Dynamically Inconsistent Preferences[1]

Xiaoxiao Hu (Wuhan University)

April 15, 2020

[1]Based on Chapter 2 and 4 of *Bounded Rationality and Industrial Organization* (Spiegler, 2011)

Introduction

Introduction

- In the previous lectures, we have learned sampling-based reasoning and coarse reasoning.
  - Sampling-based reasoning: examine a small part in detail and extrapolate naively
  - Coarse reasoning: exhibit "coarse representation" of the entire available data
- These consumers are boundedly rational.

Introduction

- Facing those bounded rational consumers, firms may use complex pricing strategies or spurious variety to explore the bounded rational consumers.
- Complex pricing strategies:
  - Firms deliberately confuse consumers, or *obfuscate*.
  - Firms use complex price patterns as a discrimination device.
- Spurious variety:
  - Firms have an incentive to create an impression of a differentiated product in order to weaken competitive pressures.

Introduction

- Complex pricing strategies and spurious variety are very common when firms face (these and other types of) bounded-rational consumers.
- This week, we will introduce another type of bounded rationality: dynamic inconsistency.
- Complex pricing strategies and spurious variety are also results of firms' profit maximizing strategies.

Dynamically Inconsistent Preferences

Introduction

- "Dynamically Inconsistent Preferences" simply means changes of tastes from time to time.
- In real-life, such phenomena are very common.
- The change in preferences could be due to
  - the mere passage of time, or
  - the presence of a particular contingency.
- The following examples illustrate this idea.

Example: Present bias

- Present-biased individual always place more weight on the current welfare compared to the future welfare.
- As time goes by, and tomorrow becomes today, such individual would change the preference ranking.
- For instance, such individual may say
  - I will start eating healthy next week.
  - I will wake up early tomorrow morning and do exercises.
  - I will do the assignments tomorrow.
  - etc.

Example: Temptations

- Consider a two-stage decision problem.
- In the first stage, you choose a restaurant.
- In the second stage, while at the restaurant, you decide which dish to order.
- You are on a diet, so from the point of view of the first stage, eating steak is inferior to eating salad.
- If you could commit yourself to a particular dish ex-ante, you would commit to eating salad rather than steak.
- However, once at the restaurant, facing a menu that contains both salad and steak, you are tempted by the latter and go for it.

Example: Dynamic implications of reference-point effects

- Imagine that you are considering signing up for a service.
- Before you sign up, you are willing to pay at most \$20.
- Suppose that at the time you are offered the service contract, the stated price is \$20.
- After you sign up for it, the firm raises the price unexpectedly to \$25.
- You are still able to cancel the deal at no cost.
- From your point of view before you signed up, you would prefer to cancel.
- However, having signed up, your point of view has changed; you weigh the pros and cons differently, and decide that you do not want to cancel.

The Multi-selves Model

- Dynamic inconsistent individuals have changing tastes.
- The most widely practiced modeling tool is the "multi-selves model."
- Decision maker is modelled as a collection of different players ("selves") having idiosyncratic preferences.
- We assign a self to a decision node, in the same way that we would assign a player to a node in an extensive-form game.

The Multi-selves Model

- We will apply this tool to two-period decision problems to analyze the Temptation problem.
- Recall the example of temptatation:
  - In period 1, you prefer salad to steak;
  - However, in period 2, if facing a menu containing steak, you cannot resist it.
  - In period 1, you choose which restaurant to go to;
  - In period 2, you choose one dish from the menu.

The Multi-selves Model

- In period 1, consumer chooses a menu A, which contains a non-empty subset of the set of consumption decisions Z.
- In period 2, he chooses an element from A.
- Self $j \in \{1, 2\}$ moves in period j and has preference relation $\succsim_j$ over Z.
- In the previous example,
  - $Z = \{steak, salad\}$
  - $salad \succ_1 steak$
  - $steak \succ_2 salad$

Sophisticated Consumers

- Sophisticated consumers are aware of the taste change.
- In our example, self 1 will choose the menu $\{salad\}$ in period 1, and consequently, self 2 will be forced to eat salad in period 2.

Naivete

- Full naivete means that in period 1, self 1 believes that self 2 has identical preferences.
- In our example, self 1 is indifferent between the menu $\{salad\}$ and $\{steak, salad\}$ in period 1.
- Partial naivete is in-between sophistication and full naivete.
- We will discuss two types of partial naivete later.

Contracting with Different types of consumers

- Consumers of different levels of sophistication display different preferences over menus in preriod 1.
- We will see how firm(s) would contract with each type of consumers.
- The main focus is the monopoly model.

Monopoly Pricing

The Monopoly Model

- Consider a monopolistic firm that interacts with a population of consumers with changing tastes.
- In period 1
  - Firm sets price scheme $t : X \to \mathbb{R}$.
  - X is the set of actions that a consumer can take in period 2.
  - t specifies a transfer (possibly negative) from consumer to firm.
  - Consumer chooses whether to accept the scheme.
  - Consumer's outside option is 0.
- In period 2
  - If consumer accepts a price scheme, he is committed to it.
  - Consumer chooses one action $x \in X$ and pays the transfer $t(x)$.

Consumer's preference

To capture the time-inconsistency,

- In period 1, consumer's willingness to pay for any action is given by the function $u : X \to \mathbb{R}$;
- In period 2, his willingness to pay is given by the function $v : X \to \mathbb{R}$.

Facing a pricing scheme t, consumer's evaluation is

- In period 1: $u(x) - t(x)$;
- In preiod 2: $v(x) - t(x)$.

The Monopoly Model

- Let $c(x)$ denote the cost that firm incurs when consumer takes action x.
- Assume that each of the functions $u - c$, $v - c$, and $u - v$ attains a non-negative maximum at a unique point.

Sophisticated Consumers

- Sophisticated consumer is aware that self 2's preferences will be given by v.
- In period 1, firm and consumer agree on v, and thus which action would be taken given a price scheme.
- All firm needs to fix is a pair (x^*, T^*), where x^* is the action consumer chooses in period 2 and $T^* = t(x^*)$ is the payment he makes.
- Therefore, we can set $t(x)$ to be arbitrarily large for any $x \neq x^*$.

Sophisticated Consumers

- Firm's problem is

$$\max_x T - c(x)$$
$$\text{s.t. } u(x) - T \geq 0.$$

- The solution is

$$x^* = \arg\max_x (u(x) - c(x))$$
$$T^* = u(x^*)$$

Full Naivete

- In period 1, naive consumer believes that self 2's preferences will still be given by u.
- In period 1, Firm and consumer disagree on which action would be taken in period 2.
- The price scheme can be summarized by a 4-tuple (x^u, T^u, x^v, T^v).
  - (x^u, T^u) is the action-payment pair in the event that consumer's willingness to pay is u;
  - (x^v, T^v) is the action-payment pair in the event that consumer's willingness to pay is v.
- Consumer believes in the former event; firm believes in the latter.
- We can ignore all other actions, because firm can set $t(x) = +\infty$ for any $x \neq x^u, x^v$.

Full Naivete

Firm's problem is

$$\begin{aligned} \max_{x^u, T^u, x^v, T^v} \; & T^v - c(x^v) \\ \text{s.t. } & v(x^v) - T^v \geq v(x^u) - T^u && (IC_2V) \\ & u(x^u) - T^u \geq u(x^v) - T^v && (IC_2U) \\ & u(x^u) - T^u \geq 0. && (IR) \end{aligned}$$

- (IC_2V) ensures that consumer would choose x^v when his preferences are given by v, as firm expects;
- (IC_2U) ensures that consumer would choose x^u when his preferences are given by u, as consumer expects;
- (IR) is consumer's participation constraint, determined by consumer's first-period preferences and what he expects will happen in period 2.

Full Naivete

$$
\begin{aligned}
&\max_{x^u, T^u, x^v, T^v} T^v - c(x^v) \\
\text{s.t. } & v(x^v) - T^v \geq v(x^u) - T^u && (IC_2V) \\
& u(x^u) - T^u \geq u(x^v) - T^v && (IC_2U) \\
& u(x^u) - T^u \geq 0. && (IR)
\end{aligned}
$$

- (IR) must be binding.
- Otherwise, we could raise T^v and T^u by the same arbitrarily small $\varepsilon > 0$.
  - Constraints are still satisfied.
  - Profit is larger.
- (IC_2V) must be binding.
- Otherwise, we could raise T^v by an arbitrarily small $\varepsilon > 0$.

Full Naivete

The binding constraints could be written as

$$\begin{cases} v(x^v) - T^v = v(x^u) - T^u \\ u(x^u) - T^u = 0 \end{cases} \Longrightarrow \begin{cases} T^v = v(x^v) + u(x^u) - v(x^u) \\ T^u = u(x^u) \end{cases}$$

The problem becomes

$$\max_{x^u, x^v} [v(x^v) - c(x^v)] + [u(x^u) - v(x^u)]$$
$$\text{s.t. } u(x^u) - v(x^u) \geq u(x^v) - v(x^v). \qquad (IC_2U)$$

Full Naivete

$$\max_{x^u, x^v} [v(x^v) - c(x^v)] + [u(x^u) - v(x^u)]$$
$$\text{s.t. } u(x^u) - v(x^u) \geq u(x^v) - v(x^v). \tag{IC_2U}$$

- Solution to the unconstraint maximization problem is

$$x^v = \arg\max_x (v(x) - c(x))$$
$$x^u = \arg\max_x (u(x) - v(x))$$

- The constraint holds automatically since $x^u = \arg\max_x (u(x) - v(x))$

Comparison between Sophisticates and Naifs

- For sophisticates, the optimal price scheme induces the same action as optimal pricing with a dynamically consistent consumer whose willingness-to-pay function is u.
- For naifs, the optimal price scheme induces the same action as optimal pricing with some dynamically consistent consumer, but the willingness-to-pay function is v.

Comparison between Sophisticates and Naifs

- Firm's earn more profit from naifs.
  - Profit from sophisticates is $T^* - c(x^*) = u(x^*) - c(x^*) = \max_x (u(x) - c(x))$
  - Profit from naifs is $T^v - c(x^v) = [v(x^v) - c(x^v)] + [u(x^u) - v(x^u)] = \max_x (v(x) - c(x)) + \max_x (u(x) - v(x))$

Return to Slide 61

Screen Consumer's Type

- Now consider the population consisting of both naifs and sophisticates.
- "Cognitive type" is consumer's private information.
- Firm can offer different price schemes to screen consumers.

Screen Consumer's Type

- Optimal menu consist of two types of price schemes:
  - Perfect commitment devices aimed at sophisticates (x^*, T^*)
  - "Betting" price scheme aimed at naifs (x^u, T^u, x^v, T^v)
- Both types of consumers evaluate the first scheme at 0.
- Naifs evaluates the second scheme at 0, whereas the sophisticates knows that it is a "betting" scheme that yields a negative payoff.
  - $T^v = v(x^v) + u(x^u) - v(x^u)$ is strictly higher than consumer's u-willlingness to pay for x^v.
- Consumers will choose the contracts designed for them.

Partial Naivete

- Now we consider Partial Naifs who hold beliefs in between sophisticates and full naifs.
- We consider two types of partial naivetes:
  - Magnitude Naivete: underestimates the magnitude of change of preference.
  - Frequency Naivete: underestimates the likelihood of v.

Partial Naivete: Magnitude Naivete

- First, consider Magnitude Naivete: believe the second-period preferences will be, *with probability 1*, $w = \alpha u + (1 - \alpha)v$, for some $\alpha \in (0, 1)$.
- α is consumer's type; higher α represents greater naivete.
- A partially naive consumer is "almost fully sophisticated" if his value of α is close to zero, such that w is close to v.
- However, note that even such a consumer assigns zero probability to the true state v.

Magnitude Naivete

- Suppose firm knows it faces a partially naive consumer of type α.
- Let us solve for firm's optimal price scheme.
- As in the case of full naivete, without loss of generality, consider (x^w, T^w, x^v, T^v)
  - (x^w, T^w) is the action-payment pair in the event that the consumer's willingness to pay is w;
  - (x^v, T^v) is the action-payment pair in the event that the consumer's willingness to pay is v.
- Similarly, we can ignore all other actions, because the firm can set $t(x) = +\infty$ for any $x \neq x^w, x^v$.

Magnitude Naivete

Firm's problem is

$$\max_{x^u, T^u, x^v, T^v} T^v - c(x^v)$$

$$\text{s.t. } v(x^v) - T^v \geq v(x^w) - T^w \qquad (IC_2V)$$

$$w(x^w) - T^w \geq w(x^v) - T^v \qquad (IC_2W)$$

$$u(x^w) - T^w \geq 0. \qquad (IR)$$

- (IC_2V) ensures that consumer would choose x^v when his preferences are given by v, as firm expects;
- (IC_2W) ensures that consumer would choose x^w when his preferences are given by w, as consumer expects;
- (IR) is consumer's participation constraint, determined by consumer's first-period preferences and what he expects will happen in period 2.

Magnitude Naivete

The solution of this maximization problem turns out to be *exactly the same as for the fully naive consumer*!

- Same as in the full-naivete case, (IC_2V) and (IR) are binding.
- The problem becomes

$$\max_{x^w, x^v} [v(x^v) - c(x^v)] + [u(x^w) - v(x^w)]$$

$$\text{s.t. } w(x^w) - v(x^w) \geq w(x^v) - v(x^v). \qquad (IC_2W)$$

- The solution is

$$x^v = \arg\max_x (v(x) - c(x))$$

$$x^w = \arg\max_x (w(x) - v(x))$$

$$T^w = u(x^w)$$

$$T^v = v(x^v) + u(x^w) - v(x^w)$$

Magnitude Naivete

$$x^w = \arg\max_x (w(x) - v(x))$$
$$x^u = \arg\max_x (u(x) - v(x))$$

- $x^w = x^u$ since w is a convex combination of u and v:

$$\begin{aligned}
&\arg\max_x (w(x) - v(x)) \\
=&\arg\max_x [(\alpha u(x) + (1-\alpha)v(x)) - v(x)] \\
=&\arg\max_x [\alpha(u(x) - v(x)) + (1-\alpha)(v(x) - v(x))] \\
=&\arg\max_x (u(x) - v(x))
\end{aligned}$$

Magnitude Naivete

- The result shows that optimal price scheme pool together all consumers with $\alpha > 0$, even if a consumer is "almost fully" sophisticated.
- Note that there is a discontinuity at $\alpha = 0$.
- $\alpha = 0$ corresponds to sophistication.

Magnitude Naivete: Perfect Screening Result

- Consider the case where "cognitive type" is consumer's private information.
- Following the same logic as we dicussed before, the perfect screening result persists for an arbitrary population consisting of diversely (magnitude) naive consumers.

Partial Naivete: Frequency Naivete

- Now consider frequency naivete: assign probability $\theta \in (0, 1)$ to u and $(1 - \theta)$ to v.
- θ is consumer's type; higher θ represents greater naivete.

Frequency Naivete

- Suppose firm knows it faces a partially naive consumer of type θ.
- Let us solve for firm's optimal price scheme.

$$\begin{aligned} & \max_{x^u, T^u, x^v, T^v} T^v - c(x^v) \\ \text{s.t. } & v(x^v) - T^v \geq v(x^u) - T^u && (IC_2V) \\ & u(x^u) - T^u \geq u(x^v) - T^v && (IC_2U) \\ & \theta \left[u(x^u) - T^u\right] + (1-\theta)\left[u(x^v) - T^v\right] \geq 0. && (IR) \end{aligned}$$

- (IC_2V) ensures that consumer would choose x^v when his preferences are given by v ;
- (IC_2U) ensures that consumer would choose x^u when his preferences are given by u ;
- (IR) is consumer's participation constraint .

Frequency Naivete

Same as in the previous cases, (IC_2V) and (IR) are binding.

- (IR) must be binding.
- Otherwise, we could raise T^v and T^u by the same arbitrarily small $\varepsilon > 0$.
  - Constraints are still satisfied.
  - Profit is larger.
- (IC_2V) must be binding.
- Otherwise, we could raise T^v by an arbitrarily small $\varepsilon > 0$, and decrease T^u by $\frac{(1-\theta)}{\theta}\varepsilon$.
  - Constraints are still satisfied.
  - Profit is larger.

Frequency Naivete

- The problem becomes

$$\max_{x^u, x^v} \theta v(x^v) + (1-\theta)u(x^v) - c(x^v) + \theta[u(x^u) - v(x^u)]$$
$$\text{s.t. } u(x^u) - v(x^u) \geq u(x^v) - v(x^v). \qquad (IC_2U)$$

- The solution is

$$\begin{aligned}
x^v &= \arg\max_x (\theta v(x) + (1-\theta)u(x) - c(x)) \\
x^u &= \arg\max_x (u(x) - v(x)) \\
T^v &= \theta v(x^v) + (1-\theta)u(x^v) + \theta[u(x^u) - v(x^u)] \\
T^u &= \theta u(x^u) + (1-\theta)v(x^u) + (1-\theta)[u(x^v) - v(x^v)]
\end{aligned}$$

Frequency Naivete

- Consumer ends up taking an action that maximizes the *weighted* surplus: $x^v = \arg\max_x(\theta v(x) + (1-\theta)u(x) - c(x))$.
- The more sophisticated the consumer, the closer the action is to maximizing $u(x) - c(x)$.
- As θ approaches zero, the action-payment pair (x^v, T^v) converges to the action-payment pair induced by sophisticated consumer's price scheme.
- Thus, the solution is continuous with respect to consumer's degree of naivete, unlike the case of magnitude naivete.

Frequency Naivete: Failure of Perfect Screening

- Now consider the case in which the degree of consumer's frequency naivete is his private information.
- Unlike the case of magnitude naivete, perfect screening breaks down when consumers display partial frequency naivete.
- That is, firm could not extract all types' surplus.

Frequency Naivete: Failure of Perfect Screening

- To see why, consider two types θ, ϕ where $\phi > \theta$.
- Suppose firm offers two schemes t_θ and t_ϕ aiming at θ and ϕ, extracting all surpluses.
- First consider scheme t_θ, the first best solution (x^u, T^u, x^v, T^v) applies.

$$
\begin{aligned}
&\theta[u(x^u) - T^u] + (1-\theta)[u(x^v) - T^v] = 0 \\
&u(x^u) - T^u = (1-\theta)[(u(x^u) - u(x^v)) - (v(x^u) - v(x^v))] > 0 \\
&u(x^v) - T^v = \theta[(v(x^u) - v(x^v)) - (u(x^u) - u(x^v))] < 0
\end{aligned}
$$

- For type $\phi > \theta$, scheme t_θ generates positive expected payoff: $\phi[u(x^u) - T^u] + (1-\phi)[u(x^v) - T^v] > 0$.
- Thus, type ϕ would want to choose the contract designed for θ
- Firm cannot fully extract both types' surplus.

Competitive Pricing

Competitive Pricing

- We will only discuss the cases with sophisticates and full naifs.
- Assume that two identical firms face the same population of consumers as before:
  - all consumers have first-period preference u and second-period preference v,
  - but they may differ in their ability to predict future preferences.

Competitive Pricing

- In period 1
  - Two firms play a simultaneous-move game in which each firm offers a menu of price schemes.
  - Subsequently, each consumer chooses a price scheme from the union of firms' menus.
- In period 2
  - Consumer chooses an action x and makes the transfer.
- To simplify our exposition, assume that when consumer is indifferent among several actions in either period, we (as analysts) are free to break the tie at our will.

Competitive Pricing: Result

Proposition 1

There is a symmetric Nash equilibrium in which firms offer the menu of price schemes $\{t_s, t_n\}$*, such that*

(i) $x_s^* = \arg\max_x (u(x) - c(x))$, $T_s^* = c(x_s^*)$.

(ii) $x_n^v = \arg\max_x (v(x) - c(x))$, $T_n^v = c(x_n^v)$,
$x_n^u = \arg\max_x (u(x) - v(x))$, $T_n^u = c(x_n^v) + v(x_n^u) - v(x_n^v)$.

(iii) $t_s(x) = \infty$ *for every* $x \neq x_s^*$*, and* $t_n(x) = \infty$ *for every* $x \neq x_n^v, x_n^u$.

(iv) *Sophisticated consumers choose* t_s*, naive consumers choose* t_n*, and firms earn zero profits.*

Competitive Pricing: Analysis

- Suppose that both firms offer the menu $\{t_s, t_n\}$
- If a consumer chooses t_s, he must take the action x_s^* in period 2, because t_s is a perfect commitment device.
- If a consumer chooses t_n, he would take action x_n^v in period 2.
  - If choose x_n^v, gets $v(x_n^v) - c(x_n^v)$;
  - If choose x_n^u, gets $v(x_n^u) - [c(x_n^v) + v(x_n^u) - v(x_n^v)] = v(x_n^v) - c(x_n^v)$.
- Both price schemes generate zero profits for firms:
  - $t_s(x_s^*) = T_s^* = c(x_s^*)$
  - $t_n^v(x_n^v) = T_n^v = c(x_n^v)$

Competitive Pricing: Analysis

- Next, we check that consumers would stick to the price scheme designed for them; and firms would not deviate to offer another price scheme.
    - t_s is designed for sophisticated consumers.
    - t_n is designed for naive consumers.

Competitive Pricing: Analysis

- For t_s, the pair (x_s^*, T_s^*) solves

$$\max_x u(x) - T$$
$$\text{s.t. } T - c(x) \geq 0$$

- All surplus goes to consumers.
- Sophisticated consumers necessarily prefer t_s to t_n in period 1.
- No firm can deviate to a price scheme t_s' that sophisticated consumers will prefer to t_s and earn strictly positive profits.

Competitive Pricing: Analysis

- For t_n, $(x_n^u, T_n^u, x_n^v, T_n^v)$ solves

$$\max_{x^u, T^u, x^v, T^v} u(x^u) - T^u$$

$$\text{s.t. } v(x^v) - T^v \geq v(x^u) - T^u \qquad (IC_2V)$$

$$u(x^u) - T^u \geq u(x^v) - T^v \qquad (IC_2U)$$

$$T^v - c(x^v) \geq 0.$$

- t_n maximizes the perceived first-period payoff of naive consumers, subject to the constraint that the price scheme generates non-negative profits for firms.
- Naive consumers necessarily prefer t_n to t_s in period 1.
- No firm can deviate to a price scheme t_n' that naive consumers will prefer to t_n and earn strictly positive profits.

Competitive Pricing: Perfect Screening Result

- The perfect screening result continues to hold in the competitive case.
- That is, the fact that consumers' type is their private information does not change the price schemes they are offered in equilibrium.

Welfare Analysis

Welfare Analysis

- As we have already learned in the previous lectures, the walfare analysis for bounded rational consumers is always problematic.
- Here we perceive the consumer as a collection of selves with different preferences over the set of consequences.
  - Should we view the different selves as genuinely separate entities, and conduct welfare analysis in the same way that we do in standard, interpersonal games?
  - If so, what would constitute a proper "social" welfare function?
- Economists who apply the multi-selves model often adopt one self's preference relation as the welfare criterion.

Welfare Analysis: Monopoly

- We adopt self 1's preference relation as the welfare criterion.
- In the monopoly case,
  - Under the price scheme chosen by sophisticates, the outcome is efficient, because it induces an action that maximizes $u(x) - c(x)$.
    - The price scheme fully extracts consumer's surplus according to u.
  - As to the price scheme chosen by naive consumer types, the outcome induces an action that maximizes $v(x) - c(x)$
    - It is efficient in terms of self 2's preference relation.
    - $T^v = v(x^v) + u(x^u) - v(x^u)$ is strictly higher than consumer's u-willlingness to pay for x^v.
    - T^v is also higher than his v-willingness to pay for x^v
    - Thus, the price scheme aimed at naive consumers is unambiguously exploitative ex-post.

Welfare Analysis: Competition

Question: Does competition eliminate the element of exploitation inherent in the price schemes aimed at naive consumer types?

- Given that $x_n^v = \arg\max(v(x) - c(x))$ and $T_n^v = c(x_n^v)$, the equilibrium price scheme for naifs is clearly not exploitative according to v.
- However, when $u(x_n^v) < c(x_n^v)$, this price scheme is exploitative according to u.
- Thus, when second-period willingness to pay is significantly higher than first-period willingness to pay, it is possible that competition will not eliminate exploitation.

Educating Naive Consumers

Educating Naive Consumers: Monopoly

- Again, we restrain our discussion on fully naive consumers.
- In the case of monopoly, the optimal price scheme aimed at naifs generates a higher profit than the optimal price scheme aimed at sophisticates. Go to Slide 29
- Therefore, it is clear that the monopolist would not want to correct consumers' naivete.

Educating Naive Consumers: Competition

- If the consumer population consists entirely of naifs, equilibrium price schemes satisfy $x_n^v = \arg\max(v - c)$ and $T_n^v = c(x_n^v)$. (zero profit)
- A firm could deviate by turning all consumers into sophisticates and simultaneously offering them a perfect commitment device that implements the action $x^* = \arg\max(u - c)$ for the payment $T^* = c(x^*) + \varepsilon$. (positive profit)
  - By definition, $u(x^*) - c(x^*) > u(x_n^v) - c(x_n^v)$.
  - If $\varepsilon > 0$ is sufficiently small, all naifs-turned-sophisticates would strictly prefer the new price scheme.
- Thus, firms' ability to de-bias naive consumers destabilizes the competitive market outcome.

Educating Naive Consumers: Competition

- If there are already sophisticated consumers in the population, we saw that in the competitive model, the equilibrium menu contains a price scheme aimed at sophisticates, which generates, like the price scheme aimed at naifs, zero profits.
- In this case, no firm has an incentive to educate naive consumers, because it cannot attract them with a price scheme that generates strictly positive profits.

Two Applications

Credit card "teaser" rates

- Credit card companies often try to attract consumers with low "teaser rates" on small-size loans and other "welcome benefits," and then switch to post-introductory interest rates far above marginal cost.
- In period 1, when consumer contemplates acquiring a credit card, he would like to utilize it at a low level, whereas in period 2, after the credit card has been issued, consumer is tempted to use the credit card for additional borrowing.

Negative option offers

- A prevalent marketing device is to offer a product, typically accompanied by some immediate benefit such as a free trial period or a gift voucher, and require consumer to explicitly reject it later on in order to avoid additional charges.
- Consumers' attitudes to the cancellation option may change over time.
- For instance, consumer may exhibit a "reference point effect" that causes his evaluation of a price scheme to depend on whether he has already accepted it.
- Or he may find it too time-consuming to cancel.

(β, δ) Model

(β, δ) Model

- (β, δ) Model aims at the present bias problem.
- This is a popular model to analyze the time-inconsistency problem.
- This model modifies conventional discounting:

Period	1	2	3	4	...	n	...
conventional:	1	δ	δ^2	δ^3	...	δ^{n-1}	...
(β, δ):	1	$\beta\delta$	$\beta\delta^2$	$\beta\delta^3$	...	$\beta\delta^{n-1}$	...

- The conventional discounting is called *exponential discounting*; (β, δ) discounting is called *hyperbolic discounting*.

(β, δ) Model

- We provide a toy model.
- Consider doing exercise example that we discussed at the beginning of the lecture.
- Doing exercise itself is painful; but you will gain the benefit of being healthy in the future.
- The present-biased individual would have the utility function as follows: at time t,

$$U_t(\tau) = \begin{cases} -c_\tau + \beta\delta v & \text{if } \tau = t \\ -\beta\delta^{\tau-t}c_\tau + \beta\delta^{\tau-t+1}v & \text{if } \tau > t \end{cases}$$

  - τ is the period that the individual exercises.
  - c_t is the cost incurred at time τ if exercising at t, v is the future benefit.
  - We allow c_t to vary across period.

(β, δ) Model

- Consider a special case with three period.
- Parameters: $c_1 = 3$, $c_2 = 5$, $c_3 = 8$, $v = 10$, $\beta = 0.5$, $\delta = 1$.

$$U_t(\tau) = \begin{cases} -c_\tau + 5 & \text{if } \tau = t \\ -0.5c_\tau + 5 & \text{if } \tau > t \end{cases}$$

where $c_1 = 3, c_2 = 5, c_3 = 8$

- We could examine naifs, sophisticates and partial naifs.

(β, δ) Model: Naifs

$$U_t(\tau) = \begin{cases} -c_\tau + 5 & \text{if } \tau = t \\ -0.5c_\tau + 5 & \text{if } \tau > t \end{cases}$$

where $c_1 = 3, c_2 = 5, c_3 = 8$

Naifs are not aware of the preference change.

- $t = 1$, since $U_1(1) = 2$, $U_1(2) = 2.5$, $U_1(3) = 1$, procrastinating to $t = 2$;
- $t = 2$, since $U_2(2) = 0$, $U_2(3) = 1$, procrastinating to $t = 3$.
- $t = 3$ is the last period, and have to do it.

(β, δ) Model: Sophisticates

$$U_t(\tau) = \begin{cases} -c_\tau + 5 & \text{if } \tau = t \\ -0.5c_\tau + 5 & \text{if } \tau > t \end{cases}$$

where $c_1 = 3, c_2 = 5, c_3 = 8$

Sophisticates are aware of the change and thus do backward induction.

- In the eye of self 1,
  - $t = 3$ is the last period, and have to do it.
  - $t = 2$, $U_2(2) = 0$, $U_2(3) = 1$, procrastinate to $t = 3$;
  - $t = 1$, the individual compares doing it now and in $t = 3$
    - he knows that if he procrastinates to $t = 2$, the future self would further procrastinate to $t = 3$.
    - $U_1(1) = 2$, $U_1(3) = 1$ so he chooses to do it in $t = 1$.

(β, δ) Model: Partial Naivete

- Naifs believe the future selves would have $\hat{\beta} = 1$.
- Sophisticates hold the correct belief $\hat{\beta} = \beta$.
- Partial Naivete is in-between: they believe $\hat{\beta} \in (\beta, 1)$.

(β, δ) Model: Partial Naivete

- Consider the previous example with $\hat{\beta} = 0.6$.

$$U_t(\tau) = \begin{cases} -c_\tau + 10\beta & \text{if } \tau = t \\ -\beta c_\tau + 10\beta & \text{if } \tau > t \end{cases}$$

$$\text{where } c_1 = 3, c_2 = 5, c_3 = 8, \beta = 0.5, \hat{\beta} = 0.6$$

For partial naifs, they conduct backward induction.

- In the eye of self 1,
  - $t = 3$ is the last period, and have to do it.
  - $t = 2$, $U_2(2|\hat{\beta}) = 1$, $U_2(3|\hat{\beta}) = 1.2$, procrastinate to $t = 3$;
  - $t = 1$, the individual compares doing it now and in $t = 3$
    - he thinks that if he procrastinates to $t = 2$, the future self would further procrastinate to $t = 3$.
    - $U_1(1|\beta) = 2$, $U_1(3|\beta) = 1$ so he chooses to do it in $t = 1$.

(β, δ) Model: Partial Naivete

- Consider the previous example with $\hat{\beta} = 0.8$.

$$U_t(\tau) = \begin{cases} -c_\tau + 10\beta & \text{if } \tau = t \\ -\beta c_\tau + 10\beta & \text{if } \tau > t \end{cases}$$

where $c_1 = 3, c_2 = 5, c_3 = 8, \beta = 0.5, \hat{\beta} = 0.8$

For partial naifs, they conduct backward induction

- In the eye of self 1
  - $t = 3$ is the last period, and have to do it.
  - $t = 2$, $U_2(2|\hat{\beta}) = 3$, $U_2(3|\hat{\beta}) = 1.6$, do it in $t = 2$;
  - $t = 1$, the individual compares doing it now and in $t = 2$.
    - he thinks that if he procrastinates to $t = 2$, the future self would do it in $t = 2$.
    - $U_1(1|\beta) = 2$, $U_1(2|\beta) = 2.5$ so he will procrastinates to $t = 2$.
- In $t = 2$, when β is realized, he will procrastinate to $t = 3$ since $U_2(2|\beta) = 0$, $U_2(3|\beta) = 1$.

Conclusion

Conclusion

- We examined optimal pricing schemes when consumers have dynamically inconsistent preferences.
- Price schemes aimed at sophisticated consumers act as commitment devices, whereas price schemes aimed at naifs are flexible contracts that essentially act as bets over consumer's consumption decision.
- It is possible to perfectly screen sophisticates, full naifs, and the magnitude naifs. However, perfect screening fails when consumers are frequency naifs.

Conclusion

- The optimal flexible price scheme aimed at naive consumers is unambiguously exploitative under monopoly. Competition need not eliminate exploitation of naifs.
- When the consumer population consists entirely of naifs, competitive firms would have the incentive to educate the consumers.
- We also briefly discussed (β, δ) model which is aimed to analyze present bias.

Conclusion

- Complex Pricing Strategies:
  - Subtle contract renewal policies such as negative options are an optimal response to consumer naivete regarding the future costs of switching or cancelling.
- Spurious Variety:
  - Multiplicity of price plan is a way of discriminating between consumers according to their degree of naivete.
  - If all consumers had correct beliefs of their future preferences, it would disappear.