Chapter 8. Second-Order Conditions (Exercises)

Exercise 8.1: Production Theory

In Exercise 6.2, we examined a firm's profit function

$$\Pi(q,p) = \max_{x,y}\{qy - px \mid G(x,y) \leq 0\},$$

where q and p are respectively the vectors of prices of outputs and inputs, y and x the corresponding quantity vectors, and the constraint reflects technological feasibility. We have proved that Π is a convex function of (q,p).

Question 1: Show that the optimum choices of y and x are given in terms of the partial derivatives of Π by

$$y = \Pi_q(q,p), \quad x = -\Pi_p(q,p).$$

Question 2: Show that output supply curves are upward-sloping and input demand curves are downward-sloping:

$$\frac{\partial y_j}{\partial q_j} \geq 0, \text{ and } \frac{\partial x_k}{\partial p_k} \leq 0,$$

for all j,k.

Exercise 8.3: Minimization

Question 1: Develope second-order sufficient conditions for the unconstrained miminization problem.

You may use the definitions and the determinantal tests of *positive (semi-)definite* matrix in the Lecture Notes directly. If you want, you could also develope the determinantal test from the tests for *negative (semi-)definite* matrix. In your derivation, you will find the following result useful: $\det(-A) = (-1)^n \det(A)$ for an $n \times n$ matrix A.

Question 2: Use Theorem 8.4 in the Lecture Notes to develope the second-order sufficient condition for the constrained minimization problem. (Hint: $\det(-A) = (-1)^n \det(A)$ for an $n \times n$ matrix A.)