Micro Theory I Review

Chapter 1: Preference and Choice

Preference-based Approach

- Set of alternatives X
- Preference relation $\succsim$, Strict preference relation $\succ$, Indifference relation $\sim$
- Rational preference: complete and transitive
- Utility functions: *only if* $\succsim$ is rational

Choice-based Approach

- Choice structure $(\mathscr{B}, C(\cdot))$
- Revealed preference: $x \succsim^* y \iff \exists B \in \mathscr{B}$ s.t. $x, y \in B$ and $x \in C(B)$
- W.A.R.P

Relationship between preference relations and choice rules

- Rational preference implies W.A.R.P.: $(\mathscr{B}, C^*(B, \succsim))$ satisfies W.A.R.P
- When $\mathscr{B}$ includes all 2 & 3-element subsets of X (and $C(B) \neq \emptyset$), then W.A.R.P implies rational preference. (Prop 1.D.2)

Chapter 2: Consumer Choice

- Consumption set: $X = \mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_l \geq 0 \text{ for } l = 1, 2, ..., L\}$.
- Walrasian budget set: $B_{p,w} = \{x \in \mathbb{R}^L_+ : p \cdot x \leq w\}$
- Walrasian demand correspondence: $x(p, w)$
- Assumptions on $x(p, w)$: HD0, Walras' law
- Given that $x(p, w)$ satisfies HD0 and Walras' law, then

  W.A.R.P $\iff$ Compensated Law of Demand (Prop 2.F.1)

Chapter 3: Classical Demand Theory

- Assumptions on $\succsim$: monotone, locally non-satiated, convex, continuous, etc.
- (Debreu's Thm) Rational + continuous $\implies$ $\exists$ continuous utility representation

Utility maximization problem (UMP)

$$v(p, w) = \max_{x \in \mathbb{R}^L_+} u(x)$$

$$\text{s.t. } p \cdot x \leq w$$

- Properties of $x(p, w)$ (Prop 3.D.2)
- Properties of indirect utility function $v(p, w)$ (Prop 3.D.3)

Expenditure minimization problem (EMP)

$$e(p, u) = \min_{x \in \mathbb{R}^L_+} p \cdot x$$

$$\text{s.t. } u(x) \geq u$$

- Properties of expenditure function $e(p, u)$ (Prop 3.E.2)
- Properties of Hicksian demand correspondence $h(p, u)$ (Prop 3.E.3)
- Hicksian demand function satisfies compensated law of demand (Prop 3.E.4)

UMP and EMP are dual problems (Prop 3.E.1)

$$\boldsymbol{e(p, v(p, w)) = w} \quad \text{and} \quad \boldsymbol{v(p, e(p, u)) = u} \tag{3.E.1}$$

$$\boldsymbol{h(p, u) = x(p, e(p, u))} \quad \text{and} \quad \boldsymbol{x(p, w) = h(p, v(p, w))} \tag{3.E.4}$$

Relationships between Demand, Indirect Utility, and Expenditure Functions

- $h(p, u)$ and $e(p, u)$: $h(p, u) = \nabla_p e(p, u)$
- $h(p, u)$ and $x(p, w)$: $D_p h(p, u) = D_p x(p, w) + D_w x(p, w) x(p, w)^T$ (Slutsky equation)
- $x(p, w)$ and $v(p, w)$: $x(\bar{p}, \bar{w}) = -\frac{1}{\nabla_w v(\bar{p}, \bar{w})} \nabla_p v(\bar{p}, \bar{w})$ (Roy's identity)

Chapter 5: Production

- Production set: $Y = \{y \in \mathbb{R}^L : F(y) \leq 0\}$
- Assumptions on Y: nonempty, closed, free disposal, etc.

Profit maximization problem (PMP)

$$\pi(p) = \max_{y \in \mathbb{R}^L} p \cdot y$$

$$\text{s.t. } y \in Y(\text{ or } F(y) \leq 0)$$

- Properties of $\pi(p)$ and $y(p)$ (Prop 5.C.1)
- law of supply

Single-output production

$$\max_{z \geq 0, q \geq 0} pq - w \cdot z$$

$$\text{s.t. } q \leq f(z)$$

- **Cost minimization problem (CMP)**

$$c(w, q) = \min_{z \geq 0} w \cdot z$$

$$\text{s.t. } f(z) \geq q$$

  - Properties of $c(w, q)$ and $z(w, q)$ (Prop 5.C.2)

- From cost minimization to profit maximization

$$\max_{q \geq 0} pq - c(w, q)$$

Geometry, aggregation, efficient production

- Geometry: total cost, marginal cost, average cost
- Aggregation: merger does not affect supply behavior when firms are price takers
- Efficient production: relationships between efficient production and profit-maximizing production (Prop 5.F.1 and Prop 5.F.2)