. Modify the model by assuming that consumers draw a sequence of two sample points $(p_i^1, p_i^2)$ from each firm $i$'s *cdf* $G_i$. If $p_i^k < p_j^k$ for both $k = 1, 2$ (i.e., if firm $i$ dominates firm $j$ in the consumer's sample) the consumer chooses firm $i$. If no firm dominates another in the sample, the consumer chooses each firm with probability $\frac{1}{2}$. Formulate the interaction as a strategic game and show that there is a Nash equilibrium in which $G_1 = G_2 \equiv U[0, 1]$.

### Exercise 7.3 (p.106) "Simple" Options

Suppose that there is only one firm in the market. The firm chooses a *cdf* over $[c, 1]$. In addition to this firm, the consumer has access to a simple option at a fixed price $p_0$. For simplicity, assume that the consumer breaks ties in favor of the firm. Show how the firm's optimal *cdf* varies with the firm's marginal cost $c$.

### Exercise 8.1 (p.115) DeBruijn Sequence

Construct a DeBruijn sequence of order 4.

### Exercise 8.2 (p.118) Complex Price Patterns

Assume $T_h = 2$. Construct a cyclic price sequence, such that after every price history, type $h$ can only predict that the probability of $L$ is $\frac{2}{3}$. Find a value $T_1^*$ such that whenever type 1 has $T_1 \geq T_1^*$, he can make a perfect prediction of the price in each period.

### Exercise 8.3 (p.122) "Partially" Coarse Buyers

Let $b = 0$ and $\Pi = \{[0, d), [d, 1]\}$. Show that if $d \in (0, \frac{1}{6})$, the buyer's optimal action given this partition is $p^* = \frac{1}{4}(1 + d)$, such that trade occurs if and only if $\omega \leq \frac{1}{4}(1 + d)$.