Biased Beliefs without Dynamic Inconsistency[1]

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[1] Chapter 5 of *Bounded Rationality and Industrial Organization* (Spiegler, 2011)

Introduction

Introduction

- In the last lecture, we considered dynamically inconsistent preferences.
  - Consumers had definite first-period preferences over their second-period consumption options; these preferences changed in the second period.
- We discussed how “naivete”, or biased beliefs that consumers hold regarding their future preferences, affects consumers’ choices and firms’ pricing strategeis.

Introduction

- In this chapter, we discuss situations in which consumers hold biased beliefs regarding their future preferences, without any element of dynamic inconsistency.
- In the two-period models we shall examine,
  - Consumer's second-period preferences over consumption alternatives are "state-dependent."
  - In the first period, he holds a (biased) belief over the possible states.

Introduction: Over-optimism

- Beliefs are over-optimistic if they systematically assign excessive probability to the good states.
- For instance, a consumer who is about to sign up for a cable TV package may be over-optimistic about the amount of leisure that he will have, and therefore exaggerate the value of having more channels on the package.

Introduction: Over-confidence

- The consumer may over-estimate the precision with which he estimates his second-period valuation of various consumption decisions.
- For instance, a consumer who is about to sign up for mobile phone services considers evidence from past consumption (his own and others') and reaches an estimate that he will demand 1,000 minutes of airtime, with little uncertainty around this estimate.
- In reality, his demand for airtime will vary more than he anticipates.

Introduction: Unforeseen contingencies

- The consumer may assign zero probability to certain states because he is simply unaware of them at the time of purchase.
- For instance, a consumer who is about to acquire a vacation package may fail to think about various expenses he will need to incur during the vacation.
- Alternatively, a consumer who considers buying a home entertainment product may fail to think about various additional components (add-ons) he will perceive as necessary or valuable after the purchase.

Over-optimism

Over-optimism: Model

- This model is closely related to the model of monopoly pricing with partially (frequency) naive consumers.
- Consider a monopolistic firm offering a service to a single consumer.
- In period 1
  - Firm offers a price scheme $t : X \to \mathbb{R}$ where $X = [0, 1]$
  - Consumer decides whether to accept or not. (outside option 0)
- In period 2
  - If accepted, consumer choose an action x.
  - Here, we interpret x as consumption quantity.
  - Firm's cost of providing the service is given by a continuously increasing function $c : X \to \mathbb{R}$.

Model

- Consumer has state-dependent preferences over second-period outcomes.
- State space is $\{u, v\}$.
  - In state u, consumer's willingness to pay for any action is given by a continuous function $u : X \to \mathbb{R}$
  - In state v, it is given by a continuous function $v : X \to \mathbb{R}$.
  - Assume that $u(0) = v(0) = 0$ and $u(x) > v(x)$ for all $x > 0$.
  - Thus, u and v can be viewed as "good" and "bad" states.
- Assume that each of the functions $u - c$, $v - c$, and $u - v$ attains a non-negative maximum at a unique point.

Model

To facilitate comparison with the previous model, assume that

- monopolist's prior belief assigns probability one to the state v;
- consumer's prior on the state u is denoted θ.
- We interpret monopolist's prior belief as correct and unbiased.
- If $\theta > 0$, we say that consumer's belief is over-optimistic, because it assigns excessive weight to the "good" state u.

Analysis

- Monopolist's first-period maximization problem is thus to find a 4-tuple (x^u, T^u, x^v, T^v)
  - where (x^u, T^u) and (x^v, T^v) are the action-payment pair in the states u and v
- Monopolist's problem is

$$\max_{x^u, T^u, x^v, T^v} T^v - c(x^v)$$

$$\text{s.t. } v(x^v) - T^v \geq v(x^u) - T^u \qquad (IC_2V)$$

$$u(x^u) - T^u \geq u(x^v) - T^v \qquad (IC_2U)$$

$$\theta\left[u(x^u) - T^u\right] + (1-\theta)\left[v(x^v) - T^v\right] \geq 0. \qquad (IR)$$

Analysis

The difference between this model and the previous model for frequency naifs lies in consumer's participation constraint.

- Since we now assume that consumer is dynamically consistent, his first-period net evaluation of the action x in state $\omega \in \{u, v\}$ is $\omega(x) - t(x)$.
- In contrast, in the previous model, consumer's first-period evaluation of x was $u(x) - t(x)$ in both states, because he had a distinct first-period willingness-to-pay function u over his second-period consumption decisions.

Analysis

- When $\theta = 0$, the model is reduced to a standard problem of monopoly pricing, as both parties agree that consumer's preferences over second-period consumption are given by v.
- Therefore, a price scheme that commits consumer to $x^* = \arg\max_x (v(x) - c(x))$ is optimal.
- The following result characterizes optimal price schemes when the consumer is over-optimistic.

Analysis

- Now, let us solve firm's problem:

$$\max_{x^u,T^u,x^v,T^v} T^v - c(x^v)$$

$$\text{s.t. } v(x^v) - T^v \geq v(x^u) - T^u \qquad (IC_2V)$$

$$u(x^u) - T^u \geq u(x^v) - T^v \qquad (IC_2U)$$

$$\theta\left[u(x^u) - T^u\right] + (1-\theta)\left[v(x^v) - T^v\right] \geq 0. \qquad (IR)$$

- The problem could be solved following the same procedure as we did in the previous lecture.
- First, (IC_2V) and (IR) are binding, implying

$$T^v = v(x^v) + \theta \cdot \left[u(x^u) - v(x^u)\right]$$

$$T^u = \theta \cdot u(x^u) + (1-\theta) \cdot v(x^u)$$

Analysis

- The problem becomes

$$\max_{x^u, x^v} [v(x^v) - c(x^v)] + \theta [u(x^u) - v(x^u)]$$
$$\text{s.t. } u(x^u) - v(x^u) \geq u(x^v) - v(x^v). \tag{IC_2U}$$

- The solution to the unconstraint maximization problem is

$$x^v = \arg\max_x (v(x) - c(x))$$
$$x^u = \arg\max_x (u(x) - v(x))$$

- The constraint (IC_2U) is satisfied automatically.

Analysis

Proposition 1

Let $\theta > 0$. Under any optimal price scheme, the consumer's second-period consumption decisions are

$$x^v = \arg\max_x (v(x) - c(x))$$
$$x^u = \arg\max_x (u(x) - v(x))$$

and the payment he makes are

$$T^v = v(x^v) + \theta \cdot [u(x^u) - v(x^u)]$$
$$T^u = \theta \cdot u(x^u) + (1 - \theta) \cdot v(x^u)$$

Analysis

- When $\theta > 0$,
  - $v(x^v) - T^v = -\theta \cdot [u(x^u) - v(x^u)] < 0$
  - $u(x^u) - T^u = (1 - \theta) \cdot [u(x^u) - v(x^u)] > 0$
- That is, the payment consumer ends up making in state v (u) is higher (lower) than his willingness to pay for the action he takes in that state.
- A consumer who shares firm's (unbiased) prior belief would not accept these price schemes.
- In this sense, the price scheme is exploitative.

Comparison with Related Models

- We will compare the optimal monopoly pricing with
  1. over-optimistic consumers; and
  2. partially (frequency) naive, dynamically inconsistent consumers.
- Health Club Example
  - many of the consumers who chose the membership option attended the gym irregularly.
  - It is possible that consumer does not have such a clear perception of the amount of physical exercise he wants to engage in. (over-optimistic)
  - It is also possible that consumer has self-control problem. (dynamic inconsistency)
- Since both assumptions are reasonable a priori, it is interesting to see if they have different implications for consumer behavior.

Consumption decisions

- The actual second-period action x^v
  - over-optimism: $x^v = \arg\max_x (v(x) - c(x))$
  - dynamic inconsistency: $x^v = \arg\max_x (\theta v(x) + (1-\theta)u(x) - c(x))$, approaches $x^v = \arg\max_x (v(x) - c(x))$ when $\theta = 1$ (perfect naivete).
- The "imaginary" action x^u that the consumer believes he will take in state u
  - same: $x^u = \arg\max_x (u(x) - v(x))$

Consumption decisions: welfare evaluation

- Even if the two models' predictions of x^v coincide in the case $\theta = 1$, welfare evaluation vary.
- In current model, there is no change of tastes, and we unequivocally use v to evaluate the outcome.
- For the dynamic inconsistency model (full naivete)
  - x^v is ex-post efficient from the point of view of self 2,
  - but x^v is inefficient from self 1's perspective.

Contractual Forms

When consumer holds biased beliefs, the qualitative structure of the optimal price scheme is the same in both models.

- It induces two relevant second-period actions, x^u and x^v.
- Firm believes that consumer will take the latter, while consumer assigns positive probability to the former.
- Consumer's net surplus is positive (negative) in state u (v).

Contractual Forms

When consumer's belief over second-period preferences is unbiased ($\theta = 0$)

- Over-optimism: induces $x^* = \arg\max_x (v(x) - c(x))$
- Dynamic inconsistency: induces $x^* = \arg\max_x (u(x) - c(x))$
  - A fully sophisticated, dynamically inconsistent consumer seeks a commitment device that will implement his first-period preferences.

Screening consumer's type

- In the previous lecture, we have seen that when there are sophisticated consumers and full naifs, monopolist is perfectly able to screen consumers.
- This property is no longer true when consumers are dynamically consistent.
  - Unbiased consumer: $(x^*, T^*) = (\arg\max_x (v(x) - c(x)), v(x^*))$
  - Over-optimistic consumer with $\theta > 0$: evaluates the price scheme for unbiased consumer at $\theta \cdot [u(x^*) - v(x^*)] > 0$
  - Over-optimistic consumer would strictly prefer this price scheme to the contract aimed at him.

Over-confidence

Over-confidence

- The bias referred to above as *over-optimism* captured the tendency to over-estimate the **expected future benefit** from firm's product.
- Another bias, referred to as *over-confidence*, is defined as underestimation of the **variance** of the product's future benefit.

Model

- In period 1
  - Firm offers a price scheme $t : X \to \mathbb{R}$.
  - Consumer decides whether to accept or not. (outside option 0)
- In period 2
  - State $s \in S = [0, 1]$ is realized.
  - In state s, consumer's second-period willingness to pay is given by $u_s(x) = \min(x, s)$
  - Consumer choose an action x given u_s and t.
- Interpretation of $u_s(x) = \min(x, s)$: consumer has random satiation point, yet as long as his consumption quantity falls below the satiation point, his marginal willingness to pay is equal to 1 in all states.
- Monopolist's prior belief is uniform over S.
- Monopolist incurs zero costs.

Unbiased Consumer

- When consumer's belief is unbiased, his prior over S is uniform.
- The price scheme $t(x) = x$ maximizes firm's profit.
    - $u_s(x) = \min(x, s)$: as long as his consumption quantity falls below the satiation point, his marginal willingness to pay is equal to 1 in all states.
    - Consumer's consumption quantity is equal to his satiation point in every state, i.e., $x_s = s$, hence it is ex-post efficient.

Extreme Overconfidence

- Now assume that consumer's belief displays an extreme form of overconfidence.
- Specifically, it assigns probability one to some $s^* \in S$.
- Thus, consumer completely ignores the noisiness of his satiation point, and regards his point estimate as a sure prediction.

Extreme Overconfidence

Firm's problem is

$$\max_{t,(x_s)_{s\in S}} \int_0^1 t(x_s)\mathrm{d}s$$

$$\text{s.t. } u_{s^*}(x_{s^*}) - t(x_{s^*}) \geq 0 \tag{IR}$$

$$x_s \in \arg\max_x [u_s(x) - t(x)], \text{ for every } s \in [0,1] \tag{IC}$$

Extreme Overconfidence

- It turns out that in this simple example, there is an optimal price scheme with the structure of a *three-part tariff*.
- Such a tariff consists of
  1. a lump-sum payment,
  2. zero marginal price for quantities below some critical level, and
  3. a strictly positive, constant marginal price for quantities above it.
- Three-part tariffs are commonly observed in a variety of industries, mainly telecommunication.

Extreme Overconfidence

Proposition 2

There is a solution $(t^*, (x_s^*)_{s\in[0,1]})$ *to the monopolist's problem, such that*

(i) $x_s^* = s$ *for all* $s \in [0,1]$.

(ii) $t^*(x) = \max(x, s^*)$

Extreme Overconfidence

- $(t^*, (x_s^*)_{s\in[0,1]}) = (\max(x, s^*), s)$ satisfies IR: $u_{s^*}(x_{s^*}) - t(x_{s^*}) = s^* - s^* = 0$
- As to IC, we need to show that for every s, $u_s(x_s^*) - t(x_s^*) \geq u_s(x) - t(x)$ for all x.
- This inequality can be rewritten as follows: $s - \max(s, s^*) \geq \min(x, s) - \max(x, s^*)$.
  - When $x \geq s$, the inequality is $s - \max(s, s^*) \geq s - \max(x, s^*) \iff \max(s, s^*) \leq \max(x, s^*)$
  - When $x < s$, the inequality is $s - \max(s, s^*) \geq x - \max(x, s^*)$ $\iff s - x \geq \max(s, s^*) - \max(x, s^*)$
- The inequality holds for all x, s, s^*.

Extreme Overconfidence

- In order to prove the optimality of t^*, consider some $(t, (x_s)_{s\in[0,1]})$ that satisfies IR and IC.
- We will show that $t(x_s) \leq \max(s, s^*)$ for all s.
  - $t^*(x_s^*) = \max(s, s^*)$
- By IC, $x_s = \arg\max_x [u_s(x) - t(x)]$ and $u_s(x) = \min(x, s) \leq s$:

$$
\begin{aligned}
& u_s(x_s) - t(x_s) \geq u_s(x_{s^*}) - t(x_{s^*}) \\
\underbrace{\Longrightarrow}_{u_s(x)\leq s} \; & s - t(x_s) \geq u_s(x_{s^*}) - t(x_{s^*})
\end{aligned}
\tag{1}
$$

Extreme Overconfidence: $t(x_s) \leq \max(s, s^*)$

Case I: $s \geq s^*$

- Recall (1): $s - t(x_s) \geq u_s(x_{s^*}) - t(x_{s^*})$
- $u_s(x) = \min(x, s)$ and $s \geq s^*$ implies $u_s(x) \geq u_{s^*}(x)$ for all x.
- RHS of (1) $= u_s(x_{s^*}) - t(x_{s^*}) \geq u_{s^*}(x_{s^*}) - t(x_{s^*}) \underbrace{\geq}_{\text{IR}} 0$
- LHS of (1) ≥ 0
- $t(x_s) \leq s$
- $t(x_s) \leq \max(s, s^*)$

Extreme Overconfidence: $t(x_s) \leq \max(s, s^*)$

Case II: $s < s^*$

- Subcase (i): $x_{s^*} < s$
  - Recall (1): $s - t(x_s) \geq u_s(x_{s^*}) - t(x_{s^*})$
  - $x_{s^*} < s < s^*$ and $u_s(x) = \min(x, s) \implies u_s(x_{s^*}) = x_{s^*}, u_{s^*}(x_{s^*}) = x_{s^*}$
  - RHS of (1) $= u_s(x_{s^*}) - t(x_{s^*}) = u_{s^*}(x_{s^*}) - t(x_{s^*}) \underbrace{\geq}_{\text{IR}} 0$
  - LHS of (1) ≥ 0
  - $t(x_s) \leq s$
  - $t(x_s) \leq \max(s, s^*)$

Extreme Overconfidence: $t(x_s) \leq \max(s, s^*)$

Case II: $s < s^*$

- Subcase (ii): $x_{s^*} \geq s$
  - Recall (1): $s - t(x_s) \geq u_s(x_{s^*}) - t(x_{s^*})$
  - $x_{s^*} \geq s$ and $u_s(x) = \min(x, s) \implies u_s(x_{s^*}) = s$
  - RHS of (1) $= u_s(x_{s^*}) - t(x_{s^*}) = s - t(x_{s^*})$
  - (1) becomes $s - t(x_s) \geq s - t(x_{s^*}) \implies$ $t(x_s) \leq t(x_{s^*}) \underbrace{\leq}_{\text{IR}} u_{s^*}(x_{s^*}) = \min(x_{s^*}, s^*) \leq s^*$
  - $t(x_s) \leq s^*$
  - $t(x_s) \leq \max(s, s^*)$

Extreme Overconfidence: $t(x_s) \leq \max(s, s^*)$

- Combining the cases, we have $t(x_s) \leq \max(s, s^*)$ for all s
- Therefore, t^* is optimal.

Intuition: Three-part tarrif

- Since consumer underestimates the probability of consuming below s^*, he does not mind paying the lump sum because the average price is not prohibitively high under the plan to consume s^*.
- The average price is prohibitively high if consumer chooses $x < s^*$, but he fails to take this scenario into account.
- On the other hand, when consumer learns that he wants to consume more than s^*, the lump sum is a sunk cost, and he is willing to pay the added payment for increased consumption.

Intuition: Three-part tarrif

- Consumer regrets having signed the contract whenever $s < s^*$ because he ends up paying more than his willingness to pay in these states.
- However, when $s \geq s^*$, consumer's net ex-post payoff is zero.
- Thus, the three-part tariff exploits consumers only to the extent that their over-confident belief reflects over-optimism.

Unforeseen Contingencies

Unforeseen Contingencies

Another common example of a belief bias is unawareness of future contingencies that affect willingness to pay for a product.

Unforeseen Contingencies

- We consider a two-period model of a competitive market:
  - In period 1, firms offer a basic product.
  - In period 2, consumers who bought the product may purchase an additional component that complements the basic product, referred to as an "add-on."
- Examples of "add-on":
  - Mini-bars and telecommunication services in hotel rooms
  - Many electric appliances are sold "bare" with a number of missing features, which are offered as add-ons only after the sale

Model

- There are two firms and a continuum of consumers.
- Firms offer a basic product and an add-on.
- For simplicity, set the costs of both components to zero.
- In period 1
  - Each firm i simultaneously chooses a strategy (p_i^1, p_i^2), where p_i^1 is the price of firm i's basic product and p_i^2 is the price of firm i's add-on.
  - Consumers choose whether to buy the basic product, and from which firm. (outside option is 0)
- In period 2
  - Conditional on buying the basic product from firm i in period 1, consumer decides in period 2 whether to buy the firm's add-on at the price it charges.
  - Consumer cannot buy the add-on from firm j, because each firm's add-on is compatible only with the firm's own basic product.

Model

- All consumers are willing to pay 1 for the basic product, independently of the firm from which it is bought.
- In contrast, there is heterogeneity among consumers in their evaluation of the add-on.
- Conditional on having purchased the basic product, consumer's willingness to pay for the add-on is distributed over $[0, 1]$ according to a strictly increasing and differentiable *cdf* F.
- Firm's revenue from selling add-ons at a price p to consumers: $R(p) = p \cdot [1 - F(p)]$
- Assume that R is hump-shaped.
- Monopoly price for add-on is $p^m = \arg\max_p R(p)$.

Case 1: Sophisticated consumers

- This is a rational-consumer benchmark: consumers are fully aware of the future demand for add-on, and in particular they know their own valuation.
- Consumers choose according to the prices that firms charge for both basic product and add-on.
- If consumer with valuation of add-on v purchases from firm i with (p_i^1, p_i^2), his payoff is

$$\underbrace{1 - p_i^1}_{\text{base product}} + \underbrace{\max(v - p_i^2, 0)}_{\text{add-on}}$$

  - In period 2 the consumer buys the add-on if and only if $p_i^2 \leq v$.

Case 1: Sophisticated consumers (Symmetric Equil.)

Proposition 3

When consumers are sophisticated, firms charge $p^2 \geq 0$ *and* $p^1 = -p^2$ *in symmetric Nash equilibrium.*

Case 1: Sophisticated consumers (Equil. Construction)

First, firms earn zero profits in symmetric Nash equilibrium.

- Consider an equilibrium strategy (p^1, p^2).
- By the equilibrium symmetry, each firm attracts half the consumer population, thus earning a profit of $\frac{1}{2} \cdot [p^1 + R(p^2)]$.
- Suppose that this profit is strictly positive.
- Consider a deviation to the strategy $(p^1 - \varepsilon, p^2 - \varepsilon)$, where $\varepsilon > 0$.
- Because the deviating firm offers a strictly lower price for both components, it will attract all consumers.
- If ε is sufficiently small, the deviation is profitable.

Case 1: Sophisticated consumers (Equil. Construction)

Next, we show that $p^2 \leq 0$ must hold.

- Consider a putative equilibrium strategy (p^1, p^2) such that $p^2 > 0$.
- By the zero-profit result, $p^1 + R(p^2) = 0$.
- Since $p^2 > 0$, $R(p^2) = p^2 \cdot [1 - F(p^2)] < p^2$, hence $p^1 + p^2 > p^1 + R(p^2) = 0$.
- Suppose that firm 1 deviates to $(p^1 + \varepsilon, p^2 - 2\varepsilon)$, where $\varepsilon > 0$ is arbitrarily small. ($p^1 + \varepsilon + p^2 - 2\varepsilon > 0$ still holds.)
- Then, firm 1 will attract all consumers for whom $v \geq p^2 - 2\varepsilon$ and will generate a strictly positive profit from them.
- At the same time, firm 1 will fail to attract all consumers for whom $v < p^2 - 2\varepsilon$. (earn zero from these consumers)
- Therefore, the deviation is profitable.

Case 1: Sophisticated consumers (Equil. Construction)

Finally, we show that $p^1 = -p^2$ must hold.

- Therefore, (p^1, p^2) must satisfy
  - $p^1 + R(p^2) = 0$ (zero-profit)
  - $p^2 \leq 0$
- $p^2 \leq 0 \implies F(p^2) = 0 \implies R(p^2) = p^2 \cdot [1 - F(p^2)] = p^2$
- $p^1 + p^2 = p^1 + R(p^2) = 0 \implies p^1 = -p^2$.

Case 1: Sophisticated consumers

Now, we check any strategy (p^1, p^2) satisfying $p^2 = -p^1 \leq 0$ is an equilibrium strategy.

- Under current strategy, every consumer purchases both base product and add-on.
- Total payment is $p^1 + p^2 = 0$.
- Deviation to higher $p^1 + p^2$ will lead to no clientele.
- Deviation to lower $p^1 + p^2$ leads to negative profit.

Case 2: Naive consumers

- In this case, consumers are unaware of the possibility that they will demand an add-on.
- They choose in period 1 entirely according to the basic product's price: buy from firm i if $p_i^1 < p_j^1$ and $p_i^1 \leq 1$.
- Ties between firms are resolved symmetrically.
- In period 2, consumers become aware of the add-on, learn their valuation, and buy the add-on if and only if $p_i^2 \leq v$.

Case 2: Naive consumers

Proposition 4

When consumers are naive, firms charge $p^2 = p^m$ and $p^1 = -R(p^m)$ in symmetric Nash equilibrium.

Case 2: Naive consumers

- Firms earn zero profits in symmetric Nash equilibrium. (Proof for sophisticated consumers applies.)
- Therefore, any symmetric equilibrium strategy (p^1, p^2) satisfies $p^1 = -R(p^2)$.
- Next, we show that $p^2 = p^m$ must hold.
  - Assume that $p^2 \neq p^m$.
  - Suppose that firm 1, say, deviates to the strategy $(p^1 - \varepsilon, p^m)$.
  - Since consumers are naive and choose only according to the price of basic product, firm 1 will attract all consumers as a result of the deviation.
  - By assumption, $p^2 \neq p^m$, $R(p^m) > R(p^2)$.
  - If $\varepsilon > 0$ is sufficiently small, $p^1 - \varepsilon + R(p^m) > 0$.
  - Deviation is profitable.

Case 2: Naive consumers

- When the consumer population consists entirely of naifs, firms set the price of add-on at the monopolistic level p^m, and basic product's price is negative.
  - The loss from the basic product is exactly offset by the profit from the add-on.
- This simple result provides an account of ultra-competitive pricing of "bare" products coupled with non-competitive pricing of their add-ons.
- Add-on cross-subsidizes basic product.

Case 3: A mixed consumer population

Now suppose that consumer population consists of both naifs (probability $\lambda \in (0,1)$) and sophisticates (probability $(1-\lambda)$).

Case 3: A mixed consumer population

Proposition 5

There exists no symmetric pure-strategy Nash equilibrium when the consumer population consists of both naifs and sophisticates.

Case 3: A mixed consumer population

- Firms earn zero profits in symmetric Nash equilibrium. (Proof for sophisticated consumers applies.)
- Therefore, any symmetric equilibrium strategy (p^1, p^2) satisfies $p^1 = -R(p^2)$.
- If $p^2 > 0$
  - Firm 1 deviating to $(p^1 + \varepsilon, p^2 - 2\varepsilon)$, where $\varepsilon > 0$ is arbitrarily small, is profitable. ($p^1 + \varepsilon + p^2 - 2\varepsilon > 0$ still holds.)
  - Firm 1 can attract part of sophisticated customers and earn positive profit.
  - Naifs will not purchase since p^1 is higher.
- If $p^2 \leq 0$, and thus $p^1 + p^2 = 0$
  - Firm 1 deviating to $(p^1 - \varepsilon, p^m)$ is profitable.
  - Firm 1 can attract all naifs and earn positive profit.
  - Sophisticates will not purchase since aggergate price is higher.

Case 3: A mixed consumer population

- We consider a variant of the basic model.
- In many situations, awareness of future contingencies that generate a demand for add-ons also allows consumer to obtain a cheaper substitute for firm's add-on.
- For instance, a consumer anticipating a need for a cold drink after checking into a hotel will get a beverage in advance, thus mitigating "mini-bar exploitation."
- To take an extreme case, assume that sophisticated consumers have costless access to a perfect substitute for add-on.

Case 3: A mixed consumer population

Proposition 6

Assume that sophisticates have access to a costless, perfect substitute for the add-on. Then, there is a symmetric Nash equilibirum where firm charges $(p^1, p^2) = (-\lambda R(p^m), p^m)$.

Case 3: A mixed consumer population

- Firms earn zero profits in symmetric Nash equilibrium. (Proof for sophisticated consumers applies.)
- In this case, sophisticated consumers buy the basic product only (as long as its price does not exceed 1).
- Price for basic product $p_1 \leq 0$.
  - If $p^1 > 0$, firm 1 deviating to $(p^1 - \varepsilon, p^m)$ is profitable.
  - Attract both sophisticates and naifs; earn positive profit.

Case 3: A mixed consumer population

- Since sophisticates would not purchase add-on and $p_1 \leq 0$, firms are not interested in attracting sophisticates.
- Firms can afford to set p^2 at the monopoly level, thus extracting full monopoly profits from naifs.
- Zero-profit implies $p^1 = -\lambda R(p^m)$.
  - Firms price basic product below marginal cost as part of a competitive strategy designed to attract naifs.
  - Firms are mindful that sophisticates will exploit this pricing strategy.

Case 3: A mixed consumer population

Main Message: Rational consumers can exert a negative externality over boundedly rational ones.

- In the present context, the reason is that firms are unable to discriminate explicitly between the two types of consumers.
- As a result, any attempt to exploit naifs is met with counter-exploitation by the sophisticates, who buy the basic product.
- As the fraction of sophisticates increases, the equilibrium price of basic product goes up toward zero.
  - $p^1 = -\lambda R(p^m) \uparrow 0$ as $\lambda \downarrow 0$
  - Intuitively, total monopoly profit from selling add-on to naifs, which is meant to cover the loss from basic product, goes down.
- This means that the total price that naifs end up paying rises.

Educating unaware consumers

- When consumers are unaware of a future contingency, a firm can fix this by explicitly alerting consumer's attention to the contingency, or by advertising the price it charges in that contingency.
- In this way, consumer immediately becomes aware of the price that the rival firm charges for the add-on as well.

Question: Will firms have an incentive to do it?

Educating unaware consumers

- Let us modify the model by enabling each firm to turn all naive consumers into sophisticates, unilaterally and at no cost.
- Firm's decision whether to educate consumers in this fashion is simultaneous with its pricing decision.

Educating unaware consumers

- Consider the original model in which sophisticates do not have a substitute for the add-ons.
- Assume all consumers are naifs.
- In Nash equilibrium, at least one firm will educate consumers, such that all consumers will act as sophisticates.
  - Poposition 4: $p^2 = p^m, p^1 = -R(p^m)$
  - If any firm could costlessly make consumers aware of the add-on, it could at the same time raise p^1 by ε and lower p^2 by 2ε, where $\varepsilon > 0$ is arbitrarily small.
  - This is precisely the profitable deviation that we used to knock out putative equilibria with $p^2 > 0$ when consumers are sophisticated.

Educating unaware consumers

- Let us now turn to the extended model in which $\lambda \in (0,1)$ and sophisticates have access to a costless perfect substitute for add-ons.
- In this case, even when firms are able to educate consumers, the previous equilibrium survives where firms choose not to educate naifs.
  - Proposition 6: $(p^1, p^2) = (-\lambda R(p^m), p^m)$
  - If a firm deviates by turning all naifs into sophisticates, this has an unwarranted by-product that these consumers will not buy the add-on unless it is negatively priced.
  - Firm will only be able to attract consumers if
    - $p^2 > 0$ and $p^1 \leq -\lambda R(p^m)$ (consumers only purchase basic product) or
    - $p^2 \leq 0$ and $p^1 + p^2 \leq -\lambda R(p^m)$ (consumers purchase both basic product and add-on).

"Uninformative information"

- The difference between "unawareness" and "zero probability" is important when we discuss "educating" consumers.
- Unaware consumer can update his beliefs in a way that a Bayesian rational consumer never would.
- Bayesian rational consumer:
  1. No Bayesian who assigns zero probability to an event would ever change this belief.
  2. Bayesian would never update his beliefs when confronted with statements that convey no information.
- Unaware consumer: A consumer who is initially unaware of an event E may change his beliefs when encountering the statement "E is either true or false".
  - This statement, while tautological and therefore completely uninformative in the usual sense, may heighten his awareness of E.

Can the model be rationalized?

- Our basic model of competitive add-on pricing when the consumer population consists entirely of naifs can be reformulated as a model with conventionally rational consumers.
- Assume that consumers are fully aware of all contingencies.
- However, instead of assuming that firms commit to (p^1, p^2) at the outset, suppose that firms can only commit to p^1 in period 1, and they determine the add-on price p^2 only in period 2.

Can the model be rationalized?

- Now firms' monopoly power in the provision of add-on does not arise from consumer naivete, but from their inability to commit to the add-on price as part of their competitive strategy in period 1.
- This is a classic "hold-up" problem.
- In sub-game perfect equilibrium of the ensuing extensive-form game, firms will set $p^2 = p^m$ in period 2, and competitive pressures will drive profits to zero, such that $p^1 = -R(p^m)$.

Can the model be rationalized?

- This reformulation does not survive some of the extensions of the basic model.
- The analysis of the mixed population case is nonsensical if firms cannot commit to p^2 in period 1.
- In addition, our discussion of educating consumers, that is, using "uninformative information" to bring add-ons to their awareness, becomes irrelevant under this reformulation.

Conclusion

Conclusion

- When consumers have over-optimistic prior beliefs regarding their future preferences, a monopolist wants to offer an exploitative contract - that is, a contract that a consumer with unbiased beliefs would not accept.
- Unlike the model with dynamically inconsistent consumers, unbiased consumers may exert an informational externality on consumers with biased beliefs.
- Pricing effects such as add-on pricing and three-part tariffs can be interpreted as consequences of plausible belief biases.

Conclusion

- Complex Pricing Strategies:
  - Three-part tariffs is optimal when consumers have biased beliefs of their future preferences (over-confidence).
  - Highly competitive pricing of basic products coupled with monopolistic pricing of add-ons emerges when consumers are unaware of contingencies that create a demand for add-ons.