Sampling-Based Reasoning I: Price Competition and Product Differentiation[1]

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[1] Chapter 6 of *Bounded Rationality and Industrial Organization* (Spiegler, 2011)

Introduction

Motivation

Consumers need to form beliefs about various events:

- prices offered by stores fluctuate;
- service qualities offered by experts vary;
- the value of insurance policies or financial products depends on assessments of future contingencies.

However, consumers are not usually sophisticated in drawing correct inferences from information.

Motivation

We consider a particular cognitive limitation: **Consumers examine carefully a small part of the environment and extrapolating naively from the sampled part.**

Psychological foundation for this limitation:

- People tend to reason anecdotally (concrete stories filled with vivid details), rather than probabilistically, about random variables.
- "the law of small numbers": people's tendency to exaggerate the informational content of a small sample.

Model

Model: Sampling-based choice procedure

Primitives:

- Set of outcomes: Z
- Consumer's preference relation over Z: $\succsim$
- Probability distribution over Z: $(F_1, ..., F_n)$

Model: Sampling-based choice procedure

A simple illustration of the primitives:

- 2 outcome $\{G, B\}$
- Consumer prefers G to B: $G \succsim B$
- 3 probability distributions:
  - F_1: probability $1/4$ of G, and probability $3/4$ of B;
  - F_2: probability $1/2$ of G, and probability $1/2$ of B;
  - F_3: probability $3/4$ of G, and probability $1/4$ of B.

A digression: EU

- In standard microeconomic theory, rational consumers with complete information adopt *Expected Utility (EU)* for decision making.
- In the previous example,

$$U(F_1) = 1/4U(G) + 3/4U(B)$$
$$U(F_2) = 1/2U(G) + 1/2U(B)$$
$$U(F_3) = 3/4U(G) + 1/4U(B)$$

- Then, the consumer's preference over lotteries F_1, F_2, F_3 are $F_3 \succsim F_2 \succsim F_1$. (This is because $G \succsim B$, i.e., $U(G) \geq U(B)$.)
- Therefore, he would choose F_3.
- And the outcome is a draw from F_3.

Model: Sampling-based choice procedure

Bounded-rational consumers use Sampling procedure:

- The consumer draws a *single* sample point from the joint distribution $(F_1, ..., F_n)$: $(z_1, ..., z_n)$.
- He chooses the alternative i^* for which z_{i^*} is $\succsim$-maximial in the sample (with a symmetric tie-breaking rule).
- The outcome is a new, independent draw from F_{i^*}.

Model: Sampling-based choice procedure

In our example,

- A possible sample point is (G, B, G)
- Since $G \succsim B$, and according to the tie-breaking rule, the alternative 1 or 3 will be drawn, each with $1/2$ probability. Suppose 1 is the alternative chosen.
- Then, the outcome will be a new, independent draw from F_1.

Another digression: conventional I.O. models with consumer search

- Consumer search is also based on sampling.
- However, at the end, he gets the sample, z_{i^*}.
- In our current model, the actual outcome is a new, independent drawn from F_{i^*}.

A few remarks

- The reliance on samples is not necessarily an aspect of intrinsic bounded rationality, it could also be lack of knowledge, for instance, lack of market experience.
  Thus, the distributions F_i are not known to the consumer.
- We only consider a single draw from the distribution. A natural generalization is to have multiple draws. When the number of draws $K \to \infty$, consumer behavior converges to the rational choice benchmark.

Price competition and technology adoption

Model Setup

Here, we provide a basic model of sampling-based reasoning.

- n identical firms and a continuum of identicial consumers
- Consumer's payoff is 1 if his need is satisfied. (0 if not satisfied)
- Marginal cost is 0 for all firms.
- Each firm's product satisfy customer's need with probability $\alpha \in (0, 1)$.
- Consumer's outside option ("doing nothing") satisfies the need with probability $\alpha_0 \in [0, 1)$.
- Let $p_i \in [0, 1]$ be the prices posted by firms; and $p_0 = 0$ be the price of outside option.

Sampling procedure

We reiterated the sampling procedure.

- Each consumer independently samples each of the $n + 1$ market alternatives (including outside option).
- A consumer selects the alternative i that maximizes $x_i - p_i$, where
  $x_i = \begin{cases} 1 \text{ if need is satisfied in the sample} \\ 0 \text{ otherwise} \end{cases}$
- Ties are resolved symmetrically between firms and in favor of the outside option.

Firm's profit

- The consumers' choice procedure thus induces a complete-information, simultaneous-move game played by the firms.
- Nash equilibrium is used as the solution concept.
- To illustrate firm's payoff function, suppose $p_n > p_{n-1} > ... > p_1 > p_0 = 0$.
- Then firm k's profit is

$$p_k \cdot \underbrace{\alpha}_{\text{Firm } k \text{ good}} \cdot \underbrace{(1-\alpha_0) \cdot (1-\alpha)^{k-1}}_{\text{cheaper alternatives all bad}}.$$

Nash Equilibrium

- We focus on Symmetric equilibrium.
- First, there is no pure strategy equilibrium.
  - Suppose all other firms are setting $p > 0$, then a firm could profit by undercutting p.
  - Suppose all other firms are setting $p = 0$, then a firm could deviate to $p = 1$ and get $\alpha \cdot (1 - \alpha_0) \cdot (1 - \alpha)^{n-1}$.
- Thus, we search for mixed strategy equilibrium.

Mixed-strategy Nash Equilibrium

Proposition 1

In symmetric Nash equilibrium, firms play the mixed strategy given by the cdf:

$$G(p) = \frac{1}{\alpha} - \frac{1-\alpha}{\alpha} \cdot p^{-1/(n-1)} \tag{1}$$

defined over the support $[(1-\alpha)^{n-1}, 1]$.

Mixed-strategy Nash Equilibrium

Sketch of the proof:

- Suppose all firms price according to the cdf $G(p)$ over $[p^L, p^H]$.
- Since the firm is willing to mix, it must be indifferent between choosing any price $p \in [p^L, p^H]$.
- The firm's profit when it chooses p is

$$\Pi = p \cdot \underbrace{\alpha}_{\substack{\text{Firm } k \\ \text{good}}} \cdot \underbrace{(1-\alpha_0)}_{\substack{\text{outside option} \\ \text{bad}}} \cdot \underbrace{\left[1-\alpha G(p)\right]^{n-1}}_{\substack{\text{A firm is not} \\ \text{both cheaper and good}}}$$

Mixed-strategy Nash Equilibrium

Sketch of the proof (continued):

- p^H must be equal to 1, otherwise, sticking to p^H gives

$$\Pi(p^H) = p^H \cdot \alpha \cdot (1 - \alpha_0) \cdot \underbrace{[1 - \alpha]}_{G(p^H)=1}^{n-1}$$

  and deviating to 1 gives $\Pi(1) = 1 \cdot \alpha \cdot (1 - \alpha_0) \cdot [1 - \alpha]^{n-1}$.
  $\Pi(1) > \Pi(p^H)$ since $1 > p^H$.
- Then,

$$\Pi = \Pi(1) = \alpha \cdot (1 - \alpha_0) \cdot [1 - \alpha]^{n-1}. \tag{2}$$

Mixed-strategy Nash Equilibrium

Sketch of the proof (continued):

- We could calculate p^L, utilizing $G(p^L) = 0$ and $\Pi(p^L) = \Pi$:

$$p^L \cdot \alpha \cdot (1 - \alpha_0) = \alpha \cdot (1 - \alpha_0) \cdot [1 - \alpha]^{n-1}$$
$$\Longrightarrow p_L = [1 - \alpha]^{n-1}.$$

- We are also able to calculate the whole distribution $G(p)$, utilizing $\Pi_(p) = \Pi$:

$$p \cdot \alpha \cdot (1 - \alpha_0) \, [1 - \alpha G(p)]^{n-1} = \alpha \cdot (1 - \alpha_0) \cdot [1 - \alpha]^{n-1}$$
$$\Longrightarrow G(p) = \frac{1}{\alpha} - \frac{1-\alpha}{\alpha} \cdot p^{-1/(n-1)}.$$

Mixed-strategy Nash Equilibrium

Given $G(p)$, we could calculate the expected equilibrium price

$$
\begin{aligned}
E(p) &= \int_{(1-\alpha)^{n-1}}^{1} p \cdot G'(p)\mathrm{d}p \\
&= \begin{cases} -\frac{1-\alpha}{\alpha}\ln(1-\alpha) & \text{for } n=2 \\ \frac{1-\alpha}{\alpha(n-2)}\left[1-(1-\alpha)^{n-2}\right] & \text{for } n \geq 2. \end{cases}
\end{aligned}
$$

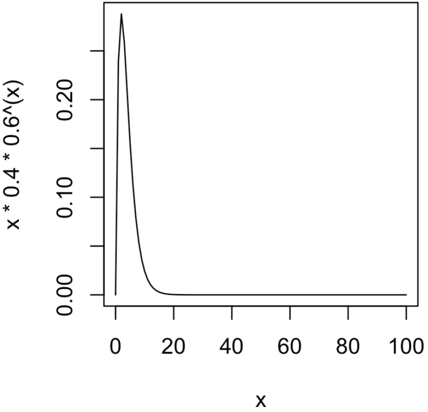
- Note that $E(p)$ is strictly decreasing with α.
- $E(p) \to 0$ as $\alpha \to 1$; and $E(p) \to 1$ as $\alpha \to 0$.
- Intuition: When α is lower, a consumer's sample is less likely to contain multiple successes. This weakens competitive pressures and causes prices to go up.

Market for Quacks

- The market exists independent of the relative magnitude of α and α_0.
- When $\alpha_0 \geq \alpha$, the firms charge a positive price for a product that that has no value relative to the outside option.
- That is, we could have an active "market for quacks."
- The reason for such market is that consumers' anecdotal reasoning causes them to attribute to skill a good outcome that is due to sheer luck.

Welfare Analysis: $\alpha = \alpha_0$

- When $\alpha = \alpha_0$, it is a "market for quacks".
- The industry profit $n\Pi = n\alpha(1-\alpha)^n$ is thus a measure of welfare loss that the industry inflicts on consumers.
- Note that the welfare loss is hump-shaped with respect to n.
- Figure below is an illustration with $\alpha = 0.4, n = x$



Welfare Analysis: $\alpha = \alpha_0$

Intuition

- standard "competitive" effect: $n \uparrow \implies$ incentive to cut prices.
- "exploitative" effect: $n \uparrow \implies$ demand for the industry $\uparrow$ ($\because$ a higher chance of a good anecdote about some product)
- Fixing α, "competitive effect" outweighs "exploitative effect" when n is sufficiently large (but the critical value of n increases as α decreases)
- Welfare loss vanishes completely as $n \to \infty$.

Welfare Analysis: $\alpha > \alpha_0$

- When consumers are rational, their expected utility is α.
- "Our" consumers' equilibrium expected utility is

$$U = \underbrace{\alpha \cdot A + \alpha_0 \cdot (1 - A)}_{\text{benefit from satisfaction of need}} - \underbrace{n\alpha(1-\alpha_0)(1-\alpha)^{n-1}}_{\text{firms' profits}}$$

where

$$\underbrace{A}_{\substack{\text{Prob. of}\\ \text{choosing firm}}} = \underbrace{1 - \alpha_0}_{\substack{\text{outside option}\\ \text{bad}}} \cdot \underbrace{[1 - (1-\alpha)^n]}_{\geq 1 \text{ firm good}}$$

- "Our" consumers' welfare is lower: $U < \alpha$
- If consumer does not enter the market, the utility is α_0.
- If α_0 and α are sufficiently close to 0, consumer welfare can fall below α_0. ("market exploitation")

Welfare Analysis: Competition policy

- We already saw that the most conventional competition policy, $n \uparrow$, may have an adverse impact on consumer welfare, because of "exploitative" effect.
- Another competition policy is to introduce a high-quality competitor into the market.
- When consumers follow the sampling-based procedure, merely adding a single high-quality competitor does not eliminate the problem of "active quacks." (See Exercise 6.1)
- Conclusion: market interventions that are viewed as proper competition policies in a world with rational consumers are ineffective when consumers behave according to the sampling-based procedure.

Welfare Analysis: Exercise 6.1

Exercise 6.1

Let $n = 2$. Modify the model by letting the success rate of firm 2 be $\alpha_2 > \alpha = \alpha_1 = \alpha_0$.

1. Derive the firms' Nash equilibrium profits. Show that firm 1's profit is independent of α_2. (Hint: When the two firms have different success rates, their equilibrium pricing strategies have the same support, but firm 2's cdf has an atom on $p = 1$.)
2. Suppose that in a stage prior to the price competition game, firms choose their success rates simultaneously and at no cost. Which profiles of success rates are consistent with sub-game perfect equilibrium in this two-stage game?

Spurious Product Differentiation

Spurious Product Differentiation

- This is a vairation of the basic model.
- *spurious product differentiation*: firms have an incentive to create an *impression* of a differentiated product in order to weaken competitive pressures.

Model Setup

- n identical firms and a continuum of identicial consumers
- Let A be a finite set of actions that firms may recommend to consumers.
- $|A| = m$ and $m > 1$
- Each action $a \in A$ satisfies Consumer's need with probability $\frac{1}{m}$.
- The actions' successes are mutually exclusive.
- Consumer's payoff is 1 if his need is satisfied. (0 if not satisfied)
- Firm $i \in \{1, ..., n\}$ chooses a pair $(p_i \in [0, 1], a_i)$
  - p_i is the price;
  - a_i is the action recommendation. (success rate $\frac{1}{m}$ regardless of action)
- Outside option has known value 0.[2]

[2]The analysis below is easily extendible to the case in which the outside option is (p_0, a_0), where $p_0 = 0$ and a_0 is drawn from the uniform distribution over A. According to the forecasting interpretation, the outside option corresponds to a "lay prediction."

Sampling procedure

- Each consumer recalls a random past episode in which firms made predictions, and isolates those firms whose predictions proved correct in that episode.
  $$x_i = \begin{cases} 1 \text{ if recommendation is correct} \\ 0 \text{ otherwise} \end{cases}$$
- If none of the firms was successful, the consumer sticks to the outside option $x_0 = p_0 = 0$.
- Among those firms with correct recommendation, the consumer selects the firm that charges the lowest price.
- Effectively, consumer maximizes $x_i - p_i$ in the sample.
- Assume symmetric tie-breaking rule.

Properties of the model

- Product differentiation is **spurious**: the firms' recommendations do not matter for consumer welfare.
- However, from firms' point of view, it is benefitial to differentiate: differentiation induces a higher chance of being chosen.
  - If no differentiation, the most expensive firm is never chosen;
  - The most expensive firm could benefit from differentiating, securing a probability of $\frac{1}{m}$ of being chosen.

Pure-strategy Nash Equilibrium: $n \leq m$

When $n \leq m$ (No. of firms $\leq$ No. of actions):

- there is a pure-strategy Nash equilibrium in which each firm recommends a **distinct action** and charges **monopoly price** $p = 1$.
- Intuition:
  - market share: the highest market share of a firm is $\frac{1}{m}$, and in this equilibrium, firms each have a market share of $\frac{1}{m}$.
  - price: each firm is charging a monopoly price.
  - Thus, no profitable deviation.
- Industry profit: $\frac{n}{m}$

Pure-strategy Nash Equilibrium: $n > m$

When $2m > n > m$, no pure-strategy equilibrium exists.

- At least one action is recommended by only 1 firm or not recommended at all. Denote one such action a_1.
- At least one action is recommended by two or more firms. Denote one such action a_2.
- Price competition in recommendation of a_2 drives the price down to 0. And firms recommending a_2 earns 0 profit.
- Suppose a_1 is not recommended at all, the firm recommending a_2 could unilaterally deviate to recommending a_1 and charging $p = 1$, earning positive profit.
- Suppose a_1 is recommended by only 1 firm, the firm must charge $p = 1$ and earn positive profit. Then, there is still a profitable deviation for the firm recommending a_2. (recommending a_1 and undercutting $p = 1$.)

Pure-strategy Nash Equilibrium: $n > m$

When $n \geq 2m$, a pure-strategy equilibrium exists: for each a, there are at least two firms recommend a and charge $p = 0$.

- On the equilibrium path, all firms earn 0 profit.
- If a firm deviates to a higher price, it will not gain any market share and the profit remains at 0.
- If a firm deviates to another action, since the current price for that action is also 0, the firm could not earn a positive profit.

Mixed-strategy Nash Equilibrium: $n \geq 2$

Proposition 2

There exists a symmetric mixed-strategy equilibrium, each firm recommends each action with probability $\frac{1}{m}$, and randomizes independently over prices according to the cdf :

$$G(p) = m - (m-1) \cdot p^{-1/(n-1)}.$$

defined over the support $[(1 - \frac{1}{m})^{n-1}, 1]$.

Mixed-strategy Nash Equilibrium: $n \geq 2$

To see why this construction constitutes an equilibrium:

- Firms' recommendation (each action with probability $\frac{1}{m}$):
  - Firm would not deviate in recommendation: its opponents mix over A uniformly and independently of their price.
- Firms' pricing strategy (cdf $G(p)$):
  - For $p \in [(1-\frac{1}{m})^{n-1}, 1]$, firm's profit is

$$\Pi = p \cdot \underbrace{\frac{1}{m}}_{\substack{\text{prob. of}\\\text{good}}} \cdot \underbrace{\left[1 - \frac{1}{m}G(p)\right]^{n-1}}_{\substack{\text{A firm is not}\\\text{both cheaper and good}}}.$$

  - Firm's profit is constant: $\Pi = \frac{1}{m}\left[1-\frac{1}{m}\right]^{n-1}$, given the cdf $G(p)$.
  - The equilibrium pricing strategy is exactly the same as in the basic model, except that $\frac{1}{m}$ replaces α.

Mixed-strategy Nash Equilibrium: $n \geq 2$

Product differentiation:

- Product differentiation ex-post: the probability that all firms make the same recommendation is $\left(\frac{1}{m}\right)^{n-1}$
- No product differentiation ex-ante: all firms play the same mixed strategy.

Mixed-strategy Nash Equilibrium: Comparative statics

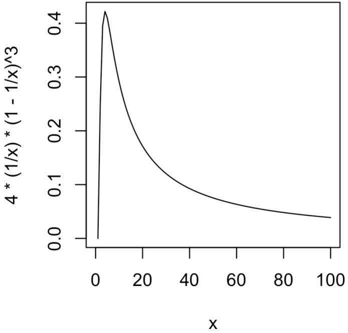
Given $G(p)$, we could calculate the expected equilibrium price

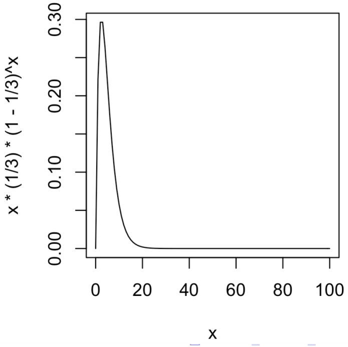
$$E(p) = \begin{cases} -(m-1)\ln(1-\frac{1}{m}) & \text{for } n = 2 \\ \frac{m-1}{n-2}\left[1-(1-\frac{1}{m})^{n-2}\right] & \text{for } n \geq 2. \end{cases}$$

- $m \uparrow \implies E(p) \uparrow$ and eventually $E(p) \to 1$ as $m \to \infty$.
- Intuition: When m is higher, it is more likely that fewer firms would recommend the same action. This weakens competitive pressures and causes prices to go up.

Mixed-strategy Nash Equilibrium: Comparative statics

- Firm profit: $\Pi = \frac{1}{m} \cdot \left(1 - \frac{1}{m}\right)^{n-1}$
  - Same as Equation (2), with $\alpha = \frac{1}{m}$ and $\alpha_0 = 0$
- Industrial profit: $n\Pi = n \cdot \frac{1}{m} \cdot \left(1 - \frac{1}{m}\right)^{n-1}$
- $n\Pi$ behave non-monotonically in m and n.
  - Figure on the left: $n = 4, m = x$
  - Figure on the right: $m = 3, n = x$





Asymmetric Mixed-strategy Equilibria: Exercise 6.2

Exercise 6.2

Construct an asymmetric mixed-strategy Nash equilibrium when $n > m$ and both n and m are even.

Heterogeneous Success Rate

Exercise 6.3

Let $A = \{a_1, a_2\}$. Assume that with probability α (or $(1-\alpha)$) the action a_1 (or a_2) alone satisfies the consumer's need. Assume $\alpha > \frac{1}{2}$. Consider a symmetric Nash equilibrium in which firms randomize over prices and recommendations independently. What is the equilibrium probability that the sub-optimal action a_2 is recommended? What happens to this probability as n tends to infinity?

Takeaways:

- Some firms make inferior recommendations as a differentiation strategy.
- This is an effect reminiscent of the "favorite long-shot bias" observed in gambling markets: the market's tendency to over-bet on low-probability outcomes.

Product Complexity as a Differentiation Device

In this subsection, we consider **case-specific recommendations**.

The following modifications are made to the previous model.

- A set of *cases* C.
- Let $t : C \to A$ be a case-specific recommendation (CSR).
- Each firm offers (p_i, t_i).
- We say that Firm i makes an exclusive recommendation in case c if $t_j(c) \neq t_i(c)$ for all $j \neq i$.
- A state is (c, a), uniformly distributed over $C \times A$.

Sampling procedure

Each consumer independently samples a state and chooses the best-performing alternative in that state.

Case-specific recommendations as pure complexity

Since all actions are equally likely to satisfy the consumers' need in each case, case-specific recommendations involve a complexity, which is redundant in terms of consumer welfare.

Firm's profit

- Strategy profile: $(p_i, t_i)_{i=1,\dots,n}$
- To illustrate firm's payoff function, suppose $p_n > p_{n-1} > \dots > p_1$.
- Define $z_k(c) = \begin{cases} 1 & \text{if } t_j(c) \neq t_k(c) \text{ for all firms } j < k \\ 0 & \text{otherwise} \end{cases}$
- Then firm k's payoff is

$$\sum_{c \in C} \frac{p_k}{\underbrace{m}_{\substack{\text{prob. of}\\ \text{good}}} \cdot \underbrace{|C|}_{\substack{\text{prob. of}\\ \text{sampled}}}} \underbrace{z_k(c)}_{\substack{\text{diff. from}\\ \text{cheaper}}} = \frac{p_k}{m \cdot |C|} \sum_{c \in C} z_k(c)$$

Degenerate CSR

Consider degenerate CSR:

- Each firm i chooses a_i such that $t_i(c) = a_i$ for all c.
- Then, the present model is immediately reduced to the simpler model of the previous sub-section.
- Therefore, all pure strategy equilibria identified in the previous sub-section survive the current extension.

Case-specific Recommendations: *Hybrid* Equilibria

When $2m > n > m$, the extended model gives rise to new equilibria, referred to as *Hybrid* Equilibria: each firm's strategy consists of a **pure, non-degenerate CSR** and a **mixed pricing strategy**.

Case-specific Recommendations: *Hybrid* Equilibria

Let $C = [0,1]$, the constructed equilibria have the following structure:

- For each firm, the fraction of cases in which it makes an exclusive recommendation is $\mu = \frac{2m-n}{n}$.
- In each case, $2m - n$ actions are recommended by one firm each, and the remaining $n - m$ actions are recommended by two firms each.
- All firms play the pricing strategy given by the cdf

$$G(p) = \frac{p - \mu}{p - p\mu} \tag{3}$$

over the support $[\mu, 1]$.

Case-specific Recommendations: *Hybrid* Equilibria

More specifically, the construction could be as follows:

- Partition C into $\binom{n}{2m-n}$ equal intervals.
- Associate a distinct subset of $2m-n$ firms with each interval, and assume that these firms make exclusive recommendations in all cases that belong to the interval.

Case-specific Recommendations: *Hybrid* Equilibria

For example, $n = 3, m = 2$

- Partition C into $\binom{3}{1} = 3$ equal intervals.
- Associate a distinct subset of 1 firm with each interval, and assume that these firms make exclusive recommendations in all cases that belong to the interval.

	$c \in [0, 1/3]$	$c \in (1/3, 2/3]$	$c \in (2/3, 1]$
a_1	Firm 1	Firm 2	Firm 3
a_2	Firm 2 and Firm 3	Firm 1 and Firm 3	Firm 1 and Firm 2

Case-specific Recommendations: *Hybrid* Equilibria

To see why this construction constitutes an equilibrium:

- Firms' recommendation:
  - Firm would not deviate in recommendation: Prior to the deviation, the firm shares a recommendation with at most one other firm. After the deviation, it shares a recommendation with one or two other firms.
- Firms' pricing strategy (cdf $G(p)$):
  - For $p \in [\mu, 1]$, firm's profit is

$$p \cdot \underbrace{\frac{1}{m}}_{\substack{\text{prob. of} \\ \text{good}}} \cdot \Bigg[\underbrace{\mu}_{\substack{\text{firm } k \\ \text{exclusive}}} + \underbrace{(1-\mu)\,(1-G(p))}_{\substack{\text{firm } k \\ \text{not exclusive but cheaper}}} \Bigg].$$

  - Firm's profit is constant: $\Pi = \frac{\mu}{m} = \frac{2m-n}{nm}$, given the cdf $G(p)$.

Case-specific Recommendations: *Hybrid* Equilibria

Industrial profit:

- *Hybrid* Equilibria: $n\Pi = \frac{2m-n}{m}$
- Symmetric mixed strategy Equilibrium: $n\Pi = n \cdot \frac{1}{m} \cdot \left(1 - \frac{1}{m}\right)^{n-1}$
- When n is relatively close to m, ***Hybrid* Equilibria generates higher industry profit.**
- Intuition: when n is close to m, the multiple cases function as "sunspots" that allow firms to coordinate their recommendations in a way that increases the probability that each firm makes an exclusive recommendation.

Can the Market Educate Consumers?

Can the Market Educate Consumers?

- From the examples in the previous sections, we know that consumers who follow sampling-based reasoning make a systematic inference error.
- In this section, we examine whether market forces alone can provide firms with an incentive to "de-bias" the consumers.

Model

- Reconsider the basic model with α.
- Relax the assumption that all firms have the same success rate, and allow $\alpha_i \in (0,1)$ for each market alternative $i = 0, 1, ..., n$.
- Assume that a firm could **credibly** disclose its success rate.
- If a firm discloses α_i and charges p_i, all consumers evaluate this market alternative at $\alpha_i - p_i$.
- If a firm does not disclose, **consumers infer nothing from the lack of disclosure**, and rely on the sampling-based procedure to evaluate the firm.

Firm's strategies

- A strategy for firm i is a pair (p_i, r_i), where $r_i = Y(N)$, indicating that the firm discloses (does not disclose) α_i.
- x_i denotes the consumer's evaluation.
- When $r_i = Y$, $x_i = \alpha_i$ with probability 1;
- When $r_i = N$, $x_i = \begin{cases} 1 & \text{with probability } \alpha_i \\ 0 & \text{with probability } 1 - \alpha_i \end{cases}$
- Consumer chooses the alternative that maximizes $x_i - p_i$ in the sample.

No disclosure

Proposition 3

For every p, the strategy (p, Y) for firm i is weakly dominated by some other strategy (p', N).

No disclosure

Suppose Firm i plays (p, Y).

Case I: suppose $p \geq \alpha_i$

- If Firm i sticks to equilibrium, $x_i - p \leq 0$ with probability 1 and no consumer would pick i. Firm i always earn 0 profit.
- If Firm i deviates to (p', N), where $p' \in (0, 1)$. Then $x_i - p' > 0$ with probability $\alpha_i > 0$. Therefore, Firm i earns strictly positive profits for some strategy profiles of its opponents.

No disclosure

Case II: suppose $p < \alpha_i$

- If Firm i sticks to equilibrium, its profit is bounded from above[3] by

$$\overline{\Pi}_Y = p \cdot \Pi_{j \neq i} \Pr(x_j - p_j \leq \alpha_i - p).$$

- If Firm i deviates to (p', N), its profit is bounded from below[2] by

$$\underline{\Pi}_N = p' \cdot \alpha_i \cdot \Pi_{j \neq i} \Pr(x_j - p_j < 1 - p').$$

- Let $p' = \frac{p}{\alpha_i}$. Then $\alpha_i - p < 1 - p'$. Thus,

$$\underline{\Pi}_N \geq \overline{\Pi}_Y.$$

- The inequality is strict if $p_j \in (p', 1 - \alpha_i + p)$ for some $j \neq i$.
  - $\alpha_i - p < 1 - p_j < 1 - p'$

[3] Because of possibility of ties

No disclosure

The lesson from this result is that “market education” will not take place when

1. (i) consumers use the sampling-based procedure to evaluate market alternative (probabilistically naive);
2. (ii) and in addition they do not infer anything from lack of disclosure (strategically naive).

Summary

Summary

- Consumers' tendency to over-infer from anecdotal evidence about firms' quality may result in a thriving market for a product of little intrinsic value.
- In a price competition model, equilibrium prices rise as the success rate that characterizes the industry falls.
- Consumers' inference error could contribute to spurious price discrimination.
- Case-specific recommendations coordinate the firms, and further hurt consumers.
- When consumers are strategically naive as well, firms will not disclose their quality in market equilibrium.