Chapter 2. Lagrange's Method (Exercises)

Exercise 2.1: Cobb-Douglas Utility Function

Consider a consumer choosing between two goods x and y, with prices p and q respectively. His income is I, so the budget constraint is

$$px + qy = I.$$

Suppose the utility function is

$$\widetilde{U}(x, y) = x^{\alpha} y^{\beta}.$$

What is the consumer's optimal bundle (x, y)? Compare your answer to the answers in Example 2.1.

Exercise 2.2: The Linear Expenditure System

Consider a consumer choosing between two goods x and y, with prices p and q respectively. His income is I, so the budget constraint is

$$px + qy = I.$$

Suppose the utility function is

$$\widehat{U}(x, y) = \alpha \ln(x - x_0) + \beta \ln(y - y_0).$$

What is the consumer's optimal bundle (x, y)?

Exercise 2.3: Production and Cost-Minimization

Part I Consider a producer who rents machines K at r per year and hires labor L at wage w per year to produce output Q, where

$$Q = \sqrt{K} + \sqrt{L}.$$

Suppose he wishes to produce a fixed quantity Q at minimum cost.

Find his factor demand function, that is, the optimal amount of K and L.

Part II Now let p denote the price of output. Suppose the producer can vary the quantity of output, and seeks to maximize profit. Factor prices and the production function are the same as in Part I.

Find the optimal output.