

Using Customer Service to Build Clients' Trust^{*}

Yuk-fai Fong[†], Xiaoxiao Hu[‡], Ting Liu[§] and Xiaoxuan Meng[¶]

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Abstract

It is well known in the credence-good literature that in an expert-client relationship, under the *Liability* assumptions, clients have to reject the expert's serious-treatment recommendations with a positive probability to ensure that the expert honestly recommends treatments. Inefficiency arises because some socially efficient treatments are not provided. We show that the expert can enhance clients' trust, or acceptance rate of the serious treatment, by providing intrinsically socially inefficient customer service upon recommending the serious treatment. Enhanced clients' trust leads to higher efficiency and higher profit for the expert. However, trust cannot be enhanced by providing customer service with different timing.

Key Words: Expert, Credence Good, Customer Service, Customers' Trust

JEL Codes: D4, D8

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[†]Hong Kong University of Science & Technology

[‡]Hong Kong University of Science & Technology

[§]Stony Brook University

[¶]Lingnan University of Hong Kong

1 Introduction

Customers' trust is a primary concern to the firm.¹ Gaining customers' trust is particularly important for sellers of credence goods because the benefits of their products are difficult or impossible to ascertain even after the products are consumed. The notion of credence goods was introduced by Darby and Karni (1973). Services provided by experts such as doctors and car mechanics who both diagnose and treat their clients' problems are commonly cited examples of credence goods exactly because even after the expert's service is provided, the client may not know whether the service is appropriate or necessary. It is well known in the literature of credence goods that in an expert-client relationship, the expert will overcharge the client if the client fully trusts him and always accepts his recommended treatment. In response, the client will reject the serious treatment recommendation with a positive probability to ensure that the expert honestly recommends the appropriate treatment for the client's problem. The client's rejection of the expert's service is an indicator of her distrust of the expert.

As an example of low customer trust in credence goods market, a survey conducted by the American Automobile Association (AAA) in 2016 suggested that two-thirds of drivers don't trust auto repairers in general.² The top two reasons for drivers' lack of trust are auto repairers' behavior of "recommending unnecessary services" (76 percent) and "overcharging for services" (73 percent).

Although less known to economists, customer service is considered by services-marketing practitioners and scholars as an important factor of customers' trust of a company. An article in Forbes recognized customer service as the first among three cited ways to build customers' trust, stating that "[e]arning a customer's trust starts with giving great service."³ According to an article at Fivestars.com, "[o]ne of the best ways to gain trust is to offer your customers service they can't find anywhere else."⁴ On how to boost dental patients' treatment compliance, an article on dentaltown.com similarly advised dentists to "[g]ive [patients] customer service that rivals what they would get at Nordstrom."⁵ A study of the

¹For an excellent explanation for the importance of reputation and trust to firms, please see Bar-Isaac and Tadelis (2008).

²See "Most U.S. Drivers Leery of Auto Repair Shops," AAA (2016), available at <http://newsroom.aaa.com/2016/12/u-s-drivers-leery-auto-repair-shops/>.

³Please see "Three Ways To Build Customer Trust," Forbes (2014), available at <https://www.forbes.com/sites/yec/2014/04/22/three-ways-to-build-customer-trust/#4a5e701922b8>.

⁴See "4 Ways to Build Trust with Your Customers," Fivestars.com (2015), available at <https://blog.fivestars.com/4-ways-to-build-trust-with-your-customers/>.

⁵See Geier J. (2016), "Seven Easy Ways to Boost Treatment Compliance," dentaltown.com, available at <http://www.dentaltown.com/magazine/articles/5746/seven-easy-ways-to-boost-treatment-compliance>.

Institute of Customer Service identified customer service as “central to the issue of trust.”⁶ Dasu and Chase (2013) also considered building of trust as a vital channel through which customer service drives companies’ sales.

Our paper is inspired by this well recognized link between customer service and customer trust. It is also motivated by the fact that in the existing literature, this link is mostly established based on behavioral and psychological arguments which are often descriptive in nature. As economists, we set out to derive this link in a game theoretical setting where players have standard payoffs.

We build upon Fong (2005) which belongs to a class of models of expert services which assumes that the expert is required to fix the client’s problem once the treatment he recommends is accepted by the client, but the type of goods or services provided is unobservable or non-verifiable. This combination of assumptions is termed *Liability* by Dulleck and Kerschbamer (2006). Other papers adopting the *Liability* assumptions include Pitchik and Schotter (1987), Wolinsky (1993), and Liu (2011).

Under the *Liability* assumptions, it is a common equilibrium property that to prevent the expert from overcharging the client, the client has to reject his recommendation for the serious treatment with a high enough probability. The rejection of the serious treatment leads to inefficiency, either in the form of under-treatment of the serious problem (in a monopolistic setting) or costly search of second opinions (in a competitive setting). See Pitchik and Schotter (1987), Wolinsky (1993), and Fong (2005) for examples of such efficiency loss.

In this paper, we define customer service as an action taken by the expert which is costly to him, beneficial to the client, and that the expert does not charge the client for. Based on this definition, we show that by providing customer service at the right time, the expert can improve the client’s equilibrium level of trust, measured by the acceptance rate of the serious treatment. Interestingly, for customer service to be effective in promoting the client’s trust of the expert, it has to be offered when the serious treatment is recommended, and it is provided irrespective of the client’s acceptance or rejection of the expert’s recommended treatment. Offering customer service at any other moment during the client’s visit would not help.

By tying provision of customer service with recommendation of the serious treatment, the expert reduces the attractiveness of overcharging the client when she has a minor problem. This in turn allows the client to be more trusting of the expert, i.e., the client can accept the serious treatment with a

⁶See "Can Customer Service Create Trust?," The Institute of Customer Service (2014), available at <https://www.instituteofcustomerservice.com/research-insight/research-library/can-customer-service-create-trust>.

higher probability, without compromising the expert's incentive to honestly recommend the appropriate treatment. The increased acceptance rate of the serious treatment improves both the efficiency of the expert's service and translates into a higher profit for the expert. However, provision of customer service may be costly to the expert-client relationship if customer service is intrinsically socially inefficient. The expert needs to trade off between the efficiency gain from increased acceptance rate and the cost of providing socially inefficient customer service. We show that it is optimal for the expert to provide customer service as long as customer service is not too socially inefficient intrinsically.

It has been shown that the prices the expert posts for the minor and serious treatments can influence the honesty of the expert's recommendation strategy. For example, Fong (2005) showed that the expert can make himself report honestly by setting the price of the serious treatment equal to the client's reservation value for getting her serious problem fixed. Our paper goes beyond Fong's (2005) analysis by considering how provision of customer service can raise the client's acceptance rate of the serious treatment while maintaining the expert's honest reporting. As a result, the expert in our setting achieves a profit level not attainable in Fong's (2005) setting where customer service is not considered.

Sinking costly customer service to improve profit is reminiscent of forward induction in money burning games, but the mechanisms are rather different. Forward induction is an equilibrium selection criterion which does not result in actual money burning on the equilibrium path, whereas in our setting, customer service gives rise to new equilibria which do not exist in its absence and is provided on the equilibrium path. Our paper is more closely related to Austen-Smith and Banks (2000) and Kartik (2007) which showed that allowing the sender in a cheap-talk game to burn money when he sends messages can give rise to more revealing equilibria than those characterized by Crawford and Sobel (1982). There are important differences in terms of our focuses and main findings. The focus of their contributions is on how money burning gives rise to more informative equilibria, whereas in our setting, customer service does not lead to more revealing equilibria as the expert is fully revealing in equilibrium whether he provides customer service or not. Our focus is instead on how provision of customer service allows the expert to improve both efficiency and profit. Such improvements are possible only because, unlike money burning, customer service is not purely dissipative and that transfer payments are possible in our setting.

With somewhat less related mechanisms, Biglaiser (1993) and Inderst and Ottaviani (2013) also study sellers' strategy of promoting buyers' trust, respectively, by way of selling through a middleman

and by way of implementing a generous return policy.

2 Model

This model builds upon Fong (2005). We generalize it to allow the expert to charge an upfront diagnostic fee and to provide costly *customer service*. We define provision of customer service as *an action taken by the expert which is costly to him, beneficial to the client, and free of charge to the client*.

There is a continuum of clients with measure 1. Each client has a problem that may be either minor (m) or serious (s). If problem $i \in \{m, s\}$ is left untreated, a client (henceforth she) has to bear a loss of l_i , where $0 < l_m < l_s$. It is common knowledge that with probability α the client's problem is serious. There is an expert monopolist (henceforth he) who provides diagnosis and treatment services.⁷ Diagnosis technology is costless. Each problem requires a specific treatment to repair. These treatments are not substitutable. Following Fong, Liu, and Wright (2014), we call the treatments for the minor problem and the serious problem, respectively, the minor treatment and the serious treatment, and denote their repair costs by $r_m \in (0, l_m)$ and $r_s \in (0, l_s)$.

We assume that the existence of clients' problems is verifiable; however, clients do not know which of the two treatments has been actually provided as long as the problem is repaired. In other words, when the problem is actually i but the expert has recommended treatment j , $j \neq i$, and the client has accepted his treatment recommendation, the expert must provide the necessary repair at repair cost r_i .⁸ Once the problem is fixed, the client would never know that her problem was actually i and the benefit of the treatment is actually l_i . This explains why the expert's service is a credence good.

For ease of exposition, for the time being, we only consider customer service which is provided at the same time (or equivalently right before) the expert recommends the serious treatment. We also assume that the quality of customer service is verifiable. In Section 4.2, we will endogenize the timing of customer service by formally arguing that it is not profitable for the expert to provide customer service with different timing. We will also show that this result remains true whether the quality of customer service is verifiable or not.

The next two paragraphs describe the timing of the game. At the beginning of the game, the expert

⁷Generalization of our main insights to a competitive setting is discussed in the Conclusion section.

⁸For the expert's liability to repair the client's problem, this combination of assumptions is termed *Liability* by Dulleck and Kerschbamer (2006).

chooses and announces (d, p_m, p_s, c) , where d is a diagnostic fee, p_m and p_s are prices for the minor and serious treatments, and $c \geq 0$ is the quality of *customer service* to be provided to clients if he recommends the serious treatment.⁹ Let c also be the cost of customer service per client. In other words, we use the cost spent on customer service as the measure of the quality of customer service. Clients have unit demand for customer service. The benefit of customer service to a client is $b = \theta c$ with the intrinsic efficiency of customer service measured by $\theta \in [0, 1]$. In other words, except for $\theta = 1$, provision of customer service is intrinsically social inefficient.¹⁰

Upon the observation of the fees and the quality of customer service announced by the expert, clients decide whether to visit him. If a client visits the expert, she must pay d . Following Fong (2005), we call the rest of the game from the moment the client has paid the diagnostic fee the *recommendation subgame*. The equilibrium of the recommendation subgame is determined by (p_m, p_s, c) , but is unaffected by the sunk payment d . Upon seeing the client, the expert first finds out whether her problem is minor or serious. Then he recommends a minor treatment at the price p_m , recommends a serious treatment at the price p_s , or refuses to provide any treatment.¹¹ If the expert does recommend a treatment, then the client decides whether to accept the recommended treatment or not. If the recommended treatment is serious, the expert must provide customer service at quality c , irrespective of whether the client accepts the recommended treatment or not. If the client accepts a treatment recommendation, the expert must provide the appropriate treatment that fixes the actual problem at the price he has quoted. As stated earlier, this treatment, however, may not have to be the one he has claimed to provide.

Now, we define mixed strategies in the recommendation subgame. Let β_i and ρ_i denote respectively the probability that the expert recommends a serious treatment at p_s and the probability that he refuses to provide treatment given that the problem is diagnosed to be i . So $(1 - \beta_i - \rho_i)$ is the probability that he recommends a minor treatment at p_m . Let γ_m (γ_s) be the probability that the client follows the treatment recommendation at p_m (p_s).

The expert's strategy consists of (d, p_m, p_s, c) and the recommendation policy characterized by $\beta_m(p_m, p_s, c)$, $\rho_m(p_m, p_s, c)$, $\beta_s(p_m, p_s, c)$, and $\rho_s(p_m, p_s, c)$ for all $p_m, p_s, c \geq 0$. The client's strat-

⁹Once again, the timing of provision of customer service will be endogenized and the assumption of verifiable customer-service quality will be relaxed in Section 4.2.

¹⁰The case of $\theta > 1$ is less interesting to analyze because in this case, it is trivial that it is optimal for the expert to provide customer service and then capture the value created by charging a high diagnostic fee.

¹¹Following Wolinsky (1993) and Fong (2005), we assume that at this point of the game the expert offers to treat the client's problem at either p_m or p_s only if doing so is profitable in expectation.

egy consists of $\gamma_m(p_m, p_s, c)$ and $\gamma_s(p_m, p_s, c)$ for all $p_m, p_s, c \geq 0$.

Throughout this paper, we restrict our attention to situations in which the following commonly assumed conditions are satisfied:

$$\begin{aligned} 0 < r_m < l_m, \quad 0 < r_s < l_s, \\ \alpha l_s + (1 - \alpha) l_m < r_s. \end{aligned} \tag{R}$$

One immediate implication of (R) is $0 < r_m < l_m < r_s < l_s$. The first line of (R) states that the expert has cost-effective technologies to treat both problems and the second line rules out uninteresting cases.¹²

The appropriate equilibrium concept is Perfect Bayesian Equilibrium (PBE). We restrict attention to the optimal equilibrium, or the PBE which gives the expert the highest profit. Moreover, for ease of exposition, we adopt the tie-breaking rule that the expert does not provide customer service when he earns the same profit whether he provides it or not.¹³

3 Benchmark: Disallowing Customer Service

In this section, we consider the case in which the expert does not have the ability to offer customer service to the client, i.e., we set $c = 0$ exogenously. This standard case serves as a useful benchmark for us to see the impact of allowing customer service, which we explore in the next section.

Lemma 1. *Suppose (R) holds and c is exogenously fixed at zero. In the optimal equilibrium, the expert sets $d = 0$, $p_m = l_m$, $p_s = l_s$. When the problem is minor, he always offers to treat the problem at $p_m = l_m$; when the problem is serious, he always offers to treat the problem at $p_s = l_s$. That is, $\beta_m = 0$, $\beta_s = 1$ and $\rho_m = \rho_s = 0$. Clients accept a treatment offered at $p_m = l_m$ with probability $\gamma_m = 1$, and a treatment offered at $p_s = l_s$ with probability $\gamma_s = (l_m - r_m) / (l_s - r_m)$. The expert's profit is*

$$\Pi^0 = \alpha \frac{l_m - r_m}{l_s - r_m} (l_s - r_s) + (1 - \alpha) (l_m - r_m). \tag{1}$$

¹²Without the second restriction, the expert will set $d = c = 0$ and a single price for both problems at the clients' *ex ante* expected loss, i.e., $p_m = p_s = \alpha l_s + (1 - \alpha) l_m$. Since this price is higher than both r_m and r_s , the expert is willing to repair both problems at this price. Knowing that the problem is always fixed, clients are willing to visit the expert and the expert captures all the surplus. This outcome is also Pareto optimal as all services are provided. Therefore, the second restriction of (R) ensures that the problem is nontrivial.

¹³The only effect of this tie-breaking rule is that, in its absence, in the knife-edge case of $\theta = \frac{r_s - r_m}{l_s - r_m}$ in Proposition 1, while customer service does not enhance efficiency or profit, equilibria with and without customer service coexist.

The proof of Proposition 1 of Fong (2005) can be easily adapted for Lemma 1. The two results are almost identical except that we allow the expert to charge a diagnostic fee here but Fong (2005) does not. According to Lemma 1, in the benchmark case, allowing a diagnostic fee has no impact on the outcomes as the expert does not charge a positive diagnostic fee in equilibrium. On the other hand, the diagnostic fee plays an important role in the analysis in the next section where customer service is allowed.

It is easy to see that the source of inefficiency in the benchmark setting is not the expert's cheating, as the expert is fully honest in equilibrium. Instead, the inefficiency arises due to the clients' rejection of the expert's recommendation of the serious treatment. We use the acceptance rate of the serious treatment γ_s as a measure of the clients' trust because it captures how frequently the clients entrust their problems to the expert. Although the clients' level of trust does not directly reflect the expert's level of honesty on the equilibrium path, it captures the clients' (correct) beliefs of the expert's best response function against γ_s in the recommendation subgame, namely, the expert would cheat if they accept at $\gamma_s > \frac{l_m - r_m}{l_s - r_m}$. In the benchmark setting, $\gamma_s = \frac{l_m - r_m}{l_s - r_m}$ is the clients' maximum acceptance rate consistent with this best response function.

4 Allowing Customer Service upon Recommendation of Serious Treatment

Now we reintroduce customer service which is provided whenever the expert recommends the serious treatment. Before we formally show how provision of customer service may enhance clients' trust, it is useful to illustrate the main idea using an example.¹⁴ Suppose that *ex ante*, the client is equally likely to have a serious problem and a minor problem. If left untreated, the client suffers a loss of 600 when the problem is serious and 240 when the problem is minor. The expert could perfectly diagnose client's problem at no cost and charge a diagnostic fee d . To treat the problem, it costs the expert 480 to perform the serious treatment and 120 to perform the minor treatment. *Liability* is assumed.

First, consider the case without customer service. In the optimal equilibrium, the expert charges no diagnostic fee, 600 for the serious treatment and 240 for the minor treatment. And in equilibrium, the expert honestly reports the client's problem. Since the expert is tempted to recommend the serious

¹⁴Special thanks to an anonymous referee for providing this illuminating example.

treatment when the problem is minor (The expert could claim that the serious treatment is needed and charge 600, but perform the minor treatment at cost 120, earning 480. By honestly making the minor treatment recommendation, the expert instead earns $240 - 120 = 120$.), the client has to reject the serious recommendation $\frac{1}{4}$ of the time to keep the expert honest. The expert, who is kept honest, earns an expected profit of $\frac{1}{2}(240 - 120) + \frac{1}{2}\frac{1}{4}(600 - 480) = 75$.

Now suppose that the expert can spend 40 to provide customer service which is only worth 36 to the client, and suppose that such customer service is provided whenever the serious treatment is recommended (although not necessarily accepted by the client). Given the same equilibrium pricing strategy for the serious and minor treatments, now to keep the expert honest, the client could accept the serious recommendation $\frac{1}{3}$ of the time. To see this, given a minor problem, the expert obtains $240 - 120 = 120$ when honestly reporting the problem; and earns $-40 + \frac{1}{3}(600 - 120) = 120$ when reporting the problem as serious. The indifference renders the expert no incentive to lie. In addition, the expert could charge a diagnostic fee of 18 since the client is expected to collect customer service valued at 36 half of the time. Thus the expert's profit increases to $18 + \frac{1}{2}(240 - 120) + \frac{1}{2}[-40 + \frac{1}{3}(600 - 480)] = 78 > 75$. But if the customer service is only worth 24 to the client, by providing customer service, the expert's profit is $12 + \frac{1}{2}(240 - 120) + \frac{1}{2}[-40 + \frac{1}{3}(600 - 480)] = 72 < 75$. Therefore, the expert needs to trade off between the increased acceptance rate and the intrinsic inefficiency of customer service.

4.1 When Does Customer Service Enhance Efficiency and Profitability

As in the benchmark case, the diagnostic fee d does not affect the equilibrium of the recommendation subgame. However, the quality of customer service, c , does. Moreover, the provision of customer service in turn affects the diagnostic fee the expert charges.

Proposition 1. *Suppose (R) holds.*

(i) *When $\theta \in \left[0, \frac{r_s - r_m}{l_s - r_m}\right]$, in the optimal equilibrium, no customer service is provided, i.e., $c = 0$.*

Lemma 1 in the benchmark case describes the remaining properties of the optimal equilibrium, and the expert's profit remains at Π^0 .

(ii) *When $\theta \in \left(\frac{r_s - r_m}{l_s - r_m}, 1\right]$, in the optimal equilibrium, customer service is provided. The optimal quality of customer service is $c = \frac{(l_m - r_m)(l_s - r_s)}{r_s - r_m}$, which is the level that makes the expert just break even recommending the serious treatment when the problem is serious. The expert sets $d = \alpha\theta \frac{(l_m - r_m)(l_s - r_s)}{r_s - r_m}$, $p_m = l_m$, $p_s = l_s$. When the problem is minor, he always offers to treat the problem at $p_m = l_m$; when*

the problem is serious, he always offers to treat the problem at $p_s = l_s$. That is, $\beta_m = 0$, $\beta_s = 1$ and $\rho_m = \rho_s = 0$. Clients accept a treatment offered at $p_m = l_m$ with probability $\gamma_m = 1$, and a treatment offered at $p_s = l_s$ with probability

$$\gamma_s = \frac{l_m - r_m}{r_s - r_m} > \frac{l_m - r_m}{l_s - r_m}.$$

The expert's profit is

$$\Pi = \alpha \theta \frac{l_m - r_m}{r_s - r_m} (l_s - r_s) + (1 - \alpha) (l_m - r_m) > \Pi^0. \quad (2)$$

The main message of Proposition 1 is that when customer service is not too socially inefficient intrinsically, i.e., for $\theta \in \left(\frac{r_s - r_m}{l_s - r_m}, 1\right]$, by providing customer service whenever he recommends the serious treatment, the expert raises clients' acceptance rate of the serious treatment from $\frac{l_m - r_m}{l_s - r_m}$ to $\frac{l_m - r_m}{r_s - r_m}$ and his profit from Π^0 to Π . To see why this is the case, notice that there is a difference in the expert's best response functions against γ_s in the recommendation subgame under the two scenarios. And the difference in the clients' (correct) beliefs of the experts' best response functions in the recommendation subgame explains the difference between the acceptance rates when they go to an expert who provides customer service and when they go to an expert who does not provide customer service. In the absence of customer service, the clients' maximum acceptance rate is $\gamma_s = \frac{l_m - r_m}{l_s - r_m}$ because the expert would cheat if they accept at $\gamma_s > \frac{l_m - r_m}{l_s - r_m}$. When the optimal quality of customer service is provided, the clients' maximum acceptance rate rises to $\gamma_s = \frac{l_m - r_m}{r_s - r_m}$ because the expert cheats if and only if $\gamma_s > \frac{l_m - r_m}{r_s - r_m}$. When customer service is too socially inefficient, i.e., for $\theta \in \left[0, \frac{r_s - r_m}{l_s - r_m}\right]$, the expert will not provide customer service.

Proof. The logic in Fong (2005) can be adapted to show that even when $c > 0$ is allowed, it is still without loss of generality to restrict attention to the intuitive price region: $(p_m, p_s) \in [r_m, l_m] \times [r_s, l_s]$.

If the serious treatment is never provided in equilibrium, the expert's profit is bounded by the total surplus of $(1 - \alpha) (l_m - r_m)$. Therefore, the expert may earn higher profit than Π^0 only if the serious problem is treated with a positive probability. This requires that the expert at least breaks even recommending the serious treatment when the client's problem is serious, i.e.,

$$-c + \gamma_s (p_s - r_s) \geq 0. \quad (3)$$

Otherwise, he could refuse to provide any treatment, i.e., set $\rho_s = 1$.

The logic in Fong (2005) can be applied to show that no pure strategy equilibrium exists. And the

equilibrium profile is characterized by the probabilities,

$$\begin{aligned}\rho_m &= \rho_s = 0, \beta_s = 1, \gamma_m = 1, \\ \beta_m &= \frac{\alpha(l_s - p_s)}{(1 - \alpha)(p_s - l_m)},\end{aligned}\tag{4}$$

$$\gamma_s = \frac{p_m - r_m + c}{p_s - r_m}.\tag{5}$$

Since the expert can extract all the surplus of his service by charging a high enough diagnostic fee, the profit-maximization problem is equivalent to the inefficiency-minimization problem. Note that the expected efficiency loss is

$$\begin{aligned}L &= (\alpha(l_s - r_s) + (1 - \alpha)\beta_m(l_m - r_m))(1 - \gamma_s) + (\alpha + (1 - \alpha)\beta_m)(c - b) \\ &= \alpha \left((l_s - r_s) + \frac{(l_s - p_s)}{(p_s - l_m)}(l_m - r_m) \right) \left(\frac{p_s - p_m - c}{p_s - r_m} \right) + \alpha \frac{(l_s - l_m)}{(p_s - l_m)}(1 - \theta)c.\end{aligned}$$

Differentiating L with respect to c gives

$$\frac{dL}{dc} = \alpha \left[\frac{(r_s - r_m)}{(p_s - r_m)} - \frac{(l_s - l_m)}{(p_s - l_m)}\theta \right] \geq 0 \text{ if and only if } \theta \leq \theta^* \equiv \frac{(r_s - r_m)(p_s - l_m)}{(p_s - r_m)(l_s - l_m)}.$$

We have

$$\frac{d\theta^*}{dp_s} = \frac{(l_m - r_m)(r_s - r_m)}{(l_s - l_m)(p_s - r_m)^2} > 0.$$

Case I: $\theta \leq \frac{(r_s - l_m)}{(l_s - l_m)}$

In this case, $\theta \leq \frac{(r_s - l_m)}{(l_s - l_m)} \leq \theta^*$ for all $p_s \in [r_s, l_s]$ and L weakly increases with c . Applying the tie-breaking rule, in the optimal equilibrium, $c = 0$. The optimal equilibrium is thus the same as that in the benchmark case.

Case II: $\theta > \frac{(r_s - r_m)}{(l_s - r_m)}$

In this case, $\theta > \frac{(r_s - r_m)}{(l_s - r_m)} \geq \theta^*$ for all $p_s \in [r_s, l_s]$ and L decreases with c . So in the optimal equilibrium c is set at the highest possible level according to (3):

$$c = \left(\frac{p_m - r_m + c}{p_s - r_m} \right) (p_s - r_s) \Leftrightarrow c = \frac{(p_m - r_m)(p_s - r_s)}{(r_s - r_m)}.$$

Plugging this back into L , and differentiating with respect to p_m , we have

$$\begin{aligned}\frac{\partial L}{\partial p_m} &= -\alpha \left((l_s - r_s) + \frac{(l_s - p_s)}{(p_s - l_m)}(l_m - r_m) \right) \frac{1}{(r_s - r_m)} + \alpha(1 - \theta) \frac{(l_s - l_m)(p_s - r_s)}{(p_s - l_m)(r_s - r_m)} \\ &< -\alpha \frac{(l_s - p_s)}{(p_s - l_m)} \leq 0.\end{aligned}$$

The first inequality follows $\theta > 0$. It implies that in the optimal equilibrium $p_m = l_m$. Plugging this into L , and differentiate with respect to p_s , we have

$$\frac{\partial L}{\partial p_s} = -\alpha\theta \frac{(l_s - l_m)(l_m - r_m)(r_s - l_m)}{(r_s - r_m)(p_s - l_m)^2} < 0.$$

So, it further follows that in the optimal equilibrium, $p_s = l_s$.

Given that $p_s = l_s$ and $p_m = l_m$, the only surplus the client receives in the recommendation subgame is the benefit from customer service

$$b = \theta c = \theta \frac{(l_m - r_m)(l_s - r_s)}{(r_s - r_m)},$$

which she receives with probability α . The optimal diagnostic fee is set at a level such that the client's participation constraint just binds:

$$d = \alpha b = \frac{\alpha\theta(l_m - r_m)(l_s - r_s)}{(r_s - r_m)}.$$

Plugging the values of p_m, p_s , and c into (4) and (5), we have

$$\begin{aligned}\beta_m &= \frac{\alpha(l_s - l_m)}{(1 - \alpha)(l_s - l_m)} = 0, \\ \gamma_s &= \frac{l_m - r_m}{r_s - r_m} > \frac{l_m - r_m}{l_s - r_m},\end{aligned}$$

where γ_s is higher than the corresponding acceptance rate in the absence of customer service.

Recall that c is set at a level such that the expert just breaks even when the problem is serious, so

$$\begin{aligned}\Pi &= d + (1 - \alpha)(l_m - r_m) \\ &= \alpha\theta \frac{l_m - r_m}{r_s - r_m} (l_s - r_s) + (1 - \alpha)(l_m - r_m) \\ &> \alpha \frac{l_m - r_m}{l_s - r_m} (l_s - r_s) + (1 - \alpha)(l_m - r_m) = \Pi^0.\end{aligned}$$

The inequality follows $\theta > \frac{(r_s - r_m)}{(l_s - r_m)}$.

Case III: $\theta \in \left(\frac{(r_s - l_m)}{(l_s - l_m)}, \frac{(r_s - r_m)}{(l_s - r_m)}\right)$

In this case, there exists p_s^* such that p_s^* is defined by $\theta = \frac{(r_s - r_m)(p_s^* - l_m)}{(p_s^* - r_m)(l_s - l_m)}$,

(1) when $p_s \leq p_s^*$, L decreases with c for $p_s < p_s^*$, and is independent of c for $p_s = p_s^*$. It is without loss to consider c at the highest level, i.e., $c = \frac{(p_m - r_m)(p_s - r_s)}{(r_s - r_m)}$;

(2) when $p_s > p_s^*$, L increases with c and c is set at the lowest level, i.e., $c = 0$.

There are two candidate optimal equilibria with $p_s = p_s^*$ and $c = \frac{(p_m - r_m)(p_s - r_s)}{(r_s - r_m)}$, and with $p_s = l_s$ and $c = 0$. In either case, it is optimal to set $p_m = l_m$. Let the loss from the first candidate equilibrium

be L_1 and the second be L_2 .

$$\begin{aligned} L_1 &> L_1|_{p_s=l_s} > \alpha(l_s - r_s) \frac{(r_s - l_m)}{(r_s - r_m)} + \alpha \left(1 - \frac{(r_s - r_m)}{(l_s - r_m)} \right) \frac{(l_m - r_m)(l_s - r_s)}{(r_s - r_m)} \\ &= \alpha \frac{(l_s - r_s)(l_s - l_m)}{(l_s - r_m)} = L_2 \end{aligned}$$

Therefore, in the optimal equilibrium, c is set at 0.

Case IV: $\theta = \frac{(r_s - r_m)}{(l_s - r_m)}$

In this case, L decreases with c for $p_s \in [r_s, l_s)$ and L is independent of c for $p_s = l_s$. For $p_s < l_s$, in the optimal equilibrium, c is set at the highest possible level, i.e., $c = \frac{(p_m - r_m)(p_s - r_s)}{(r_s - r_m)}$. For $p_s = l_s$, due to the independence, it is without loss to consider $c = \frac{(p_m - r_m)(p_s - r_s)}{(r_s - r_m)}$. Given the result from Case II, when $c = \frac{(p_m - r_m)(p_s - r_s)}{(r_s - r_m)}$, it is optimal to set $p_m = l_m$ and $p_s = l_s$. Therefore, in the optimal equilibrium, L is independent of c . Applying the tie-breaking rule, the expert sets $c = 0$. \square

Here we provide some intuition for Proposition 1. Providing costly customer service upon the recommendation of the serious treatment makes it less attractive for the expert to recommend the serious treatment whether the problem is serious or minor. Since the expert has a strict incentive to honestly report when the problem is serious in the absence of customer service, as long as the quality of customer service is not set too high, the expert's incentive to honestly report a serious problem is unaffected. On the other hand, and more importantly, making recommendation of the serious treatment less attractive allows clients to accept the serious-treatment recommendation at a higher rate, i.e., to trust the expert more often, while maintaining the expert's incentive to honestly report a minor problem.

The increased acceptance rate of the serious treatment improves the overall efficiency of the expert's service since the clients' problems are less likely to be left untreated. So clients value the expert's service more. When customer service is not intrinsically inefficient, i.e., $\theta = 1$, clearly the expert will provide customer service. However, when $\theta < 1$, there is a tradeoff. Notice that the efficiency gain from increased acceptance rate is independent of θ whereas the efficiency loss of providing customer service decreases with θ . Therefore, there exists a unique cutoff value of intrinsic efficiency above which the benefit of increased acceptance rate outweighs the efficiency loss from provision of intrinsically inefficient customer service and below which inducing higher acceptance rate is socially too costly. As a result, customer service should only be provided when the intrinsic efficiency of customer service is higher than the cutoff value. Improved efficiency of the expert's service translates into higher profit for the expert because the expert can raise the diagnostic fee to capture the surplus clients derived in the

recommendation subgame.

It is worth noting that the trust-enhancing role of customer service is rather different from that of a costly signal in the standard signaling game. Notice that providing customer service is equally costly whether the client's problem is minor or serious, i.e., independent of the expert's type; yet the expert benefits more from raising clients' acceptance rate when the problem is minor, i.e., when the expert's type is bad, than when the problem is serious. There is room for enhancement of clients' trust mainly due to the fact that, in the absence of customer service, there is slackness in the expert's incentive to truthfully report a serious problem instead of refusing to treat the client. In the optimal equilibrium, the expert raises the quality of the customer service up to the level where he is indifferent between truthfully reporting a serious problem and sending the client away.

Our finding that costly customer service can be used to promote acceptance of the serious treatment is broadly in line with practitioners' advice for experts. On how to address dental patients' lack of trust for dentists, the CEO of a dental consulting company based in California gives dentists this advice, "Before you present patients with large treatment plans, *take the time* to earn their trust. Show them you care about their well-being by providing education. Build connections by asking about their jobs, their families and their oral health goals and encourage your team members to do the same."¹⁵ The owner of another dental consulting firm also advises dentists to "*take the time* to treat these patients the way they should be treated" and argues that "the practices with the most case acceptance are the practices that are good at asking questions and focusing on what their patients want... It's the dentists and staff members who act as coaches rather than salesmen who are the most successful."¹⁶ On how to boost treatment compliance, another dental consultant advises dentists "to educate their patients during an exam... [m]ake sure [the patient] receives the [proposed] treatment plan in writing at the end of the appointment, and be careful to include any home-care instructions."¹⁷

Note that the gain in efficiency and the gain in profit from the proposed pricing strategy are the same in this setting since clients get zero surplus. A nice feature of our analysis is that the comparative

¹⁵See McKenzie, S. (2016), "Don't Scare New Patients Off with Expensive Treatment Plans," available at <http://www.sallymckenzieblog.com/index.php/dont-scare-new-patients-off-expensive-treatment-plans/>.

¹⁶See Knight, R. (2012), "Are you too much of a car salesman? Why selling treatments to your patients doesn't work," available at <http://www.dentalproductsreport.com/dental/article/are-you-too-much-car-salesman>.

¹⁷See Geier J. (2016), "Seven Easy Ways to Boost Treatment Compliance," [dentaltown.com](http://www.dentaltown.com/magazine/articles/5746/seven-easy-ways-to-boost-treatment-compliance), available at <http://www.dentaltown.com/magazine/articles/5746/seven-easy-ways-to-boost-treatment-compliance>.

statics of this gain can be fully characterized.

Proposition 2. *Let $G = \Pi - \Pi^0$ be the expert's gain from offering customer service. Then*

- (a) *G is increasing in α (the probability that the client's problem is serious);*
- (b) *G is increasing in l_s and l_m (the loss that the client bears if problem $i \in \{s, m\}$ is left untreated);*
- (c) *G is decreasing in r_s (the cost of the serious treatment); there is a cutoff \hat{l} , which is independent of α , such that G is increasing in r_m (the cost of the minor treatment) if and only if l_m is above \hat{l} , i.e., the minor treatment is very valuable;*
- (d) *G is increasing in θ (the intrinsic efficiency of customer service).*

Proof. *See the Appendix. ■*

The gain from offering customer service is monotone in α , l_s , l_m , r_s , and θ . Although the gain is nonmonotone in r_m , we are still able to show that it increases in r_m when l_m is relatively large and decreases in r_m otherwise.¹⁸ Suppose the expert has to incur an upfront setup cost before he can start offering customer service. Then the comparative statics informs us of when the expert is more likely to pay this setup cost. This tends to happen with high α, l_m, l_s, θ and/or low r_s , and high r_m in the case of high l_m and low r_m in case of low l_m .

4.2 Timing and Verifiability of Customer Service

In the above analysis, we have considered provision of customer service of verifiable quality and the timing of which coincides with the recommendation of the serious treatment. According to Wikipedia, customer service “is the provision of service to customers before, during and after a purchase.” So it is natural to ask whether providing customer service at a different moment during a client's consultation visit can also make the client more trusting. We will show below that providing customer service at any other point in time would not promote trust. Furthermore, we will show that relaxing the assumption of verifiable customer-service quality will not affect our main findings regarding the optimal provision and timing of customer service.

¹⁸Note despite our assumption that $r_s > \alpha l_s + (1 - \alpha) l_m$, the admissible range of l_m is (r_m, r_s) . This is because for any $l_m \in (r_m, r_s)$ and $l_s > r_s$, there exists α such that $r_s > \alpha l_s + (1 - \alpha) l_m$ is satisfied.

4.2.1 Timing of Customer Service

If customer service is provided (sunk) before recommendation of any treatment, it does not affect the equilibrium of the recommendation subgame. Similarly, providing customer service following the recommendation of the minor treatment also would not affect the client's trust as the client would accept the minor treatment with or without customer service. For these two timings, when $\theta \in [0, 1)$, i.e., $b < c$, customer service is intrinsically socially inefficient and it should not be provided. When $\theta = 1$, i.e., $b = c$, the expert's profit is unaffected by provision of customer service. Therefore, providing customer service before recommendation of treatment or at the moment of recommending the minor treatment is neither efficiency enhancing nor profit improving.

If customer service is provided conditional on the acceptance of the serious treatment, the effective amount paid by the client for the serious treatment is $\hat{p}_s \equiv p_s - b = p_s - \theta c$, and the effective amount received by the expert is $\tilde{p}_s \equiv p_s - c$. Customer service does not affect other aspects of the game. Since $b = \theta c \leq c$, customer service is a (weakly) inefficient form of transfer payment from the expert to the client, with an efficiency loss of $(1 - \theta)c$. The client will accept the serious treatment recommendation with a positive probability if $\hat{p}_s \leq l_s$. Modifying expressions of β_m and γ_s according to the effective price paid and the effective amount received, we have

$$\begin{aligned}\beta_m &= \frac{\alpha(l_s - \hat{p}_s)}{(1 - \alpha)(\hat{p}_s - l_m)}, \\ \gamma_s &= \frac{p_m - r_m}{\tilde{p}_s - r_m} = \frac{p_m - r_m}{\hat{p}_s - (1 - \theta)c - r_m}.\end{aligned}$$

The last equality follows $\hat{p}_s - \tilde{p}_s = (1 - \theta)c$.

The expected efficiency loss is

$$L = [\alpha(l_s - r_s) + (1 - \alpha)\beta_m(l_m - r_m)](1 - \gamma_s) + (\alpha + (1 - \alpha)\beta_m)\gamma_s(1 - \theta)c.$$

Substituting the expressions of β_m and γ_s into L and then differentiating L with respect to c , we have

$$\frac{\partial L(p_m, \hat{p}_s, c)}{\partial c} = \alpha \frac{(1 - \theta)(p_m - r_m)(r_s - r_m)}{(\hat{p}_s - (1 - \theta)c - r_m)^2} > 0.$$

Therefore, for any given p_m , p_s , and c which satisfy the constraints $p_s - c \geq r_s$ and $p_s - \theta c \leq l_s$, lowering c by Δc and lowering p_s by $\theta \Delta c$ leads to lower L while ensuring that the constraints remain satisfied. This implies that in the optimal equilibrium, c is set at the lowest possible level, i.e., $c = 0$ if it is provided contingent on the acceptance of the serious treatment. This concludes that providing customer service at any other moment does not promote trust.

Customer service in our setting serves the single purpose of promoting clients' trust. In many real-life settings, customer service is also provided for other reasons. If the expert provides customer service to promote trust as well as for other reasons, then our theory implies that he will provide a higher level of customer service when he tries to upsell the client.

To establish this point more formally, let us assume that the expert is required to provide a minimum quality of customer service \underline{c} to every client who visits him.¹⁹ Let c be the incremental customer service to be provided upon recommendation of the serious treatment. To be consistent with our main model, we still assume that customer service is socially inefficient. The benchmark case can be easily modified to include the minimum customer service requirement. Assume that the inefficiency caused by the minimum customer service is small enough so that the expert will not be driven out of business, i.e.,

$$(1 - \theta) \underline{c} < \Pi^0.$$

Since the minimum customer service has to be provided to every client, it can be viewed as sunk when the *recommendation subgame* begins. And thus the recommendation subgame will not be affected²⁰ and it is still optimal to provide the same level of additional customer service, $c = \frac{(l_m - r_m)(l_s - r_s)}{r_s - r_m}$, whenever a serious treatment is recommended for $\theta \in \left(\frac{r_s - r_m}{l_s - r_m}, 1\right]$. Since the client receives the minimum customer service valued at $\theta \underline{c}$, the expert could fully extract such benefit by raising the diagnostic fee d to $\alpha \theta c + \theta \underline{c}$ (to $\theta \underline{c}$ in the modified benchmark case where the incremental customer service is not provided). The expert's profit Π decreases by $(1 - \theta) \underline{c}$ due to the inefficiency of the (minimum) customer service and since the minimum customer service is required to be provided in the benchmark case as well, Π^0 also decreases by $(1 - \theta) \underline{c}$. Therefore, in a setting where a minimum level of customer service has to be provided, providing the incremental customer service whenever a serious treatment is recommended still has the same positive effect on efficiency and profit.

¹⁹There exist laws that govern the minimum customer service standards. For instance, according to *U.S. Code Title 47 § 552 - Consumer protection and customer service (b) Commission Standards*, "The Commission shall, within 180 days of October 5, 1992, establish standards by which cable operators may fulfill their customer service requirements. Such standards shall include, at a minimum, requirements governing — (1) cable system office hours and telephone availability; (2) installations, outages, and service calls; and (3) communications between the cable operator and the subscriber (including standards governing bills and refunds)." Moreover, some practitioners also advocate the implementation of "Minimum Standards of Customer Service". A Google search of the exact term, with quotation marks included returned 44,800 results.

²⁰The break-even condition with the sunk cost included is $-\underline{c} - c + \gamma_s (p_s - r_s) \geq -\underline{c}$, which is equivalent to (3).

4.2.2 Non-verifiable Customer Service

Now suppose that provision of customer service is observable but not verifiable. In this case, if customer service is provided at the moment the serious treatment is recommended, the client could still make her acceptance decision contingent on the quality of customer service provided, and the highest acceptance rate of the serious-treatment recommendation is still captured by (5), i.e., $\gamma_s = \frac{p_m - r_m + c}{p_s - r_m}$. Understanding that, under the condition of Part (ii) of Proposition 1, it is still optimal for the expert to set $c = \frac{(l_m - r_m)(l_s - r_s)}{r_s - r_m}$, and doing so still results in the same profit level Π .

As in the case of verifiable customer service, the expert still does not benefit from providing customer service when he recommends the minor treatment. However, it is no longer credible for the expert to promise any customer service after the client accepts his treatment. Such promise will never be delivered because when the expert shirks on the quality of customer service, he saves cost and there is no consequence, for this being a one-shot game. Therefore, compared to the case of verifiable customer service, it is even clearer that it is optimal to provide customer service upon the recommendation of the serious treatment.

5 Conclusion

In this paper, we formalize the trust-enhancing role of customer service which was identified by service-marketing practitioners and researchers. Tying provision of customer service with recommendation of the serious treatment makes it more costly for the expert to overcharge clients and thus promote clients' trust. Provision of customer service allows clients to accept the expert's treatment at a higher probability without compromising the expert's incentive to honestly report clients' problems. The increased acceptance rate of the serious treatment leads to both higher efficiency and higher profitability of the expert's service. When the gain from increased acceptance rate outweighs the loss from potentially intrinsically inefficient customer service, the expert should provide customer service to capture the net gain.

Our sharp prediction on timing of provision of customer service is due to our focus on the efficiency- and trust-enhancing role of customer service and our assumption that players have standard payoffs. Firms may provide customer service for various other reasons, and for these purposes, it may be more effective to provide customer service before recommendation of a treatment or after provision of the

treatment. Also, if clients are motivated by reciprocity, then providing customer service to clients any time before the recommendation of the serious treatment can potentially help boost the acceptance rate of the serious treatment.

Our model could be extended to allow competition among experts where clients are allowed to search for second opinions at a cost. In the competitive market, by providing customer service, the expert can similarly induce clients to accept his recommendations at a higher probability, and thus reduce the expected search cost. The main difference from the monopoly setting is that, due to competition, the improved efficiency will likely translate into higher consumer surplus instead of higher firm profits. A potential issue when competitive firms offer customer service is that clients may visit multiple experts to collect free customer service. This issue will not arise in our setting because we assume that clients have unit demand for customer service. As a more general point, this issue can be addressed if the experts coordinate to provide identical or very similar customer service the marginal benefit of which falls substantially after clients have already received it from one expert. For example, in the context of doctor-patient relationships, the staff at a medical clinic can provide good customer service by listening patiently to the patient's concern, showing empathy, and teaching the patient how to make appropriate life-style changes following the diagnosis of a serious illness. It is unlikely patients would visit multiple clinics to receive such similar services.

We believe customer service is an interesting topic which is underexplored by economists, and hope to see more economic and game-theoretic studies on the topic in the future.

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Appendix: Proof of Proposition 2

Π is attained when $\theta > \frac{(r_s - r_m)}{(l_s - r_m)}$. From the expressions of Π and Π^0 , we obtain

$$G = \alpha \theta \frac{l_m - r_m}{r_s - r_m} (l_s - r_s) - \alpha \frac{l_m - r_m}{l_s - r_m} (l_s - r_s).$$

It follows from direct calculations that

$$\begin{aligned} \frac{\partial G}{\partial \alpha} &= \theta \frac{l_m - r_m}{r_s - r_m} (l_s - r_s) - \frac{l_m - r_m}{l_s - r_m} (l_s - r_s) > \frac{l_m - r_m}{l_s - r_m} (l_s - r_s) - \frac{l_m - r_m}{l_s - r_m} (l_s - r_s) = 0, \\ \frac{\partial G}{\partial l_s} &= \alpha \theta \frac{l_m - r_m}{r_s - r_m} - \alpha \frac{(l_m - r_m)(r_s - r_m)}{(l_s - r_m)^2} > \alpha \frac{(l_m - r_m)(l_s - r_s)}{(l_s - r_m)^2} > 0, \\ \frac{\partial G}{\partial l_m} &= \alpha \theta \frac{(l_s - r_s)}{(r_s - r_m)} - \alpha \frac{(l_s - r_s)}{(l_s - r_m)} > \alpha \frac{(l_s - r_s)}{(l_s - r_m)} - \alpha \frac{(l_s - r_s)}{(l_s - r_m)} = 0, \\ \frac{\partial G}{\partial r_s} &= -\alpha \theta \frac{(l_m - r_m)(l_s - r_m)}{(r_s - r_m)^2} + \alpha \frac{l_m - r_m}{l_s - r_m} < -\alpha \frac{(l_m - r_m)(l_s - r_s)}{(r_s - r_m)(l_s - r_m)} < 0, \\ \frac{\partial G}{\partial \theta} &= \alpha \frac{l_m - r_m}{r_s - r_m} (l_s - r_s) > 0, \\ \frac{\partial G}{\partial r_m} &= -\alpha \theta \frac{(r_s - l_m)}{(r_s - r_m)^2} (l_s - r_s) + \alpha \frac{(l_s - l_m)}{(l_s - r_m)^2} (l_s - r_s) \end{aligned}$$

The sign of $\partial G / \partial r_m$ depends on the magnitude of l_m :

$$\begin{aligned} \frac{\partial \left(\frac{\partial G}{\partial r_m} \right)}{\partial l_m} &= \alpha \theta \frac{1}{(r_s - r_m)^2} (l_s - r_s) - \alpha \frac{1}{(l_s - r_m)^2} (l_s - r_s) > \alpha \frac{(l_s - r_s)^2}{(l_s - r_m)^2 (r_s - r_m)} > 0, \\ \lim_{l_m \rightarrow r_m} \frac{\partial G}{\partial r_m} &= -\alpha \theta \frac{(l_s - r_s)}{(r_s - r_m)} + \alpha \frac{(l_s - r_s)}{(l_s - r_m)} < -\alpha \frac{(l_s - r_s)}{(l_s - r_m)} + \alpha \frac{(l_s - r_s)}{(l_s - r_m)} = 0, \\ \lim_{l_m \rightarrow r_s} \frac{\partial G}{\partial r_m} &= \alpha \frac{(l_s - r_s)^2}{(l_s - r_m)^2} > 0. \end{aligned}$$

Therefore, there exists $\hat{l} \in (r_m, r_s)$ such that $\partial G / \partial r_m > 0$ for $l_m \in (\hat{l}, r_s)$ and $\partial G / \partial r_m < 0$ for $l_m \in (r_m, \hat{l})$. Finally, \hat{l} is independent of α since the solution to $\frac{\partial G}{\partial r_m} = 0$ is independent of α . \square