

Chapter 9. Uncertainty (Exercises)

Exercise 9.1: Taxation of Risky Income Consider the model of one safe and one risky asset. Now, we introduce taxation of interest income on the risky asset (with deduction allowed for losses) at the rate $\tau \in (0, 1)$. Thus, the final random wealth is

$$W = W_0 + (1 - \tau)xr.$$

Question 1: Show that the first-order condition for an interior optimum x is

$$\int_{\underline{r}}^{\bar{r}} rU'(W_0 + (1 - \tau)xr)f(r)dr = 0.$$

Question 2: Deduce that if τ changes, the optimum x changes keeping $(1 - \tau)x$ constant. Therefore, if the tax rate on risky income increases, so does the amount of wealth held in the risky asset. Suggest an economic intuition.

Exercise 9.2: Saving with Uncertainty A consumer lives for two periods. Income in period 1 is sure and equal to Y_1 . The income Y_2 in period 2 can be random. If he saves S from his period-1 income, he gets total return (principal plus interest) of rS in period 2, where r can be random. His objective is to maximize the expected present value of the utility of consumption in the two periods:

$$U(Y_1 - S) + \delta \mathbb{E}[U(Y_2 + rS)].$$

where $U' > 0$ and $U'' < 0$.

Question 1: Write down the first- and second- order conditions. Show that as Y_1 increases, S also increases but at a smaller rate, that is, the marginal propensity to save lies between 0 and 1.

Question 2: Next, suppose that Y_2 is sure but r is random, and examine the effect of an increase in Y_2 .

Question 3: Finally, suppose r is sure but Y_2 is random, and examine the effect of an increase in r .