Notes on Applied Nonlinear Control

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1 Introduction

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Linear Systems

Linear control theory has been predominantly concerned with the study of linear time-invariant (LTI) control systems, of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \tag{1}$$

with $\mathbf{x}$ being a vector of states and $\mathbf{A}$ being the system matrix. LTI systems have quite simple properties, such as

- a linear system has a *unique equilibrium point* if $\mathbf{A}$ is nonsingular;
- the equilibrium point is stable if all eigenvalues of $\mathbf{A}$ have negative real parts, *regardless of initial conditions*;
- the transient response of a linear system is composed of the natural modes of the system, and the general solution can be solved analytically;
- in the presence of an external input $\mathbf{u}(t)$, *i.e.*, with

  $$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

  the system response has a number of interesting properties. First, it satisfies the *principle of superposition*. Second, the asymptotic stability of the system (1) implies bounded-input bounded-output stability in the presence of $\mathbf{u}$. Third, a sinusoidal input leads to a sinusoidal output of the same frequency.

Remark: superposition

1.1.2 Multiple Equilibrium Points

For the linearized system, stability is seen by noting that for any initial condition, the motion always converges to the equilibrium point $x = 0$.

1.1.3 Limit Cycles

Limit cycles in nonlinear systems are different from linear oscillations in a number of fundamental aspects. First, the amplitude of the self-sustained excitation is independent of the initial condition, while the oscillation of a marginally stable linear system has its amplitude determined by its initial conditions. Second, marginally stable linear systems are very sensitive to changes in system parameters (with a slight change capable of leading either to stable convergence or to instability), while limit cycles are not easily affected by parameter changes.

1.1.4 Bifurcations

1.1.5 Chaos

2 Phase Plane Analysis

2.1 Concepts of Phase Plane Analysis

2.1.1 Phase Portraits

2.1.2 Singular Points

2.2 Constructing Phase Portraits

2.3 Phase Plane Analysis of Nonlinear Systems

Phase plane analysis of nonlinear systems is related to that of linear systems, because the local behavior of a nonlinear system can be approximated by a linear system behavior. Yet, nonlinear systems can display much more complicated patterns in the phase plane, such as multiple equilibrium points and limit cycles.

2.3.1 Limit Cycles

In the phase plane, a limit cycle is defined as an isolated closed curve.

Depending on the motion patterns of the trajectories in the vicinity of the limit cycle, one can distinguish three kinds of limit cycles

1. all trajectories in the vicinity of the limit cycle converge to it as $t \to \infty$;

2. all trajectories in the vicinity of the limit cycle diverge from it as $t \to \infty$;
3. some of the trajectories in the vicinity converge to it, while the others diverge from it as $t \to \infty$;

3 Fundamentals of Lyapunov Theory

3.1 Nonlinear Systems and Equilibrium Points

3.1.1 Autonomous and Non-autonomous Systems

In fact, adaptive controllers for linear time-invariant plants usually make the closed-loop control systems nonlinear and non-autonomous.

3.2 Concepts of Stability

3.2.1 Local and Global Stability

Linear time-invariant systems are either asymptotically stable, or marginally stable, or unstable, as can be seen from the modal decomposition of linear system solutions; linear asymptotic stability is always global and exponential, and linear instability always implies exponential blow-up.

3.3 Lyapunov's Direct Method

Comparing the definition of stability and mechanical energy, one can easily see some relations between the mechanical energy and the stability concepts described earlier:

- zero energy corresponds to the equilibrium point $(\mathbf{x} = \mathbf{0}, \dot{\mathbf{x}} = \mathbf{0})$
- asymptotic stability implies the convergence of mechanical energy to zero
- instability is related to the growth of mechanical energy

Faced with a set of nonlinear differential equations, the basic procedure of Lyapunov's direct method is to generate a scalar "energy-like" function for the dynamic system, and examine the time variation of that function.

3.3.1 Positive Definite Functions and Lyapunov Functions

3.3.2 Equilibrium Point Theorems