Chapter 6. Convex Sets and Their Separations (Exercises)

Exercise 6.1: Commodities that Cause Disutility. How is Figure 6.1 altered when

(a) one of the choice variable is labor, which gives disutility to consumers and is an input to production,

(b) when one of the goods is pollution, which gives disutility to consumers and is the by-product of an economically desirable good which is the other choice variable?

Interpret the associated prices in each of these contexts.

Exercise 6.2: Convexity of a Firm's Profit Function. A firm chooses vectors x of inputs and y of outputs subject to a production possibility constraint $G(x, y) \leq 0$, to maximize profit $qy - px$, where q denotes the row vector of output prices and p that of input prices. Let $\Pi(q, p)$ be the maximized profit expressed as a function of the prices. Prove that Π is a convex function of (q, p).

Exercise 6.3: Corner Solutions. Consider an economy with labor endowment L. It can produce two goods x_1 and x_2. A unit of good j needs a fixed amount of a_j units of labor, so the production possibility constraint is

$$a_1 x_1 + a_2 x_2 \leq L.$$

The world prices of the two goods are p_1 and p_2, independent of the levels of production chosen by this country. The aim is to maximize the value of national product, $(p_1 x_1 + p_2 x_2)$.

Question 1: Draw a figure and solve the problem by separation of two convex sets. You will need to consider two cases separately, depending on which of p_1/p_2 and a_1/a_2 is larger.

Question 2: Having the solutions in the figure, verify Lagrange's conditions. Find and interpret the Lagrange multiplier. Show that the maximized national product, expressed as a function of the prices (the revenue function or the GNP function) is

$$R(p_1, p_2) = \max\left\{\frac{p_1}{a_1}, \frac{p_2}{a_2}\right\} L.$$