Advanced Microeconomics Final, 2021 Fall

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Question 1. (30')

(a) (15') Show that the production set Y is a convex cone *if and only if* it is additive and satisfies the nonincreasing returns condition.

Note:

- A set $S \subseteq \mathbb{R}^n$ is *additive* if for any $x, x' \in S$, we have $x + x' \in S$.
- S is called a *convex cone* if for any $x, x' \in S$ and any $\alpha, \alpha' \geq 0$ we have

$$\alpha x + \alpha' x' \in S.$$

(b) (15') Suppose that $\pi(\cdot)$ is the profit function of the production set Y and that $y(\cdot)$ is the associated supply correspondence. Assume also that Y is closed and satisfies the free disposal property. Show that

(i) (7') $\pi(\cdot)$ is homogeneous of degree one.

(ii) (8') $\pi(\cdot)$ is convex.

Question 2. (40') Consider the utility function

$$u(x_1, x_2) = \sqrt{x_1} + x_2.$$

Let $(p_1, p_2, w) \in R^3_{++}$, where p_1 and p_2 denote the prices of goods 1 and 2 respectively and w denotes consumer's wealth.

(a) (5') Formally state the consumer's utility maximization problem (UMP). Briefly show that a solution to the UMP exists.

(b) (25') Use the Kuhn-Tucker procedure to solve the UMP (e.g., find Walrasian demand $x(p,w)$).

(c) (10') Given the Walrasian demand $x(p,w)$ and indirect utility function $v(p,w)$ you solved in (2), please derive the corresponding Hicksian demand $h(p,u)$ and expenditure correspondence $e(p,u)$. (Hint: UMP and EMP are "dual" problems. Thus, try to use the definition of inverse function.)

Question 3. (30') Attention: Please choose to answer either A or B. You may also chooose to answer both. Your score for Question 3 will be max{A,B}.

A. Suppose that $u(x)$ is differentiable and strictly quasi-concave and that the Walrasian demand function $x(p,w)$ is differentiable. Show that

(a) (15') If $u(x)$ is homogeneous of degree one, then $x(p,w)$ and $v(p,w)$ are homogeneous of degree one in w.

(b) (10') The wealth expansion path is a straight line through the origin. (Note: wealth expansion path is $E_{\overline{p}} = \{x(\overline{p}, w) : w > 0\}$)

(c) (5') What does this imply about the wealth elasticities of demand? (Note: wealth elasticity of demand for good l is $\varepsilon_{lw} = \frac{\partial x_l / x_l}{\partial w / w}$)

B. People are insensitive to small changes. For example, Alice is indifferent between two cups of bubble tea when one cup differs from the other in having only one more grain of sugar. However, such an indifference is not transitive, because eventually, after many grains of sugar are added, Alice will become able to tell one cup is sweeter than the other.

Formally, suppose the set of alternatives is $X = [0, \infty)$. Given $x, y \in X$, Alice forms her preference according to the following two rules:

- she strictly prefers x to y (i.e., $x \succ y$) if and only if $x - y \geq 1$;
- she is indifferent between x and y (i.e., $x \sim y$) if and only if $|x - y| < 1$;

Based on the definitions of $\succ$ and $\sim$ above, we say Alice prefers x to y (i.e., $x \succsim y$) if (i) $x \succ y$ or (ii) $x \sim y$.

(a) (8') Is Alice's preference relation (i.e., $\succsim$) complete? If yes, show your arguments formally; otherwise, provide a counter-example.

(b) (8') Is Alice's preference relation (i.e., $\succsim$) transitive? If yes, show your arguments formally; otherwise, provide a counter-example.

(c) (6') Suppose that for any set $B \subseteq X$, Alice's choice rule is:

$$C(B) = \{x \in B \mid x \succsim y \text{ for all } y \in B\}.$$

Consider $B = \{1, 1.5, 2\}$. What is Alice's choice rule $C(B)$?

(d) (8') Let $\mathscr{B}$ be a collection of $B \subset X$. Given Alice's choice rule in part (c), does there exist a choice structure $(\mathscr{B}, C(\cdot))$ such that Alice's choices violate WARP? If yes, show an example; if not, show your argument formally. (Hint: examine Alice's choice in the last question closely)