Sampling-Based Reasoning II: Obfuscation[1]

Xiaoxiao Hu (Wuhan University)

March 18, 2020

[1] Chapter 7 of *Bounded Rationality and Industrial Organization* (Spiegler, 2011)

Introduction

Introduction

- Last Chapter: exogenous randomness
  - Firms are restricted in the amount of randomness they can inflict on consumers.
- This Chapter: endogenous randomness
  - Firms can deliberately confuse consumers, or *obfuscate*.
  - Examples of obfuscation:
    - Firms can change price and quality over time.
    - Firms can discriminate among consumers on an arbitrary basis.
    - Firms can adopt complicated multi-dimensional pricing schemes.
  - Consumers relying on a sampling-based procedure are prone to making *inference errors* in the face of such obfuscation strategies.

Introduction

Main questions:

- Does sampling-based reasoning give firms an incentive to obfuscate?
- How does this motive respond to competitive pressures?
- What are the welfare implications of obfuscation?

Model of Competitive Obfuscation

Model of Competitive Obfuscation

- A set $\{1, ..., n\}$ of identical profit-maximizing firms that offer a homogeneous product
- A continuum of identical consumers with unit demand
- Consumers' willingness to pay normalized to 1
- Consumers have no outside option.
- Firms have a constant marginal cost of $c \in [0, 1)$.
- Firms play a simultaneous-move game in which a strategy for firm i is a *cdf* $G_i(p)$.

Sampling Procedure

The sampling procedure is the same as described in last Chapter.

- Each consumer independently draws a *single* sample from each of the n *cdf*s.
- Consumer chooses the alternative that gives him the highest payoff in the sample. (Assume a symmetric tie-breaking rule.)
- The outcome is a new, independent draw from the *cdf* associated with the chosen alternative.
- A firm incurs its cost only when it sells the product.

Pricing Strategy

- Permissible prices are $\in (-\infty, 1]$
  - The upper bound 1: customers' willingness to pay
  - No lower bound: in particular, firms are allowed to price below marginal cost c
- Recall the firms pricing strategy follow a *cdf* $G_i(p)$. Justifications include:
  - Firms often change their product prices over time, by means of sales and special offers.
  - Firms also discriminate on the basis of criteria that are not always transparent to the consumer.

Pricing Strategy

- The firms' strategies are probability distributions.
  - Such strategies are different from "mixed strategies."
  - In the current scenario, "mixed strategy" would mean mixing over different probability distributions.

Firm i's payoff function

- Support of G_i: $T_{G_i} \subset (-\infty, 1]$
- Expected price according to G_i: μ_{G_i}
- Probability of being chosen when realization is p: $H_i(p|G_{-i})$, where $G_{-i} = (G_j)_{j \neq i}$
- Ex-ante expected probability of being chosen: $\mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right]$
- For continuous *cdf* and well-defined densities:

$$H_i(p|G_{-i}) = \prod_{j \neq i} \left[1 - G_j(p)\right]$$

$$\mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right] = \int_{-\infty}^{1} \prod_{j \neq i} \left[1 - G_j(p)\right] g_i(p) \mathrm{d}p$$

Firm i's payoff function

Firm i's payoff function is

$$\pi_i(G_1, ..., G_n) = \underbrace{[\mu_{G_i} - c]}_{\text{expected markup}} \cdot \underbrace{\mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right]}_{\text{size of clientele}}.$$

Randomization as "obfuscation device": An example

- Consider an example with $n = 3$ and $c = 0$.
- Suppose firms 1 and 2 both play *cdf* $G \equiv U[0, 1]$.
- Consider the two different pricing strategies for firm 3:
  - Assigning probability 1 to some price $\beta \in (0, 1)$ gives payoff

$$\pi_3 = \underbrace{\beta}_{\text{markup}} \cdot \underbrace{(1-\beta)^2}_{\text{clientele}}$$

  - Assigning probability β to $p = 1$ and probability $1 - \beta$ to $p = 0$ gives

$$\pi_3 = \underbrace{[\beta \cdot 1 + (1-\beta) \cdot 0]}_{\text{markup}} \cdot \underbrace{(1-\beta)}_{\text{clientele}} = \beta \cdot (1-\beta).$$

  - Strategy 2 is better.

Randomization as "obfuscation device": An example

Why is Strategy 2 better?

- The price is "competitive" (0) in some situations and "monopolistic" (1) in others.
- The former situations generate a clientele by creating an impression that the firm is cheap;
- The latter situations generate revenue by exploiting this impression.
- The obfuscation is extreme:
    - Consumer ends up choosing firm 3 if and only if $p_3 = 0$ in the sample;
    - The actual price is 1 with probability β.

Nash Equilibrium

Nash Equilibrium: Illustration

Claim

Let $n = 2$ and $c = 0$. The uniform distribution over the intercal $[0, 1]$ is a symmetric equilibrium strategy.

Nash Equilibrium: Illustration

- $H_2(p|G_1) = 1 - p$ for every $p \in [0, 1]$.
  - $p > 1$: no clientele
  - $p < 0$: lower expected revenue condition on being chosen, without changing market share
- Firm 2's profit function

$$\pi_2 = \mu_{G_2} \cdot \mathbb{E}_{G_2}\left[H_2(p|G_1)\right] = \mu_{G_2} \cdot \mathbb{E}_{G_2}(1 - p) = \mu_{G_2} \cdot (1 - \mu_{G_2}).$$

- π_2 is only a function of expected price.
- $\mu_{G_2} = \frac{1}{2}$ maximizes π_2.
- Any $G_2(p)$ with $\mu_{G_2} = \frac{1}{2}$ is a best response.
- $U[0, 1]$ is such a distribution.

Nash Equilibrium

Proposition 1

For every $n \geq 2$, the game has a unique symmetric Nash equilibrium. Each firm plays the cdf

$$G(p) = 1 - \sqrt[n-1]{\frac{2(1-p)}{n(1-c)}} \tag{1}$$

over the support $\left[1 - \frac{n}{2}(1-c), 1\right]$.

Step 1: The support of the equilibrium strategy

Result

$G_i(p)$ is continuous function over an interval with 1 as the upper bound.

Step 1: The support of the equilibrium strategy

- Continuity: if G places an atom on some p, then it is profitable for any firm to deviate by shifting the weight from p slightly downward.

Step 1: The support of the equilibrium strategy

- Upper bound 1:
  - Suppose upper bound is $\overline{p} \neq 1$.
  - Consider a deviation by shifting all the weights assigned to $(\overline{p} - \varepsilon, \overline{p})$ to $p = 1$.
  - $\pi_i = [\mathbb{E}(p_i) - c] \cdot \mathbb{E}_{G_i}[H_i(p|G_{-i})]$
  - Such change raises $\mathbb{E}(p_i)$ by more than $(1 - \overline{p}) \cdot (1 - G(\overline{p} - \varepsilon))$. (previous average price for the range $(\overline{p} - \varepsilon, \overline{p})$ is less than $\overline{p}$, but calculated as if it is $\overline{p}$ here)
  - Reduces $\mathbb{E}_{G_i}[H_i(p|G_{-i})]$ by less than $(1 - G(\overline{p} - \varepsilon))^2$ (calculated as if there is only 1 opponent and all previous weights are on $\overline{p} - \varepsilon$)
  - When ε is small, the reduction in $\mathbb{E}_{G_i}[H_i(p|G_{-i})]$ compared to the increase in $\mathbb{E}(p_i)$ is negligible.
- A similar argument establishes that the support of G contains no holes.

Step 2: Linearity of the residual demand function

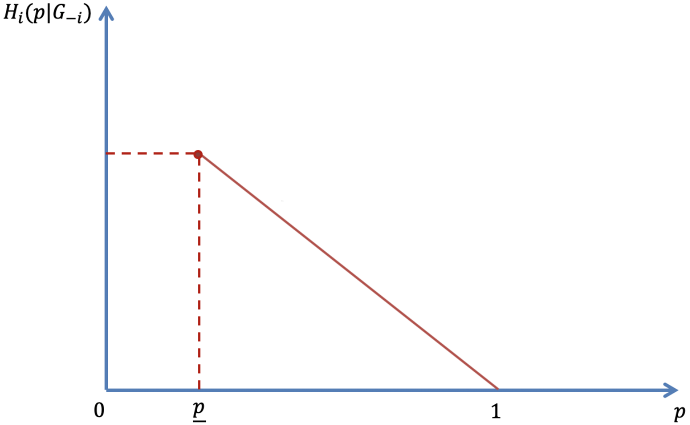
This is the **key step** in the proof.

Result

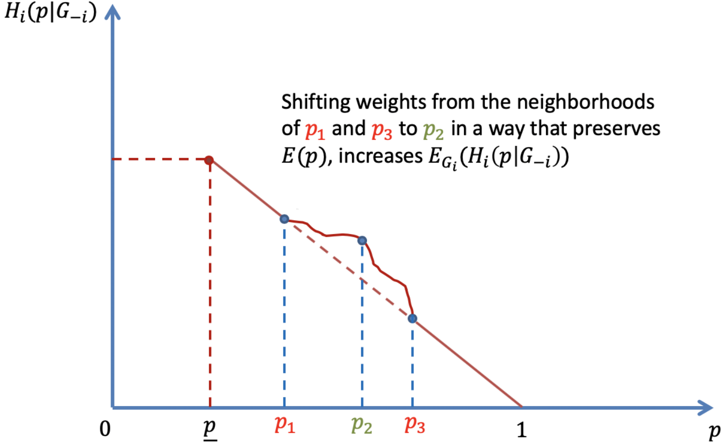
$H_i(p|G_{-i})$ could be written in the form $H_i(p|G_{-i}) = a(1-p)$, where $a > 0$ is a constant to be determined.

Step 2: Linearity of the residual demand function

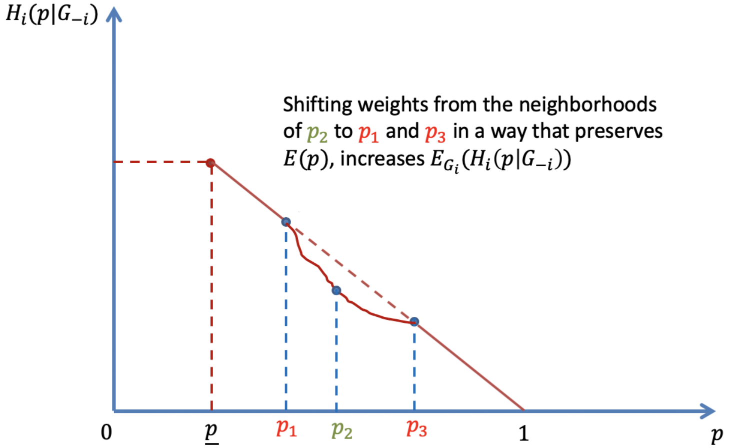
- $H_i(p|G_{-i})$ is decreasing in p.
- $H_i(p=1|G_{-i})=0$.
- Next two slides will show that $H_i(p|G_{-i})$ must be linear.



Step 2: Linearity of the residual demand function



Step 2: Linearity of the residual demand function



Step 2: Linearity of the residual demand function

Restate the result:

Result

$H_i(p|G_{-i})$ could be written in the form $H_i(p|G_{-i}) = a(1-p)$, where $a > 0$ is a constant to be determined.

- $H_i(p|G_{-i})$ is linear in p, and $H_i(p = 1|G_{-i}) = 0$ implies

$$H_i(p|G_{-i}) = a(1-p).$$

- $a > 0$ captures the fact that $H_i(p|G_{-i})$ is a decreasing function.

Step 3: Deriving the equilibrium formula

- Size of clientele: $\mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right] = \mathbb{E}_{G_i}\left[a(1-p)\right] = a(1-\mu_{G_i})$
- Firm i's payoff function:

$$\pi_i = [\mu_{G_i} - c] \cdot \mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right] = [\mu_{G_i} - c] \cdot a(1-\mu_{G_i})$$

$$\overset{FOC}{\Longrightarrow} \ \mu_{G_i} = \frac{1+c}{2}.$$

- By symmetry of equilibrium, each Firm i gets $\frac{1}{n}$ of the clientele: $\mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right] = \frac{1}{n}$
- $\mathbb{E}_{G_i}\left[H_i(p|G_{-i})\right] = a(1-\mu_{G_i}) = \frac{1}{n} \implies a = \frac{2}{n(1-c)}$

Step 3: Deriving the equilibrium formula

- From $H_i(p|G_{-i}) = a(1-p)$ and $a = \frac{2}{n(1-c)}$:

$$H_i(p|G_{-i}) = a(1-p) = \frac{2(1-p)}{n(1-c)}$$

- On the other hand,

$$H_i(p|G_{-i}) = \prod_{j \neq i} \left[1 - G_j(p)\right] = \left[1 - G(p)\right]^{n-1}$$

- Therefore,

$$G(p) = 1 - \sqrt[n-1]{\frac{2(1-p)}{n(1-c)}}$$

- Lower bound $\underline{p}$:

$$G(\underline{p}) = 0 \implies \underline{p} = 1 - \frac{n}{2}(1-c)$$

Corollary

Corollary 1

The expected price according to $G(p)$ is $\mu_G = \frac{1+c}{2}$, independently of the number of firms n.

- This result is a by-product of the proof.
- See Slide 26.

Corollary

Corollary 2

For all $n \geq 2$, $G(p; n+1)$ is a mean preserving spread of $G(p; n)$.

- "$G(p; n+1)$ is a *mean preserving spread* of $G(p; n)$" is equivalent to "For every $p \in (-\infty, 1)$, $\int_p^1 G(w; n)\mathrm{d}w$ is decreasing with n." (see Mas-Colell, Whinston & Green (1995, p. 198))
- Thus, an increase in n results in an increase in the variance of the equilibrium *cdf*, without affecting expected price.

Intuition

- When n increases, there are 2 strategic considerations:
  - Competitive motive: offer an attractive price distribution;
  - Incentive to confuse the consumer: introduce greater variance into price distribution.
- It turns out that in equilibrium, firms respond to greater competition by cultivating the “obfuscatory effect” only.

Strict incentive to obfuscate?

Question: Is the firms' equilibrium preference for obfuscation strict?

- For all n, firms are indifferent between the equilibrium strategy $G(p; n)$ and the degenerate *cdf* that assigns probability one to $p = \frac{1+c}{2}$.
- In this sense, the incentive to obfuscate is weak in equilibrium.

Positive probability assigned to $p < c$

Prices fall below marginal cost with positive probability.

- When $n > 2$, lower bound $\underline{p} = 1 - \frac{n}{2}(1-c) < c$
- Probability of $p < c$:

$$G(c) = 1 - \sqrt[n-1]{\frac{2}{n}} \tag{2}$$

- The expression (2) does not behave monotonically with n.
  - It attains a maximum of approximately 20 % when $n = 4$, and then decreases monotonically.
  - As n tends to infinity, the probability of below-marginal-cost prices vanishes.

Exercise 7.1

Exercise 7.1

Let $n = 2$ and $c = 0$. Modify the model by allowing prices to get values in $\{0, 1\}$ only. In addition, assume that for every firm $k = 1, 2$, the consumer observes $m_k = k$ independent sample points from the firm's price distribution and chooses the firm with the lowest average price in the sample. In case of a tie, he chooses firm 2. If the average price of both firms in the consumer's sample is 1, he opts out and chooses none of the firms. Formulate the interaction between the firms as a strategic game and find its Nash equilibrium.

Exercise 7.2

Exercise 7.2

Let $n = 2$ and $c = 0$. Modify the model by assuming that consumers draw a sequence of two sample points (p_i^1, p_i^2) from each firm i's *cdf* G_i. If $p_i^k < p_j^k$ for both $k = 1, 2$ (i.e., if firm i dominates firm j in the consumer's sample) the consumer chooses firm i. If no firm dominates another in the sample, the consumer chooses each firm with probability $\frac{1}{2}$. Formulate the interaction as a strategic game and show that there is a Nash equilibrium in which $G_1 = G_2 \equiv U[0, 1]$.

Lower bound on prices

- The main role of the assumption that prices have no lower bound is to simplify the proof and obtain a simple formula for the symmetric equilibrium strategy.
- If we add a lower bound $p = c$, the result of $\mu_G = \frac{1+c}{2}$, and $G(p; n+1)$ is a *mean-preserving spread* of $G(p; n)$ for all $n \geq 2$ would continue to hold.
- In this case, as $n \to \infty$, the equilibrium strategy converges to a simple distribution that assigns probability $\frac{1}{2}$ to each of the extreme values $p = c$ and $p = 1$.
- This limit distribution has the maximal possible variance subject to the constraint that $p \in [c, 1]$.
- For proof, see Appendix B of
Spiegler, R. (2006). Competition Over Agents With Boundedly Rational Expectations. *Theoretical Economics*, 1(2), 207-231.

Welfare Analysis

Welfare Criteria

- If consumers' true preference only concerns the **expected value**, then raising the number of competitors in the market does not make consumers better or worse off.
- If consumers' true preferences over probability distributions display risk aversion, then our equilibrium analysis implies that raising the number of competitors makes consumers strictly worse off, because $G(p; n+1)$ is a *mean-preserving spread* of $G(p; n)$.
  - However, any definition of consumer welfare that incorporates an explicit assumption regarding the consumers' risk attitudes is problematic in this model.
  - Our consumer is unaware of the actual structure of the *cdf*s he is facing.
  - At the time he makes a decision, there is no reason to assume that the risk he perceives has anything to do with the actual risk.

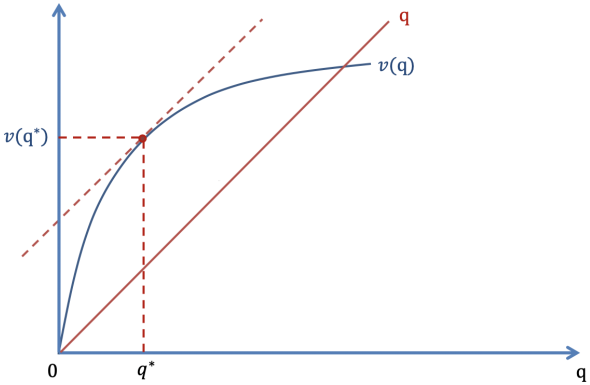
Production Inefficiencies

Model of production inefficiencies

- Now assume that firms choose not only the prices of their product but also its quality.
- p denotes price; $q \in [0, \infty)$ denotes product *quality*, measured in cost units.
- $v(q)$ is consumers' willingness to pay of quality q: v is increasing and strictly concave, with $v(0) = 0$
- Consumer's payoff for (q, p) is $v(q) - p$.
- Firm's payoff is $p - q$.
- Total surplus $v(q) - q$ attains maximum at a unique $q^* > 0$; normalize $v(q^*) - q^* = 1$.

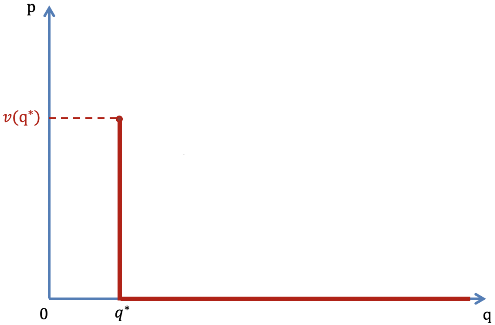
Bound on prices and Pareto efficiency

- We impose a lower bound on prices: $p \in [0, v(q)]$.
- Pareto-efficient pairs (q, p):
  - $q = q^*$ and $p \in [0, v(q^*)]$; or
  - $q > q^*$ and $p = 0$ (Consumer: $v(q)$; Firm: $-q$)



Firm's strategy

- Firm's strategy is a probability distribution over pairs (q, p).
- Simplification: it is always sub-optimal for firms to assign positive probability to quality-price pairs below the Pareto frontier.
  - $q = q^*$ and $p \in [0, v(q^*)]$; or
  - $q > q^*$ and $p = 0$
- Firm i's strategy can be reduced to a *cdf* G_i over $\pi = p - q$.
- The support of π is $(-\infty, 1]$.



Firm's payoff function

- Probability of being chosen when (p, q) induces π: $H_i(\pi|G_{-i})$
- $H_i(\pi|G_{-i})$ decreases with π
- Firm i's payoff function is $\mathbb{E}_{G_i}(\pi_i) \cdot \underbrace{\mathbb{E}_{G_i}\left[H_i(\pi|G_{-i})\right]}_{\text{size of clientele}}$
- For continuous *cdf* and well-defined densities:

$$H_i(\pi|G_{-i}) = \prod_{j \neq i} \left[1 - G_j(\pi)\right]$$

$$\mathbb{E}_{G_i}\left[H_i(\pi|G_{-i})\right] = \int_{-\infty}^{1} \prod_{j \neq i} \left[1 - G_j(\pi)\right] g_i(\pi) \mathrm{d}\pi$$

Nash Equilibrium

Proposition 2

For every $n \geq 2$, the game has a unique symmetric Nash equilibrium. Each firm plays the cdf

$$G(\pi) = 1 - \sqrt[n-1]{\frac{2(1-\pi)}{n}}$$

over the support $\left[1 - \frac{n}{2}, 1\right]$.

Welfare Implications

Proposition 3

The symmetric equilibrium has the following properties

1. $\mathbb{E}\pi = \frac{1}{2}$ *for every* $n \geq 2$.
2. *For every* $n > 2v(q^*)$, $q > q^*$ *with positive probability.*
3. *The expectation of* $v(q) - q$ *is strictly decreasing with* n *in the range* $n > 2v(q^*)$.

Constant mean

1. $\mathbb{E}\pi = \frac{1}{2}$ for every $n \geq 2$.
   - Same as in the previous model.
   - Detailed calulation, see Slide 26.

Positive probability on $q > q^*$

2. For every $n > 2v(q^*)$, $q > q^*$ with positive probability.
   - $q > q^*$ is equivalent to $\pi = -q < -q^*$.
   - The probabiliity of $\pi < -q^*$ is

$$G(-q^*) = 1 - \sqrt[n-1]{\frac{2(1+q^*)}{n}}$$

   - $G(-q^*) > 0$ implies $n > 2(1+q^*) = 2v(q^*)$
   - This result is analogous to $\Pr(p < c) > 0$ when $n > 2$.

Expected total surplus

3. The expectation of $v(q) - q$ is strictly decreasing with n in the range $n > 2v(q^*)$.
   - Total surplus is a concave function of π.

$$v(q) - q = \begin{cases} v(q^*) - q^* & \text{if } \pi \in [-q^*, 1] \\ v(-\pi) + \pi & \text{if } \pi \in (-\infty, -q^*) \end{cases}$$

   - Since $G(\cdot; n+1)$ is a *mean-preserving spread* of $G(\cdot; n)$, the expected value of any concave function of π is lower under $G(\cdot; n+1)$ than under $G(\cdot; n)$. (See Mas-Colell, Whinston & Green (1995, p.198).)

Interpretation of the Results

- The number of competitors does not affect equilibrium industrial profits.
- However, when the number of firms is sufficiently high, expected total surplus falls below the maximal level 1.
- The efficiency loss increases with n, and is borne entirely by the consumers.
- In this regard, greater competition has a strictly negative effect on both consumer welfare adn social welfare.

Forces driving the inefficiency result

- *Mean-preserving spread* property of G: the firms' increased effort to obfuscate in response to greater competition
- Diminishing marginal utility from quality

Market Intervention: Introducing "Simple" Options

Introducing “Simple” Options

- A market intervention that that a priori seems to benefit consumers is to endow them with an attractive additional option, which is *simple to understand*.
- In the context of a market for financial services, say, this “plain vanilla” option can be provided by the government (or by a government-regulated provider).
- Alternatively, a regulator can force firms to offer a simple option in addition to their “regular,” possibly complex product.

First model of “simple” option

- Same model as our first model: *Model of Competitive Obfuscation*
- Add a “simple” option, denoted 0
- Consumer now faces $n+1$ alternatives $(G_0, G_1, ..., G_n)$
- G_0 is the “simple” option: degenerate *cdf*, assign probability 1 to some price p_0
- For simplicity, assume that the support of firms’ *cdf*s is restricted to $[c, 1]$

Special Case: $p_0 = c$

- Suppose the "simple" outside option is priced at marginal cost (i.e., $p_0 = c$).
- Because $H_i(p) = 0$ for all $p > c$, firm i has no incentive to assign any weight to prices in $(c, 1)$.
- Thus, firm i's best-reply assigns some probability α to $p = c$ and probability $1 - \alpha$ to $p = 1$, yielding an expected payoff of:

$$\underbrace{[\alpha \cdot 0 + (1-\alpha) \cdot (1-c)]}_{\text{expected markup}} \cdot \underbrace{\alpha H_i(c)}_{\text{clientele}}.$$

- The value of α that maximizes this expression is $\frac{1}{2}$.

General result for $p_0 \in [c, \frac{1+c}{2})$

Proposition 4

Let $n > 2$. If $p_0 \in [c, \frac{1+c}{2})$, then in symmetric Nash equilibrium, firms play a distribution that assigns probability $\frac{1}{2}$ to each of the extreme prices $p = c$ and $p = 1$.

For detailed proof, see Proposition 6 of
Spiegler, R. (2006). Competition Over Agents With Boundedly Rational Expectations. *Theoretical Economics*, 1(2), 207-231.

Comparison with distribution in the basic model

- Adding a simple and attractively priced option to the consumers' set of available market options causes firms to react by maximizing the variance of their *cdf*.
- It does not, however, affect the expected equilibrium price.
- In other words, firms respond to the simple option by cultivating the obfuscatory motive without trying to behave more competitively in expectation.

Welfare loss compared to "simple" outside option only

- When $p_0 \in (c, \frac{1+c}{2})$, the probability that the consumer ends up choosing the simple outside option is $\left(\frac{1}{2}\right)^n$.
- This expression decreases with n, and converges to zero as $n \to \infty$.
- Expected price difference between the market alternative and the simple outside option: $\frac{1+c}{2} - p_0$
- The consumer's expected welfare loss is thus

$$\underbrace{\left[1 - \left(\frac{1}{2}\right)^n\right]}_{\text{prob. of choosing firms}} \cdot \underbrace{\left[\frac{1+c}{2} - p_0\right]}_{\text{expected price difference}}$$

- This expression increases with n.

Exercise 7.3

Exercise 7.3

Suppose that there is only one firm in the market. The firm chooses a *cdf* over $[c, 1]$. In addition to this firm, the consumer has access to a simple option at a fixed price p_0. For simplicity, assume that the consumer breaks ties in favor of the firm. Show how the firm's optimal *cdf* varies with the firm's marginal cost c.

Second model of "simple" option

Rather than offering a simple additional option, the regulator forces each firm to offer a simple alternative, modeled as a degenerate *cdf*, in addition to its regular, possibly complex product.

Firm's strategy

A strategy for firm i is a pair (p_i, G_i)

- $p_i \in [c, 1]$ is the price of the firm's simple product
- G_i is a *cdf* over $[c, 1]$ which represents the firm's complex product

Result

- The consequences of this intervention for symmetric Nash equilibrium are very similar to those of the first intervention.
- Because consumers never err in evaluating the simple products, competitive forces push their prices to marginal cost.
- Thus, when firms design the pricing strategy for their complex product, they are in the same strategic situation as in the first regulatory intervention.
- Each firm assigns probability $\frac{1}{2}$ to each of the extreme prices, $p = c$ and $p = 1$.

Summary

Summary

- When consumers use sampling-based reasoning to evaluate market alternatives and firms can generate essentially any probability distribution over prices, market competition gives firms incentive to "obfuscate": to generate noisy price distributions.
- This incentive responds strongly to the intensity of competition, at the expense of "proper" competitive behavior.
  - In particular, increasing the number of competitors induces a *mean-preserving spread* of the distribution over prices.
  - Regardless of the number of competitors, consumers always get half the surplus in equilibrium.

Summary

- When firms determine **both price and quality**, and the production of quality has diminishing marginal returns, the welfare conclusions become even more drastic: increasing the number of competitors may strictly reduce total expected surplus and consumer welfare.
- A regulatory intervention that introduces **"simple" options** into the market (either by offering them through a government agency or by forcing firms to offer them) may be ineffective, because firms respond to these policies with obfuscation rather than with "proper" competitive behavior.