Chapter 7. Concave Programming (Exercises)

Exercise 7.1: Minimization. Develop the theory of minimization of a convex function along lines parallel to those used in this chapter for maximization of a concave function.

Exercise 7.2: Convexity of Maximum Value Function. Let θ be a vector of parameters and consider the problem of choosing x to maximize $F(x, \theta)$ subject to $G(x) \leq c$. Let $V(\theta)$ denote the maximum value as a function of the parameters.

Question 1: Prove that if F is convex as a function of θ for each fixed x, then V is convex.

Question 2: In Chapter 5, we saw geomatrically that the minimum cost of producing a given quantity of output, regarded as a function of input prices, is concave. Derive that formally as a corollary of the above general result.

Exercise 7.3: More on Linear Programming.

Question 1: Show that the optimal solution x^* of the linear-programming problem of Example 7.1, and the corresponding vector of multipliers λ^* are such that

$$\mathcal{L}(x, \lambda^*) \leq \mathcal{L}(x^*, \lambda^*) \leq \mathcal{L}(x^*, \lambda)$$

for all non-negative x and λ. In other words, x^* maximizes the Lagrangian when $\lambda = \lambda^*$, and λ^* minimizes the Lagrangian when $x = x^*$. In other words, the graph of the Lagrangian in (x, λ) space is shaped like a saddle. Therefore, (x^*, λ^*) is said to be a *saddle-point* of the Lagrangian.

Question 2: Let $V(a, c)$ denote the maximum value function of the linear-programming problem. Show that V is convex in a for each fixed c, and concave in c for each fixed a.