Coarse Reasoning[1]

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[1] Chapter 8 of *Bounded Rationality and Industrial Organization* (Spiegler, 2011)

Introduction

Introduction

- Chapter 6 and Chapter 7: examine a small part of the object in detail and extrapolate naively
- This Chapter:
  - process the entire available data on the market objects
  - create a "coarse representation" of the data: may fail to perceive patterns that underlie the data

Modeling Framework

- Pricing strategy for a firm: $f : \Omega \to P$
  - Ω: a set of *situations* (states of Nature, histories in a dynamic model, etc.)
  - P: a set of prices

Modeling Framework

Boundedly rational consumer: *coarse representation* of f

- *Perceptual partition* $\Pi = \{\Pi_1, ..., \Pi_K\}$ of Ω
- $\Pi(\omega)$: the partition cell that contains the situation ω
- *Coarse representation* of f: mixed strategy σ^{Π}
- Probability that σ^{Π} assigns to a given price p conditional on a situation ω: the fraction of situations $\omega' \in \Pi(\omega)$ for which $f(\omega') = p$.

Modeling Framework

An example of coarse representation:

- Situations $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$
- Prices $P = \{p_1, p_2\}$
- Pricing strategy:
    - $f(\omega_1) = f(\omega_2) = p_1$;
    - $f(\omega_3) = f(\omega_4) = f(\omega_5) = p_2$.
- *Perceptual partition* $\Pi = \{\Pi_1 = \{\omega_1, \omega_3\}, \Pi_2 = \{\omega_2, \omega_4, \omega_5\}\}$
- *Coarse representation* (consider ω_1):
    - probability assigns to p_1 conditional on $\omega_1 = \frac{1}{2}$;
    - probability assigns to p_2 conditional on $\omega_1 = \frac{1}{2}$.
    - Because $\Pi(\omega_1) = \Pi_1 = \{\omega_1, \omega_3\}$ and $f(\omega_1) = p_1, f(\omega_3) = p_2$

Modeling Framework

- Rational consumer: maximizes his (expected) utility given a *perfect perception* of f
- A fully rational consumer is a special case that corresponds to a maximally fine perceptual partition: $\Pi = \{\{\omega\}\}_{\omega \in \Omega}$.

An Illustrative Example

- A consumer is interested in purchasing a single unit of some product from a monopolist.
- The consumer's willingness to pay takes two values, h and l ($h > l > 0$), with equal probability.
- The monopolist incurs no costs.
- The consumer makes his consumption decision *before* observing the price.

An Illustrative Example

- Suppose that both parties observe the consumer's willingness to pay.
- In particular, the monopolist can condition the product price on it.
- Assume that the monopolist always charges a price equal to the consumer's willingness to pay.

An Illustrative Example

- Situations: $\Omega = \{h, l\}$
- Prices: $P = \{p_h = h, p_l = l\}$
- Pricing strategy: $f(h) = h; f(l) = l$.

An Illustrative Example: Raional Consumer

- Maximally fine perceptual partition: $\Pi = \{\{h\}\{l\}\}$
- Rational consumer would always buy the product.

An Illustrative Example: Boundedly Raional Consumer

- Suppose that the bounded rational consumer's perceptual partition is "fully coarse": $\Pi = \{\{h, l\}\}$
- That is, he believes that the firm charges h or l with equal probability, independently of his willingness to pay.

An Illustrative Example: Boundedly Raional Consumer

Interpretation of the bounded-rational behavior.

- One interpretation for this coarse belief is that when the consumer enters the market, his access to historical data is restricted to the monopolist's past prices.
- An alternative interpretation is that the consumer is unaware that the monopolist knows his willingness to pay.

An Illustrative Example: Boundedly Raional Consumer

- Assume that the consumer is risk-neutral.
- Since the expected price is $\frac{1}{2}(h+l)$, the consumer will purchase the product in the high state only.
- Thus, a pricing strategy that extracts the whole surplus from a rational consumer leads to inefficient trade.

Complex Price Patterns as a Discimination Device

Introduction

In this section, we discuss a monopolistic firm creating a complex price pattern to discriminate between consumers with diverse degrees of coarseness, interpreted as heterogeneity in consumers' ability to perceive patterns.

Model: Firm

- Time is discrete: $t = 1, 2, ...$
- There is a long-lived monopolistic firm that offers a product in each period.
- The firm can serve any number of consumers at no cost.
- The firm's pricing strategy is an infinite price sequence $(p^1, p^2, ...)$, to which it commits at the outset.
- The firm's objective is to maximize its long-run average per-period profit.

Model: Consumers

- In every period t, a large population of short-lived consumers choose whether or not to enter the market.
- The decision is taken before they get to observe p^t.
- If a consumer enters the market in period t, he incurs a fixed entry cost $\varepsilon > 0$ and learns p^t upon entry.
- If he subsequently purchases the product, he pays p^t, consumes the product, and exits the market.
- In the next period, a new generation of consumers faces a similar dilemma.

Model: Consumers

- The consumer population is divided into two groups of equal size.
- A consumer of type θ is willing to pay $\theta \in \{h, l\}$, where $h > l > \varepsilon$.
- θ is consumer's *private information*.
- Thus, the consumer's payoff is

$$\begin{cases} \theta - p - \varepsilon & \text{if enters and buys;} \\ -\varepsilon & \text{if enters and does not buy;} \\ 0 & \text{if does not enter.} \end{cases}$$

Model: Coarse Reasoning

- Each consumer type θ is also characterized by an integer T_θ.
- Consumer type θ form a prediction of the price in period $t > T_\theta$ on the basis of the T_θ *most recent prices*, $(p^{t-T_\theta}, ..., p^{t-1})$.
- Specifically, the probability that the consumer assigns to $p^t = p$ is equal to the *long-run frequency* with which p follows the same sequence of T_θ most recent prices.
- The consumer chooses to enter in period t if and only if the expected price, given his prediction, does not exceed $\theta - \varepsilon$.

Model: Coarse Reasoning (Example)

- Consider $t = 8$
- Price history:
  $(p^1 = H, p^2 = L, p^3 = L, p^4 = H, p^5 = L, p^6 = H, p^7 = L)$
- $T_l = 2, T_h = 3$
- Then, consumer of type l assigns
  - probability $1/2$ to $p^8 = H$;
  - probability $1/2$ to $p^8 = L$.
- Then, consumer of type h assigns
  - probability 1 to $p^8 = H$

Model: Coarse Reasoning (Explanation)

- Although all consumers have access to the same actual price history, they differ in their ability to process the data.
- When a consumer mines a long price history in order to make a prediction, he looks for patterns: T_θ is an indicator of the maximal complexity of price patterns his data mining is able to elicit.
- An alternative interpretation: T_θ is a measure of the maximal complexity of price patterns that the consumer is willing to attribute to the firm.

Model: Perceptual Partition

- Partition Π_θ: for $\theta \in \{l, h\}$, two histories belong to the same partition cell if and only if they share the same sequence of T_θ most recent prices.
- As T_θ increases, the perceptual partition becomes finer.
- The correlation between the consumer's "preference type" θ and his "cognitive type," as captured by T_θ, will be important for the analysis.

Analysis

We will now see that when consumers are "diversely coarse," the firm may have an incentive to create complex price patterns as a discrimination device.

Analysis

From now on, assume (without loss of generality) that prices can take only two values, L and H.

“DeBruijn” Price Sequences (Example 1)

Assume that the firm's strategy induces a price sequence that follows the **cycle** (L, L, H, H).

- If $T_\theta = 2$, the consumer can perfectly predict the price at each period: $\begin{cases} p^t = H & \text{whenever}(p^{t-2}, p^{t-1}) \in \{(L, L), (L, H)\}; \\ p^t = L & \text{whenever}((p^{t-2}, p^{t-1}) \in \{(H, H), (H, L)\} \end{cases}$
- If $T_\theta = 1$, historical prices have no informational value: p^t is equally likely to be L or H, independent of whether $p^{t-1} = L$ or $p^{t-1} = H$

"DeBruijn" Price Sequences (Example 2)

Similarly, suppose that the firm's strategy induces a price sequence that follows the **cycle** (L, L, L, H, L, H, H, H).

- If $T_\theta = 3$, the consumer can perfectly predict the price at each period: $\begin{cases} p^t = H & \text{whenever}(p^{t-3}, p^{t-2}, p^{t-1}) \in \\ & \{(L,L,L),(L,H,L),(H,L,H),(L,H,H)\}; \\ p^t = L & \text{whenever}(p^{t-3}, p^{t-2}, p^{t-1}) \in \\ & \{(L,L,H),(H,H,H),(H,H,L),(H,L,L)\}. \end{cases}$
- If $T_\theta = 2$, historical prices have no informational value: p^t is equally likely to be L or H, independent of (p^{t-2}, p^{t-1}).

"DeBruijn" Price Sequences

In both examples, we constructed a cyclic price sequence with the following properties:

1. a consumer with $T_\theta = d$ can perfectly perceive the pattern and make correct predictions;
2. a consumer with $T_\theta < d$ can perceive no pattern, and in fact cannot even make a statistically valuable prediction that will make use of his access to historical price data.

Such magical sequences are called *DeBruijn sequences*.

DeBruijn Sequence

Definition 1

An infinite price sequence $(p^1, p^2, ...)$ is a DeBruijn sequence of order T if for every period $t > T$:

1. The T - truncated history $(p^{t-T}, ..., p^{t-1})$ uniquely determines p^t.
2. For every $p \in \{L, H\}$, the long-run frequency of $p^t = p$ conditional on any possible $(T-1)$ - truncated history $(p^{t-T+1}, ..., p^{t-1})$ is $\frac{1}{2}$.

DeBruijn Sequence

Proposition 1

For every $T = 1, 2, \ldots,$ *there exists a DeBruijn sequence of order* T.

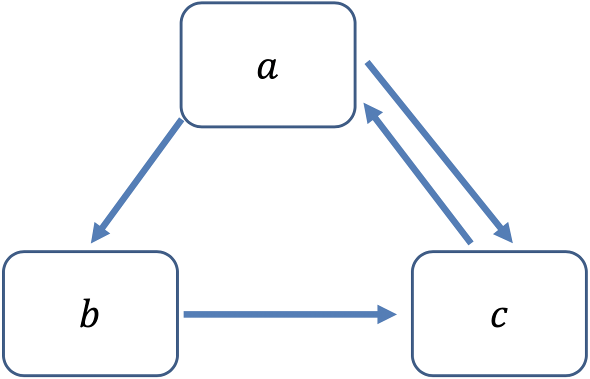
DeBruijn Sequence: Proof of Proposition 1

- For $T = 1$, a DeBruijn sequence is simply $(L, H, L, H, ...)$.
- We have already demonstrated the existence of a DeBruijn sequence for $T = 2, 3$.
- For $T \geq 4$, we need to introduce a few concepts and use the result in Graph Thoery.

Directed Graph

- A *directed graph* (or digraph) is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them.
- Formally, a directed graph is a pair (V, E), where
  - V is a finite set of *nodes*;
  - $E \subseteq V \times V$ is a set of *directed links*, where $(x, y) \in E$ means that there is a link from the node x in to the node y.

Directed Graph



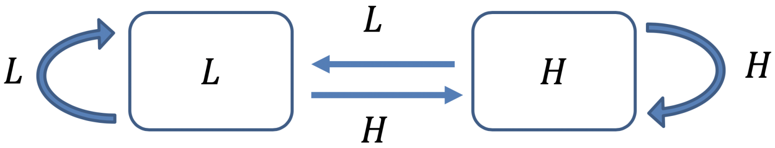
$$V = \{a, b, c\}; E = \{(a, b), (b, c), (a, c), (c, a)\}$$

DeBruijn Graph

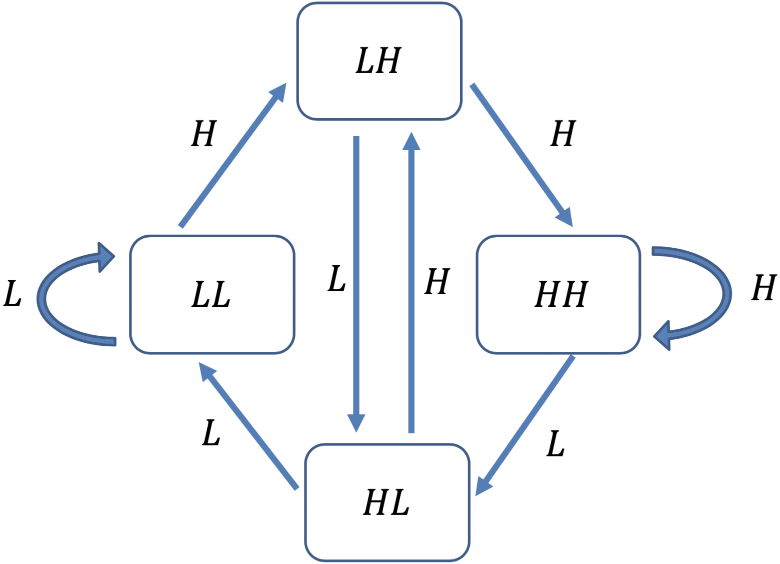
We need a $(T-1)$ - dimensional *DeBruijn Graph* of 2 symbols.

- 2 symbols are H and L
- $V = \{L, H\}^{T-1}$, for example,
  - when $T = 2$, $V = \{L, H\}$.
  - when $T = 3$, $V = \{LL, LH, HL, HH\}$.
- For every node $x = (a_1, ..., a_{T-1})$ and any $a \in \{L, H\}$, assume that there is a link from x into $(a_2, ..., a_{T-1}, a)$. For example,
  - when $T = 2$, $E = \{(\underbrace{L}_{a_1}, \underbrace{L}_{a}), (\underbrace{L}_{a_1}, \underbrace{H}_{a}), (H, L), (H, H)\}$.
  - when $T = 3$, $E = \{(\underbrace{L}_{a_1}\underbrace{L}_{a_2}, \underbrace{L}_{a_2}\underbrace{L}_{a}), (\underbrace{L}_{a_1}\underbrace{L}_{a_2}, \underbrace{L}_{a_2}\underbrace{H}_{a}), ...\}$.

DeBruijn Graph: $T = 2$



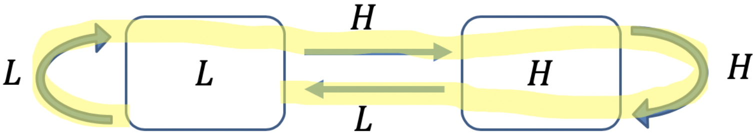
DeBruijn Graph: $T = 3$



DeBruijn Graph

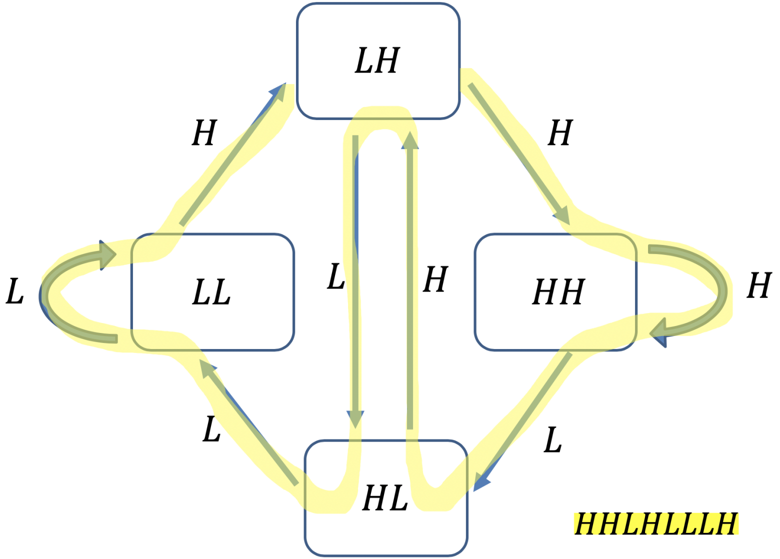
- This graph is connected.
- It is also "regular" – for every node x, there are two links that go into x and two links that go out of x.
- A basic result in Graph Theory states that connectedness and regularity are necessary and sufficient for the existence of an "Euler cycle," that is, a path that passes through every link exactly once.
- We can now define an infinite, cyclic price sequence (p^t) entirely in terms of the links in the Euler cycle, where the t^{th} link in the Euler cycle is from $x = (a_1, ..., a_{T-1})$ into $(a_2, ..., a_{T-1}, \underbrace{p^t}_{a})$.

DeBruijn Graph: $T = 2$



LHHL

DeBruijn Graph: $T = 3$



DeBruijn Sequence

- The constructed sequence satisfies the two conditions of a DeBruijn sequence.
- First, since the length of the Euler cycle is 2^T, this is also the length of the cycle in the price sequence.
- Second, by the definition of an Euler cycle, it passes through every link in the graph exactly once.
- Knowledge of the T most recent price realizations is sufficient since there are exactly 2^T distinct sequences of length T.
- Knowledge of the $T - 1$ most recent price realizations conveys no information about the next price, because it is equivalent to knowing only that the next link in the cycle will originate from a particular node.

DeBruijn Sequence

Exercise 8.1

Construct a DeBruijn sequence of order 4.

DeBruijn Sequence: Current Context

- Suppose that the two consumer types differ in their degree of coarseness: $T_h \neq T_l$.
- The monopolist can adopt a pricing strategy that induces a DeBruijn sequence of order $T = \max(T_h, T_l)$.
- The consumer type θ with the higher T_θ will be perfectly able to predict prices in each period.
- The other consumer type will be completely unable to make valuable predictions because he will regard both price levels to be equally likely after every history.
- In other words, the complex price pattern will confuse only the coarser consumer, in a way that may enable the monopolist to implement profitable price discrimination.

Conditions for Profitability of Complex Price Patterns

The correlation between the consumer's preference and cognitive types is crucial for this analysis.

Case I: $T_h \geq T_l$

When the consumer type with the highest willingness to pay is also the more sophisticated consumer type, obfuscation is unprofitable.

Case I: $T_h \geq T_l$

Proposition 2

If $T_h \geq T_l$, then a constant pricing strategy (i.e., $f(\omega) = p$ for all $\omega \in \Omega$) maximizes the monopolist's long-run profit.

Proof of Proposition 2

- Π_h is a finer than Π_l.
- Now consider a cell in Π_l, in which type l chooses to enter the market.
- The long-run average price in the cell is at most $l - \varepsilon$.
- Therefore, the monopolist's long-run average profit conditional on that cell is at most $l - \varepsilon$.

Proof of Proposition 2

- Similarly, consider a cell in Π_l, in which type l chooses **not** to enter the market.
- Since Π_h is a refinement of Π_l, we can partition the cell into sub-cells, where each sub-cell belongs to Π_h.
- If the average price in any of these sub-cells exceeds $h - \varepsilon$, type h's decision at that sub-cell is not to enter.
- Since the fraction of type l in the consumer population is $\frac{1}{2}$, the monopolist's long-run average profit conditional on a cell in which l chooses not to enter is at most $\frac{1}{2}(h - \varepsilon)$.

Proof of Proposition 2

- We conclude that the monopolists long-run average profit cannot exceed $\max\{l - \varepsilon, \frac{1}{2}(h - \varepsilon)\}$.
- Constant price $f(\omega) = l - \varepsilon$ for all ω gives $l - \varepsilon$.
- Constant price $f(\omega) = h - \varepsilon$ for all ω gives $\frac{1}{2}(h - \varepsilon)$.
- One of the two constant pricing strategies is necessarily optimal.

Proposition 2: Intuition

- When type h is more sophisticated than type l, type h will be able to make perfect price predictions whenever type l can.
- This means that in each period, the monopolist effectively faces a choice between charging $l - \varepsilon$ from both consumer types and charging $h - \varepsilon$ from type h.
- This is the same dilemma that the monopolist faces when constrained to constant pricing strategies.
- Hence, the monopolist has no incentive to obfuscate.

Case II: $T_h < T_l$

When the consumer type with the lowest willingness to pay is the more sophisticated type, then the monopolist has an incentive to obfuscate if entry costs are sufficiently high.

Case II: $T_h < T_l$

Proposition 3

Let $T_h < T_l$ and $\varepsilon > \max(h - l, 2l - h)$. Then, a DeBruijn sequence of order T_l, consisting of the prices $L = l - \varepsilon$ and $H = (h - \varepsilon) + (h - l)$, generates a long-run average profit strictly above $\max[l - \varepsilon, \frac{1}{2}(h - \varepsilon)]$, hence it is strictly better for the monopolost than any constant pricing strategy.

Proof of Proposition 3

By the definition of a DeBruijn sequence of order T_l,

- type l is perfectly able to predict the price in each period;
- type h is completely unable to predict prices and believes that in each period the price is equally likely to be L or H.

Proof of Proposition 3

Type l's behavior (prices $L = l - \varepsilon$ and $H = (h - \varepsilon) + (h - l)$):

- Type l is able to predict the price.
- Type L enters and buys if $p = L$
- Profit from type L: $\frac{1}{2}L = \frac{1}{2}(l - \varepsilon)$

Proof of Proposition 3

Type h's behavior (prices $L = l - \varepsilon$ and $H = (h - \varepsilon) + (h - l)$):

- Type h is unable to predict the price.
- Type h enters since the expected price $\frac{1}{2}(H + L) = (h - \varepsilon)$ does not exceed $h - \varepsilon$.
- After enterance, type h buys in both occasions since $H < h$ under the assumption $\varepsilon > h - l$.
- Profit from type H: $\frac{1}{2}L + \frac{1}{2}H = h - \varepsilon$

Proof of Proposition 3

- Total profit: $\frac{1}{2}\Big[\underbrace{\frac{1}{2}(l-\varepsilon)}_{\text{from type } L}+\underbrace{(h-\varepsilon)}_{\text{from type } H}\Big]=\frac{1}{4}(l-\varepsilon)+\frac{1}{2}(h-\varepsilon)$.
- Under the assumption $\varepsilon > 2l-h$, the optimal constant pricing strategy is to set $p = h-\varepsilon$ in every period since $\frac{1}{2}(h-\varepsilon) > l-\varepsilon$.
- Maximum profit under constant pricing strategy: $\frac{1}{2}(h-\varepsilon)$.
- The constructed strategy generates higher profit than the maximum profit under constant pricing strategy.

Proposition 3: Intuition

- The idea behind this construction is to use a price pattern that confuses type h only.
- The existence of DeBruijn sequences means that we can construct such a price pattern.
- Type l understands the price pattern and enters the market only when this is the optimal action.
- In contrast, type h fails to perceive any pattern and reasons only in terms of the long-run average price.
- Complex price pattern effectively achieves price discrimination.

Discussions: Entry Cost

- The existence of entry costs is also crucial for this analysis.
- It creates a wedge between the consumer's optimal decisions
  - before he learns the price and
  - after he observes the price realization.
- This is what gives consumers with high predictive skills the advantage over less sophisticated consumers.

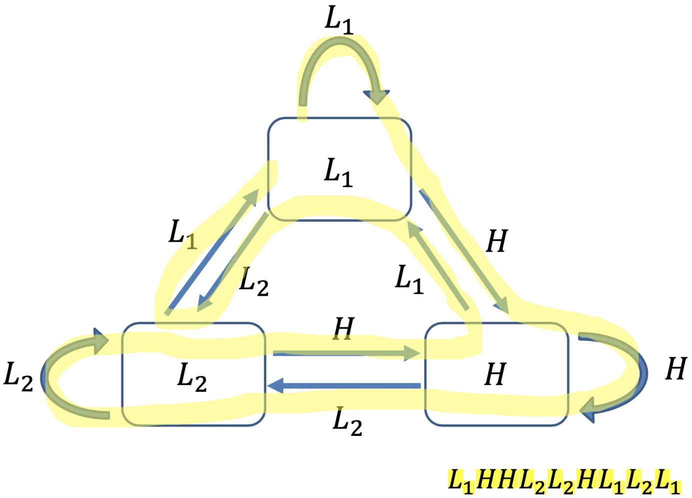
Limitations

- First, it only works under particular restrictions on entry costs.
- Second, it merely shows that the monopolist has a strict incentive to obfuscate, but it does not derive the optimal obfuscatory pricing strategy: in a DeBruijn price sequence, both L and H are realized with a long-run frequency of $\frac{1}{2}$.
    - Long-run frequency of L being some arbitrary rational fraction α, in which type l can perfectly predict the price whereas type h could only predict that L would occur with probability α, is attainable as long as T_l is sufficiently large relative to T_h.

Probability of $L = \alpha \neq \frac{1}{2}$: An Example

- Let $T_h = 1$, and suppose that we whish to sustain $\alpha = \frac{2}{3}$.
- Extend the idea behind the construction of DeBruijn sequences as follows: Create two "copies" of the low price realization and denote them L_1 and L_2.
- The copies are relevant only for the monopolist's construction of the price sequence; consumers who observe the price sequence cannot distinguish between L_1 and L_2.
- Construct the complete directed graph in which the set of nodes is $\{L_1, L_2, H\}$.

Probability of $L = \alpha \neq \frac{1}{2}$: An Example



- Therefore, one such sequence is *LHHLLHLLL*.

Probability of $L = \alpha \neq \frac{1}{2}$: An Example

- It is easy to verify that since $T_h = 1$, type h can only predict after every price history that the probability of L in the next period is $\frac{2}{3}$.
- Knowing 1 price is equivalent to knowing only that the next link in the cycle will originate from a particular node.
- In contrast, as long as $T_l \geq 4$, type l will be able to predict the price in every period.

Exercise 8.2

Exercise 8.2

Assume $T_h = 2$. Construct a cyclic price sequence, such that after every price history, type h can only predict that the probability of L is $\frac{2}{3}$. Find a value T_l^* such that whenever type l has $T_l \geq T_l^*$, he can make a perfect prediction of the price in each period.

Probability of $L = \alpha \neq \frac{1}{2}$

- In general, if T_l is sufficiently high relative to T_h, we can reduce the problem of optimally discriminating between the two consumer types into the following constrained maximization problem:

$$
\begin{aligned}
&\max_{\alpha,L,H} \alpha L + [\alpha L + (1-\alpha)H] \\
\text{s.t. } & L \leq l - \varepsilon \\
& \alpha L + (1-\alpha)H \leq h - \varepsilon \\
& H \leq h
\end{aligned}
$$

Limited Understanding of Adverse Selection

Adverse Selection

- Akerlof (1970, QJE) *The Market for "Lemons": Quality uncertainty and the Market Mechanism.*
- Goods of different quality are sold in the market place.
- Informaion asymmetry: Seller of the good knows more information about the quality compared to Buyer.
- The sophistication of Buyer, who are aware of this information asymmetry, may cause efficient trade to break down.

Adverse Selection: An Example

	Fraction	Seller's Valuation	Buyer's Valuation
Good Car	50%	$80K$	$100K$
Lemon	50%	$40K$	$50K$

- Seller knows the true quality but Buyer does not.
- Fraction of lemons conditional on price p posed by Seller:

$$\begin{cases} 1 & \text{if } 40K \leq p < 80K \\ 0.5 & \text{if } p \geq 80K \end{cases}$$

- Buyer's valuation conditional on p:

$$\begin{cases} 50K & \text{if } 40K \leq p < 80K & \text{trade possible} \\ 75K & \text{if } p \geq 80K & \text{trade not possible} \end{cases}$$

- Only Lemon is sold at the market at $p = 50K$.

Adverse Selection: An Example

- More generally, if fraction of lemons is higher than 40%, then good used cars are not sold.
- That is, if the fraction of lemons is high enough, the bad cars will drive out the good ones, even when trade is efficient for both types of cars.

Adverse Selection: An Example

Question: How does this classical result change when consumers' understanding of the market equilibrium is less than perfect, in the sense of being "coarse"?

A Buyer-Seller Example

- A seller enters the market aiming to sell a single object.
- A buyer enters with the intent to buy the object.
- The seller's valuation of the object, denoted ω, is drawn uniformly from $\Omega = [0, 1]$ and represents the object's quality.
- The value of ω is the seller's private information.
- The buyer's valuation of the object is $v = \omega + b$, where $b \in (0, 1)$ is a parameter that measures the gains from trade.
- Since $b > 0$, trade is always efficient, independently of the state.

A Buyer-Seller Example

- The market mechanism is a simple two-sided auction.
- The buyer and seller submit a bid p and an ask price a, respectively.
- If $p \geq a$, trade takes place at the price p.
- If $p < a$, trade does not occur.
- It follows that the seller has a weakly dominant strategy: to submit an ask price equal to his valuation.
- From now on, we will assume that the seller follows this strategy (formally, $f(\omega) = \omega$) and focus on the buyer's considerations.

A Buyer-Seller Example

- The buyer's coarse reasoning is captured by a *perceptual partition* $\Pi = \{\Pi_1, ..., \Pi_K\}$ of Ω.
- The buyer's coarse representation of the seller's strategy f is a mixed strategy σ^{Π} defined as follows: for every $\omega \in [0, 1]$, σ^{Π} mixes uniformly over $\Pi(\omega)$.

A Benchmark: Bayesian-Rational Buyer

- Suppose that the buyer understands the market model and knows the seller's strategy: $\Pi(\omega) = \{\omega\}$ for all $\omega \in \Omega$.
- Then buyer chooses p to maximize

$$\underbrace{\Pr(\omega \leq p)}_{\text{trade prob.}} \cdot [\underbrace{\mathbb{E}(v|\omega \leq p)}_{\substack{\text{expected valuation} \\ \text{conditional on trade}}} - p]$$

- Since $v = \omega + b$ and ω is uniformly distributed over $[0, 1]$
  - $\Pr(\omega \leq p) = p$
  - $\mathbb{E}(v|\omega \leq p) = \frac{1}{2}p + b$
- Optimal p: $p_r^* = b$.
- Adverse Selection causes market failure: while the efficient outcome is to have trade in every state, in this market equilibrium trade occurs only when the seller's valuation is weakly below b.

"Fully Coarse" Buyer

- For a "fully coarse" buyer, $\Pi(\omega) = \Omega$ for all $\omega \in \Omega$.
- Given that the seller's strategy is $f(\omega) = \omega$, the buyer's coarse perception of f is that the seller uniformly randomizes over the set of ask prices $[0, 1]$, independently of ω.
- Then buyer chooses p to maximize

$$\underbrace{\Pr(f(\omega) \leq p)}_{\text{trade prob.}} \cdot [\underbrace{\mathbb{E}(v)}_{\substack{\text{expected}\\\text{valuation}}} - p] = p \cdot \left[\frac{1}{2} + b - p\right]$$

- Optimal p: $p_c^* = \frac{1}{2}(b + \frac{1}{2})$

Comparison between "Coarse" and Rational Buyer

- Recall $p_r^* = b$ and $p_c^* = \frac{1}{2}(b + \frac{1}{2})$
- Then $p_c^* > p_r^*$ iff $b < \frac{1}{2}$
- Thus, when the gains from trade are relatively small, the "fully coarse" buyer submits a higher bid and trades with higher probability than a Bayesian rational buyer.
- However, when the gains from trade are large, the comparison is reversed.
- Special case of $b = 0$:
  - Bayesian-rational benchmark implies no trade.
  - "Fully coarse" buyer trades with probability $\frac{1}{4}$.

Intuition

A fully coarse perception of the seller's strategy has two contradictory effects:

- On one hand, the coarse buyer's expected valuation of the object is higher than in the benchmark case because he fails to take adverse selection into account. This raises the buyer's bid relative to the benchmark.
- On the other hand, the buyer does not realize that if he raises his bid, this will enhance the expected quality of the traded object. This lowers the buyer's bid relative to the benchmark.
- When the gains from trade are small, the former consideration outweighs the latter.

“Partially Coarse” Buyer

- Next, we consider “Partially Coarse” Buyer.
- In these examples, we fix $b = 0$.
- Suppose that the buyer’s partition divides Ω into K equal invervals.

“Partially Coarse” Buyer

Proposition 4

Let $\Pi = \left\{ \left[0, \frac{1}{K}\right), \left[\frac{1}{K}, \frac{2}{K}\right), ..., \left[\frac{K-1}{K}, 1\right] \right\}$. *The buyer's optimal action given this partition is* $p^* = \frac{1}{4K}$.

Proof of Proposition 4

- For every $k = 1, ..., K$, denote $\pi_k = \left[\frac{k-1}{K}, \frac{k}{K}\right)$.
- Expected value conditional on the state being π_k:

$$\mathbb{E}(v|\pi_k) = \frac{1}{2}\left(\frac{k-1}{K} + \frac{k}{K}\right) = \frac{2k-1}{2K}$$

- The probability of trade conditional on the state being π_k:

$$\Pr[f(\omega \leq p)|\pi_k] = \begin{cases} 1 & \text{if } p \geq \frac{k}{K} \\ \frac{p-\frac{k-1}{K}}{\frac{1}{K}} = pK - (k-1) & \text{if } p \in \pi_k \\ 0 & \text{if } p < \frac{k-1}{K} \end{cases}$$

Proof of Proposition 4

- Buyer chooses p to maximize

$$U = \frac{1}{K} \sum_{k=1}^{K} \left\{ \Pr[f(\omega) \leq p | \pi_k] \cdot \left[\frac{2k-1}{2K} - p \right] \right\}.$$

- Suppose $p \in \pi_{l+1}$ for some interger $l > 0$, then

$$U = \frac{1}{K} \left[\underbrace{(pK - l)\left(\frac{2l+1}{2K} - p \right)}_{\text{partition } \pi_{l+1}} + \sum_{k=1}^{l} \left(\frac{2k-1}{2K} - p \right) \right]$$

$$= \frac{1}{2K^2} [-2p^2K^2 + pK(2l+1) - l(l+1))].$$

- $U < 0$ for $l = 1, 2, \ldots$.
- It follows that $l = 0$.

Proof of Proposition 4

- Then $U = p\left(\frac{1}{2K} - p\right)$
- Optimal p: $p^* = \frac{1}{4K}$.
- The probability of trade decreases with K and converges to zero as K tends to infinity.
- In this case, the amount of trade decreases monotonically with the buyer's level of sophistication.
- The following exercise shows that this effect is not general.

"Partially Coarse" Buyer

Exercise 8.3

Let $b = 0$ and $\Pi = \{[0, d), [d, 1]\}$. Show that if $d \in (0, \frac{1}{6})$, the buyer's optimal action given this partition is $p^* = \frac{1}{4}(1 + d)$, such that trade occurs if and only if $\omega \leq \frac{1}{4}(1 + d)$.

Recall,

- Bayesian-rational benchmark implies no trade.
- "Fully coarse" buyer trades with probability $\frac{1}{4}$.

Action-Dependent Feedback

- Suppose that the buyer has access to a very large "database" containing historical records of previous buyer-seller interactions.
- These records include all past sellers' ask prices and buyers' bid prices.
- However, the records of past buyers' valuations of the object are incomplete: they are available only for past periods in which trade actually took place.

Action-Dependent Feedback

- Under this scenario, the buyer can correctly evaluate $\Pr(\omega \leq p)$ because he has access to a large, unbiased sample of randomly generated ask prices.
- Let us examine a "steady state" of the learning process, in which the buyer's processors all follow a constant bid p^*.
- The object's expected value in the buyer's incomplete "database" is $\mathbb{E}(v|\omega \leq p^*)$.
- We assume that the buyer's extrapolation from this sample is *naive*: he uses $\mathbb{E}(v|\omega \leq p^*)$ to evaluate the object, independently of the bid p he considers.

Action-Dependent Feedback

- Buyer chooses p to maximize

$$\Pr(\omega \leq p) \cdot [\mathbb{E}(v|\omega \leq p^*) - p] = p\left[\frac{1}{2}p^* + b - p\right].$$

- Optimal $p = \frac{1}{4}p^* + \frac{1}{2}b$.
- Consistency with steady state: $p = p^* \implies p^* = \frac{2}{3}b < b = p_r^*$

Action-Dependent Feedback

- The conclusion is that the buyer's bounded rationality exacerbates the market failure caused by adverse selection.
- The reason is that of the two effects highlighted in the previous sub-section, only one remains: the buyer's failure to perceive that if he changes his bid, the expected quality of the traded object will change.
- The other effect – namely, the buyer's failure to perceive adverse selection – is effectively eliminated, because the buyer's evaluation of the object's expected quality conditional on trade is correct, as it is based only on feedback from those periods in which the seller found it optimal to trade.

Summary

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- When consumers differ in the degree of coarseness of their representation of a complex pricing strategy, firms may have an incentive to create complex price patterns as a discriminatory device.
- However, whether such obfuscation is profitable depends on the correlation between consumers' "preference type" and "cognitive type."

Summary

- When consumers have a coarse perception of pricing strategies in markets with asymmetric information, this gives rise to two opposing effects.
  - On one hand, consumers underestimate the negative relation between the firms' displayed willingness to sell and the quality of its product.
  - On the other hand, they underestimate the positive relation between product quality and their own displayed willingness to buy.
- This ambiguity means that the probability of Pareto-efficient trade is not monotone with respect to the magnitude of consumers' coarse reasoning, and it is not necessarily higher than in the rational-consumer benchmark.