Notes on Applied Nonlinear Control

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2017/10/10

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Linear Systems

Linear control theory has been predominantly concerned with the study of linear time-invariant (LTI) control systems, of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \tag{1}$$

with $\mathbf{x}$ being a vector of states and $\mathbf{A}$ being the system matrix. LTI systems have quite simple properties, such as

- a linear system has a *unique equilibrium point* if $\mathbf{A}$ is nonsingular;
- the equilibrium point is stable if all eigenvalues of $\mathbf{A}$ have negative real parts, *regardless of initial conditions*;
- the transient response of a linear system is composed of the natural modes of the system, and the general solution can be solved analytically;
- in the presence of an external input $\mathbf{u}(t)$, *i.e.*, with

  $$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

  the system response has a number of interesting properties. First, it satisfies the *principle of superposition*. Second, the asymptotic stability of the system (1) implies bounded-input bounded-output stability in the presence of $\mathbf{u}$. Third, a sinusoidal input leads to a sinusoidal output of the same frequency.

Remark: superposition

1.1.2 Multiple Equilibrium Points

For the linearized system, stability is seen by noting that for any initial condition, the motion always converges to the equilibrium point $x = 0$.

1.1.3 Limit Cycles

Limit cycles in nonlinear systems are different from linear oscillations in a number of fundamental aspects. First, the amplitude of the self-sustained excitation is independent of the initial condition, while the oscillation of a marginally stable linear system has its amplitude determined by its initial conditions. Second, marginally stable linear systems are very sensitive to changes in system parameters (with a slight change capable of leading either to stable convergence or to instability), while limit cycles are not easily affected by parameter changes.

1.1.4 Bifurcations

1.1.5 Chaos

2 Phase Plane Analysis

2.1 Concepts of Phase Plane Analysis

2.1.1 Phase Portraits

2.1.2 Singular Points

2.2 Constructing Phase Portraits