Dynamic Optimization

Assignment 1

Due date: March 23, 2020 (Before class)

Submission method: Please submit your assignment to your TA Zeng, Zixuan via E-mail: 2251681954@qq.com and cc me: sherryecon@qq.com.

Other requirements:

1. Your assignment should be in **pdf** format.

2. The title of your submission email should be "Optimization Assignment 1 - your student ID - your name", for example, "Optimization Assignment 1 - 201901010101 - Zhang, San"

There are 4 questions in total. Please present your work in a neat and organized manner.

Question 1: Exercise 2.3

Part I Consider a producer who rents machines K at r per year and hires labor L at wage w per year to produce output Q, where $Q = \sqrt{K} + \sqrt{L}$. Suppose he wishes to produce a fixed quantity Q at minimum cost.

a. Formally state the problem and find the optimal amount of K^* and L^*.

b. Interpret the Lagrange multiplier.

Part II Now let p denote the price of output. Suppose the producer can vary the quantity of output, and seeks to maximize profit. Factor prices and the production function are the same as in Part I.

a. Formally state the problem and find the optimal output Q^*.

b. Relate this to your interpretation of the Lagrange multiplier.

Question 2: Exercise 3.2

There is a fixed total Y of goods at the disposal of society. There are two consumers who envy each other. if consumer 1 gets $Y_1 \geq 0$ and consumer 2 gets $Y_2 \geq 0$, their utilties are

$$U_1 = Y_1 - kY_2^2, \quad U_2 = Y_2 - kY_1^2,$$

where k is a positive constant. The allocation must satisfy $Y_1 + Y_2 \leq Y$, and maximize $U_1 + U_2$.

a. Formally state the maximization problem and write down the first order necessary conditions.

b. Show that if $Y > 1/k$, the resource constraint will be slack at the optimum.

Question 3

Stafford spends his weekly income $I > 0$ on hot dogs (x) and cigarettes (y).

Stafford's preferences are represented by the following utility function:

$$u(x, y) = x^{\alpha} y^{1-\alpha}$$

where $x \geq 0$, $y \geq 0$ and $\alpha \in (0, 1)$. The price per hot dog ($p > 0$) and price per cigarettes ($q > 0$) are fixed. The store where he buys cigarettes has a policy of selling no more than $M > 0$ units[1] of cigarettes per week to a customer, and this store is the only store in the town.

a. Formally state Stafford's utility maximization problem and derive Stafford's optimal consumption bundle (x, y).

b. Assume that Stafford' weekly income I is such that the store's policy is a binding constraint. How much in utility terms is Stafford willing to pay to (marginally) increase the cigarette quota (M)?

[1] M is a given parameter.

Question 4: Exercise 4.3

Consider a consumer planning his consumption over two years. He will have income I_1 during the first year and I_2 during the second. In each year, there are two goods to consume. In year 1, the prices are p_1 and q_1, and the corresponding quantities x_1 and y_1. In year 2, we similarly have p_2, q_2 and x_2, y_2. The utility function is

$$u_1 = \alpha_1 \ln(x_1) + \beta_1 \ln(y_1) + \alpha_2 \ln(x_2) + \beta_2 \ln(y_2).$$

This is to be maximized subject to two budget constraints, one for each year.

a. Solve the problem, and find the multipliers λ_1 and λ_2 for the two constraints.

b. How much more of year-2 income will the consumer require if he is to give up $\mathrm{d}I_1$ of year-1 income? In other words, what is the rate of return needed to induce him to save a little?

c. You would expect borrowing and lending institutions arise in an economy populated by such consumers. What governs who will borrow and who will lend?